$$P(\gamma > \gamma_{th}) = \Gamma\left(m, \frac{m\gamma_{th}}{\Gamma(m}\right)$$

$$P_{out} = 1 - P(\gamma_1 > \gamma_{th}; \gamma_2 > \gamma_{th}; \gamma_3 > \gamma_{th}, \dots, \gamma_N > \gamma_{th})$$

$$\boxed{P_{out} = 1 - \prod_{n=1}^{N} \frac{\Gamma\left(m_n, \frac{m_n\gamma_{th}}{\bar\gamma_n}\right)}{\Gamma(m_n)}}$$

$$\boxed{p_{GAUS}(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-m)^2}{2\sigma^2}}}$$

- ~~IZRAZ~~ IZRAZ PREKO $\gamma\left(m, \frac{m\gamma_{th}}{\bar\gamma}\right)$

$$P(\gamma > \gamma_{th}) = \frac{1}{\Gamma(m)} \int_{\frac{m\gamma_{th}}{\bar\gamma}}^{\infty} t^{m-1} e^{-t}\, dt =$$

$$= \frac{1}{\Gamma(m)}\left[-\int_0^{\frac{m\gamma_{th}}{\bar\gamma}} t^{m-1} e^{-t}\, dt + \underbrace{\int_0^{\infty} t^{m-1} e^{-t}\, dt}_{\Gamma(m)}\right] =$$

$$= 1 - \frac{1}{\Gamma(m)} \int_0^{\frac{m\gamma_{th}}{\bar\gamma}} t^{m-1} e^{-t}\, dt = 1 - \frac{\gamma\left(m, \frac{m\gamma_{th}}{\bar\gamma}\right)}{\Gamma(m)}$$

$$\boxed{P_{out} = 1 - \prod_{n=1}^{N} \left(1 - \frac{\gamma\left(m_n, \frac{m_n\gamma_{th}}{\bar\gamma}\right)}{\Gamma(m)}\right)}$$

~~IZ TEMA~~ TIVEN IZRA.

- MGF OF $1/\gamma$ $\quad p(\gamma) = \frac{m^m \gamma^{m-1}}{\Gamma(m)\cdot\bar\gamma^m} e^{-\frac{m\gamma}{\bar\gamma}}$ NAKAG. SNR DISTRO.

$$M_{1/\gamma}(s) = E\left[e^{sx}\right] = E\left[e^{\frac{s}{\gamma}}\right] = \int_0^\infty e^{\frac{s}{\gamma}} \cdot p(\gamma)\, d\gamma$$

$$= \int_0^\infty e^{\frac{s}{\gamma}} \cdot \frac{m^m \cdot \gamma^{m-1}}{\Gamma(m)\cdot\bar\gamma^m} e^{-\frac{m\gamma}{\bar\gamma}}\, d\gamma = \frac{m^m}{\Gamma(m)\cdot\bar\gamma^m} \underbrace{\int_0^\infty \gamma^{m-1} e^{\frac{s}{\gamma} - \frac{m\gamma}{\bar\gamma}}\, d\gamma}_{\circledast}$$

GRADSTHATN 3.471.9

$$\boxed{\int_0^\infty x^{\nu-1} e^{-\frac{\beta}{x} - \gamma x}\, dx = 2\left(\frac{\beta}{\gamma}\right)^{\frac{\nu}{2}} K_\nu\left(2\sqrt{\beta\gamma}\right) \qquad \begin{array}{l} \nu \equiv m \\ \beta \equiv s \\ \gamma \equiv \frac{m}{\bar\gamma} \end{array}}$$

$$M_{1/\gamma}(s) = \frac{m^m}{\Gamma(m)\cdot\bar\gamma^m} \cdot 2\left(\frac{s}{m/\bar\gamma}\right)^{m/2} K_m\left(2\sqrt{s\cdot\frac{m}{\bar\gamma}}\right) =$$

