Outage Probability of Multi-hop Relay Systems in Various Fading Channels

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Abstract. In this paper we study the end-to-end outage performance of multi-hop cooperative communication systems employing amplify and forward (AaF) relaying under Rayleigh, Nakagami, Rician and Weibull fading channels. The outage probability performances of multi-hop systems with fixed gain and variable gain relays is compared. The outage probability for multi-hop systems under Rayleigh, Nakagami and Weibull fading models can be determined only by combining analytical results with numerical integration techniques. We show that fixed gain system has a better outage performance compared to the variable gain for all fading scenarios. This performance gap increases by increasing the number of hops.

Key words: Wireless cooperative communications**,** outage probability, multipath fading, multi-hop relay systems

1 Introduction

The newest trend in the contemporary wireless networks is based on the paradigm of partner cooperation, which has already occupied an entire new area of research in the wireless communications, called cooperative communications. Cooperative terminals exploit the properties of the multipath transmission of the radio signal in order to increase the efficiency and robustness of their communication. That means that the neighboring wireless stations, which are in the area of one transmitter-receiver pair, are “assisting” the communication between them in their “leisure time” by performing the function of relay, thus achieving the effect of virtual diversity (e.g. virtual multi-antenna array). The effect of spatial diversity is generated because the cooperating partners on different locations are resending independent copies of the signal over orthogonal channels. The receiver is combining the signal’s replicas from the source and other partners in a constructive manner in order to provide reliable decision of the transmitted symbol.

In such cooperative scenario, the outage probability (OP) is the key performance measure of the cooperative relaying system. In this paper, we study the end-to-end OP performance of the multi-hop relay systems operating over independent Rayleigh, Nakagami, Rician and Weibull fading channels. We consider amplify and forward (AaF) multi-hop systems with fixed and variable gains. A fixed gain relay requires knowledge of the average fading signal-to-noise ratio (SNR) of the previous hop, while the variable gain relay requires knowledge of the instantaneous channel state information (CSI) of each hop.

The remainder of this paper is organized as follows. Next Section presents the system model and presents our novel analytical results. Numerical results are presented in Section 3, and Section 4 concludes the article.

2 System And Channel Models

Fig. 1 presents the studied non-regenerative multi-hop communication system, which consists of the source T, the destination D and (N–1) AaF relays. Each hop is subjected to the independent but non-identical Rayleigh, Nakagami, Rician or Weibull fading, for which the per-hop SNR γ is distributed according to the probability distribution functions (PDFs) given by [5]:

$$p_{ray}(\gamma)=\frac{1}{\bar{\gamma}}\exp\left(-\frac{\gamma}{\bar{\gamma}}\right), \tag{1}$$

$$p_{nak}(\gamma)=\frac{m^m\gamma^{m-1}}{\bar{\gamma}^m\,\Gamma(m)}\exp\left(-\frac{m\gamma}{\bar{\gamma}}\right), \tag{2}$$

$$p_{ric}(\gamma)=\frac{(1+K)}{\bar{\gamma}}\exp\left(-\frac{\gamma}{\bar{\gamma}}-K\right)I_0\left(2\sqrt{\frac{K(1+K)\gamma}{\bar{\gamma}}}\right), \tag{3}$$

$$p_{wei}(\gamma)=\frac{c}{2}\left(\frac{\Gamma\left(1+\frac{2}{c}\right)}{\bar{\gamma}}\right)^{\frac{c}{2}}\gamma^{\frac{c}{2}-1}\exp\left(-\left(\frac{\gamma}{\bar{\gamma}}\Gamma\left(1+\frac{2}{c}\right)\right)^{\frac{c}{2}}\right), \tag{4}$$

respectively, where m in (2) is the Nakagami fading parameter, $\bar{\gamma}$ is the average per-hop SNR, K in (3) is the Rician factor, c in (4) is the Weibull parameter, $I_0(\cdot)$ in (3) is the modified zero-th order Bessel function of first kind, and $\Gamma(\cdot)$ in (2) and (4) is the Gamma function. In case of AaF, relays amplify and then forward the received signal from the previous node, while, in case of decode and forward (DaF), the relays fully decode the received signal and then forward it to the next hop. The variable gain and fixed gain relaying is modeled according to concepts presented in [1] and [2], respectively.

