☐ PROAKIS (APPENDIX A): THE LEVINSON-DURBIN ALGORITHM

$$\boxed{\Phi_p \cdot a_p = \phi_p}$$

Φ_p $p \times p$ TOEPLITZ MATRIX (DIAGONAL CONSTANT MATRIX)

a_p – VECTOR PREDICTOR COEFFICIENT

$$a_p' = [a_{p1}, a_{p2}, \dots a_{pp}]$$
$$\phi_p' = [\phi(1), \phi(2), \dots, \phi(p)]$$

$$\boxed{a_p = \Phi_p^{-1} \phi_p}$$

– FIRST ORDER PREDICTOR

$$\boxed{\phi(0)\, a_{11} = \phi(1)} \qquad \boxed{a_{11} = \phi(1)/\phi(0)}$$

VIDI: N7.19.11...

– RESIDUAL MSE FOR FIRST-ORDER PREDICTOR IS

$$\varepsilon_1 = \phi(0) - a_{11}\phi(1) = \phi(0) - a_{11}^2 \phi_0 = \phi_0(1 - a_{11}^2)$$

– COEFFICIENT OF m-ORDER PREDICTOR IN TERMS OF (m-1)th-ORDER PREDICTOR

$$a_m = \begin{bmatrix} a_{m1} \\ a_{m2} \\ \vdots \\ a_{mm} \end{bmatrix} = \begin{bmatrix} a_{m-1} \\ \hline 0 \end{bmatrix} + \begin{bmatrix} d_{m-1} \\ \hline k_m \end{bmatrix}$$

$$\Phi_m = \begin{bmatrix} \Phi_{m-1} & \phi_{m-1}' \\ \phi_{m-1}' & \phi(0) \end{bmatrix} \qquad \boxed{\phi_{m-1}' = \mathrm{fliplr}(\phi_{m-1})}$$

$$\boxed{\left[\begin{array}{c|c} \Phi_{m-1} & \phi_{m-1}' \\ \hline \phi_{m-1}' & \phi(0) \end{array}\right] \left(\begin{bmatrix} a_{m-1} \\ \hline 0 \end{bmatrix} + \begin{bmatrix} d_{m-1} \\ \hline k_m \end{bmatrix} \right) = \begin{bmatrix} \phi_{m-1} \\ \hline \phi(m) \end{bmatrix}}$$

1) $\Phi_{m-1} \cdot a_{m-1} + \Phi_{m-1} \cdot d_{m-1} + \phi_{m-1}' \cdot k_m = \phi_{m-1}$

$$\Phi_{m-1} \cdot a_{m-1} = \phi_{m-1} \Rightarrow \Phi_{m-1} d_{m-1} + \phi_{m-1}' k_m = 0$$

$$d_{m-1} = -k_m \underbrace{\Phi_{m-1}^{-1} \phi_{m-1}'}_{a_{m-1}'}$$

ϕ_m IN REVERSE ORDER

a_{m-1}' } a_{m-1} IN REVERSE ORDER

$$d_{m-1} = -k_m \begin{bmatrix} a_{m-1\,m-1} \\ a_{m-1\,m-2} \\ \cdots \\ a_{m-1\,1} \end{bmatrix}$$

$$\Phi'_{m-1} \cdot a_{m-1} + \Phi'_{m-1} \cdot d_{m-1} + \phi(0) \cdot k_m = \phi(m)$$

$$\Phi'_{m-1} \cdot a_{m-1} \cancel{k_m} \Phi'_{m-1} a'_{m-1} + \phi(0) \cdot k_m = \phi(m) \quad (*1)$$

$$k_m = \frac{\phi(m) - \Phi'_{m-1} \cdot a_{m-1}}{\phi(0) - \Phi'_{m-1} a'_{m-1}} = \frac{\phi(m) - \Phi'_{m-1} a_{m-1}}{\varepsilon_{m-1}}$$

- $\hat{x} = a_{11} \cdot x(n-1)$

$$MSE_1 = \varepsilon_1 = E\left[(\hat{x} - x)^2\right] = E\left[\left(a_{11} x(n-1) - x(n-1)\right)^2\right] =$$

$$a_{11}^2 \underbrace{E[x^2(n-1)]}_{\phi(0)} - 2a_{11} \underbrace{E[x(n-1) \cdot x(n-1)]}_{\phi(0)} + \underbrace{E[x^2(n-1)]}_{\phi(0)} =$$

$$= \phi(0)\left[a_{11}^2 - 2a_{11} + 1\right] = \phi(0)\left(a_{11} - 1\right)^2$$

- $\hat{x} = a_{21} x(n-1) + a_{22} x(n-2)$

$$\varepsilon_2 = E\left[(\hat{x} - x)^2\right] = E\left[\left(a_{21} x(n-1) + a_{22} x(n-2) - x(n-1)\right)^2\right] =$$

$$= E\left[\left(a_{21} x(n-1) - x(n-1)\right)^2 + 2\left(a_{21} x(n-1) - x(n-1)\right) \cdot a_{22} x(n-2) + a_{22}^2 x^2(n-2)\right]$$

$$= \varepsilon_1 + 2E\left[a_{21} a_{22} x(n-1) \cdot x(n-2)\right] - 2a_{22} E\left[x(n-1) x(n-2)\right] + a_{22}^2 \phi(0)$$

$$= \varepsilon_1 + a_{21} a_{22} \cdot \phi(1) - 2a_{22} \phi(1) + a_{22}^2 \phi(0) =$$

$$= \varepsilon_1 + a_{22}(a_{21} - 2)\phi(1) + a_{22}^2 \phi(0)$$

$$= \varepsilon_1 + 2E\left[\left(a_{21} x(n-1) - x(n-1)\right)\right] \cdot E\left[a_{22} x(n-2)\right] + a_{22}^2 \phi(0)$$

$$\varepsilon_{m-1} = \phi(0) - a'_{m-1} \cdot \Phi_{m-1} \Rightarrow \text{RESIDUAL MSE}$$

$$a_m = \begin{bmatrix} a_{m1} \\ a_{m2} \\ \cdots \\ a_{mm} \end{bmatrix} = \begin{bmatrix} a_{m-1} \\ \cdots \\ 0 \end{bmatrix} + \begin{bmatrix} d_{m-1} \\ \cdots \\ k_m \end{bmatrix} \qquad d_{m-1} = -k_m \begin{bmatrix} a_{m-1\,m-1} \\ a_{m-1\,m-2} \\ \vdots \\ a_{m-1\,1} \end{bmatrix}$$

$$\boxed{\begin{aligned} a_{mk} &= a_{m-1k} - k_m a_{m-1\,m-k} & k &= 1, 2, \dots m-1 \\ a_{mm} &= k_m & m &= 1, 2, \dots p \end{aligned}} \quad (**)$$

$$\boxed{\varepsilon_m = \phi(0) - \sum_{k=1}^{m} a_{mk}\phi(k)}$$

$$\varepsilon_m = \phi(0) - \sum_{k=1}^{m-1} a_{m-1\,k}\phi(k) - a_{mm}\left[\phi(m) - \sum_{k=1}^{m-1} a_{m-1\,m-k}\phi(k)\right]$$

$$\varepsilon_m = \phi(0) - \sum_{k=1}^{m-1} a_{mk}\phi(k) - a_{mm}\cdot\phi(m) =$$

$$= \phi(0) - \sum_{k=1}^{m-1}\left(a_{m-1k} - k_m a_{m-1\,m-k}\right)\phi(k) - a_{mm}\cdot\phi(m) =$$

$$= \phi(0) - \sum_{k=1}^{m-1} a_{m-1k}\phi(k) + \sum_{k=1}^{m-1} \underbrace{k_m}_{\equiv a_{mm}} a_{m-1\,m-k}\phi(k) - a_{mm}\phi(m)$$

$$= \phi(0) - \sum_{k=1}^{m-1} a_{m-1k}\phi(k) - a_{mm}\underbrace{\left(\phi(m) - \sum_{k=1}^{m-1} a_{m-1\,m-k}\phi(k)\right)}_{\underset{=a_{mm}}{k_m}\cdot\,\varepsilon_{m-1}}$$

(*1)

MMV

$$\varepsilon_m = \varepsilon_{m-1} - a_{mm}^2\,\varepsilon_{m-1}$$

$$\boxed{\varepsilon_m = \varepsilon_{m-1}\left(1 - a_{mm}^2\right)}$$

$$\boxed{a_{mm} = \frac{\phi(m) - \sum_{k=1}^{m-1} a_{m-1\,m-k}\,\phi(k)}{\varepsilon_{m-1}}}$$

(*)

$\boxed{m=1}$ $\boxed{MMV}$

$$a_{11} = \frac{\phi(1)}{\phi(0)}$$

$$\boxed{\varepsilon_1 = \phi_0\left(1 - a_{11}^2\right)}$$

(*1) $\Rightarrow$ $a_{mk} = a_{m-1k} - a_{mm}\,a_{m-1\,m-k}$ $\quad k = 1 \ldots m-1 = 1 \ldots 1$

$\boxed{m=2}$ $\boxed{a_{21} = a_{11} - a_{22}\cdot a_{11}}$ $\quad a_{22} = \dfrac{\phi(2) - a_{11}\cdot\phi(1)}{\varepsilon_1}$

$$\boxed{\varepsilon_2 = \varepsilon_1\left(1 - a_{22}^2\right)}$$

$\boxed{m=3}$ $_{k=1}$ $a_{31} = a_{21} - a_{33}\cdot a_{22}$; $\quad \boxed{\varepsilon_3 = \varepsilon_2\left(1 - a_{33}^2\right)}$

$_{k=2}$ $a_{32} = a_{22} - a_{33}\,a_{21}$

$$\hat{x} = a_{31}\cdot x(n-1) + a_{32}\,x(n-2) + a_{33}\,x(n-3)$$

$$a_{33} = \frac{\phi(3) - \sum_{k=1}^{2} a_{n-1\,n-k}\phi(k)}{\varepsilon_2} = \frac{\phi(3) - a_{22}\cdot\phi(1) - a_{21}\cdot\phi(2)}{\varepsilon_2}$$

$$x_n = \sum_{k=1}^{p} a_k x_{n-k} + G\,v_n \qquad H(z) = \frac{G}{1 - \sum_{k=1}^{p} a_k z^{-k}}$$

v_n – INPUT SEQUENCE

– PREDICTION OF x_n WHEN INPUT IS AWGN

$$\hat{x}_n = \sum_{k=1}^{p} a_k x_{n-k} \qquad e_n = x_n - \hat{x}_n = x_n - \sum_{k=1}^{p} a_k x_{n-k} = G v_n$$

$$\mathcal{E}_n = E[e_n^2] = E\left[\left(x_n - \sum_{k=1}^{p} a_k x_{n-k}\right)^2\right] =$$

$$= E[x_n^2] - 2E\left[x_n \sum_{k=1}^{p} a_k x_{n-k}\right] + E\left[\left(\sum_{k=1}^{p} a_k x_{n-k}\right)^2\right] =$$

$$= \phi(0) - 2 \sum_{k=1}^{p} a_k \phi(k) + \sum_{k=1}^{p} \sum_{m=1}^{p} a_k a_m \phi(k-m)$$

$$\downarrow$$

$$\boxed{\mathcal{E}_p = G^2 = \phi(0) - \sum_{k=1}^{p} a_k \phi(k)} \qquad \sum_{i=1}^{p} a_i \phi(i-j) = \phi(j)$$

$$\boxed{\hat{\phi}(n) = \frac{1}{N} \sum_{i=1}^{N-n} x_i x_{i+n}} \qquad n = 0, 1, 2, \ldots p$$

MATLAB SIG. PROCESS. TIME DOMAIN BASED MODELING

– LINEAR PREDICTION

$$x(k) = -a(2)\,x(k-1) - a(3)\,x(k-2) - \ldots - a(n+1)\,x(k-n)$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 3 & 2 & 1 & 0 \\ 4 & 3 & 2 & 1 \end{bmatrix} \qquad \begin{bmatrix} 3 & 2 & 1 & 0 \\ 4 & 3 & 2 & 1 \\ 0 & 4 & 3 & 2 \\ 0 & 0 & 4 & 3 \end{bmatrix}$$

1

$(2 \cdot Nt - 1)$ $\qquad 2Nt - 1 - Nt + 2 = \underline{\underline{Nt + 1}}$

$Nt = 3 \qquad L = 2 \cdot 3 - 1 = 6 - 1 = 5$ $\qquad y = 2Nt - 1 - \underbrace{(Nt - 2)}_{1} =$

$x = Nt - 1 = 3 - 1 = 2$ $\qquad = 5 - 1 = 4$

$c(:, x:7)$

$Nt = 4 \qquad L = 2Nt - 1 = 2 \cdot 4 - 1 = 7$ $\qquad y = 7 - 2 = 5$

$x = Nt - 1 = 3$?? $\qquad y = Nt + 1$

$x = Nt + 1 - Nt + 1 = 2$

• ALTERNATIVNA FORMA NA (*)

$$a_{mm} = \frac{\phi(m) - \sum_{k=1}^{m-1} a_{m-1,k}\,\phi(m-k)}{\varepsilon_{m-1}}$$

ALTERNATIVNOSTA E VO RAZLIKATA ~~VO~~ DEFINIRANJE NA INDEKS.

$m=2$ $$a_{22} = \frac{\phi(2) - a_{11}\cdot\phi(1)}{\varepsilon_1}$$

$m=3$ $$a_{33} = \frac{\phi(3) - \sum_{k=1}^{2} a_{2k}\,\phi(3-k)}{\varepsilon_2} = \frac{\phi(3) - a_{21}\cdot\phi(2) - a_{22}\phi(1)}{\varepsilon_2}$$

$$\varepsilon_m = \varepsilon_{m-1}(1-a_{mm}^2)$$

$$a_{mm} = \frac{\phi(m) - \sum_{k=1}^{m-1} a_{m-1,k}\,\phi(m-k)}{\varepsilon_{m-1}}$$

$$a_{mk} = a_{m-1,k} - a_{mm}\,a_{m-1,m-k}$$

$m=1,2,\ldots p$
$k=1,2,\ldots m-1$

$$a_{11} = \frac{\phi(1)}{\phi(0)}$$

$$\varepsilon_1 = \phi(0)(1-a_{11}^2)$$

SAMO OVA TI TREBA ZA IMPLEMENTACIJA VO MATLAB!!!

$m=4$ $$a_{44} = \frac{\phi(4) - a_{31}\phi(3) - a_{32}\phi(2) - a_{33}\phi(1)}{\varepsilon_3}$$

$$\varepsilon_4 = \varepsilon_3(1-a_{33}^2)$$

$$a_{41} = a_{31} - a_{44}a_{33}$$
$$a_{42} = a_{32} - a_{44}a_{32}$$
$$a_{43} = a_{33} - a_{44}a_{31}$$

- LINEAR PREDICTION MODELING ASSUMES THAT EACH OUTPUT SAMPLE OF SIGNAL, $x(k)$, IS LINEAR COMBINATION OF THE PAST „N" OUTPUTS (I.E. IT CAN BE LINEARY PREDICTED FROM THESE OUTPUTS, AND COEFFICIENTS ARE CONSTANT FROM SAMPLE TO SAMPLE:

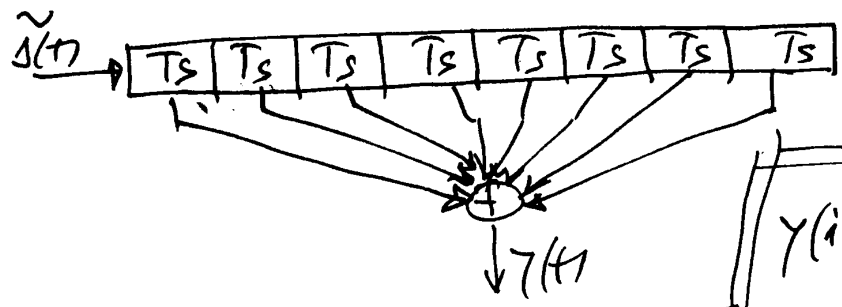
$$\boxed{x(k) = -a(2)x(k-1) - a(3)x(k-2) - \ldots - a(n+1)x(k-n)}$$

OVA NE E BAŠ VO RAKE RECEIVEROT KADE SE ZEMA VO DESNATA STRANA 1, $x(k)$ = VO PREDVID!!!

• NAČELNO MULTIPATH FADING-OT MOŽEŠ DA GO SMETAŠ KAKO LPC I ZATOA ESTIMACIJATA NA KANALITE KOEFICIENTI MOŽEŠ DA GI PRAVIŠ SO LPC() T.E. levinson()

MMM!!!

• JERUCHIM'S MULTIPATH FADING MODEL



$$\boxed{y(i) = \sum_{n=-N_1}^{N_2} s(i-n)\,g(n)}$$

CONVOLUCIJA (PATH GAIN ⊛ INPUT SIGNAL)

$x(k) = a(2)x(k-1) - a(3)x(k-2) - \dots - a(n+1)x(k-n)$ LPC FROM DSP USER'S GUIDE

$(2N-1-N)+1 = 2N-1+1 = N$

⊙ SINGULAR VALUE DECOMPOSITION

M – IS AN $m \times n$ MATRIX COMING FROM THE FIELD K.

$\boxed{M = U \Sigma V^*}$ ⇒ THERE EXIST FACTORIZATION OF THIS FORM

U – UNITARY MATRIX OVER K $(m \times m)$

– UNITARY MATRIX $(n \times n)$

$$\boxed{U^t U = U \cdot U^t = I_n}$$

IDENTITY MATRIX

U^t – CONJUGATE TRANSPOSE (HERMITIAN ADJOINT)

– $\boxed{U^{-1} = U^t}$ ⇒ MATRIX U IS UNITARY ONLY IF IT HAS INVERSE WHICH IS EQUAL TO ITS CONJUGATE TRANSPOSE !!!

V^* – CONJUGATE TRANSPOSE OF „V", V – $n \times n$ UNITARY MATRIX OVER „K"

Σ – DIAGONAL MATRIX $(m \times n)$ WITH NONNEGATIVE (WITH DECREASING VALUES) REAL NUMBERS ON THE DIAGONAL (DIAGONAL ENTRIES OF Σ ARE CALLED SINGULAR VALUES OF M

$$\boxed{[M]_{m \times n} = [U]_{m \times m} [\Sigma]_{m \times n} [V^*]_{n \times n}}$$

⊙ EIGEN VALUE DECOMPOSITION

$\boxed{A \cdot v = \lambda \cdot v}$ λ – SKALAR (EIGEN VALUE) v – VECTOR (EIGEN VALUE)

$A \cdot V = V \cdot \boldsymbol{\lambda}$ $\boxed{A = V \cdot \lambda \cdot V^{-1}}$

VIDI MATLAB FILE LinAlgExamples.m

„U" E EIGENVALUE NA „A" SAMO AKO $A^{n\times n}$ ~~MATRICA~~ DA MENUVA „NEGOVATA MAGNITUDA" AKO SE POMNOŽI SO NEGO. FAKTOROT ZA KOJ DA MENUVA NEGOVATA MAGNITUDA SE VIKA <u>EIGENVALUE.</u>

$$x = [1 \; 2 \; 3 \; 4 \; 5]$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 3 & 2 & 1 & 0 \\ 4 & 3 & 2 & 1 \\ 5 & 4 & 3 & 2 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & 1 & 2 & 3 & 4 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

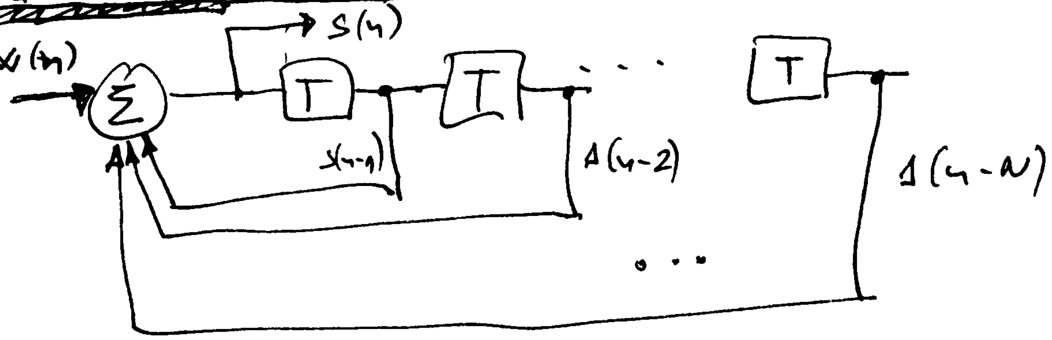
VIDI corrTemp.m

XCORR 1 2 3 4 5

5
4 5
3 4 5
2 3 4 5
1 2 3 4 5

~~CHANNEL ESTIMATION MODELING (MARKKU PUKKILA)~~

- <u>RAHUL SAFAYA THESIS FOR CHANNEL ESTIMATION</u>



- REPRESENTATION OF COMPLEX GAUSSIAN PROCESS BY GENERAL AUTOREGRESIVE (AR) MODEL

$$S(n) = \sum_{i=1}^{N} \phi_i S(n-i) + w(n)$$

$S(n)$ – COMPLEX GAUSSIAN
ϕ_i – PARAMETERS OF THE MODEL
N – NUMBER OF DELAYS IN AR MODEL
W – SEQUENCE OF IID ZERO-MEAN COMPLEX GAUSSIAN

$$f_{w(n)} = \frac{1}{\sqrt{2\pi\sigma_n^2}} e^{-x^2/2\sigma^2}$$

ZNAČI NE+ KOMPLEKSEN I NAMU JE IMA RAYLEIGH DISTRO

$\bar{S}(n) = F\,\bar{S}(n-1) + \bar{W}(n)$ – STATE MODEL

$\bar{S}, \bar{W}$ – COLUMN VECTORS $(N \times 1)$

F – $N \times N$ MATRIX

▪ MEAN, VARIANCE & AUTOCORRELATION OF AR PROCESS

- $\mu_s = E[S(n)] = E\left[\sum_{i=1}^{N} \phi_i S(n-i) + W(n)\right] = 0$

- $\sigma_s^2 = E[S(n) \cdot S(n)] = E\left[S(n) \sum_{i=1}^{N} \phi_i S(n-i) + W(n)\right] =$

$$= \sum_{i=1}^{N} \phi_i R_{ss}(i) + \sigma_n^2$$

- AUTOCORRELATION

$$R_{ss}(m) = E\left[S(n-m)\, S(n)\right] = E\left[\sum_{i=1}^{N} \phi_i S(n-i) \cdot S(n-m)\right] =$$

$$= \sum_{i=1}^{N} \phi_i R_{ss}(m-i) \qquad \boxed{R_{ss}(m) = \sum_{i=1}^{N} \phi_i R_{ss}(m-i)}$$

$$\begin{bmatrix} 1 & r_{ss}(1) & r_{ss}(2) & \cdots & r_{ss}(N-1) \\ r_{ss}(1) & 1 & r_{ss}(1) & & r_{ss}(N-2) \\ \vdots & & & & \\ r_{ss}(N-1) & 1 & \cdots & & 1 \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_N \end{bmatrix} = \begin{bmatrix} r_{ss}(1) \\ r_{ss}(2) \\ \vdots \\ r_{ss}(N) \end{bmatrix}$$

$$\boxed{r_{ss}(m) = \frac{R_{ss}(m)}{R_{ss}(0)} = \sum_{i=1}^{N} r_{ss}(m-i) \quad m \geq 1}$$

$\bar{R} \cdot \phi = r_{ss}$ — MATRIX

$$\boxed{\phi = R^{-1} r_{ss}}$$

GO PROVERIV I OTOD TMSTAP. RAK ZA $N \geq 4$ MI GREŠI ESTIMATOROT!!

▪ DATA BASED CHANNEL ESTIMATOR FORMULATION

- TRAINING SEQUENCE KNOW TO THE RECEIVER IS SENT OVER THE CHANNEL (ONLY FOR FLAT FADING)

$\boxed{\bar{x} = [x_0 x_1 \cdots x_{M-1}]^T}$ – TRAINING SEQUENCE

$\bar{h} = [\tilde{h}_0\, \tilde{h}_1\, \tilde{h}_2 \cdots \tilde{h}_{L-1}]^T$ — L – CHANNEL IMPULSE RESPONSE LENGTH

$\bar{Y} = \bar{X} * \bar{h} + n_c$ – SIGNAL RECEIVED

$$\boxed{\bar{Y} = \bar{X} \cdot \bar{h} + \bar{n}_c} \qquad \bar{X} = \text{toeplitz}(x, [x(1), \text{zeros}(1, N_t-1)])$$

- SNR OF THE CHANEL IF $E_b = 1$

$$\frac{E_b}{N_0} = \frac{1}{2\sigma_c^2}$$

- FOLLOWING LINEAR REGRESSION METHOD

$$\boxed{\hat{h} = (\bar{X}^T \bar{X})^{-1} (\bar{X}^T \bar{Y})}$$

OVA DEFINITIVNO TREBA !!!

MMV

- THE PERFORMANCE OF THE ESTIMATOR WILL DEPEND ON THE CHANNEL NOISE.

$$\hat{h} = (\bar{X}^T \bar{X})^{-1} (\bar{X}^T (\bar{X} \cdot \bar{h} + \bar{n}_c)) = (\bar{X}^T \bar{X})^{-1} (\bar{X}^T \cdot \bar{X} \bar{h} + \bar{X}^T \bar{n}_c)$$

$$= (\bar{X}^T \bar{X})^{-1} \cdot (X^T \cdot \bar{X}) \bar{h} + (\bar{X}^T \bar{X})^{-1} \cdot \bar{X}^T \bar{n}_c = \bar{h} + (\bar{X}^T \bar{X})^{-1} \bar{X}^T \bar{n}_c$$

error is!

$$\boxed{e = \hat{h} - \bar{h} = (\bar{X}^T \bar{X})^{-1} (\bar{X}^T \bar{n}_c)}$$

$$N + \cancel{N_t} - 1 - \cancel{N_t} + 1 = N$$

- PROPERTIES OF DATA ESTIMATOR

$$E[e] = E[(\bar{X}^T \bar{X})^{-1} (\bar{X}^T \bar{n}_c)] = (\bar{X}^T \bar{X})^{-1} (\bar{X}^T) \underbrace{E(\bar{n}_c)}_{\to 0} = 0$$

- ERROR COVARIANCE

$$P_D = E[e \cdot e^H] = E[(\bar{X}^T X)^{-1} (\bar{X}^T \bar{n}_c) \cdot [(\bar{X}^T X)^{-1} (\bar{X}^T \bar{n}_c)]^H] =$$

$$= E[(\bar{X}^T \bar{X})^{-1} (\bar{X}^T \bar{n}_c)(\bar{n}_c^H \bar{X}) \cdot (\bar{X}^T \bar{X})^{-1}] =$$

$$= (\bar{X}^T \bar{X})^{-1} \bar{X}^T \underbrace{E[\bar{n}_c \bar{n}_c^H]}_{\sigma_c^2 I} \bar{X} \cdot (\bar{X}^T \bar{X})^{-1} =$$

$$= \sigma_c^2 [(\bar{X}^T \bar{X})^{-1} \cdot \bar{X}^T \cdot \bar{X} (\bar{X}^T \bar{X})^{-1}] = \underline{\sigma_c^2 (\bar{X}^T \bar{X})^{-1}}$$

$$(\bar{X}^T\cdot\bar{X}) = \overbrace{\begin{bmatrix} x_0 & x_1 & \dots & x_{M-1} & 0 & \dots & 0 \\ 0 & x_0 & x_1 & \dots & x_{M-1} & \dots & 0 \\ & & & & & & \\ 0 & 0 & 0 & \dots & x_0 & x_1 \dots & x_{M-1} \end{bmatrix}}^{\bar{X}^T} \underbrace{\begin{bmatrix} x_0 & 0 & \dots & 0 \\ x_1 & x_0 & \dots & 0 \\ \vdots & \vdots & & x_0 \\ x_{M-1} & x_{M-2} & \dots & x_1 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \dots & x_{M-1} \end{bmatrix}}_{\bar{X}} =$$

$$= \begin{bmatrix} \sum_{i=0}^{M-1} x_i^2 & \sum_{i=0}^{M-1} x_i x_{i-1} & \dots & \sum_{i=0}^{M-1} x_i x_{i-L+1} \\ & & & \\ \sum_{i=0}^{M-1} x_{i-L+1} x_i & \dots & \sum_{i=0}^{M-1} x_i x_{i-1} & \sum_{i=0}^{M-1} x_i^2 \end{bmatrix}$$

$$x_i = \pm 1 \Rightarrow \sum_{i=0}^{M-1} x_i^2 = M$$

$$(\bar{X}^T\cdot\bar{X}) = M \begin{bmatrix} 1 & \frac{1}{M}\sum_{i=0}^{M-1} x_i x_{i-1} & \dots & \frac{1}{M}\sum_{i=0}^{M-1} x_i x_{i-L+1} \\ \dots & & & \\ \frac{1}{M}\sum_{i=0}^{M-1} x_i x_{i-L+1} & \dots & \frac{1}{M}\sum_{i=0}^{M-1} x_i x_{i-1} & 1 \end{bmatrix}$$

$$\boxed{r_{xx}(k) = \frac{1}{R_{xx}(0)} \sum_{i=0}^{M-1} x_i x_{i-k}}$$

NORMALIZIRANA FORMA NA AVTOKORELACIJATA OD TRAINING SEKVENCATA !!!

$$\bar{X}^T\cdot\bar{X} = M \begin{bmatrix} 1 & r_{xx}(1) & \dots & r_{xx}(L-1) \\ \vdots & & & \vdots \\ r_{xx}(L-1) & & r_{xx}(1) & 1 \end{bmatrix}$$

FOR IDEAL AUTOCORRELATION

$$P_D = \sigma_c^2 \cdot (\bar{X}^T \bar{X})^{-1} = \sigma_c^2 \, \frac{1}{M} \begin{bmatrix} 1 & r_{xx}(1) & \dots & r_{xx}(L-1) \\ r_{xx}(L-1) & \dots & & 1 \end{bmatrix} = \frac{\sigma_c^2}{M} \cdot \bar{I}$$

• FOR SINGLE PROCESS ESTIMATE

$$\boxed{P_D = \frac{\sigma_c^2}{M}}$$

- INVERSE RELATIONSHIP BETWEEN COVARIANCE AND LENGTH OF THE DATA ESTIMATE. ALSO DATA ESTIMATE WORSENS AS CHANNEL NOISE INCREASES.

MITIGATING THE EFFECTS OF FRQ. SELECTIVE FADING

$$v = \frac{\lambda/2}{T} \qquad T = \frac{\lambda/2}{v} = \frac{0.33/2}{55.56\,m/s} \doteq 3\,msec$$

ILIJA HERSONSKI

$\boxed{3212672}$

$[1\ 2\ 3\ 4\ 5]$

$$\begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \left[\begin{array}{ccccc} - & - & - & - & - \\ - & - & - & - & \\ & & & & \\ \end{array}\right]_{50000\times 5} \end{array}$$

$$f_s = 2 f_{max}$$

$$f_{max} = 200\,kHz$$

$$\boxed{f_s \geq 400\,kHz}$$

$\boxed{\text{bitrate: } 271\ kbps}$

- rms delay spread FOR URBAN ENVIRONMENT

$\boxed{\sigma_\tau \doteq 2\,\mu s} \Rightarrow$ COHERENCE BANDWIDTH $f_0 = \frac{1}{5\sigma_\tau}$

$$f_0 = \frac{1}{10 \cdot 10^{-6}} = 0.1 \cdot 10^6 = \underline{100\,kHz}$$

$\boxed{f_0 < W} \Rightarrow$ <u>FRQ. SELECTIVE FADING</u>

- VITERBI ALGORITHM TO COMPUTE <u>MLSE</u> OF MESSAGE BITS.

$r_{tr}(t) = s_{tr}(t) * h_c(t)$ „tr" – TRAINING SEQUENCE

ESTIMATE OF „h_c" BY THE MATCHED FILTER $\Rightarrow h_e$

$$h_e(t) = r_{tr}(t) * h_{mf}(t) = \underbrace{s_{tr}(t) * h_c(t) * h_{mf}(t)}_{R_s(t)}$$

$$\boxed{h_e(t) = R_s(t) * h_c(t)}$$

$$\boxed{L_0 = L_{c(ISI)} + L_c}$$

$L_{c(ISI)}$ – CHANNEL INDUCED ISI

L_c – CONTROLED ISI CAUSED BY GAUSSIAN FILTERING

- BINGHAMTON CHANEL ESTIMATION/EQUILIZATION

$$X = \begin{bmatrix} 21 & 22 & 23 & \dots & 999 & 1000 \\ \vdots & \vdots & & & \vdots & \vdots \\ 2 & 3 & 4 & \dots & 980 & 981 \\ 1 & 2 & 3 & \dots & 979 & 980 \end{bmatrix}$$

$T = 1000$
$M = 500$
$N = 20$ — smothing length $= N+1$
$L_H = 5$ — CHANNEL LENGHT $\boxed{L_H + 1 = 6}$

```
hb = zeros(N+1,1)
for i = 1:M-10
    hb = hb + x(:,i+10) * conj(s(i+10+N-P))
end
```

$\underline{\underline{i=1}}$ $\qquad$ $\underline{\underline{i=2}}$

$$\begin{bmatrix} (31) \\ \vdots \\ (12) \\ (11) \end{bmatrix} \cdot \left(s(18)\right)^* + \begin{bmatrix} (32) \\ \vdots \\ (13) \\ (12) \end{bmatrix} \left(s(19)^*\right) + \dots = \underline{\underline{hb}}$$

SCALAR MMSE EQUILIZER (BY USING TRAINING SEQUENCE)

$\bar{Z} = \bar{X} \cdot \bar{C} \qquad \bar{X}^T * \bar{Z} = \left(\bar{X}^T * \bar{X}\right) \cdot C \qquad R_{xz} = R_{xx} \cdot C$

$$\boxed{C = R_{xx}^{-1} \cdot R_{xz}}$$

$\boxed{MMV}$

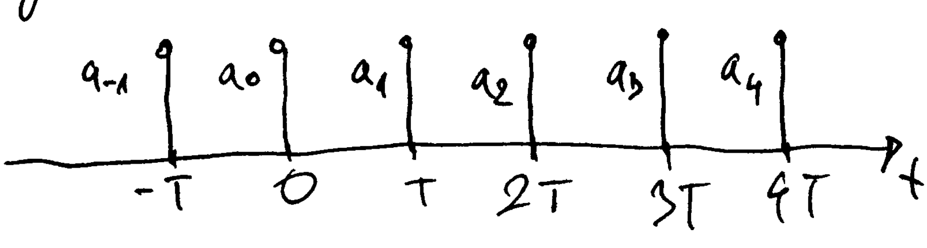
- DSPLog's : BER for BPSK IN ISI CHANNEL WITH $\boxed{\text{MMSE EQUILIZATION}}$

$$\boxed{s(t) = \sum_{n=-\infty}^{\infty} a_n g(t - nT)}$$

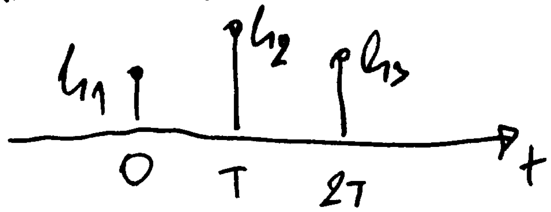
$g(t)$ – TRANSMIT FILTER

- NO TRANSMIT PULSE SHAPING FILTER i.e.:

$g(t) = \delta(t) \quad ; \quad s(k) = a_n$



- CHANNEL MODEL:



(3 TAP MULTIPATH)

DESIRED SIGNAL

$$y(k) = s(k) \circledast h(k) + n$$

AWGN $\quad f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

$$\boxed{\sigma^2 = \frac{N_0}{2}}$$

- MMSE EQUILIZATION (THEORY)

FOR EACH SAMPLE TIME „k" WE WOLD LIKE TO FIND A SET OF COEFFICIENTS $c[k]$ WHICH MINIMIZES ERROR BETWEEN THE DESIRED SIGNAL AND EQUILIZED SIGNAL $c[k] \circledast y[k]$

$$E[e[k]]^2 = E\left[\left(s[k] - c(k) \circledast y(k)\right)^2\right] =$$

$$\boxed{\begin{aligned}(A+B)^T &= A^T + B^T \\ (A \cdot B)^T &= B^T \cdot A^T\end{aligned}}$$

$$= E\left[\left(s(k) - c^T y\right)\left(s(k) - c^T y\right)^T\right] =$$

$$= E\left[s^2(k)\right] - E\left[c^T y\, s(k)\right] - E\left[s(k) \cdot y^T \cdot c\right] + E\left[c^T \cdot y \cdot y^T \cdot c\right]$$

$$= E\left[s^2(k)\right] - c^T R_{ys} - R_{sy} \cdot c + c^T R_{yy} c$$

$e[k]$ - error at sample time „k"

c - column vector $[K \times 1]$

y - –//– $[K \times 1]$

K - NUMBER OF TAPS IN EQUILIZER

R_{ys} - CROSSCORELATION BETWEEN RECEIVED AND INPUT SEQUENCE

R_{yy} - AUTOCORRELATION OF RECEIVED SEQUENCE

- WHE SHOULD FIND „c" THAT MINIMIZES $\underline{E(e(k))^2}$

- DEFFERENTIATION WITH RESPECT TO „c"

$\boxed{MMV}$

$$\frac{\partial}{\partial c}\left[E[s^2[k]] - c^T R_{ys} - R_{sy} c + c^T R_{yy} c\right] = 0$$

$$-R_{sy} + R_{yy} c = 0 \qquad \boxed{c = R_{yy}^{-1} \cdot R_{sy}}$$

ISTOTO GO VELI I SKLAR NA STR. 180 !!!

$$R_{sy} = E\left[s(k)\, y^T\right] = E\left[s(k) \cdot (h \cdot s(k) + n)^T\right] =$$

$$h\, E[s^2(k)] + E[s(k) \cdot n] = \underline{h}$$

$$R_{yy} = E[y y^T] = E\left[[h s(k)+n]\cdot[h s(k)+n]^T\right] =$$

$$= E\left[(h s(k)+n)(s^T(k)\cdot h^T + n^T)\right] = E(hh^T) E(s^2(k)) + E[h s(k) n^T]$$

$$+ E[n s^T(k)\cdot h^T] + E(n^2)$$

$$\sigma^2 = \frac{N_0}{2} \qquad N_0 = 1 = 2\sigma^2 \Rightarrow \sigma^2 = \frac{1}{2} \qquad \boxed{\sigma = \frac{1}{\sqrt{2}}}$$

$$= E[hh^T]\,\underline{E[s^2(k)]} + h\,\underbrace{E[s(k) n^T]}_{\text{non-correl}} + \underbrace{E[n s^T(k)]}_{\text{noncorr}} h^T + E(n^2)$$

$$\boxed{R_{yy} = E[hh^T] + E(n^2)} \quad \text{MMV} \qquad \boxed{c = \frac{h}{E[hh^T]+E(n^2)}} \quad (*1)$$

$$E[s^2] = 1$$

$$E[ns] = 0$$

$$\boxed{ht = [0.2\ 0.9\ 0.3]} \quad ([1, 2, 3])$$

ZERO FORCING EQUILIZATION (dsplog) $(K=3)$

DEFAULT TAP POSITION $= 2$

$$hM = \begin{bmatrix} 2 & 1 & 0 & 0 & 0 & 0 & 0 \\ 3 & 2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 3 & 2 & 1 & 0 & 0 & 0 \\ 0 & \cdots & 0 & & & & \\ 0 & 0 & 0 & & 0 & 3 & 2 \end{bmatrix}$$

$$\boxed{hM * c\_zf = d' = \begin{bmatrix}0\\0\\0\\1\\0\\0\\0\end{bmatrix}} \quad \text{MMV}$$

ZA DEFAULT TAP POSITION = 1 PROVJSOT ZA PRVATA KOLONA !!!

$$d = [0\ 0\ 0\ 1\ 0\ 0\ 0]$$

$$\boxed{c\_zf = [inv(hM) * d']}$$

$$yFilt\_zf = conv(y, c\_zf)$$

$$yFilt\_zf = (K+2 : end)$$

$$\downarrow$$

$$yFilt\_zf = conv(yFilt\_zf, ones(1,1))$$

$$\boxed{ySampl\_zf = yFilt\_zf(1:1:N)}$$

- DSPLog's BER FOR BPSK IN ISI CHANNEL WITH ZERO FORCING EQUILIZATION.

- FIND SET OF FILTER coefficients $c[k]$ which:

$$\boxed{h[k] \circledast c[k] = \delta(k)}$$

VO FREKVENTEN DOMEN SAKA FREKVENTNATA KARAKTERISTIKA NA REZULTANTNATA PRENOSNA K-KA BIDE KONSTANTNA ZA SEKOJA FREKVENCIJA !!!

- AFTER EQUILIZATION

$$\hat{y}_{ZF}[k] = c[k] \circledast y[k] = c[k]\left(s[k] \circledast h[k] + n\right) =$$

$$= s[k] + c[k] \circledast n$$

- MMSE EQUILIZATION (DSPLOG)

hAutoCorr = xcorr(ht)

hM = toeplitz (hAutoCorr ...)

POSITIVE COEFFICIENTS

$$ht = [1, 2, 3] \qquad hAutoCorr = [\overset{-2}{3}\ \overset{-1}{8}\ \overset{0}{14}\ \overset{1}{8}\ \overset{2}{3}]$$

$\boxed{d = zeros[1, 2*K+1]}$ $\qquad d = [\overset{1}{0}\overset{2}{0}\overset{3}{0}\overset{4}{\underline{0}}\overset{5}{0}\overset{6}{0}\overset{7}{0}]$

d[3,4,5] = d[3:5] = fliplr(ht);

d = [0032100]

$\boxed{c\_mmse = [inv(hM) * d.']. ;}$ → SE DOBIVA NA FORMULATA (*1)

yfilt_mmse = conv(y, c_mmse);

$\boxed{yfilt\_mmse = yfilt\_mmse(K+2 : K+2+N-1)}$

(*1) $\quad R_{yy} = E[h \cdot h^T] + E[n^2]$ $\qquad E[n^2] = 2\sigma^2 = N_0$

$E[n^2] = \boxed{2\sigma^2} = N_0$ $\qquad \boxed{\frac{Eb}{N_0}\Big|_{dB} = 25\,dB}$ $\qquad$ NE TREBA!!!

$$\boxed{hM = hM + I \cdot 10^{-0.1\, EbN_0\_dB} / 2}$$

NE ZNAM ZOŠTO GO DELI SO 2

$$10^{0.1\, EbN_0\_dB}$$

$$\frac{Eb}{N_0}\Big|_{dB} = 10 \log \frac{Eb}{N_0} \qquad \frac{Eb}{N_0} = 10$$

$$\boxed{N_0 = Eb \cdot 10^{-0.1\, EbN_0\_dB}} \qquad \boxed{I = eye(2K+1)}$$

awgn (x, SNR|dB)

$$SNR = \frac{Eb}{N_0} = 1 \qquad SNR|_{dB} = 10 \log 1 = 0$$

$N_0 = 2 \cdot \sigma^2$ $\qquad n = \frac{1}{\sqrt{2}}\, randn(1,N) \qquad E(n^2) = \sigma^2 = 0.5$

$$\boxed{N_0 = 2 \cdot \sigma^2 = 1}$$

VIDI! awgn Vs randn $\qquad$ awgn(x, 0) = randn(1,N) + x

- VO GENERALNO SLUČAJ: awgn(x, SNR)

NE E EKVIVALENTNO NA: $\quad x + \frac{sqrt(N_0)}{\sqrt{2}}\, randn$

OVA DAVA 2x POMALA SREDNA SNAGA NA SIGNALOT!!

$\gamma = \alpha^2 \cdot \frac{E_s}{N_0}$ $\quad \bar{\gamma} = E[\gamma] = E[\alpha^2] \cdot \frac{E_s}{N_0} = \Omega \frac{E_s}{N_0}$

$$\boxed{\alpha = \oplus\sqrt{\gamma \frac{N_0}{E_s}}} \qquad \alpha^2 = \frac{N_0}{E_s}\gamma \qquad p_\alpha = \frac{\alpha}{\sigma^2} e^{-\frac{\alpha^2}{2\sigma^2}} = \frac{2\alpha}{\Omega} e^{-\frac{\alpha^2}{\Omega}}$$

$$p_\gamma = \frac{p_\alpha}{\frac{\partial \gamma}{\partial \alpha}} \Bigg|_{\alpha = f(\gamma)} = \Bigg/ \quad \frac{\Omega}{\bar{\gamma}} = \frac{N_0}{E_s} \quad \frac{\bar{\gamma}}{\Omega} = \frac{E_s}{N_0}$$

$$p_\gamma = \frac{1}{\cancel{2\alpha} \cdot \frac{\bar{\gamma}}{\cancel{\Omega}}} \cdot \frac{\cancel{2}\alpha}{\cancel{\Omega}} \cdot e^{-\frac{\gamma}{\cancel{\Omega}} \cdot \frac{\Omega}{\bar{\gamma}}} = \frac{1}{\bar{\gamma}} \cdot e^{-\gamma/\bar{\gamma}}$$

RAYLEIGH

MMSE akACM

akAutoCorr = $[3 \; 8 \; \underbrace{14 \; 8 \; 3}_{\text{POZITIVNI COEFICIENTI}}]$

- MMSE (DEFAULT TAP = 3) (K=2 2K+1=5) | MMSE DEFAULT (2K+1=5, TAP = 1)

$$\begin{bmatrix} 14, & 8, & 3, & 0, & 0 \\ 8 & 14, & 8, & 3, & 0 \\ 3 & 8 & 14 & 8 & 3 \\ 0 & 3 & 8 & 14 & 8 \\ 0 & 0 & 3 & 8 & 14 \end{bmatrix} \qquad \begin{bmatrix} 14 & 0 & 0 & 0 & 0 \\ 8 & 14 & 0 & 0 & 0 \\ 3 & 8 & 14 & 0 & 0 \\ 0 & 3 & 8 & 14 & 0 \\ 0 & 0 & 3 & 8 & 14 \end{bmatrix}$$

$$\begin{bmatrix} a_{k1} \\ a_{k2} \\ a_{k3} \end{bmatrix} \qquad [a_{k1} \; a_{k2} \; a_{k3}] \begin{bmatrix} t_1 & t_2 & t_3 \\ 0 & t_1 & t_2 \\ 0 & 0 & t_1 \end{bmatrix} = \begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix}$$

$r_1 = a_{k1} \cdot t_1 \qquad r_2 = a_{k1} \cdot t_2 + a_{k2} \cdot t_1 \qquad r_3 = a_{k1} t_3 + a_{k2} t_2 + a_{k3} t_1$

- dsplog BER FOR BPSK IN OFDM WITH RAYLEIGH MULTIPATH CHANNEL

FFT SIZE nFFT = 64 (nFFT = 64)
NUMBER OF SUBCARRIERS = 52 (nDSC = 52)
FFT SAMPLING FREQUENCY 20MHz
SUBCARRIER - SPACING (312.5 kHz)

SUBCARRIER INDEX $\{-26 \div -1, 1 \div 26\}$
Cyclic Prefix Duration $T_{cp} = 0.8\mu s$
DATA SYMBOL -II- $T_d = 3.2\mu s$
TOTAL SYMBOL -II- $T_s = 4\mu s$

• dsplog : UNDERSTANDING OFDM TRANSMISSION

32
+31
63+1=64

$$g_k(t) = \frac{1}{\sqrt{T}} e^{j\frac{2\pi k t}{T}} w(t)$$

$k = 0, 1, \dots, K-1$ – FREQUENCY OF THE SINUSOIDAL

$w(t) = u(t) - u(t-T)$ – RECTANGULAR WINDOW OVER [0 T]

– EACH SINUSOIDAL GETS MODULATED BY INDEPENDENT INFORMATION

a_k – INFORMATION

$$\boxed{s(t) = a_0 \cdot g_0(t) + a_1 \cdot g_1(t) + \dots + a_{K-1} \cdot g_{K-1}(t) = \sum_0^{K-1} a_k g_k(t)}$$

$$s(t) = \frac{1}{\sqrt{T}} \sum_0^{K-1} a_k e^{j\frac{2\pi k t}{T}} w(t)$$

SAMPLED VERSION

$$s(nT) = \frac{1}{\sqrt{T}} \sum_0^{K-1} a_k e^{j\frac{2\pi k n T}{T}} w(nT) = \frac{1}{\sqrt{T}} \sum_0^{K-1} a_k e^{-j2\pi k n}$$

DFT

$$X(k) = \sum_{n=0}^{N-1} x(n) \cdot e^{-j\frac{2\pi k n}{N}} = \sum_{n=0}^{N-1} x(n) \cdot W_N^{nk} \qquad k = 0, 1, \dots N-1$$

$$\boxed{W_N = e^{-j\frac{2\pi}{N}}}$$

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) \cdot e^{j\frac{2\pi}{N} nk} = \frac{1}{N} \sum_{k=0}^{N-1} X(k) \cdot W_N^{-nk} \qquad n = 0, 1, \dots N-1$$

THE OPERATION PERFORMED IN OFDM CORRESPONDS TO: IDFT

○ dsplog OFDM transmiter script

nFFTSize = 64   nBitPerSymbol = 52

ifftshift   GO IFFTA $-26 \div -1$ NA $38 \div 63$

CYCLIC PREFIX : [outputiFFT(49:64), outputiFFT]

fftshift[0 0 0 0 0 0 1 2 3 .... 52 0 0 0 ... 0 0 0]

1 2 3 4 5 6 7 8 9 ... 58 ... 62 63 64

-31 -30 -29 -28 -26 ... -23

32 − 6 = 28

= [28 29 30 .... 52 0 0 0 ... 0 0 0 0 0 0 0 0 0 1 2 3 .. 27]

$T_{cp} = 0.8\mu s \quad T_d = 3.2\mu s \quad T_s = T_{cp} + T_d = 4\mu s$

$$\frac{T_{cp}}{T_s} = \frac{0.8}{4} = 0.2$$

- BPSK BER WITH OFDM MODULATION

$E_s(T_d + T_{cp}) = E_b \cdot T_d \qquad E_s = \frac{T_d}{T_d + T_{cp}} \cdot E_b$

- USED CARRIERS:

$-8{,}1250\,MHz\ (-26/64 \cdot 20MHz) \div 8{,}1250\,MHz\ (26/64 \cdot 20MHz)$

SIGNAL ENERGY IS SPREAD OVER 16.25MHz

NOISE IS PREAD OVER 20MHz

$20M \cdot E_s = 16.25M\, E_b$

$$\boxed{E_s = \frac{nDSC}{nFFT} E_b} \qquad \boxed{Q(z) = \frac{1}{2}\operatorname{erfc}\frac{z}{\sqrt{2}}}$$

$$\boxed{\frac{E_s}{N_0} = \frac{E_b}{N_0}\,\frac{nDSC}{nFFT}\,\frac{T_d}{T_d + T_{cp}}} \qquad E_b/N_0 \text{ vs } E_s/N_0$$

$$\frac{E_s}{N_0}\Big|_{dB} = \frac{E_b}{N_0}\Big|_{dB} + 10\log\frac{nDSC}{nFFT} + 10\log\frac{T_d}{T_d + T_{cp}}$$

$-1 \quad -1 \quad -1 \quad 1 \quad -1 \quad 1$

$$P_b(bpsk) = \frac{1}{2}\operatorname{erfc}\sqrt{\frac{E_b}{N_0}} = \frac{1}{2}\operatorname{erfc}\sqrt{\frac{2E_b}{N_0}} \cdot \frac{1}{\sqrt{2}} =$$

$$= Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = Q\left(\sqrt{2\gamma}\right)$$

- BER for BPSK in OFDM WITH RAYLEIGH MULTIPATH CHANNEL.

$$h(t) = \frac{1}{\sqrt{n}}\left[h_1(t-t_1) + h_2(t-t_2) + \dots + h_n(t-t_n)\right]$$

$h_1(t-t_1)$ - CHANNEL coefficient of the first tap

$h_2(t-t_2)$ - CHANNEL coefficient of the second tap

$1/\sqrt{n}$ - NORMALIZE the average POWER TO „1"

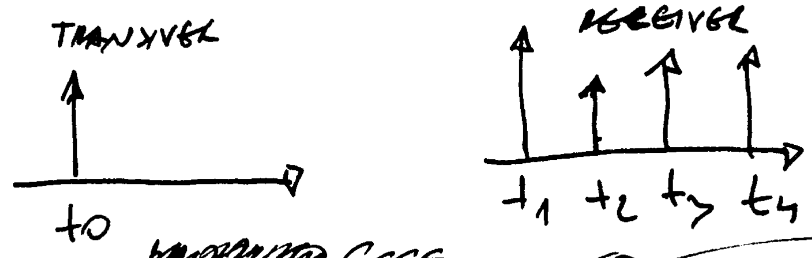


FIGURE: IMPULSE RESPONSE OF A MULTIPATH CHANN.

