Performance analysis of dual-hop dual-antennas MIMO systems in Rayleigh fading

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Abstract - **In this paper we study the error and outage probability (OP) performance of dual-hop multiple input multiple output (MIMO) relay systems utilizing Alamouti coding and modified amplify and forward (MAF) relaying in flat Rayleigh fading channels. The error performances of dual-hop MIMO systems with variable gain relays is compared with dual-hop single antenna systems and regenerative i.e. decode and forward (DF) dual-hop MIMO system. Results obtained by means of simulation show that MAF MIMO systems achieve significantly better bit error performance than dual-hop single-antenna systems and comparable performance with DF dual-hop MIMO systems. The performance gap increases with usage of dual antenna in relay and the receiver. The OP performances of these systems are compared with dual-hop single-antenna and dual-antennas point-to-point systems. We show significant improvement of OP performance compared to dual-hop single-antenna and comparable performance with dual-antennas point-to-point systems. Moreover, for OP performance we show exact fit of numerical results with the results obtained by mathematical analysis.**

Keywords - **Cooperative wireless communications; MIMO; Alamouti's coding; dual-hop relay systems; bit error probability; outage probability; Rayleigh fading;**

I. INTRODUCTION

User cooperation (a.k.a. cooperative diversity) has recently emerged as a very important research area in today's wireless communications [1], known as the cooperative communications. The concept of cooperation is based on the multipath propagation properties of the radio channel utilized to increase the efficiency and robustness of their communication. The neighboring wireless nodes (also called relays or partners) assist each other's communication process by dedicating some of their resources to transmit part (or all) of the partners' information.

By properly coordinating different spatially distributed nodes in a wireless system, one can effectively synthesize a virtual antenna array that emulates the operation of a multi-antenna transceiver. The distributed nature of such a communication process provides a unique opportunity for cooperation, distributed signal processing and of gaining the same advantages as those found in MIMO systems.

In wireless communications systems the bit error probability (BEP) and outage probability (OP) are most important performance measures. In this paper, we study the BEP and OP performance of the dual-hop relay system consisted of a source (with two antennas), a relay (with one or two antennas) and a destination (with one or two antennas), which utilizes the Alamouti's space time block coding (STBC) over independent Rayleigh fading hops [2]. The Alamouti's STBC is a highly efficient technique that utilizes the available degrees of freedom of a communication channel (or hop) with 2 transmit antennas by doubling its capacity and its diversity gain [2].

This paper proposes a novel relaying scheme, called *modified amplify and forward* (MAF) relaying, based on which the relay decouples the STBC signal received from the source into two independent data streams, amplifies each of them separately and transmits them over the relay-destination hop. We study the performance of this relaying scheme analytically and then compare it with the classic amplify and forward (AF) and decode and forward (DF) relaying schemes. This particular implementation of the MAF scheme uses variable gain relays [3] which require the instantaneous channel state information (CSI) at the receiving end of each hop. We also analyzed OP performance of these systems and compared it with dual-hop single-antenna and dual-antennas point-to-point systems.

Error performance analysis of dual-hop MAF system with two antennas at source, relay and destination can be found in [4]. The Error performance analysis for dual-hop MAF system with multiple antennas at source and destination and single antenna at the relay can be found in [5].

The remainder of this paper is organized as follows. Next Section presents the system and channel model. In Section III we derive expressions for the end-to-end SNR and the Moment Generating Function (MGF) needed for successful analysis of outage probability of the systems. Results are presented in Section IV, and Section V concludes the article.

II. SYSTEM AND CHANNEL MODELS

In this paper we analyze MIMO relay systems utilizing Alamouti scheme in three different dual-hop configurations: 2x1x1 MIMO relay system (where only the source is

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equipped with two antennas), 2x2x1 MIMO relay system (where source and relay are each equipped with two antennas), and 2x2x2 MIMO relay system (where source, relay and destination are each equipped with two antennas).

Fig. 1 presents 2x2x2 MIMO relay system (as the most general configuration that incorporates the other two as special cases), where the source, the relay and the destination are denoted by *S*, *R* and *D*, respectively.

The *S-R* hop and the *R-D* hop are assumed to be independent 2x2 MIMO Rayleigh channels with respective channel matrices $H = [h_{11}, h_{12}; h_{21}, h_{22}]$ and $G = [g_{11}, g_{12}; g_{21}, g_{22}]$. The elements h_{ij} and g_{ij} of these matrices are the channel coefficients between the respective pairs of *i*-th transmit antenna and *j*-th receive antenna, and are considered as independent circularly-symmetric complex Gaussian random processes with zero mean and unit variance. Therefore, the squared envelope of signal transmitted over channel $h_{ij}\,(g_{ij})$ follows the exponentially decaying probability distribution function (PDF) [6] with identical mean values $E\left[\left|h_{ij}\right|^2\right] = E\left[\left|g_{ij}\right|^2\right]$. The transmission powers from each transmit antenna are also assumed the same and equal to *E*. The relay and the receiver have perfect CSI available of the previous hop and the direct communication between the source and the destination is unavailable.