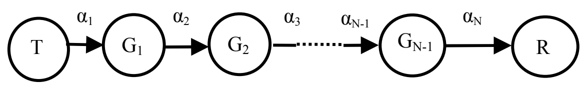

Fig. 1. *N*-hop system model

Fig. 1 presents the *N*-hop relaying scenario, with the (lowpass-equivalent) signal at the input of the *n*-th relay expressed as

$$r_n(t) = G_{n-1} \cdot \alpha_n \cdot \sqrt{\varepsilon_n} \cdot r_{n-1}(t) + w_n(t) , \tag{5}$$

where G_{n-1} is the gain of the (*n-1*)-th relay, α_n is the fading amplitude of the *n*-th hop, ε_n is the transmitting power of the (*n-1*)-th relay, r_{n-1} is the signal at the input of the (*n-1*)-th relay, and $w_n(t)$ is an additive white Gaussian noise (AWGN) with average power N_{0n}. Based on (5), the powers of the useful signal and the noise at the receiver output are respectively given by:

$$\begin{aligned} &P_S = (G_1^2 G_2^2 \cdots G_{N-1}^2)(\varepsilon_1 \alpha_1^2 \varepsilon_2 \alpha_2^2 \cdots \varepsilon_N \alpha_N^2) , \\ &P_N = (G_1^2 G_2^2 \cdots G_{N-1}^2)(\varepsilon_2 \alpha_2^2 \varepsilon_3 \alpha_3^2 \cdots \varepsilon_N \alpha_N^2) N_{01} + \\ &(G_2^2 \cdots G_{N-1}^2)(\varepsilon_3 \alpha_3^2 \varepsilon_4 \alpha_4^2 \cdots \varepsilon_N \alpha_N^2) N_{02} + \cdots + N_{0N} . \end{aligned} \tag{6}$$

By using (6), it can be shown that the signal-to-noise ratio (SNR) at the output of an *N*-hop multi-hop system, i.e., the receiver output, is expressed as

$$\gamma_{eq}^{-1} = \sum_{n=1}^{N} \left(\prod_{t=1}^{n-1} G_t^2 N_{0t} \prod_{t=1}^{n} \gamma_t \right)^{-1} . \tag{7}$$

For the case of variable gain relays, the *n*-th relay gain is set to [1]:

$$G_n^2 = \frac{1}{{\alpha_n}^2} , \tag{8}$$

where α_n is the *n*-th hop fading amplitude. In fixed gain case, the *n*-th relay gain is constant:

$$G_n = \frac{1}{C_n} , \tag{9}$$

where, according to [2], $C_n = \bar{\gamma}_n \left(e^{1/\bar{\gamma}_n} E_1(1/\bar{\gamma}_n)\right)^{-1}$. Note that $E_1(\cdot)$ is exponential integral function defined by [4, eq. (5.1.1)], and $\bar{\gamma}_n$ is average SNR of the n-th hop.

The OP is defined as the probability that the instantaneous output SNR falls below a predetermined threshold ratio γ_{th},

$$P_{out} = P[\gamma_{eq} < \gamma_{th}]. \tag{10}$$

To derive the analytical expression for the OP for the variable gain case, we used Moment Generating Function (MGF) based approach, devised in [1] and [5],

$$P_{out} = P(\gamma_{eq} < \gamma_{th}) = 1 - P\left(\frac{1}{\gamma_{eq}} < \frac{1}{\gamma_{th}}\right) = 1 - \mathcal{L}^{-1}\left(\frac{\mathcal{M}_{1/\gamma_{eq}}(-s)}{s}\right)\Big|_{1/\gamma_{th}}, \tag{11}$$