CYCLIC PREFIX LENGTH

~~[illegible]~~ CECE

$\boxed{3122220}$

$\frac{0.8}{16} = 0.05\,\mu s$ 10 samples $= 0.5\,\mu s$

- BER FOR BPSK IN RAYLEIGH FADING

$$\boxed{P_b = \frac{1}{2}\left(1 - \sqrt{\frac{E_b/N_0}{E_b/N_0 + 1}}\right) = \frac{1}{2}\left(1 - \sqrt{\frac{\bar{\gamma}}{\bar{\gamma}+1}}\right)}$$

$\bar{\gamma} = E[\gamma]$ $\gamma = \alpha^2 \cdot \frac{E_b}{N_0}$ $\bar{\gamma} = \frac{E_b}{N_0} \cdot E(\alpha^2) = \frac{E_b}{N_0} \cdot \Omega$

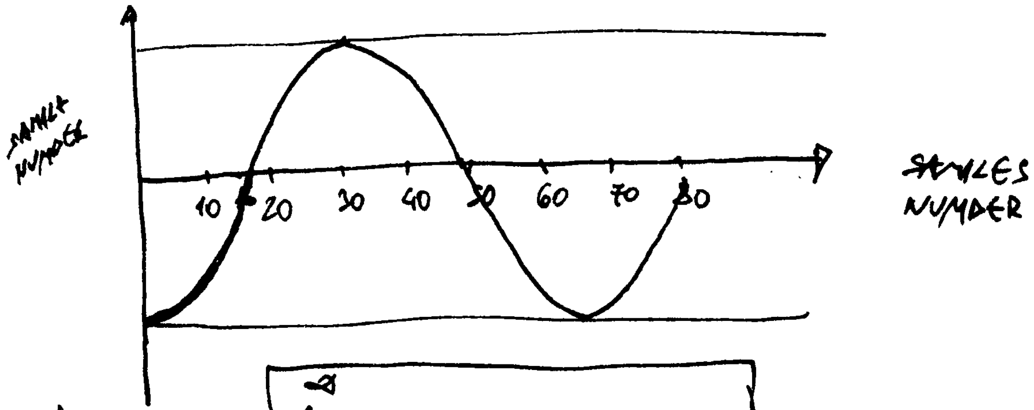
$h_t = [1, 2, 3, 4, 5]$

$$\begin{bmatrix} t_1 \\ t_2 \\ \vdots \\ t_9 \end{bmatrix}^T \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 & 4 & 5 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 3 & 4 & 5 & 0 & 0 \\ 0 & 0 & 0 & 1 & 2 & 3 & 4 & 5 & 0 \\ 0 & 0 & 0 & 0 & 1 & 2 & 3 & 4 & 5 \\ \vdots & & & & & & & \vdots & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} t_1 \\ t_2 \\ \vdots \\ t_9 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 3 & 2 & 1 & 0 & 0 & 0 & 0 & 0 \\ 5 & 4 & 3 & 2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 5 & 4 & 3 & 2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 5 & 4 & 3 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 5 & 4 & 3 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 5 & 4 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} t_1 \\ t_2 \\ \vdots \\ \vdots \\ t_9 \end{bmatrix}$$

$$\begin{bmatrix} \; \end{bmatrix} \begin{bmatrix} h_1 & h_2 & h_3 & h_4 & h_5 \\ h_1 & h_2 & h_3 & h_4 & h_5 \\ & & & & \\ h_1 & h_2 & h_3 & h_4 & h_5 \end{bmatrix} \begin{bmatrix} t_1 & t_2 & t_3 & 0 & 0 \\ 0 & t_1 & t_2 & t_3 & 0 \\ 0 & 0 & t_1 & t_2 & \\ & & & & \end{bmatrix}$$

- CYCLIC PREFIX IN ORTHOGONAL FDM



$$\boxed{z(\tau) = \int_{-\infty}^{\infty} x(t)\, y(\tau - t)\, dt}$$

$z(t) = x(t) * y(t)$

EFFECT OF PASSING SINUSOIDAL SIGNAL THROUGH MULTIPATH CHANNEL

$$\boxed{h(t) = a_1 \delta(t - t_1) + a_2 \delta(t - t_2)}; \quad x = e^{j 2\pi f_1 t}$$

$$y(t) = h(t) * x(t) = \int_{-\infty}^{\infty} h(\mu) \cdot x(t - \mu)\, d\mu =$$

$\left.\begin{matrix}\mu = t_1 \\ \mu = t_2\end{matrix}\right\} \Rightarrow$ PULSE!!

$$= \int_{-\infty}^{\infty} \left[a_1 \delta(\mu - t_1) + a_2 \delta(\mu - t_2)\right] \cdot x(t - \mu)\, d\mu = \underline{a_1 \cdot e^{j 2\pi f_1 (t - t_1)} + a_2 e^{j 2\pi f_1 (t - t_2)}}$$

$$\boxed{\tilde{h}(t) = \sum_{k=1}^{N} a_k \cdot \delta(t - \tau_k)}$$

$$\boxed{y(t) = e^{j 2\pi f_1} \left(a_1 \cdot e^{-j 2\pi t_1} + a_2 e^{-j 2\pi f_1 t_2}\right)}$$

$$\boxed{\delta(\mu - t_1) = \begin{cases} \infty & \mu = t_1 \\ 0 & \mu \neq t_1 \end{cases}}$$

$$\boxed{\delta(\mu - t_2) = \begin{cases} \infty & \mu = t_2 \\ 0 & \text{OTHERS} \end{cases}}$$

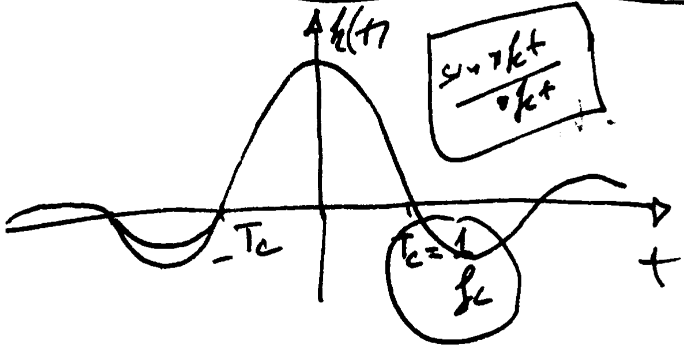
ORIGINAL SINUSOID $x(t)$ albeit with modifications in amplitude and phase.

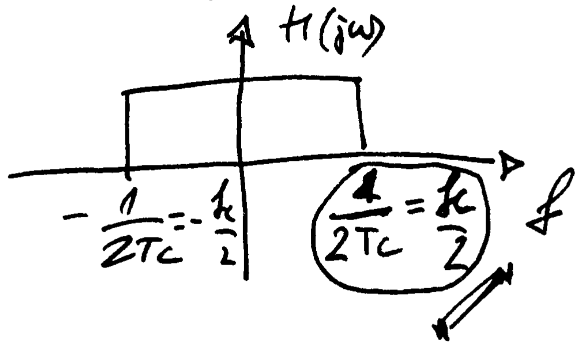
- <u>TRANSMIT BEAMFORMING</u> (display)

$$x = \frac{\sin \pi f_c t}{\pi f_c t} \qquad t = (0, 1, 2, \dots N-1) \frac{1}{f_c} = \underline{(0 : N-1) T_c}$$

$$x(j\omega) = \int_{-\infty}^{\infty} \frac{\sin \pi f_c t}{\pi f_c t} e^{-j\omega t}\, dt = \frac{1}{\pi f_c} \int_{-\infty}^{\infty} \frac{\sin \pi f_c t}{\pi f_c t} e^{-j 2\pi f_c t}\, d(\pi f_c t)$$

$T_c = 0.1\mu s$ $\quad \boxed{f_c = \frac{1}{T_c} = 10MHz} \quad \boxed{f_g = \frac{f_c}{2} = 5MHz}$

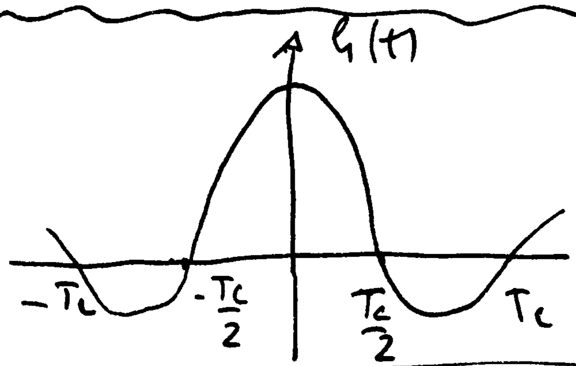


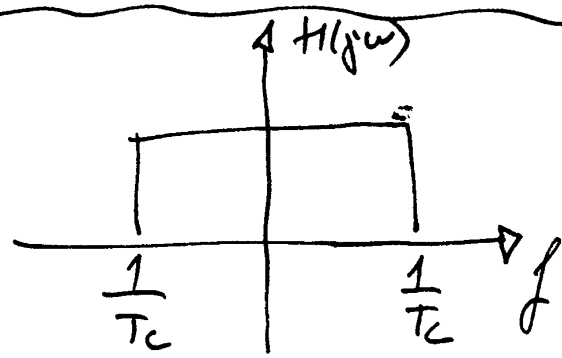


$$h(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{+j2\pi f t}\, df = \frac{1}{2\pi}\int_{-f_c/2}^{f_c/2} e^{j2\pi f t}\, df =$$

$$= \frac{1}{2\pi}\,\frac{1}{(j2\pi t)}\cdot e^{j2\pi f t}\Big|_{-f_c/2}^{f_c/2} = \frac{1}{2\pi}\,\frac{e^{j2\pi\frac{f_c t}{2}} - e^{-j\frac{2\pi f_c t}{2}}}{j2\pi t}$$

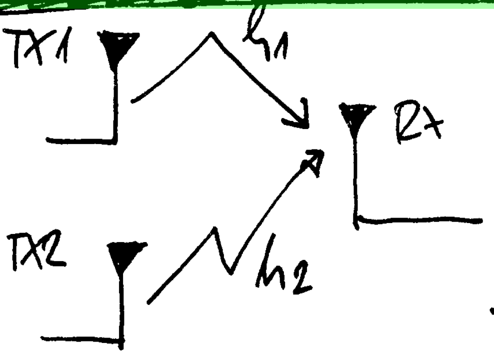
$$= \frac{1}{2\pi}\,\frac{\sin(\pi f_c t)}{\pi f_c t}\cdot f_c = \frac{f_c}{2\pi}\,\frac{\sin \pi f_c t}{\pi f_c t} = \frac{f_c}{2\pi}\,\underline{\underline{\sin(\pi f_c t)}}$$





$T_c = 0.2\mu s \quad \boxed{\frac{1}{0.2}\cdot 10^6 = 5MHz}$

CONTINUE (TRANSMIT BEAMFORMING)



FLAT FADING CHANNEL

$$\boxed{y = \begin{bmatrix} h_1 & h_2 \end{bmatrix}\begin{bmatrix} x \\ x \end{bmatrix} = (h_1 + h_2)x + n}$$

RECEIVED SYMBOL

$$y = \begin{bmatrix} h_1 & h_2 \end{bmatrix}\begin{bmatrix} e^{-j\theta_1} \\ e^{-j\theta_2} \end{bmatrix} x + n$$

MMV

$$y = [h_1 \; h_2] \begin{bmatrix} x \cdot e^{-j\theta_1} \\ x \, e^{-j\theta_2} \end{bmatrix} \qquad h_1 = |h_1| e^{j\theta_1} \quad h_2 = |h_2| e^{j\theta_2}$$

TRANS. BEAMFORM

$$y = \left(h_1 \cdot e^{-j\theta_1} + h_2 \cdot e^{-j\theta_2}\right) \cdot x + n = \left(|h_1| + |h_2|\right) \cdot x + n$$

- FOR EQUILIZATION DEVIDE THE RECEIVED SYMBOL WITH NEW EFECTIVE CHANNEL

$$\boxed{\hat{y} = \frac{y}{|h_1| + |h_2|} = x + \frac{n}{|h_1| + |h_2|}}$$

$\boxed{070200885}$ IVICA [redacted]

- NO BEAMFORMING

$$\boxed{y = [h_1 \; h_2] \cdot \begin{bmatrix} x \\ x \end{bmatrix} = (h_1 + h_2) x}$$

WITH NOISE

$$y = (h_1 + h_2) x + n$$

- SISO Equalization:

FROM DSPLOG THREAD FOR BER IN RAYLEIGH CHANNEL

- ZERO FORSING

$$\boxed{yHat\_zf = \frac{y}{h}}$$

- MMSE Equalization

$$\boxed{yHat\_mmse = \frac{conj(h).*y}{\left(h.*conj(h) + 10^{-0.1 EbNo\_dB}\right)}}$$

- CHANNEL ESTIMATION ~~EQUALIZATION~~ WITH TRAINING SEQUENCE (OFDM)

X - KNOWN

$$Y = HX + N$$

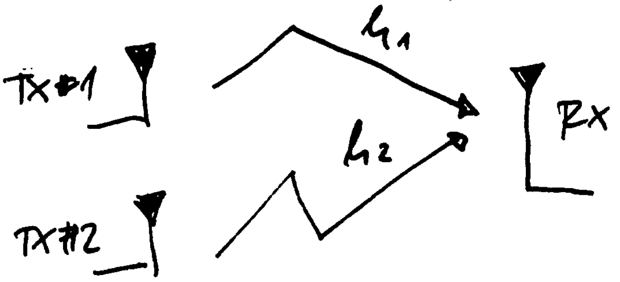
$$\boxed{\hat{H} = \frac{Y}{X}}$$

MMV

- ALAMOUTI STBC (SPACE-TIME BLOCK CODING)
- TRANSMIT SEQUENCE $\{x_1, x_2, x_3, \ldots, x_n\}$

1 TIME SLOT x_1 & x_2 FROM FIRST & SECOND ANTENNA

2 TIME SLOT $-x_2^*$ & x_1^* FROM FIRST & SECOND ANTENNA



SPACE (ANTENNA) \ TIME →		
	x_1	$-x_2^*$
	x_2	x_1^*

IMPLEMENTACIJA NA PRIEMNIKOT NA PP. 26÷27 MMV

- FIRST TIME SLOT RECEIVED SIGNAL

$$\boxed{y_1 = h_1 x_1 + h_2 x_2 + n_1 = [h_1\ h_2]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_1}$$

- SECOND TIME SLOT RECEIVED SIGNAL

$$\boxed{y_2 = -h_1 x_2^* + h_2 x_1^* + n_2 = [h_1\ h_2]\begin{bmatrix} -x_2^* \\ x_1^* \end{bmatrix} + n_2}$$

MMV

11

KANALOT TREBA DA E IST ZA VREMETRAENJETO NA DVATA SIMBOLI.

$$E\left\{\begin{bmatrix} n_1 \\ n_2^* \end{bmatrix}[n_1^*\ n_2]\right\} = \begin{bmatrix} |n_1|^2 & 0 \\ 0 & |n_2|^2 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2^* \end{bmatrix} = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2^* \end{bmatrix}$$

$$y_2^* = -h_1^* x_2 + h_2^* x_1 + n_2^* = h_2^* x_1 - h_1^* x_2$$

- PSEUDOINVERSE FOR GENERAL $m \times n$ MATRIX

$$H^+ = (H^H H)^{-1} \cdot H^H \qquad H = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix}$$

$$H^H = \begin{bmatrix} h_1^* & h_2^* \\ h_2 & -h_1 \end{bmatrix}^T = \begin{bmatrix} h_1^* & h_2 \\ h_2^* & -h_1 \end{bmatrix} \qquad \boxed{(H \cdot H^+ \cdot H) = H} \quad \text{WIKI}$$

$$\boxed{(H^H \cdot H) = \begin{bmatrix} h_1^* & h_2 \\ h_2^* & -h_1 \end{bmatrix}\begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix} = \begin{bmatrix} |h_1|^2 + |h_2|^2 & 0 \\ 0 & |h_2|^2 + |h_1|^2 \end{bmatrix}}$$

$$(H^H \cdot H)^{-1} = \begin{bmatrix} 1/(|h_1|^2 + |h_2|^2) & 0 \\ 0 & 1/(|h_1|^2 + |h_2|^2) \end{bmatrix}$$

INVERSE OF THE DIAGONAL MATRIX IS JUST INVERSE OF THE DIAGONAL ELEMENTS.

- THE ESTIMATE OF THE TRANSMITED SYMBOL IS:

$$\widehat{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}} = (H^H H)^{-1} \cdot H^H \cdot \begin{bmatrix} y_1 \\ y_2^* \end{bmatrix} = (H^H H)^{-1} H^H \left(H \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2^* \end{bmatrix}\right) = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + (H^H H)^{-1} H^H \begin{bmatrix} n_1 \\ n_2^* \end{bmatrix}$$

$$\boxed{\begin{bmatrix}\hat{x}_1\\ \hat{x}_2\end{bmatrix} = \begin{bmatrix}x_1\\ x_2\end{bmatrix} + (H^H H)^{-1} H^H \begin{bmatrix}n_1\\ n_2^*\end{bmatrix}}$$

ISTATA FORMULA KAKO I ZA **MRC** (N7.9977)

MMV

- BER WITH Alamouti STBC
- FOR MRC WITH 2 RECEIVER ANTENNAS

$$\boxed{P_{e,MRC} = p_{MRC}^2 \left(1 + 2(1 - p_{MRC})\right)}$$

$$p_{MRC} = \frac{1}{2} - \frac{1}{2}\left(1 + \frac{1}{E_b/N_0}\right)^{-1/2}$$

Osgood FORMA

TAJ ZA KORISTAM FORMULATA OD PROAKIS

$$\boxed{P_e = \left[\frac{1}{2}(1-\mu)\right]^L \sum_{k=0}^{L-1} \binom{L-1+k}{k} \left[\frac{1}{2}(1+\mu)\right]^k \qquad \mu = \sqrt{\frac{\gamma}{1+\gamma}}}$$

PROAKIS FORMA

$$\boxed{\gamma \triangleq \frac{E_b}{N_0}}$$

$$p_{MRC} = \frac{1}{2}\left[1 - \left(\frac{E_b/N_0}{1+E_b/N_0}\right)^{1/2}\right] = \frac{1}{2}\left[1 - \sqrt{\frac{\gamma}{1+\gamma}}\right]$$

$$\boxed{p_{MRC} = \frac{1}{2}(1-\mu)} \Rightarrow \boxed{\mu = 1 - 2p_{MRC}}$$

$$\boxed{L=2} \quad P_e = \left[\frac{1}{2}(1-\mu)\right]^2 \sum_{k=0,1}^{1} \binom{2-1+k}{k}\left[\frac{1}{2}(1+\mu)\right]^k$$

$$= \left[\frac{1}{2}(1-\mu)\right]^2 \cdot \left\{\binom{1}{0}\left[\frac{1}{2}(1+\mu)\right]^0 + \binom{2}{1}\frac{1}{2}(1+\mu)\right\}$$

$$\binom{2}{1} = \frac{2!}{1!\cdot 1!} = 2$$

$$P_e = \underbrace{\left[\frac{1}{2}(1-\mu)\right]^2}_{p_{MRC}} \left[1 + (1+\mu)\right] =$$

$$= p_{MRC}^2 \left[1 + 1 + 1 - 2p_{MRC}\right] = p_{MRC}^2\left(1 + 2(1 - p_{MRC})\right)$$

DRUGA ALTERNATIVNA FORMA ZA 2 PRIEMNI ANTENI:

$$\boxed{P_e = \frac{1}{4}(1-\mu)^2(2+\mu)} \Rightarrow$$

VNIMAVAJ VAZI SAMO ZA DVE PRIEMNI ANTENI !!! ...

MRC 2 ANTENNAS

$\bar{x} = \{x_1, x_2, x_3, \dots x_N\}$

$\bar{h}_1 = \{h_{11}, h_{12}, h_{13}, \dots h_{1N}\}$ $\quad x_1 \;\; -x_2^* \;\; x_3 \;\; -x_4^* \dots$ ANT 1

$\bar{h}_2 = \{h_{21}, h_{22}, h_{23}, \dots h_{2N}\}$ $\quad x_2 \;\; x_1^* \;\; x_4 \;\; x_3^* \dots$ ANT 2

$y_1 = h_{11} \cdot x_1 + h_{21} \cdot x_2$ $\quad y_3 = h_{13} \cdot x_3 + h_{23} x_4$

$y_2 = -h_{12} x_2^* + h_{22} x_1^*$ $\quad y_4 = -h_{14} x_4^* + h_{24} x_3^* \dots$

- MATLAB KANSKA SCRIPT

$hEg = \begin{bmatrix} h_1^* & h_2^* & \dots \\ h_2 & -h_1 & \dots \end{bmatrix}$ $\quad hEgPow = sum(hEg.*conj(hEg),1)$

$hEg^* = \begin{bmatrix} h_1 & h_2 & \dots \\ h_2^* & -h_1^* & \dots \end{bmatrix}$ $\quad hEgPow = [|h_1|^2+|h_2|^2, |h_2|^2+|h_1|^2 \dots]$

$\hat{y} = \dfrac{sum(hEg.*yMod)}{hEgPow}$ $\quad \hat{y}(2:2:end) = conj(\hat{y}(2:2:end))$

KANAROT TREBA DA SE NAPRAVI DA DOJDE IST ZA DVA POSLEDOVATELNI SIMBOLI T.E

$\bar{h} = \begin{bmatrix} h_1 & h_1 & h_3 & h_3 & \dots & h_{N-1} \\ h_2 & h_2 & h_4 & h_4 & \dots & h_N \end{bmatrix}$ $\quad \bar{x} = \begin{bmatrix} x_1; -x_2^*; x_3; -x_4^* \dots \\ x_2; x_1^*; x_4; x_3^* \dots \end{bmatrix}$

- TOGAŠ:

$\boxed{\begin{array}{ll} y_1 = h_1 \cdot x_1 + h_2 x_2 & y_3 = h_3 x_3 + h_4 y_4 \\ y_2 = -h_1 \cdot x_2^* + h_2 x_1^* & y_4 = -h_3 x_4^* + h_4 \cdot x_3^* \end{array}}$

ODGOVARA NA: AL

$y = sum(h.'.*x.')$ ~~dot(h, x)~~

(AKO SAKAS DA DOBIES VECTOR COLONA HERMITIAN!!!)

- VNIMAVAJ !!! $\boxed{dot(A,B) = A^H.*B}$

AKO A I B SE VEKTORI.

OVA MI BEŠE GOLEMA GREŠKA! KOJ ZNAE KADE SE ~~NE SE~~ PROVLEKLA !!!

$\boxed{\mu = \sqrt{\dfrac{EbNo/2}{1+EbNo/2}} = \sqrt{\dfrac{EbNo}{2+EbNo}}}$ } ZA ALAMOUTI TRANSMIT DIVERSITY

$\boxed{P_e = 0.25(1-\mu)^2(2+\mu)}$

• povtoruvanje

$$\begin{bmatrix} y_1 \\ y_2^* \end{bmatrix} = \underbrace{\begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix}}_{H} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

$$\boxed{\begin{aligned} y_1 &= h_1 x_1 + h_2 x_2 \\ y_2 &= h_2 x_1^* - h_1 x_2^* \\ y_2 &= -h_1 x_2^* + h_2 x_1^* \end{aligned}}$$

$$H = \begin{bmatrix} h_1 & h_2 \\ h_2^* & h_1^* \end{bmatrix} \qquad (H^H H)^{-1} = \begin{bmatrix} \frac{1}{|h_1|^2 + |h_2|^2} & 0 \\ 0 & \frac{1}{|h_1|^2 + |h_2|^2} \end{bmatrix}$$

$$\begin{bmatrix} \hat{x}_1 \\ x_2 \end{bmatrix} = H^+ \cdot \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \qquad \boxed{H^+ = (H^H H)^{-1} H^H}$$

$$\begin{bmatrix} \hat{x}_1 \\ x_2 \end{bmatrix} = (H^H H)^{-1} H^H \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = (H^H H)^{-1} H^H \cdot \left(H \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2^* \end{bmatrix} \right)$$

$$= \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + (H^H H)^{-1} H^H \begin{bmatrix} n_1 \\ n_2^* \end{bmatrix} = \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \frac{H^H}{H^H H} \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}}_{\triangleq \text{ MRC}}$$

$$yMod = \begin{bmatrix} y_1 & y_1 & y_3 & y_3 & \cdots & y_{N-1} & y_{N-1} \\ y_2^* & y_2^* & y_4^* & y_4^* & \cdots & y_N^* & y_N^* \end{bmatrix}$$

pomegju sebe se ednakvi zošto taka e definirano za Alamouti.

$$hEq = \begin{bmatrix} h_1^* & h_2^* & h_3^* & h_4^* & \cdots & h_{N-1} & h_N \\ h_2 & -h_1 & h_4 & -h_3 & \cdots & h_N & -h_{N-1} \end{bmatrix}$$

$$hEqPow = \begin{bmatrix} h_1^* & h_2^* & \cdots \\ h_2 & -h_1 & \cdots \end{bmatrix} \begin{bmatrix} h_1 & h_2 & \cdots \\ h_2^* & -h_1^* & \cdots \end{bmatrix} = \left[|h_1|^2 + |h_2|^2, |h_1|^2 + |h_2|^2 \cdots \right]$$

(1)   MRC

yHat = sum(hEq.*yMod,1)./hEqPow

$$yHat_1 = (y_1 \cdot h_1^* + y_2^* \cdot h_2) / (|h_1|^2 + |h_2|^2); \quad yHat_3 = (y_3 \cdot h_3^* + y_4 \cdot h_4) / (|h_3|^2 + |h_4|^2)$$

$$yHat_2^* = (y_1 \cdot h_2^* - y_2^* \cdot h_1) / (|h_1|^2 + |h_2|^2); \quad yHat_4^* = (y_3 \cdot h_4^* - y_4^* \cdot h_3) / (|h_3|^2 + |h_4|^2)$$

$$yHat_2 = (y_1^* h_2 - y_2 h_1^*) / (|h_1|^2 + |h_2|^2); \quad yHat_4 = (y_3^* h_4 - y_4 h_3^*) / (|h_3|^2 + |h_4|^2)$$

(2)

• ~~Moe mislenje dolu treba vaka:~~

MMV

$$\cancel{yMod = \begin{bmatrix} y_1 & y_1 & y_3 & y_3 & \cdots & y_{N-1} & y_{N-1} \\ y_2 & y_2^* & y_4 & y_4^* & \cdots & y_N & y_N^* \end{bmatrix}}$$

$$\cancel{hEq = \begin{bmatrix} h_1 & h_2^* & h_3 & h_3^* \\ h_2 & -h_1^* & h_4 & -h_4^* \end{bmatrix}}$$

$$\cancel{yHat_1 = h_1 y_1 + h_2 y_2}$$

$$\cancel{yHat_2 = h_2^* \cdot y_1 - h_1^* y_2^* \Rightarrow yHat_2 = h_2 \cdot y_1^* - h_1 y_2}$$

Za finalna smetaj za formulata na pp. 27 (*$)

Dokaz za toa e implementacijata od Matlab na pp. 37

$$② \Rightarrow yHat_2 = \frac{y_1^* h_2 - y_2 \cdot h_1^*}{|h_1|^2+|h_2|^2} = \frac{(h_1 x_1 + h_2 x_2)^* h_2 - (h_2 x_1^* - h_1 x_2^*) h_1^*}{|h_1|^2+|h_2|^2} =$$

$$= \frac{1}{k}\left(\cancel{h_1^* x_1^* h_2} + h_2^* x_2^* \cdot h_2 - \cancel{h_2 x_1^* h_1^*} + h_1 x_2^* h_1^* \right) = \qquad \boxed{k = |h_1|^2 + |h_2|^2}$$

$$= \frac{1}{|h_1|^2+|h_2|^2}\left(|h_2|^2 x_2^* + |h_1|^2 x_2^* \right) = \underline{\underline{x_2^*}}$$

$$① \Rightarrow yHat_1 = \frac{y_1 h_1^* + y_2^* \cdot h_2}{k} = \frac{(h_1 x_1 + h_2 x_2) h_1^* + (h_2^* x_1 - h_1^* x_2) h_2}{k} =$$

$$= \frac{1}{k}\left(|h_1|^2 x_1 + \cancel{h_2 x_2 h_1^*} + |h_2|^2 x_1 - \cancel{h_1^* x_2 h_2} \right) = \frac{\cancel{|h_1|^2+|h_2|^2}}{\cancel{k}} x_1 = \underline{\underline{x_1}}$$

PO SE IZGLEDA NEMA POTREBA DA SE VRŠI KONJUGACIJA VO ② OVDE ZOŠTO:

$$\boxed{yHat_2 = \frac{y_1 h_2^* - y_2^* h_1}{k}} = \frac{(h_1 x_1 + h_2 x_2) h_2^* - (h_2 x_1^* - h_1 x_2^*)^* h_1}{k}$$

$$yHat_2 = \frac{1}{k}\left(\cancel{h_1 x_1 h_2^*} + |h_2|^2 x_2 - \cancel{h_2^* x_1 h_1} + |h_1|^2 x_2 \right) = \frac{\cancel{|h_1|^2+|h_2|^2}}{\cancel{k}} x_2 = x_2$$

MNOGU VAŽNO IZVEDUVANJE !!! **MMV**

- FOR 4 PHASE PSK & DPSK MRC BER

MMV
$$P_b = \frac{1}{2}\left[1 - \frac{\mu}{\sqrt{2-\mu^2}} \sum_{k=0}^{L-1} \binom{2k}{k} \left(\frac{1-\mu^2}{4-2\mu^2}\right)^k \right] =$$

VNIMAVAJ SEKADE GO IMAM VKLUČENO ŠUMOT, A TOJ TREBA DA FIGURIRA VO KRAJNATA FORMULA

IZVEDUVANJATA SO VKLUČEN ŠUM SE NA NB. p.149

$$= \frac{1}{2}\left[1 - \frac{\mu}{\sqrt{2-\mu^2}}\left(1 + 2\,\frac{1-\mu^2}{4-2\mu^2}\right)\right]$$

$$P_b = \frac{1}{2}\left[1 - \frac{\mu}{\sqrt{2-\mu^2}} - \frac{\mu(1-\mu^2)}{\sqrt{2-\mu^2}\,(2-\mu^2)}\right]$$

$$\boxed{P_b = 0.5\left[1 - \frac{\mu}{\sqrt{2-\mu^2}} - \frac{\mu(1-\mu^2)}{\sqrt{(2-\mu^2)^3}}\right]}$$

- KAJ MCR $yHat = \dfrac{sum(conj(h) .* y)}{|h_1|^2 + |h_2|^2}$

ZA $L=2$:
$$yHat1 = \frac{[h_1^* \; h_2^*] \cdot \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}}{|h_1|^2 + |h_2|^2} = \frac{h_1^*(h_1 x_1 + n_1) + h_2^*(h_2 x_1 + n_2)}{|h_1|^2 + |h_2|^2}$$

$$\text{yHat}_1 = \frac{h_1^*(h_1 x + n_1) + h_2^*(h_2 x + n_2)}{|h_1|^2 + |h_2|^2} =$$

$$= \left(|h_1|^2 x + h_1^* n_1 + |h_2|^2 x + h_2^* n_2\right) / \left(|h_1|^2 + |h_2|^2\right)^2$$

$$\text{yHat}_1 = \frac{\cancel{(|h_1|^2 + |h_2|^2)} x}{\cancel{|h_1|^2 + |h_2|^2}} + \frac{h_1^* n_1 + h_2^* n_2}{|h_1|^2 + |h_2|^2}$$

$$\boxed{\text{yHat}_1 = x + \frac{h_1^* n_1 + h_2^* n_2}{|h_1|^2 + |h_2|^2}}$$

IZVEDUVANJE ZA EQUALIZIRANIOT SIGNAL VO MRC !!!

$$\boxed{\text{yHat} = x + \frac{h^H \cdot n}{h^H \cdot h}}$$

$$h^H = [h_1^* \; h_2^*] \qquad n = \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

$$\hat{y} = \frac{h^H \cdot y}{h^H \cdot h} = \frac{h^H (h \cdot x + n)}{h^H \cdot h} = x + \frac{h^H \cdot n}{h^H \cdot h}$$

POVNEZ!!!

$$Q(z) = \frac{1}{2} \operatorname{erfc} \frac{z}{\sqrt{2}}$$

MPSK

$$\boxed{P_b = 2\, Q\left(\underbrace{\sqrt{2 \cdot \log M}}_{a} \; \sin\left(\frac{\pi}{M}\right) \cdot \sqrt{\bar{\gamma}}\right)}$$

AWGN

$$M = 4 \quad P_b = \operatorname{erfc}\left(\sqrt{2}\; \underbrace{\sin\left(\frac{\pi}{4}\right)}_{\sqrt{2}/2} \sqrt{\bar{\gamma}}\right) = \operatorname{erfc}\left(\sqrt{\bar{\gamma}}\right)$$

- RAYLEIGH $\qquad a = \text{sqrt}(4) \cdot \frac{\sqrt{2}}{2} = \sqrt{2}$

$$P_b = 0.5\left(1 - \text{sqrt}\,\frac{0.5(\sqrt{2})^2 \bar{\gamma}}{1 + 0.5(\sqrt{2})^2 E_b/N_0}\right) = 0.5\left(1 - \sqrt{\frac{\bar{\gamma}}{1+\bar{\gamma}}}\right)$$

TREBA DA GO SKRATI?

- SPORED MENE P_b ZA RAYLEIGH KANAL VO SLUČAJ NA MPSK TREBA DA E/PB

$$\cancel{P_b = \left(1 - \sqrt{\frac{0.5 a^2 \bar{\gamma}}{1 + 0.5 a^2 \bar{\gamma}}}\right)}$$

- DCoFC FORMULA ZA MPSK VO RAYLEIGH FADING

$$\boxed{P_b = \frac{M-1}{M}\left\{1 - \sqrt{\frac{a^2\bar{\gamma}/2}{1+a^2\bar{\gamma}/2}}\left(\frac{M}{(M-1)\pi}\right) \cdot \left[\frac{\pi}{2} + \tan^{-1}\left(\sqrt{\frac{a^2\bar{\gamma}/2}{1+a^2\bar{\gamma}/2}} \cot\frac{\pi}{M}\right)\right]\right\}}$$

(*II)

• VO DCoFC UNION BOUND ~~ZA~~ MPSK E ~~DADEN~~ ZA

$$\boxed{d^2 = 2\sin^2\frac{\pi}{M}} \quad \text{T.E} \quad \boxed{d = \sqrt{2}\sin\frac{\pi}{M}}$$

– UNION BOUND FOR MPSK

$$P_S(\epsilon) \le 2Q\left(\sqrt{\frac{2E_s}{N_0}}\sin\frac{\pi}{M}\right)$$

(HMV UNION BOUND)

$$E_s = ld(M) \Rightarrow \boxed{P_b \le 2Q\left(\sqrt{\frac{2\,ld M\, E_b}{N_0}}\sin\frac{\pi}{M}\right)}$$

ZNAČI FORMULATA ~~O~~ SKLATE ŠTO JA KORISTAM E TOČNA

– EQUATION 8.71

$$\boxed{P_b(\epsilon) = \frac{1}{2}(P_1 + 2P_2 + P_3) \quad M=4}$$

AWGN

• DCoFC AVERAGE SER FOR RAYLEIGH (EQ. 8.112)

$$\boxed{P_s = \left(\frac{M-1}{M}\right)\left\{1 - \underbrace{\sqrt{\frac{\sin^2\frac{\pi}{M}\,ld M\cdot E_b/N_0}{1+\sin^2\frac{\pi}{M}\,ld M\, E_b/N_0}}}_{\text{"}b\text{"}=}\left(\frac{M}{(M-1)\pi}\right)\left(\frac{\pi}{2} + \operatorname{atan}\left(b\cdot\cot\left(\frac{\pi}{M}\right)\right)\right)\right\}}$$

DCoFC | HMV

ISTA E SO *II AKO SE ZEME DEKA: $\boxed{d = \sqrt{ld M}\sin\frac{\pi}{M}}$

→ OVAA FORMULA E ~~ZA~~ ~~NESMETKA~~ NA SER NO KOJA ĆE ZA ~~SPOREDBA~~ SO P_s-SIM NE MI SE SOVPAĐA. ŠE SOVPAĐA SO P_b-SIM !?

• DCoFC AVERAGE ~~BER~~

$$\boxed{g_{psk} = \sin^2\frac{(2k+1)\pi}{M}}$$

$$\overline{P_k} \triangleq \int_0^\infty P_k\, p_\gamma(\gamma)\, d\gamma = K_+ - K_- \qquad k = 0,1,2,\ldots M-1$$

$$K_\pm = \frac{1}{2}\left(\frac{2k\pm1}{M}\right)\left[1 - \sqrt{\frac{g_{psk}(k\pm)\,\overline{\gamma}_s}{1+g_{psk}(k\pm)\,\overline{\gamma}_s}}\left(\frac{M}{(2k\pm1)\pi}\right)\cdot \operatorname{atan}\left(\sqrt{\frac{1+g_{psk}(k\pm)\,\overline{\gamma}_s}{g_{psk}(k\pm)\,\overline{\gamma}_s}}\tan\frac{(2k\pm1)\pi}{M}\right)\right]$$

8.117

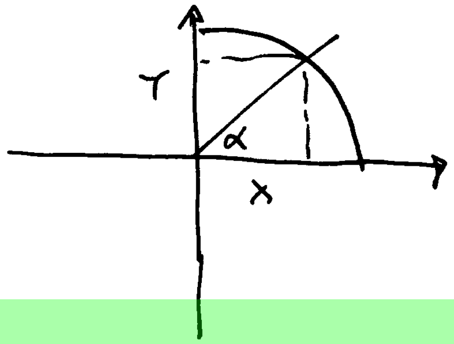
NE MI IZLEGUVA VO MATLAB

HMV

$$P_b(\epsilon) = \frac{1}{2}(P_1 + 2P_2 + P_3) \qquad M=4$$

$$P_b(\epsilon) = \frac{1}{3}(P_1 + 2P_2 + P_3 + 2P_4 + 3P_5 + 2P_6 + P_7) \qquad M=8$$

$$P_b(\epsilon) = 0.5\left(\sum_{k=1}^{3} P_k + \sum_{k=1}^{5} P_k + P_5 + 2P_6 + P_7\right) \qquad M=16$$



$$tg\,\alpha = \frac{Y}{X}$$
$$\alpha = tg^{-1}\left(\frac{Y}{X}\right)$$

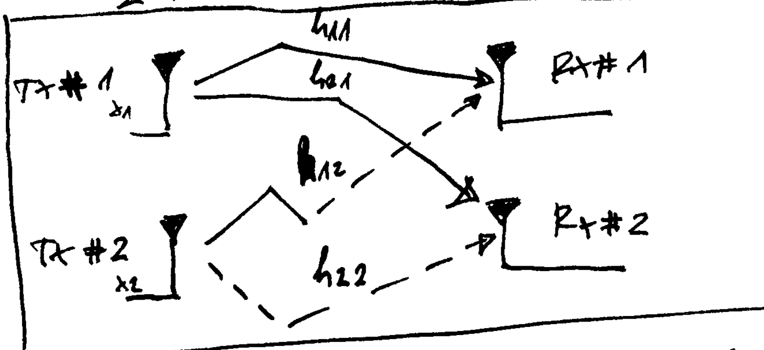
VO MATLAB PRIMEROT introMIMOSystems.m IMPLEMENTIRAN E MIMO SO ALAMOUTI KOD I EDNOSTAVNA DEMODULACIJA. (PART 2)

• DsyLog MIMO WITH ~~[illegible]~~ Zero Forcing Equalizer

- SIMO $\triangleq$ RECEIVE DIVERSITY (SELECTION COMBINING, EGC & MRC)
- MISO $\triangleq$ TRANSMIT DIVERSITY (Alamouti 2x1 STBC)

1. TRANSMIT SEQUENCE $\{x_1, x_2, x_3, \dots, x_n\}$

1 ANTENA $x_1 \; x_3 \; x_5 \; \dots$
2 ANTENA $x_2 \; x_4 \; x_6 \; \dots$ } DOUBLING OF DATA RATE



(2x2) MIMO CHANNEL

$$y_1 = h_{11} \cdot x_1 + h_{12} \cdot x_2 + n_1 = \begin{bmatrix} h_{11} & h_{12} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_1$$

$$y_2 = h_{21} x_1 + h_{22} x_2 + n_2 = \begin{bmatrix} h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_2,$$

$$\boxed{\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}}$$

DATA PODOCNA PRESMETANI I OD MIMO WITH ML ~~ZATOA~~ ŠTO IMA SLOW FADING!!!

$$Y = \bar{H} x + n$$

- TO SOLVE FOR x WE NEED TO FIND MATRIX W WHICH SATISFIES $WH = I$ (ZERO FORCING LINEAR DETECTOR)

$$\boxed{W = (H^H H)^{-1} H^H}$$ } PSEUDOINVERSE FOR GENERAL $m \times n$ MATRIX

$$H^H \cdot H = \begin{bmatrix} h_{11}^* & h_{21}^* \\ h_{12}^* & h_{22}^* \end{bmatrix} \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} |h_{11}|^2 + |h_{21}|^2 & h_{11}^* h_{12} + h_{21}^* h_{22} \\ h_{12}^* h_{11} + h_{22}^* h_{21} & |h_{12}|^2 + |h_{22}|^2 \end{bmatrix}$$

- BER FOR MIMO 2x2 WITH ZERO FORCING IS EQUIVALENT TO 1x1 CHANNEL IN RAYLEIGH FADING

$$\boxed{P_b = \frac{1}{2}\left(1 - \sqrt{\frac{\gamma}{1+\gamma}}\right)}$$

($*)

$$\begin{bmatrix} h_{11} & h_{12} & h_{13} & h_{14} & \dots & h_{1N/2} \\ h_{21} & h_{22} & h_{23} & h_{24} & \dots & h_{2N/2}\end{bmatrix}^T \begin{bmatrix} x_1 & x_3 & x_5 & \dots & x_{N-1} \\ x_2 & x_4 & x_6 & \dots & x_N\end{bmatrix}$$

$\boxed{\text{G.992.5 annex B}}$

$$\begin{bmatrix} y_{11}, y_{12} \dots \\ y_{21}, y_{21} \dots \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} x_1 & x_3 & x_5 & \dots & x_{N-1} \\ x_2 & x_4 & x_6 & \dots & x_N\end{bmatrix}$$

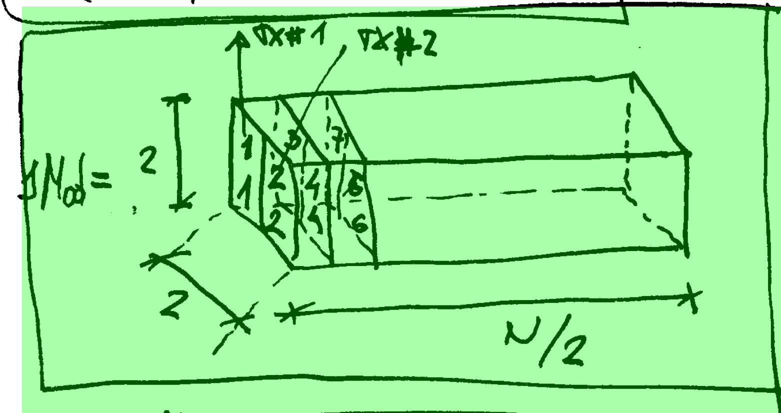
$\boxed{\text{IC0106}}$

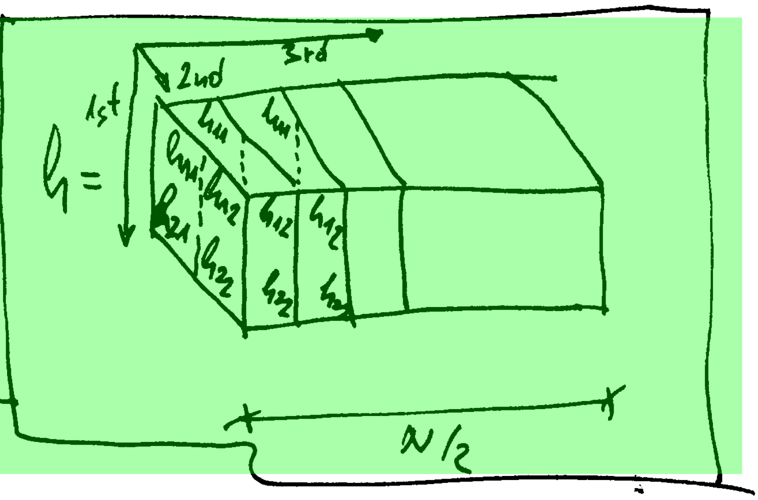
$$y_{11} = h_{11} \cdot x_1 + h_{12} \cdot x_2 \;; \quad y_{12} = h_{11} \cdot x_3 + h_{12} \cdot x_4 \dots$$

$$y_{21} = h_{21} x_1 + h_{22} x_2 \;; \quad y_{22} = h_{21} \cdot x_3 + h_{22} x_4 \dots$$

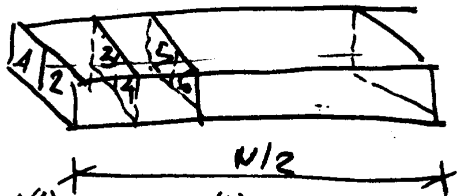
+4nd

$\boxed{size(sMod) = 2 \times 2 \times N/2}$ $\quad size(h) = 2 \times 2 \times N/2$





$\boxed{size(y) = 2 \times N/2}$

$y =$ 

$$sum(h.*sMod, 2) \Rightarrow \quad y_1 = h^{(1)}_{11} \cdot x_1 + h^{(1)}_{12} \cdot x_2$$

$$y_2 = h^{(1)}_{21} x_1 + h^{(1)}_{22} \cdot x_2$$

$$y_3 = h^{(2)}_{11} x_3 + h^{(2)}_{12} x_4 \dots$$

$$y_4 = h^{(2)}_{21} x_3 + h^{(2)}_{22} x_4 \dots$$

$$\begin{bmatrix} a & b \\ c & c \end{bmatrix}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$H^{-1} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}^{-1} = \frac{1}{h_{11} h_{22} - h_{12} h_{21}} \begin{bmatrix} h_{22} & -h_{12} \\ -h_{21} & h_{11} \end{bmatrix}$$

$$hCof(1,1,:) = sum(h(:,2,:).*conj(h(:,2,:)),1) = |h_{12}|^2 + |h_{22}|^2 \quad \text{„d"}$$

$$hCof(2,2,:) = sum(h(:,1,:).*conj(h(:,1,:))) = |h_{11}|^2 + |h_{21}|^2 \quad \text{„a"}$$

$$hCof(2,1,:) = -sum(h(:,2,:).*conj(h(:,1,:))) = h_{12} h_{11}^* + h_{22} \cdot h_{21}^* \quad \text{„c"}$$

$$hCof(1,2,:) = -sum(h(:,1,:).*conj(h(:,2,:))) = h_{11} \cdot h_{12}^* + h_{21} h_{22}^* \quad \text{„d"}$$

GO PRAVI $(H^H . H)^{-1}$ VIDI (**)

$$hDen = (hCof(1,1,:).*hCof(2,2,:)) - (hCof(1,2,:).*hCof(2,1,:)) \quad \boxed{ad - bc}$$

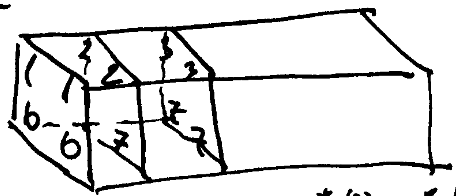
DETERMINANTATA NA $(H^H H)$ (VIDI $*)

- SE BARA DA SE DOBIE $\boxed{W = (H^H H)^{-1} H^H}$

$$\boxed{yHat = inv(H^H H * H) * (H^H * Y)} \rightarrow \text{EQUALIZATION FOR MIMO}$$

$$Y = \begin{bmatrix} 1 & 3 & 5 & \cdots & N/2-1 \\ 2 & 4 & 6 & \cdots & N/2 \end{bmatrix} \qquad yMod = \begin{bmatrix} 1 & 1 & 3 & 3 & \cdots \\ 2 & 2 & 4 & 4 & \cdots \end{bmatrix}$$

$$H = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$$

c = 

$$reshape(c,2,10) = \begin{matrix} 1 & 1 & 2 & 2 & 3 & 3 \\ 6 & 6 & 7 & 7 & 7 & 7 \end{matrix}$$

$$hMod = \begin{bmatrix} h_{11}^{*(1)} & h_{12}^{*(1)} & h_{11}^{*(2)} & h_{12}^{*(2)} & \cdots & h_{11}^{*(N)} & h_{12}^{*(N)} \\ h_{21}^{*(1)} & h_{22}^{*(1)} & h_{21}^{*(2)} & h_{22}^{*(2)} & \cdots & h_{21}^{*(N)} & h_{22}^{*(N)} \end{bmatrix}$$

$$yMod1 = \begin{bmatrix} y_1 & y_1 & y_3 & y_3 & \cdots & y_{N-1} & y_{N-1} \\ y_2 & y_2 & y_4 & y_4 & \cdots & y_N & y_N \end{bmatrix}$$

$$y_1 = h_{11}^{(1)} x_1 + h_{12}^{(1)} x_2$$
$$y_2 = h_{21}^{(1)} x_1 + h_{22}^{(1)} x_2$$

$$yMod^{(2)} = sum(hMod.*yMod1,1) = \begin{bmatrix} h_{11}^* & h_{12}^* \\ h_{21}^* & h_{22}^* \end{bmatrix} .* \begin{bmatrix} y_1 & y_1 \\ y_2 & y_2 \end{bmatrix} = \begin{bmatrix} h_{11}^* y_1 + h_{21}^* y_2 \\ h_{12}^* y_1 + h_{22}^* y_2 \end{bmatrix}$$

- MNOZENJE NA MATRICI

$$\begin{bmatrix} h_{11}^* & h_{12}^* \\ h_{21}^* & h_{22}^* \end{bmatrix}^{T} * \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{11} y_1 + h_{21}^* y_2 \\ h_{12} y_1 + h_{22} y_2 \end{bmatrix}$$

EVE ZOSTO VO GORNATA IMPLEMENTACIJA NE BEVI TRANSPONIRANJE

$$yMod^{(2)} = \begin{bmatrix} h_{11}^{(1)*} y_1 + h_{21}^{(1)*} y_2 ; & h_{12}^{(1)*} y_1 + h_{22}^{(1)*} y_2 ; & h_{11}^{(2)*} y_3 + h_{21}^{(2)*} y_4 ; & h_{12}^{(2)*} y_3 + h_{22}^{(2)*} y_4 \cdots \end{bmatrix}$$

$$size(yMod^{2}) = 1 \times N$$

$$size(yMod) = 2 \times N$$

$$yMod = kron(reshape(yMod2,2,N/nTx),ones(1,2))$$

$$yMod = \begin{bmatrix} h_{11}^{(1)*} y_1 + h_{21}^{(1)*} y_2 ; & h_{11}^{(1)*} y_1 + h_{21}^{(1)*} y_2 ; & h_{11}^{(2)*} y_3 + h_{21}^{(2)*} y_4 ; & h_{11}^{(2)*} y_3 + h_{21}^{(2)*} y_4 & \cdots \\ h_{12}^{(1)*} y_1 + h_{22}^{(1)*} y_2 ; & h_{12}^{(1)*} y_1 + h_{22}^{(1)*} y_2 ; & h_{12}^{(2)*} y_3 + h_{22}^{(2)*} y_4 ; & h_{12}^{(2)*} y_3 + h_{22}^{(2)*} y_4 & \cdots \end{bmatrix}$$

$$reshape(hInv,2,N) = \begin{bmatrix} hCof11^{(1)} & hCof12^{(1)} & hCof11 & hCof12 & \cdots \\ hCof21^{(1)} & hCof22^{(1)} & hCof21 & hCof22 & \cdots \end{bmatrix} \cdot \frac{1}{hDen}$$

yHat = sum(reshape(h(hu, 2, N) .* yMod, 1));

$$yHat = \left[hCof_{11}^{(1)} \cdot yMod_1 + hCof_{21}^{(1)} yMod_2 ;\ hCof_{12} \cdot yMod_1 + hCof_{22} yMod_2 ; \dots \right] \cdot \frac{1}{hDen}$$

$\hookrightarrow \boxed{yMod_1 = h_{11}^{*} y_1 + h_{21}^{*} y_2}$

☐ MIMO WITH MMSE Equalizer

- Minimum Mean Square Error (MMSE) APPROACH TRIES TO FIND A MATRIX W WHICH MINIMIZES:

$$E\left\{ [Wy - x][Wy - x]^H \right\}$$

$$\boxed{W = [H^H H + N_0 I]^{-1} H^H}$$

VIDI PP. 14

hCof(1,1,:) = sum(h(:,2,:).* conj(h(:,2,:)),1) + 10^(-EbN0_dB(ii)/10);

hCof(2,2,:) = sum(h(:,1,:).* conj(h(:,1,:)),1) + 10^(-EbN0_dB(ii)/10);

VIDI ($*) ŠUMOT GO PODAVA VO DIJAGONALATA !!!

