Outage probability approximations for dual-hop Amplify-and-Forward MIMO relay systems in Rayleigh fading

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***Abstract*—In this paper we present tight approximations of the outage probability (OP) for the entire signal-to-noise ratio (SNR) range of practical interest for an amplify and forward (AF) relaying system with channel state information (CSI) at the relay and destination, which employs multiple antennas at the nodes and orthogonal space-time block coding (OSTBC) transmission over a flat Rayleigh fading. In the AF relay, the incoming signal is decoupled, amplified and forwarded to the destination. The results for the outage probability obtained by the approximations are compared with exact results of the outage probability obtained by numeric inversion of the Laplace transform of the Moment Generating Function (MGF) and with the results obtained by the Monte Carlo simulations. The comparison shows close matching of the results.**

I. Introduction

Multiple-input multiple-output (MIMO) technology is becoming commonplace in the contemporary communication systems since they offer significant performance improvements in terms of their capacity and reliability, achieved through exploiting the multipath propagation in the wireless medium. Wireless systems with 2 to 4 antennas are currently used for local area networks (802.11n, 802.11ac) or developed for cellular systems such as 3GPP-LTE and LTE advanced.

Partly motivated by the MIMO concept, a user cooperation has recently emerged as an additional breakthrough concept in wireless communications, called cooperative diversity, which has the potential to revolutionize the next generation communication systems by offering additional capacity and reliability improvements with small additional signal processing and cost [1]. Some cooperative (i.e. relay) techniques are already part of the standard for LTE-Advanced [2]. The neighbouring wireless nodes (also called relays or partners) assist each other's communication process by dedicating some of their resources to transmit part (or all) of the partners' information. By properly coordinating different spatially distributed nodes in a wireless system, one can effectively synthesize a virtual antenna array to achieve spatial diversity, similarly as in MIMO. The distributed nature of such a communication process provides a unique opportunity for cooperation, distributed signal processing and of gaining the same advantages as those found in the MIMO systems. The combination of MIMO and cooperative relaying schemes merge the benefits of MIMO system's diversity and multiplexing gains with benefits of relaying for overcoming shadowing, reducing unnecessary high transmission power and alleviating radio frequency interference.

In this paper we will focus on the MIMO AF relaying systems. There is abundance of research papers related to MIMO AF relaying systems. Some focus on single-antenna relays [3] - [5] and some on multiple-antenna relays [6] - [9]. In paper [3] the authors have presented error performance and outage probability analysis for the cooperative relay system with transmit diversity with and without direct link transmission. The paper [5] tackles the problem of finding exact closed-form and asymptotic expressions of the outage and error performance in a single-antenna relay system where the source and the destination have multiple-antenna by using OSTBC transmissions in Nakagami-m fading. In the letter [7] the authors has presented a closed-form expression for outage probability of cooperative MIMO relay system in which the source and the relay use OSTBC for transmission whereas the relay and the destination use Maximum Ratio Combining (MRC) for reception. The paper [8] provides accurate expressions and closed-form bounds of outage probability (OP) of a MIMO AF relay system using OSTBC where each of its nodes is equipped with two antennas over a Nakagami-m fading environment. Most of the papers in the literature (e.g. [4] - [6], [8] - [9]) for the implementation of the MIMO AF relaying system use a specific *amplify and forward* (AF) relaying scheme called Decouple-and-Forward (DCF) relaying that has recently been proposed by Lee et al. in [6]. DCF is linear processing technique by which the relay converts multiple spatial streams of the received OSTBC signal into a single spatial stream signal without symbol decoding. As a result of this linear decoupling process, if the additive channel noise is neglected, the estimate of the transmitted symbol can be mathematically expressed as product of the transmitted symbol and the sum of the squared modulus of the MIMO channel coefficients. After the relay decouples the OSTBC signal it re-encodes the decoupled symbols by usage of OSTBC, amplifies each of them separately and transmits them over the relay-destination hop.