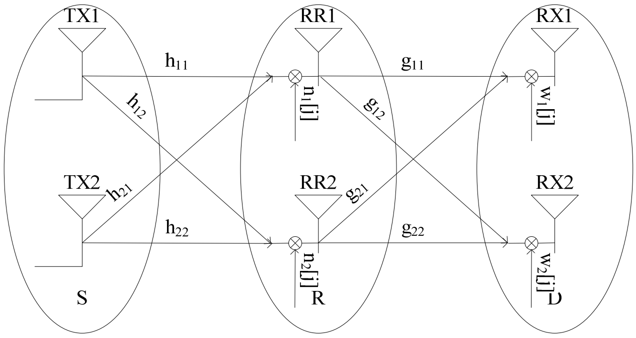

Fig. 1. Dual-hop MIMO system model

According to Fig. 1, we consider a MIMO relay system operating in half-duplex time-orthogonal relaying where the communication process is divided in two phases (phase 1 and phase 2). The source *S* transmits towards *R* during phase 1, then *R* transmits towards *D* during phase 2. We assume that the source *S* employs the Alamouti STBC encoding. In particular, groups of 2 independent symbols (x_1 and x_2) are transmitted over the two transmit antennas (using the Alamouti orthogonal transmission matrix $[x_1, x_2; -x_2^*, x_1^*]$) in two successive sub-slots of the phase 1 (sub-slot 1 and sub-slot 2). During phase 2, *R* transmits the two amplified decoupled signals during sub-slot 3 and sub-slot 4.
The subscript index in the noise terms $n_i[j], w_i[j]$ denotes receiving antenna, and the index in the squared brackets denotes the sub-slot.

A. Dual-hop 2x1x1 MIMO system

For analysis of 2x1x1 system we assume that only a single antenna at the relay and a single antenna at the destination are active. In sub-slots 1 and 2, the received signals at the single relay antenna (denoted by a subscript index 1) are given by:

$$y_1[1] = \sqrt{E}\,(h_{11}\cdot x_1 + h_{21}\cdot x_2) + n_1[1]\,, \tag{1}$$

$$y_1[2] = \sqrt{E}\,(-h_{11}\cdot x_2^* + h_{21}\cdot x_1^*) + n_1[2]\,, \tag{2}$$

where $n_1[1]$ and $n_1[2]$ denote the additive complex-valued white Gaussian noise (AWGN) in the sub-slot 1 and 2 in the relay antenna, and E is transmission power of each of the two source antennas. The AWGN process has a zero mean and variance N_0.

Using $y_1[1]$ and $y_1[2]$, the relay demodulates and decouples the independent data streams x_1 and x_2 (by multiplication with the decoupling matrix $[h_{11}^*, h_{21};\ h_{21}^*, h_{11}]$) but does not perform detection. After the decoupling, the relay amplifies the decoupled data streams $\tilde{x}_1$ and $\tilde{x}_2$ by the amplification factor G_1 and forward them towards destination over the *R-D* hop in sub-slots 3 and 4.

We denote this scheme as the modified AF (MAF) relaying. Compared to the DF scheme, the MAF scheme has a simpler implementation but comparable performance, as presented in the following section. Decoupled signals in the relay are given by:

$$\tilde{x}_1 = h_{11}^* y_1[1] + h_{21} y_1^*[2] = \sqrt{E}\,\Delta_1 x_1 + \xi_1\,, \tag{3}$$

$$\tilde{x}_2 = h_{21}^* y_1[1] - h_{11} y_1^*[2] = \sqrt{E}\,\Delta_1 x_2 + \xi_2\,, \tag{4}$$

where:

$$\Delta_1 = |h_{11}|^2 + |h_{21}|^2\,, \qquad \xi_1 = h_{11}^* n_1[1] + h_{21} n_1^*[2], \\ \xi_2 = h_{21}^* n_1[1] - h_{11} n_1^*[2]\,. \tag{5}$$

In (5) Δ_1 represents squared Forbenius norm of the channel 2x1 channel vector of the *S-R* hop, and ξ_1 and ξ_2 are the Gaussian noise components added to each of the data symbols with zero mean and conditional variances equal to $\Delta_1 N_0$.

After amplification, *R* forwards the decoupled signals $\tilde{x}_1$ and $\tilde{x}_2$ serially over its single antenna towards *D* in the two successive sub-slots 3 and 4, so the received signal in the single destination antenna is given by:

$$r_1[3] = G_1\, g_{11}\, \tilde{x}_1 + w_1[3]\,, \\ r_1[4] = G_1\, g_{11}\, \tilde{x}_2 + w_1[4]\,, \tag{6}$$

where G_1 is the relay amplification factor and $w_1[j]$ denotes the AWGN at the single receive antenna at the destination *D* with zero mean and variance N_0.