☐ ZERO FORCING WITH SUCCESSIVE INTERFERENCE CANCELLATION (ZF-SIC)

$$\begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix} = (H^H H)^{-1} H^H \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

GLAVNATA FORMULA !!! ($) SO KOJA PRAKTIČNO JA KANCELIRAŠ INTERFERENCIJATA ZA NEPARNITE ČLENOVI OD $\hat{x}$

$$\boxed{\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} y_1 - h_{12}\hat{x}_2 \\ y_2 - h_{22}\hat{x}_2 \end{bmatrix}} = \begin{bmatrix} h_{11}x_1 + \cancel{h_{12}x_2} + n_1 - \cancel{h_{12}\hat{x}_2} \\ h_{21}x_1 + \cancel{h_{22}x_2} + n_2 - \cancel{h_{22}\hat{x}_2} \end{bmatrix}$$

$$\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} h_{11}x_1 + n_1 \\ h_{21}x_1 + n_2 \end{bmatrix} \qquad \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} h_{11} \\ h_{21} \end{bmatrix} x_1 + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

RECEIVE DIVERSITY

- EQUALIZED SYMBOL IS:

$$\boxed{\hat{x}_1 = \frac{h_1^H r}{h_1^H h_1}}$$

ODD SYMBOLS

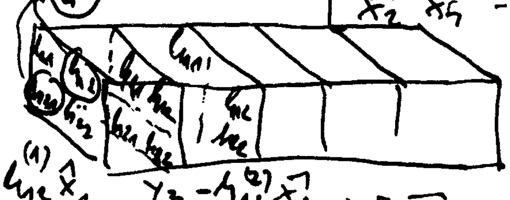
$h_1 \Rightarrow h(:,1,:)$

$$xHatMod^{D} = \begin{bmatrix} \hat{x}_2 & \hat{x}_4 & \dots & \hat{x}_N \\ \hat{x}_2 & \hat{x}_4 & \dots & \hat{x}_N \end{bmatrix}$$

$$xHatMod^{*} = \begin{bmatrix} \hat{x}_1 & \hat{x}_3 & \dots & \hat{x}_{N-1} \\ \hat{x}_2 & \hat{x}_4 & \dots & \hat{x}_N \end{bmatrix}$$

$$yHat = xHat = [\hat{x}_1\ \hat{x}_2\ \dots\dots\ \hat{x}_N]$$

$$y = \begin{bmatrix} y_1 & y_3 & \dots & y_{N-1} \\ y_2 & y_4 & \dots & y_N \end{bmatrix}$$

h = 

$$r = y - \underbrace{xHatMod}_{\equiv yHatMod} .* h(:,2,:) = \begin{bmatrix} y_1 - h_{12}^{(1)}\hat{x}_1 & y_3 - h_{12}^{(2)}\hat{x}_1 & \dots \\ y_2 - h_{22}^{(1)}\hat{x}_2 & y_4 - h_{21}^{(2)}\hat{x}_2 & \dots \end{bmatrix}$$

- MOŽE DA SE NAPRAVI ~~SIC~~ I ZA PARNITE ČLENOVI OD PREDMETOT SIGMA. SE DOBIVAAT OŠTE PODOBRI REZULTATI

$$\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} y_1 - h_{11}\hat{x}_1 \\ y_2 - h_{21}\hat{x}_1 \end{bmatrix} = \begin{bmatrix} \cancel{h_{11}x_1} + h_{12}x_2 + n_1 - \cancel{h_{11}\hat{x}_1} \\ \cancel{h_{21}x_1} + h_{22}x_2 + n_2 - \cancel{h_{21}\hat{x}_1} \end{bmatrix} = \begin{bmatrix} h_{12} \\ h_{22} \end{bmatrix} x_2 + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

$h(:,1,:)$

$$\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} h_{12} \\ h_{22} \end{bmatrix} x_2$$

EQUALIZATION (MRC)

$$\hat{x}_2 = \frac{h_2^H \cdot r}{h_2^H \cdot h_2}$$

$h_2 = h(:,2,:)$
EVEN SYMBOLS

MIMO WITH ZF SIC AND OPTIMAL ORDERING

- RECEIVED POWER AT THE BOTH ANTENNAS CORRESPONDING TO THE TRANSMITTED SYMBOL x_1 IS:

$$P_{x_1} = |h_{11}|^2 + |h_{21}|^2.$$

- RECEIVED POWER AT THE BOTH ANTENNAS CORRESPONDING TO THE TRANSMITTED SYMBOL x_2 IS:

$$P_{x_2} = |h_{12}|^2 + |h_{22}|^2$$

PROVERKA ZA PRVIOT ZA SEKOJ TIMESLOT!

- IF $\boxed{P_{x_1} > P_{x_2}}$ THEN

$$\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} y_1 - h_{11}\hat{x}_1 \\ y_2 - h_{21}\hat{x}_1 \end{bmatrix} = \begin{bmatrix} h_{12} \\ h_{22} \end{bmatrix} x_2 + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

$$\boxed{\bar{r} = \bar{h}_1 x_2 + \bar{n}}$$

$$\hat{x}_2 = \frac{h_2^H \cdot r}{h_2^H \cdot h_2}$$

- ELSE IF $\boxed{P_{x_1} < P_{x_2}}$ THEN

$$\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} y_1 - h_{12}x_2 \\ y_2 - h_{22}x_2 \end{bmatrix} = \begin{bmatrix} h_{11} \\ h_{21} \end{bmatrix} x_1 + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

$$\hat{x}_1 = \frac{h_1^H \cdot r}{h_1^H \cdot h_1}$$

- GO IMPLEMENTIRAV SO SWAP. JA IMAM LOGIKATA NO NE MI E 100% DOKAŽANO

MIMO WITH ML EQUALIZATION

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

- MAXIMUM LIKELIHOOD RECEIVER TRIES TO FIND $\hat{x}$ WHICH MINIMIZES:

$$J = |y - H\hat{x}|^2 \qquad J = \left\| \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} - \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix} \right|^2$$

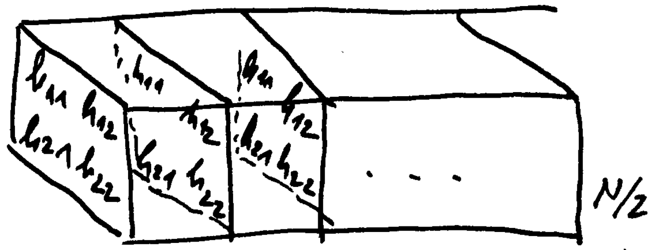
$$J_{1,1} = \left| \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} - \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right|^2 \qquad J_{1,-1} = \left| \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} - \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right|$$

$J_{-1,1} = \left| \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} - \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix} \right|^2$ $J_{-1,-1} = \left| \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} - \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} -1 \\ -1 \end{bmatrix} \right|^2$

$\min[J_{11}, J_{1,-1}, J_{-1,1}, J_{-1,-1}] = MIN$

if $MIN = J_{11} \Rightarrow [x_1\ x_2] = [1, 1]$;

if $MIN = J_{-1-1} \Rightarrow [x_1\ x_2] = [-1, -1]$; T.e $[0,0]$

$h =$ 

$x_1 = \underbrace{\begin{bmatrix} 1 & 1 & \cdots \\ 1 & 1 & \cdots \end{bmatrix}}_{N}$ $x_2 = \underbrace{\begin{bmatrix} 1 & 1 & \cdots \\ -1 & -1 & \cdots \end{bmatrix}}_{N}$

$x_3 = \underbrace{\begin{bmatrix} -1 & -1 & \cdots \\ 1 & 1 & \cdots \end{bmatrix}}_{N}$ $x_4 = \underbrace{\begin{bmatrix} -1 & -1 & \cdots \\ -1 & -1 & \cdots \end{bmatrix}}_{N}$

$h_{MOD} = \underbrace{\begin{bmatrix} h_{11}^{(1)} & h_{21}^{(1)} & h_{11}^{(2)} & h_{21}^{(2)} & \cdots \\ h_{12}^{(1)} & h_{22}^{(1)} & h_{12}^{(2)} & h_{22}^{(2)} & \cdots \end{bmatrix}}_{N}$

OVA SE PRAVI VAKA ZA DA ODI SO „DOT" PRODUCT A NA MNOŽENJE NA MATRICI

$J_{11} = sum(hMod .* x_1) = [h_{11}^1 + h_{12}^{(1)}, h_{21}^{(1)} + h_{22}, h_{11}^{(2)} + h_{12}^{(2)} \cdots]$

$J_{1-1} = sum(hMod .* x_2) = [h_{11}^1 - h_{12}^1, h_{21}^1 - h_{22}^1, h_{11}^2 - h_{12}^2 \cdots]$

$J_{-11} = sum(hMod .* x_3) = [-h_{11}^1 + h_{12}^1, -h_{21}^1 + h_{22}^1, -h_{11}^2 + h_{12}^2 \cdots]$

$J_{-1,-1} = sum(hMod .* x_4) = [-h_{11}^1 - h_{12}^1, -h_{21}^1 - h_{22}^1, -h_{11}^2 - h_{12}^2]$

3, 2, 4

[-1, 1, 1, -1, -1, -1]

- STAVI GI DOBIENITE REZULTATI ZA MIMO ML VO GRUPATA SLIKA. ✓

⊡ INTRODUCTION TO MIMO Systems (Matlab Help)

- PART 1: TRANSMIT DIVERSITY VS. RECEIVE DIVERSITY

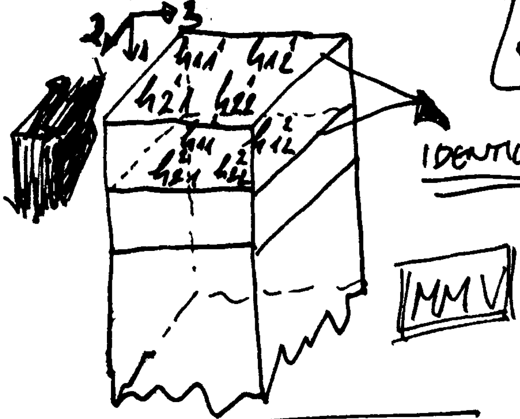
```
frmLen = 100;            % FRAME LENGTH
numPackets = 1000;       % NUMBER OF PACKETS
N=2; M=2                 % NUMBER OF TX, NUMBER OF RX ANTEN.
tx2 = zeros(frmLen,N);   H = zeros(frmLen, N, M)
error11 = error21 = error12 = zeros(1, numPackets)
BER11 = zeros(1, length(EbNo));
```

- LOOP OVER NUMBER OF PACKETS

for packetIdx = 1 : numPack

H = zeros(frmLen, N, M)   100×2×2

- I VO MATLAB PRIMEROT GENERIRANJETO NA ~~RAYLEIGH~~ FEDINGOT I ŠUMOT E VO EbN₀ ~~JELZATA~~



$\boxed{EbN_0 = 0:2:20}$

for Idx = 1 : length(EbNo)
  for packetIdx = 1 : numPackets
  end
end

$\boxed{size(tx2) = 100 \times 2}$ $\quad tx2 = \begin{bmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \\ - & - \\ - & - \end{bmatrix}$

H(1:N:end, :, :) = (randn(frmLen/2, N, M) + j randn(frmLen/2, N, M))

H(2:N:end, :, :) = H(1:N:end, :, :)

$\boxed{r_{11} = H(:,1,1).*tx + 10^{-0.05 EbN_0} \cdot n}$

TOTAL TRANSMIT POWER IS CONSTANT

$\boxed{r_{21} = \mathrm{sum}(H(:,:,1).*tx2, 2)\frac{1}{\sqrt{N}} + 10^{-0.005 EbN_0} \cdot n}$

ALAMOUTI

EQUIVALENT

$H(:,:,1) = \begin{bmatrix} h_{11}^1 & h_{12}^1 \\ h_{11}^2 & h_{12}^2 \\ h_{11}^3 & h_{12}^3 \\ h_{11}^4 & h_{12}^4 \\ - & - \end{bmatrix} = \begin{bmatrix} h_1^{(1)} & h_2^{(1)} \\ h_1^{(1)} & h_2^{(1)} \\ h_1^{(2)} & h_2^{(2)} \\ h_1^{(2)} & h_2^{(2)} \\ \cdots \end{bmatrix}$

$H(:,:,1).*tx_2 = \begin{bmatrix} h_1^{(1)} & h_2^{(1)} \\ h_1^{(1)} & h_2^{(1)} \\ h_1^{(2)} & h_2^{(2)} \\ h_1^{(2)} & h_2^{(2)} \end{bmatrix} \begin{bmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \\ x_3 & x_4 \\ -x_4^* & x_3 \end{bmatrix}$

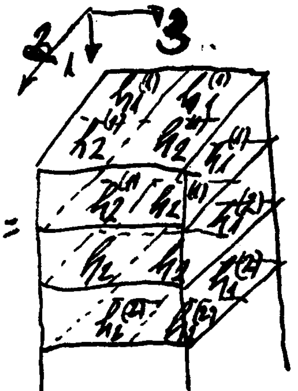
$\boxed{r_{12}(:,i) = H(:,1,i).*tx \qquad i = 1:M}$ MRC

- FRONT-END COMBINERS

z_{21-1} = r21(1:N:end).* conj(H(hidx,1,1)) + conj(r21(2:N:end)).*H(hidx,2,1)

z_{21-2} = r21(1:N:end).*conj(H(hidx,2,1)) conj(r21(2:N:end)).* H(hidx,1,1))

$\boxed{hidx = 1:N:length(H)}$

$H=$ 

$$tx2 = \begin{bmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \\ x_3 & x_4 \\ -x_4^* & x_3^* \\ --- & \end{bmatrix}$$

$$\boxed{\begin{aligned} r_{21}^1 &= h_1^1 \cdot x_1 + h_2^1 \cdot x_2 \\ r_{21}^2 &= -h_1^1 \cdot x_2^* + h_2^1 x_1^* \end{aligned}}$$

ALAMOUTI

sum(H(:,:,1).*tx2,2)

$$r_{21}(1:N:end) = \begin{bmatrix} r_{21}^1 \\ r_{21}^2 \\ r_{21}^3 \\ \vdots \end{bmatrix} \text{frmlen}$$

$$H(hidx,1,1) = \begin{bmatrix} h_1^1 \\ h_1^2 \\ h_1^3 \\ \vdots \end{bmatrix} \text{frmlen} \qquad H(hidx,2,1) = \begin{bmatrix} h_2^1 \\ h_2^2 \\ h_2^3 \\ \vdots \end{bmatrix}$$

OVA E ANALOGNO NA FORMULITE OD DUPLOA NA 11. 26/27

$$\boxed{z_{21-1}^1 = r_{21}^1 \cdot h_1^{1*} + r_{21}^{2*} \cdot h_2^1 \quad ; \quad z_{21-2}^1 = r_{21}^1 \cdot h_2^{1*} - r_{21}^{2*} \cdot h_1^1}$$

$$\cancel{z_{21-1}^1 + z_{21-2}^1 = r_{21}^1 (h_1^{1*} + h_2^{1*}) + r_{21}^{2*} (h_1^1 + h_2^1) =}$$

$$\cancel{= (h_1^1 x_1 + h_2^1 x_2)(h_1^{1*} + h_2^{1*}) + (-h_1^1 x_2^* + h_2^1 x_1^*)^* (h_1^1 + h_2^1) =}$$

$$\cancel{= |h_1^1|^2 x_1 + h_1^1 h_2^{1*} x_1 + h_2^1 h_1^{1*} x_2 + |h_2^1|^2 x_2 - |h_1^1|^2 x_2 - h_1^{1*} h_2^1 x_2 +}$$

$$\cancel{+ h_1^1 h_2^{1*} x_1 + |h_2^1|^2 x_1}$$

$$\cancel{z_{21-1}^1 = |h_1^1|^2 x_1 + h_1^1 h_2^1 x_1 + h_2^1 h_1^1 x_2 + |h_2^1|^2 x_2 -}$$

$$\cancel{- |h_1^1|^2 x_2 - h_1^{1*} h_2^1 x_2 + h_1^1 h_2^1 x_1 + |h_2^1|^2 x_1}$$

$$\boxed{z_{21}^1(1:N:end) = z_{21-1}^1 \qquad z_{21}^1(2:N:end) = z_{21-2}^1}$$

DOKAZ!!!

$$z_{21-1}^1 = (h_1^1 x_1 + h_2^1 x_2)(h_1^{1*} + (-h_1^1 x_2^* + h_2^1 x_1^*)^* h_2^1 =$$

$$= |h_1^1|^2 x_1 + \cancel{h_2^1 h_1^{1*} x_2} - \cancel{h_1^{1*} h_2^1 x_2} + |h_2^1|^2 \cdot x_1 = \left(|h_1^1|^2 + |h_2^1|^2\right) x_1$$

$$z_{21-2}^2 = (h_1^1 x_1 + h_2^1 x_2) \cdot h_2^{1*} + (-h_1^1 x_2^* + h_2^1 x_1^*)^* h_1^1 =$$

$$= \cancel{h_1^1 h_2^{1*} x_1} + |h_2^1|^2 x_2 + |h_1^1|^2 x_2 - \cancel{h_2^{1*} h_1^1 x_1} = \left(|h_1^1|^2 + |h_2^1|^2\right) x_2$$

- FORMULITE NA 11 26/27 SE POMNOZENI ZOSTO DELAT SO $|h_1|^2 + |h_2|^2$!!!

• HADAMARD MATRIX

$H' * H = n I$; $n \times n = size(H)$

eg. hadamard(4) =

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

→ VAKA SE GENERIRA EQUALIZER BIRANATA. $\hat{G}$ = IMPULSNOT ODZIV NA KANALOT.

SE KORISTI ZA ORTHOGONAL PILOTS !!! ...

$\boxed{h_e = r' \cdot pilots \quad r = h \cdot tx + n \quad tx = [pilots; t]}$

⊡ Intro MIMO Systems P2 (G2-coded 2x2 System)

W = hadamar(plen)

plen = 8 - NUMBER OF PILOT SYMBOLS PER FRAME

frmLen = 100

maxNumPackets = 2000

$\boxed{\text{pilots} = W(:, 1:N);}$ → estimated CHANNEL

H = zeros(plen + frmLen, M); H_e = zero(frmLen, N, M);

Z_e = zeros(frmLen, M); Z1_e = zero(frmLen/N, M);

Z2_e = Z1_e; Z = Z_e; Z1 = Z1_e; Z2 = Z2_e;

$\boxed{H(1,:,:) = (randn(N,M) + 1i * randn(N,M)) / sqrt(2)}$

- HOLD IT CONSTANT FOR WHOLE FRAME AND PILOT SYMBOLS.

H(ones(plen + frmLen, 1), :, :)

• CODE RATES:
- CODE RATE OF CONVOLUTIONAL CODE

$\boxed{\text{k/n - FOR EVERY "k" bits of useful THE CODER GENERATES TOTAL "n" bits OF DATA, of WHICH "n-k" ARE REDUNDANT}}$

1/2 - code: one redundant bit is inserted after every SINGLE BIT

2/3 - code: one redundant bit is inserted after every second bit

3/4 - code: 1 redundant after every third & etc.

- ORTHOGONAL SPACE TIME CODING FO 4 ANTENNAS (RATE HALF CODE)

$$C_{4,1/2} = \begin{bmatrix} c_1 & c_2 & c_3 & c_4 \\ -c_2 & c_1 & -c_4 & c_3 \\ -c_3 & c_4 & c_1 & -c_2 \\ -c_4 & -c_3 & c_2 & c_1 \\ c_1^* & c_2^* & c_3^* & c_4^* \\ -c_2^* & c_1^* & -c_4^* & c_3^* \\ -c_3^* & c_4^* & c_1^* & -c_2^* \\ -c_4^* & -c_3^* & c_2^* & c_1^* \end{bmatrix}$$

FROM WIKIPEDIA

3080178

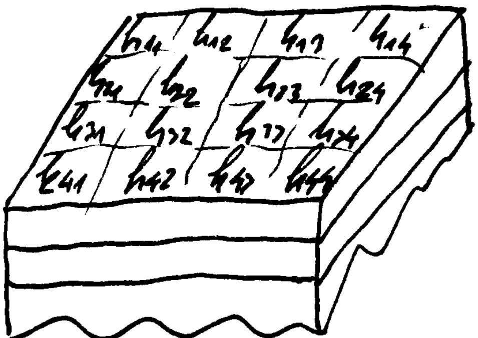
□ MATLAB SCRIPT . ostbc4m.m

N = 4; - NUMBER OF TRANSMIT ANTENNAS

rate = 0.5; inc = N/rate = 8 ; repFactor = 8

$$z_1 = r_1 \cdot h_1^* + r_2 \cdot h_2^* + r_3 \cdot h_3^* + r_4 h_4^* + r_5^* h_1 + r_6^* h_2 + r_7^* h_3 + r_8^* h_4 ;$$
$$z_2 = r_1 \cdot h_2^* - r_2 h_1^* - r_3 h_4^* + r_4 h_3^* + r_5^* h_2 - r_6^* h_1 - r_7^* h_4 + r_8^* h_3 ;$$
$$z_3 = r_1 h_3^* + r_2 h_4^* - r_3 h_1^* - r_4 h_2^* + r_5^* h_3 + r_6^* h_4 - r_7^* h_1 - r_8 h_2 ;$$
$$z_4 = r_1 h_4^* - r_2 h_3^* + r_3 h_2^* - r_4 h_1^* + r_5^* h_4 - r_6^* h_3 + r_7^* h_2 - r_8^* h_1 ;$$

4xM STBC COMBINER IMPLEMENTATION

H = 

length(EbNo) = 10

wb = 100/length(EbNo) = 10

wb = wb + 100/length(EbNo)

- VO SCRIPTATA jsImplementationP3.m SE REALIZI-RANI SITE MIMO SYSTEMI ELABORIRANI VO MATLAB. TAA NEMAM FUNKCII KOI MOŽEŠ DA GI KORISTIŠ ZA GENERIRANJE UROVREDA.

□ LEVEL CROSSING RATE → POSITIVE GOING LEVEL CROSSING RATE

- EXPECTED RATE AT WHICH RAYLEIGH FADING ENVELOPE CROSSES SPECIFIC LEVEL IN POSITIVE-GOING DIRECTION. NUMBER OF LEVEL CROSSINGS PER SECOND IS:

$$N_R = \int_0^\infty \dot{r}\, p(R, \dot{r})\, d\dot{r} = \sqrt{2\pi} f_m \rho e^{-\rho^2}$$

EXAMPLE 4.6

RAYLEIGH FADING SIGNAL $f_m = 20Hz$

$$N_R = \sqrt{2\pi} f_m \rho e^{-\rho^2} \qquad \rho = \frac{R}{R_{rms}} = 1$$

$$N_R = \sqrt{2\pi}\cdot 20 \cdot 1 \cdot e^{-1} = \underline{\underline{18.4427}}$$

$f_c = 900 MHz$ $\boxed{070200170}$ ← EXIČO

- GO IMPLEMENTIRAV VO MATLAB SO KORISTENJE NA rayleighchan()

NATASA BENDESKA

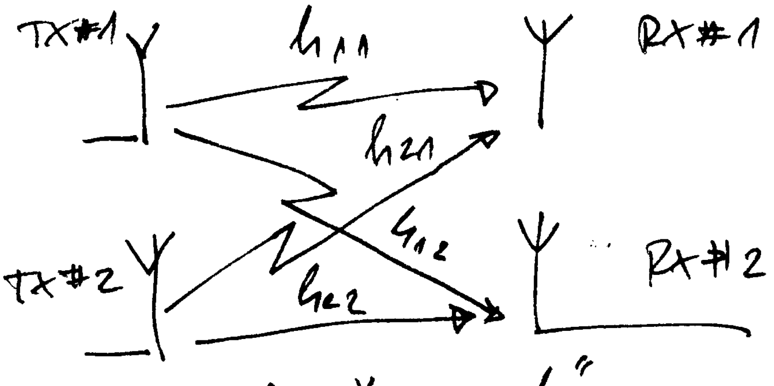
$\boxed{3090600}$

ZA $\rho = 1$ DOBIV $N_R = 14$

ZA $\rho = 1/\sqrt{2}$ DOBIV $N_R = 20$

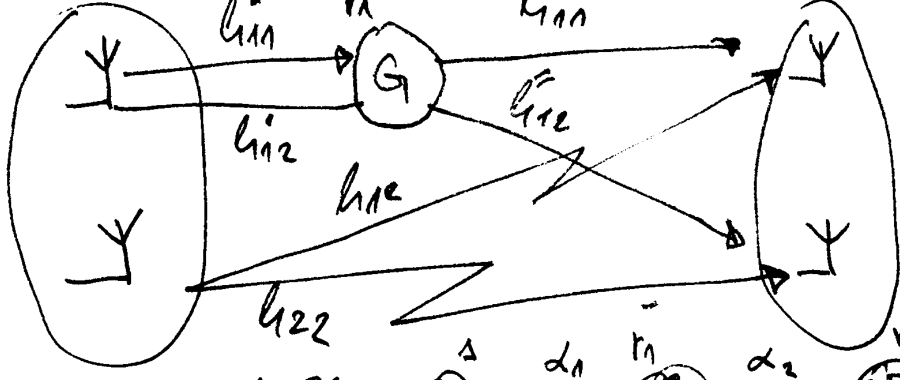
☐ ICTI 2010

- COMBINATION MIMO + MULTIHOP
- COMBINATION MULTIPATH + MULTIHOP



$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\boxed{\begin{aligned} y_1 &= h_{11}x_1 + h_{12}x_2 \\ y_2 &= h_{21}x_1 + h_{22}x_2 \end{aligned}}$$



DALI IMA REZON?

• POVTORUVANJE

$$r_1 = \alpha_1 s \sqrt{E_1} + n_1$$

$$r_2 = \sqrt{E_2}\,\alpha_2 \cdot (r_1 \cdot G) + n_2 = \sqrt{E_2}\cdot\alpha_2\cdot\alpha_1\cdot\sqrt{E_1}\cdot G\cdot s + G\sqrt{E_2}\alpha_2\cdot n_1 + n_2$$

$$r_2 = \sqrt{\epsilon_1 \epsilon_2}\,\alpha_1 \alpha_2 \cdot G \cdot s + G\sqrt{\epsilon_2}\,\alpha_2 \cdot n_1 + n_2$$

$$\cancel{\gamma_{eq} = \frac{\sqrt{\epsilon_1\epsilon_2}\,\alpha_1\alpha_2 \cdot G}{G\sqrt{\epsilon_2}\,\alpha_2 \cdot n_1 + n_2}}$$

$$r_1 = \alpha_1\sqrt{\epsilon_1}\cdot s + n_1$$

$$\boxed{E(s^2) = 1}$$

$$\boxed{E(n_1) = N_1}$$

$$\gamma_1 = \frac{\alpha_1^2 \epsilon_1^2}{N_1} \qquad \gamma = \frac{\alpha^2 \epsilon_b}{N_0} \qquad \bar{\gamma} = E(\alpha^2)\cdot\frac{\epsilon_b}{N_0}$$

$$\boxed{\bar{\gamma} = \Omega \cdot \frac{\epsilon_b}{N_0}}$$

$$\gamma_{eq} = \frac{\epsilon_1^2 \cdot \epsilon_2 \cdot \alpha_1^2 \cdot \alpha_2^2 \cdot G^2}{G^2 \cdot \epsilon_2 \cdot \alpha_2^2 \cdot N_1 + N_2}$$

$$\boxed{E(n_2) = N_2}$$

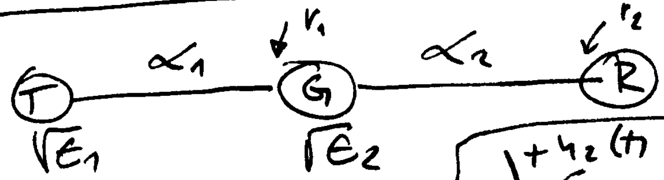
$$\gamma_{eq} = \frac{\frac{\epsilon_1 \cdot \epsilon_2 \alpha_1^2 \cdot \alpha_2^2}{N_1 \cdot N_2}\cdot G^2}{G^2 \frac{\epsilon_2 \alpha_2^2}{N_2} + \frac{1}{N_1}} = \frac{\gamma_1 \cdot \gamma_2 \cdot G^2}{G^2 \gamma_2 + \frac{1}{N_1}}$$

$$\boxed{\gamma_{eq} = \frac{\gamma_1 \gamma_2}{\gamma_2 + \frac{1}{G^2 N_1}}}$$

$$G = \frac{1}{\sqrt{\epsilon_1}\,\alpha_1} \Rightarrow$$

$$\gamma_{eq} = \frac{\gamma_1 \gamma_2}{\gamma_2 + \frac{\alpha_1^2 \epsilon_1}{N_1}} = \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2}$$

INSTANTENOUS SNR

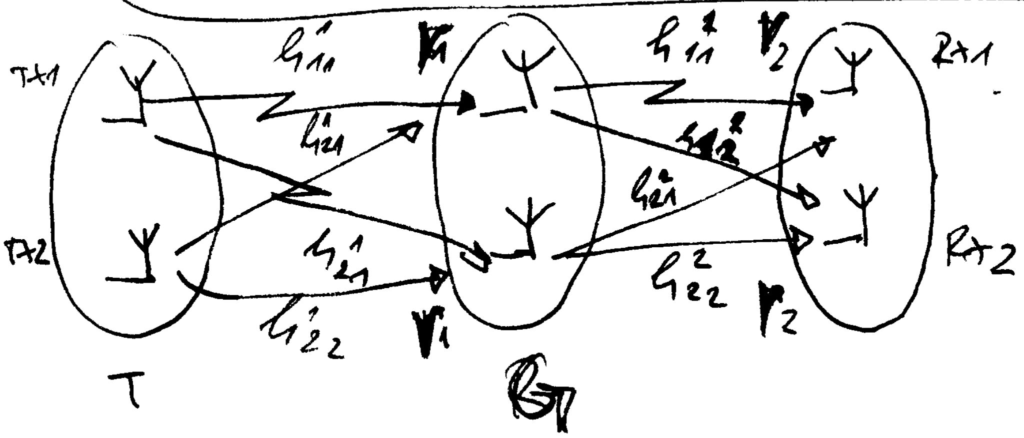


$$r_1 = \alpha_1 \cdot \sqrt{\epsilon_1}\cdot s + n_1(t)$$

$$r_2 = G \cdot \alpha_2 \cdot \left(\sqrt{\epsilon_2}\cdot r_1\right) + n_2(t)$$

$$\boxed{r_2 = \alpha_2 G \sqrt{\epsilon_2}\left(\alpha_1\sqrt{\epsilon_1}\cdot s + n_1\right) + n_2(t) = G\alpha_1\alpha_2\sqrt{\epsilon_1}\sqrt{\epsilon_2}\cdot s + G\alpha_2\sqrt{\epsilon_2}\cdot n_1 + n_2}$$

$$\boxed{10^{-0.05\,\epsilon_b N_0\,dB}\cdot n = \frac{n}{\left(10^{0.1\,\epsilon_b N_0\,dB}\right)^{1/2}} = \frac{n}{\sqrt{\epsilon_b N_0 L_n}}}$$



VARIABLE GAIN:

$$\boxed{G^2 = \frac{\epsilon_2}{\epsilon_1 \alpha_1^2 + N_{01}}}$$

$$\boxed{\begin{aligned} P_{e,BPSK} &= \frac{1}{2}\,e^{-\bar{\gamma}} \\ P_{e,RAYLEIGH} &= \frac{1}{2(\bar{\gamma}+1)} \end{aligned}}$$

• $P_b = ?$ FOR DUALHOP FIXGAIN 2030200

$$P_e = \frac{1}{\pi}\int_0^{\pi/2} M_\gamma\left(-\frac{a^2}{2\sin^2\theta}\right)d\theta = \int_0^\infty Q(a\sqrt{\gamma})\,p_\gamma(\gamma)\,d\gamma$$

BPSK: $P_b = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = Q\left(\sqrt{2\gamma}\right)$

$$P_e = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\gamma}\right) \qquad Q(z) = \frac{1}{2}\,\mathrm{erfc}\,\frac{z}{\sqrt{2}}$$

$$P_b = \frac{1}{2}\,\mathrm{erfc}\,\frac{\sqrt{\gamma}\cdot\sqrt{2}}{\sqrt{2}} = \frac{1}{2}\,\mathrm{erfc}\,\frac{\sqrt{2\gamma}}{\sqrt{2}} = Q\left(\sqrt{2\gamma}\right)$$

$\boxed{a = \sqrt{2}}$

$$\boxed{P_e = \frac{1}{\pi}\int_0^{\pi/2} M_\gamma\left(-\frac{1}{\sin^2\theta}\right)d\theta}$$

INTEGRAL TO BE SOLVED FOR BPSK

MGF FOR DUALHOP FIXGAIN SYST. WITH BPSK

$$\boxed{M_{\gamma_{eq}}(s) = \frac{1}{\bar{\gamma}_1 s+1} + \frac{C\bar{\gamma}_1\cdot s\cdot e^{\left(C/\bar{\gamma}_2(\bar{\gamma}_1 s+1)\right)}}{\bar{\gamma}_2\left(\bar{\gamma}_1 s+1\right)^2}\,E_1\left(\frac{C}{\bar{\gamma}_2\left(\bar{\gamma}_1 s+1\right)}\right)}$$

$$M_\gamma(s) = \frac{1}{1-\frac{\bar{\gamma}_1}{\sin^2\theta}} - \frac{C\cdot\bar{\gamma}_1\cdot e^{\frac{C}{\bar{\gamma}_2\left(-\frac{\gamma_1}{\sin^2\theta}+1\right)}}}{\sin^2\theta\cdot\bar{\gamma}_2\left(1-\frac{\gamma_1}{\sin^2\theta}\right)^2}\,E_1\left(\frac{C}{\bar{\gamma}_2\left(\cancel{1}-\frac{\bar{\gamma}_1}{\sin^2\theta}\right)}\right)$$

$\bar{\gamma}_1 = \bar{\gamma}_2 = \bar{\gamma}$

$$= \frac{\sin^2\theta}{\sin^2\theta-\bar{\gamma}} - \frac{C\cancel{\bar{\gamma}}\;e^{\frac{C\cdot\sin^2\theta}{\bar{\gamma}(\sin^2\theta-\bar{\gamma})}}}{\sin^2\theta\cancel{\bar{\gamma}}\left(\frac{\sin^2\theta-\bar{\gamma}_1}{\sin^2\theta}\right)^2}\cdot E_1\left(\frac{C\cdot\sin^2\theta}{\bar{\gamma}\left(\sin^2\theta-\bar{\gamma}\right)}\right)$$

$$= \frac{\sin^2\theta}{\sin^2\theta-\bar{\gamma}} - \frac{C\cdot\sin^{2}\theta}{\left(\sin^2\theta-\bar{\gamma}_1\right)^2}\exp\left(\frac{C\sin^2\theta}{\bar{\gamma}(\sin^2\theta-\bar{\gamma})}\right)E_1\left(\frac{C\sin^2\theta}{\bar{\gamma}(\sin^2\theta-\bar{\gamma})}\right)$$

$$\boxed{M_\gamma(s) = \frac{\sin^2\theta}{\sin^2\theta-\bar{\gamma}} - \frac{C\sin^2\theta}{\left(\sin^2\theta-\bar{\gamma}\right)^2}\exp\left(\frac{C\cdot\sin^2\theta}{\bar{\gamma}(\sin^2\theta-\bar{\gamma})}\right)E_1\left(\frac{C\sin^2\theta}{\bar{\gamma}(\sin^2\theta-\bar{\gamma})}\right)}$$

▣ MGF APPROACH OF CALCULATING BER (SIMON & ALOUINI) CHAPTER 1.1.3

$$\boxed{P_b(E/\gamma) = C_1 \exp(-a_1 \gamma)}$$

DPSK $\boxed{P_b(E/\gamma) = \frac{1}{2} e^{-\gamma}}$

$$P_b = \int_0^\infty P_b(E/\gamma)\, p_\gamma(\gamma)\, d\gamma = \int_0^\infty C_1 \exp(-a_1\gamma)\, p_\gamma(\gamma)\, d\gamma = \underline{\underline{C_1 M(-a_1)}}$$

AVERAGE BER OF DPSK IN RAYLEIGH FADING

DPSK $\boxed{P_b = 0.5 \cdot M(+1)}$

TOKMU KAKO ŠTO & KAZANO VO:
M.O. Hasna, A Performance Analysis of 2hop system

· BPSK $\boxed{P_b(E/\gamma) = \frac{1}{2} \operatorname{erfc}(\sqrt{\gamma})}$

– KONVERZIJA DTK SKRAR $\boxed{\hat{\beta} = \sqrt{2\gamma}}$ $\boxed{\operatorname{erfc} = \operatorname{erfc}/2}$

$$\hat{\beta} \gg 1 \quad P_b(E/\gamma) = \frac{1}{\hat{\beta}\sqrt{2\pi}} e^{-\hat{\beta}^2/2}$$

$$\boxed{P_b(E/\gamma) = \frac{1}{\sqrt{2\gamma}\sqrt{2\pi}} e^{-\gamma} = \frac{1}{2\sqrt{\pi\gamma}} e^{-\gamma}}$$

$$\cdot \quad P_b(E/\gamma) = \int_{\xi_1}^{\xi_2} c_2\, h(\xi) \exp(-a_2\, g(\xi)\, \gamma)\, d\xi$$

$$\frac{1}{2}\operatorname{erfc}(\sqrt{\gamma}) = \frac{1}{2}\frac{2}{\sqrt{\pi}} \int_{\sqrt{\gamma}}^{\infty} e^{-\frac{\gamma}{2}} d\gamma_1 = \frac{1}{\sqrt{\pi}} \int_{\sqrt{\gamma}}^{\infty} e^{-\frac{\gamma}{2}} d\gamma$$

$$\cdot \quad P_b(E/\gamma) = \int_{\xi_1}^{\xi_2} c_2\, h(\xi) \exp(-a_2 g(\xi)\gamma)\, d\xi \Rightarrow$$

$$P_b = \int_0^\infty P_b(E/\gamma)\, p_\gamma(\gamma)\, d\gamma = C_2 \int_{\xi_1}^{\xi_2} h(\xi)\, M_\gamma(-a_2 g(\xi))\, d\xi$$

$$P_b = C_2 \int_{\xi_1}^{\xi_2} h(\xi)\, M_\gamma\left(-a_2\, g(\xi)\right) d\xi$$

MMV

$$P_b(E/\gamma) = \int_{\xi_1}^{\xi_2} C_2\, h(\xi) \exp\left(-a_2\, g(\xi)\, \gamma\right) d\xi$$

BPSK $\quad P_b(E/\gamma) = \frac{1}{2}\operatorname{erfc}(\sqrt{\gamma}) = \frac{1}{\pi}\int_{\sqrt{\gamma}}^{\infty} e^{-\frac{x^2}{2}}\, dx$

$$P_b(E/\gamma) = Q(\sqrt{2\gamma}) = \frac{1}{\sqrt{2\pi}}\int_{\sqrt{2\gamma}}^{\infty} \exp\left(-\frac{x^2}{2}\right) dx = \frac{1}{\pi}\int_0^{\pi/2} \exp\left(-\frac{(\sqrt{2\gamma})^2}{2\sin^2\theta}\right) d\theta$$

$$P_b(E/\gamma) = \frac{1}{\pi}\int_0^{\pi/2} \exp\left(-\frac{\gamma}{\sin^2\theta}\right) d\theta$$

$C_2 = \frac{1}{\pi}$; $\quad h(\xi) = 1$;

$a_2 = +1 \quad g(\xi) = \frac{1}{\sin^2\theta}$.

$\xi_1 = 0 \quad \xi_2 = \pi/2$

$$\boxed{P_b = \frac{1}{\pi}\int_0^{\pi/2} M_\gamma\left(-\frac{1}{\sin^2\theta}\right) d\theta}$$

ISTOTO ŠTO DIREKTNO SE DOBIVA SO IZVOZE (5.3)

• ALAMOUTI COMBINER:

$$\boxed{yHat_1 = \frac{y_1 \cdot h_1^* + y_2^* h_2}{|h_1|^2 + |h_2|^2} \qquad yHat_2 = \frac{y_1 h_2^* - y_2^* h_1}{|h_1|^2 + |h_2|^2}}$$

3065355 BINA PETKOVSKA 48

ALAMOUTI

$$h1 = \begin{bmatrix} h_1 & h_1 & h_3 & h_3 & \cdots & h_{N-1} \\ h_2 & h_2 & h_4 & h_4 & \cdots & h_{N-1} \end{bmatrix} \qquad x = \begin{bmatrix} x_1 & -x_2^* & \cdots \\ x_2 & x_1^* & \cdots \end{bmatrix}$$

$$G = \frac{\bar{\gamma}}{\bar{\gamma}\cdot \alpha^2 + 1} = \frac{\bar{\gamma}}{\bar{\gamma}\cdot h^2 + 1} \Bigg\} \text{ VARIABLE GAIN}$$

$$y_1 = h_1 \cdot x_1 + h_2 \cdot x_2 \qquad y_2 = -h_1 \cdot x_2^* + h_2 x_1^*$$

$$\boxed{y_1Hat = y_1 \cdot h_1^* + y_2^* h_2; \quad y_2Hat = y_1 h_2^* - y_2^* h_1}$$

$$y_1Hat = \hat{y}_1 \qquad y_2Hat = \hat{y}_2$$

$\hat{y}_1 = (h_1x_1+h_2x_2)\cdot h_1^* + (-h_1x_2^*+h_2x_1^*)^* h_2 = |h_1|^2\cdot x_1 + \cancel{h_2h_1^*x_2} - \cancel{h_1^*h_2x_2} + h_2^* x_1 h_2 = |h_1|^2 x_1 + |h_2|^2 x_1 = (|h_1|^2+|h_2|^2) x_1$

$\hat{y}_2 = y_1\cdot h_2^* - y_2^* h_1 = (h_1x_1+h_2x_2)\cdot h_2^* - (-h_1x_2^*+h_2x_1^*)^*\cdot h_1 = \cancel{h_1h_2^*x_1} + |h_2|^2 x_2 - (-|h_1|^2 x_2 + \cancel{h_2^* h_1 x_1}) = (|h_1|^2+|h_2|^2) x_2$

$hA = sum(|\bar{h}_1|.^\wedge 2, 1) = h_1^2+h_2^2;\ h_1^2+h_2^2;\ h_3^2+h_4^2,\ h_3^2+h_4^2 \dots$

$\boxed{mean(hA) = 2}$ $\boxed{hA = sum(abs(\bar{h}_1).^\wedge 2, 1)}$

→ ZNAČI OD [illegible] 2 PATI POJAKEN SIGNAL SE DOBIVA NA IZLEZ NA KOMBAJNEROT.