In this paper, we study the approximations of the outage probability of a dual hop DCF relaying system, consisted of a source, a DCF half-duplex relay and a destination, each equipped with multiple antennas and utilizing an OSTBC transmission technique in a Rayleigh fading environment. We

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have arrived at simple universal approximations for the outage probability of those systems, which proves to be extremely accurate in the SNR range of a practical interest.

The remainder of this paper is organized as follows. Next section presents the system and channel model. In Section 3 we derive closed form expressions for approximation of the outage probability. The numerical analysis is presented in Section 4, and Section 5 concludes the article.

II. System And Channel Models

In this paper we analyze outage probability of a dual-hop relay system (consisted of a source, a relay and a destination), with multiple antennas at the nodes that utilize OSTBC transmission (Fig.1). We consider two system configurations: *N*x*1*x*N* configuration, where the source and the destination are equipped with *N* antennas and the relay with single antenna and *N*x*N*x*N* configuration, where the source, the relay and the destination are each equipped with *N* antennas. We assume that there is no spatial correlation between the signals transmitted or received in different antennas. The AF relay applies variable-gain amplification of its input signal, which requires the instantaneous channel state information (CSI) of the source-relay hop being available to the relay [10]. The destination is also assumed to have a full CSI of the relay-destination hop for coherent demodulation. In Fig.1 we present a MIMO dual-hop relay system (as the most general configuration that incorporates the two system configurations as special cases), where the source, the relay and the destination are denoted by *S*, *R* and *D*, respectively. The relay system is designated as *N*x*1*x*N* in the case when the relay *R* has one antenna and *N*x*N*x*N* when the relay *R* has *N* antennas. The *S-R* hop and the *R-D* hop are modeled as the independent MIMO Rayleigh channels with respective channel matrices $\mathbf{H}$ and $\mathbf{G}$. The elements h_{ij} and g_{ij} of these matrices are the channel coefficients between the *i*-th transmit antenna and *j*-th receive antenna, considered as independent circularly-symmetric complex Gaussian random processes with zero mean and unit variance. Therefore, the squared envelope of signal transmitted over channel h_{ij} (g_{ij}) follows the exponentially decaying probability distribution function (PDF) [11] with same mean squared values $E\left[|h_{ij}|^2\right] = E\left[|g_{ij}|^2\right] = 1$. The source *S* transmit power is *P* and the direct communication between the source and the destination is unavailable. The utilized OSTBC codes are designated as three digit codes, *NKL*, where *N* is the number of antennas, *K* is the number of code symbols transmitted in a code block, and *L* is the number of required time slots for single codeword [12].

We assume that the communication system operates in half-duplex mode, divided in two phases (phase 1 and phase 2). The source *S* transmits towards *R* during phase 1, then *R* transmits towards *D* during phase 2. Since we assume that the source *S* employs the OSTB encoding in the phase 1, group of *K* information symbols $\mathbf{X} = [x_1,\ x_2,\ ..x_K]^T$ are transmitted over the *N* transmit antennas in *L* successive time slots. During the phase 2 the *N*x1x*N* system's relay decouples, amplifies and transmits the *K* received symbols to the destination. In case of the *N*x*N*x*N* system, during the phase

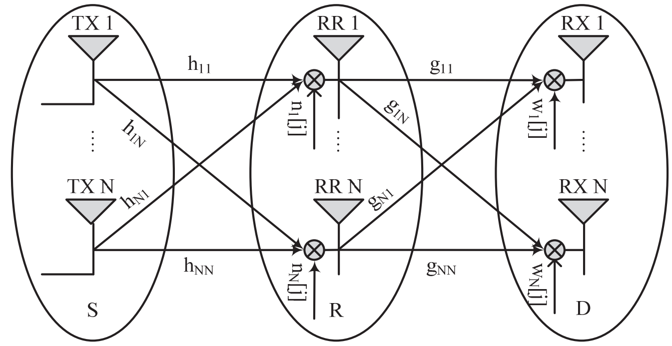

Fig. 1. Dual-hop MIMO relay system.

