Performance analysis of dual-hop MIMO systems

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Abstract. In this paper we study the end-to-end bit error and outage probability (OP) performance of dual-hop multiple input multiple output (MIMO) systems with Alamouti's coding using modified amplify and forward (MAF) relaying under flat Rayleigh fading channels. The bit error performances of dual-hop MIMO systems with variable gain relays is compared with dual-hop single antenna systems and regenerative i.e. decode and forward (DF) dual-hop MIMO system. We show that MAF MIMO systems achieve significantly lower bit error probability then dual-hop single-antenna systems and comparable performance with DF systems. The performance gap increases with usage of dual antenna in relay and the receiver. The OP performances of these systems are compared with single-antenna dual-hop and dual-antenna single-hop systems. We show significant improvement of OP performance compared to single-antenna dual-hop and comparable performance with dual-antenna single-hop systems.

Key words: Cooperative wireless communications**,** MIMO, Alamouti space time block coding, dual-hop relay systems, bit error probability, outage probability, Rayleigh fading

1 Introduction

The new hot topic in the contemporary wireless communications is user and infrastructure-based cooperation, which has already occupied an entire new area of research in the wireless communications, called cooperative communications. Cooperative terminals exploit the properties of the multipath transmission of the radio signal in order to increase the efficiency and robustness of their communication. That means that the neighboring wireless stations, which are in the area of one transmitter-receiver pair, are "assisting" the communication between them in their "leisure time" by performing the function of relay. Beside cooperation of the user terminals i.e. user cooperation, another cooperative scenario is cooperation of base stations i.e. infrastructure-based cooperation. In this paper the proposed multiple antennas systems are intended for such scenario, mostly due to the space and cost limitations of the mobile stations. Namely, in order to provide transmit or receive diversity and sufficient decorrelation of the transmitted signals, antennas in the mobile station should be separated about three wavelengths. The sufficient separation of antennas in

the base station is about ten wavelengths but there is no space limitation. Additionally, taking in account that each base station serves many mobile stations it is more cost efficient to add on complexity in the base station.

In wireless communications systems the bit error probability (BEP) and outage probability (OP) are most important performance measure of the cooperative relaying system. Therefore, we investigated the end-to-end BEP and OP performance of the dual-hop relay systems using multiple antennas with Alamouti's space time block coding (STBC), operating over independent Rayleigh fading channels. We analyzed modified amplify and forward (MAF) dual-hop dual-antenna systems with variable gains, and compared their BER performance to regenerative DF systems as well as with dual-hop single antenna systems. The MAF scheme compared to pure non-regenerative amplify and forward (AF) scheme requires implementation of Alamouti decoder in the relay. Moreover we analyzed OP performance of these systems and compared it with single-antenna dual-hop and dual-antenna single-hop systems. We used variable gain relays [1] which require knowledge of the instantaneous channel state information (CSI) of each hop. The fixed gain relays which require knowledge of the average fading signal-to-noise ratio (SNR) of the previous hop were not considered due to the fact that for decoding of Alamouti's space time block code knowledge of CSI is required.

The remainder of this paper is organized as follows. Next Section presents the system and channel model. In Section 3 we derive expressions for end-to-end SNR needed for successful analysis of outage probability of the systems. Results are presented in Section 4, and Section 5 concludes the article.

2 System And Channel Models

In this paper we analyzed MIMO relay systems utilizing Alamouti scheme in three different configurations: 2x1x1 MIMO system where only the source is equipped with two antennas, 2x2x1 MIMO system where source and relay are equipped with two antennas, and 2x2x2 MIMO where source, relay and destination are equipped with two antennas. Fig. 1 presents the studied dual-hop MIMO communication system, which consists of the source S, the destination D and MAF relay R. It is assumed that each hop is subjected to the independent but non-identical Rayleigh fading, for which the per-hop SNR γ is distributed according to the probability distribution function (PDF) given by [4]:

$$p(\gamma) = \frac{1}{\bar{\gamma}} \, e^{-\frac{\gamma}{\bar{\gamma}}}, \ \ \gamma \geq 0 \, . \tag{1}$$

where $\bar{\gamma}$ is the average per-hop SNR. We assumed that average per-hop SNR is equal for each hop i.e. $\bar{\gamma} = \bar{\bar{\gamma}}$. It is also assumed that the amplitudes of fading from each transmit antenna to each receive antenna are mutually uncorrelated, Rayleigh distributed and that the average signal powers at each receive antenna from each transmit antenna are the same. Further, we assumed that the relay and the receiver have perfect knowledge of the channel.

In case of MAF system, the relay amplifies and forwards the received signal, while in case of decode and forward (DF) the relay fully decodes the received signal and then forwards it to the next hop. The variable gain relaying is modeled according to concepts presented in [1], and the dual-antenna systems with transmit diversity are designed by using of Alamouti's scheme given in [2].
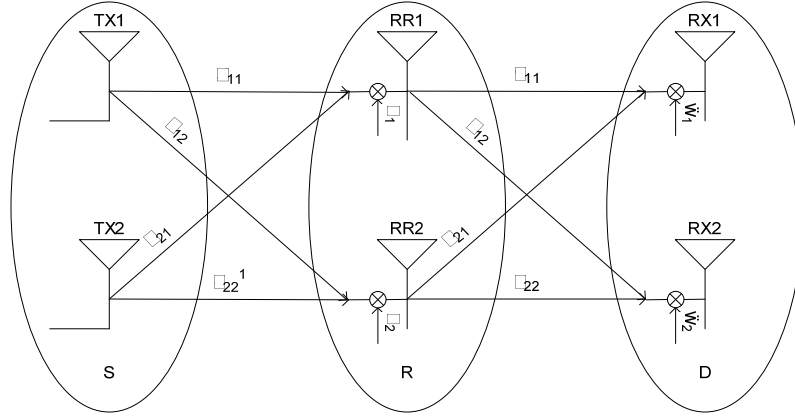

Fig. 1. Dual-hop MIMO system model.

In following sections of the paper we will use notation given in the Fig. 1. Namely all variables related to the first hop will be notated with dot above the symbol and all variables related to the second hop will be notated with double dot above the symbol. The first index in the subscript of channel coefficients identifies the transmitting antenna and the second index identifies the receiving antenna.

2.1 Dual-hop 2x1x1 MIMO system

For analysis of 2x1x1 system we assume that only the first antennas of the relay and the destination depicted on Fig.1 are active. The transmitted signal at the source S is given in following form:

$$x = [x_1, x_2, x_3, \ldots, x_N] . \tag{2}$$

The received signals in first antenna of the relay R in the first and second time slots are given with:

$$\dot{y_1} = \sqrt{E} \left(\dot{h}_{11} \cdot x_1 + \dot{h}_{21} \cdot x_2 \right) + \dot{n}_{11} , \tag{3}$$

$$\dot{y_2} = \sqrt{E} \left(-\dot{h}_{11} \cdot x_2^* + \dot{h}_{21} \cdot x_1^* \right) + \dot{n}_{21} , \tag{4}$$

where x_1 and x_2 are the transmitted symbols, h_{ij} are the channel coefficients, n_{i1} are noise components in the first and second time slot in the first relay antenna, and E is radiating power of one source antenna. It is assumed that radiating power of any

antenna at the source and the relay are equal to E. The noise in the first hop and first antenna of the relay can be presented in following form:

$$\dot{w}_1 = \left[\dot{n}_{11}, \dot{n}_{21}, \dot{n}_{31}, \ldots \dot{n}_{N1}\right]. \tag{5}$$