~~mean~~ tt = reshape(x(:, 1:2:end), 1, N)

tt = reshape(x(:, 1:2:end), 1, N)

mean

ZVONKO	M
070234685	

$x = [1, 2, 3, 4]$

$P_x = x.^2 = [1, 4, 9, 16]$

$mean(P_x) = \frac{1+4+9+16}{4} = \frac{16+14}{4} = \frac{30}{4} = 7{,}5$

$y = \frac{x}{7{,}5} = \frac{1}{7{,}5}[1, 2, 3, 4]$

$y^2 = \left(\frac{1}{7.5}\right)^2 [1, 4, 9, 16] = P_y$

$mean[P_y] = \left(\frac{1}{7.5}\right)^2 \cdot 7.5 = \frac{1}{7.5}$ ⇒ TREBA DA DELAM SO sqrt(7.5)

• MISLAM DEKA NEMA REZON OD VAR G

$$y_1 = G_1 \cdot h_1 x_1 + G_2 h_2 \cdot x_2 \qquad y_2 = -G_1 h_1 \cdot x_2^* + G_2 h_2 \cdot x_1^*$$

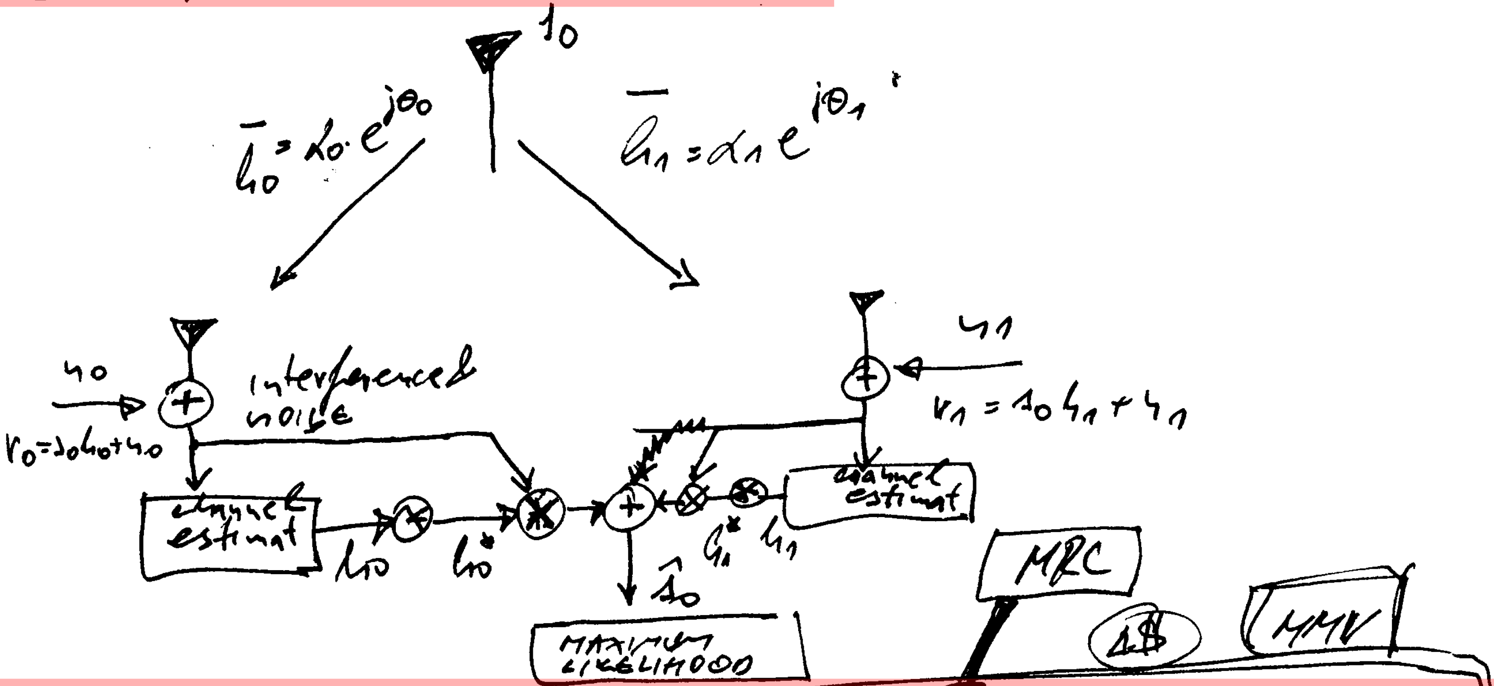
$$\hat{y}_1 = \frac{y_1 h_1^* + y_2^* h_2}{k} = \frac{(G_1 h_1 x_1 + G_2 h_2 x_2) \cdot h_1^* + (-G_1 h_1 x_2^* + G_2 h_2 x_1^*)^* h_2}{k}$$

$$\hat{y}_1 = \frac{1}{k}\left(G_1 |h_1|^2 x_1 + G_2 h_2 h_1^* \cdot x_2 - G_1^* h_1^* x_2 h_2 + G_2^* |h_2|^2 x_1 \right)$$

$$= \frac{1}{k}\left[(G_1 |h_1|^2 + G_2^* |h_2|^2) x_1 + \underbrace{(G_2 h_1^* h_2 - G_1^* h_1^* h_2)}_{\to 0} x_2 \right]$$

⊙ A SIMPLE TRANSMIT DIVERSITY TECHNIQUE FOR WIRELESS COMMUNICATIONS (Siavash M. Alamouti)

I CLASICAL MRC SCHEME



$$\hat{s}_0 = r_0 \cdot h_0^* + r_1 \cdot h_1^* = h_0^*(s_0 h_0 + n_0) + h_1^*(s_0 h_1 + n_1) =$$
$$= |h_0|^2 s_0 + n_0 h_0^* + |h_1|^2 s_0 + n_1 h_1^* = (|h_0|^2 + |h_1|^2) s_0 + \underline{n_0 h_0^* + n_1 h_1^*}$$

$$d^2(x, y) = (x - y)(x^* - y^*) \quad (*\Delta)$$

$$d^2(r_0, h_0 s_i) + d^2(r_1, h_1 s_i) \le d^2(r_0, h_0 s_k) + d^2(r_1, h_1 s_k) \quad \forall i \neq k$$

$$(x - y)(x^* - y^*) = |x|^2 - x y^* - y x^* + |y|^2$$

$$\hat{s}_0 = (|h_0|^2 + |h_1|^2) s_0 + n_0 h_0^* + n_1 h_1^* =$$
$$= (\alpha_0^2 + \alpha_1^2) s_0 + h_0^* n_0 + h_1^* n_1$$

$$d^2(r_0, h_0 s_i) = (r_0 - h_0 s_i)(r_0^* - h_0^* s_i^*) =$$
$$= |r_0|^2 - r_0 h_0^* s_i^* - r_0^* h_0 s_i + |h_0 s_i|^2$$
$$d^2(r_1, h_1 s_i) = (r_1 - h_1 s_i)(r_1^* - h_1^* s_i^*) =$$
$$= |r_1|^2 - r_1 h_1^* s_i^* - r_1^* h_1 s_i + |h_1|^2 |s_i|^2$$
$$d^2(r_0, h_0 s_i) + d^2(r_1, h_1 s_i) = |r_0|^2 + |r_1|^2 - (r_0 h_0^* + r_1 h_1^*) s_i^* -$$
$$- (r_0^* h_0 + r_1^* h_1) s_i + (\alpha_0^2 + \alpha_1^2)|s_i|^2 =$$
$$= \underline{|r_0|^2 + |r_1|^2 - \hat{s}_0 s_i^* - \hat{s}_0^* s_i + (\alpha_0^2 + \alpha_1^2)|s_i|^2}$$
$$d^2(r_0, h_0 s_k) + d^2(r_1 h_1 s_k) = \underline{|r_0|^2 + |r_1|^2 - \hat{s}_0 s_k^* - \hat{s}_0^* s_k + (\alpha_0^2 + \alpha_1^2)|s_k|^2}$$
$$d^2(r_0, h_0 s_i) + d^2(r_1, h_1 s_i) \leq d^2(r_0, h_0 s_k) + d^2(r_1, h_1 s_k)$$
$|r_0|^2 + |r_1|^2$ - SE KRATI / OSTAJU SAMO

$$\boxed{(\alpha_0^2 + \alpha_1^2)|s_i|^2 - \hat{s}_0 s_i^* - \hat{s}_0^* s_i \leq (\alpha_0^2 + \alpha_1^2)|s_k|^2 - \hat{s}_0 s_k^* - \hat{s}_0^* s_k \quad \boxed{\forall i \neq k}}$$
MMV

- ML schema (ponavljanje od prije log)

$$\bar{Y} = \begin{bmatrix} y_1 & y_3 & y_5 & \cdots \\ y_2 & y_4 & y_6 & \cdots \end{bmatrix} \qquad F_{11} = \begin{bmatrix} 1 & 1 & 1 & \cdots \\ 1 & 1 & 1 & \cdots \end{bmatrix}$$

$$J_{11} = \mathrm{sum}\left(\begin{bmatrix} |y_1 - 1| & |y_3 - 1| & \cdots \\ |y_2 - 1| & |y_4 - 1| & \cdots \end{bmatrix}, 1\right) \qquad F_{1-1} = \begin{bmatrix} 1 & 1 & 1 & \cdots \\ -1 & -1 & -1 & \cdots \end{bmatrix}$$

$y_2^2 - 1^2$ (za kompleksni vaj euclidian distance) $\rightarrow y_{1R} + j y_{1I} - 1 - j0$

$(y_{1R} - 1)^2 - (y_{1I} + 0)^2$

$$J_{11} = \left[\, |y_1 - 1| + |y_2 - 1|,\ |y_3 - 1| + |y_4 - 1|, \cdots \right]$$

- simple euclidean distance

WIKIPED.

$A(x_1, y_1, z_1) \qquad B(x_2, y_2, z_2)$

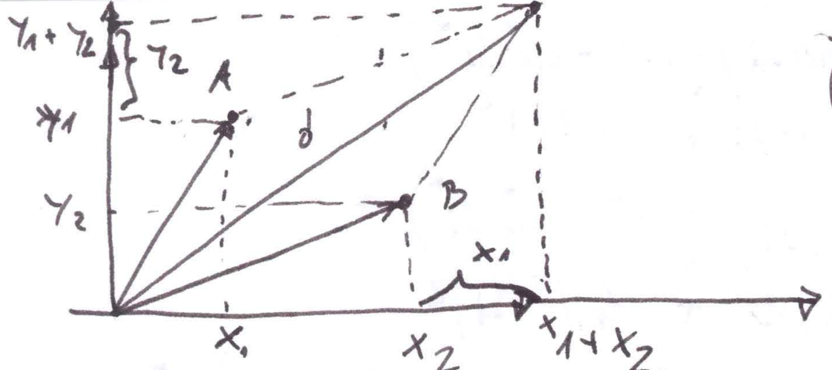
$$d(A, B) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z - z_1)^2}$$

$d(z_1, z_2) \qquad z_1 = x_1 + j y_1 \qquad z_2 = x_2 + j y_2$

$$d(z_1, z_2) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$d^2 \Rightarrow (x_1 + j y_1 - x_2 - j y_2)(x_1^* - j y_1^* - x_2^* + j y_2) =$$
$$= (x_1 - x_2 + j(y_1 - y_2))(x_1 - x_2 - j(y_1 - y_2)) =$$
$$= (x_1 - x_2)^2 + (y_1 - y_2)^2 = d^2(z_1, z_2)$$

DOKAZANO!



$$\boxed{d^2 = (x_1+x_2)^2 + (y_1+y_2)^2}$$

$$(\alpha_0^2+\alpha_1^2)|s_i|^2 - \hat{s}_0 s_i^* - \hat{s}_0^* s_i \le (\alpha_0^2+\alpha_1^2)|s_k|^2 - \hat{s}_0 s_k^* - \hat{s}_0^* s_k$$

$$d^2(\hat{s}_0, s_i) = (\hat{s}_0 - s_i)(\hat{s}_0^* - s_i^*) = |\hat{s}_0|^2 - \hat{s}_0 s_i^* - s_i \hat{s}_0^* + |s_i|^2$$

$$(\alpha_0^2+\alpha_1^2)|s_i|^2 + d^2(\hat{s}_0, s_i) + |s_0|^2 + |s_i|^2 =$$

$$= (\alpha_0^2+\alpha_1^2+1)|s_i|^2 + d^2(\hat{s}_0, s_i) + |s_0|^2$$

ĆE SE SKRATI!!!

$$\boxed{(\alpha_0^2+\alpha_1^2+1)|s_i|^2 + d^2(\hat{s}_0, s_i) \le (\alpha_0^2+\alpha_1^2+1)|s_k|^2 + d^2(\hat{s}_0, s_k) \quad \forall i \ne k}$$

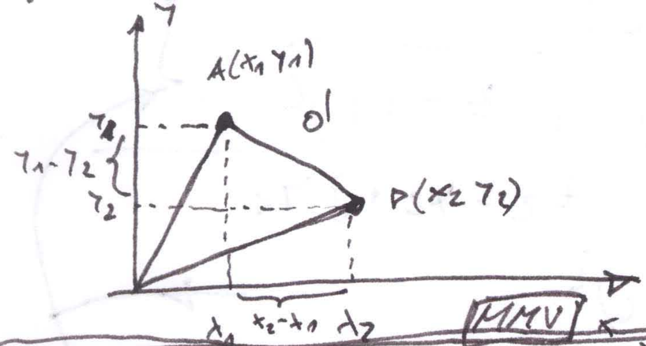
FOR PSK SIGNALS (EQUAL ENERGY CONSTELLATIONS)

$$\boxed{|s_i|^2 = |s_k|^2 = E_s \quad \forall i, k}$$

- HENCE **DECISION RULE** CAN BE SIMPLIFIED TO:

$$\boxed{d^2(\hat{s}_0, s_i) \le d^2(\hat{s}_0, s_k)}$$

→ AKO SE SEĆAVAŠ PRAVILO ZA ~~BPSK~~ ~~ZA~~ QAM (~~NE SE SEĆAVAŠ TOČNO~~) GO KORISTEŠE OVA ZA IMPLEMENTACIJA NA ~~PRIEMNIKOT~~ ODLUČUVAČOT VO PRIEMNIKOT. TOGAŠ NAJRAZBIRLIVO OBJAŠ GRAFIČKOTO TOLKUVANJE !!!



$$\boxed{d^2 = (x_2-x_1)^2 + (y_2-y_1)^2}$$

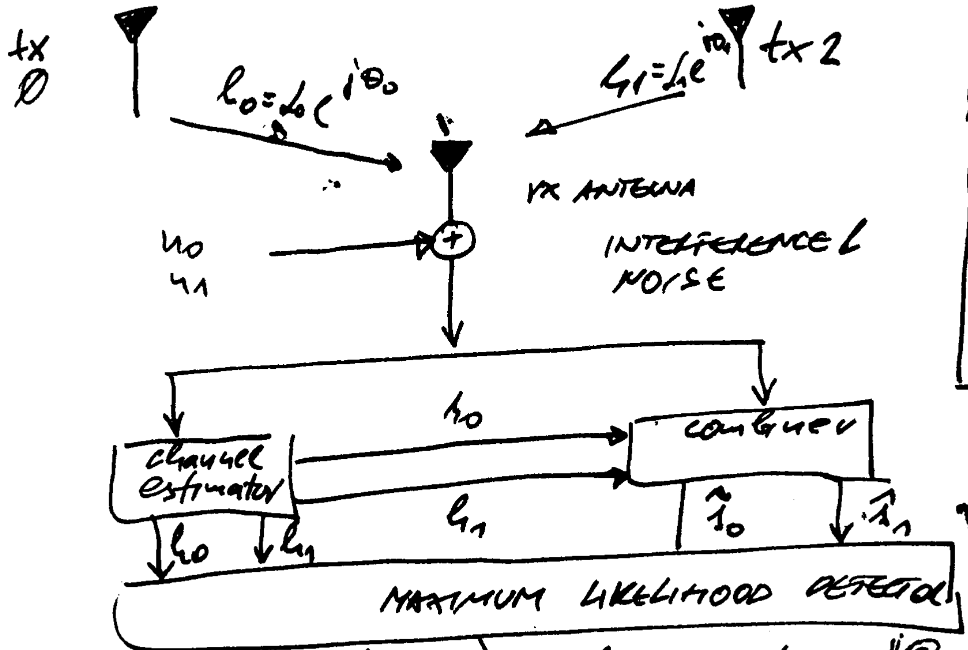
EUCLIDIAN DISTANCE

KOLKU d E POMALO TOLKU ESTIMIRANIOT SIMBOL E POBLIZOK NA ORIGINALNIOT!!

MMV ←

OD TIE PRIČINI ODLUČUVAČOT OSVEN MAXIMUM LIKELIHOOD MOŽE DA SE NAREČE MINIMUM EUCLIDIAN DISTANCE

TWO BRANCH TRANSMIT DIVERSITY WITH ONE RECEIVER



	ANTENNA 1	ANTENNA 2
time t	s_0	s_1
time t+T	$-s_1^*$	s_0^*

$$h_0(t) = h_0(t+T) = h_0 = \alpha_0 e^{j\theta_0}$$
$$h_1(t) = h_1(t+T) = h_1 = \alpha_1 e^{j\theta_1}$$

$$r_0 = h_0 s_0 + h_1 s_1 + n_0$$
$$r_1 = -h_0 s_1^* + h_1 s_0^* + n_1$$

$$\boxed{\begin{aligned}\hat{s}_0 &= h_0^* r_0 + h_1 r_1^* \\ \hat{s}_1 &= h_1^* r_0 - h_0 r_1^*\end{aligned}}$$

• COMBINING SCHEME

$$\hat{s}_0 = h_0^*(h_0 s_0 + h_1 s_1 + n_0) + h_1(-h_0 s_1^* + h_1 s_0^* + n_1)^* =$$
$$= |h_0|^2 s_0 + \cancel{h_0^* h_1 s_1} + h_0^* n_0 - \cancel{h_0^* h_1 s_1} + |h_1|^2 s_0 + h_1 n_1^*$$
$$\boxed{\hat{s}_0 = (\alpha_0^2 + \alpha_1^2) s_0 + h_0^* n_0 + h_1 n_1^*}$$

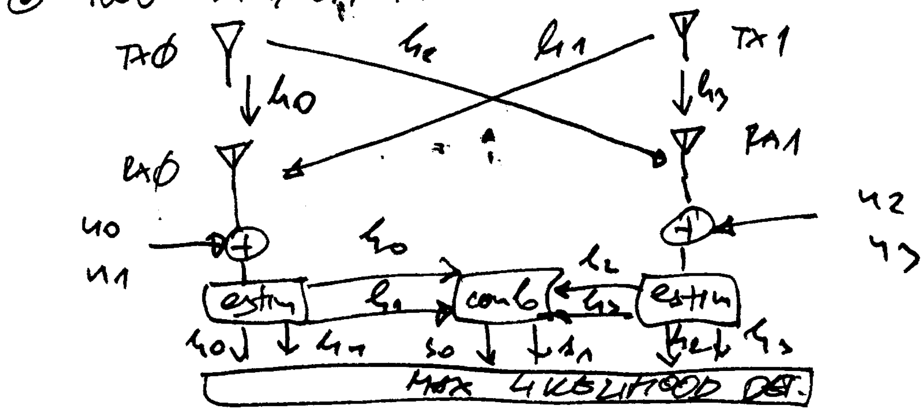
$$\hat{s}_1 = h_1^*(h_0 s_0 + h_1 s_1 + n_0) - h_0(-h_0^* s_1 + h_1^* s_0 + n_1^*) =$$
$$= \cancel{h_1^* h_0 s_0} + |h_1|^2 s_1 + h_1^* n_0 + |h_0|^2 - \cancel{h_0 h_1^* s_0} - h_0 n_1^*$$
$$\boxed{\hat{s}_1 = (\alpha_0^2 + \alpha_1^2) s_1 + h_1^* n_0 - h_0 n_1^*}$$

$\hat{s}_0, \hat{s}_1$ ARE THEN ~~SENT~~ TO MAXIMUM LIKELIHOOD DETECTOR

- COMPARE WITH MRC (48):

$$\hat{s}_0 = (\alpha_0^2 + \alpha_1^2) s_0 + h_0^* n_0 + h_1^* n_1$$

⊙ TWO-BRANCH TRANSMIT DIVERSITY WITH M RECEIVERS



	RXØ	RX1
TXØ	h_0	h_2
TX1	h_1	h_3

	RXØ	RX1
time t	r_0	r_2
time t+T	r_1	r_3

$$\boxed{\begin{aligned} r_0 &= h_0 s_0 + h_1 s_1 + n_0 \\ r_1 &= -h_0 s_1^* + h_1 s_0^* + n_1 \\ r_2 &= h_2 s_0 + h_3 s_1 + n_2 \\ r_3 &= -h_2 s_1^* + h_3 s_0^* + n_3 \end{aligned}}$$

	TX0	TX1
t	s_0	s_1
$t+T$	$-s_1^*$	s_0^*

	h_0	h_1
TX0	h_0	h_2
TX1	h_1	h_3

	RX0	RX1
t	r_0	r_2
$t+T$	r_1	r_3

$$\boxed{\begin{aligned} \hat{s}_0 &= h_0^* r_0 + h_1 r_1^* + h_2^* r_2 + h_3 r_3^* \\ \hat{s}_1 &= +h_1^* r_0 - h_0 r_1^* + h_3^* r_2 - h_2 r_3^* \end{aligned}}$$

POLOGIZMO E SO DUPLI INDEXI VIDI PP. 52

$$\hat{s}_0 = |h_0|^2 s_0 + \cancel{h_0^* h_1 s_1} + h_0^* n_0 - \cancel{h_1 h_0^* s_1} + |h_1|^2 s_0 + h_1 n_1^* +$$
$$+ |h_2|^2 s_0 + \cancel{h_2^* h_3 s_1} + h_2^* n_2 - \cancel{h_3 h_2^* s_1} + |h_3|^2 s_0 + h_3 n_3^*$$

$$\boxed{\hat{s}_0 = (\alpha_0^2 + \alpha_1^2 + \alpha_2^2 + \alpha_3^2)\, s_0 + h_0^* n_0 + h_1 n_1^* + h_2^* n_2 + h_3 n_3^*}$$

ANALOGNO:

$$\boxed{\hat{s}_1 = (\alpha_0^2 + \alpha_1^2 + \alpha_2^2 + \alpha_3^2)\, s_1 - h_0 n_1^* + h_1^* n_0 - h_2 n_3^* + h_3^* n_2}$$

$\hat{s}_0$ & $\hat{s}_1$ ARE SENT TO MAXIMUM LIKELIHOOD DETECTOR

choose s_i iff:

$$\boxed{d^2(\hat{s}_0, s_i) \leq d^2(\hat{s}_0, s_k)} \rightarrow \text{FOR PSK } \forall k \neq i$$

$$\boxed{\begin{aligned} &(\alpha_0^2 + \alpha_1^2 + \alpha_2^2 + \alpha_3^2)|s_i|^2 + d^2(\hat{s}_0, s_i) \leq \\ &\leq (\alpha_0^2 + \alpha_1^2 + \alpha_2^2 + \alpha_3^2)|s_k|^2 + d^2(\hat{s}_0, s_k) \quad \forall k \neq i \end{aligned}}$$

$\rightarrow$ ANY MODULATION

- FOR s_1:

$$\boxed{d^2(\hat{s}_1, s_i) \leq d^2(\hat{s}_1, s_k)} \rightarrow \text{FOR PSK } \forall k \neq i$$

$$\lambda = \left(\frac{f[\text{MHz}]}{300}\right)^{-1} = \frac{300}{2500} = \frac{3}{25} = \frac{3}{0.25} \cdot 10^{-2} = 12 \cdot 10^{-2} = \underline{0.12}$$

- SOFT FAILURE

$r_0 = h_0 \cdot s_0 + h_1 \cdot s_1 + n_0$ IF $h_1 = 0$ $\quad r_0 = h_0 \cdot s_0 + n_0$

$r_1 = -h_0 \cdot s_1^* + h_1 \cdot s_0^* + n_1$ $\quad r_1 = -h_0 \cdot s_1^* + n_1$

$$\hat{s}_0 = h_0^* r_0 + h_1 r_1^* = h_0^* r_0 = h_0^* h_0 s_0 + h_0^* n_0 = \alpha^2 s_0 + h_0^* n_0$$
$$\hat{s}_1 = -h_1^* r_0 + h_0 r_1^* = -h_0 r_1^* = +h_0 h_0^* s_1 - h_0 n_1^* = +\alpha_0^2 s_1 - h_0 n_1^*$$

$\underline{\text{noisePower}} = \text{sigPower} - \text{reqSNR}$ $\quad x = \text{randn}(1,N) + j\,\text{randn}(1,N)$

$\text{noisePower} = 2 - 10 = \underline{-8\,\text{dB}}$

$\boxed{wgn(x, N, -8\,dB)} \rightarrow$ AWGN ГО КОРИСТИ ОВА

- ВАРИЈ. 2x2x2

$$Y_1 = h_1 x_1 + h_2 x_2 \qquad \hat{Y}_1 = Y_1 h_1^* + Y_2^* h_2$$

$$Y_2 = -h_1 x_2^* + h_2 x_1^* \qquad \hat{Y}_2 = Y_1 h_2^* - Y_2^* h_1$$

$$\boxed{G = \frac{1}{h_1}} \qquad Y_1 = G_1 h_1 x_1 + G_2 h_2 \cdot x_2 = x_1 + x_2$$

$$Y_2 = -x_2^* + x_1^*$$

$$\hat{Y}_1 = (x_1 + x_2) h_1^* + (-x_2^* + x_1^*)^* h_2 = \underline{\underline{h_1^* x_1}} + \underline{h_1^* x_2} - \underline{h_2 x_2^*} + \underline{\underline{h_2 x_1}}$$

$$\hat{Y}_1 = (h_1^* + h_2) x_1 + (h_1^* - h_2) x_2$$

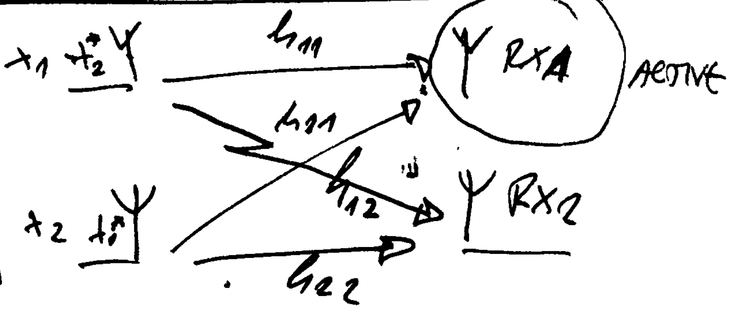
$$\boxed{G = \frac{EbN_0}{EbN_0(|h_1|^2 + |h_2|^2 + |h_3|^2 + |h_4|^2) + 1}}$$

2x2x2 ВАКА ЌЕ ГО ДЕФИНИРАМ G ЗА ДА ГО ИНВЕРТИРА КАНАЛОТ!!!

$$\boxed{G = \frac{EbN_0}{EbN_0(|h_1|^2 + |h_2|^2) + 1}}$$

- ЗА ЧЛАНОТ

$$\boxed{\begin{aligned} Y_1 &= h_{11} x_1 + h_{21} x_2 + n_1 \\ Y_2 &= -h_{11} x_2^* + h_{21} x_1^* + n_2 \end{aligned}}$$



$\boxed{2 \times 1 \times 1 \text{ SYSTEM}}$

$$\hat{Y}_1 = h_{11}^* Y_1 + h_{21} Y_2^*$$

$$\hat{Y}_2 = h_{21}^* Y_1 - h_{11} Y_2^*$$

$$\hat{Y}_1 = h_{11}^*(h_{11} x_1 + h_{21} x_2 + n_1) + h_{21}(-h_{11}^* x_2 + h_{21}^* x_1 + n_2^*)$$

$$\hat{Y}_1 = |h_{11}|^2 x_1 + \cancel{h_{11}^* h_{21} x_2} + h_{11}^* n_1 - \cancel{h_{21} h_{11}^* x_2} + |h_{21}|^2 x_1 + h_{21} n_2^*$$

$$\boxed{\hat{Y}_1 = (|h_{11}|^2 + |h_{21}|^2) x_1 + h_{11}^* n_1 + h_{21} n_2^*}$$

$$P_S = (|h_{11}|^2 + |h_{21}|^2)^2$$
$$P_N = |h_{11}|^2 + |h_{21}|^2$$

$$\hat{Y}_2 = h_{21}^*(h_{11} x_1 + h_{21} x_2 + n_1) - h_{11}(-h_{11}^* x_2 + h_{21}^* x_1 + n_2^*) =$$

$$= \cancel{h_{11} h_{21}^* x_1} + |h_{21}|^2 x_2 + h_{21}^* n_1 + |h_{11}|^2 x_2 - \cancel{h_{21}^* h_{11} x_1} - h_{11} n_2^*$$

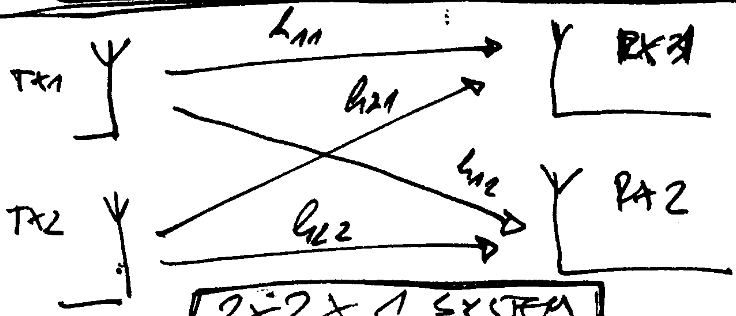
$$= \underline{(|h_{21}|^2 + |h_{11}|^2) x_2 + h_{21}^* n_1 - h_{11} n_2^*}$$

$$\boxed{r_2 = G \cdot r_1 + n}$$

$$\boxed{G = \sqrt{\frac{E}{E|h|^2 N_0}} = \sqrt{\frac{E}{E\cdot(|h_1|^2+|h_2|^2)+N_0}}}$$

STARA ŠEMA

	RX1	RX2
TX1	h_0	h_2
TX2	h_1	h_3



	TX1	TX2
t	x_1	x_2
$t+T$	$-x_2^*$	x_1^*

	RX1	RX2
t	y_{11}	y_{12}
$t+T_1$	y_{21}	y_{22}

$\boxed{2\times2\times1 \text{ SYSTEM}}$

	FIRST TIME SLOT	SECOND TIME SLOT
A1	$y_{11} = h_{11}x_1 + h_{21}x_2 + n_{11}$	$y_{21} = -h_{11}x_2^* + h_{21}x_1^* + n_{21}$
A2	$y_{12} = h_{12}x_1 + h_{22}x_2 + n_{12}$	$y_{22} = -h_{12}x_2^* + h_{22}x_1^* + n_{22}$

$\boxed{\text{MMV}}$

$$\hat{y}_1 = h_{11}^* y_{11} + h_{21} y_{21}^* + h_{12}^* y_{12} + h_{22} y_{22}^*$$

$$\hat{y}_2 = \underbrace{h_{21}^* y_{11} - h_{11} y_{21}^*}_{\text{FIRST ANTENNA}} + \underbrace{h_{22}^* y_{12} - h_{12} y_{22}^*}_{\text{SECOND ANTENNA}}$$

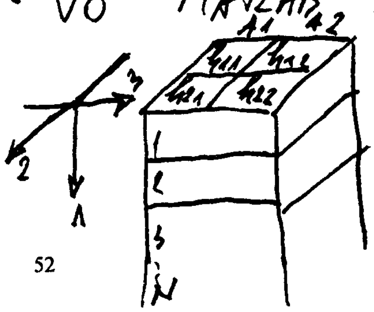
$$\hat{y}_1 = |h_{11}|^2 x_1 + h_{11}^* n_{11} + \cancel{h_{11}^* h_{21} x_2} - \cancel{h_{21} h_{11}^* x_2} + |h_{21}|^2 x_1 + h_{21} n_{21}^* +$$
$$+ |h_{12}|^2 x_1 + \cancel{h_{12}^* h_{22} x_2} + h_{12}^* n_{12} - \cancel{h_{22} h_{12}^* x_2} + |h_{22}|^2 x_1 + h_{22} n_{22}^*$$

$$\boxed{\hat{y}_1 = (|h_{11}|^2 + |h_{12}|^2 + |h_{21}|^2 + |h_{22}|^2)x_1 + \underset{0}{h_{11}^* n_{11}} + \underset{1}{h_{21} n_{21}^*} + \underset{2}{h_{12}^* n_{12}} + \underset{3}{h_{22} n_{22}^*}}$$

$$\hat{y}_2 = \cancel{h_{21}^* h_{11} x_1} + |h_{21}|^2 x_2 + h_{21}^* n_{11} + |h_{11}|^2 x_2 - \cancel{h_{11} h_{21}^* x_1} - h_{11} n_{21}^* +$$
$$\cancel{h_{22}^* h_{12} x_1} + |h_{22}|^2 x_2 + h_{22}^* n_{12} + |h_{12}|^2 x_2 - \cancel{h_{12} h_{22}^* x_1} - h_{12} n_{22}^*$$

$$\boxed{\hat{y}_2 = (|h_{11}|^2 + |h_{12}|^2 + |h_{21}|^2 + |h_{22}|^2)x_2 + \underset{1\ 0}{h_{21}^* n_{11}} - \underset{0\ 1}{h_{11} n_{21}^*} + \underset{3\ 2}{h_{22}^* n_{12}} - \underset{1\ 3}{h_{12} n_{22}^*}}$$

- Vo MATLAB, MATRICATA h ZA FRAVAM VAKA:



$$\begin{array}{r} 34 \\ -15 \\ \hline 22 \end{array} \qquad \begin{array}{r} 37 \\ -21 \\ \hline \boxed{16} \end{array}$$

$$\boxed{\gamma_{eq}^{-1} = \sum_{n=1}^{N} \frac{1}{\prod_{t=1}^{n-1} N_{0t} \prod_{t=1}^{N-1} G_t^2 \prod_{t=1}^{n} \gamma_t}}$$

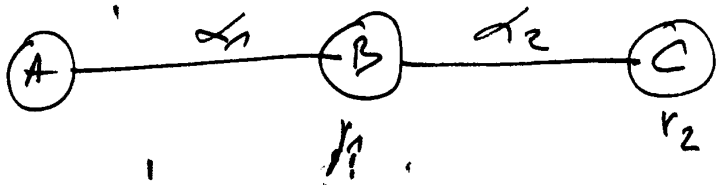
$$\gamma_{eq}^{-1} = \frac{1}{\gamma_1} + \frac{1}{N_1 \cdot G \cdot \gamma_1 \cdot \gamma_2}$$

$$\gamma_{eq} = \frac{1}{\frac{1}{\gamma_1} + \frac{1}{G\gamma_1\gamma_2}} = \frac{1}{\frac{G\gamma_2 + 1}{G\gamma_1\gamma_2}} = \frac{G\cdot\gamma_1\gamma_2}{G\cdot\gamma_2 + 1}$$

$$\boxed{\gamma_{eq} = \frac{\gamma_1\gamma_2}{\gamma_2 + \frac{1}{G}}} \Longrightarrow \text{Fixed GAIN}$$

• VARIABLE GAIN

$$\gamma_{eq} = \frac{\gamma_1\gamma_2}{\gamma_1 + \gamma_2 + 1} \qquad G = \sqrt{\frac{E_2}{E_1\alpha^2 + N_0}}$$



$$r_1 = \sqrt{E_1}\cdot\alpha_1\cdot s(t) + n_1(t) \ ; \qquad r_2 = G\cdot\sqrt{E_2}\,\alpha_2\, r_1 + n_2(t)$$

$$r_2 = G\sqrt{E_2}\,\alpha_2\left(\sqrt{E_1}\,\alpha_1\cdot s(t) + n_1(t)\right) + n_2(t) =$$

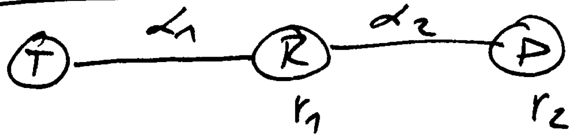
$$= G\,\alpha_1\alpha_2\sqrt{E_1}\sqrt{E_2}\cdot s(t) + G\sqrt{E_2}\,\alpha_2\cdot n_1(t) + n_2(t)$$

$$\gamma_{eq} = \frac{G^2\cdot\alpha_1^2\cdot\alpha_2^2\cdot E_1\cdot E_2}{G^2 E_2\cdot\alpha_2^2\cdot N_{01} + N_{02}} = \frac{G^2\alpha_1^2\alpha_2^2 E_1 E_2}{N_{01}N_{02}\left(\frac{G^2E_2\alpha_2^2}{N_{02}} + \frac{1}{N_{01}}\right)}$$

$$\boxed{\gamma_{eq} = \frac{\frac{E_1\alpha_1^2}{N_{01}}\cdot\frac{E_2\alpha_2^2}{N_{02}}}{\frac{E_2\alpha_2^2}{N_{02}} + \frac{1}{G^2N_{01}}} = \frac{\gamma_1\cdot\gamma_2}{\gamma_2 + \frac{1}{G^2\cdot N_{01}}}}$$

$$G^2 = \frac{\varepsilon_2}{\varepsilon_1 \cdot \alpha_1^2 + N_{01}} \qquad \gamma_{eq} = \frac{\gamma_1 \gamma_2}{\gamma_2 + \frac{1}{G^2 \cdot N_{01}}}$$

$$\gamma_{eq} = \frac{\gamma_1 \gamma_2}{\gamma_2 + \frac{1}{\frac{\varepsilon_2 N_{01}}{\varepsilon_1 \alpha_1^2 + N_{01}}}} =$$



$$r_1 = \sqrt{\varepsilon_1} \cdot \alpha_1 \cdot s(t) + n_1(t) \qquad r_2 = \alpha_2 \cdot G \, \alpha_2 \cdot r_1 + n_2(t)$$

$$r_2 = \alpha_2 G \left(\sqrt{\varepsilon_1} \alpha_1 s(t) + n_1(t) \right) + n_2(t)$$

$$r_2 = \alpha_1 \alpha_2 \cdot G \cdot \sqrt{\varepsilon_1} s(t) + \alpha_2 G \, n_1(t) + n_2(t)$$

$$\boxed{\gamma_{eq} = \frac{\alpha_1^2 \alpha_2^2 G^2 \cdot \varepsilon_1}{\alpha_2^2 \cdot G^2 N_{01} + N_{02}}} = \frac{G^2 \cdot \frac{\varepsilon_1 \alpha_1^2}{N_{01}} \cdot \frac{\alpha_2^2}{N_{02}}}{\frac{\alpha_2^2 G^2}{N_{02}} + \frac{1}{N_{01}}} =$$

$$= \frac{\frac{\varepsilon_1 \alpha_1^2}{N_{01}} \frac{\alpha_2^2}{N_{02}}}{\frac{\alpha_2^2}{N_{02}} + \frac{1}{G^2 N_{01}}} = \qquad G^2 = \frac{\varepsilon_2}{\varepsilon_1 \alpha_1^2 + N_{01}}$$

$$= \frac{\frac{\varepsilon_1 \alpha_1^2}{N_{01}} \frac{\alpha_2^2}{N_{02}}}{\frac{\alpha_2^2}{N_{02}} + \frac{1}{\frac{\varepsilon_2 N_{01}}{\varepsilon_1 \alpha_1^2 + N_{01}}}} = \frac{K}{\frac{\alpha_2^2}{N_{01}} + \frac{\varepsilon_1 \alpha_1^2 + N_{01}}{\varepsilon_2 N_{01}}} =$$

$$= \frac{\frac{\varepsilon_1 \alpha_1^2}{N_{01}} \frac{\alpha_2^2}{N_{02}}}{\frac{1}{\varepsilon_2} \left[\frac{\varepsilon_2 \alpha_2^2}{N_{01}} + \frac{\varepsilon_1 \alpha_1^2}{N_{01}} + 1 \right]} = \frac{\frac{\varepsilon_1 \alpha_1^2}{N_{01}} \frac{\varepsilon_2 \alpha_2^2}{N_{02}}}{\frac{\varepsilon_2 \alpha_2^2}{N_{01}} + \frac{\varepsilon_1 \alpha_1^2}{N_{01}} + 1}$$

$$\boxed{\gamma_{eq} = \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2 + 1}} \qquad \boxed{G^2 = \frac{\varepsilon_2}{\varepsilon_1 \alpha_1^2 + N_{01}}}$$

• OUTAGE PROBABILITY OF 2×1×1 SYSTEM

$$\gamma_1 = \frac{\epsilon \cdot (|h_{11}|^2 + |h_{21}|^2)^2}{\cancel{(|h_{11}|^2 + |h_{21}|^2)} N_0} = \frac{P_{S1}}{P_{N1}}$$

$$r_2 = G \cdot |h_2|^* \underbrace{\left(\sqrt{\epsilon}\left(|h_{11}|^2 + |h_{21}|^2\right) x_1 + h_{11}^* u_1 + h_{21} u_2\right)}_{\hat{y}_1} + w_{u2}(t)$$

$$r_2 = G \cdot h_{12}^* \left(\sqrt{\epsilon}\left(|h_{11}|^2 + |h_{21}|^2\right)\right) x_1 + G \cdot h_{12}^* \cdot \sqrt{P_{N1}} + \underline{\underline{w_{u2}(t)}}$$

$$\gamma_2 = \frac{P_{S2}}{P_{N2}} = \frac{\epsilon \cdot G^2 |h_2|^2 \left(|h_{11}|^2 + |h_{21}|^2\right)^2}{G^2 |h_2|^2 \cdot P_{N1} N_0 + N_0} = \frac{\epsilon G^2 \cdot |h_2|^2 \mathring{\Delta}_1^2}{G^2 \cdot |h_2|^2 \cdot \mathring{\Delta}_1 + 1}$$

• OUTAGE PROBABILITY OF 2×2×1 SYSTEM

$$\boxed{h_1 Abs = |h_{11}|^2 + |h_{12}|^2 + |h_{21}|^2 + |h_{22}|^2}$$

$$\gamma_1 = \frac{\epsilon \left(|h_{11}|^2 + |h_{12}|^2 + |h_{21}|^2 + |h_{22}|^2\right)^2}{N_0 \cancel{\left(|h_{11}|^2 + |h_{21}|^2 + |h_{12}|^2 + |h_{22}|^2\right)}} = \frac{\epsilon \cdot \mathring{\Delta}_2}{N_0}$$

$$r_2 = G \cdot h_2 \left(\sqrt{\epsilon}\left(|h_{11}|^2 + |h_{12}|^2 + |h_{21}|^2 + |h_{22}|^2\right) x_1 + h_{11}^* u_{11} + h_{21} u_{21}^* + h_{12}^* u_{12} + h_{22} u_{22}^*\right) + w_{u2}(t)$$

$\hat{y}_1^1$

$$\boxed{\gamma_2 = \frac{\epsilon G^2 \cdot |h_2|^2 \left(|h_{11}|^2 + |h_{12}|^2 + |h_{21}|^2 + |h_{22}|^2\right)}{G^2 |h_2|^2 \left(|h_{11}|^2 + |h_{21}|^2 + |h_{12}|^2 + |h_{22}|^2\right) + 1}}$$

?

$$r_2 = G\left(\left(|h_{11}^2|^2 + |h_{21}^2|^2\right)\hat{z}_1 + h_{11}^* u_1^* + h_{21}^* u_2^*\right) =$$

$$= G\left(|h_{11}^2|^2 + |h_{11}^2|^2\right)\left(\sqrt{\epsilon}\left(h_1 Abs\right) x_1 + h_{11}^* u_{11} + h_{21} u_{21}^* + h_{12}^* u_{12} + h_{22} u_{22}^*\right) + w_{u2}(t)$$

$$P_S = \epsilon G^2 \left(|h_{11}^2|^2 + |h_{11}^2|^2\right)^2 h_1 Abs^2$$

$$\boxed{P_N = G^2 \underbrace{\left(|h_{11}^2|^2 + |h_{11}^2|^2\right)}_{\underline{\underline{h_2 Abs}}} \left(h_1 Abs\right) + 1}$$

DIAGRE ?

$$\boxed{\gamma_1 = \frac{\epsilon}{N_0} \cdot \mathring{\Delta}_2}$$

ϵ – POWER OF THE ANTENNA

$$\boxed{\begin{aligned}\ddot{y}_1 &= G(\ddot{h}_{11}\hat{z}_1 + \ddot{h}_{21}\hat{z}_2) + \ddot{n}_{11}\\ \ddot{y}_2 &= \hat{G}(-\ddot{h}_{11}\hat{z}_2^* + \ddot{h}_{21}\hat{z}_1^*) + \ddot{n}_{21}\end{aligned}}$$

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$$\cancel{\ddot{y}_1 = \ddot{h}_{11}(\dot{h}_{11} x_1 + \dot{h}_{21}^* \hat{z}_2^* + \dots)}$$

$$\ddot{y}_1 = \ddot{h}_{11}(\dot{\Delta}_1 x_1 + \dot{h}_{11}^* n_1 + \dot{h}_{21} n_2^*) + \ddot{h}_{21}(\dot{\Delta}_1 x_2 + \dot{h}_{21}^* n_1 - \dot{h}_{11} n_2^*) + \ddot{n}_{11}$$

$$\boxed{\hat{\ddot{y}}_1 = \ddot{h}_{11}^* \ddot{y}_1 + \ddot{h}_{21} \ddot{y}_2^* = (*)}$$

$$\ddot{y}_2 = -\ddot{h}_{11}(\dot{\Delta}_1 x_2 + \dot{h}_{21}^* n_1 - \dot{h}_{11} n_2^*)^* + \ddot{h}_{21}(\dot{\Delta}_1 x_1 + \dot{h}_{11}^* n_1 + \dot{h}_{21} n_2^*)^* + \ddot{n}_{21}$$

$$\hat{\ddot{y}}_1 = \ddot{h}_{11}^*\left[\ddot{h}_{11}(\dot{\Delta}_1 x_1 + \dot{h}_{11}^* n_1 + \dot{h}_{21} n_2^*) + \ddot{h}_{21}\dot{\Delta}_1 x_2 + \ddot{h}_{21}\dot{h}_{21}^* n_1 - \ddot{h}_{21}\dot{h}_{11} n_2^* + \ddot{n}_{11}\right]$$
$$+ \ddot{h}_{21}\left[-\ddot{h}_{11}\dot{\Delta}_1 x_2^* - \ddot{h}_{11}\dot{h}_{21} n_1^* + \ddot{h}_{11}\dot{h}_{11}^* n_2 + \ddot{h}_{21}\dot{\Delta}_1 x_1^* + \ddot{h}_{21} h_{11} n_1^* + \ddot{h}_{21} h_{21}^* n_2 + \ddot{n}_{21}\right]^*$$

$$\hat{\ddot{y}}_1 = |\ddot{h}_{11}|^2 \dot{\Delta}_1 x_1 + |\ddot{h}_{11}|^2 \dot{h}_{11}^* n_1 + |h_{11}|^2 \dot{h}_{21} n_2^* + \ddot{h}_{11}^* \dot{\Delta}_1 x_2 + \ddot{h}_{11}^* \ddot{h}_{21} \dot{h}_{21} n_1 -$$
$$\ddot{h}_{11}^* \ddot{h}_{21} h_{11} n_2 + \ddot{h}_{21}\left[-\ddot{h}_{11}^* \dot{\Delta}_1 x_2 - \ddot{h}_{11} \dot{h}_{21}^* n_1 + \ddot{h}_{11} h_{11} n_2^* + \dot{h}_{21}^* \Delta_1 x_1 +\right.$$
$$\left.+ \ddot{h}_{21}^* \dot{h}_{11}^* n_1 + \dot{h}_{21}^* \dot{h}_{21}^* n_2 + \ddot{n}_{21}^*\right] =$$

$$= \underline{|\ddot{h}_{11}|^2 \Delta_1 x_1} + |\ddot{h}_{11}|^2 \dot{h}_{11}^* n_1 + |h_{11}|^2 h_{21} n_2^* + \cancel{\ddot{h}_{11}^* \Delta_1 x_2} + \cancel{\dot{h}_{11}^* \ddot{h}_{21} h_{21} n_1} - \cancel{\ddot{h}_{11}^* \ddot{h}_{21} h_{11} n_2^*} + \ddot{h}_{11}^* \ddot{n}_{11}$$
$$- \cancel{\ddot{h}_{21} \ddot{h}_{11}^* \Delta_1 x_2} - \cancel{\ddot{h}_{21} \ddot{h}_{11}^* h_{21} n_1} + \cancel{\ddot{h}_{21} \ddot{h}_{11}^* h_{11} n_2^*} + \underline{|h_{21}|^2 \Delta_1 x_1} + |\ddot{h}_{21}|^2 h_{11} n_1 +$$
$$|\ddot{h}_{21}|^2 h_{21}^* n_2 + \ddot{n}_{21}^* \ddot{h}_{21}$$

$$\boxed{\ddot{y}_1 = G(\ddot{h}_{11}\hat{z}_1 + \ddot{h}_{21}\hat{z}_2) + \ddot{n}_{11} \qquad \ddot{y}_2 = G(-\ddot{h}_{11}\hat{z}_2^* + \ddot{h}_{21}\hat{z}_1^*) + \ddot{n}_{21}}$$

$$\hat{z}_1 = \dot{\Delta}_2 x_1 + \dot{\eta}_1 \qquad \underline{\dot{\eta}_1 = \dot{h}_{11}^* n_{11} + \dot{h}_{21} n_{21}^* + \dot{h}_{12}^* n_{12} + \dot{h}_{22} n_{22}^*}$$

$$\hat{z}_2 = \dot{\Delta}_2 x_2 + \dot{\eta}_2 \qquad \dot{\eta}_2 = \dot{h}_{21}^* n_{11} - \dot{h}_{11} n_{21}^* + \dot{h}_{22}^* n_{12} - \dot{h}_{12} n_{22}^*$$

$$\ddot{y}_1 = G(\ddot{h}_{11}\dot{\Delta}_2 x_1 + \ddot{h}_{11}\dot{\eta}_1 + \ddot{h}_{21}\dot{\Delta}_2 x_2 + \ddot{h}_{21}\dot{\eta}_2) + \ddot{n}_{11}$$

$$\ddot{y}_2 = G(-\ddot{h}_{11}(\dot{\Delta}_2 x_2^* + \dot{\eta}_2^*) + \ddot{h}_{21}(\dot{\Delta}_2 x_1^* + \dot{\eta}_1^*)) + \ddot{n}_{21}$$

$$\hat{\ddot{y}}_1 = G\ddot{h}_{11}^*(\ddot{h}_{11}\dot{\Delta}_2 x_1 + \ddot{h}_{11}\dot{\eta}_1 + \ddot{h}_{21}\dot{\Delta}_2 x_2 + \ddot{h}_{21}\dot{\eta}_2 + \ddot{n}_{11}) +$$
$$G\ddot{h}_{21}\left(-\ddot{h}_{11}^*(\dot{\Delta}_2 x_2 + \dot{\eta}_2) + G\ddot{h}_{21}^*(\dot{\Delta}_2 x_1 + \dot{\eta}_1) + \ddot{n}_{21}^*\right) =$$

$$= \left[\underline{|\ddot{h}_{11}|^2 \Delta_2 x_1} + |\ddot{h}_{11}|^2 \dot{\eta}_1 + \cancel{\ddot{h}_{21}\ddot{h}_{11}^* \Delta_2 x_2} + \cancel{\ddot{h}_{11}^* \ddot{h}_{21} \dot{\eta}_2} + \ddot{h}_{11}^* \ddot{n}_{11}/G\right.$$
$$\left.- \cancel{\ddot{h}_{21}\ddot{h}_{11}^* \Delta_2 x_2} - \cancel{\ddot{h}_{11}^* \ddot{h}_{21} \dot{\eta}_2} + \underline{|\ddot{h}_{21}|^2 \Delta_2 x_1} + |\ddot{h}_{21}|^2 \dot{\eta}_1 + \ddot{h}_{21} \ddot{n}_{21}^*/G\right] \cdot G$$

$$\hat{\ddot{y}}_1 = G_2\Big(\underbrace{|\ddot{h}_{11}|^2 + |\ddot{h}_{21}|^2}_{\ddot{\Delta}_1}\Big)\dot{\Delta}_2 x_1 + G_2\Big(\underbrace{|\ddot{h}_{11}|^2 + |\ddot{h}_{21}|^2}_{\ddot{\Delta}_1}\Big)\dot{\eta}_1 + \ddot{h}_{11}^* \ddot{n}_{11} + \ddot{h}_{21}\ddot{n}_{21}^*$$

$$\boxed{\gamma_{eq_2} = \frac{\epsilon \cdot G_2^2 \ddot{\Delta}_1^2 \cdot \dot{\Delta}_2^2}{\dot{\Delta}_2 G_2^2 \ddot{\Delta}_1^2 N_0 + \underbrace{\left(|\ddot{h}_{11}|^2 + |\ddot{h}_{21}|^2\right)}_{\ddot{\Delta}_1} N_0} = \frac{\epsilon \cdot G_2^2 \ddot{\Delta}_1^{\cancel{2}} \dot{\Delta}_2^2}{\cancel{\ddot{\Delta}_1}\big(G_2^2 \ddot{\Delta}_1 \dot{\Delta}_2 N_0 + N_0\big)} = \frac{\epsilon G_2^2 \ddot{\Delta}_1 \dot{\Delta}_2^2}{N_0 G_2^2 \ddot{\Delta}_1 \dot{\Delta}_2 + N_0}}$$