2 the relay decouples, amplifies, OSTB encodes and transmits the *K* received symbols to the destination. The received signal in the single relay antenna at the end of the phase 1 is:

$$\mathbf{Y} = \sqrt{E}\ \mathbf{C}\ \mathbf{H} + \mathbf{N}\ ,\quad \mathbf{Y} = [y_1,\ y_2,\ \ldots y_L]^T\ , \qquad (1)$$

where $\mathbf{C}$ is LxN codeword matrix of the OSTB code, $\mathbf{H}$ is Nx1 channel vector of the *S-R* hop $\mathbf{H} = [h_1,\ h_2,\ ..h_N]^T$ and $\mathbf{N} = [n_1,\ n_2,\ ..n_L]^T$ is *S-R* hop's Lx1 additive white Gaussian noise (AWGN) vector whose elements have zero mean and variance N_0. The superscript operator $\mathbf{T}$ denotes matrix transpose operation, and E is the average transmitted power per symbol. In [13] it is shown that the particular decoupled symbol at the single antenna at the relay is given by:

$$\tilde{x}_k =\ \sqrt{E}\ \|\mathbf{H}\|_F^2\ x_k + \xi_k,\quad k = 1, 2, ..K\ , \qquad (2)$$

where $\|\mathbf{H}\|_F^2 = \sum_{i=1}^{N} |h_i|^2$ is the squared Frobenius norm of the matrix $\mathbf{H}$, ξ is complex-valued AWGN random variable with zero mean and variance $\|\mathbf{H}\|_F^2 N_0$. In case where the relay has *N* antennas (*N*x*N*x*N* system), the decoupled symbol at the relay is again given by (2) where $\|\mathbf{H}\|_F^2 = \sum_{\mathbf{i=1}}^{\mathbf{N}} \sum_{\mathbf{j=1}}^{\mathbf{N}} |h_{ij}|^2$. We have chosen to amplify the decoupled symbol $\tilde{x}_k$ with the following amplification factor:

$$A = \sqrt{E_R / \left(E_S \|\mathbf{H}\|_F^4 + \|\mathbf{H}\|_F^2\ N_0 \right)}\ , \qquad (3)$$

where transmitted symbol energies from the source and the relay are equal i.e. $E_R = E_S = E$. For the *N*x*1*x*N* system the destination *D* combines the signals from its multiple antennas by using Maximum Ratio Combining (MRC). In order to compare the two systems fairly, we assume same transmit power from the relay in both cases. Therefore, for the *N*x*1*x*N* system we should normalize the relay gain in (3) with power normalization factor c which will increase the single-antenna relay average transmit power proportionally to the number of the transmit antennas at *S*. Taking that in the consideration and approximating (3) for analytical tractability we define the relay gain as:

$$A \approx 1 / \left(\sqrt{b}\ \|\mathbf{H}\|_F^2 \right)\ , \qquad (4)$$

where b is the power normalization factor. Namely, in (4) we select $b = c$ for the *N*x*1*x*N* and $b = 1$ for the *N*x*N*x*N* system.

The decision variable for each of the decoupled symbols at the destination D is expressed as:

$$\hat{x}_k = A \, \|\mathbf{G}\|_F^2 \, \tilde{x}_k + \mu_k, \quad k = 1, 2, \ldots K \tag{5}$$

where $\|\mathbf{G}\|_{\mathrm{F}}^2 = \sum_{\mathbf{i=1}}^{\mathbf{N}} |g_i|^2$ and $\|\mathbf{G}\|_{\mathrm{F}}^2 = \sum_{\mathbf{i=1}}^{\mathbf{N}} \sum_{\mathbf{j=1}}^{\mathbf{N}} |g_{ij}|^2$ are the corresponding squared Frobenius norms of the channel matrix $\mathbf{G}$ for the *N*x*1*x*N* and *N*x*N*x*N* systems, and μ is complex-valued AWGN random variable with zero mean and variance $\|\mathbf{G}\|_F^2 N_0$. If we introduce (2) in (5) we obtain the expression that represents the decoupled symbols for both the *N*x*1*x*N* and *N*x*N*x*N* configurations at the destination *D*:

$$\hat{x}_k = \sqrt{E} \; A \, \|\mathbf{H}\|_F^2 \, \|\mathbf{G}\|_{\mathrm{F}}^2 \, x_k + A \, \|\mathbf{G}\|_{\mathrm{F}}^2 \, \xi_k + \mu_k \; . \tag{6}$$