where $\mathcal{M}_{1/\gamma_{eq}}$ is MGF of the reciprocal of γ_{eq}. For Laplace transform inversion, we used Euler numerical technique given in [5, appendix 9B]:

$$P\left(\frac{1}{\gamma_{eq}} < \frac{1}{\gamma_{th}}\right) = \frac{2^{-k} e^{\frac{A}{2}}}{1/\gamma_{th}} \sum_{k=0}^{K} \binom{K}{k} \sum_{n=0}^{N+k} \frac{(-1)^n}{\alpha_n} \mathrm{Re}\left(\frac{M_{1/\gamma}\left(-\frac{A+2\pi jn}{2/\gamma_{th}}\right)}{\frac{A+2\pi jn}{2/\gamma_{th}}} \right) + E(A,K,N), \tag{12}$$

$$E(A,K,N) = \frac{e^{-A}}{1-e^{-A}} + \frac{2^{-k} e^{\frac{A}{2}}}{1/\gamma_{th}} \sum_{k=0}^{K} (-1)^{N+k+1} \binom{K}{k} \mathrm{Re}\left(\frac{M_{1/\gamma}\left(-\frac{A+2\pi j(N+k+1)}{2/\gamma_{th}}\right)}{\frac{A+2\pi j(N+k+1)}{2/\gamma_{th}}} \right), \tag{13}$$

where $\alpha_n = 1$ for $n = 1,2,\ldots,N$ and $\alpha_n = 2$ for $n = 0$. We set $A = 10\ln(10)$ in order to have discretization error less then 10^{-10}.

In case of Nakagami and Rayleigh (m_n=1) fading, we used the closed form expression for the reciprocal SNR of the n-th hop ($1/\gamma_n$):

$$\mathcal{M}_{1/\gamma_n}(-s) = \frac{2}{\Gamma(m)} \cdot \left(\frac{m_n s}{\bar{\gamma}_n}\right) K_{m_n}\left(2\sqrt{\frac{m_n s}{\bar{\gamma}_n}}\right). \tag{14}$$

Under the assumption that the hops are subjected to independent fading and the variable relay gain is chosen according (8), the MGF of $1/\gamma_{eq}$ is product of the MGF's of $1/\gamma_n$, $n = 1,2,\ldots N$ [1].

In case of Rician PDF, it is impossible to find $\mathcal{M}_{1/\gamma_{eq}}$ in closed form, neither derive it with CAS (Computer Algebra System), therefore, we resorted to numerical integration. For the Weibull PDF, we modified PDF (4) in a more convenient form:

$$p_{\gamma\, wei}(\gamma) = bA^{-b}\gamma^{b-1}\exp\left(-\left(\frac{\gamma}{A}\right)^{b}\right), \tag{15}$$

where A and b are given by

$$b = \frac{c}{2}; \quad A = \frac{\bar{\gamma}}{\Gamma\left(1+\frac{2}{c}\right)}. \tag{16}$$

and then, we used symbolic integration for those values of the argument of s that are required by the Euler numeric technique, i.e.,

$$\mathcal{M}_{1/\gamma}(-s) = \int_0^{\infty} b\cdot A^{-b}\cdot\gamma^{b-1}e^{-\frac{s}{\gamma}-\left(\frac{\gamma}{A}\right)^{b}}d\gamma\ . \tag{17}$$

3 Numerical Results

In this Section, we present some illustrative figures which depict the excellent match between the numerical results and the corresponding results obtained by Monte Carlo simulation implemented in Matlab. For the dual-hop ($N = 2$) fixed gain relaying under Rayleigh fading, we use the closed form expression for the OP

$$P_{out} = 1 - 2\sqrt{\frac{C\gamma_{th}}{\bar{\gamma}_1\bar{\gamma}_2}}\exp\left(-\frac{\gamma_{th}}{\bar{\gamma}_1}\right)K_1\left(2\sqrt{\frac{C\gamma_{th}}{\bar{\gamma}_1\bar{\gamma}_2}}\right). \tag{18}$$

where $K_1(\cdot)$ is first-order modified Bessel function of the second kind.