MMV

$$P_{\dot{\eta}_1} = \left(|\dot{h}_{11}|^2 + |\dot{h}_{21}|^2 + |\dot{h}_{12}|^2 + |\dot{h}_{22}|^2\right) N_0 = \dot{\Delta}_2 \cdot N_0$$

- ЗА ДА ИДЕ НАТАМУ СЕГАДЕ ВО ФОРМУЛИТЕ КАДЕ ИМАШ X ТРЕБА ДА МНОЖИШ СО $\sqrt{\epsilon}$
- POUT T.C. EQUIVALENT SNR FOR $2\times2\times2$

$$\ddot{y}_{11} = G\big(\ddot{h}_{11}\hat{\dot{z}}_1 + \dot{h}_{21}\hat{\dot{z}}_2\big) + \ddot{n}_{11} \qquad \ddot{y}_{21} = G\big(-\ddot{h}_{11}\hat{\dot{z}}_2^* + \ddot{h}_{21}\hat{\dot{z}}_1^*\big) + \ddot{n}_{21}$$

$$\ddot{y}_{12} = G\big(\ddot{h}_{12}\hat{\dot{z}}_1 + \dot{h}_{21}\hat{\dot{z}}_2\big) + \ddot{n}_{12} \qquad \ddot{y}_{22} = G\big(-\ddot{h}_{12}\hat{\dot{z}}_2^* + \ddot{h}_{22}\hat{\dot{z}}_1^*\big) + \ddot{n}_{22}$$

$$\hat{\ddot{z}}_1 = G_2 \ddot{\Delta}_2 \cdot \hat{\dot{z}}_1 + \ddot{\eta}_1 \quad i \quad \ddot{\eta}_1 = \ddot{h}_{11}^* \ddot{n}_{11} + \ddot{h}_{21}\ddot{n}_{21}^* + \ddot{h}_{12}^* \ddot{n}_{12} + \ddot{h}_{22}\ddot{n}_{22}^*$$

$$\hat{\ddot{z}}_2 = G_2 \ddot{\Delta}_2 \hat{\dot{z}}_2 + \ddot{\eta}_2$$

$$\hat{\dot{z}}_1 = \sqrt{\epsilon}\dot{\Delta}_2 x_1 + \dot{\eta}_1 \qquad \hat{\dot{z}}_2 = \dot{\Delta}_2 x_2 + \dot{\eta}_2$$

$$\hat{\ddot{z}}_1 = G_2\ddot{\Delta}_2\left(\sqrt{\epsilon}\dot{\Delta}_2 x_1 + \dot{\eta}_1\right) + \ddot{\eta}_1 = \underline{G_2\ddot{\Delta}_2\sqrt{\epsilon}\dot{\Delta}_2 x_1 + G\ddot{\Delta}_2\dot{\eta}_1 + \ddot{\eta}_1}$$

$$\gamma_{eq_3} = \frac{G_2^2 \ddot{\Delta}_2^2 \epsilon \cdot \dot{\Delta}_2^2}{G_2^2 \ddot{\Delta}_2^2 \dot{\Delta}_2 \cdot N_0 + \ddot{\Delta}_2 N_0} = \frac{\epsilon}{N_0}\,\frac{\ddot{\Delta}_2^{\cancel{2}} \cdot \dot{\Delta}_2^2 \cdot G_2^2}{\cancel{\ddot{\Delta}_2}\big(G_2^2 \ddot{\Delta}_2 \dot{\Delta}_2 + 1\big)}$$

$$\boxed{P_{\ddot{\eta}_1} = \ddot{\Delta}_2 \cdot N_0} \qquad \boxed{\gamma_{eq_3} = \frac{\epsilon}{N_0}\,\frac{\ddot{\Delta}_2 \dot{\Delta}_2^2 G_2^2}{G_2^2 \ddot{\Delta}_2 \dot{\Delta}_2 + 1}}$$

- ЕДНОСТАВНО ИЗВЕДУВАЊЕ ЗА $2\times2\times1$ ГО ВКЛУЧУВА $\sqrt{\epsilon}$

$$\hat{\dot{y}}_1 = G\ddot{\Delta}_1 \hat{\dot{z}}_1 + \underbrace{\ddot{h}_{11}^*\ddot{n}_1 + \ddot{h}_{21}\ddot{n}_2^*}_{\ddot{\xi}_1} = G\ddot{\Delta}_1\left(\dot{\Delta}_2 x_1 + \dot{\eta}_1\right) + \ddot{\xi}_1 =$$

АНАЛОГНО НА ФОРМУЛАТА (5)

$$= G\ddot{\Delta}_1\dot{\Delta}_2 \cdot \sqrt{\epsilon}x_1 + G\cdot\ddot{\Delta}_1\dot{\eta}_1 + \ddot{\xi}_1$$

$$\boxed{\gamma_{eq_2} = \frac{G^2 \ddot{\Delta}_1^2 \dot{\Delta}_2^2 \epsilon}{G^2 \ddot{\Delta}_1^2 \dot{\Delta}_2 N_0 + \ddot{\Delta}_1 \cdot N_0} = \frac{\epsilon}{N_0}\,\frac{G^2 \ddot{\Delta}_1 \dot{\Delta}_2^2}{G^2 \ddot{\Delta}_1 \dot{\Delta}_2 + 1}}$$

$$y_1 = \sqrt{E} h_{11} x_1 + \sqrt{E} h_2 x_2 \qquad y_2 = -\sqrt{E} h_1 x_2^* + \sqrt{E} h_2 x_1^*$$

$$\hat{y}_1 = h_1^* y_1 + h_2 y_2^* = \sqrt{E} |h_1|^2 x_1 + \sqrt{E} h_1^* h_2 x_2 +$$
$$+ h_2 \left(-\sqrt{E} h_1^* x_2 + \sqrt{E} h_2^* x_1\right) = \sqrt{E} |h_1|^2 x_1 + \cancel{\sqrt{E} h_1^* h_2 x_2} +$$
$$\cancel{h_2 \sqrt{E} h_1^* h_2 x_2} + \sqrt{E} |h_2|^2 x_1 = \sqrt{E} \left(|h_1|^2 + |h_2|^2\right) x_1$$

$$\begin{array}{r} 38 \\ -22 \\ \hline 16\,dB \end{array} \qquad \begin{array}{r} 38 \\ 14 \\ \hline 24 \end{array}$$

- SECOND SYMBOL IN THE DESTINATION FOR 2x2x1 SYSTEM

$$\hat{\ddot{y}}_2 = G_2 \ddot{\Delta}_1 \hat{\dot{x}}_2 + \ddot{\xi}_2 = G_2 \ddot{\Delta}_1 \left(\sqrt{E} \dot{\Delta}_2 x_2 + \dot{\eta}_2\right) + \ddot{\xi}_2$$

$$= \sqrt{E} G_2 \ddot{\Delta}_1 \dot{\Delta}_2 x_2 + G_2 \ddot{\Delta}_1 \dot{\eta}_2 + \ddot{\xi}_2$$

$$\gamma_{eq_2} = \frac{E \cdot G_2^2 \ddot{\Delta}_1^2 \dot{\Delta}_2^2}{\left(G_2^2 \ddot{\Delta}_1^2 \dot{\Delta}_2 + \ddot{\Delta}_1\right) N_0} = \frac{E}{N_0} \frac{G_2^2 \ddot{\Delta}_1^{\cancel{2}} \dot{\Delta}_2^2}{\cancel{\ddot{\Delta}_1}\left(G^2 \ddot{\Delta}_1 \dot{\Delta}_2 + 1\right)}$$

$$\boxed{\gamma_{eq_2} = \frac{E}{N_0} \frac{G_2^2 \ddot{\Delta}_1 \dot{\Delta}_2^2}{G_2^2 \ddot{\Delta}_1 \dot{\Delta}_2 + 1}}$$

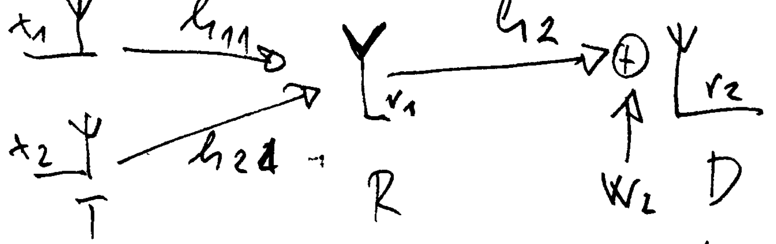
$$\hat{y}_1 = \sqrt{E} \dot{\Delta}_1 x_1 + \dot{\xi}_1$$

$$\boxed{G_1 = \sqrt{\frac{\ddot{E}}{\dot{E} \ddot{\Delta}_1^2 + \Delta_1 N_0}}}$$

PRAVILNA FORMULA ~~ZA~~ 2x1x1 SYSTEM.

$$\begin{array}{r} 38 \\ 22 \\ \hline 16 \end{array} \qquad \begin{array}{r} 38 \\ 13 \\ \hline 25 \end{array}$$

2x1x1 Wout COMBINER IN RELAY



$$y_1 = h_{11} x_1 + h_{21} x_2 + u_{11}$$
$$y_2 = -h_{11} x_2^* + h_{21} x_1^* + u_{21}$$

$$r_1 = [y_1, y_2, y_3 \ldots y_N]$$

$$r_{21} = G_1 h_2 r_1 + W_2 = G_1 \cdot h_2 \left(h_{11} x_1 + h_{21} x_2 + u_{11}\right) + W_2$$
$$r_{22} = G_1 \cdot h_2 \left(-h_{11} x_2^* + h_{21} x_1^* + u_{21}\right) + W_2$$

$r_2 = [r_{21}, r_{22}, r_{23}, \dots r_{2N}]$

- ALAMOUTI DECODER

$\hat{y}_1 = h_{11}^* \underbrace{y_1}_{r_{21}} + h_{21} \underbrace{y_2^*}_{r_{22}} \qquad \hat{y}_2 = h_{21}^* y_1 - h_{11} y_2^*$

$$\hat{y}_1 = h_{11}^* \left(G \cdot h_2 h_{11} x_1 + G h_2 h_{21} x_2 + G h_2 n_{11} + w_2 \right) +$$
$$+ h_{21} \left(-G \cdot h_2 h_{11} x_2^* + \underbrace{h_{21} x_1}_{G h_2} + G h_2 n_{21} + w_2 \right)^* =$$
$$= \underline{G \cdot h_2 \cdot |h_{11}|^2 x_1} + G h_2 h_{21} h_{11}^* x_2 + G h_{11}^* h_2 n_{11} + h_{11}^* w_2 +$$
$$- G h_{21} h_2^* h_{11}^* x_2^* + \underline{G h_2^* |h_{21}|^2 x_1} + G \cdot h_{21} h_2^* n_{21}^* + h_{21} w_2^*$$
$$= G \left(h_2 |h_{11}|^2 + h_2^* |h_{21}|^2 \right) x_1 + G h_{21} h_{11}^* \left(h_2 - h_2^* \right) \cdot x_2 + \xi$$

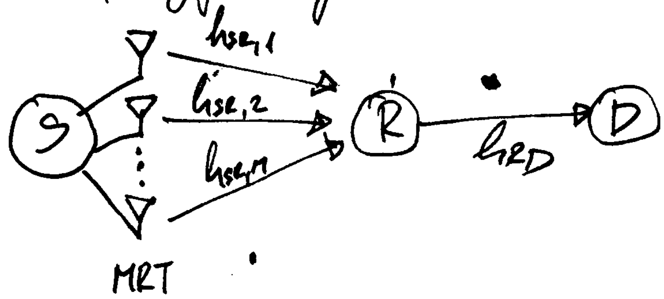
- ŠTO SE SLUČUVA AKO ZEMAM $\boxed{r_2 = v_2 / h_2}$

$$r_{21} = G \cdot y_1 + \frac{w_2}{h_2} = G \left(h_{11} x_1 + h_{21} x_2 + n_{11} \right) + w_2 / h_2$$
$$r_{22} = G \cdot y_2 + \frac{w_2}{h_2} = G \left(-h_{11} x_2^* + h_{21} x_1^* + n_{21} \right) + w_2 / h_2$$
$$\hat{y}_1 = h_{11}^* \left(G h_{11} x_1 + G h_{21} x_2 + G n_{11} + w_2 / h_2 \right) +$$
$$+ h_{21} \left(-G h_{11}^* x_2 + G h_{21}^* x_1 + G n_{21}^* + w_2^* / h_2^* \right) =$$
$$= |h_{11}|^2 G x_1 + \cancel{G h_{11}^* h_{21} x_2} + G h_{11}^* n_{11} + h_{11}^* w_2 / h_2 +$$
$$\cancel{- G h_{21} h_{11}^* x_2} + G |h_{21}|^2 x_1 + G h_{21} n_{21}^* + h_{21} w_2^* / h_2^* =$$
$$= G \left(|h_{11}|^2 + |h_{21}|^2 \right) x_1 + G h_{11}^* n_{11} + G h_{21} n_{21}^* + \frac{h_{11}^* w_2}{h_2} + \frac{h_{21} w_2^*}{h_2^*}$$

– NEUPOTREBLJIV E ZATOA ŠTO NEMA NAČIN (D) DA GO NAUČI KANALOT OD PRVIOT HOP ŠTO E PREMISA VO MOETO IZVEDUVANJE. KAKO I DA E GO IMAM SIMULIRANO NO NE E KORISNO!!!

- AKO GO IZBEREŠ $G = \dfrac{1}{|h_{11}|^2 + |h_{21}|^2}$

☐ H. Min, Effect of Multiple Antennas at Source ...



$$w = \left(\frac{h_{SR}^H}{\sqrt{h_{SR}^H h_{SR}}} \right)^T$$

$$h_{SR} = [h_{SR,1}; h_{SR,2}, \dots, h_{SR,M}]^T$$

$$\boxed{r_R(t) = \sqrt{E_S}\, h_{SR}^T \cdot w \cdot s(t) + n_{SR}(t)}$$

$$r_D(t) = \sqrt{G_R}\, h_{RD} \cdot G \left(\sqrt{E_S} \frac{h_{SR}^H h_{SR}\, s(t)}{\sqrt{h_{SR}^H h_{SR}}} + n_{SR}(t) \right) + n_{RD}(t) =$$

$$= \frac{\sqrt{G_R}\, h_{RD} \cdot G \cdot \sqrt{E_S}\, h_{SR}^H h_{SR}}{\sqrt{h_{SR}^H h_{SR}}} s(t) + \sqrt{G_R}\, h_{RD}\, G\, n_{SR}(t) + n_{RD}(t)$$

$$\gamma_{eq} = \frac{\dfrac{G_R \cdot |h_{RD}|^2 \cdot G^2 \cdot E_S \cdot |h_{SR}^H \cdot h_{SR}|^2}{|h_{SR}^H h_{SR}|}}{G_R |h_{RD}|^2 \cdot G^2 N_0 + N_0} =$$

$$= \frac{G^2 N_0 \cdot \overbrace{\dfrac{G_R \cdot |h_{RD}|^2}{N_0} \dfrac{E_S \cdot h_{SR}^H h_{SR}}{N_0}}^{K}}{G_R |h_{RD}|^2 \cdot G^2 + 1} = \qquad \left| \; G^2 = \frac{1}{\cancel{G_R |h_{RD}|^2}\; h_{SR}^H h_{SR} E_S + N_0} \right|$$

$$\cancel{\frac{K}{\dfrac{G_R|h_{RD}|^2}{h_{SR}^H h_{SR} E_S + N_0} + 1} = \frac{N_0}{G_R |h_{RD}|^2 + h_{SR}^H h_{SR} E_S + N_0} \quad h_{SR}^H h_{SR} E_S + N_0}$$

$$= \frac{N_0 \overbrace{\dfrac{G_R |h_{RD}|^2}{N_0} \cdot \dfrac{E_S h_{SR}^H h_{SR}}{N_0}}^{K}}{G_R |h_{RD}|^2 + \dfrac{1}{G^2}} = \frac{K}{\dfrac{G_R |h_{RD}|^2}{N_0} + \dfrac{1}{N_0 G^2}}$$

$$= \frac{K}{\dfrac{G_R |h_{RD}|^2}{N_0} + \dfrac{h_{SR}^H h_{SR} E_S + N_0}{N_0}} = \frac{K}{\dfrac{G_R |h_{RD}|^2}{N_0} + \dfrac{h_{SR}^H h_{SR} E_S}{N_0} + 1}$$

$$\boxed{\gamma_{eq} = \frac{\dfrac{E_R |h_{RD}|^2}{N_0} \cdot \dfrac{E_S h_{SR}^H h_{SR}}{N_0}}{\dfrac{E_R |h_{RD}|}{N_0} + \dfrac{h_{SR} h_{SR} E_S}{N_0} + 1}}$$

- T. Fan, et al, On the Performance of MIMO Spatial ...

$r = \sqrt{\eta}\, \bar{H} \cdot s + n_r \qquad r\ (L \times 1) \qquad H\ (L \times M)$

$\bar{H} = \sqrt{\alpha}\, \tilde{H} \qquad \boxed{\alpha = x^{-\gamma} 10^{\xi/10}}$ lognormal shadowing

x - distance
γ - pathloss exponent

ξ - random variable with normal distrib with standard deviation $\delta = 8\,dB$

$y = G \cdot d + n_d \qquad G$ - chanel between R&D

$\bar{G} = \sqrt{\beta} \cdot \tilde{G} \rightarrow$ i.i.d. complex Gaussian random var.

- ~~Wideo~~ I. H. Lee - Decouple-and-Forward Relaying

$H^R = \{h^R_{ij}\}_{2\times 2} \qquad H^P = \{h^P_{ij}\}_{2\times 2}$

$$\begin{bmatrix} y^R_{11} & y^R_{12} \\ y^R_{21} & y^R_{22} \end{bmatrix} = \begin{bmatrix} h^R_{11} & h^R_{12} \\ h^R_{21} & h^R_{22} \end{bmatrix} \begin{bmatrix} x_1 & -x_2^* \\ x_2 & +x_1 \end{bmatrix} + \begin{bmatrix} e^R_{11} & e^R_{12} \\ e^R_{21} & e^R_{22} \end{bmatrix}$$

y_{it} : i - receive antenna
t - t-th time slot

$E[x_1 x_1^*] = E[x_2 x_2^*] = P/2 \qquad P$ - total transmit power

	TS1	TS2
ANT1	$y^R_{11} = h^R_{11} x_1 + h^R_{12} x_2$	$y_{12} = -h_{11} x_2^* + h^R_{12} x_1$
ANT2	$y^R_{21} = h^R_{21} x_1 + h^R_{22} x_2$	$y^R_{22} = -h^R_{21} x_2^* + h^R_{22} x_1$

$$\hat{x}_1 = h^{R*}_{11} y^R_{11} + h^R_{12} y^{*R}_{12} + h^*_{21} \cdot y^R_{21} + h_{22} \cdot y^{*R}_{22}$$

$$\hat{x}_2 = h^*_{12} y^R_{11} + h^R_{11} y^{*R}_{12} + h^*_{22} y^R_{21} - h^R_{21} y^{*R}_{22}$$

$$\begin{bmatrix} y_{11}^D & y_{12}^D \\ y_{21}^D & y_{22}^D \end{bmatrix} = \begin{bmatrix} h_{11}^D & h_{12}^D \\ h_{21}^D & h_{22}^D \end{bmatrix} \begin{bmatrix} G\tilde{x}_1 & -G\tilde{x}_2^* \\ G\tilde{x}_2 & G\tilde{x}_1^* \end{bmatrix} + \begin{bmatrix} e_{11}^D & e_{12}^D \\ e_{21}^D & e_{22}^D \end{bmatrix}$$

MOJATA FORMULA VEZI:

$$\gamma_{eq3} = \frac{\epsilon}{N_0} \frac{G_2^2 \ddot{\Delta}_2 \dot{\Delta}_2^2}{G_2^2 \ddot{\Delta}_2 \dot{\Delta}_2 + 1}$$

TRANSFORMIRANJE VO NOTACIJATA OD ČLANAKOT NA LEE:

$$\gamma_{eq3} = \frac{\epsilon}{N_0} \frac{G^2 \cdot (H^{2D})(H^{2R})^2}{G^2 \cdot H^{2D} \cdot H^{2R} + 1} = \frac{\epsilon}{N_0} \left[\frac{1}{H^{2R^{-1}} + (G^2 \cdot H^{2D} (H^{2R})^2)^{-1}} \right]$$

$$\gamma_{eq3} = \frac{\epsilon}{N_0} \left(\frac{1}{H^{2R}} + \frac{1}{G^2 \cdot H^{2D} (H^{2R})^2} \right)^{-1}$$

$$\epsilon_s = \epsilon_b \cdot l d M \qquad \epsilon = \epsilon_b = \frac{\epsilon_s}{l d M} \qquad \boxed{\epsilon_b = \frac{P}{2}}$$

$$\gamma_{eq3} = \frac{\epsilon_b}{N_0 l d M} \left(\frac{1}{H^{2R}} + \frac{1}{G^2 \cdot H^{2D} (H^{2R})^2} \right)$$

$$G^2 = \frac{\epsilon}{\epsilon \cdot (H_R^2)^2 + H_R^2 \cdot N_0} = \frac{1}{(H_R^2)^2 + \frac{H_R^2}{\epsilon / N_0}}$$

$$G_{2P}^2 = \left[(H_R^2)^2 + \frac{H_R^2}{\frac{P}{2 \cdot N_0}} \right]^{-1} \doteq \left[(H_R^2)^2 + \frac{2 H_R^2}{\rho} \right]^{-1} \rightarrow \text{POWER SCALED RELAY GAIN}$$

- If $\epsilon / N_0 \gg 1$:

$$\boxed{G_A^2 \doteq \frac{1}{(H_R^2)^2}} \rightarrow \text{APPROXIMATE RELAY GAIN.}$$

- $z_1 = \frac{1}{\|H_R\|^2}$ $\quad z_2 = \frac{1}{\|H_D\|^2}$ $\qquad f_{z_i}(z) = \frac{1}{6 \beta_i^4 z^5} e^{-\frac{1}{\beta_i z}}$

VO MOJOT SLUČAJ: $\boxed{\beta_1 = \beta_2 = 2} \rightarrow$ VARIANCE NA h_1 I h_2

$\|H\|^2 = |h_{11}|^2 + |h_{12}|^2 + |h_{21}|^2 + |h_{22}|^2$ randn(N,1) + 1i randn(N,1)

- INVERSE OF CHI-SQUARE DISTRIBUTION

POGLEJ str. 75

$$f(x;\nu) = \frac{2^{-\nu/2}}{\Gamma(\nu/2)} x^{-\nu/2-1} e^{-1/(2x)}$$

$$f(x;4) = \frac{1}{2^2\,\Gamma(4/2)} x^{-3} e^{-1/2x} = \underline{\frac{1}{4\cdot x^3}\cdot e^{-\frac{1}{2x}}}$$

$\Gamma(n) = (n-1)!$ $\quad \Gamma(2) = (2-1)! = 1$

$$\int_0^\infty x^{\nu-1} e^{-\frac{\beta}{x}-\gamma x}\,dx = 2\left(\frac{\beta}{\gamma}\right)^{\frac{\nu}{2}} K_\nu\left(2\sqrt{\beta\gamma}\right)$$

POTSETNAUDE

$$M(-s) = \int_0^\infty p(\gamma)\cdot e^{-\gamma s}\,d\gamma$$

$$p(\gamma) = \frac{\partial M(s)}{ds}$$

$$P_{out}(\gamma < x) = \int_0^x p(\gamma)\,d\gamma \qquad \frac{dP}{dx} = p(\gamma)\;(\;)\mathcal{L}$$

$$\int_0^\infty \frac{dP(\gamma)}{d\gamma} e^{-s\gamma}\,d\gamma = \int_0^\infty p(\gamma)e^{-s\gamma}\,d\gamma = \underline{M(-s)}$$

$$s\cdot \hat{P}_{out}(s) = M(-s)$$

$$\boxed{s\,\hat{P}_{out} = M(-s)}$$

$$P_{out} = \mathcal{L}^{-1}\left[\frac{M(-s)}{s}\right]$$

$$\boxed{P_{out} = \frac{1}{2\pi j}\int_{-\sigma-j\infty}^{\sigma+j\infty}\left(\frac{M(-s)}{s}\right)e^{s\gamma}\,ds}$$

$$P_{out} = \mathcal{L}^{-1}\left[\frac{M(-s)}{s}\right]$$

$$\int_0^\infty x^{\nu-1} e^{-\frac{\beta}{x}-\gamma x}\,dx = 2\left(\frac{\beta}{\gamma}\right)^{\nu/2} K_\nu\left(2\sqrt{\beta\gamma}\right)$$

$$\boxed{f(x,4) = \frac{1}{4x^3}\, e^{-1/2x}}$$

$$M(-s) = \int_0^\infty p(\gamma)\, e^{-\gamma s}\, d\gamma = \int_0^\infty \frac{1}{4x^3} e^{-\frac{1}{2x}} \cdot e^{-xs}\, dx$$

$$= \frac{1}{4}\int_0^\infty x^{-2-1} e^{-\frac{0.5}{x} - sx}\, dx = 2\left(\frac{0.5}{s}\right)^{-\frac{2}{2}} K_{-2}\left(2\sqrt{\frac{s}{2}}\right)$$

$$\boxed{M(-s) = 2\left(\frac{1}{2s}\right)^{-1} K_{-2}\left(\sqrt{2s}\right)}$$

$$\boxed{\beta = 2} \qquad \boxed{M(-s) = 4s\, K_{-2}(\sqrt{2s})}$$

MOJA FORMULA

ZEROK NE KONTAM OD KADE GO VADI PDF.

$$M(-s) = 2\left(\frac{1}{\beta s}\right)^2 K_4\left(2\sqrt{\frac{s}{\beta}}\right)$$

$$\boxed{f_{Z_i}(z) = \frac{1}{6\beta_i^4 z^5}\, e^{-\frac{1}{\beta_i z}}}$$

ZOŠTO IZLEGUVA z^5 !??

$$M_{Z_i}(-s) = \int_0^\infty \frac{1}{6\beta_i^4 \cdot z^5}\, e^{-\frac{1}{\beta_i z}} \cdot e^{-zs}\, dz = \frac{1}{6\beta_i^4}\int_0^\infty z^{-5} e^{-\frac{1}{\beta_i z}} e^{-zs}\, dz$$

$$M_{Z_i}(-s) = \frac{1}{6\beta_i^4}\int_0^\infty z^{-4-1} e^{-\frac{1}{\beta_i z} - zs}\, dz = \frac{1}{\cancel{6}\,3\,\beta_i^4}\, \cancel{2}\left(\frac{1}{\beta_i s}\right)^{-2} K_{-4}\left(2\sqrt{\frac{s}{\beta_i}}\right)$$

$$M_{Z_i}(-s) = \frac{1}{3\beta_i^4}\, \beta_i^2 s^2 K_{-4}\left(2\sqrt{\frac{s}{\beta_i}}\right) = \frac{s^2}{3\beta_i^2} K_{-4}\left(2\sqrt{\frac{s}{\beta_i}}\right)$$

– AKO NAMESTO $\frac{\nu}{2}$ STAVIŠ "ν" VO FORMULATA OD WIKIPEDIJA

$$f(x;\nu) = \frac{2^{-\nu}}{\Gamma(\nu)}\, x^{-\nu-1} e^{-\frac{1}{2x}}$$

$$\nu = 4 \qquad f(x;\nu) = \frac{2^{-4}}{24}\, x^{-5} e^{-\frac{1}{2x}}$$

$$EbNo\_dB = 10\log \boxed{EbNo} \rightarrow \text{PER ANTENNA}$$

$$EbNo^{(1)}\_dB = 10\log\left(\frac{EbNo}{2}\right) = 10\log(EbNo) - 10\log 2$$

$$\boxed{P_{out} = \mathcal{L}^{-1}\left[\frac{M(-s)}{s}\right]}$$

$$\boxed{M(\gamma) = \frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty} M(-s)\, e^{s\gamma}\, ds}$$

ABRAMOWITZ :

9.6.6 $\boxed{K_{-\nu}(z) = K_{\nu}(z)}$

(4)

$$\boxed{M(-s) = \frac{s^2}{3\beta_i^2} K_{-4}\left(2\sqrt{\frac{s}{\beta_i}}\right) = \frac{s^2}{3\beta_i^2} K_4\left(2\sqrt{\frac{s}{\beta_i}}\right)}$$

$\beta_i = 2 \Rightarrow$ $\boxed{M(-s) = \frac{s^2}{12} K_4\left(\sqrt{2s}\right)}$

$$\boxed{M(\gamma) = \mathcal{L}^{-1}[M(-s)] = \frac{1}{96}\cdot\frac{e^{-\frac{1}{2t}}}{t^5}}$$

OVA E VSUŠNOST PDF

- **BPSK**

$$P_b = \frac{1}{\pi}\int_0^{\pi/2} M_\gamma\left(-\frac{1}{\sin^2\theta}\right) d\theta = \frac{1}{12\pi}\int_0^{\pi/2} s^2 K_4\left(\sqrt{2s}\right) d\theta$$

(1)

$$= \frac{1}{12\pi}\int_0^{\pi/2} \frac{1}{\sin^4\theta} K_4\left(\sqrt{\frac{2}{\sin^2\theta}}\right) d\theta$$

$$\boxed{\beta_i = \frac{1}{\sqrt{EbNo}}}$$

$$10^{-0.05\cdot EbNo\_dB} = \left|10^{-0.1\, EbNo\_dB}\right|^{1/2} = \frac{1}{EbNo}$$

$$EbNo\_dB = 10\log EbNo \qquad EbNo = 10^{+0.1\cdot EbNo\_dB}$$

$$M(-s) = \frac{s^2\, EbNo}{3} K_4\left(2\sqrt{\frac{s}{(\sqrt{EbNo})^{-1}}}\right) = \frac{s^2\gamma}{3} K_4\left(2\,\frac{\sqrt{s}\,\sqrt[4]{\gamma}}{\cancel{\ }}\right)$$

$$z_1 = \frac{1}{\|H^R\|^2} \qquad z_2 = \frac{1}{\|H^R\|^2} \qquad w = z_1 + z_2$$

$\Rightarrow$ W IMA MGF KOJ E PROIZVOD NA MGF-VI EDINEČNITE

$$M_w(s) = M_{z_1}(s) \cdot M_{z_2}(s) = \frac{s^2}{3\beta_1^2} K_4\left(2\sqrt{\frac{s}{\beta_1}}\right) \cdot \frac{s^2}{3\beta_2^2} K_4\left(2\sqrt{\frac{s}{\beta_2}}\right)$$

$$\boxed{M_w(s) = \frac{s^4}{9\beta_1^2\beta_2^2} K_4\left(2\sqrt{\frac{s}{\beta_1}}\right) K_4\left(2\sqrt{\frac{s}{\beta_2}}\right)} \qquad \beta = \frac{1}{\sqrt{E_bN_0}} = 1/\sqrt{\gamma}$$

$$M_w = \frac{\left(\frac{1}{\sin^2\theta}\right)^4}{9\beta^4}\left[K_4\left(2\sqrt{\frac{1}{\beta\sin^2\theta}}\right)\right]^2 = \frac{\gamma^2}{9\sin^8\theta} K_4^2\left(2\sqrt{\frac{\sqrt{\gamma}}{\sin^2\theta}}\right)$$

QAM-16

$E = 10 \cdot d^2$ $\qquad \boxed{snr = \frac{E_b}{N_0}} \qquad \boxed{\sigma^2 = \frac{N_0}{2}}$ MMV

$$N_0 = \frac{E_b}{snr} \qquad \boxed{\sigma^2 = \frac{E_b}{2 \cdot snr} = \frac{E_s}{2 \cdot k \cdot snr}}$$

ZA QAM-16 $\qquad s = sR + j\,sI \qquad sR \in [-3..3]$, $sI \in [-3..+3]$

$\boxed{\text{mean}(\text{abs}(s).\wedge 2) \doteq 10}$ MMV $\qquad \boxed{E_s \doteq 10}$

$$\boxed{\sigma^2 = \frac{E_s}{2 \cdot 4 \cdot snr} = \frac{E_s}{8\,snr}}$$

OVAA DVOJKA TREBA DA JA KOMPENZIRA ŠTO ZEMAS:

y = (randn(N,1) + 1i randn(N,1)) / sqrt(2)

$$\sigma^2 = \frac{N_0}{2} \qquad \frac{E_b}{N_0} = \frac{E_s}{k \cdot N_0} \qquad \boxed{N_0 = \frac{E_s}{k \cdot E_bN_0}}$$

$$\boxed{\sigma^2 = \frac{E_s}{2 \cdot k \cdot E_bN_0}} \rightarrow$$ ~~[illegible]~~ MNOGU VAŽNA FORMULA!! DA SE KORISTI ZA GENERIRANJE NA ŠUMOT ZA QAM-M!!!

MMV

VIDI: $\boxed{\text{BER\_QAM\_M\_AWGN.m}}$

• MPSK $\quad \sigma^2 = \frac{N_0}{2} \qquad \frac{E_b}{N_0} = \frac{E_s}{K \cdot N_0} \qquad \boxed{N_0 = \frac{E_s}{K \cdot E_bN_0}}$

$$\boxed{\sigma^2 = \frac{E_s}{2 \cdot K \cdot E_bN_0}} \qquad \text{PSK-4} \quad \boxed{K = 2} \quad \sigma^2 = \frac{E_s}{4 \cdot E_bN_0}$$

q/σ^2

$$\frac{E_b}{N_0} = \frac{E_s}{K \cdot N_0} \qquad 10\log\frac{E_b}{N_0} = 10\log\frac{E_s}{N_0} - 10\log K$$

$$z_1 = \frac{1}{\|H^R\|^2} \qquad z_2 = \frac{1}{\|H^P\|^2}$$

$$\int_0^\infty x^{\nu-1} e^{-\frac{\beta}{x} - \gamma x} dx = 2\left(\frac{\beta}{\gamma}\right)^{\nu/2} K_\nu(2\sqrt{\beta\gamma})$$

$$\boxed{K_{-4}(x) = K_4(x)}$$

$$f_{z_i}(z) = \frac{1}{6\beta_i^4 z^5} e^{-1/\beta_i z} \qquad i = 1, 2.$$

$M(-s)$

$$M_{z_i}(s) = E_{z_i}\left[e^{-sz}\right] = \int_0^\infty f_{z_i}(z) \cdot e^{-sz} dz = \frac{1}{6\beta_i^4}\int_0^\infty z^{-5} e^{-\frac{1}{\beta_i z} - sz} dz$$

$$M_{z_i}(s) = \frac{1}{6\beta_i^4}\int_0^\infty z^{-4-1} e^{-\frac{1/\beta_i}{z} - sz} dz = \frac{1}{3\beta_i^4}\left(\frac{1}{\beta_i s}\right)^{\cancel{-4/2}\ -2} K_{-4}\left(2\sqrt{\frac{s}{\beta_i}}\right)$$

$$\boxed{M_{z_i}(s) = \frac{s^2}{3\beta_i^2} \cdot K_4\left(2\sqrt{\frac{s}{\beta_i}}\right)}$$

$w = z_1 + z_2$

z_1, z_2 – INDEPENDENT $\boxed{\text{ZDRUŽEN MGF}}$

$$\boxed{M_w(s) = M_{z_1}(s) \cdot M_{z_2}(s) = \frac{s^4}{9\beta_1^2\beta_2^2} K_4\left(2\sqrt{\frac{s}{\beta_1}}\right) \cdot K_4\left(2\sqrt{\frac{s}{\beta_2}}\right)}$$

$$M_x(s) = \int_0^\infty p(x) \cdot e^{-xs} dx \quad \Big/ \frac{d}{ds} \qquad \frac{dM_x(s)}{ds} = s\int_0^\infty p(x) e^{-xs} dx$$

$$P_\gamma(\gamma) = \int_0^{\gamma_{th}} p_\gamma(x)\, dx \qquad \frac{dP(\gamma)}{d(\gamma)}\bigg|_{\gamma=\gamma_{th}} = p_\gamma(\gamma_{th})$$

$$p_\gamma(\gamma) = \frac{dP_\gamma(\gamma)}{d\gamma} \Big/ \mathcal{L} \qquad \int_0^\infty p_\gamma(\gamma) e^{-\gamma s} d\gamma = \mathcal{L}\left\{\frac{dP_\gamma(\gamma)}{d\gamma}\right\}$$

$$P_\gamma(\gamma) = \mathcal{L}^{-1}\left[\frac{MGF(-s)}{s}\right] \longleftarrow MGF(-s) = s \cdot P_\gamma(s)$$

BPSK:

$$P_b = \frac{1}{\pi}\int_0^{\pi/2} M\left(-\frac{1}{\sin^2\theta}\right) d\theta$$

$$w = \frac{1}{\|H^R\|^2} + \frac{1}{\|H^P\|^2}$$

$$C_g = \frac{E_s N_0}{L d M} = \frac{E_s N_0}{K}$$

$E_s N_0$

$$M_w(-s) = \frac{s^4}{9 p_1^2 p_2^2} K_4\left(2\sqrt{\frac{1}{p_1}}\right) K_4\left(2\sqrt{\frac{1}{p_2}}\right)$$

VO NAŠIOT SLUČAJ $p_1 = p_2 = 2$

$$\boxed{M_w(-s) = \frac{s^4}{9\cdot 4\cdot 4}\left[K_4\left(2\sqrt{\frac{s}{2}}\right)\right]^2 = \frac{s^4}{144}\cdot\left[K_4\left(\sqrt{2s}\right)\right]^2}$$

$$\gamma_A^{DCF} = c_g\left[\frac{1}{\|H^R\|} + \frac{1}{\|H^P\|^2\,\|H^R\|^4\circ G_A^2}\right]^{-1} = \quad G_A^2 = \frac{1}{\|H^R\|^4}$$

$x_1^2 + x_2^2 + x_3^2 + x_4^2$

$$= c_g\left[\frac{1}{\|H^R\|^2} + \frac{1}{\|H^P\|^2\cdot\cancel{\|H^R\|^2}}\right]^{-1}$$

$$\frac{1}{\gamma_A^{DCF}} \frac{1}{c_g^{-1}} = \frac{1}{\|H^R\|^2} + \frac{1}{\|H^P\|^2} = \frac{1}{\gamma_1} + \frac{1}{\gamma_2}$$

KAKO VO ČLANAKOT OD HASNA

$$P(\gamma_{eq} < \gamma_{th}) = P\left(\frac{1}{\gamma_{eq}} > \frac{1}{\gamma_{th}}\right) = P\left(\frac{1}{\gamma_A^{DCF}} > \frac{1}{\gamma_{th}}\right)$$

$$= 1 - P\left(\frac{1}{\gamma_A^{DCF}} < \frac{1}{\gamma_{th}}\right) = 1 - \mathcal{L}^{-1}\left[\frac{M_w(-s)}{s}\right]$$

$$\boxed{\text{CDF} = 1 - \mathcal{L}^{-1}\left[\frac{M_w(-s)}{s}\right]} = P_{out}$$

$$P_{out} = 1 - \mathcal{L}^{-1}\left\{\frac{s^3}{144}\left[K_4\left(\sqrt{2s}\right)\right]^2\right\}$$

MILAN ZOKI L PEŠAN

070860870

DARKO JANEVSKI DEJAN TIMKOVSKI (ŠEF)

SIMON OD PRODUKT

• ZA $\gamma = \gamma_1 = \gamma_2 = \frac{1}{\sqrt{E_b N_0}} = \frac{1}{\sqrt{\delta}}$

ZA SLUČAJ KOGA GO MENUVAM γ e $r = 1\cdot 1 \cdot 4/\sqrt{E_b N_0}$

$$M_W(-1) = \frac{1^4\, E_bN_0^2}{9}\left[K_4\left(2\sqrt{1\sqrt{E_bN_0}}\right)\right]^2 = \frac{1^4\cdot\delta^2}{9}\left[K_4\left(2\sqrt{1\sqrt{\delta}}\right)\right]^2$$

• ZA $\gamma = \gamma_1 = \gamma_2 = \sqrt{E_b N_0}$

$$\boxed{M_W(-1) = \frac{1^4}{9\cdot E_bN_0^2}\cdot\left[K_4\left(2\sqrt{\frac{1}{\sqrt{E_bN_0}}}\right)\right]^2}$$

$\boxed{\delta = E_bN_0^{-1}}$

VO FIZIČKA SMISLA $\delta = E_bN_0$ NO VO SLUČAJOV δ e NEKOJA DRUGA [illegible]

$$p_Z(z) = \frac{1}{6\beta^4 z^5}\cdot e^{-\frac{1}{\beta\cdot z}} \qquad z = \frac{1}{\|H^R\|^2}$$

$$\gamma = z\cdot\delta \Rightarrow \boxed{\gamma = \frac{\delta}{\|H^R\|^2}}$$

$$p_\gamma(\gamma) = \frac{p_Z(z)}{\frac{d\gamma}{dz}}\Bigg|_{z=\frac{\gamma}{\delta}} = \frac{\frac{\delta^5}{6\beta^4\cdot\gamma^5}\cdot e^{-\frac{\delta}{\beta\cdot\gamma}}}{\delta}$$

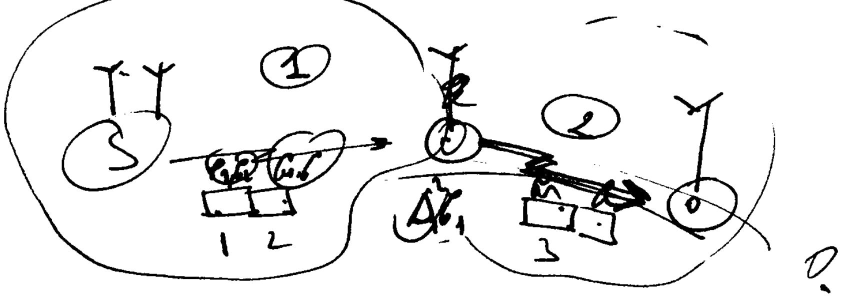
$$p_\gamma(\gamma) = \frac{\delta^4}{6\beta^4\cdot\gamma^5}\cdot e^{-\frac{\delta}{\beta\cdot\gamma}} = \frac{1}{6\alpha^4\cdot\gamma^5}\cdot e^{-\frac{1}{\alpha\cdot\gamma}}$$

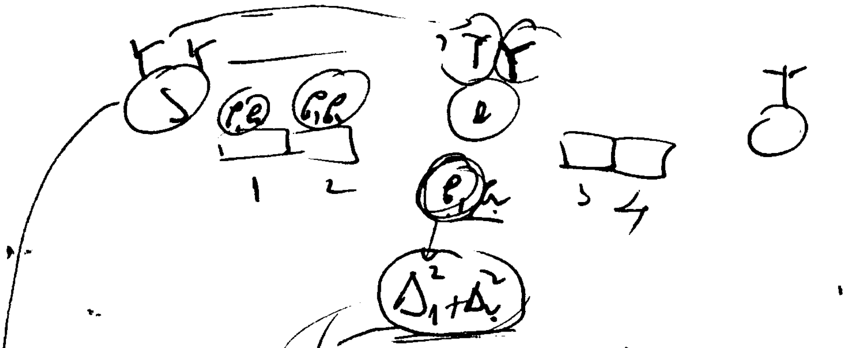
$$\boxed{\alpha = \frac{\beta}{\delta}} \qquad \boxed{\alpha = \beta\cdot E_bN_0} \Rightarrow \boxed{M_W = \frac{1^4}{9\alpha_1^2\alpha_2^2}K_4\left(2\sqrt{\frac{1}{\alpha_1}}\right)K_4\left(2\sqrt{\frac{1}{\alpha_2}}\right)}$$

$$\cancel{\beta_1 = \beta_2 = \beta = 2 \quad M_W = \frac{1^4\delta^4}{9\cdot\beta^4}K_4\left(2\sqrt{\frac{1\cdot\delta}{\beta}}\right)}$$

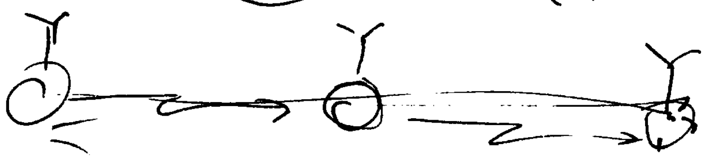
$$\cancel{M_W = \frac{1^4\, E_bN_0^4}{144}K_4\left(\sqrt{2\,1\,E_bN_0}\right)}$$

$$M_W = \frac{\delta^4}{144\cdot E_bN_0^4}\cdot\left[K_4\left(2\sqrt{\frac{1}{2E_bN_0}}\right)\right]^2 = \frac{1^4}{144E_bN_0^4}\left[K_4\left(\sqrt{\frac{2\,1}{E_bN_0}}\right)\right]^2$$



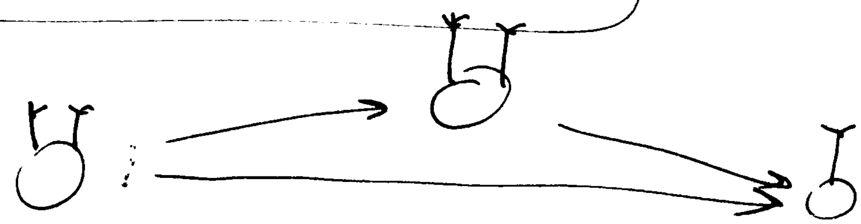


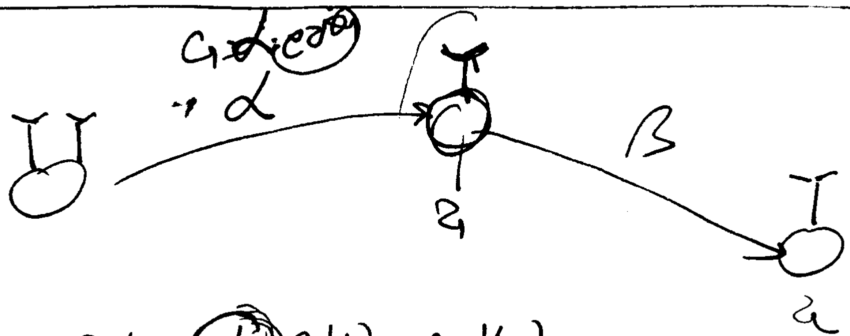
$$(|h_{11}|^2+|h_{12}|^2+|h_{21}|^2+|h_{22}|^2$$



$$\alpha^2 \rightarrow \Delta_1^2$$

$$\alpha^2 \rightarrow \Delta_1^2+\Delta_2^2$$





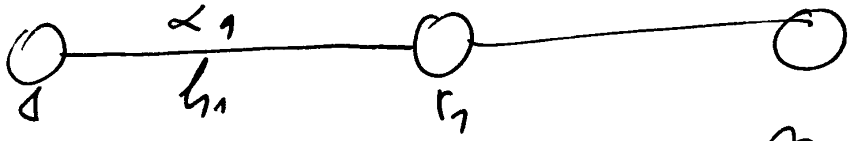
$$r_1(t) = \alpha S(t) + n_1(t)$$

$$r_2(t) = G \cdot \beta\, r_1(t) + n_2(t)$$

$$= G\alpha\beta\, S(t) + \underbrace{G\beta\, n_1(t) + n_2(t)}$$

$$G = \frac{E_2}{E_1\alpha^2 + N_0}$$

$$\Rightarrow \gamma = \frac{\gamma_1 \gamma_2}{\gamma_1+\gamma_2+1}$$



$r_1 = \alpha \cdot 1(t) + n$

$r_1 = h_1 \cdot 1(t) + n(t)$

$\boxed{r_1' = r_1 / h_1 \Rightarrow}$

$y_1 = h_{11} x_1 + h_{12} x_2 + n_{11}$

$\sigma^2 = \frac{E_2 h_2}{E_1 \Delta_1^2 + N_0 \cancel{2}}$ $\quad \Delta_1^2 = |h_{11}|^2 + |h_{12}|^2$

$$\gamma = \frac{E}{N_0} \frac{\sigma^2 \alpha \beta^2}{\sigma^2 \alpha \beta + 1}$$