III. Outage probability approximations for arbitrary SNR

The instantaneous end-to-end SNR per information symbol of the signal before the receiver at D for both system configurations is:

$$\gamma = \frac{P_S}{P_N} = \frac{E}{N_0} \cdot \frac{A^2 \|\mathbf{G}\|_{\mathrm{F}}^2 \, \|\mathbf{H}\|_F^4}{A^2 \|\mathbf{G}\|_{\mathrm{F}}^2 \, \|\mathbf{H}\|_F^2 + 1} \; . \tag{7}$$

Introducing (4) in (7) we can present the approximated end-to-end SNR (γ) for both systems in the following form [4]:

$$W = 1/\mathit{\Gamma} = 1/\left(\overline{\gamma} \, \|\mathbf{H}\|_F^2\right) + b/\left(\overline{\gamma} \, \|\mathbf{G}\|_F^2\right) = U + V \;, \tag{8}$$

where $W = 1/\mathit{\Gamma}$, $U = 1/(\overline{\gamma} \, \|\mathbf{H}\|_F^2)$, $V = b/(\overline{\gamma}\|\mathbf{G}\|_F^2)$, $\overline{\gamma} = E/N_0 = c \cdot \rho$ is the average transmit SNR per symbol and ρ is the total average transmit SNR per symbol at *S*. The expression (8) is applicable for the *N*x*1*x*N* if we substitute $b = c$ and for the *N*x*N*x*N* system if we substitute $b = 1$. U and V are random variables that follow the inverse gamma distribution [9]. Since the *S-R* and *R-D* hops are subject to an independent Rayleigh fading, U and V are independent and the MGF of their sum is product of their MGF's:

$$M_W(-s) = \frac{4\sqrt{b^m}}{\Gamma^2(m)} \left(\frac{s}{\overline{\gamma}}\right)^m K_m\left(\sqrt{\frac{4s}{\overline{\gamma}}}\right) K_m\left(\sqrt{\frac{4bs}{\overline{\gamma}}}\right) \;, \tag{9}$$

where K_m ([14, eq.(8.432.1)]) denote m-th order modified Bessel function of second kind. For the *N*x*1*x*N* system configuration, the MGF of the $1/\Gamma$ is expressed as (9), where $m = N$ and $b = c$, whereas, for the *N*x*N*x*N* system configuration, $m = N^2$ and $b = 1$. The CDF of the random variable $\mathit{\Gamma}$ is obtained according to [15, eq.(31)]:

$$F_{\mathit{\Gamma}}(\gamma) \; = 1 - \mathcal{L}^{-1}\left[M_w(-s)/s\right]\big|_{w=1/\gamma} \;, \tag{10}$$

where $\mathcal{L}^{-1}$ denotes the inverse Laplace transform. In order to find (10) in closed form [9], we approximated the function K_m using its power series expansion [14, eq.(8.446)]. For small arguments as $z \to 0$, the infinite sum in [14, eq.(8.446)] can be neglected and kept only the finite sum i.e.:

$$K_m(z) \approx \frac{1}{2} \cdot \left(\frac{2}{z}\right)^m \cdot \sum_{k=0}^{m-1} (-1)^k \cdot \frac{(m-k-1)!}{k!} \cdot \left(\frac{z}{2}\right)^{2\,k} . \tag{11}$$