In the relay we used reduced complexity receiver with Alamouti decoder and without detector. Such implementation of the relay reduces the complexity (especially for more advanced modulation schemes) and versatility of the system. The decoded signal is:

$$\hat{y}_1 = \dot{h}_{11}^* \dot{y}_1 + \dot{h}_{21} \dot{y}_2^* = \sqrt{E}\ \dot{\Delta}_1 x_1 + \dot{\xi}_1 , \tag{6}$$

$$\hat{y}_2 = \dot{h}_{21}^* y_1 - \dot{h}_{11} \dot{y}_2^* = \sqrt{E}\ \dot{\Delta}_1 x_2 + \dot{\xi}_2 , \tag{7}$$

where:

$$\dot{\Delta}_1 = \left|\dot{h}_{11}\right|^2 + \left|\dot{h}_{21}\right|^2 , \qquad \dot{\xi}_1 = \dot{h}_{11}^* \dot{n}_1 + \dot{h}_{21} \dot{n}_2^*,$$
$$\dot{\xi}_2 = \dot{h}_{21}^* \dot{n}_1 - \dot{h}_{11} \dot{n}_2^* . \tag{8}$$

Decoded signal can be presented in following form:

$$\dot{r}_1 = \left[\hat{y}_1, \hat{y}_2, \hat{y}_3 \ldots \hat{y}_N\right]. \tag{9}$$

where N represents total number of transmitted symbols. The decoded signal is amplified and forwarded towards the destination. Since we assume that in the destination only the first antenna is active ($\ddot{h}_{12} = \ddot{h}_{22} = 0$), the received signal is given with:

$$\ddot{r}_1 = G_1 \ddot{h}_{11}\ \hat{y}_1(t) + \ddot{w}_1(t). \tag{10}$$

Where G_l is the gain of the relay and $\ddot{w}_1(t)$ is an additive white Gaussian noise in the second hop of the system and the first destination antenna with average power N_0. It is helpful to stress that subscript index for G_i, r_i, w_i and Δ_i is related to the type of Alamouti's code used in the given hop i.e. to the number of the receiving antennas. If the code is with two transmit antenna and one receive antenna $i = 1$, and if code is with two transmit antenna and two receive antenna $i = 2$.

Taking in consideration that the CSI for the first hop are required in order to implement the Alamouti decoder, we have chosen variable gain in the relay in order to cancel the effect of the channel in the first hop. If we change (6) or (7) in (10) and following the definition of G for variable gain relays in (4) from [1] the equation for the gain of 2x1x1 system can be expressed in following form:

$$G_1 = \sqrt{\frac{\ddot{E}}{\ddot{E}\ \dot{\Delta}_1^2 + \dot{\Delta}_1\ N_0}} , \tag{11}$$

where $\dot{E}$ and $\ddot{E}$ are the radiating powers of single antenna at the source S and relay R. However, it is assumed: $\dot{E} = \ddot{E} = E$. In the destination D the $\ddot{r}_1$ signal is equalized and detected with maximum likelihood detector.

2.2 Dual-hop 2x2x1 and 2x2x2 MIMO systems

If we consider 2x2x1 system the received signals in first relay antenna in the first and second time slots are given with:

$$\dot{y}_{11} = \sqrt{E}\,\dot{h}_{11} \cdot x_1 + \sqrt{E}\,\dot{h}_{21} \cdot x_2 + \dot{n}_{11}, \tag{12}$$

$$\dot{y}_{21} = -\sqrt{E}\,\dot{h}_{11} \cdot x_2^* + \sqrt{E}\,\dot{h}_{21} \cdot x_1^* + \dot{n}_{21}. \tag{13}$$

The signals in second antenna of the relay in the first and second time slots are:

$$\dot{y}_{12} = \sqrt{\dot{E}}\,h_{12} \cdot x_1 + \sqrt{E}\,\dot{h}_{22} \cdot x_2 + \dot{n}_{12}, \tag{14}$$

$$\dot{y}_{22} = -\sqrt{E}\,\dot{h}_{12} \cdot x_2^* + \sqrt{E}\,\dot{h}_{22} \cdot x_1^* + \dot{n}_{22}, \tag{15}$$

where n_{ij} are noise components in the first and second time slot in the first and second antenna. The noise in the first hop and the first antenna of the relay is presented with equation (5) and the noise in the first hop and the second antenna of the relay can be presented in following form:

$$\dot{w}_2 = [\dot{n}_{12,}\dot{n}_{22}, \dot{n}_{32}, \dots \dot{n}_{N2}]. \tag{16}$$