$f_\alpha(x) \leftarrow$ Гама со SHAPE PARAMETAR n, SCALE PARAMETAR θ моќ

$f_\beta(x) \leftarrow$ Гам. — m

$N_{awgn}^2 \rightarrow$ Гама $\boxed{n_k \sim N(0,\theta)}$

$\sum_{k=1}^{N} n_k^2 \rightarrow$ chi-square with N DOF

$\downarrow$ || Гама со shape param $N/2$

$\sum_{k=1}^{2} n_k^2 \rightarrow$ exp или chi-sq со 2 DOF

Darko Ore 075296917

I-H. Lee, et. al, END-TO-END ANALYSIS FOR DUAL-HOP

$$f_{\gamma_A^{DCF}}(\gamma) = \left(\frac{1}{c\rho}\right)^6 \frac{\gamma^5}{(\beta_1\beta_2)^2} e^{-\frac{(\beta_1+\beta_2)\gamma}{c\rho\beta_1\beta_2}} \left[\left(\frac{\beta_1+\beta_2}{\beta_1\beta_2}\right)^4 \frac{\gamma^2}{18(c\rho)^2} K_4\left(\frac{2\gamma}{c\rho\sqrt{\beta_1\beta_2}}\right)\right.$$

$$+ \left\{\left(\frac{\beta_1+\beta_2}{\beta_1\beta_2}\right)^3 \frac{4\gamma^2}{9(c\rho)^2\sqrt{\beta_1\beta_2}} - \left(\frac{\beta_1+\beta_2}{\beta_1\beta_2}\right)^2 \frac{2\gamma}{3c\rho\sqrt{\beta_1\beta_2}}\right\} K_3\left(\frac{2\gamma}{c\rho\sqrt{\beta_1\beta_2}}\right)$$

$$+ \left\{\left(\frac{\beta_1+\beta_2}{\beta_1\beta_2}\right)^2 \frac{8\gamma^2}{(c\rho)^2\beta_1\beta_2} - \left(\frac{\beta_1+\beta_2}{\beta_1\beta_2}\right)^2 \frac{8\gamma}{3c\rho\beta_1\beta_2} + \frac{2}{3\beta_1\beta_2}\right\} K_2\left(\frac{2\gamma}{c\rho\sqrt{\beta_1\beta_2}}\right)$$

$$\left.+ \left\{\left(\frac{\beta_1+\beta_2}{\beta_1\beta_2}\right) \frac{16\gamma^2}{9(c\rho)^2(\beta_1\beta_2)^{3/2}} - \frac{8\gamma}{3c\rho(\beta_1\beta_2)^{3/2}}\right\} K_1\left(\frac{2\gamma}{c\rho\sqrt{\beta_1\beta_2}}\right) + \frac{8\gamma^2}{9(c\rho\beta_1\beta_2)^2} K_0\left(\frac{2\gamma}{c\rho\sqrt{\beta_1\beta_2}}\right)\right]$$

- USTE EDNAJ IZVEDUVANJE NA PDUT ~~ZA~~ 2+2+2

$$z_1 = \frac{1}{\|H^R\|^2} \qquad z_2 = \frac{1}{\|H^D\|^2} \qquad W = z_1 + z_2 = \frac{1}{\|H^R\|^2} + \frac{1}{\|H^D\|^2}$$

$$\frac{c\rho}{\gamma_A^{DCF}} = \frac{1}{\|H^R\|^2} + \frac{1}{\|H^D\|^2} = W \qquad c\rho = \delta = \epsilon_b N_0$$

$$p_z(z) = \frac{1}{6\beta^4 z^5} e^{-\frac{1}{\beta z}} \qquad Y_1 = \frac{1}{c\rho \cdot \|H_R\|^2} = \frac{1}{\delta \cdot \|H_R\|^2} = \frac{z_1}{\delta}$$

$$p_Y(y) = \left.\frac{p_z(z)}{\frac{\partial y}{\partial z}}\right|_{z=f(y)} \qquad \boxed{y = \frac{z}{\delta}} \qquad \frac{dy}{dz} = \frac{1}{\delta} \qquad \boxed{z = \delta \cdot y}$$

$$p_Y(y) = \frac{1}{6\beta^4 \cdot \delta^4 \cdot y^5} e^{-\frac{1}{\beta\cdot\delta\cdot y}} = \left|\beta\cdot\delta = \alpha\right| = \frac{1}{6\alpha^2 y^5} \cdot e^{-\frac{1}{\alpha\cdot y}}$$

$$M_Y(s) = \frac{s^2}{3\alpha_1^2} K_4\left(2\sqrt{\frac{s}{\alpha_1}}\right) \qquad Q = Y_1 + Y_2 = \frac{1}{\delta\|H_R\|^2} + \frac{1}{\delta\|H_D\|^2}$$

$$M_Q = \frac{s^4}{9\alpha_1^2\alpha_2^2} K_4\left(2\sqrt{\frac{s}{\alpha_1}}\right) K_4\left(2\sqrt{\frac{s}{\alpha_2}}\right) \qquad \alpha_1 = \alpha_2 = \alpha$$

$$M_Q = \frac{s^4}{9\alpha^4}\left[K_4\left(2\sqrt{\frac{s}{\alpha}}\right)\right]^2 = \left|\begin{matrix}\alpha = \beta\cdot\delta \\ = 2\delta\end{matrix}\right| = \frac{s^4}{144\,\delta^4}\left[K_4\left(\sqrt{\frac{2s}{\delta}}\right)\right]^2$$

~~ZA~~ $\beta = 2$

ZA $p=1 \Rightarrow \alpha = p \cdot \delta = \delta$ $\boxed{\alpha=\delta}$

$$\boxed{M_g = \frac{s^4}{g\,\delta^4}\left[K_4\left(2\sqrt{\frac{s}{\delta}}\right)\right]^2}$$

OVA TREBA DA SE KORISTI „p" e z's ($z=x+iy$) VAZHNE PER REAL DIMENSION (31.74)

SKOEVSKA 200171 $\boxed{\text{0P200403}}$ JOSE

$$\cancel{\begin{bmatrix} x_1 & x_2 \\ -x_2 & x_1 \end{bmatrix} * \begin{bmatrix} x_1 & x_2 \\ -x_2 & x_1 \end{bmatrix} = \begin{bmatrix} x_1^2 - x_2^2 ; & x_1x_2 + x_2x_1 \\ -x_2x_1 - x_1x_2 ; & x_1^2 - x_2^2 \end{bmatrix}}$$

$$\cancel{\begin{bmatrix} x_1 & x_2 \\ -x_2 & x_1 \end{bmatrix} .* \begin{bmatrix} x_1 & x_2 \\ -x_2 & x_1 \end{bmatrix} = \begin{bmatrix} x_1^2 & x_2^2 \\ -x_2^2 & x_1^2 \end{bmatrix}}$$

$$\cancel{\begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix} \begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix} = \begin{bmatrix} x_1^2 - |x_2|^2 ; & -x_1x_2^* + |x_1|^2 \\ x_1x_2 + x_1^*x_2 ; & -|x_2|^2 + (x_1^*)^2 \end{bmatrix}}$$

$$\cancel{\begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix} \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix} = \begin{bmatrix} h_1^2 + |h_2|^2 ; & h_1h_2 - h_2h_1^* \\ h_2^*h_1 - h_1^*h_2 ; & |h_2|^2 - |h_1|^2 \end{bmatrix}}$$

$$\boxed{\begin{bmatrix} x_1 & -x_2 \\ x_2 & x_1 \end{bmatrix} * \begin{bmatrix} x_1 & x_2 \\ -x_2 & x_1 \end{bmatrix} = \begin{bmatrix} x_1^2 + x_2^2 & \cancel{x_1x_2} - \cancel{x_1x_2} \\ \cancel{x_2x_1} - \cancel{x_1x_2} & x_1^2 + x_2^2 \end{bmatrix}}$$

$$x = \begin{bmatrix} x_1 & x_2 \\ -x_2 & x_1 \end{bmatrix} \quad \boxed{x' * x = \Delta I \qquad \Delta = x_1^2 + x_2^2}$$

$\Rightarrow$ ORTOGONAL SIGNAL T.E Hadamard MATRIX

GENERALIZACIJA E SO HERMITIAN TRANSPOSE !!! VIDI ZA KNIGATA OD: H. JAFARKHANI

☐ I-H. Lee END-TO-END ANALYSIS OF DUAL-HOP ...

ORTHOGONAL DESIGN E GENERALIZACIJA OD ALAMOUTI !!! (ORTHOGONAL SPACE-TIME CODING) $\boxed{\text{MMV}}$

· KOMPLEKSNITE INDEXI OD ALAMOUTI SE POTREBNI ZA DA IMAJ MODUL eg. $|x_1|^2$ A NE SAMO x_1^2 STO E DOVOLNO ZA REAL ORTHOGONAL DESIGN

$$\sigma^2 = \overline{\left(\xi^2 - \cancel{\bar{\xi}^2}\right)} = \text{mean}\left(\xi^2\right) = E\left[\xi^2\right]$$

$\downarrow 0$

- COMPLEX GAUSSIAN

$z = x + iy$ $\quad (x,y) \rightarrow$ JOINTLY GAUSSIAN i.e. FOLLOWS 2 DIMENSIONAL GAUSSIAN

$$\boxed{m_z = m_x + i\, m_y} \qquad m_z = E[z]$$

$$\sigma^2 = E\left[(x-m_x)^2\right] = E\left[(y-m_y)^2\right] = \frac{1}{2}\left[|z-m_z|^2\right]$$

$\hookrightarrow$ $\underline{z\text{'s VARIANCE PER REAL DIMENSION}}$

$$p(x,y) = \frac{1}{2\pi\sigma^2}\, e^{-\frac{(x-m_x)^2-(y-m_y)^2}{2\sigma^2}}$$

IF: $z = x + iy \rightarrow$

$$\boxed{p(z) = \frac{1}{2\pi\sigma^2}\, e^{-\frac{|z-m_z|^2}{2\sigma^2}}}$$

MMV

$$\boxed{m_z = E[z]} = m_x + i m_y$$

$$y^R = h^R G_{n_t^s} + e^R \qquad y^R = h^R G_{n_t^s} + e^R$$

$$\boxed{y^R = \{y_\ell^R\}_{1\times L} \qquad e^R = \{e_\ell^R\}_{1\times L}} \rightarrow$$

MOZE VEKTOR NOTACIJA DO CLANKOV

A4: top 1.9; bot 4.3 side 1.3
column: 8.8 cm; space betw. colum: 4mm; paragraf 3.5mm

$$Y^D = h^D x^R + E^D \quad \Big\} \quad \underline{\text{SIGNAL AT THE DESTINATION}}$$

$$Y^D = \{y_{i\ell}^D\}_{n_r^D\times L} \qquad E^D = \{e_{i\ell}^D\}_{n_r^D\times L} \qquad x^R = \{\alpha\, y_\ell^R\}_{1\times L}$$

$$\boxed{\alpha = \sqrt{\frac{n_t^s}{\sum_{i=1}^{n_t^s} |h_i^R|^2}}}$$

SIGNAL POWER AT „D"

$$\boxed{P_R \stackrel{\text{def}}{=} \alpha^4 \left(\sum_{i=1}^{n_r^D} |h_i^D|^2\right)^2 \left(\sum_{j=1}^{n_t^s} \left(|h_j^R|^2\right)\right)^2 E\left[|x_k|^2\right]}$$

• NOISE POWER AT DESTINATION

$$n_k = \alpha^2 \left(\sum_{i=1}^{n_r^p} |h_i^p|^2\right)\left(\sum_{i=1}^{n_t^s} |h_i^R|^2\right)\left[\alpha^2\left(\sum_{j=1}^{n_r^p}|h_i^p|^2\right)\sigma^2 + \sigma^2\right]$$

• OVERALL END-END SNR

$$\gamma^{NS}(\rho) = \frac{r_k}{n_k}\cdot\frac{1}{\log_2 M} = c\rho\left[\frac{1}{\sum_{j=1}^{n_t^s}|h_i^R|^2} + \frac{1}{n_t^s \sum_{i=1}^{n_r^p} |h_i^p|^2}\right]^{-1}$$

ρ - TRANSMIT SNR

• END TO END BER ANALYSIS

$$z^R = \frac{1}{\sum_{i=1}^{n_t^s} |h_i^R|^2} \qquad z^P = \frac{1}{n_t^s \sum_{j=1}^{n_r^p} |h_j^p|^2}$$

WIKI: $f(x;\nu) = \dfrac{2^{-\nu/2}}{\Gamma(\nu/2)} x^{-\nu/2-1} e^{-1/2x}$ } INVERSE CHI-SQUARE

OBIČNA chi square

$$\boxed{f(x;k) = \frac{1}{2^{k/2}\Gamma(k/2)} x^{k/2-1} e^{-\frac{x}{2}}}$$

$$f\left(\frac{1}{x};k\right) = \frac{1}{2^{k/2}\Gamma(k/2)}\left(\frac{1}{x}\right)^{\frac{k}{2}-1} e^{-\frac{1}{2x}} = \frac{1}{2^{k/2}\Gamma(k/2)} x^{-\frac{k}{2}+1} e^{-\frac{1}{2x}}$$

~~KRIV~~

$\boxed{y = \frac{1}{x}}$

$$f(y;k) = \frac{f(x;k)}{\left|\frac{\partial y}{\partial x}\right|}\Bigg|_{x=f(y)}$$

$y = \frac{1}{x} \to x = \frac{1}{y}$

$\frac{dy}{dx} = -\frac{1}{x^2} = -y^2$

$$f(y;k) = +\frac{1}{y^2}\cdot\frac{1}{2^{k/2}\Gamma(k/2)}\left(\frac{1}{y}\right)^{\frac{k}{2}-1} e^{-\frac{1}{2y}}$$

$$= \frac{1}{2^{k/2}\Gamma(k/2)}\cdot\left(\frac{1}{y}\right)^{\frac{k}{2}-1+2} e^{-\frac{1}{2y}} = \frac{1}{2^{k/2}\Gamma(k/2)} y^{-\frac{k}{2}-1} e^{-1/2y}$$

DOKAZANO !!!

$$\boxed{f(y;k) = \frac{1}{2^{k/2}\Gamma(k/2)} y^{-\frac{k}{2}-1} e^{-1/2y}} \longrightarrow \text{PDF FOR INVERSE CHI SQUARE}$$

$$f_Z(z) = \frac{1}{2^{k/2}\Gamma(k/2)} \cdot z^{-\frac{k}{2}-1} \cdot e^{-\frac{1}{2z}} \qquad Z = \frac{1}{\sum_{i=1}^{k/2} |h_i|^2}$$

IF $k = 2 \cdot n_t^s = 2 \cdot 4 = 8$

$$f_Z(z) = \frac{1}{2^4 \Gamma(4)} \cdot z^{-4-1} e^{-\frac{1}{2z}} = \frac{1}{16 \cdot 6} z^{-5} \cdot e^{-\frac{1}{2z}}$$

$$\boxed{\int_0^\infty x^{\nu-1} e^{-\frac{\beta}{x} - \gamma x} dx = 2\left(\frac{\beta}{\gamma}\right)^{\nu/2} K_\nu\left(2\sqrt{\beta\gamma}\right)}$$

$$f_Z(z) = \frac{1}{6\beta^4 z^5} e^{-1/\beta z} \qquad M_Z(-s) = E\left[e^{-sz}\right]$$

$$M_Z(-s) = \frac{1}{6\beta^4} \int_0^\infty z^{-5} e^{-\frac{1}{\beta z} - sz} dz = \frac{1}{6\beta^4} \int_0^\infty z^{-4-1} e^{-\frac{1}{\beta z} - sz} dz$$

$$= \frac{2}{6\beta^4} \left(\frac{1}{s\beta}\right)^{-2} \cdot K_4\left(2\sqrt{\frac{s}{\beta}}\right) = \frac{1}{3\beta^2} \cdot s^2 K_4\left(2\sqrt{\frac{s}{\beta}}\right)$$

$$\boxed{M_Z(-s) = \frac{s^2}{3\beta^2} K_4\left(2\sqrt{\frac{s}{\beta}}\right)}$$

$$Z^R = \frac{1}{\sum_{i=1}^{n_t^s} |h_i^R|^2} = \left| n_t^s = 2 \right| = \frac{1}{|h_1^R|^2 + |h_2^R|^2} \qquad \boxed{k = 2}$$

$$f_{Z^R}(z) = \frac{1}{2^1 \cdot 1} \cdot z^{-2} e^{-\frac{1}{2z}} = \frac{1}{2z^2} \cdot e^{-\frac{1}{2z}} = 0.5 z^{-2} e^{-\frac{1}{2z}}$$

$$M_Z(-s) = 0.5 \int_0^\infty z^{-2} e^{-\frac{1}{2z} - sz} dz = 0.5 \cdot \left(\frac{1}{2s}\right)^{-1} \cdot K_2\left(2\sqrt{\frac{s}{2}}\right)$$

$$\boxed{M_Z(-s) = s \cdot K_2\left(\sqrt{2s}\right)}$$

• I-H. Lee

$$M_{Z^R}(s) = \frac{2}{\beta_1^{n_t^s} \cdot \Gamma(n_t^s)} \left(\frac{1}{\beta_1 s}\right)^{-n_t^s/2} K_{n_t^s}\left(2\sqrt{\frac{s}{\beta_1}}\right)$$

76 $n_t^s = 2 \;\; \beta_1 = 1$

$$M_Z^R = \frac{2}{1 \cdot 1} \cdot s \, K_2\left(2\sqrt{s}\right)$$

IF $\beta_1 = 2$

$$M_{z^2}(s) = \frac{2}{4\cdot 1}\cdot 2\cdot s\, K_2\left(\sqrt{2s}\right) = \underline{s\, K_2\left(\sqrt{2s}\right)}$$

- <u>PROAKIS CHI-SQUARE</u>

$$p_Y(y) = \frac{1}{\sigma^n 2^{n/2}\,\Gamma\left(\frac{n}{2}\right)}\, y^{\frac{n}{2}-1} e^{-\frac{y}{2\sigma^2}} \qquad \boxed{\sigma^2 = \frac{\beta}{2}}$$

- INVERSE CHI-SQUARE

$x = \frac{1}{y}$ $\qquad p_x(x) = \frac{p_Y(y)}{\frac{dx}{dy}}$ $\quad y = f(x)$ $\quad \frac{dx}{dy} = -\frac{1}{y^2} = x^2$

$y = 1/x$

$$p_x(x) = \frac{1}{x^2}\,\frac{1}{\sigma^n 2^{n/2}\,\Gamma\left(\frac{n}{2}\right)}\left(\frac{1}{x}\right)^{\frac{n}{2}-1} e^{-\frac{1}{2\sigma^2 x}}$$

$$p_x(x) = \frac{1}{\sigma^n 2^{n/2}\,\Gamma\left(\frac{n}{2}\right)}\cdot\left(\frac{1}{x}\right)^{\frac{n}{2}-1+2} e^{-\frac{1}{2\sigma^2 x}} = \frac{1}{\sigma^n 2^{n/2}\,\Gamma\left(\frac{n}{2}\right)}\, x^{-\frac{n}{2}-1} e^{-\frac{1}{2\sigma^2 x}}$$

$\sigma^2 = \frac{\beta}{2}$ $\quad \sigma = \sqrt{\frac{\beta}{2}}$

$$p_x(x) = \frac{1}{\left(\frac{\beta}{2}\right)^{\frac{n}{2}}\cdot 2^{n/2}\,\Gamma\left(\frac{n}{2}\right)}\cdot x^{-\frac{n}{2}-1} e^{-\frac{1}{\beta x}}$$

$$\boxed{p_x(x) = \frac{1}{\beta^{n/2}\cdot\Gamma\left(\frac{n}{2}\right)}\cdot x^{-\frac{n}{2}-1} e^{-\frac{1}{\beta x}}}$$

INVERSE CHI

<u>I-CHI-SQUARE</u> WITH n-degrees OF FREEDOM

$$\boxed{f(x,a,b) = \frac{1}{b^a\,\Gamma(a)}\, x^{a-1} e^{-\frac{x}{b}}}$$

GAMMA DISTRO

"a"

$\boxed{\text{CHI-SQUARE} = \text{GAMMA DISTRO WITH } \underline{\text{SHAPE FACTOR}}\ \frac{n}{2}}$

- ZA $\boxed{n=2}$ INVERSE CHI-SQUARE

$$p_x(x) = \frac{1}{\beta^1\cdot\Gamma(1)}\cdot x^{-2}\cdot e^{-\frac{1}{\beta x}} = \frac{1}{\beta x^2}\cdot e^{-\frac{1}{\beta x}}$$

$$\frac{1}{\beta}\int_0^\infty x^{-2} e^{-\frac{1}{\beta x}}\cdot e^{-sx}\,dx = \frac{2}{\beta}\cdot\left(\frac{1}{s\cdot\beta}\right)^{-1}\cdot K_2\left(2\sqrt{\frac{s}{\beta}}\right) =$$

$$\boxed{M(-s) = 2\cdot s\, K_2\left(2\sqrt{\frac{s}{\beta}}\right)}$$

$$M_{Z^e}(s) = \frac{2}{\beta_1^{n_t^s}\,\Gamma(n_t^s)}\left(\frac{1}{\beta_1 s}\right)^{-n_t^s/2} K_{n_t^s}\left(2\sqrt{\frac{s}{\beta_1}}\right)$$

$$\boxed{p_{z^e}(z) = \frac{1}{\beta_1^{n_t^s/2}\,\Gamma\left(\frac{n_t^s}{2}\right)} \cdot z^{-\frac{n_t^s}{2}-1}\, e^{-\frac{1}{\beta_1 z}}}$$

(*$)

$$\int_0^\infty x^{\nu-1} e^{-\frac{\beta}{x}-\gamma x}\, dx = 2\left(\frac{\beta}{\gamma}\right)^{\nu/2} K_\nu\left(2\sqrt{\beta\gamma}\right)$$

$$M(-s) = \int_0^\infty \frac{1}{\beta_1^{\frac{n_t^s}{2}}\,\Gamma\left(\frac{n_t^s}{2}\right)} \cdot z^{-\frac{n_t^s}{2}-1}\, e^{-\frac{1}{\beta_1 z}} \cdot e^{-sz}\, dz =$$

TOČNA FORMULA ~~ZA~~ INVERSE CHI-SQUARE

MMV !!!

$$= \frac{1}{\beta_1^{\frac{n_t^s}{2}}\,\Gamma\left(\frac{n_t^s}{2}\right)} \cdot 2\left(\frac{1}{\beta_1 s}\right)^{-\frac{n_t^s}{4}} K_{n_t^s}\left(2\sqrt{\frac{s}{\beta_1}}\right)$$

$$\boxed{M(-s) = \frac{2\cdot s^{+n_t^s/4}}{\beta_1^{n_t^s/4}\,\Gamma\left(\frac{n_t^s}{2}\right)} \cdot K_{\frac{n_t^s}{2}}\left(2\sqrt{\frac{s}{\beta_1}}\right)}$$

$\underline{n_{ts} = 4}$

$$\boxed{M(-s) = \frac{s}{\beta_1} K_2\left(2\sqrt{\frac{s}{\beta_1}}\right)} = \frac{2s}{\beta_1} K_2\left(2\sqrt{\frac{s}{\beta_1}}\right)$$

$$w = \frac{1}{z^e} + \frac{1}{z^p} \Rightarrow \boxed{M_w(-s) = \frac{4s^2}{\beta_1^2}\left[K_2\left(2\sqrt{\frac{s}{\beta_1}}\right)\right]^2}$$

$$p_{z^e}(z) = \frac{1}{\beta_1^2\,\Gamma(2)} \cdot z^{-3} e^{-\frac{1}{\beta_1 z}} = \frac{1}{\beta_1^2} z^{-3} e^{-\frac{1}{\beta_1 z}}$$

$$M(-s) = \frac{1}{\beta_1^2}\int_0^\infty z^{-3} e^{-\frac{1}{\beta_1 z} - sz}\, dz = \frac{1}{\beta_1^2} \cdot 2\left(\frac{1}{\beta_1 s}\right)^{-1} K_{+2}\left(2\sqrt{\frac{s}{\beta_1}}\right)$$

$$M(-s) = \frac{2s}{\beta_1} \cdot K_2\left(2\sqrt{\frac{s}{\beta_1}}\right) \qquad \boxed{M_w(-s) = \frac{4s}{\beta_1^2}\left[K_2\left(2\sqrt{\frac{s}{\beta_1}}\right)\right]^2}$$

(*$)

• I-H. Lee ODI SO FORMULATA I ZEMA $n_t^s = 8$

$$p_{z^e}(z) = \frac{1}{\beta^{n_t^s/2}\,\Gamma\left(\frac{n_t^s}{2}\right)} z^{-\frac{n_t^s}{2}-1} e^{-\frac{1}{\beta_1 z}} \qquad \Gamma\left(\frac{n_t^s}{2}\right) = \Gamma(4) = 6$$

ZOL: $n_t^s = 8$ $\boxed{p_{z^e}(z) = \frac{1}{\beta^4 \cdot 6} \cdot z^{-5} e^{-\frac{1}{\beta_1 z}}}$

NE ZNAM ZOŠTO [illegible] LEE $n_t^s = 8$!?

$$\boxed{P_{out} = 1 - \mathcal{L}^{-1}\left[\frac{M(-s)}{s}\right]} \qquad p_x(x) = \frac{d P_x(x)}{dx} \Big/ \mathcal{L}$$

laplace Λ $P(X<x)$

$$\int_{-\infty}^{\infty} p_x(x)\cdot e^{-sx}\,dx = \int_{-\infty}^{\infty} \frac{dP_x(x)}{dx}\, e^{-sx}\,dx$$

$$\underbrace{\qquad}_{\boxed{M(-s)}} \qquad \underbrace{\qquad}_{s\cdot \hat{P}_x(s)}$$

$$\hat{P}_x(s) = \frac{M(-s)}{s}$$

$$P_{out_{1/x}} = P(X \leq x) = \int_0^x p_x(x)\,dx$$

$$P_{out\,1/x} = P\left(\frac{1}{X} < \frac{1}{x}\right) = P(X > x) = 1 - P(X<x)$$

$$\boxed{P_{out\,1/x} = 1 - P(X<x) = 1 - \mathcal{L}^{-1}\left[\frac{M(-s)}{s}\right]}$$

- $$p_{z^R} = \frac{1}{\beta^{n_t^s/2}\,\Gamma\left(\frac{n_t^s}{2}\right)} \cdot z^{-\frac{n_t^s}{2}-1}\, e^{-\frac{1}{\beta_1 z}} \qquad \boxed{n_t^s = 8}$$

$$\boxed{p_z^R = \frac{1}{\beta^4\cdot \cancel{\Gamma(4)}_{6}} \cdot z^{-5}\cdot e^{-\frac{1}{\beta_1 z}}} \qquad \boxed{\begin{aligned} z_1 &= \frac{1}{\|H^R\|^2} \\ z_2 &= \frac{1}{\|H^D\|^2}\end{aligned}}$$

γ

$$\frac{1}{z} = \frac{1}{\gamma\|H_R\|^2} + \frac{1}{\gamma\|H_D\|^2} \qquad \cancel{\frac{z_1}{\gamma} + \frac{z_2}{\gamma}}$$

$$\frac{\gamma}{z} = \frac{1}{\|H_R\|^2} + \frac{1}{\|H_D\|^2}$$

$$\frac{1}{z} = \frac{1}{\gamma\|H_R\|^2} + \frac{1}{\gamma\|H_D\|^2} = \frac{z_1}{\gamma} + \frac{z_2}{\gamma} = \mu_1 + \mu_2$$

$$p_{z_1} = \frac{1}{6\beta^4\cdot \cancel{\ }}\, z_1^{-5}\, e^{-\frac{1}{\beta_1 z}} \qquad p_{z_2} = \frac{1}{6\beta^4}\, z_2^{-5}\, e^{-\frac{1}{\beta_1 z_2}}$$

$$p_{\mu_1} = \frac{p_{z_1}(z_1)}{\frac{d\mu_1}{dz_1}} \left|\begin{aligned} &\frac{d\mu_1}{dz_1} = \frac{1}{\gamma} \\ &z_1 = f(\mu_1) \\ &z_1 = \gamma\mu_1 \end{aligned}\right| = \gamma\cdot \frac{1}{6\beta^4(\gamma\cdot\mu_1)^5}\cdot e^{-\frac{1}{\beta_1(\gamma\mu_1)}} =$$

$$= \frac{1}{6\cdot\beta^4\gamma^4\mu_1^5}\, e^{-1/\beta_1\gamma\mu_1} = \frac{1}{6\alpha^4\mu_1^5}\cdot e^{-\frac{1}{\alpha_1\mu_1}}$$

$$M(-s) = \frac{1}{6\alpha^4}\int_0^{\infty} \mu_1^{-4-1}\, e^{-\frac{1}{\alpha_1\mu_1} - \mu_1 s}\, d\mu_1 = \frac{\cancel{2}}{\cancel{6}_3\,\alpha^4}\cdot\left(\frac{1}{\alpha_1 s}\right)^{-2}\cdot K_4\left(2\sqrt{\frac{s}{\alpha_1}}\right)$$

$$M(-1) = \frac{1}{3\alpha_1^4} \cdot \alpha_1^2 1^2 K_4\left(2\sqrt{\frac{1}{\alpha_1}}\right) = \frac{1^2}{3\alpha_1^2} \cdot K_4\left(2\sqrt{\frac{1}{\alpha_1}}\right)$$

$$M_W(-1) = M_1(-1) \cdot M_2(-1) = \frac{1^4}{9\alpha_1^2\alpha_2^2} \cdot K_4\left(2\sqrt{\frac{1}{\alpha_1}}\right) K_4\left(2\sqrt{\frac{1}{\alpha_2}}\right)$$

$\alpha_1 = \alpha_2$

$$M_W(-1) = \frac{1^4}{9\alpha^4} \cdot \left[K_4\left(2\sqrt{\frac{1}{\alpha}}\right)\right]^2 = \left| \begin{array}{l} \alpha = \beta \cdot \gamma \\ \beta = 1 \\ \alpha = \gamma \end{array} \right| = \frac{1^4}{9\gamma^4}\left[K_4\left(2\sqrt{\frac{1}{\gamma}}\right)\right]^2$$

STO AKO:

$$P_{z^2} = \frac{1}{\beta^{n_t^s} \cdot \Gamma(n_t^s)} z^{-n_t^s - 1} e^{-\frac{1}{\beta_1 z}}$$

$$\int_0^\infty z^{-\nu-1} e^{-\frac{\beta}{z} - \gamma z} dz = 2\left(\frac{\beta}{\gamma}\right)^{-\frac{\nu}{2}} K_\nu(2\sqrt{\beta\gamma})$$

$$\frac{1}{\beta^{n_t^s} \Gamma(n_t^s)} \int z^{-n_t^s - 1} e^{-\frac{1}{\beta_1 z} - 1 z} dz = \frac{2}{\beta_1^{n_t^s} \Gamma(n_t^s)} \left(\frac{1}{\beta_1 1}\right)^{-\frac{n_t^s}{2}} \cdot K_{n_t^s}\left(2\sqrt{\frac{1}{\beta_1}}\right)$$

- PROVERI ~~ZA~~ KOJA DISTRIBUCIJA SE DOBIVA AKO SE SUMIRAAT (COMPLEX GAUSSIAN)^2 ???

- DA TOA E SUŠTINATA !!!

{DA OVA E OK NO H-I LEE TVRDI DEKA E CHI-SQUARE TOJ GREŠI !!!

|COMPLEX GAUSIAN| $\triangleq$ RAYLEIGH

[RAYLEIGH]^2 ~ ~~GAMMA~~ EXPONENTIAL DISTRO

$$\Delta_2 = \underbrace{|h_{11}|^2}_{\text{EXPON.D}} + \underbrace{|h_{21}|^2}_{\text{EXP.D}} + |h_{12}|^2 + |h_{22}|^2$$

SUM OF EXP. DISTROS $\triangleq$ GAMMA DISTRO WITH 4 DEGREES

NMU

MOREOVER: NAKAGAMI^2 IID GAMMA$_3$ (NG.PP B)

- OVA E ODGOVOR NA TOA ZOŠTO H.-M. LEE GO KORISTI IZRAZOT (10) VO DCF CLANAKOT

RAYLEIGH:

$$p_\alpha(\alpha) = \frac{\alpha}{\sigma^2} e^{-\frac{\alpha^2}{2\sigma^2}} = \left| \begin{array}{l} \Omega = 2\sigma^2 \\ \sigma^2 = \frac{\Omega}{2} \end{array} \right| = \frac{2\alpha}{\Omega} e^{-\frac{\alpha^2}{\Omega}}$$

$$\gamma = \frac{E_b}{N_0} \cdot \alpha^2 \qquad \bar{\gamma} = \frac{E_b}{N_0} \cdot \Omega \qquad \underline{\underline{\Omega = E[\alpha^2]}}$$

$$\frac{E_b}{N_0} = \frac{\bar{\gamma}}{\Omega} \qquad \gamma = \frac{\bar{\gamma}}{\Omega} \cdot \alpha^2 \qquad \alpha = \pm\sqrt{\frac{\Omega\gamma}{\bar{\gamma}}} \qquad \alpha^2 = \frac{\Omega}{\bar{\gamma}} \cdot \gamma \qquad -\frac{1}{\Omega} \cdot \frac{\Omega}{\bar{\gamma}} \cdot \gamma$$

$$p_\gamma(\gamma) = \left.\frac{p_\alpha(\alpha)}{\left|\frac{d\gamma}{d\alpha}\right|}\right|_{\alpha = +\sqrt{\frac{\Omega\gamma}{\bar{\gamma}}}} = \frac{1}{\frac{\bar{\gamma}}{\cancel{\Omega}} \cdot \cancel{2\alpha}} \frac{\cancel{2\alpha}}{\cancel{\Omega}} \cdot e$$

$$\boxed{p_\gamma(\gamma) = \frac{1}{\bar{\gamma}} \cdot e^{-\gamma/\bar{\gamma}}}$$

<u>EXP. DISTR</u>

- ~~DEFINITION~~ DEFINITION OF EXPONENTIAL DISTRO

$$p(x;\lambda) = \lambda e^{-\lambda x} \quad x \geq 0$$

$$\boxed{\bar{x} = \frac{1}{\lambda} = \bar{\gamma}} \qquad \boxed{\sigma^2 = \frac{1}{\lambda^2} = \bar{\gamma}^2}$$

$$\boxed{M(s) = \frac{1}{1 - s/\lambda} = \frac{1}{1 - s\bar{\gamma}}}$$

- ~~DA~~ IMAM ZA 2 STEPENI NA SLOBODA:

$$\boxed{\Delta_1 = |h_{11}|^2 + |h_{21}|^2} \qquad \Delta_1 = X_1 + X_2$$

EXPONENTIAL

$$\boxed{p_{x_i}(x_i) = \frac{1}{\bar{\gamma}} \cdot e^{-x_i/\bar{\gamma}}} \qquad E[x_1] = E[x_2] = \bar{\gamma}$$

θ PARAMETER OF GAMMA DISTRO

k - SHAPE PARAM.

θ - SCALE PARAM.

$$\boxed{Y \sim \Gamma(k, \theta) \quad \text{OR} \quad Y \sim \text{Gamma}(k, \theta)}$$

$$f(x; k, \theta) = \frac{x^{k-1}}{\theta^k \Gamma(k)} e^{-x/\theta} = \frac{x^{a-1}}{b^a \Gamma(a)} e^{-\frac{x}{b}}$$

MATLAB NOTACIJA

- INVERSE GAMMA

$$f(x; k, \bar{\gamma}) = \frac{x^{k-1}}{\bar{\gamma}^k \Gamma(k)} \cdot e^{-\frac{x}{\bar{\gamma}}}$$

$$\boxed{f(\gamma; k, \bar{\gamma}) = \frac{\gamma^{k-1}}{\bar{\gamma}^k \Gamma(k)} e^{-\gamma/\bar{\gamma}}}$$

$$\boxed{f(\gamma;k,\bar\gamma) = \frac{\gamma^{k-1}}{\bar\gamma^k\,\Gamma(k)}\, e^{-\gamma/\bar\gamma}} \quad \boxed{\text{GAMMA}}$$

$$\int_0^\infty x^{\nu-1} e^{-\frac{\beta}{x}-x\cdot\gamma}\,dx = 2\left(\frac{\beta}{\gamma}\right)^{\nu/2} K_\nu(2\sqrt{\beta\gamma})$$

- INVERSE

$$z = \frac{1}{\gamma} \qquad p(z) = \frac{f(\gamma;k,\bar\gamma)}{\left|\frac{dz}{d\gamma}\right|}\Bigg|_{\gamma=\frac{1}{z}} \qquad \frac{dz}{d\gamma} = -\frac{1}{\gamma^2}$$

$$p_z(z) = \gamma^2 f(\gamma;k,\bar\gamma) = \frac{1}{z^2}\cdot\frac{1}{\bar\gamma^k\,\Gamma(k)}\cdot\left(\frac{1}{z}\right)^{k-1} e^{-\frac{1}{z\bar\gamma}}$$

$$\boxed{p_z(z) = \frac{1}{\bar\gamma^k\,\Gamma(k)}\, z^{-k-1}\, e^{-1/z\bar\gamma}} \quad \$\#$$

INVERSE GAMMA $\$\Delta$

$$\boxed{M_z(s) = \frac{2}{\Gamma(k)}\left(\frac{s}{\bar\gamma}\right)^{k/2} K_k\left(2\sqrt{\frac{s}{\bar\gamma}}\right)}$$

$$MGF_z(-s) = \frac{1}{\bar\gamma^k\,\Gamma(k)}\int_0^\infty z^{-k-1} e^{-\frac{1}{z\bar\gamma}-sz}\,dz = \frac{1}{\bar\gamma^k\,\Gamma(k)}\, 2\left(\frac{1}{\bar\gamma s}\right)^{-k/2} K_{-k}\left(2\sqrt{\frac{s}{\bar\gamma}}\right)$$

$$= \frac{2}{\bar\gamma^k\,\Gamma(k)}(\bar\gamma s)^{k/2} K_k\left(2\sqrt{\frac{s}{\bar\gamma}}\right)$$

- NAVRAĆANJE NA $M_{1/\gamma}(s)$ ZA GAMMA FIG NAKAGAMI

$$p_\gamma(\gamma) = \frac{m^m\gamma^{m-1}}{\Gamma(m)\,\bar\gamma^m}\, e^{-\frac{m\gamma}{\bar\gamma}} \qquad \text{SO ZAMENA} \quad \bar\gamma_1 = \frac{\bar\gamma}{m} \qquad \boxed{\text{MMV}}$$

$$\boxed{p_\gamma(\gamma) = \frac{\gamma^{m-1}}{\Gamma(m)\,\bar\gamma_1^m}\, e^{-\frac{\gamma}{\bar\gamma_1}}} \Longrightarrow \text{ČISTA GAMMA}$$

- SO UPOTREBA NA $\$\Delta$ ZA MGF SE DOBIVA ✓

$$M_z(-s) = \frac{2}{\Gamma(m)}\left(\frac{s}{\bar\gamma_1}\right)^{m/2} K_m\left(2\sqrt{\frac{s}{\bar\gamma_1}}\right) = \frac{2}{\Gamma(m)}\left(\frac{s\cdot m}{\bar\gamma}\right)^{m/2} K_m\left(2\sqrt{\frac{s\,m}{\bar\gamma}}\right)$$

- $\Delta_2 = |h_{11}|^2 + |h_{21}|^2 + |h_{12}|^2 + |h_{22}|^2 \qquad \boxed{k=4} \qquad \boxed{z_2 = \frac{1}{\Delta_2}}$

$$M_z(-s) = \frac{2}{\Gamma(4)}\left(\frac{s}{\bar\gamma}\right)^2 K_4\left(2\sqrt{\frac{s}{\bar\gamma}}\right) = \frac{s^2}{3\bar\gamma^2} K_4\left(2\sqrt{\frac{s}{\bar\gamma}}\right)$$

- $\Delta_1 = |h_{11}|^2 + |h_{21}|^2 \qquad k=2 \qquad \boxed{z_1 = \frac{1}{\Delta_1}}$

$$\boxed{M_z(-s) = \frac{2s}{\bar\gamma}\cdot K_2\left(2\sqrt{\frac{s}{\bar\gamma}}\right)}$$

$$\boxed{M_{z_1+z_2}(-s) = \frac{2s^3}{3\bar\gamma^3} K_4\left(2\sqrt{\frac{s}{\bar\gamma}}\right) K_2\left(2\sqrt{\frac{s}{\bar\gamma}}\right)} \quad ✓$$

$$\boxed{M_{z_1+z_2}(-s) = \frac{s^4}{9\bar{\gamma}^4} \cdot K_4\left(2\sqrt{\frac{\gamma}{\bar{\gamma}}}\right)}$$

- **Pout FOR 2+2x1 SYSTEM**

$$\boxed{P_{out\_2+2\times1} = 1 - \mathcal{L}^{-1}\left[\frac{M_{z_1+z_2}(-s)}{s}\right]} \qquad \delta$$

- **Pout FOR 2+2 SYSTEM**

$$\dot{\gamma}_2 = \frac{\epsilon}{N_0}\cdot \dot{\Delta}_2 = \frac{\epsilon}{N_0}\left(\underbrace{|h_{11}|^2 + |h_{21}|^2 + |h_{12}|^2 + |h_{22}|^2}_{\text{GAMMA } 4\text{ DoF}}\right)$$

exp exp exp exp

$$p_{\dot{\Delta}_2}(\delta) = \frac{\delta^{n-1}}{\bar{\delta}^n \Gamma(n)} e^{-\delta/\bar{\delta}} = \Big/ n=4 \Big/ = \frac{\delta^3}{\bar{\delta}^4 6} \cdot e^{-\delta/\bar{\delta}}$$

$$\boxed{p_{\dot{\Delta}_2}(x) = \frac{\delta^3}{6\bar{\delta}^4} e^{-\delta/\bar{\delta}}}$$

$$\frac{\dot{\gamma}_2}{\bar{\gamma}} = \dot{\Delta}_2 \qquad \frac{\gamma}{\bar{\gamma}} = \delta \qquad \gamma = \bar{\gamma}\cdot\delta$$

$$p_{\dot{\gamma}_2}(\gamma) = \frac{p_{\dot{\Delta}_2}(\delta)}{\left|\frac{\partial \gamma}{\partial \delta}\right|}\Bigg|_{\delta = \frac{\gamma}{\bar{\gamma}}} = \frac{1}{\bar{\gamma}} \cdot \frac{\delta^3}{6\bar{\delta}^4} \cdot e^{-\delta/\bar{\delta}}$$

$$p_{\dot{\gamma}_2}(\gamma) = \frac{1}{\bar{\gamma}} \cdot \frac{\gamma^3}{\bar{\gamma}^3 \cdot 6 \cdot \bar{\delta}^4} \cdot e^{-\gamma/\bar{\gamma}^2} = \frac{\gamma^3}{6\bar{\gamma}^8} e^{-\gamma/\bar{\gamma}^2}$$

$$P(\gamma < \gamma_0) = \int_0^{\gamma_0} p_{\dot{\gamma}_2}(\gamma)\, d\gamma = \frac{1}{6\bar{\gamma}^8}\int_0^{\gamma_0} \gamma^3 e^{-\gamma/\bar{\gamma}^2}$$

$$P(\gamma < \gamma_0) = \frac{1}{6\bar{\gamma}^6}\left(6\bar{\gamma}^6 e^{\gamma_0/\bar{\gamma}^2} - 6\bar{\gamma}^6 - 3\bar{\gamma}^2\gamma_0^2 - 6\bar{\gamma}^4\gamma_0 - \gamma_0^3\right)e^{-\frac{\gamma_0}{\bar{\gamma}^2}}$$

$$= 1 - \left(\frac{6\bar{\gamma}^6 + 3\bar{\gamma}^2\gamma_0^2 + 6\bar{\gamma}^4\gamma_0 + \gamma_0^3}{6\bar{\gamma}^6}\right)e^{-\gamma_0/\bar{\gamma}^2}$$

$$\boxed{P(\gamma < \gamma_0) = 1 - \left(6\bar{\gamma}^6 + 3\bar{\gamma}^2\gamma_0^2 + 6\bar{\gamma}^4\gamma_0 + \gamma_0^3\right)\frac{e^{-\gamma_0/\bar{\gamma}^2}}{6\bar{\gamma}^6}}$$

GO SMOLEDIV ! IMA OTSTAPUVANJE OD EMPIRISKITE REZULTATI !!!