After some algebraic manipulations [9], it can be shown that approximate CDF for the *N*x*1*x*N* and *N*x*N*x*N* systems is:

$$\begin{aligned} F_{\Gamma_{\mathrm{a}}}(\gamma) \approx 1 + \frac{1}{\Gamma(m)} \cdot \sum_{k=0}^{m-1} \sum_{n=0}^{m-1} \frac{(-1)^{m+k+n} \Gamma(m-k)}{\Gamma(m-n)} \\ \cdot \frac{(2\,k+n-m+1)_{m-n-1} (b+1)^n \; b^k}{\Gamma(k+1) \; \Gamma(n+1)} \\ \cdot \left(\frac{\gamma}{\overline{\gamma}}\right)^{n+2\;k} \cdot \exp\left(-\frac{(b+1)\gamma}{\overline{\gamma}}\right) \;, \end{aligned} \tag{12}$$

where $(...)_{...}$ represents pochhammer symbol.

The OP is defined as the probability that the instantaneous SNR falls below a predetermined threshold ratio γ_0:

$$P_{\mathrm{out}} = P(\gamma < \gamma_0) = F_{\Gamma}(\gamma)\big|_{\gamma=\gamma_0} \approx F_{\Gamma_a}(\gamma)\big|_{\gamma=\gamma_0} \; . \tag{13}$$

By introducing (9) in the (10) we can find the exact values of the OP by means of numerical inversion of the Laplace transform, and by usage of (13) we can obtain a tight approximation for the outage probability of both system configurations. In order to simplify the equation (12) we take in the consideration only the terms where $k = 0$. This simplification results in a loose approximation of the outage probability:

$$P_{\mathrm{out}} \approx 1 - \sum_{n=0}^{m-1} \frac{(b+1)^n}{n!} \cdot \left(\frac{\gamma_0}{\overline{\gamma}}\right)^n \cdot \exp\left(-\frac{(b+1)\gamma_0}{\overline{\gamma}}\right) \; . \tag{14}$$

The previous expressions can be used for calculation of the tight (13) and loose (14) approximations of the outage probability for the *N*x*1*x*N* system ($m = N$ and $b = c$) and for the *N*x*N*x*N* system ($m = N^2$ and $b = 1$)

IV. Numerical results

In this section we illustrate the accuracy of our approximations. At the beginning, we will validate our approximations of OP for different number of antennas N by comparison with: (a) the exact values obtained by numerical inversion of the Laplace transform of the MGF, and (b) the values obtained by Monte Carlo simulations. For Laplace transform inversion we used Euler numerical technique given in [11, eq.(9B.9)].

We have focused on several practical OSTBC schemes, such as 222, 334 and 434, and established their respective exact and approximate outage probabilities when applied in the considered system (Fig.1). According to [12] and [16] the codeword matrices for these schemes are given by:

$$\mathbf{C}_{222} = \begin{bmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{bmatrix}, \; \mathbf{C}_{334} = \begin{bmatrix} x_1 & x_2 & x_3 \\ -x_2^* & x_1^* & 0 \\ x_3^* & 0 & -x_1^* \\ 0 & x_3^* & -x_2^* \end{bmatrix},$$

$$\mathbf{C}_{434} = \begin{bmatrix} x_1 & x_2 & \frac{x_3}{\sqrt{2}} & \frac{x_3}{\sqrt{2}} \\ -x_2^* & x_1^* & \frac{x_3}{\sqrt{2}} & \frac{-x_3}{\sqrt{2}} \\ \frac{x_3}{\sqrt{2}} & \frac{x_3}{\sqrt{2}} & \frac{(-x_1-x_1^*+x_2-x_2^*)}{2} & \frac{(-x_2-x_2^*+x_1-x_1^*)}{2} \\ \frac{x_3^*}{\sqrt{2}} & \frac{-x_3^*}{\sqrt{2}} & \frac{(x_2+x_2^*+x_1-x_1^*)}{2} & -\frac{(x_1+x_1^*+x_2-x_2^*)}{2} \end{bmatrix}. \tag{15}$$