In the following, the relay amplification factor and the Forbenius norms of the *S-R* and the *R-D* channels of each system configuration are denoted by the appropriate index *k*, i.e., by G_k, Δ_k and Λ_k, respectively. The particular values of the index *k* (*k* = 1, 2 or 3) denote the 2x1x1, 2x2x1 and 2x2x2 system configurations, respectively (Fig. 1).

Considering that the first hop CSI is required for the implementation of the Alamouti decoder, in the relay R we have chosen variable gain relaying method to invert the fading effect of the S-R hop while limiting the output power of the relay if the signal over that hop is in deep fade. Based on (3), (4) and the choice of the gain in variable gain relays in [3, Eq. (4)] the gain of 2x1x1 system is selected as:

$$G_1 = \sqrt{\frac{E_R}{E_S\, \Delta_1^2 + \Delta_1\, N_0}}, \tag{7}$$

where E_S and E_R are the transmission powers of a single antenna at the source S and relay R.

In D, the received signal (6) is passed through the matched filter [7], [8] (since D knows the channel coefficient over the R-D hop), after which the receiver detects the two independent symbols ($\tilde{x}_1$ and $\tilde{x}_2$) in their respective sub-slots 3 and 4.

B. Dual-hop 2x2x1 and 2x2x2 MIMO systems

For both of these two system configurations, the received signals in relay antenna 1 in sub-slots 1 and 2 are given by:

$$y_1[1] = \sqrt{E}\, h_{11} \cdot x_1 + \sqrt{E}\, h_{21} \cdot x_2 + n_1[1], \tag{8}$$

$$y_1[2] = -\sqrt{E}\, h_{11} \cdot x_2^* + \sqrt{E}\, h_{21} \cdot x_1^* + n_1[2]. \tag{9}$$

The signals in relay antenna 2 in sub-slots 1 and 2 are:

$$y_2[1] = \sqrt{E}\, h_{12} \cdot x_1 + \sqrt{E}\, h_{22} \cdot x_2 + n_2[1], \tag{10}$$

$$y_2[2] = -\sqrt{E}\, h_{12} \cdot x_2^* + \sqrt{E}\, h_{22} \cdot x_1^* + n_2[2], \tag{11}$$

where $n_i[j]$ are AWGN components in the first and second antenna in the sub-slots 1 and 2. The decoupled signals in the relay are given by:

$$\begin{aligned}\tilde{x}_1 &= h_{11}^* y_1[1] + h_{21} y_1^*[2] + h_{12}^* y_2[1] + h_{22} y_2^*[2] \\ &= \sqrt{E}\, \Delta_2 x_1 + \eta_1 ,\end{aligned} \tag{12}$$

$$\begin{aligned}\tilde{x}_2 &= h_{21}^* y_1[1] - h_{11} y_1^*[2] + h_{22}^* y_2[1] - h_{12} y_2^*[2] \\ &= \sqrt{E}\, \Delta_2 x_2 + \eta_2 ,\end{aligned} \tag{13}$$

where:

$$\Delta_2 = \Delta_3 = |h_{11}|^2 + |h_{12}|^2 + |h_{21}|^2 + |h_{22}|^2 , \tag{14}$$

$$\eta_1 = h_{11}^* n_1[1] + h_{21} n_1^*[2] + h_{12}^* n_2[1] + h_{22} n_2^*\, [2], \tag{15}$$

$$\eta_2 = h_{21}^* n_1[1] - h_{11} n_1^*[2] + h_{22}^* n_2[1] - h_{12} n_2^*[2] . \tag{16}$$

Note that η_1 and η_2 denote the complex-valued AWGN with zero mean and variances $\Delta_2 N_0 = \Delta_3 N_0$. The signals $\tilde{x}_1$ and $\tilde{x}_2$ are then amplified by the appropriate relay amplification factor.

Similarly to the 2x1x1 configuration, we consider (12), (13) and [3, Eq. (4)] and select relay amplification factor of the 2x2x1 and the 2x2x2 systems as:

$$G_2 = G_3 = \sqrt{\frac{E_R}{E_S \cdot \Delta_2^2 + \Delta_2\, N_0}}. \tag{17}$$

After the amplification, in sub-slots 3 and 4, the relay R transmit the symbols $\tilde{x}_1$ and $\tilde{x}_2$ towards destination by utilizing the Alamouti's STBC technique over its two antennas. The destination D demodulates and decouples the signal received from R in these two sub-slots. These decoupled signals are then fed into the detector of D, whose performance is measured in terms of OP and BER.