- P_{out} FOR 2x1 SYSTEM

$$\boxed{\begin{matrix}\gamma = \bar{\gamma}\cdot\delta \\ \delta = \frac{\gamma}{\bar{\gamma}}\end{matrix}}$$

$$\dot{\gamma}_1 = \frac{\epsilon}{N_0}\cdot\dot{\Delta}_1 \qquad \dot{\Delta}_1 = |h_{11}|^2 + |h_{21}|^2$$

$$p_{\dot{\Delta}_1}(\delta) = \frac{\delta^{k-1}}{\bar{\gamma}^k\,\Gamma(k)}\cdot e^{-\delta/\bar{\gamma}} = \left|k=2\right| = \frac{\delta}{\bar{\gamma}^2}\cdot e^{-\delta/\bar{\gamma}}$$

$$p_{\dot{\gamma}_1}(\gamma) = \frac{p_{\dot{\Delta}_1}(\delta)}{\frac{d\gamma}{d\delta}}\Bigg|_{\gamma=\bar{\gamma}\cdot\delta} = \frac{1}{\bar{\gamma}}\cdot\frac{\delta}{\bar{\gamma}^2}\cdot e^{-\delta/\bar{\gamma}}\Bigg/_{\delta=\frac{\gamma}{\bar{\gamma}}} = \cancel{\bar{\gamma}}\cdot\frac{\gamma}{\bar{\gamma}^4}\cdot e^{-\gamma/\bar{\gamma}^2}$$

$$\boxed{p_{\dot{\gamma}_1}(\gamma) = \frac{\gamma}{\bar{\gamma}^4}\,e^{-\gamma/\bar{\gamma}^2}}$$

$$P_{out} = P(\gamma<\gamma_0) = \int_0^{\gamma_0} p_{\dot{\gamma}_1}(\gamma)\,d\gamma$$

$$P_{out-2x1} = 1 - e^{-\frac{\gamma_0}{\bar{\gamma}^2}} \cdot \frac{\gamma_0}{\bar{\gamma}^2}\,e^{-\frac{\gamma_0}{\bar{\gamma}^2}}$$

MANCA

$$\boxed{P_{out\_2x1} = 1 - \left(1+\frac{\gamma_0}{\bar{\gamma}^2}\right)e^{-\gamma_0/\bar{\gamma}^2}}$$

DA TO PROVERAM INVERZNOM $\mathcal{L}$ TR.

$$p_x(x) = \frac{dP_x(x)}{dx}\ \Big/\ \mathcal{L} \qquad M(-s) = s\cdot\hat{P}_x(s)$$

$$\boxed{\hat{P}_x(s) = \frac{M(-s)}{s}} \qquad \boxed{s = \frac{A+2\pi j y}{2x}} \qquad \boxed{x = \frac{1}{\gamma_0}}$$

- SE VRAĆAM PONOVO NA 2x2x2

$$\hat{z}_1 = \sqrt{\epsilon}\,\dot{\Delta}_2 x_1 + \dot{\eta}_1 \qquad \dot{\Delta}_2 = |h_{11}|^2 + |h_{21}|^2 + |h_{12}|^2 + |h_{22}|^2$$

$$\dot{\eta}_1 = h_{11}^*\eta_{11} + h_{21}\eta_{21}^* + h_{12}^*\eta_{12} + h_{22}\eta_{22}^*$$

$$\hat{\hat{z}}_1 = G_2\ddot{\Delta}_2\hat{z}_1 + \ddot{\eta}_1 = G_2\ddot{\Delta}_2\left(\sqrt{\epsilon}\,\dot{\Delta}_2 x_1 + \dot{\eta}_1\right) + \ddot{\eta}_1 =$$

$$= G_2\ddot{\Delta}_2\sqrt{\epsilon}\,\dot{\Delta}_2 x_1 + G_2\ddot{\Delta}_2\dot{\eta}_1 + \ddot{\eta}_1$$

$$\gamma_{eq3} = \frac{G_2^2\ddot{\Delta}_2^2\cdot\epsilon\cdot\dot{\Delta}_2^2}{G_2^2\ddot{\Delta}_2^2\dot{\Delta}_2 N_0 + \ddot{\Delta}_2 N_0} = \frac{\epsilon}{N_0}\,\frac{G_2^2\ddot{\Delta}_2\cdot\dot{\Delta}_2^2}{G_2^2\ddot{\Delta}_2\dot{\Delta}_2 + 1}$$

$$G_2^2 = \frac{\epsilon}{\epsilon\dot{\Delta}_2^2 + \dot{\Delta}_2 N_0}$$

$$\boxed{G_2^2 \doteq \frac{1}{\dot{\Delta}_2^2}}$$

$$\gamma_{eq_3} = \frac{\epsilon}{N_0} \frac{\cancel{\frac{1}{\dot{\Delta}_2^2}}\, \ddot{\Delta}_2 \cancel{\dot{\Delta}_2^2}}{\frac{1}{\dot{\Delta}_2^2} \cdot \ddot{\Delta}_2 \cancel{\dot{\Delta}_2} + 1}$$

$$\boxed{\gamma_{eq_3} = \frac{\epsilon}{N_0} \frac{\dot{\Delta}_2 \ddot{\Delta}_2}{\ddot{\Delta}_2 + \dot{\Delta}_2}}$$

$$\gamma_{eq_3} = \frac{\epsilon}{N_0} \left(\frac{1}{\dot{\Delta}_2} + \frac{1}{\ddot{\Delta}_2} \right)^{-1}$$

$$\frac{\epsilon_B N_0}{\gamma_{eq}} = \frac{1}{\dot{\Delta}_2} + \frac{1}{\ddot{\Delta}_2}$$

$$\boxed{\frac{1}{\gamma_{eq}} = \frac{1}{\epsilon_B N_0 \dot{\Delta}_2} + \frac{1}{\epsilon_B N_0 \ddot{\Delta}_2}} \quad \#*$$

$$\frac{1}{\gamma_{eq}} = \frac{1}{\bar{\gamma} \cdot \dot{\Delta}_2} + \frac{1}{\bar{\gamma} \cdot \ddot{\Delta}_2} = z_1' + z_2'$$

$$z_1' = \frac{z_1}{\bar{\gamma}} \qquad z_2' = \frac{z_2}{\bar{\gamma}} \qquad z_i' = \frac{z_i}{\bar{\gamma}}$$

$$p_{z_i}(z) = \frac{1}{\bar{\gamma}^n \Gamma(n)} \cdot z_i^{-n-1} e^{-\frac{1}{z_i \bar{\gamma}}}$$

$$p_{z_i'}(z_i') = \frac{p(z_i)}{\left| \frac{d z_i'}{d z_i} \right|} = \bar{\gamma} \cdot \frac{1}{\bar{\gamma}^n \Gamma(n)} z_i^{-n-1} \cdot e^{-\frac{1}{z_i \bar{\gamma}}}$$

$$z_i = \bar{\gamma} \cdot z_i'$$

$$n - 1 + n + 1 = 2n$$

$$p_{z_i'}(z_i') = \frac{1}{\bar{\gamma}^{n-1} \Gamma(n)} \cdot \bar{\gamma}^{-n-1} z_i'^{-n-1} \cdot e^{-\frac{1}{z_i' \bar{\gamma}^2}}$$

$$\boxed{p_{z_i'}(z_i') = \frac{1}{\bar{\gamma}^{2n} \Gamma(n)} \cdot z_i'^{-n-1} \cdot e^{-\frac{1}{z_i' \bar{\gamma}^2}}}$$

$$\boxed{\int_0^\infty x^{\nu-1} e^{-\frac{\beta}{x} - \gamma x} dx = 2 \left(\frac{\beta}{\gamma} \right)^{\nu/2} K_\nu \left(2\sqrt{\beta \cdot \gamma} \right)}$$

$$M(-s) = \frac{1}{\bar{\gamma}^{2n} \Gamma(n)} \int_0^\infty z_i'^{-n-1} e^{-\frac{1}{z_i' \bar{\gamma}^2} - s z_i'} d z_i' = \underline{\frac{2}{\bar{\gamma}^{2n} \Gamma(n)} \left(\frac{1}{s \bar{\gamma}^2} \right)^{-n/2} \cdot K_n \left(2 \sqrt{\frac{s}{\bar{\gamma}^2}} \right)}$$

$$\boxed{n = 4}$$

$$M(-s) = \frac{\cancel{2} \cdot s^2 \cancel{\bar{\gamma}^4}}{3\cancel{6}\, \bar{\gamma}^8 4} \cdot K_4 \left(2 \sqrt{\frac{s}{\bar{\gamma}^2}} \right) = \underline{\frac{s^2}{3 \bar{\gamma}^4} \cdot K_4 \left(2 \sqrt{\frac{s}{\bar{\gamma}^2}} \right)}$$

$$\boxed{M_{z_1' + z_2'}(s) = \frac{s^4}{9 \bar{\gamma}^8} \left[K_4 \left(2 \sqrt{\frac{s}{\bar{\gamma}^2}} \right) \right]^2}$$

$$\gamma_{eq_3} = \frac{\epsilon}{N_0} \frac{G_2^2 \ddot{\Delta}_2 \dot{\Delta}_2^2}{G_2^2 \ddot{\Delta}_2 \dot{\Delta}_2 + 1} \qquad G_2 = \sqrt{\frac{\epsilon}{\epsilon \dot{\Delta}_2^2 + \dot{\Delta}_2 N_0}}$$

$$\gamma_{eq_3} = \frac{\epsilon}{N_0} \cdot \cancel{\frac{\epsilon}{\epsilon \Delta_2}} \cdot \frac{\ddot{\Delta}_2 \dot{\Delta}_2^2}{\ddot{\Delta}_2 \dot{\Delta}_2 + \frac{\epsilon \dot{\Delta}_2^2 + \dot{\Delta}_2 N_0}{\epsilon}}$$

$$\gamma_{eq_3} = \frac{\epsilon}{N_0} \frac{\ddot{\Delta}_2 \dot{\Delta}_2^2}{\ddot{\Delta}_2 \dot{\Delta}_2 + \dot{\Delta}_2^2 + \dot{\Delta}_2 \cdot \frac{N_0}{\epsilon}} = \frac{\epsilon}{N_0} \frac{\ddot{\Delta}_2 \dot{\Delta}_2}{\ddot{\Delta}_2 + \dot{\Delta}_2 + \frac{N_0}{\epsilon}}$$

$$\gamma_{eq_3} = \frac{\epsilon}{N_0} \left[\frac{1}{\dot{\Delta}_2} + \frac{1}{\ddot{\Delta}_2} + \frac{N_0}{\epsilon} \right]^{-1} \qquad \frac{\epsilon 6 N_0}{\gamma_{eq_3}} = \frac{1}{\dot{\Delta}_2} + \frac{1}{\ddot{\Delta}_2} + \frac{1}{\epsilon 6 N_0 \ddot{\Delta}_2 \dot{\Delta}_2}$$

$$\boxed{\frac{1}{\gamma_{eq_3}} = \frac{1}{\bar{\gamma} \dot{\Delta}_2} + \frac{1}{\bar{\gamma} \cdot \ddot{\Delta}_2} + \frac{1}{\ddot{\Delta}_2 \dot{\Delta}_2}}$$

SCALE ~~PARAM~~

$$W = \underbrace{\dot{\Delta}_2}_{\text{GAMMA}} \cdot \underbrace{\ddot{\Delta}_2}_{\text{GAMMA}}$$

SHAPE ~~PARAMETER~~

(PAPER FROM JOHN MALIK)

- GENERALIZED GAMMA DISTRO:

$$f(x; a, d, \gamma) = \gamma \frac{1}{a^d \, \Gamma\left(\frac{d}{\gamma}\right)} x^{d-1} e^{-\left(\frac{x}{a}\right)^{\gamma}} \qquad \begin{array}{l} x > 0 \\ a, d, \gamma > 0 \end{array}$$

$$U = X \cdot Y$$

$$f(u) = 2\gamma u^{d_2 - 1} \left[\Gamma\left(\frac{d_1}{\gamma}\right) \Gamma\left(\frac{d_2}{\gamma}\right) (a_1 a_2)^{d_2} \right]^{-1} \cdot \left[\left(\frac{u}{a_1 a_2}\right)^{\frac{\gamma}{2}} \right]^{-\left(\frac{d_2}{\gamma} - \frac{d_1}{\gamma}\right)}$$

$$\cdot K_{\frac{d_2}{\gamma} - \frac{d_1}{\gamma}} \left[2 \left(\frac{u}{a_1 a_2}\right)^{\gamma/2} \right]$$

$$d_1 = d_2 = d \qquad a_1 = a_2 = a \qquad \gamma = 1$$

$$f(u) = \frac{2 u^{d-1}}{\Gamma^2(d) \cdot a^{2d}} \left(\frac{u}{a^2}\right)^{0} K_0 \left[2 \left(\frac{u}{a^2}\right)^{\frac{1}{2}} \right]$$

$$f(u) = \frac{2 u^{d-1}}{\Gamma^2(d) \cdot a^{2d}} \cdot K_0 \left[2 \left(\frac{u}{a^2}\right)^{\frac{1}{2}} \right]$$

$$\boxed{f(u) = \frac{2 \cdot u^{n-1}}{\Gamma^2(n) \, \bar{\gamma}^{2d}} K_0 \left[2 \sqrt{\frac{u}{\bar{\gamma}^2}} \right]}$$

DISTRIBUTION OF PRODUCTS OF GAMMA VARIATES

$\boxed{n=4}$ $\boxed{f(\mu) = \frac{2\mu^3}{36\cdot\bar{\gamma}^8}\cdot K_0\left[2\sqrt{\frac{\mu}{\bar{\gamma}^2}}\right]}$

MGF МОЖЕ ДА СЕ НАЈДЕ $\boxed{6.611.9}$ GRADSHT.

6.624.1

$w = \frac{1}{\mu}$ $f_w(w) = \left.\frac{f(\mu)}{\left|\frac{dw}{d\mu}\right|}\right|_{\mu=\frac{1}{w}}$ $\frac{dw}{d\mu} = -\frac{1}{\mu^2}$

$$f_w(w) = \mu^2 \cdot \frac{\cancel{2}\mu^3}{\underset{18}{\cancel{36}}\bar{\gamma}^8} K_0\left[2\sqrt{\frac{1}{w\bar{\gamma}^2}}\right] = \frac{1}{18w^5\cdot\bar{\gamma}^8} K_0\left[2\sqrt{\frac{1}{w\bar{\gamma}^2}}\right]$$

$$M(-s) = \frac{1}{18\bar{\gamma}^8}\int_0^\infty w^{-5} K_0\left[2\sqrt{\frac{1}{w\bar{\gamma}^2}}\right] e^{-sw}\,dw$$

6.621.3 ВО GRADSHTEYN Е СЛИЧЕН ИНТЕГРАЛ САМО ШТО НЕМА 1/w

$$\boxed{M(-s) = \frac{s^2}{36\bar{\gamma}^{10}}\cdot \mathrm{MeijerG}\left([[\,],[\,]],[[-1,-1,-5],[\,]],\frac{s}{\bar{\gamma}^2}\right)}$$

I. H. Lee END-TO-END BER ANALYSIS (CONTINUE...)

$$Z_R = \frac{1}{\sum_{i=1}^{n_t^s}|h_i^R|^2} \qquad Z_D = \frac{1}{n_t^s\sum_{j=1}^{n_t^D}|h_j^D|^2}$$

$\boxed{36\cdot 4 = 144}$

$$M_{Z^R}(s) = \frac{2}{\beta_1^{n_t^s}\Gamma(n_t^s)}\left(\frac{1}{\beta_1 s}\right)^{-\frac{n_t^s}{2}} K_{n_t^s}\left(2\sqrt{\frac{s}{\beta_1}}\right)$$

$$M_{Z^D}(s) = \frac{2}{\beta_2^{n_t^D}\Gamma(n_t^D)(n_t^s)^{n_t^D}}\left(\frac{1}{n_t^s\beta_2 s}\right)^{-n_t^D/2} K_{n_t^D}\left(2\sqrt{\frac{s}{n_t^s\beta_2}}\right)$$

$$w = Z^R + Z^D \qquad M_w(s) = \frac{4}{(N\beta_1\beta_2)^{\frac{N}{2}}\Gamma^2(N)} s^N K_N\left(2\sqrt{\frac{s}{\beta_1}}\right) K_N\left(2\sqrt{\frac{s}{N\beta_2}}\right)$$

$N = n_t^s = n_t^D$

- IF $\boxed{N=4}$ $\beta_1 = \beta_2 = \bar{\gamma}$

$$M_w(s) = \frac{4\, s^4}{(4\cdot\bar{\gamma})^{2}\underbrace{\Gamma^2(4)}_{36}} K_4\left(2\sqrt{\frac{s}{\bar{\gamma}}}\right) K_4\left(2\sqrt{\frac{s}{4\bar{\gamma}}}\right)$$

$$M_w(s) = \frac{s^4}{144\cdot\bar{\gamma}^2} K_4\left(2\sqrt{\frac{s}{\bar{\gamma}}}\right)\cdot K_4\left(2\sqrt{\frac{s}{4\bar{\gamma}}}\right)$$

$$M_W(s) = \frac{4\, s^N}{(N \bar{\gamma}^2)^{\frac{N}{2}}\, \Gamma^2(N)} K_N\left(2\sqrt{\frac{s}{\bar{\gamma}}}\right) K_N\left(2\sqrt{\frac{s}{N\bar{\gamma}}}\right)$$

$\boxed{N=2}$

$$M_W(s) = \frac{2\cancel{4}\cdot s^2}{\cancel{2}\cdot\bar{\gamma}^2\cdot 1} K_2\left(2\sqrt{\frac{s}{\bar{\gamma}}}\right) K_2\left(2\sqrt{\frac{s}{2\bar{\gamma}}}\right)$$

$$\boxed{M_W(s) = \frac{2 s^2}{\bar{\gamma}^2} K_2\left(2\sqrt{\frac{s}{\bar{\gamma}}}\right) K_2\left(\sqrt{\frac{2s}{\bar{\gamma}}}\right)}$$

OVA MISLAM DEKA E ZA $\boxed{2\times1\times2}$

$$\boxed{\frac{M_W(s)}{s^N} = \frac{4}{(N \beta_1\beta_2)^{\frac{N}{2}}\, \Gamma^2(N)} K_N\left(2\sqrt{\frac{s}{\beta_1}}\right) K_N\left(2\sqrt{\frac{s}{N\beta_2}}\right)}$$

$$F_{\gamma^{ENS}_{(\gamma)}}(\gamma) = 1 - \left[\frac{d^{(N-1)}}{dw^{(N-1)}} \mathcal{L}^{-1}\left[\frac{M_W(s)}{s^N}\right]\right]\Bigg|_{w=\frac{c_0}{\gamma}}$$

$$\boxed{\mathcal{L}^{-1}\left[\frac{M_W(s)}{s^N}\right] = \frac{2 e^{-(\beta_1+N\beta_2)/(N\beta_1\beta_2 w)}}{(N\beta_1\beta_2)^{N/2}\,\Gamma^2(N)\, w} K_N\left(\frac{2}{\sqrt{N\beta_1\beta_2}\, w}\right)}$$

$$f(t) \to F(s) = \int_{-\infty}^{\infty} f(t)\, e^{-st}\, dt$$

$$\frac{d^k f(t)}{dt^k} \Rightarrow s^k F(s) \qquad \mathcal{L}\left\{\frac{d^k f(t)}{dt^k}\right\} = s^k F(s)$$

- DA PROBAM DA GI NAJDAM Pout ~~ZA~~ 2×1 & 2×2 AKO NAMESTO GAMMA KORISTAM CHI-SQUARE

$$p_Y(y) = \frac{1}{\sigma^n 2^{n/2}\, \Gamma\left(\frac{n}{2}\right)} y^{\frac{n}{2}-1} e^{-\frac{y}{2\sigma^2}}$$

$2\sigma^2 \rightarrow$ VARIANCE OF GAUSSIAN VARIABLE

$$\gamma = \frac{\epsilon_b}{N_0} \underset{\parallel}{X^2} \; (= Y) \qquad \bar{\gamma} = \frac{\epsilon_b}{N_0}\cdot\Omega \qquad \frac{\epsilon_b}{N_0} = \frac{\bar{\gamma}}{\Omega}$$

$$\boxed{\Omega = 2\sigma^2} \qquad \sigma = \sqrt{\frac{\Omega}{2}} \qquad \boxed{\gamma = \frac{\bar{\gamma}}{\Omega}\cdot y} \qquad \frac{d\gamma}{dy} = \frac{\bar{\gamma}}{\Omega} \qquad y = \frac{\Omega}{\bar{\gamma}}\cdot\gamma$$

$$p_\gamma(\gamma) = \frac{p_Y(y)}{\left|\frac{d\gamma}{dy}\right|}\Bigg|_{y=\frac{\Omega}{\bar{\gamma}}\gamma} = \frac{\Omega}{\bar{\gamma}}\cdot\frac{1}{\left(\frac{\Omega}{2}\right)^{\frac{n}{2}}\cdot 2^{\frac{n}{2}}\,\Gamma\left(\frac{n}{2}\right)}\left(\frac{\Omega}{\bar{\gamma}}\cdot\gamma\right)^{\frac{n}{2}-1} e^{-\frac{\gamma}{\bar{\gamma}}}$$

$$= \frac{\Omega}{\bar{\gamma}}\,\frac{\Omega^{\cancel{\frac{n}{2}-1}}}{\cancel{\Omega}^{\frac{n}{2}}\cdot\Gamma\left(\frac{n}{2}\right)\cdot\bar{\gamma}^{\frac{n}{2}-1}}\cdot\gamma^{\frac{n}{2}-1}\cdot e^{-\frac{\gamma}{\bar{\gamma}}} = \frac{\gamma^{\frac{n}{2}-1}}{\bar{\gamma}^{\frac{n}{2}}\,\Gamma\left(\frac{n}{2}\right)}\cdot e^{-\frac{\gamma}{\bar{\gamma}}}$$

$$p_\gamma(\gamma) = \frac{\gamma^{\frac{n}{2}-1}}{\bar\gamma^{\frac{n}{2}}\Gamma(\frac{n}{2})} \cdot e^{-\frac{\gamma}{\bar\gamma}}$$

— ZNAČI, TREBA MESTO $\frac{n}{2}$ DA STAVAŠ 4 =

NO RABOTI ZA 2×1

$n=4$

$$\boxed{p_\gamma(\gamma) = \frac{\gamma}{\bar\gamma^2} e^{-\frac{\gamma}{\bar\gamma}}}$$

OVA E TOČEN REZULTAT!!! ...

$$P_{out} = 1 - e^{-\frac{\gamma_0}{\bar\gamma}} - \frac{\gamma_0}{\bar\gamma} e^{-\gamma_0/\bar\gamma} = 1 - \left(1 + \frac{\gamma_0}{\bar\gamma}\right) e^{-\gamma_0/\bar\gamma}$$

- DAVA POLOŠI REZULTATI OD GAMMA PRISTAPOT !!.

$\boxed{2\times1\ 2\times2}$ FINAL — $\boxed{\text{WORKING FINAL!!!}}$ $\boxed{\text{MMV}}$

- GAMMA APPROACH REVISITED

$$\gamma_2 = \bar\gamma \cdot \left(\underbrace{|h_{11}|^2}_{exp} + \underbrace{|h_{21}|^2}_{exp} + \underbrace{|h_{12}|^2}_{exp} + \underbrace{|h_{22}|^2}_{exp}\right) = \bar\gamma \cdot \Delta_2$$

$$P_{\Delta_{21}}(\Delta_{21}) = \frac{1}{\lambda} \cdot e^{-\frac{\delta}{\lambda}} \qquad \boxed{E(\delta) = \lambda} \qquad \delta^2 = \frac{1}{\lambda^2}$$

VO MOJOT SLUČAJ $mean(|h_{11}|^2) = 1 = \lambda$

GAMMA DISTRO WITH $\theta = \lambda = 1$

$$f(\delta; n, \lambda) = \frac{x^{n-1}}{\lambda^n \Gamma(n)} \cdot e^{-\frac{x}{\lambda}} \qquad \lambda = 1 \Rightarrow$$

$$\boxed{f(\delta; 4, 1) = \frac{\delta^{4-1}}{\Gamma(4)} e^{-\delta}}$$

$$\boxed{\gamma = \bar\gamma \cdot \delta}$$

$$\delta = \frac{\gamma}{\bar\gamma}$$

$$\frac{d\gamma}{d\delta} = \bar\gamma$$

$$\boxed{p_\gamma(\gamma; 4, 1) = \frac{1}{\bar\gamma} \frac{\gamma^{4-1}}{\Gamma(4) \cdot \bar\gamma^{4-1}} e^{-\frac{\gamma}{\bar\gamma}}}$$

A TOA E GAMMA!!!

ZNAČI:

$$(\#☆)\quad \boxed{p_\gamma(\gamma; 4, 1) = \frac{\gamma^{4-1}}{\Gamma(4)\bar\gamma^4} e^{-\frac{\gamma}{\bar\gamma}}}$$

$\boxed{\text{MMV FINAL}}$ ČISTA GAMMA

$\boxed{n=4}$

$$p_\gamma(\gamma; 4, 1) = \frac{\gamma^3}{6 \cdot \bar\gamma^4} e^{-\gamma/\bar\gamma}$$

$\boxed{n=2}$

$$\boxed{p_\gamma(\gamma; 2, 1) = \frac{\gamma}{\bar\gamma^2} e^{-\gamma/\bar\gamma}}$$

$$\boxed{P_{out} = 1 - \left(1 + \frac{\gamma_0}{\bar\gamma}\right) e^{-\frac{\gamma_0}{\bar\gamma}}}$$

PDF OF GAMMA

$$\boxed{P_{out} = 1 - \left(1 + \frac{\gamma_0}{\bar\gamma} + \frac{\gamma_0^2}{2\bar\gamma^2} + \frac{\gamma_0^3}{6\bar\gamma^3}\right) e^{-\gamma/\gamma_0}}$$

$$F(x; k, \theta) = 1 - \sum_{i=0}^{k-1} \frac{\left(\frac{x}{\theta}\right)^i}{i!} e^{-\frac{x}{\theta}}$$

WIKIPEDIA

$$\boxed{p_\gamma(\gamma;4;1) = \frac{\gamma^{4-1}}{\Gamma(4)\cdot\bar{\gamma}^4}\cdot e^{-\gamma/\bar{\gamma}}} \quad \boxed{\text{FINAL}}$$

UNIVERZALNA PDF ZA MXL MIMO SYSTEM !!!

→ ČISTA GAMMA DISTRIBUCIJA

- 2×2 MIMO: $p_\gamma(\gamma;4,1) = \dfrac{\gamma^3}{6\cdot\bar{\gamma}^4}\, e^{-\gamma/\bar{\gamma}}$
- 2×1 MIMO: $p_\gamma(\gamma;2,1) = \dfrac{\gamma}{\bar{\gamma}^2}\, e^{-\gamma/\bar{\gamma}}$

- MGF FOR MXL MIMO SYSTEM

$$\cancel{M(-s) = \frac{1}{\Gamma(4)\bar{\gamma}^4}\int \gamma^{4-1} e^{-\frac{\gamma}{\bar{\gamma}}-s\gamma}\, d\gamma}$$

(#*) pp. 85

$$\frac{1}{\gamma_{eq}} \doteq \underbrace{\frac{1}{\bar{\gamma}\dot{\Delta}_2}}_{=1/\gamma_1} + \underbrace{\frac{1}{\bar{\gamma}\ddot{\Delta}_2}}_{=1/\gamma_2}$$

$$\boxed{p_{\gamma_i}(\gamma_i) = \frac{\gamma_i^{4-1}}{\Gamma(4)\,\bar{\gamma}^4}\, e^{-\gamma_i/\bar{\gamma}}}$$

- INVERSE GAMMA (FROM (#)) pp. 82

$$W = \frac{1}{\gamma_{eq}} = \underbrace{\frac{1}{\gamma_1}}_{z_1} + \underbrace{\frac{1}{\gamma_2}}_{z_2}$$

$$\boxed{p_{z_i}(z_i) = \frac{1}{\bar{\gamma}^4\Gamma(4)}\, z_i^{-4-1}\, e^{-\frac{1}{z_i\bar{\gamma}}}} \quad \text{FINAL MMV}$$

$$\int_0^\infty x^{\nu-1} e^{-\frac{\beta}{x}-\gamma x}\, dx = 2\left(\frac{\beta}{\gamma}\right)^{\frac{\nu}{2}} K_\nu(2\sqrt{\beta\gamma})$$

$$M_{z_i}(-s) = \frac{1}{\Gamma(4)\,\bar{\gamma}^4}\int_0^\infty z_i^{-4-1} e^{-\frac{1}{z_i\bar{\gamma}}-s\cdot z_i}\, dz_i = \frac{2}{\Gamma(4)\bar{\gamma}^4}\left(\frac{1}{s\cdot\bar{\gamma}}\right)^{-\frac{4}{2}} K_4\left(2\sqrt{\frac{s}{\bar{\gamma}}}\right)$$

$$\boxed{M_{z_i}(-s) = \frac{2(s\bar{\gamma})^{\frac{4}{2}}}{\Gamma(4)\bar{\gamma}^4} K_4\left(2\sqrt{\frac{s}{\bar{\gamma}}}\right) = \frac{2}{\Gamma(4)}\left(\frac{s}{\bar{\gamma}}\right)^{\frac{4}{2}} K_4\left(2\sqrt{\frac{s}{\bar{\gamma}}}\right)} \quad \text{MMV}$$

$\boxed{N=4}$ $$M_{z_1} = \frac{2}{6}\cdot\frac{s^2}{\bar{\gamma}^2}\cdot K_4\left(2\sqrt{\frac{s}{\bar{\gamma}}}\right) = \frac{s^2}{3\bar{\gamma}^2}\cdot K_4\left(2\sqrt{\frac{s}{\bar{\gamma}}}\right)$$

$$\boxed{M_W = M_{z_1}\cdot M_{z_2} = \frac{s^4}{9\bar{\gamma}^4}\cdot\left[K_4\left(2\sqrt{\frac{s}{\bar{\gamma}}}\right)\right]^2} \quad \boxed{\text{OK!!!}} \quad (5*)$$

→ OVA E DEFINITIVNO TAKA !!!

$$\boxed{P_{out} = 1 - \mathcal{L}^{-1}\left[\frac{M_W(s)}{s}\right]}$$

- EXACT VALUE

$$\boxed{\frac{1}{\gamma_{eq}} \doteq \frac{1}{\bar{\gamma}\dot{\Delta}_2} + \frac{1}{\bar{\gamma}\cdot\ddot{\Delta}_2} + \frac{1}{\dot{\Delta}_2\ddot{\Delta}_2}}$$

$$W = \frac{1}{\dot{\Delta}_2\ddot{\Delta}_2}$$

$$p_W = \frac{1}{18w^5} K_0\left[2\sqrt{\frac{1}{w}}\right]$$

$$\boxed{p_W = \frac{1}{18w^5} K_0\left[\frac{2}{\sqrt{w}}\right]}$$

$$M_w(-s) = \frac{1}{18}\int_0^\infty w^{-5} K_0\left[\frac{2}{\sqrt{w}}\right] e^{-sw} dw = \frac{s^5}{36}\ \text{MeijerG}([[2],[]],[[-1,-1,-5],[]]s)$$

- FOR $2\times 2\times 1$ SYSTEM

$$M_{z_1}(-s) = \frac{s^2}{3\bar{\gamma}^2} K_4\left(2\sqrt{\frac{s}{\bar{\gamma}}}\right) \qquad \boxed{M_{z_2}(-s) = \frac{2s}{\bar{\gamma}} K_2\left(2\sqrt{\frac{s}{\bar{\gamma}}}\right)}$$

$$\boxed{M_w = \frac{2s^3}{3\bar{\gamma}^3} K_4\left(2\sqrt{\frac{s}{\bar{\gamma}}}\right)\cdot K_2\left(2\sqrt{\frac{s}{\bar{\gamma}}}\right)}$$

OK!!!

PROVEREN, TOČEN IZRAZ !!! VO SIMULACIITE TREBA DA SE KORISTI: $\boxed{G = 1/\dot{\Delta}_2}$

- CONTINUE I.M. LEE END-TO-END BER ANALYSIS

$$\mathcal{L}^{-1}\left\{\frac{M_w(s)}{s^N}\right\} = \frac{2e^{-(\beta_1+N\beta_2)/(N\beta_1\beta_2 w)}}{(N\beta_1\beta_2)^{N/2}\,\Gamma^2(N)\,w} K_N\left(\frac{2}{\sqrt{N\beta_1\beta_2}}\right)$$

$\boxed{2\times 1\times 1}$ P_{out}

$$G_1^2 = \frac{\epsilon}{G\dot{\Delta}_1^2 + \dot{\Delta}_1 N_0}$$

$$\gamma_{eq_1} = \bar{\gamma}\cdot\frac{G_1^2|\ddot{l}_{11}|^2\cdot\Delta_1^2}{G_1^2|\ddot{l}_{11}|^2\cdot\dot{\Delta}_1+1} = \left| G_1^* = \frac{1}{\dot{\Delta}_1}\right|$$

MMV OK!!!

$$\gamma_{eq_1} = \bar{\gamma}\frac{|\ddot{l}_{11}|^2}{\frac{|\ddot{l}_{11}|^2}{\dot{\Delta}_1}+1} = \bar{\gamma}\frac{1}{\frac{1}{\dot{\Delta}_1}+\frac{1}{|\ddot{l}_{11}|^2}}$$

$$\frac{\bar{\gamma}}{\gamma_{eq_1}} = \frac{1}{\Delta_1} + \frac{1}{|\ddot{l}_{11}|^2} \qquad \underbrace{\frac{1}{\gamma_{eq_1}}}_{W} = \underbrace{\frac{1}{\bar{\gamma}\dot{\Delta}_1}}_{z_1} + \underbrace{\frac{1}{\bar{\gamma}\cdot|\ddot{l}_{11}|^2}}_{z_2}$$

$$\boxed{p_{z_1} = \frac{z_1^{-3}}{\bar{\gamma}} e^{-\frac{1}{z_1\bar{\gamma}}}} \qquad \boxed{M_{z_1}(s) = \frac{2s}{\bar{\gamma}} K_2\left(2\sqrt{\frac{s}{\bar{\gamma}}}\right)}$$

OVA E ISTO TAKA INVERZNA GAMA NO SO DoF $n=1$

$$Y = |\ddot{l}_{11}|^2 \qquad p_Y(Y) = \frac{1}{\bar{Y}}\cdot e^{-\frac{Y}{\bar{Y}}} \qquad \boxed{\bar{Y}=1} \quad p_Y(Y) = e^{-Y}$$

$$z_2 = \frac{1}{\bar{\gamma}Y};\ Y = \frac{1}{\bar{\gamma}z_2} \qquad p_{z_2}(z_2) = \frac{p(Y)}{\left|\frac{dz_2}{dY}\right|}\Bigg|_{Y=\frac{1}{\bar{\gamma}z_2}} = \frac{\bar{\gamma}\cdot Y^2}{1}e^{-1/\bar{\gamma}z_2} = (\bar{\gamma}\cdot z_2^2)^{-1}e^{-\frac{1}{\bar{\gamma}z_2}}$$

$$\frac{dz_2}{dY} = -\frac{1}{\bar{\gamma}Y^2} \qquad = \frac{e^{-1/\bar{\gamma}z_2}}{\bar{\gamma}z_2^2}$$

$$\boxed{M_{z_2}(-s) = 2\sqrt{\frac{s}{\bar{\gamma}}}\cdot K_1\left(2\sqrt{\frac{s}{\bar{\gamma}}}\right)} \qquad \boxed{M_w = 4\sqrt{\left(\frac{s}{\bar{\gamma}}\right)^3} K_2\left(2\sqrt{\frac{s}{\bar{\gamma}}}\right) K_1\left(2\sqrt{\frac{s}{\bar{\gamma}}}\right)}$$

OK!!!

MOŽE SO VERIFIKACIJA NA N.94

- RAYLEIGH

$$p_x(x) = \frac{x}{\sigma^2} e^{-\frac{x^2}{2\sigma^2}} \qquad \Omega = 2\sigma^2 = 1$$

$$M_{z_1}(s) = \frac{2s}{\bar\gamma} K_2\left(2\sqrt{\frac{s}{\bar\gamma}}\right)$$

$$p_x(x) = 2x \cdot e^{-x^2} \qquad y = x^2 \qquad x = \oplus\sqrt{y} \qquad \frac{dy}{dx} = 2x$$

$$p_Y(y) = \left.\frac{p_x(x)}{\frac{dy}{dx}}\right|_{x=\sqrt{y}} = \cancel{\frac{1}{2x}} \cdot \frac{2x}{\cancel{\ldots}} \cdot e^{-x^2} = \underline{\underline{e^{-y}}}$$

$$z = \frac{1}{\bar\gamma \cdot y} \qquad \frac{dz}{dy} = -\frac{1}{\bar\gamma \cdot y^2} \qquad y = \frac{1}{\bar\gamma z}$$

$$p_z(z) = \left.\frac{p_Y(y)}{\frac{1}{\bar\gamma \cdot y^2}}\right|_{y=\frac{1}{\bar\gamma z}} = \cancel{\bar\gamma} \cdot \frac{1}{\bar\gamma^{\cancel{2}} \cdot z^2} \cdot e^{-\frac{1}{\bar\gamma z}} = \underline{\underline{\frac{1}{\bar\gamma z^2} \cdot e^{-\frac{1}{\bar\gamma z}}}}$$

$$\int_0^\infty x^{+\nu-1} e^{-\frac{\beta}{x} - \gamma x} dx = 2\left(\frac{\beta}{\gamma}\right)^{\frac{\nu}{2}} K_\nu\left(2\sqrt{\beta\gamma}\right)$$

$$M_{z_2}(-s) = \frac{1}{\bar\gamma} \int_0^\infty z^{-1-1} e^{-\frac{1}{\bar\gamma z} - sz} dz = \frac{1}{\bar\gamma} \cdot 2 \left(\frac{1}{\bar\gamma \cdot s}\right)^{-\frac{1}{2}} K_1\left(2\sqrt{\frac{s}{\bar\gamma}}\right)$$

$$M_{z_2}(-s) = \frac{2\sqrt{\bar\gamma s}}{\bar\gamma} K_1\left(2\sqrt{\frac{s}{\bar\gamma}}\right) = \underline{\underline{2\sqrt{\frac{s}{\bar\gamma}} K_1\left(2\sqrt{\frac{s}{\bar\gamma}}\right)}}$$

$$M_w = M_{z_1}(s) \cdot M_{z_2}(s) = \frac{2s}{\bar\gamma} K_2\left(2\sqrt{\frac{s}{\bar\gamma}}\right) \cdot 2\sqrt{\frac{s}{\bar\gamma}} K_1\left(2\sqrt{\frac{s}{\bar\gamma}}\right)$$

$$\boxed{M_w = 4\sqrt{\left(\frac{s}{\bar\gamma}\right)^3} K_2\left(2\sqrt{\frac{s}{\bar\gamma}}\right) K_1\left(2\sqrt{\frac{s}{\bar\gamma}}\right)}$$

- REDUCED EXPRESSION (10) FROM I-H. Lee

$$P_{out} = 1 - \mathcal{L}^{-1}\left\{\frac{M_w(s)}{s^N}\right\} = 1 - \left.\frac{2 \cdot e^{-\frac{\bar\gamma + N\bar\gamma}{N \cdot \bar\gamma^2 w}}}{N^{N/2} \cancel{\bar\gamma} N \Gamma^2(N) w} \cdot K_N\left(\frac{2}{\sqrt{N\bar\gamma^2}\, w}\right)\right|_{w = \frac{\gamma}{\gamma_0}}$$

$$P_{out} = 1 - \frac{2 e^{-\frac{1+N}{N\bar\gamma w}}}{N^{N/2} \bar\gamma^N \Gamma^2(N) w} \cdot K_N\left(\frac{2}{\bar\gamma\sqrt{N}\,\frac{\gamma}{\gamma_0}}\right) =$$

$$= 1 - \underline{\frac{2\gamma_0 e^{-\frac{(1+N)\gamma_0}{N \cdot \bar\gamma^2}}}{N^{N/2} \cdot \bar\gamma^{N+1} \Gamma^2(N)} K_N\left(\frac{2\gamma_0}{\bar\gamma^2\sqrt{N}}\right)} \overset{N=4}{=} 1 - \frac{\gamma_0 e^{-\frac{5\gamma_0}{4\bar\gamma^2}}}{8\bar\gamma^5 \cancel{36}} K_4\left(\frac{\gamma_0}{\bar\gamma^2}\right)$$

$$= 1 - \frac{\gamma_0 e^{-\frac{5\gamma_0}{4\bar\gamma^2}}}{288\bar\gamma^5} K_4\left(\frac{\gamma_0}{\bar\gamma^2}\right)$$

$$z\frac{d}{dz}K_\nu(z) + \nu K_\nu(z) = -zK_{\nu-1}(z)$$

- GRADSHTEYN 6.621.3

$$\int_0^\infty x^{\mu-1} e^{-\alpha x} K_\nu(\beta x)\, dx = \frac{\sqrt{\pi}\,(2\beta)^\nu}{(\alpha+\beta)^{\mu+\nu}} \frac{\Gamma(\mu+\nu)\Gamma(\mu-\nu)}{\Gamma(\mu+\frac{1}{2})} F\left(\mu+\nu, \nu+\tfrac{1}{2}; \mu+\tfrac{1}{2}; \tfrac{\alpha-\beta}{\alpha+\beta}\right)$$

□ **AVERAGE BER ~ RAYLEIGH FADING**

$$P_b = \frac{1}{\pi}\int_0^{\pi/2} M_\gamma\left(-\frac{a^2}{2\sin^2\theta}\right)d\theta = \int_0^\infty \underbrace{Q(a\sqrt{\gamma})}_{P_b(\varepsilon/\gamma)}\, p_\gamma(\gamma)\, d\gamma$$

BPSK: $\boxed{P_e(\varepsilon/\gamma) = Q(\sqrt{2\gamma})}$ $\Rightarrow a = \sqrt{2}$ $a^2 = 2$

$$P_b = \frac{1}{\pi}\int_0^{\pi/2} M_\gamma\left(-\frac{1}{\sin^2\theta}\right)d\theta$$

2×2×2 SYSTEM

$$\boxed{M_{1/\gamma}(s) = \frac{s^4}{9\bar\gamma^4}\left[K_4\left(2\sqrt{\frac{s}{\bar\gamma}}\right)\right]^2}$$

$$M_{1/\gamma}\left(-\frac{1}{\sin\theta}\right) = \frac{1}{9\bar\gamma^4\sin^8\theta}\left[K_4\left(2\sqrt{\frac{1}{\bar\gamma\sin^2\theta}}\right)\right]^2$$

⊙ 2×2 SYSTEM (I.E. M×L SYSTEM) BER

MGF ZA GAMMA PDF

$$M_\gamma(-s) = \int_0^\infty \frac{\gamma^{n-1}}{\Gamma(4)\,\bar\gamma^4} e^{-\frac{\gamma}{\bar\gamma}} e^{-s\gamma}\, d\gamma = \frac{1}{(1-\bar\gamma s)^4}$$

PROVJERI S ILI WIKI-PEDIA!!

n = 2 $\quad M_\gamma(+s) = \frac{1}{(1-\bar\gamma s)^2}$

n = 4 $\quad M_\gamma(+s) = \frac{1}{(1-\bar\gamma s)^4}$

$\boxed{n=2}$

$$P_b = \frac{1}{\pi}\int_0^{\pi}\frac{\sin^4\theta}{(\sin^2\theta+\bar\gamma)^2}d\theta = \left| M_\gamma(-s) = \frac{1}{(1+s)^2} \right| =$$

$$\boxed{\frac{-2\bar\gamma^2 + \sqrt{\bar\gamma(1+\bar\gamma)}\,(2\bar\gamma+2\pi) - 3\bar\gamma}{4(1+\bar\gamma)\sqrt{\bar\gamma(1+\bar\gamma)}}}$$

~~n = 4 $P_b = \frac{1}{\pi}\int_0^{\pi}\frac{\sin^8\theta}{(\sin^2\theta+\bar\gamma)^4}d\theta$~~

MMV OK!!!

$\boxed{n=4}$ $M(-s) = \dfrac{1}{(1+\bar\gamma s)^4}$ $s = \dfrac{1}{\sin^2\theta}$

$$P_b = \int_0^{\pi/2} \frac{\sin^8\theta}{(\sin^2\theta+\bar\gamma)^4}\,d\theta = \frac{-16\bar\gamma^4-56\bar\gamma^3-70\bar\gamma^2-35\bar\gamma+16\sqrt{\bar\gamma(1+\bar\gamma)}\,(\bar\gamma^3+3\bar\gamma^2+3\bar\gamma+1)}{32\sqrt{\bar\gamma(1+\bar\gamma)^7}}$$

MILAN STANKOVIC

$$\boxed{P_b = \frac{-16\bar\gamma^4-56\bar\gamma^3-70\bar\gamma^2-35\bar\gamma+16\sqrt{\bar\gamma(1+\bar\gamma)}\,(\bar\gamma^3+3\bar\gamma^2+3\bar\gamma+1)}{32\sqrt{\bar\gamma(1+\bar\gamma)^7}}}$$

BER 2×2 ALAMUTI | BPSK

POTVRDENO SO SIMULACIJA

MMV OK!!!

- DEFINITIVNO NE MOŽE DA SE KORISTI 5* N8.??90 ZA PRESMETKA NA BER !!? NE ZNAM ZOŠTO? TAA JA KORISTAM ZA PRESMETKA NA P_{out}!!!
- ZA DAF END-TO-END BER

$$\boxed{P_{RS}(\bar\gamma) = P_{RS_1}(\bar\gamma) + P_{RS_2}(\bar\gamma) - 2P_{RS_1}P_{RS_2}}$$

JA NAJDOV I VO CLANCITE NA MASNA!!!

- UNIVERZALNA MGF ZA 2×1×1, 2×2×1 i 2×2×2

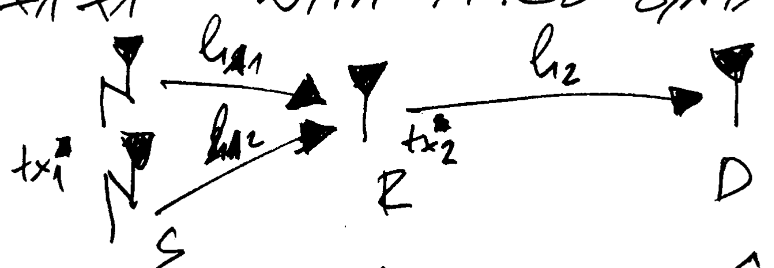
$$\boxed{M_W(-s) = \frac{4}{\Gamma(n)\Gamma(m)}\left(\frac{s}{\bar\gamma}\right)^{\frac{n+m}{2}} K_n\left(2\sqrt{\frac{s}{\bar\gamma}}\right) K_m\left(2\sqrt{\frac{s}{\bar\gamma}}\right)}$$

@$

MMV!!! OK!!!

KORISTI DA OVAA FORMULA VO SITE SLUČAI. TREBA DA NAPRAVAM EDNA MI FORMULA KOJA ĆE SI OBEDINI 2×1×1, 2×2×1, 2×2×2.