The outputs of each antenna combiner are added to each other in order to get the decoded signals in first and second timeslot:

$$\hat{z}_1 = \dot{h}_{11}^* \dot{y}_{11} + \dot{h}_{21} \dot{y}_{21}^* + \dot{h}_{12}^* \dot{y}_{12} + \dot{h}_{22} y_{22}^* = \sqrt{E}\,\dot{\Delta}_2 x_1 + \dot{\eta}_1 , \tag{17}$$

$$\hat{z}_2 = \dot{h}_{21}^* \dot{y}_{11} - \dot{h}_{11} \dot{y}_{21}^* + \dot{h}_{22}^* \dot{y}_{12} - \dot{h}_{12} \dot{y}_{22}^* = \sqrt{E}\,\dot{\Delta}_2 x_2 + \dot{\eta}_2 , \tag{18}$$

where:

$$\dot{\Delta}_2 = |\dot{h}_{11}|^2 + |\dot{h}_{12}|^2 + |\dot{h}_{21}|^2 + |\dot{h}_{22}|^2 , \tag{19}$$

$$\dot{\eta}_1 = \dot{h}_{11}^* \dot{n}_{11} + \dot{h}_{21} \dot{n}_{21}^* + \dot{h}_{12}^* \dot{n}_{12} + \dot{h}_{22} \ddot{n}_{22}^* , \tag{20}$$

$$\dot{\eta}_2 = \dot{h}_{21}^* \dot{n}_{11} - \dot{h}_{11} \dot{n}_{21}^* + \dot{h}_{22}^* \dot{n}_{12} - \dot{h}_{12} \dot{n}_{22}^* . \tag{21}$$

The decoded signal at the output of the combiner is:

$$\dot{r}_2 = [\hat{z}_1, \hat{z}_2, \hat{z}_3 \dots \hat{z}_N] . \tag{22}$$

This signal is amplified and forward towards the destination. The received signal at the destination is:

$$\ddot{r}_2 = G_2 \ddot{h}_{11}\, \hat{z}_1(t) + \ddot{w}_1(t). \tag{23}$$

The signal $\dot{r}_2$ is transmitted in same manner as the signal x in the source. We have chosen variable gain of the relay in order to reverse the effect of the channel in the first hop. Taking in consideration equations (17), (18) and (4) in [1] the selected gain in the relay is:

$$G_2 = \sqrt{\frac{E}{E \cdot \dot{\Delta}_2^2 + \dot{\Delta}_2 \, N_0}} \qquad (24)$$

In case of single antenna at the destination, similarly to the 2x1x1 MIMO system the signal is equalized and detected with maximum likelihood detector. In case of 2x2x2 MIMO system where two antennas are used at the destination the signal is decoded in same manner as in the relay i.e. by using equations (17) and (18). The output of the combiner i.e. the decoded signal is fed to the maximum likelihood detector.

3 Outage probability of dual-hop dual-antenna systems

The outage probability is defined as the probability that the instantaneous SNR falls below a predetermined threshold ratio γ_{th}

$$P_{out} = P(\gamma_{eq} < \gamma_{th}) \qquad (25)$$

where γ_{eq} represent equivalent i.e. end-to-end instantaneous SNR of the dual-hop system. In order to successfully find outage probability we derived the equivalent (end-to-end) SNR for 2x1x1, 2x2x1, and 2x2x2 dual-hop system.