In the case of the 2x2x1 system the decoupled symbols at the destination are given by:

$$\hat{x}_1 = G_2 \Lambda_2 \tilde{x}_1 + \zeta_1 , \tag{18}$$

$$\hat{x}_2 = G_2 \Lambda_2 \tilde{x}_2 + \zeta_2 , \tag{19}$$

where:

$$\begin{aligned}\Lambda_2 &= |g_{11}|^2 + |g_{21}|^2, \\ \zeta_1 &= g_{11}^* w_1[3] + g_{21} w_1^*[4], \\ \zeta_2 &= g_{21}^* w_1[3] - g_{11} w_1^*[4] .\end{aligned} \tag{20}$$

In (20) Λ_2 represents squared Forbenius norm of the 2x1 channel vector of R-D hop, ζ_1 and ζ_2 are the Gaussian noise components added to each of the data symbols with zero mean and conditional variances equal to $\Lambda_2\, N_0$.

In the case of 2x2x2 MIMO system the decoupled symbols at the destination are given by:

$$\hat{x}_1 = G_3 \Lambda_3 \tilde{x}_1 + \mu_1 , \tag{21}$$

$$\hat{x}_2 = G_3 \Lambda_3 \tilde{x}_2 + \mu_2 , \tag{22}$$

where:

$$\Lambda_3 = |g_{11}|^2 + |g_{12}|^2 + |g_{21}|^2 + |g_{22}|^2 , \tag{23}$$

$$\mu_1 = g_{11}^* w_1[3] + g_{21} w_1^*[4] + g_{12}^* w_2[3] + g_{22} w_2^*\, [4], \tag{24}$$

$$\mu_2 = g_{21}^* w_1[3] - g_{11} w_1^*[4] + g_{22}^* w_2[3] - g_{12} n_2^*[4] . \tag{25}$$

Note that μ_1 and μ_2 denote the Gaussian noise with zero mean and variance equal to $\Delta_3 N_0$, and Δ_3 is the squared Forbenius norm of the 2x2 channel matrix of the R-D hop.

III. Outage Probability Of Dual-Hop Dual-Antennas Systems

The outage probability is defined as the probability that the instantaneous SNR falls below a predetermined threshold ratio γ_0:

$$P_{out} = P(\gamma_k < \gamma_0) , \tag{26}$$

where γ_k represent equivalent end-to-end instantaneous SNR of the dual-hop system. The particular values of the index k (k = 1, 2 or 3) denote the 2x1x1, 2x2x1 and 2x2x2 system configurations, respectively (Fig. 1). We now derive the end-to-end SNR for 2x1x1, 2x2x1, and 2x2x2 dual-hop system.

For the 2x1x1 MIMO system, considering (3) and (6) the received signal at the destination in the sub-slot 3 can be presented as:

$$\begin{aligned} r_1[3] &= G_1\, g_{11}\, \tilde{x}_1 + w_1[3] \\ &= G_1\, g_{11} \left(\sqrt{E}\, \Delta_1 x_1 + \xi_1\right) + w_1[3] = \\ &= G_1\, g_{11}\, \sqrt{E}\, \Delta_1\, x_1 + G_1\, g_{11}\, \xi_1 + w_1[3]\,. \end{aligned} \tag{27}$$

The respective received signal in sub-slot 4, $r_1[4]$, encompasses the symbol x_2, but yields same end-to-end SNR. Therefore the following derivation of end-to-end SNR refers to both symbols.

The instantaneous signal power is given with:

$$P_s = \left|G_1\, g_{11}\, \sqrt{E}\, \Delta_1\right|^2 = E\, G_1^2\, |g_{11}|^2\, \Delta_1^2\,, \tag{28}$$

and instantaneous noise power is:

$$\begin{aligned} P_N &= |G_1\, g_{11}|^2 (|h_{11}|^2 N_0 + |h_{21}|^2 N_0) + N_0 = \\ &= G_1^2\, |g_{11}|^2 \Delta_1 N_0 + N_0\,. \end{aligned} \tag{29}$$

Hence end-to-end instantaneous SNR at the output of D for 2x1x1 dual-hop system is given by:

$$\gamma_1 = \frac{P_s}{P_N} = \frac{E}{N_0} \cdot \frac{G_1^2\, |g_{11}|^2\, \Delta_1^2}{G_1^2\, |g_{11}|^2 \Delta_1 + 1}. \tag{30}$$

where the index 1 in γ_1 refers to the configuration 2x1x1.

For the 2x2x1 MIMO system, if we replace (12) in (18) we will get:

$$\hat{x}_1 = G_2\, \Lambda_2\, \Delta_2\, \sqrt{E}\, x_1 + G_2\, \Lambda_2\, \eta_1 + \zeta_1\,. \tag{31}$$

By following the similar steps as those in (28), (29) and (30), starting from (31) it is straightforward to show that the end-to-end SNR of 2x2x1 system is:

$$\gamma_2 = \frac{E}{N_0} \cdot \frac{G_2^2\, \Lambda_2 \Delta_2^2}{G_2^2\, \Lambda_2 \Delta_2 + 1}, \tag{32}$$

Therefore, it can be shown that end-to-end SNR for 2x2x2 system is:

$$\gamma_3 = \frac{E}{N_0} \cdot \frac{G_3^2\, \Lambda_3\, \Delta_3^2}{G_3^2\, \Lambda_3\, \Delta_3 + 1}. \tag{33}$$