Fig. 2 presents the OP versus average per-hop SNR for dual-hop fixed gain system in Rayleigh fading. Note that $\bar{\gamma}_1 = \bar{\gamma}_2$, and the thresholds γ_{th} are set as 0, 5 and 10dB respectively.

Fig. 3 presents the comparative curves of the OP of AaF system with fixed gain, AaF system with variable gain and a DaF system (used for comparison) with multiple hops in Rayleigh fading. Note that N = 2, 3 and 4, and $\gamma_{th} = 5\ dB$. It is obvious that, for the medium to large average SNRs, the two-hop systems with variable gain relays slightly outperform two-hop system with fixed gain relays, and for low to medium SNRs, the fixed gain systems outperform variable-gain systems. However, as the number of hops increase, the fixed-gain multi-hop systems outperform the correspon-ding variable gain multi-hop systems for all regions of SNR given identical fading model is applied. We obtained the similar results for other types of channels as well. The illustrative results for the case of the Weibull PDF are presented on Fig. 4. It is important to mention that in our analysis saturation effect of the fixed gain relays is not taken in to account.

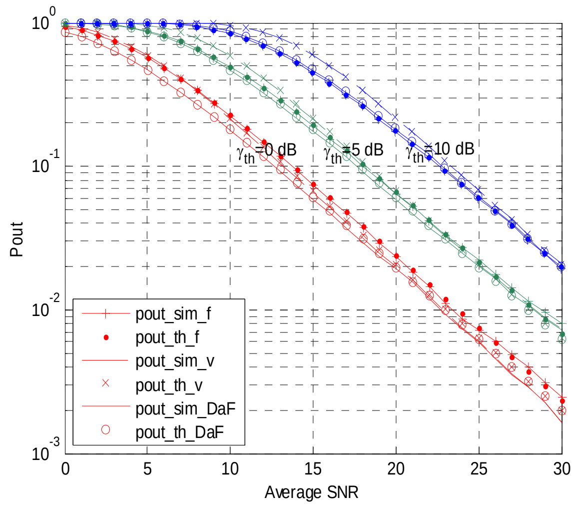

Fig. 2. OP performance of dual-hop system in Rayleigh fading with γ_{th} = 0, 5, and 10 *dB*

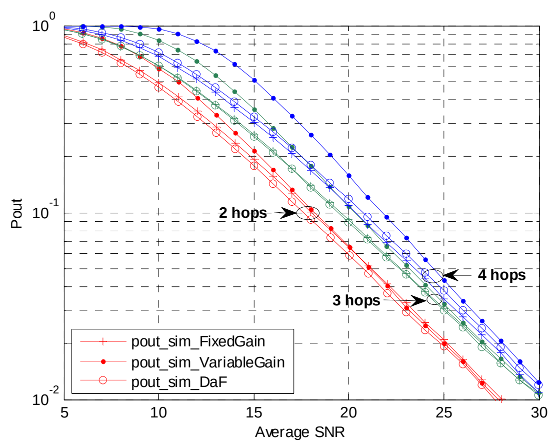

Fig. 3. OP for multi-hop system for N=2, 3 and 4 hops in Rayleigh fading when γ_{th} =5 dB

We also analyzed the multi-hop system with 4 hops and thresholds SNRs γ_{th} of 0, 5 and 10 dB under various types of fading. In Fig. 5 we present the results for a four-hop system in case when all hops are subjected to Rayleigh fading, whereas in Fig. 6 we present results for Nakagami fading. Fig. 7 and Fig. 8 present the results for Rician and Weibull fading, respectively. For this numerical analysis, we set the PDF parameters to following values: m=2.285, K=3 and b=1.5. In all cases for $\gamma_{th}=0$ (except for Rician fading), results again indicate that, for medium to large average SNRs, the variable gain systems slightly outperform the fixed gain systems. However, for $\gamma_{th}>0$, we notice better performance of the fixed gain systems for all channel models. For higher γ_{th}, the performance gap between the two systems is increased in favor of the fixed gain systems.