For 2x1x1 MIMO system the first step is to derive the instantaneous SNR ($\dot{\gamma}_1$) of the first hop i.e. the goal is to find instantaneous SNR for system with two transmit and one receive antenna with Alamouti STBC. From equations (6) and (7) it is easy to derive that the instantaneous SNR in the first hop is:

$$\dot{\gamma}_1 = \frac{E}{N_0} \cdot \dot{\Delta}_1 \, . \qquad (26)$$

The received signal at the destination in the first time slot can be presented in following form:

$$\ddot{r}_1 = G_1 \, \ddot{h}_{11} \, \hat{y}_1 + \ddot{w}_1(t) = G_1 \, \ddot{h}_{11} \, \sqrt{E} \cdot \dot{\Delta}_1 \cdot x_1 + \; G_1 \ddot{h}_{11} \dot{\xi}_1 \; + \; \ddot{w}_1(t) \, . \qquad (27)$$

Since in the second hop we deal with 1x1 system (1 transmit and 1 receive antenna) in order to depict the system from Fig.1 we assumed that all channel parameters of the second hop are set to 0 except $\ddot{h}_{11}$. Taking in consideration equation (27) it is easy to show that end-to-end SNR for 2x1x1 dual-hop system is given with:

$$\gamma_{eq1} = \frac{E}{N_0} \cdot \frac{G_1^2 \, |\ddot{h}_{11}|^2 \, \dot{\Delta}_1^2}{G_1^2 \, |\ddot{h}_{11}|^2 \dot{\Delta}_1 + 1} \qquad (28)$$

For 2x2x1 dual-hop system the first step is to derive the instantaneous SNR ($\dot{\gamma}_2$) of the first hop i.e. we should find instantaneous SNR for system with 2 transmit and 2 receive antennas with Alamouti STBC. From equations (17) and (18) it is easy to derive that the instantaneous SNR in the first hop is:

$$\dot{\gamma}_2 = \frac{E}{N_0} \cdot \dot{\Delta}_2 . \tag{29}$$

Decoded signal in first time slot of first antenna of the destination (second antenna is not active) can be present in similar manner to equation (6):

$$\hat{\ddot{y}}_1 = G_2 \, \ddot{\Delta}_1 \hat{\ddot{z}}_1 + \ddot{\xi}_1 \tag{30}$$

where analogous to (8) $\ddot{\xi}_1$ is given with:

$$\ddot{\xi}_1 = \ddot{h}_{11}^* \ddot{n}_1 + \ddot{h}_{21} \ddot{n}_2^* \tag{31}$$

If we replace (17) in (30) we will get:

$$\hat{\ddot{y}}_1 = G_2 \, \ddot{\Delta}_1 \hat{\ddot{z}}_1 + \ddot{\xi}_1 = G_2 \, \ddot{\Delta}_1 \, \dot{\Delta}_2 \, \sqrt{E} \, x_1 + G_2 \, \ddot{\Delta}_1 \, \dot{\eta}_1 + \ddot{\xi}_1 \tag{32}$$

Taking in account (32) it is straightforward to show that the end-to-end SNR is:

$$\gamma_{eq2} = \frac{E}{N_0} \cdot \frac{G_2^2 \, \ddot{\Delta}_1 \dot{\Delta}_2^2}{G_2^2 \, \ddot{\Delta}_1 \dot{\Delta}_2 + 1} . \tag{33}$$

Where analogous to (8) $\ddot{\Delta}_1$ is given with

$$\ddot{\Delta}_1 = \left| \ddot{h}_{11} \right|^2 + \left| \ddot{h}_{21} \right|^2 . \tag{34}$$