For easier mathematical analysis of dual-hop system, we now approximate G_1, G_2 and G_3 from (7) and (17) as [3]:

$$G_k \cong \frac{1}{\Delta_k}\,. \tag{34}$$

Changing (34) in either (30), (32), or (33) we can sublimate the approximated end-to-end SNR (γ_k) in the following form:

$$w_k = \frac{1}{\gamma_k} = \frac{1}{\bar{\gamma}\, \Delta_k} + \frac{1}{\bar{\gamma}\, \Lambda_k} = u_k + v_k\,, \tag{35}$$

where $\bar{\gamma} = E/N_0$ is average transmit SNR, $u_k = 1/(\bar{\gamma}\, \Delta_k)$ and $v_k = 1/\bar{\gamma}\, \Lambda_k$.

Given that instantaneous channel power follows exponential distribution, Δ_k follows the gamma distribution which can generally be expressed as:

$$f(x) = \frac{x^{\upsilon-1}}{\theta^{\upsilon}\Gamma(\upsilon)}\, e^{-\frac{x}{\theta}}, \text{for } x \geq 0, \text{and } \upsilon, \theta > 0, \tag{36}$$

with unit scale parameter $\theta = 1$ and shape parameter υ. Since $E\left[\left|h_{ij}\right|^2\right] = E\left[\left|g_{ij}\right|^2\right] = 1$, the PDFs of the squared Forbenius norms used in this paper, are respectively given by:

$$f_{\Delta_1}(x) = \frac{x}{\Gamma(2)}\, e^{-x}\,, \qquad x \geq 0\,, \tag{37}$$

$$f_{\Delta_2}(x) = f_{\Delta_3}(x) = \frac{x^3}{\Gamma(4)}\, e^{-x}\,, \qquad x \geq 0\,, \tag{38}$$

$$f_{\Lambda_1}(x) = e^{-x}\,, \qquad x \geq 0\,, \tag{39}$$

$$f_{\Lambda_2}(x) = \frac{x}{\Gamma(2)}\, e^{-x}\,, \qquad x \geq 0\,, \tag{40}$$

$$f_{\Lambda_3}(x) = \frac{x^3}{\Gamma(4)}\, e^{-x}\,, \qquad x \geq 0\,. \tag{41}$$

By using the functional transformation of the random variable $u_k = 1/(\bar{\gamma}\, \Delta_k)$ and $v_k = 1/(\bar{\gamma}\, \Lambda_k)$ it can be shown that both follow the inverse gamma distribution:

$$\begin{aligned} f(x) &= \frac{x^{-\upsilon-1}}{\bar{\gamma}^{\upsilon}\, \Gamma(\upsilon)}\, e^{-\frac{1}{x\bar{\gamma}}}, \\ &\text{for } x > 0, \text{and } \upsilon, \theta > 0, \end{aligned} \tag{42}$$

with scale parameter equal to $1/\bar{\gamma}$, therefore, their PDFs are respectively given by:

$$f_{U_1}(x) = \frac{x^{-3}}{\bar{\gamma}^2\, \Gamma(2)}\, e^{-\frac{1}{x\bar{\gamma}}}, \qquad x > 0\,, \tag{43}$$

$$f_{U_2}(x) = f_{U_3}(x) = \frac{x^{-5}}{\bar{\gamma}^4\, \Gamma(4)}\, e^{-\frac{1}{x\bar{\gamma}}}, \qquad x > 0\,, \tag{44}$$

$$f_{V_1}(x) = \frac{x^{-2}}{\bar{\gamma}}\, e^{-\frac{1}{x\bar{\gamma}}}, \qquad x > 0\,, \tag{45}$$

$$f_{V_2}(x) = \frac{x^{-3}}{\bar{\gamma}^2\, \Gamma(2)}\, e^{-\frac{1}{x\bar{\gamma}}}, \qquad x > 0\,, \tag{46}$$

$$f_{V_3}(x) = \frac{x^{-5}}{\bar{\gamma}^4\, \Gamma(4)}\, e^{-\frac{1}{x\bar{\gamma}}}, \qquad x > 0\,. \tag{47}$$

By means of [9, Eq. (3.471.9)] it is easy to find that MGFs of u_k and v_k can be generally expressed as:

$$M(-s) = \frac{2}{\Gamma(\upsilon)} \left(\frac{s}{\bar{\gamma}}\right)^{\frac{\upsilon}{2}} K_{\upsilon}\left(2\sqrt{\frac{s}{\bar{\gamma}}}\right), \tag{48}$$

where υ denotes diversity order of corresponding hop and K_υ denotes υ-th order modified Bessel function of second kind. Taking into account (48) the corresponding MGFs of u_k and v_k for 2x1x1, 2x2x1 and 2x2x2 MIMO relay system are:

$$M_{\mathrm{U}_1}(-s) = \frac{2}{\Gamma(2)} \frac{s}{\bar{\gamma}} K_2\left(2\sqrt{\frac{s}{\bar{\gamma}}}\right), \tag{49}$$