2×1×1 WITH FIXED GAIN



i

	TS1	TS2
A1	x_1	$-x_2^*$
	x_2	x_1

DCF

$\hat y_1 = h_{11}^* y_1 + h_{12} y_2^*$

$\hat y_2 = h_{12}^* y_1 + h_{11} y_2^*$

$y_1 = h_{11}x_1 + h_{12}x_2 + n_1$

$y_2 = -h_{11}x_2^* + h_{12}x_1^* + n_2$

AF

$\cancel{\hat y_1 = h_{11}x_1 - h_{11}x_2^*} + h_{12}x_2 + h_{12}x_1^* =$

$\cancel{= h_{11}(x_1 - x_2^*)}$

$tx_{21} = G \cdot y_1$ $\quad tx_{22} = G \cdot y_2$ $\quad tx_2 = [Gy_1, Gy_2, \dots Gy_N]$

$r_2 = h_2 \cdot tx_2 + n$ $\quad r_{21} = h_2 G (h_{11} x_1 + h_{12} x_2 + n_1)$

$r_{22} = h_2 G (-h_{11} x_2^* + h_{12} x_1^* + n_2)$

$\hat{y}_1 = \bar{h}_{11}^* y_1 + \bar{h}_{12} y_2^*$ $\quad \boxed{\bar{h}_{11} = h_2 \cdot G \cdot h_{11}}$

$\hat{y}_2 = \bar{h}_{12}^* y_1 + \bar{h}_{11} y_2^*$

- ORTHOGONAL PILOTS

$r_{21}^0 = h_2 G h_{11} + h_2 \cdot G \cdot h_{12}$

$r_{22}^0 = -h_2 G h_{11} + h_2 G \cdot h_{12}$

$2 \cdot h_2 G h_{12}$ $\quad +$

$2 \cdot h_2 G h_{11}$ $\quad -$

SX-268-RR

$P(\gamma < \gamma_0) = \int_0^{\gamma_0} p_\gamma(\gamma) d\gamma$ $\quad \boxed{p_\gamma(\gamma) = \frac{dP}{d\gamma}}$ $\mathcal{L}$

$M(-s) = \int_{-\infty}^{\infty} \frac{dP}{d\gamma} e^{-\gamma s} d\gamma = s \cdot \hat{P}(s)$

$P(s) = \frac{M(-s)}{s}$ $\quad \boxed{P = \mathcal{L}^{-1}\left[\frac{M(-s)}{s}\right]}$

$$\boxed{p_\gamma(\gamma) = \frac{d}{d\gamma}\left[\mathcal{L}^{-1}\left[\frac{M(-s)}{s}\right]\right]}$$

$P(\gamma_{eq} < \gamma_0) = P\left(\frac{1}{\gamma_{eq}} > \frac{1}{\gamma_0}\right) = 1 - P\left(\frac{1}{\gamma_{eq}} < \frac{1}{\gamma_0}\right) =$

$= 1 - \mathcal{L}^{-1}\left\{\frac{M_{1/\gamma_{eq}}(-s)}{s}\right]$

pp. 94 @$ ⇒ 2×2×2

$$M_{1/\gamma_{eq}}(-1) = \frac{4}{\Gamma(4)\,\Gamma(4)} \left(\frac{1}{\gamma}\right)^4 K_4\left(2\sqrt{\frac{1}{\gamma}}\right) K_4\left(2\sqrt{\frac{1}{\gamma}}\right)$$ $*

$$\boxed{F_{1/\gamma_{eq}} = 1 - \frac{d^{N-1}}{d w^{(N-1)}} \mathcal{L}^{-1}\left\{\frac{M_w(1)}{s^N}\right\} \Bigg|_{w=\frac{1}{\gamma_{eq}}}}$$

$$\boxed{\mathcal{L}^{-1}\left[\frac{M_w(s)}{s^N}\right] = \frac{2 \cdot e^{-\frac{1+N}{N\gamma w}}}{N^{N/2}\gamma^N \Gamma^2(N)\, w} K_N\left(\frac{2}{\sqrt{N}\gamma w}\right)}$$

GO OPRAVA !!!

$$F_{1/\gamma_{eq}} = 1 - \frac{d^3}{d w^3}\left\{\frac{\cancel{1}\, e^{-\frac{5}{4\gamma w}}}{18\,\gamma^4\, w} K_4\left(\frac{2}{\gamma w}\right)\right\}$$

$$F_{1/\gamma_{eq}} = 1 - \frac{1}{18\gamma^4}\frac{d^3}{dw^3}\left\{\frac{e^{-5/4\gamma w}}{w} K_4\left(\frac{2}{\gamma w}\right)\right\}$$

$$F_{1/\gamma_{eq}} = 1 - \frac{1}{18\gamma^4}\frac{d^3}{dw^3}\left\{\frac{e^{-1.25/(\gamma w)}}{w} K_4\left(\frac{2}{\gamma w}\right)\right\}\Bigg|_{w=\frac{1}{\gamma_0}}$$

$$\underbrace{(x^3)'' = (3x^2)' = 6x\Big|_{x=1} = 6}$$

GO LEŠAVAM SO MATLE TOOLBOX !!!

$$\mathcal{L}\left[\frac{1}{x} e^{-b/x} K_\nu\left(\frac{a}{x}\right)\right] = \underline{2 K_\nu(u_-)\, K_\nu(u_+)}$$

PRUDNIKOV BRYCHKOV 3.16.6.6

$$\underbrace{u_\pm = \sqrt{e}\left(\sqrt{p+a} \pm \sqrt{p-a}\right)} \qquad \underline{u_\pm = \sqrt{p}\left(\sqrt{b+a} \pm \sqrt{b-a}\right)}$$

$a = \frac{1}{8}$ $$\mathcal{L}\left[\frac{1}{x} e^{-b/x} K_\nu\left(\frac{a}{x}\right)\right] = \underline{2 K_\nu(u_-)\, K_\nu(u_+)}$$

$b = \frac{1}{8}$ $\quad u_- = u_+ = \sqrt{p}\,\sqrt{\frac{2}{8}}$

$$\mathcal{L}\left[\frac{1}{t} e^{-\frac{1}{\delta t}} K_\nu\left(\frac{1}{\delta t}\right)\right] = \boxed{2 \cdot K_\nu\left(\sqrt{\frac{2p}{\delta}}\right) \cdot K_\nu\left(\sqrt{\frac{2p}{\delta}}\right)}$$

$$(*) \Rightarrow M_W(-s) = \frac{s^\nu}{K} K_\nu\left(\sqrt{\frac{2s}{\delta/2}}\right) \cdot K_\nu\left(\sqrt{\frac{2s}{\delta/2}}\right)$$

$$\frac{M_W(-s)}{s^\nu} = \frac{4}{\Gamma(N)\cdot\Gamma(N)\cdot\delta^N}\left[K_\nu\left(\sqrt{\frac{2s}{\delta/2}}\right)\right]^2$$

$$\frac{\Gamma^2(N)\cdot\delta^N}{2} \cdot \frac{M_W(-s)}{s^\nu} = 2\left[K_\nu\left(\sqrt{\frac{2s}{\delta/2}}\right)\right]^2$$

$$\mathcal{L}^{-1}\left[2 K_\nu\left(\sqrt{\frac{2s}{\delta/2}}\right)\right] = \frac{1}{w} e^{-\frac{2}{\delta w}} \cdot K_\nu\left(\frac{2}{\delta w}\right)$$

$$\boxed{\mathcal{L}^{-1}\left[\frac{M_W(-s)}{s^\nu}\right] = \frac{2}{\Gamma^2(N)\cdot\delta^N\cdot w} e^{-\frac{2}{\delta w}} K_\nu\left(\frac{2}{\delta w}\right)}$$

MMV

$$\mathcal{L}\left[\frac{d^n}{ds^n}F(s)\right] = (-1)^n t^n f(t)$$

DIFERENCIRANJE VO s-DOMEN

VIDI pp. 99

- DIFERENCIRANJE VO t DOMEIN

$$\frac{d^n}{dt^n}\left[f(t)\right] = s^N \cdot F(s) \qquad \underline{F(s) = \hat{f}(s)}$$

$$\frac{d^{n-1}}{dt^{n-1}} P(t) = s^N \cdot \hat{f}(s) = s^N \cdot M(-s)$$

$$P(t) = \mathcal{L}^{-1}\left[\frac{M(-s)}{s}\right] \qquad f(t) = \frac{dP(t)}{dt} \qquad \int^t [\;]$$

$$M(-s) = s \cdot \hat{P}(s) \qquad \boxed{\hat{P}(s) = \frac{M(-s)}{s}}$$

$$\frac{d^n P(t)}{dt^n} = \frac{d^{n-1}}{dt^{n-1}} \cdot f(t) \;; \qquad s^{N} \cdot \hat{P}(s) = s^{n-1} \cdot MGF$$

$$P(x<x_0) = F(x) = \frac{d^{N-1}}{dx^{N-1}} \mathcal{L}^{-1}\left\{ \frac{M(-s)}{s^N} \right\}$$

$$\mathcal{L}^{+1}\left\{ \frac{d^N}{dx^N} f(x) \right\} = s^N \cdot \hat{f}(s) = \underline{s^N \cdot M(-s)}$$

$$P(s) = \frac{M(-s)}{s} \qquad M(-s) = s\, P(s)$$

$$\mathcal{L}\left\{ \frac{d^N}{dx^N} f(x) \right\} = s^{N+1} P(s)$$

$$P(s) = \frac{1}{s^{N+1}} \mathcal{L}\left\{ \frac{d^N}{dx^N} f(x) \right\}$$

$$I'_\nu(z) = I_{\nu-1}(z) - \frac{\nu}{z} I_\nu(z)$$

$$z \frac{d}{dz} K_\nu(z) = -\nu K_\nu(z) - z K_{\nu-1}(z)$$

$$\boxed{K'_\nu(z) = -\frac{\nu}{z} K_\nu(z) - K_{\nu-1}(z)}$$

$$\sum_{i=1}^{2} \sum_{j=1}^{2} |h_{ij}|^2 = \sum_{i=1}^{2} \left(|h_{i1}|^2 + |h_{i2}|^2 \right) =$$

$$= |h_{11}|^2 + |h_{12}|^2 + |h_{21}|^2 + |h_{22}|^2$$

$$\gamma_{eq,2} = \frac{E}{N_0} \cdot \frac{G_2^2\, \ddot{\Delta}_2 \dot{\Delta}_2^2}{G_2^2\, \ddot{\Delta}_2 \dot{\Delta}_2 + 1} = \frac{E}{N_0} \frac{\frac{1}{\dot{\Delta}_2^{\cancel{2}}} \cdot \ddot{\Delta}_2 \cancel{\dot{\Delta}_2^2}}{\frac{1}{\dot{\Delta}_2^{\cancel{2}}} \ddot{\Delta}_2 \cancel{\dot{\Delta}_2} + 1} =$$

$$= \frac{1}{\frac{1}{\dot{\Delta}_2} + \frac{1}{\ddot{\Delta}_2}}$$

OVA E NAJVAŽNATA!!!

$$F_{1/\gamma_{eq}} = 1 - \frac{d^{N-1}}{dw^{(N-1)}} \mathcal{L}^{-1}\left[\frac{M_w(-s)}{s^N}\right]$$

$$\mathcal{L}^{-1}\left[\frac{M_w(-s)}{s^N}\right] = \frac{2}{\Gamma^2(N)\, \gamma^N w} e^{-\frac{2}{\gamma w}} K_N\left(\frac{2}{\gamma w}\right)$$

$$p_{1/\gamma_{eq}}(w) = \frac{d F_{1/\gamma_{eq}}}{dw}$$

$$MGF = \int_0^\infty p_{1/\gamma_{eq}}(w) \cdot e^{-sw} \, dw$$

$$P_b = \int_0^{\pi/2} M\left(-\frac{1}{\sin^2\theta}\right) d\theta$$

$$P_b = \int_0^\infty Q(\sqrt{2\gamma}) \cdot \underline{p_{1/\gamma}(w)} \, d\gamma$$

$$Q(\sqrt{2\gamma}) = \left| \begin{aligned} Q(z) &= \frac{1}{2}\operatorname{erfc}\left(\frac{z}{\sqrt{2}}\right) \\ &= \frac{1}{2}\operatorname{erfc}(\sqrt{\gamma}) \end{aligned} \right|$$

$$\operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_{\cancel{x}}^\infty e^{-\cancel{x}^2} d\cancel{x}$$

$$P_b = \int_0^\infty \frac{1}{2}\operatorname{erfc}\sqrt{\frac{1}{w}}\; p_w(w)\, dw$$

$$\frac{1}{2}\operatorname{erfc}(\sqrt{\gamma}) = \frac{1}{\sqrt{\pi}} \int_{\sqrt{\gamma}}^\infty e^{-t^2} dt$$

$$P_b = \int_0^\infty P_b(E/\gamma)\, p(\gamma)\, d\gamma \qquad P_b(E/\gamma) = \frac{2}{\sqrt{\pi}} \int_{\sqrt{\gamma}}^\infty e^{-x^2} dx$$

$$P_b = \int_0^\infty \left(\frac{1}{\sqrt{\pi}} \int_{\sqrt{\gamma}}^\infty e^{-x^2} dx\right) p(\gamma)\, d\gamma$$

$$P_b(E/\gamma) = \underbrace{\frac{1}{\pi} \int_0^{\pi/2} e^{-\frac{x^2}{2\sin^2\theta}} d\theta}_{Q(x)}$$

$$P_b(\varepsilon/\gamma) = Q(\sqrt{2\gamma})$$

$$P_b = \int_0^\infty \left(\frac{1}{\pi}\int_0^{\pi/2} e^{-\frac{2\gamma}{2\sin^2\theta}}\, d\theta\right)\cdot p(\gamma)\, d\gamma =$$

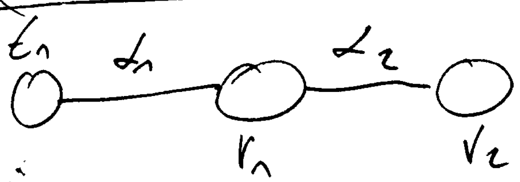
$$= \frac{1}{\pi}\int_0^{\pi/2}\int_0^\infty e^{-\frac{\cancel{2}\gamma}{\cancel{2}\sin^2\theta}}\, p(\gamma)\, d\gamma\, d\theta = \frac{1}{\pi}\int_0^{\pi/2} \underbrace{\int_0^\infty p(\gamma)\, e^{-\frac{\gamma}{\sin^2\theta}}\, d\gamma}_{M\left(-\frac{1}{\sin^2\theta}\right)}\, d\theta$$

$$P_b(\varepsilon/\gamma) = \frac{1}{\pi}\int_0^{\pi/2} e^{-\frac{\gamma}{\sin^2\theta}}\, d\theta$$

$$w = \frac{1}{\gamma} \qquad P_e(\varepsilon/w) = \frac{1}{\pi}\int_0^{\pi/2} e^{-\frac{1}{w\sin^2\theta}}\, d\theta$$

$$P_b = \int_0^\infty \left(\frac{1}{\pi}\int_0^{\pi/2} e^{-\frac{1}{w\sin^2\theta}}\, d\theta\right)\cdot p_w(w)\, dw =$$

$$= \frac{1}{\pi}\int_0^{\pi/2}\left(\int_0^\infty p_w(w)\, e^{-\frac{1}{w\sin^2\theta}}\, dw\right) d\theta \qquad \boxed{3095715}$$



$$r_1 = \varepsilon_1\alpha_1\cdot s(t) + n_1$$

$$r_2 = G_1\cdot\alpha_2(\alpha_1 s + n_1) + n_2$$

$$r_2 = \varepsilon_1 G_1\,\alpha_1\cdot\alpha_2\cdot s + G_1\alpha_2\cdot n_1 + n_2$$

$$\frac{\varepsilon_1^2 G_1^2\alpha_1^2\alpha_2^2}{G_1^2\alpha_2^2 N_{01} + N_{02}} = \frac{1}{N_{01}N_{02}}\,\frac{\varepsilon_1^2 G_1^2\alpha_1^2\alpha_2^2}{\frac{G_1^2\alpha_2^2}{N_{02}} + \frac{1}{N_{01}}} = \frac{\frac{\varepsilon_1^2\alpha_1^2}{N_{01}}\cdot\frac{\alpha_2^2}{N_{02}}}{\frac{\alpha_2^2}{N_{02}} + \frac{1}{G_1^2 N_{01}}}$$

P. Dharmawansa, et. al, Analytical Performance of AF...

521 £

$H_1 \in \mathbb{C}^{N_R \times N_S}$ – CHANNEL MATRIX BTWN SOURCE & RELAY

$H_2 \in \mathbb{C}^{N_D \times N_R}$ – –11– BTWN RELAY & DESTINATION

$\boxed{37990}$

RAYLEIGH FADING $\mathcal{CN}(0,1)$

3061666

$X \in \mathbb{C}^{N_S \times N_T}$ N_T – NUMBER OF SYMBOL PERIODS USED TO SEND OSTBC CODEWORD

6332,0	1550	6 RATI
	2880	12 RATI

618 €

558

$37990 \times X = 1550$ $x = \frac{1550}{37990} = 4\%$

$x = \frac{2880}{37990} = 7{,}6\%$

KERIM SERAFEDIN

$41.990 \cdot 0{,}04 =$

GJORGJI DIMITROV 21. SKOLO ZA SLEPI

2773504 MAGACIN GERMET

NAB KOMERC 023074278

SONY 2 3290707

- CODE RATE $R = \frac{N}{N_T}$

$X = (x_1, x_2, \dots, x_{N_T})$ $x_i \in \mathbb{C}^{N_S \times 1}$

$$X = \begin{bmatrix} x_{11}, & x_{12}, & \dots, & x_{1N_t} \\ x_{21}, & x_{22}, & \dots, & x_{2N_t} \\ \dots & & & \\ x_{N_S 1}, & x_{N_S 2}, & \dots, & x_{N_S N_t} \end{bmatrix}$$

ρ – TOTAL TRANSMIT POWER SPANNING ALL N_S SOURCE ANTENNAS.

$\boxed{E(\|x_k\|^2) = \rho}$

$$\boxed{y_k = \underbrace{H_1 x_k + n_k}_{\text{AT RELAY}} \qquad k = 1, 2, \ldots, N_T}$$

$n_k \sim CN(0, I_{N_R})$

- FIX-GAIN NON-REGENERATIVE PROTOCOL

$$\boxed{a = \sqrt{\frac{b}{N_R(1+\rho)}}, \quad b \in R^+}$$

ANHOCH
3PARČINA
2340 KINGSTON

$$E\left(\|G \cdot y_k\|^2\right) \leq b$$

$$r_k = aH_2 y_k + w_k, \quad k = 1, 2, \ldots, N_T$$

COLORED GAUSSIAN NOISE

AT DESTINATION

$$\boxed{r_k = aH_2H_1 x_k + aH_2 n_k + w_k \qquad k = 1, 2, \ldots, N_T}$$

$\boxed{\delta = \rho}$ SINCE NOISE VARIANSE = 1

29.09.2010

PASOŠ

KATIN A

12^{40} MIN

3D

- LICNA KARTA
- MOLBENO MALANE
- ADMINISTRATIVNA TAKSA

HOTEL $5200/40 = 130,0$ €

SPUTNIK: POLA SOBA: 255€ (4 noćev.)

SPUTNIK: 600 € + 255 € = 855

1200 − 855 = 345

AVION

21670/61,5 = 352€

855 + 352 = $\boxed{1207€}$ SPUTNIK

KOSTON HOTEL: 5800 RUR = 145€

1/2: 72.5€

1242€ KOLSTON

- OVERALL MIMO-OUTPUT MODEL IS EQUIVALENT TO POINT-TO-POINT SYSTEM WITH aH_2H_1 channel matrix, AND COLORED GAUSSIAN NOISE:

$$E\left\{(aH_2n_k + w_k)(aH_2n_k + w_k)^H \mid H_2\right\} = K$$

CONDITIONAL COVARIANCE

$$\boxed{K = a^2 H_2 H_2^H + I_{N_D}}$$

$$E\left[aH_2n_k \cdot w_k\right] = 0$$

$$E\left\{aH_2 n_k a^H H_2^H n_k^H + w_k \cdot w_k^H\right\} = a^2 H_2 H_2^H + I_{N_D}$$

- NOISE WHITENING OPERATION:

$$\tilde{r}_k = \left(\sqrt{K}\right)^{-1} \cdot r_k$$

$$\boxed{\tilde{r}_k = \frac{aH_2H_1}{\sqrt{K}} x_k + \frac{aH_2n_k + w_k}{\sqrt{K}}}$$

HMV

NOISE WHITENING

MORA DA GO IMPLEMENTIRAM VO MATLAB !!!

$$\boxed{\tilde{r}_k = \tilde{H} x_k + \tilde{n}_k}$$

- SE POKAZUVA DEKA VO DESTINACIJATA E DOVOLNO DA SE ZNAE H_2H_1, H_1.

$$\boxed{\tilde{H} = \frac{aH_2H_1}{\sqrt{K}} \qquad \tilde{n}_k = \frac{aH_2n_k + w_k}{\sqrt{K}}}$$

EQUIVALENT WHITENED NOISE

$\sim CN(0, I_{N_D})$

- EQUATION OF FULL TRANSMISSION FOR A GIVEN CODEWORD X:

$$\boxed{\tilde{R} = \tilde{H} \cdot X + \tilde{N}}$$

POCLE

PETAR CAKE 16
VELKO GROZDANOV

~~NAVECER, SEKOJ TRET DEN!~~
~~OD POCETOK MOZE SEKOJ DEN DODEKA NE IZLEZE NADVOR. POCLE SEKOJ TRET DEN.~~

$$G = \sqrt{\frac{E_1}{E_2 \cdot \alpha^2 + N_0}}$$

$$z_{11} = \sqrt{E} A_{11} x_1 + \xi_1$$

$$r_{31} = G_1 \cdot g_{11} \sqrt{E} A_{11} x_{1k} + G_1 g_{11} \xi_1$$

$$\frac{G_1^2 |g_{11}|^2 E \cdot A_{11}^2}{G_1^2 |g_{11}|^2 \cdot A_{11} N_0} = \frac{E A_{11}}{N_0}$$

$$\begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} \cdot \begin{bmatrix} a_1^* & a_3^* \\ a_2^* & a_4^* \end{bmatrix} = \begin{bmatrix} |a_1|^2+|a_2|^2 ; & a_1 a_3^* + a_2 a_4^* \\ a_1^* a_3 + a_2^* a_4 ; & |a_3|^2+|a_4|^2 \end{bmatrix}$$

---

1. FB 2. FixG 3. VarG

II I III

38 57 55

$$\boxed{\tilde{H} = \frac{a H_2 H_1}{\sqrt{K}}} \qquad \tilde{n}_k = \frac{a H_2 u_k + w_k}{\sqrt{K}}$$

- OVOJ PRIS PAT E PODOBAR. VO ICUMT ČLANAKOT JAS LAŠKA SI INDEKSIRAV ANTENITE, A OVDE

$\tilde{n}_k$ e column vector !!!

$\tilde{n}_k$ - EQUIVALENT WHITENED NOISE $\sim CN(0, I_{N_D})$

$$\boxed{\tilde{R} = \tilde{H} X + \tilde{N}}$$

→ TRANSMISSION EQUATION FOR GIVEN CODEWORD „X"

$\tilde{R} = (\tilde{r}_1, \ldots, \tilde{r}_{N_T})$ $\quad \tilde{N} \sim CN(0, I_{N_D} \otimes I_{N_T})$

$$\boxed{\tilde{s}_l = \|\tilde{H}\|_F^2 s_l + n_l, \quad l = 1, 2, \ldots N}$$

$n_l \sim CN(0, \|\tilde{H}\|_F^2)$

$$\boxed{\gamma_l = \|\tilde{H}\|^2 E\{|s_l|^2\} = \alpha \bar{\gamma} a^2 \mathrm{Tr}(H_1^H H_2^H K^{-1} H_2 H_1) \qquad \alpha = \frac{1}{R \cdot N_S}}$$

SLIČNO MI IZLEGUVA SNR-OT NO ZA SINGLE-HOP VO ICUMT ČLANAKOT !!!

$$\boxed{p = \max(N_R, N_D), \quad q = \min(N_R, N_D); \quad v_{ij} = i + j + p - q - 1}$$

- Statistics of the SNR (CUMULANTS AND MOMENTS)

$M_\gamma(s) = K^{-1} \det(I(s)) \qquad K = \prod_{i=1}^{q} \Gamma(p-i+1)\Gamma(q-i+1)$

$I(1)$ – Hankel MATRIX WITH (i,j) entry given as:

$$I_{ij}(1) = \frac{\Gamma(v_{ij})}{a^{2v_{ij}}(1+\alpha\bar{\delta}\cdot 1)^{v_{ij}+N_s}} \sum_{l=0}^{N_s} \binom{N_s}{l} (\alpha\bar{\delta}1)^l \times$$

$$\times\, U\left(v_{ij}; v_{ij}+1-l; \frac{1}{a^2(1+\alpha\bar{\delta}1)}\right)$$

U – confluent hypergeometric function of second kind.

Theorem 2: n-th ORDER CUMULANT OF THE SNR δ

$$\boxed{\mu_n = K^{-1}(n-1)!\, N_s (\alpha\bar{\delta}a^2)^n \sum_{l=1}^{2} \det\left(\tilde{J}(l)\right)}$$

- CUMULANT GENERATING FUNCTION

WIKIPEDIA

$$g(t) = \sum_{n=1}^{\infty} K_n \frac{t^n}{n!}$$

- MOMENT GENERATING FUNCTION

$$\boxed{E(e^{tx}) = 1 + \sum_{n=1}^{\infty} \mu_n' \frac{t^n}{n!}}$$

CONTINUE ON 107...

- WITH FORMAL POWER SERIES LOGARITHM

$$g(t) = \log(E(e^{tx}))$$

$$\sigma^2 = (\xi^2 - \bar{\xi}^2) = \bar{\xi}^2 = E(\xi^2)$$

$$u_{11} \triangleq u_{1[1]} \qquad u_{21} \triangleq u_{1[2]} \qquad r_{31} \triangleq V_{1[3]}$$

$$r_{41} \triangleq V_{1[4]}$$

$$z_{11} = \tilde{x}_{11} \qquad z_{21} = \tilde{x}_{12}$$

– NE ZABORAVAJ DA GA SMENIŠ SLIKATA!!!

JU 133 — TERMIN F

VALENTINA

VTORNIK, ČETVRTOK, PETOK, 2xSLIKI

25.09.2010

$$\int_0^\infty \frac{1}{\delta} e^{-\frac{1}{\delta x}} e^{-sx}\, dx = \frac{2}{\delta\sqrt{s\delta}} K_1\left(2\sqrt{\frac{s}{\delta}}\right)$$

$\alpha = 1$

$$M_{\mu_c}(-s) = \frac{2}{\Gamma(1)} \left(\frac{s}{\delta}\right)^{1/2} K_1\left(2\sqrt{\frac{s}{\delta}}\right)$$

$$\boxed{M_{\mu_c}(-s) = \frac{2}{\Gamma(\alpha)} \left(\frac{s}{\delta}\right)^{\frac{\alpha}{2}} K_\alpha\left(2\sqrt{\frac{s}{\delta}}\right)}$$

3167971

$$\mu = \frac{1}{\delta \cdot g_{11}} = \frac{1}{\delta x} \qquad f_{g_{11}}(x) = f_g(x) = \frac{1}{\delta} \cdot e^{-\frac{x}{\delta}}$$

$$p_\mu = \frac{p_g(x)}{\left|\frac{d\mu}{dx}\right|}\Bigg|_{x = \frac{1}{\delta\mu}} = \frac{e^{-\frac{1}{\delta\mu}}}{\left|-\frac{1}{\delta x^2}\right|} = \frac{e^{-\frac{1}{\delta\mu}}}{\frac{\delta^2\mu^2}{\delta}}$$

$$\boxed{p_\mu = \frac{1}{\delta\mu^2} e^{-\frac{1}{\delta\mu}}}$$

3248000

2304

$$\int_0^\infty \frac{1}{g x^2} e^{-\frac{1}{gx}} e^{-sx}\, dx = 2\sqrt{\frac{s}{g}}\, K_1\left(2\sqrt{\frac{s}{g}}\right)$$

		AUTOC.	
3,7 mm ≙	3700/2 = 1,8500 m	1,74	→ 10 KATASTARSKI
4,3 mm ≙	4300/2 = 2,150 m	2,26	
		AUTOCAD	
3,2 mm ≙	3200/2 = 1,6 m	1,59	→ PO STAR PREMER
3,8 mm ≙	3700/2 = 1,85 m	1,89	

1,35 m — FAKTIČKA SOSTOJBA

1,60 m

$q = \min(N_R, N_D) \qquad p = \max(N_R, N_D)$

$$\tilde{J}_{ij}(\ell) = \begin{cases} \dfrac{\Gamma(v_{ij}+n)}{a^{2(v_{ij}+n)}} U\left(v_{ij}+n; v_{ij}+1; \dfrac{1}{a^2}\right) & \text{if } \ell = j \\ \Gamma(v_{ij}) & \text{if } \ell \neq j \end{cases}$$

$\tilde{J}(\ell)$ – $q \times q$ MATRIX WITH ENTRIES

⊖ FIRST MOMENT OF SNR γ IS GIVEN:

$$m_1 = K^{-1} N_s a \cdot \bar{\gamma} \sum_{\ell=1}^{q} \det(J(\ell))$$

$$J_{ij}(\ell) = \begin{cases} \dfrac{\Gamma(v_{ij}+1)}{a^{2v_{ij}}} U\left(v_{ij}+1; v_{ij}+1; \dfrac{1}{a^2}\right) & \ell = j \\ \Gamma(v_{ij}) & \ell \neq j \end{cases}$$

$\underline{\underline{25 \times 300}} = \underline{\underline{7500}}$

2

$4m \times 3$

LAZSM: $5 \times 4m \times 30m = 20 \times 30$ MKD $= 600$

⊖ SECOND MOMENT OF γ (COROLLARY 2):

$$m_2 = \mu_2 (N_s + 1) + \frac{K^{-1}}{2!} N_s^2 (\alpha \bar{\gamma} a^2)^2 (q^2 - q) \tilde{J}(A)$$

- A $q \times q \times q$ rank-3 TENSOR WITH (i,j,k) element

$$a_{ijk} = \begin{cases} \dfrac{\Gamma(v_{ij}+1)}{a^{2(v_{ij}+1)}} U\left(v_{ij}+1; v_{ij}+1; \dfrac{1}{a^2}\right) & k = 1,2 \\ \Gamma(v_{ij}) & k \neq 1,2 \end{cases}$$

- $J(\cdot)$ IS RANK-3 TENSOR OPERATOR:

$$\boxed{J(A) = \sum_{\epsilon} \operatorname{sgn}(\epsilon) \sum_{\delta} \operatorname{sgn}(\delta) \prod_{k=1}^{2} a_{\epsilon_k \delta_k k}}$$

$\epsilon = \epsilon_1, \epsilon_2, \ldots, \epsilon_q \qquad \delta = \delta_1, \delta_2, \ldots, \delta_q$

↳ PERMUTATIONS OF INTEGERS $1, 2, \ldots, q$

Exact Probability Distributions

- Voredi rate 1 i rate 2 → ICUMT

- ~~Razlika ima vo toa što vo 2x1x1 sistemot mislam deka snagata vo relejot pak za odnos so 2 a ne trema~~

- MGF-ot vo generalen slučaj će ide

$$M_W(-s) = \frac{4\, n_S^{d_1/2} n_B^{d_2/2}}{\Gamma(d_1)\Gamma(d_2)} \left(\frac{s}{\bar{\gamma}}\right)^{\frac{d_1+d_2}{2}} K_{d_1}\left(2\sqrt{\frac{s}{\bar{\gamma}/n_S}}\right) K_{d_2}\left(2\sqrt{\frac{s}{\bar{\gamma}/n_B}}\right)$$

$$\boxed{M_W(-s) = \frac{4\, n_S^{d_1/2} n_B^{d_2/2}}{\Gamma(d_1)\Gamma(d_2)} \left(\frac{s}{\bar{\gamma}}\right)^{\frac{d_1+d_2}{2}} K_{d_1}\left(2\sqrt{\frac{n_S \cdot s}{\bar{\gamma}}}\right) K_{d_2}\left(2\sqrt{\frac{n_B \cdot s}{\bar{\gamma}}}\right)}$$

→ MGF vo generalen slučaj za K×L×M system.

pp 99 (ili ICUMT10-stosic-hadzivelkov v.4.pdf)

→ $$P_{out} = 1 - \frac{2}{\Gamma^2(d)\,\bar{\bar{\gamma}}^d}\left\{\frac{d^{d-1}}{dw^{d-1}}\left[\frac{e^{-\frac{2}{\bar{\bar{\gamma}} w}}}{w} K_d\left(\frac{2}{\bar{\bar{\gamma}} w}\right)\right]\right.$$

$$\boxed{\bar{\bar{\gamma}} \triangleq \frac{\bar{\gamma}}{n_S}} \qquad d \triangleq n$$

$$\boxed{P_{out} = 1 - \frac{2\, n_S^n}{\Gamma^2(n)\,\bar{\gamma}^n}\left\{\frac{d^{n-1}}{dw^{n-1}}\left[\frac{e^{-\frac{2 \cdot n_S}{\bar{\gamma} w}}}{w} K_n\left(\frac{2 n_S}{\bar{\gamma} w}\right)\right]\right.}$$

Exact closed form

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} u_1 & u_1^* \\ u_2 & u_2^* \end{bmatrix}$$

GENERALIZED COMPLEX ORTHOGONAL DESIGNS

$$\begin{bmatrix} y_{11} & y_{21} \\ y_{12} & y_{22} \end{bmatrix} = \begin{bmatrix} h_{11} & h_{21} \\ h_{12} & h_{22} \end{bmatrix} \begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix} + \begin{bmatrix} n_1 & n_2 \\ n_1 & n_2 \end{bmatrix}$$

$$\boxed{\begin{aligned} y_{11} &= h_{11} x_1 + h_{21} x_2 \\ y_{21} &= -h_{11} x_2^* + h_{21} x_1 \end{aligned}}$$

$$\begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix} \cdot \begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix}^H = \begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix} \begin{bmatrix} x_1^* & x_2^* \\ -x_2 & x_1 \end{bmatrix}$$

$$= \begin{bmatrix} |x_1|^2 + |x_2|^2 & \cancel{x_1 x_2^*} - \cancel{x_1 x_2^*} \\ \cancel{x_2 x_1^*} - \cancel{x_1^* x_2} & |x_2|^2 + |x_1|^2 \end{bmatrix} = \begin{bmatrix} |x_1|^2 + |x_2|^2 & 0 \\ 0 & |x_1|^2 + |x_2|^2 \end{bmatrix}$$

→ DOKAZ ZA ORTOGONALNOST NA MATRICATA +2

- GENERALIZED ORTHOGONAL DESIGN $\boxed{G_{T\times N}}$ IS

$$G^H \cdot G = c\left(|x_1|^2 + |x_2|^2 + \dots + |x_K|^2\right) I_N = c \cdot \sum_{k=1}^{K} |x_k|^2 I_N$$

UNITARY MATRIX

$$U^H \cdot U = U \cdot U^H = I_N \qquad \boxed{\{U\}_{N\times N}}$$

$C = G(s_1, s_2, \dots s_K)$ } ENTRIES OF C ARE LINEAR COMBINATIONS OF $s_1, s_2, \dots s_K$

$s_i \triangleq x_i$

$C_{t,n}$ $\quad n = 1, 2, \dots N$ } TRANSMITED SIMULTANEOUSLY AT ~~TRANSMIT~~ TIMESLOTS $t = 1, 2, \dots T$

$n = 1, 2 \dots N$ → ANTENNA

TIMESLOT $t = 1, 2, \dots, T$

TRANSMITED SYMBOL AT t TIMESLOT

- K SYMBOLS TRANSMITTED OVER T TIMESLOTS

$R = \frac{K}{T} \Rightarrow$ RATE OF THE CODE

$$[r_1, r_2^*] = [s_1, s_2]\,\Omega + [\eta_1, \eta_2^*] \qquad \Omega = \begin{bmatrix} \alpha_1 & \alpha_2^* \\ \alpha_2 & -\alpha_1^* \end{bmatrix}$$

$$r_1 = \alpha_1 s_1 + \alpha_2 s_2 + \eta_1$$

$$r_2^* = \alpha_2^* s_1 - \alpha_1^* s_2 + \eta_2^*$$

$$r_2 = \alpha_2 s_1^* - \alpha_1 s_2^* + \eta_2 = -\alpha_1 s_2^* + \alpha_2 s_1^*$$

$$G_C = \begin{pmatrix} G \\ G^* \end{pmatrix} = \begin{bmatrix} \sum_{k=1}^{K} x_k E_k \\ \sum_{k=1}^{K} x_k^* E_k \end{bmatrix}$$

$$G_C^H \cdot G_C = 2 \sum_{k=1}^{K} |x_k|^2 I_N$$

DECODING MATRIX

$$\begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2^* \end{bmatrix} = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\tilde{x}_1 = h_1 x_1 + h_2 x_2$$

$$\tilde{x}_2^* = h_2^* x_1 - h_1 x_2^*$$

$$\tilde{x}_2 = -h_1 x_2^* + h_2 x_1^*$$

$$\boxed{\Omega = \begin{bmatrix} h_1 & h_2^* \\ h_2 & -h_1^* \end{bmatrix}}$$

JAFARKHANI 4.17

RECEIVED VECTOR „r"

C – $2T \times N$ CODE WORD

$$r = C \cdot H + N = \begin{bmatrix} C_R \\ C_R^* \end{bmatrix} \cdot H + N = \begin{pmatrix} C_R \cdot H \\ C_R^* \cdot H \end{pmatrix} + \begin{pmatrix} N_1 \\ N_2 \end{pmatrix}$$

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \cdot \begin{bmatrix} x_1 & x_2 & x_3 \\ x_1 & x_2 & x_3 \\ x_1 & x_2 & x_3 \end{bmatrix}$$

$$H = [\alpha_1, \alpha_2, \ldots \alpha_N]$$

$$r' = (r_1, r_2, \ldots, r_T, r_{T+1}^*, r_{T+2}^*, \ldots r_{2T}^*)^T = \begin{bmatrix} C_R \circ H \\ C_R \cdot H^* \end{bmatrix}$$

$$\boxed{H^T \cdot C_R^T = (s_1, s_2, \ldots, s_K) \cdot \Omega_R}$$

THERE EXIST MATRIX Ω_R THAT:

$$\Omega_R \cdot \Omega_R^H = \sum_{n=1}^{N} |\alpha_n|^2 I_N$$

$$G = \sum_{k=1}^{K} x_k G_k \qquad \{G_k\}_{T\times N} \text{ - REAL MATRICES}$$

→ ELEMENTS OF NON-SQUARE GENERATOR MATRICES ARE LINEAR COMBINATIONS OF $x_1, x_2, \dots, x_K$

$$r'^T = (s_1, s_2, \dots, s_L)\cdot \Omega + [N_1^T, N_2^H]$$

$$\Omega \cdot \Omega^H = 2\sum_{n=1}^{N} |\alpha_n|^2 I_k$$

$$\boxed{r'^T \cdot \Omega^H = 2\sum_{n=1}^{N} |\alpha_n|^2 (s_1, s_2, \dots, s_k) + (N_1^T, N_2^H)\cdot\Omega^H}$$

$$\Omega = \begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 & \alpha_1^* & \alpha_2^* & \alpha_3^* & \alpha_4^* \\ \alpha_2 & -\alpha_1 & -\alpha_4 & \alpha_3 & \alpha_2^* & -\alpha_1^* & -\alpha_4^* & \alpha_3^* \\ \alpha_3 & \alpha_4 & -\alpha_1 & -\alpha_2 & \alpha_3^* & \alpha_4^* & -\alpha_1^* & -\alpha_2^* \\ \alpha_4 & -\alpha_3 & \alpha_2 & -\alpha_1 & \alpha_4^* & -\alpha_3^* & \alpha_2^* & -\alpha_1^* \end{bmatrix}$$

- APLICIRANO VO NAŠIOT ČLANAK

~~$[x_1, x_2, \dots, x_T, x_{T+1}, x_{T+2}, \dots, x_{2T}]$~~

$$Y = C\cdot H + N \qquad N = \begin{bmatrix} n_1 \\ n_1 \\ n_1 \end{bmatrix}_{J\times 1}$$

$$\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} X_1 & X_2 \\ -X_2^* & X_1^* \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = C\cdot H$$

$$Y' = [Y_1, Y_2, \dots, Y_T, Y_{T+1}^*, Y_{T+2}^*, \dots, Y_{2T}^*]$$

$$Y'^T = [x_1, x_2, \dots, x_K]\cdot\Omega + [N_1^T, N_2^H]$$

$$\Omega = \begin{bmatrix} h_1 & h_2^* \\ h_2^* & -h_1 \end{bmatrix}^T \qquad \boxed{Y'^T\cdot\Omega^H = 2\sum_{n=1}^{N} |\alpha_n|^2 (s_1, s_2, \dots, s_k) + [N_1^T N_2^H]\Omega^H}$$

$$y^T \cdot \Omega^H = 2 \sum_{n=1}^{N} |\alpha_n|^2 [s_1, s_2, \dots s_K] + (N_1^T, N_2^H) \cdot \Omega^H$$

$$[y_1, y_2^*] \cdot \overbrace{\begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix}^*}^{\Omega^T} = 2 \cdot \left[|h_1|^2 + |h_2|^2\right] \cdot [s_1, s_2] +$$

$$[n_1, n_1, n_1^*, n_1^*] \cdot \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix}^H$$

TIMESLOT

$$[y_1, y_2^*] \begin{bmatrix} h_1^* & h_2^* \\ h_2 & -h_1 \end{bmatrix} = [s_1 s_1, s_2 s_2] +$$

$$+ [n_1, n_2^*] \begin{bmatrix} h_1^* & h_2^* \\ h_2 & -h_1 \end{bmatrix} \Rightarrow$$

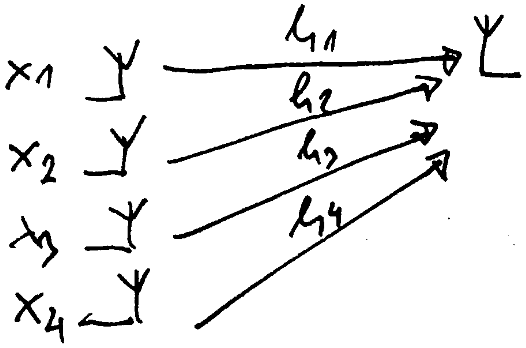
$$[y_1 h_1^* + y_2^* h_2, \; y_1 h_2^* - y_2^* h_1] = [s_1 s_1, s_2 s_2] +$$

$$+ [n_1 h_1^* + n_2^* h_2, \; n_1 h_2^* - n_2^* h_1]$$

- MNOGU VAZNO IZVEDUVANJE ŠTO GI PRETSTAVUVA FORMULITE VO FORMA NA MATRICI

$$[\tilde{x}_1, \tilde{x}_2] = [y_1, y_2^*] \cdot \overbrace{\begin{bmatrix} h_1 & h_2^* \\ h_2 & -h_1^* \end{bmatrix}}^{\Omega}{}^H = s_1 \underbrace{[x_1, x_2]}_{\boxed{MMV}} + [n_1, n_2^*] \overbrace{\begin{bmatrix} h_1 & h_2^* \\ h_2 & -h_1^* \end{bmatrix}^H}^{\Omega^H}$$

SE KORISTI POSLEDNATA NOMENKLATURA OD KANAL v7



$$C = \begin{array}{c} \left.\begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ -x_2 & x_1 & -x_4 & x_3 \\ -x_3 & x_4 & x_1 & -x_2 \\ -x_4 & -x_3 & x_2 & x_1 \\ x_1^* & x_2^* & x_3^* & x_4^* \\ -x_2^* & x_1^* & -x_4^* & x_3^* \\ -x_3^* & x_4^* & x_1^* & -x_2^* \\ -x_4^* & -x_3^* & x_2^* & x_1^* \end{bmatrix}\right. \end{array} \begin{array}{l} \left.\vphantom{\begin{matrix}a\\a\\a\\a\end{matrix}}\right\} T \\ \left.\vphantom{\begin{matrix}a\\a\\a\\a\end{matrix}}\right\} T \end{array}$$

$$H = [h_1, h_2, h_3, h_4]^T$$

$$\boxed{Y^T = G \cdot H^T + N^T}$$

$$Y^T = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \\ y_8 \end{bmatrix} = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ -x_2 & x_1 & -x_4 & x_3 \\ -x_3 & x_4 & x_1 & -x_2 \\ -x_4 & -x_3 & x_2 & x_1 \\ & \cdots & & \\ & \cdots & & \\ -x_4^* & -x_3^* & x_2^* & x_1 \end{bmatrix} \cdot \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ n_3 \\ n_4 \\ n_5 \\ n_6 \\ n_7 \\ n_8 \end{bmatrix} \begin{matrix} N_{T_s}^T \\ \\ N_{2T_s}^T \end{matrix}$$

$$Y_a^T = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_5^* \\ \vdots \\ y_8^* \end{bmatrix} = \begin{bmatrix} x_1 & x_2 & x_3 & x_4^* \\ & \cdots & & \\ x_1 & x_2 & x_3 & x_4 \\ & \cdots & & \\ -x_4 & -x_3 & x_2 & x_1 \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ n_3 \\ n_4 \\ n_5^* \\ n_6^* \\ n_7^* \\ n_8 \end{bmatrix} \begin{matrix} \} N_{T_s}^T \\ \\ \} N_{2T_s}^T \end{matrix}$$

PSM RALCEP

- REAL ORTHOGONAL DESIGN MMV

$$Y_a = [Y_1, Y_2^*] = [X_1, X_2]\,\Omega + [N_1, N_2^*] \quad / \Omega^H$$

$$\Omega = \Omega(\alpha_1, \alpha_2) = \begin{bmatrix} \alpha_1 & \alpha_2^* \\ \alpha_2 & -\alpha_1^* \end{bmatrix} = \begin{bmatrix} h_1 & h_2^* \\ h_2 & -h_1^* \end{bmatrix}$$

$$\boxed{[\tilde{X}_1, \tilde{X}_2] = [Y_1, Y_2^*]\,\Omega^H = (|\alpha_1|^2 + |\alpha_2|^2)[X_1, X_2] + \tilde{N}}$$

(*) GENERALIZACIJA

$$\tilde{X} = Y_a \cdot \Omega^H = 2\sum_{n=1}^{N} |h_n|^2 [X_1, X_2, \ldots X_K] + [N_1^*, N_{2T}^*]\,\Omega^H$$

$$\boxed{\Omega = \left[\begin{array}{cccc|cccc} h_1 & h_2 & h_3 & h_4 & h_1^* & h_2^* & h_3^* & h_4^* \\ h_2 & -h_1 & -h_4 & h_3 & h_2^* & -h_1^* & -h_4^* & h_3^* \\ h_3 & h_4 & -h_1 & -h_2 & h_3^* & h_4^* & -h_1^* & -h_2^* \\ h_4 & -h_3 & h_2 & -h_1 & h_4^* & -h_3^* & h_2^* & -h_1^* \end{array}\right]}$$

OVA MOŽE DA ZAFRKAVA VO FORMULATA ZA SNR !!!

$$\begin{bmatrix} y_1 \\ y_2^* \end{bmatrix} = [x_1, x_2] \cdot \begin{bmatrix} h_1 & h_2^* \\ h_2 & -h_1^* \end{bmatrix} \Rightarrow \begin{array}{l} y_1 = x_1 h_1 + x_2 h_2 \\ y_2^* = x_1 h_2^* - x_2 h_1^* \\ y_2 = x_1^* h_2 - x_2^* h_1 \end{array}$$

VO SIMULACIJATA NA ZOKI TOJ VAKA GO KONSTRUIRA STBC KODOT

$$\boxed{\gamma_2 = \frac{\epsilon}{N_0} \frac{G_2^2 \Lambda_2 \Delta_2^2}{G_2^2 \Lambda_2 \Delta_2 + 1}}$$

$$\begin{bmatrix} y_1 \\ y_2^* \end{bmatrix} = [x_1, x_2] \begin{bmatrix} h_1 & h_2^* \\ h_2 & -h_1^* \end{bmatrix} = [x_1, x_2] \cdot \Omega$$

$$[\tilde{x}_1, \tilde{x}_2] = [y_1, y_2] \cdot \Omega^H = [y_1, y_2] \begin{bmatrix} h_1 & h_2^* \\ h_2 & -h_1^* \end{bmatrix}^H$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix}^T \cdot \begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix}$$

$$\begin{bmatrix} y_{11} & y_{21} \\ y_{12} & y_{22} \end{bmatrix} = \begin{bmatrix} h_{11} & h_{21} \\ h_{12} & h_{22} \end{bmatrix} \cdot \begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix}$$

$$y_{11} = h_{11} x_1 + h_{21} x_2 + n_1$$

$$y_{21} = -h_{11} x_2^* + h_{21} x_1^* + n_2$$

$$r = s + n \qquad \frac{E(s^2)}{E(n^2)} = \underline{\underline{SNR}}$$

$$\tilde{x} = 2 \sum_{i=1}^{K} |h_i|^2 x + [N_1^T, N_2^T] \Omega^H \qquad \Delta_2 = 2 \sum_{i=1}^{K} |h_i|^2$$

$$2 \times 2 \times 1 \qquad P_S = G_2^2 \Lambda_2^2 \Delta_2^2 \cdot \epsilon \qquad P_N = G_2^2 \Lambda_2^2 \Delta_2 + \Lambda_2$$

$$\gamma_2 = \frac{\epsilon}{N_0} \frac{G_2^2 \Lambda_2 \Delta_2^2}{G_2^2 \Lambda_2 \Delta_2 + 1}$$

$$\Delta_2' = \sum_{i=1}^{K} |h_i|^2 \qquad \Lambda_2' = \sum_{i=1}^{K} |g_i|^2$$

$$\gamma_2 = \frac{\epsilon}{N_0} \frac{G_2^2 \, 2 \Lambda_2' \cdot 4 \cdot \Delta_2'^{2}}{4 G_2^2 \Lambda_2' \Delta_2' + 1} = \frac{\epsilon}{N_0} \frac{8 G_2^2 \Lambda_2' \Delta_2'^{2}}{4 G_2^2 \Lambda_2' \Delta_2' + 1}$$

$$P_S = 16 G_2^2 \Lambda_2^2 \Delta_2^2 \cdot \epsilon \qquad P_N = 2\left(G_2^2 \Lambda_2^2 \Delta_2 + \Lambda_2\right)$$

$$C_{3\,1/2} = \begin{bmatrix} c_1 & c_2 & c_3 \\ -c_2 & c_1 & -c_4 \\ -c_3 & c_4 & c_1 \\ -c_4 & -c_3 & c_2 \\ c_1^* & c_2^* & c_3^* \\ -c_2^* & c_1^* & -c_4^* \\ -c_3^* & c_4^* & c_1^* \\ -c_4^* & -c_3^* & c_2^* \end{bmatrix}$$

$$\Omega = \left[\begin{array}{cccc|cccc} h_1 & h_2 & h_3 & h_4 & h_1^* & h_2^* & h_3^* & h_4^* \\ h_2 & -h_1 & -h_4 & h_3 & h_2^* & -h_1^* & -h_4^* & h_3^* \\ h_3 & h_4 & -h_1 & -h_2 & h_3^* & h_4^* & -h_1^* & -h_2^* \end{array}\right]$$

?

- Real orthogonal 2×L code

$$[\tilde{x}_1, \tilde{x}_2] = [y_1, y_2^*] \cdot \Omega^H = [y_1, y_2^*] \begin{bmatrix} h_1 & h_2^* \\ h_2 & -h_1^* \end{bmatrix}$$

$$\tilde{x}_1 = y_1 h_1 + y_2^* h_2 \qquad \tilde{x}_2 = y_1 h_2^* - y_2^* h_1^*$$

- Real orthogonal 4×L code

$$[\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \tilde{x}_4] = [y_1, y_2, y_3^*, y_4^*] \begin{bmatrix} h_1 & h_2^* & h_3^* & h_4^* \\ h_2^* & -h_1^* & h_4^* & -h_3^* \\ h_3^* & -h_4^* & -h_1^* & h_2^* \\ h_4^* & h_3^* & -h_2^* & -h_1^* \end{bmatrix}$$

$$y_1 \quad y_2 \quad y_3 \quad y_4$$

$$y_1 h_1^* + y_2 h_2^* + \cancel{y_3^* h_3^*} + y_4^* h_4^* + y_1 h_2^* - y_2 h_1^* - h_4 y_3^* + h_3^* y_4^*$$
$$+ y_1 h_3^* + y_2 h_4^* - y_3^* h_1^* - y_4^* h_2^* + y_4^* h_4^* - y_2 h_3^* + y_3^* h_2 - y_4^* h_1^*$$

$$y_1 h_1^* + y_2 h_2^* + y_3^* h_3^* + y_4^* h_4^* + y_1 h_2^* - y_2 h_1^* - y_3 h_4^* + y_4 h_3$$
$$+ y_1 h_3^* + y_2 h_4^* - y_3 h_1^* - y_4 h_2^* + y_1 h_4^* - y_2 h_3^* + y_3 h_2^* - y_4 h_1^*$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = [h_1\ h_2]\begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} \qquad \begin{aligned} y_1 &= h_1x_1 + h_2x_2 + n_1 \\ y_2 &= -h_1x_2^* + h_2x_1^* + n_2 \end{aligned}$$

$$\begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{bmatrix} = [y_1\ y_2^*] \underbrace{\begin{bmatrix} h_1 & h_2^* \\ h_2 & -h_1^* \end{bmatrix}^H}_{\Omega^H} = [y_1, y_2^*] \underbrace{\begin{bmatrix} h_1^* & h_2^* \\ h_2 & -h_1 \end{bmatrix}}_{\Omega^H}$$

$$\boxed{\tilde{x}_1 = y_1 h_1^* + y_2^* h_2 \qquad \tilde{x}_2 = y_1 h_2^* - y_2^* h_1}$$

$$\tilde{x}_1 = h_1^*(h_1x_1 + h_2x_2 + n_1) + h_2(-h_1^*x_2 + h_2^*x_1 + n_2^*)$$

$$= |h_1|^2x_1 + \cancel{h_1^*h_2x_2} + h_1^*n_1 - \cancel{h_2h_1^*x_2} + |h_2|^2x_1 + h_2n_2^*$$

$$= (|h_1|^2 + |h_2|^2)x_1 + h_1^*n_1 + h_2n_2^*$$

$$\tilde{x}_2 = (|h_1|^2 + |h_2|^2)x_2 + h_2^*n_1 - h_1n_2^*$$

$$\boxed{\xi_1 = h_1^*n_1 + h_2n_2^* \qquad \xi_2 = h_2^*n_1 - h_1n_2^*}$$

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$$\begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{bmatrix} = [y_1\ y_2^*]\ \cancel{\begin{bmatrix} h_1^* & h_2^* \\ h_2 & -h_1 \end{bmatrix}} + \underbrace{[n_1\ n_2^*] \overbrace{\begin{bmatrix} h_1^* & h_2^* \\ h_2 & -h_1 \end{bmatrix}}^{\Omega^H}}_{[\xi_1,\ \xi_2]}$$

$$\begin{bmatrix} |h_1|^2+|h_2|^2 & 0 \\ 0 & |h_1|^2+|h_2|^2 \end{bmatrix}$$

$H = [h_1, h_2, h_3, h_4]$; $N = [n_1, n_2, n_3, \dots, n_8]$ $N_a = [n_1, \dots, n_4, n_5^*, \dots, n_8^*]$

$$C = [x_1, x_2, \dots x_4;\ \dots\dots\dots;\ -\bar{x}_4\ -\bar{x}_3\ \bar{x}_2\ \bar{x}_1]_{8\times4}$$

$$\Omega = [\underbrace{h_1, h_2, h_3, h_4, \bar{h}_1\ \bar{h}_2\ \bar{h}_3\ \bar{h}_4}_{1\times8};\ \dots\ h_4\ -h_3\ h_2\ -h_1\ \bar{h}_4\ -\bar{h}_3\ \bar{h}_2\ -\bar{h}_1]_{4\times8}$$

$$Y = (C\cdot H^T + N^T)^T = [y_1, y_2, y_3, \dots, y_8]$$

$$Y_a = [y_1, y_2, y_3, y_4, \bar{y}_5, \bar{y}_6, \bar{y}_7, \bar{y}_8] = X\cdot\Omega + N_a$$

$$\tilde{X} = Y_a\cdot\Omega^H = 2\cdot X\cdot\Delta_1 I_{4\times4} + \underbrace{N_a\cdot\Omega^H}_{\Xi}$$

$$\Delta_1 = \sum_{i=1}^{4} |h_i|^2 = \mathrm{rank}^2(H)$$

MM V

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