For 2x2x2 dual-hop system the decoded signal in first time slot is:

$$\hat{\ddot{y}}_1 = G_2 \, \ddot{\Delta}_2 \hat{\ddot{z}}_1 + \ddot{\eta}_1 = G_2 \, \ddot{\Delta}_2 \, \dot{\Delta}_2 \, \sqrt{E} \, x_1 + G_2 \, \ddot{\Delta}_2 \, \dot{\eta}_1 + \ddot{\eta}_1, \tag{35}$$

where:

$$\ddot{\Delta}_2 = \left| \ddot{h}_{11} \right|^2 + \left| \ddot{h}_{12} \right|^2 + \left| \ddot{h}_{21} \right|^2 + \left| \ddot{h}_{22} \right|^2 , \tag{36}$$

$$\ddot{\eta}_1 = \ddot{h}_{11}^* \ddot{n}_{11} + \ddot{h}_{21} \ddot{n}_{21}^* + \ddot{h}_{12}^* \ddot{n}_{12} + \ddot{h}_{22} \ddot{n}_{22}^*. \tag{37}$$

Therefore, that end-to-end SNR for 2x2x2 system is:

$$\gamma_{eq3} = \frac{E}{N_0} \cdot \frac{G_2^2 \, \ddot{\Delta}_2 \dot{\Delta}_2^2}{G_2^2 \, \ddot{\Delta}_2 \dot{\Delta}_2 + 1} . \tag{38}$$

4 Numerical Results

Usually, in real world applications the system radiation power is limited, therefore in the simulations we kept the total radiating power constant. In order to have the same total radiated power from two transmit antennas with the power of the single transmit antenna, the energy allocated to each symbol was halved. For the simulations we have chosen BPSK modulation scheme. In order to have good reference for analyzing the results we have chosen result for dual-hop variable gain system as upper bound and the result for single-hop receive diversity system with 4 antennas and maximum

ratio combiner as lower bound. The results were expected to be located between these two BER (Bit Error Rate) curves. Obtained bit error probabilities for the 2x1x1, 2x2x1, and 2x2x2 dual-hop dual-antenna systems are given on Fig. 2.
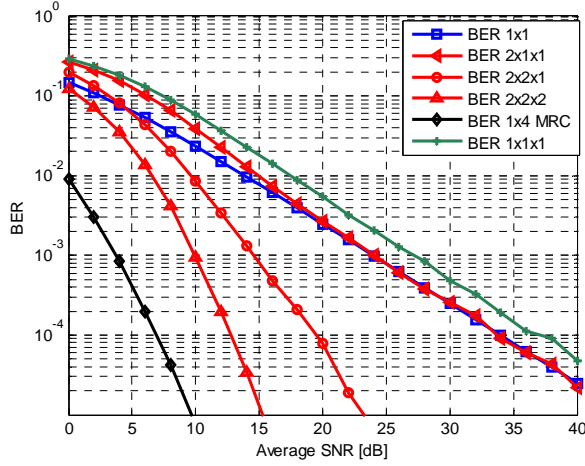

Fig. 2. BER for dual-hop dual-antenna MAF systems.

From the Fig. 2 it is obvious that in 2x2x1MIMO scheme we obtain diversity gain of 15dB at BER at 10^{-4} and for 2x2x2 MIMO scheme we obtain diversity gain of 25dB at BER of 10^{-4} which are similar to diversity gains for single-hop MIMO systems given in [2]. Furthermore, on Fig.3 we present comparison of the BER for non-regenerative MAF system with the BER performance of regenerative DF system. DF system slightly outperforms MAF system. The performance gap increases as number of used antenna at the relay and the destination increases.