$$M_{\mathrm{U}_2}(-s) = M_{\mathrm{U}_3}(-s) = \frac{2}{\Gamma(4)}\left(\frac{s}{\bar{\gamma}}\right)^2 K_4\left(2\sqrt{\frac{s}{\bar{\gamma}}}\right), \tag{50}$$

$$M_{\mathrm{V}_1}(-s) = 2\sqrt{\frac{s}{\bar{\gamma}}} K_1\left(2\sqrt{\frac{s}{\bar{\gamma}}}\right), \tag{51}$$

$$M_{\mathrm{V}_2}(-s) = \frac{2}{\Gamma(2)} \frac{s}{\bar{\gamma}} K_2\left(2\sqrt{\frac{s}{\bar{\gamma}}}\right), \tag{52}$$

$$M_{\mathrm{V}_3}(-s) = \frac{2}{\Gamma(4)}\left(\frac{s}{\bar{\gamma}}\right)^2 K_4\left(2\sqrt{\frac{s}{\bar{\gamma}}}\right). \tag{53}$$

Since the first and second hop are subject to an independent Rayleigh fading, u_k and v_k are independent and the MGF of their sum is product of their MGFs:

$$M_{W_1}(-s) = \\ = \frac{4}{\Gamma(2)}\left(\frac{s}{\bar{\gamma}}\right)^{\frac{3}{2}} K_2\left(2\sqrt{\frac{s}{\bar{\gamma}}}\right) K_1\left(2\sqrt{\frac{s}{\bar{\gamma}}}\right), \tag{54}$$

$$M_{W_2}(-s) = \\ = \frac{4}{\Gamma(4)\,\Gamma(2)}\left(\frac{s}{\bar{\gamma}}\right)^3 K_4\left(2\sqrt{\frac{s}{\bar{\gamma}}}\right) K_2\left(2\sqrt{\frac{s}{\bar{\gamma}}}\right), \tag{55}$$

$$M_{W_3}(-s) = \\ = \frac{4}{\Gamma^2(4)}\left(\frac{s}{\bar{\gamma}}\right)^4 \left(K_4\left(2\sqrt{\frac{s}{\bar{\gamma}}}\right)\right)^2 \tag{56}$$

Having the MGF of $w_k = 1/\gamma_k$ for the analyzed types of dual-hop dual-antennas systems we can find outage probability of the system by using [6]:

$$P(\gamma_k < \gamma_0) = 1 - \mathfrak{L}^{-1}\left(\frac{M_{w_k}(-s)}{s}\right)\Bigg|_{1/\gamma_0}, \tag{57}$$

which is the probability that the end-to-end SNR will fall below predetermined threshold γ_0. Operator $\mathfrak{L}^{-1}$ denotes inverse Laplace transform. By using (54), (55), and (56) and the Euler technique for numerical inversion of Laplace transform outage probabilities for any of the presented dual-hop dual-antennas MIMO systems can be found numerically [6, Appendix 9B.1] , [10].

Moreover, we have found the exact closed-form solution for OP for the 2x2x2 system configuration. From [11 Eq.3.16.6.6] we can found inverse Laplace transform for the term in the brackets in (56):

$$\frac{\Gamma^2(4)\,\bar{\gamma}^4}{2}\,\mathfrak{L}^{-1}\left[\frac{M_{W_3}(-s)}{s^4}\right] = \mathfrak{L}^{-1}\left[2\left(K_4\left(\sqrt{\frac{2s}{\frac{\bar{\gamma}}{2}}}\right)\right)^2\right] \\ = \frac{1}{w_3}\, e^{-\frac{2}{\bar{\gamma}\,w_3}} K_4\left(\frac{2}{\bar{\gamma}\,w_3}\right). \tag{58}$$

By usage of [5, Eq.9] outage probability can be presented as third derivate from the inverse Laplace of the fraction $M_{W_3}(-s)/s^4$:

$$P(\gamma_3 < \gamma_0) = 1 - \left\{\frac{d^3}{d\,w_3^3}\mathfrak{L}^{-1}\left[\frac{M_{W_3}(-s)}{s^4}\right]\right\}\Bigg|_{w_3=\frac{1}{\gamma_0}} \\ = 1 - \frac{2}{\Gamma^2(4)\,\bar{\gamma}^4}\left\{\frac{d^3}{d\,w_3^3}\left[\frac{e^{-\frac{2}{\bar{\gamma}\,w_3}}}{w_3} K_4\left(\frac{2}{\bar{\gamma}\,w_3}\right)\right]\right\}\Bigg|_{w_3=\frac{1}{\gamma_0}}. \tag{59}$$

The expression (59) can be found in closed form by using formula for derivate of modified Bessel functions [12 Eq. 9.6.29] or by using CAS (Computer Algebra System) such as, Maple or Mathematica.