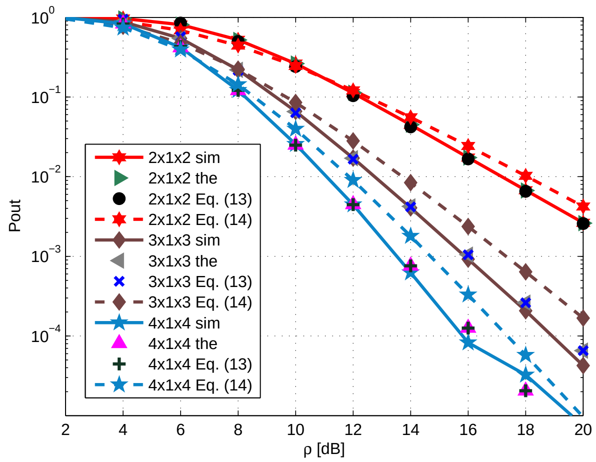

Fig. 2. OP of MIMO AF system with single-antenna relay ($\gamma_0 = 5dB$).

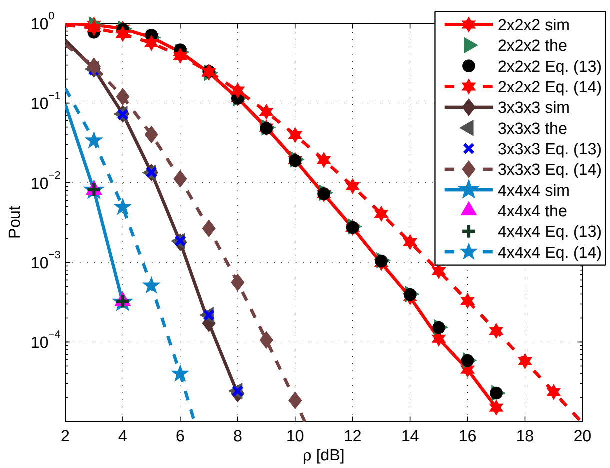

Fig. 3. OP of MIMO AF system with multiple-antenna relay for ($\gamma_0 = 5dB$).

For the OSTB codes given in (15) average power per symbol is calculated as:

$$E = P \cdot c, \qquad c = L/\left(KN\right) \ , \tag{16}$$

and received symbols in a single relay antenna for 222, 334 and 434 OSTB codes are decoupled according to [9, eq.(34)].

On Fig.2 we present OP performance results for the *N*x*1*x*N* system with up to 4 antennas at the source and the destination. The solid line curves represent the OP results obtained by Monte Carlo simulations, the triangle markers represent the exact OP results obtained by numerical inversion of Laplace transform (10), the dashed line curves represent the results obtained by loose approximation (14) and remaining markers (o, x and +) represents the results obtained by the expression for tight approximation of the outage probability (13). It should be mentioned that the respective curves for $N > 4$ could easily be obtained. Similarly, on Fig.3 we present the OP results for the *N*x*N*x*N* system with up to 4 antennas at the source, relay and the destination. The solid line curves represent the OP results obtained by Monte Carlo simulations, the triangle markers represent the exact OP results obtained by numerical inversion of Laplace transform (10), the dashed line curves represent the results obtained by loose approximation (14) and remaining markers (o, x and +) represents the results obtained by the expression for tight approximation of the outage probability (13). Both comparisons show close match of the results obtained by approximation (13), exact results obtained by the numerical inversion of the Laplace transform and the results obtained by the Monte Carlo simulation.

V. Conclusion

In this paper we have analyzed outage probability of the dual-hop relay systems with multiple antennas at the source, the relay and the destination that utilize OSTBCs and amplify-and-forward relaying schemes. For those systems we have derived generalized closed form expressions for tight (13) and loose (14) approximation of the outage probability. We have shown that the results obtained with our approximations closely match the exact results and the results obtained by Monte Carlo simulations.

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