On Fig.4 we present results of simulation of the outage probability (OP) of the analyzed dual-hop dual-antenna systems. For the sake of simple comparison on the same picture we presented results for single-hop single-antenna system, dual-hop single-antenna system, single-hop 2x1 antenna system, and single-hop 2x2 antenna system. It is obvious that 2x1x1 system has similar OP performance as dual-hop single-antenna system. If we remove the constraint of same total radiated power the OP performance of 2x1x1 system would improve around 3 dB. The OP performance for 2x2x1 and 2x2x2 systems are better than dual-hop single-antenna system for 16dB and 25dB at OP of 10^{-3}. Moreover, these two systems are lagging the OP performance of single-hop 2x1 and 2x2 system from 0dB to 4dB.
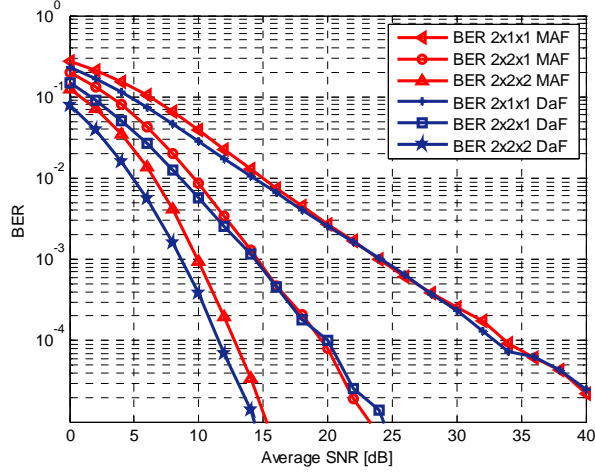
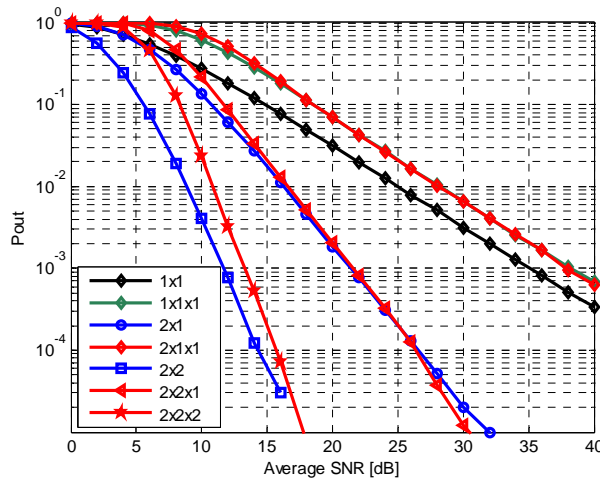**Fig. 3.** BER for dual-hop and dual-antenna MAF and DF systems.

Fig. 4. Outage probability of 2x1x1, 2x2x1 and 2x2x2 MAF system (γ_{th}=5dB).

The overall performance of 2x1x1 system is not worth the cost of implementation. However, the usage of 2x2x1 system gives substantial improvement in performance compared to the single-antenna dual-hop systems. We believe this is the most-feasible configuration to be met in the reality. One possible 2x2x1 configuration is where originating base station has two antennas, the cooperating base station acting as relay has two antennas, and the mobile station has single antenna. The 2x2x2 system gives best BER and OP performance, however its usage in future wireless communications seams less probable.

5   Conclusion

In this paper, the bit error probability and outage probability performance of three dual-hop MIMO configurations (2x1x1, 2x2x1 and 2x2x2) with modified AF variable gain relays in Rayleigh fading have been studied. The BER performances of the systems were compared with dual-hop single antenna system with variable gain relay and with corresponding configurations of dual-hop dual antenna non-regenerative DF systems. The diversity gain of dual-hop MIMO MAF system compared to single-antenna AF system is ranging from 15 to 25 dB at BER of 10^{-4} depending of the number of antennas employed in the destination. However, there is only 3dB gain at BER of 10^{-4} for 2x1x1 MIMO system. The BER performances of dual-antenna MAF systems are slightly worse than dual-antenna DF systems (0-2dB). The performance gap increases with increase of the number of antennas in the relay and the destination. The OP performances of the systems were compared with single-antenna dual-hop systems, and dual-antenna single-hop systems. While the benefit of usage of 2x1x1 system is marginal, the OP performances for 2x2x1 and 2x2x2 systems are better than dual-hop single-antenna systems for 16dB and 25dB at OP of 10^{-3}.

Taking in account the superior performance compared to dual-hop single antenna systems, their lower complexity and slightly inferior performance compared to dual-antenna DF, we have shown that usage of dual-antenna could be very beneficial in future wireless communications systems with infrastructure-based cooperation.

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