IV. Numerical Results

Usually, in real world applications the transmission power of a wireless station is limited. Therefore in the simulations we kept constant the total transmission power. In order to have the same total transmission power from two transmit antennas with the power of the single transmit antenna, the energy allocated to each symbol is divided by 2. For the simulations we have chosen BPSK modulation scheme. In order to have good reference for analyzing the results we have chosen result for single-antenna dual-hop variable gain system as upper bound and the result for point-to-point receive diversity system with 4 antennas and maximum ratio combiner as lower bound. Obtained bit error probabilities for the 2x1x1, 2x2x1, and 2x2x2 dual-hop dual-antennas systems are given on Fig. 2.

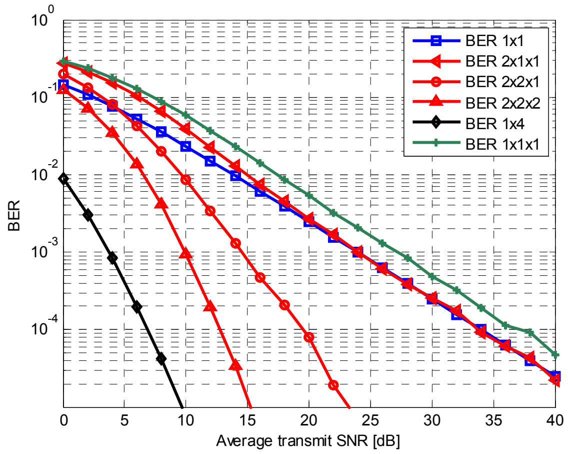

Fig. 2. BER for dual-hop dual-antennas MAF systems obtained by simulation

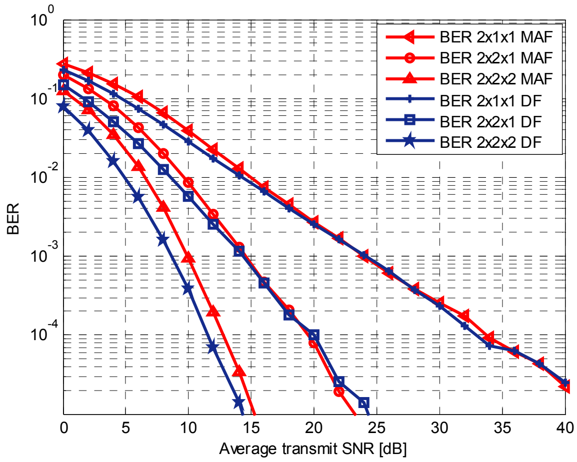

Fig. 3. BER for dual-hop and dual-antennas MAF and DF systems

From the Fig. 2 it is obvious that in 2x2x1MIMO system we obtained diversity gain of 16dB at BER at 10^{-4} and for 2x2x2 MIMO scheme we obtain diversity gain of 23dB at BER of 10^{-4}. Those results are similar to diversity gains for point-to-point MIMO systems obtained in [2]. Furthermore, on Fig.3 we present comparison of the BER for non-regenerative MAF system with the BER performance of regenerative DF system. DF system slightly outperforms MAF system. The performance gap increases as number of receive antenna at the relay and the destination increases.

On Fig.4 we present simulation and theoretic results for the outage probability (OP) of the analyzed dual-hop dual-antennas systems. For the sake of better comparison, on the same figure we presented results for point-to-point single-antenna system (denoted: 1x1), dual-hop single-antenna system (denoted: 1x1x1), point-to-point 2x1 antenna system, and point-to-point 2x2 antenna system.

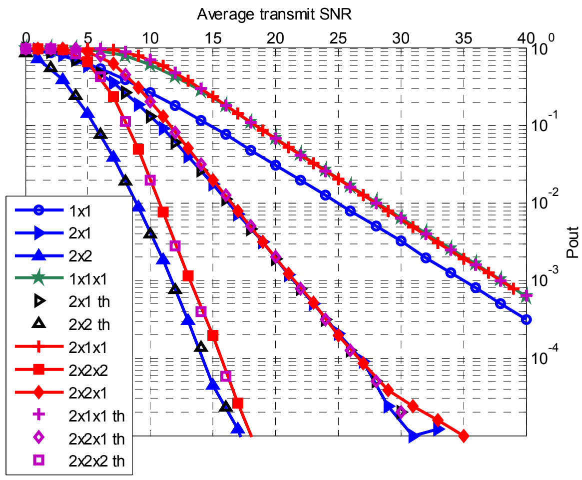

Fig. 4. Theoretical vs. simulated OP of 2x1x1, 2x2x1 and 2x2x2 MAF system ($\gamma_0 = 5dB$)