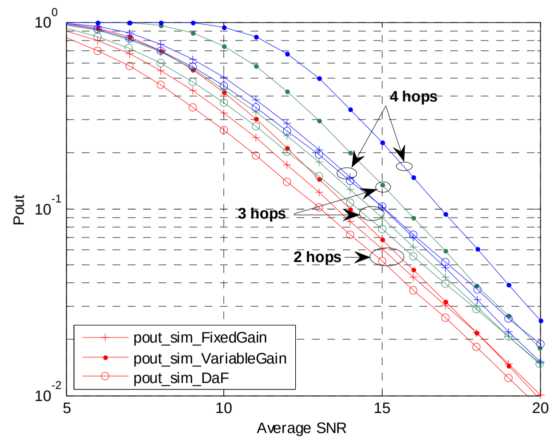

Fig. 4. OP for relaying system for N=2,3 and 4 in Weibull fading when γ_{th}=5 dB

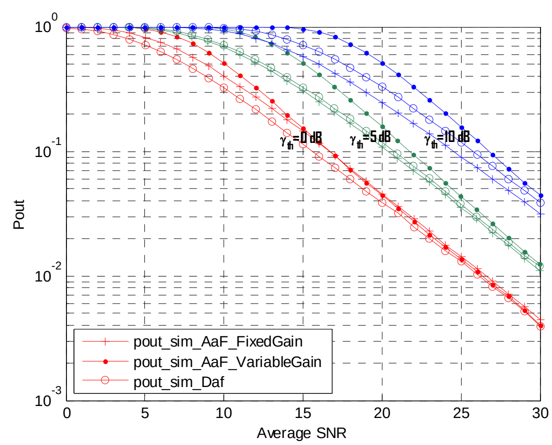

Fig. 5. OP for 4-hop system in Rayleigh fading $\gamma_{th}=0$, 5, and 10 dB

For 4-hop systems in Rician fading, the fixed gain systems outperform the variable gain system in all SNR regions regardless of the thresholds γ_{th} (Fig. 7). The gap in

OP performance increases as γ_{th} increases. We emphasize that the fixed gain relays achieve their best possible performance since the effect of the saturation sensibility of fixed gain system is not accounted for. Moreover, in all cases it can be observed that as the number of hops increase fixed gain system even slightly outperform DaF system due to sub-optimal power allocation in DaF systems [3].

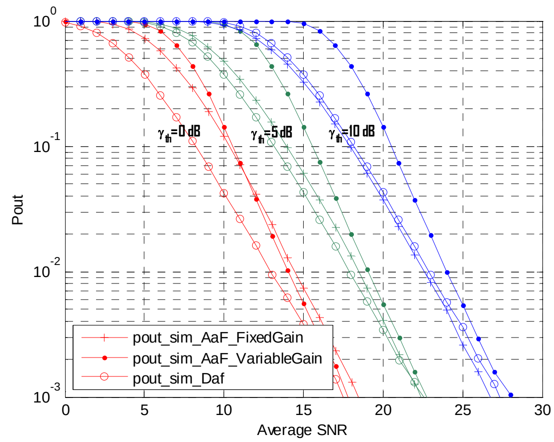

Fig. 6. OP for 4 hop system in Nakagami fading where γ_{th}=0, 5, 10 dB, and m= 2.3