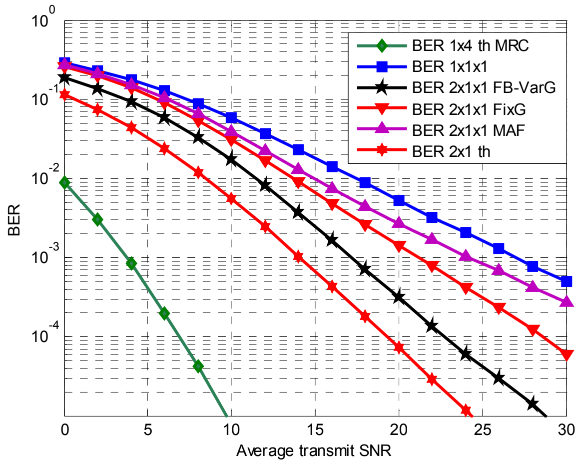

Fig.5 Comparison of 2x1x1 MAF, TDD and FixG systems

Furthermore on the same figure we present results obtained by our theoretic analysis (denoted with “th” extension). Numerical results show that the OP analysis, either by usage of numerical approach in (57) or by usage of mathematical closed form in (59), makes an exact match with the simulation results in various dual-antennas scenarios. Moreover, from Fig.4 it is obvious that 2x1x1 system has similar OP performance as 1x1x1 system. If we remove the constraint of same total transmission power the OP performance of 2x1x1 system would improve around 3 dB. The OP performance for 2x2x1 and 2x2x2 systems are better than 1x1x1 system for 16dB and 25dB at OP of 10^{-3}. However, these two systems are lagging the OP performance of point-to-point 2x1 and 2x2 systems from 0dB to 4dB

On Fig.5 we compare BER performance of 2x1x1 MAF system with system with fixed gain (FixG) where relay amplification factor is chosen according [3, Eq.15] and the CSI feedback system with variable gain (FB-VarG) i.e. the system with transmit beamforming at the relay. It is clear that FB-VarG system outperforms FixG and MAF system. MAF system shows worst BER performance. However, FixG and

FB-VarG systems have its own pros and cons. In the FixG system the relay is pure AF and hence it is very simple. However it implies higher complexity and cost in the destination since destination has to fully implement Alamouti's decoder and has full knowledge of the first and second hop channel coefficients. In the FB-VarG system relay has higher complexity and cost since it has to estimate the channel coefficients of the second hop in order to compensate second hop's channel gain and phase variation by multiplying the transmit signal with estimated channel coefficient. Moreover the destination has to estimate the channel coefficient of the first hop i.e. end-to-end channel coefficient and implement Alamouti's decoder. Taking into account the higher complexity and cost of FixG and FB-VarG systems we believe that MAF system has best performance vs. cost ratio.

However, the overall performance of 2x1x1 MAF system is not worth the cost of implementation since it has similar performance as 1x1x1 system, but the usage of 2x2x1 system gives substantial improvement in performance compared to the single-antenna dual-hop systems. We believe this is the most-feasible configuration to be met in future infrastructure cooperation. For example, one possible 2x2x1 configuration is where originating base station has two antennas, the cooperating base station acting as relay has two antennas, and the mobile station has single antenna. The 2x2x2 system gives best BER and OP performance, however its usage in future wireless communications seams less probable.

However, there are a number of important issues that either have not been addressed in our research, or have received insufficient treatment. The performance analysis may be extended for multi-hop MIMO systems using higher order STBC and different modulation schemes. Additionally, in the future, synchronization, security and distribution of CSI in those systems could be more comprehensively addressed.

V. CONCLUSION

In this paper, the error probability and outage probability performance of three types of dual-hop dual-antennas MIMO relay systems (2x1x1, 2x2x1 and 2x2x2) with modified AF (MAF) variable gain relays in Rayleigh fading have been studied. The BER performances of the systems were compared with dual-hop single-antenna system with variable gain relay and with corresponding configurations of dual-hop dual-antennas regenerative DF systems. The diversity gain of dual-hop dual-antennas MIMO MAF relay systems compared to single-antenna AF relay system is ranging from 16 to 23 dB at BER of 10^{-4} depending of the number of antennas employed in the destination. However, there is only 3dB gain at BER of 10^{-4} for 2x1x1 MIMO relay system. The BER performances of dual-antennas MIMO relay systems are slightly worse than dual-antennas DF relay systems (0-2dB). The performance gap increases with increase of the number of antennas in the relay and the destination. The OP performances of the dual-hop dual-antennas MIMO relay systems were compared with single-antenna dual-hop relay systems, and dual-antennas point-to-point systems. While the benefit of usage of 2x1x1 system is marginal, the OP performances for 2x2x1 and 2x2x2 systems are better than 1x1x1 systems for 16dB and 25dB at OP of 10^{-3}.

In the paper we have presented two approaches for theoretical analysis of OP. The first with usage of numerical integration of MGF and second with closed form expression of OP. Both approaches have shown exact match with simulation results.

Taking into account the superior performance compared to dual-hop single-antenna rely systems, their lower complexity and slightly inferior performance compared to dual-antennas DF, we have shown that usage of multiple antennas at the source, relay and/or destination can potentially be very beneficial in future wireless communications systems.

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