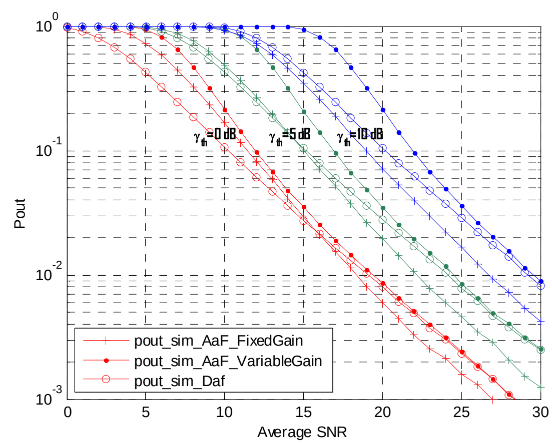

Fig. 7. OP for 4 hop system in Rician fading where γ_{th}=0, 5, 10 dB, and *K*=3

In Fig. 9 we compared the OP performances of 4-hop systems under Nakagami fading with fading parameter *m*= 2.2857 and Rician fading with the respective Rician factor *K*=3, calculated according expression (19). With such selection of the parameters similar fading effects should be expected.

$$K = \frac{\sqrt{m^2 - m}}{m - \sqrt{m^2 - m}}. \tag{19}$$

Considering Fig. 9 we find out that such approximation is relatively successful since the OP performances for the two channel models are similar, particularly for low to medium average SNRs. As average SNR increases, the Nakagami fading model foresees lower OP compared to the Rician fading model.

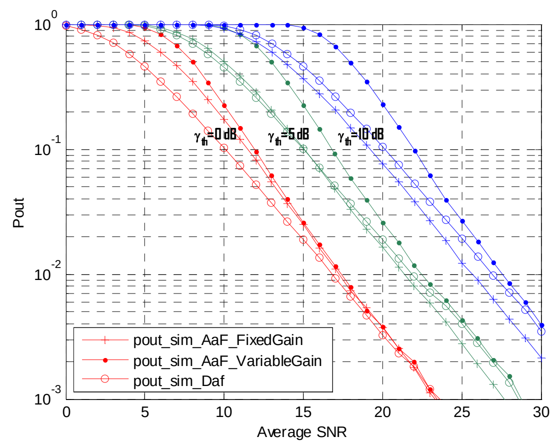

Fig. 8. OP for 4 hop system in Weibull fading (γ_{th}=0, 5, 10 dB))

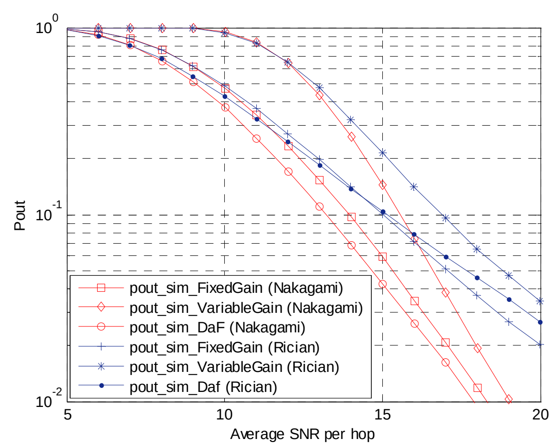

Fig. 9. OP for 4 hop system in Nakagami and Rician fading (γ_{th}= 5 dB)

4 Conclusion

In this paper, the outage performance of multi-hop AaF wireless relay system with fixed and variable gain relays had been studied by combination of analytical, numerical and simulation methods. Due to the complexity of the output SNR, the OP for multi-hop systems under Rayleigh, Nakagami, Rician and Weibull fading models can be determined only by combining analytical results with numerical integration techniques, except for the dual hop case.

Despite their lower complexity, the multi-hop system with fixed gain relays typically outperforms the variable gain system. The performance gap between these two systems increases substantially by increasing the number of hops for all considered fading channels regardless of the selection of the average per hop SNR. The increase of the SNR threshold further widens this gap.

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