$$= \frac{m^{m-\frac{m}{2}}\cdot\bar\gamma^{\frac{m}{2}}}{\Gamma(m)\cdot\bar\gamma^m} \cdot 2 s^{m/2} \cdot K_m\left(2\sqrt{\frac{sm}{\bar\gamma}}\right) = \frac{2}{\Gamma(m)\cdot\bar\gamma^{m/2}} (s\cdot m)^{\frac{m}{2}} K_m\left(2\sqrt{\frac{sm}{\bar\gamma_1}}\right)$$

$$M_{1/\bar{\gamma}}(s) = \frac{2}{\Gamma(m)} \cdot \left(\frac{s \cdot m}{\bar{\gamma}}\right)^{m/2} K_m\left(2\sqrt{\frac{sm}{\bar{\gamma}}}\right)$$ DOKAZANO!!!

K_m - MODIFIED BESSEL FUNCTION OF SECOND KIND

FOR THE n-TH HOP

$$M_{1/\bar{\gamma}_n}(s) = \frac{2}{\Gamma(m)} \left(\frac{s \cdot m}{\bar{\gamma}_n}\right)^{m/2} K_m\left(2\sqrt{\frac{sm}{\bar{\gamma}_n}}\right)$$

- CONVERT AM. RICIAN DISTRO TO SNR RICIAN DISTRO

$$p(r) = \frac{r}{\sigma^2} e^{-\frac{r^2+A^2}{2\sigma^2}} I_0\left(\frac{Ar}{\sigma^2}\right) \qquad z = \frac{r}{\sigma}$$

$$p(z) = z\, e^{-\frac{z^2+a^2}{2}} I_0(az) \qquad a = \frac{A}{\sigma}$$

$$P(z < z_0) = \int_0^{z_0} z\, e^{-\frac{z^2+a^2}{2}} I_0(az)\, dz = \underline{\underline{1 - Q(a, z_0)}}$$

---

gamma0 = -20:0.1:0

$$\text{ratio} = \frac{10^{0.1 \cdot \text{gamma0dB}}}{10^{0.1\, \text{mean-ndB}}} = [9.9;\ 10,1;\ 10,4;\ 10,6;\ 10,8;\ \ldots]e\text{-}3$$

$$\text{Pout} = 1 - e^{-\text{ratio}} = 1 - \frac{1}{e^{\text{ratio}}}$$

$$\text{gamma0} \nearrow \rightarrow e^{\text{ratio}} \nearrow \Rightarrow \text{Pout} \nearrow \Delta$$

$$p(x) = \frac{x}{\sigma^2} e^{-\frac{x^2}{2\sigma^2}} \Big\} = \underline{\underline{\text{RAYLEIGH}}} \qquad \boxed{E[x^2] = 2\sigma^2}$$

$$\gamma = \frac{x^2}{n^2} \qquad \frac{d\gamma}{dx} = \frac{2x}{n^2} \qquad \boxed{x = \sqrt{\gamma} \cdot n}$$

$$p(\gamma) = \left.\frac{p(x)}{\frac{d\gamma}{dx}}\right|_{x=\sqrt{\gamma}\cdot n} = \frac{\sqrt{\gamma} \cdot n}{\sigma^2} \cdot e^{-\frac{\gamma \cdot n^2}{2\sigma^2}} \cdot \frac{1}{\frac{2\sqrt{\gamma} \cdot n}{n^2}}$$

$$= \frac{n^2}{2\sigma^2} \cdot e^{-\frac{\gamma}{\frac{2\sigma^2}{n^2}}} = \left| \bar{\gamma} = \frac{2\sigma^2}{n^2} \right| = \frac{1}{\bar{\gamma}} \cdot e^{-\gamma/\bar{\gamma}}$$

$$P(\gamma < \gamma_0) = \int_0^{\gamma_0} \frac{1}{\bar{\gamma}} e^{-\gamma/\bar{\gamma}} d\gamma = \left| \begin{array}{ll} \frac{\gamma}{\bar{\gamma}} = \mu & \frac{d\gamma}{\bar{\gamma}} = d\mu \\ \gamma = \gamma_0 & \mu = \frac{\gamma_0}{\bar{\gamma}} \end{array} \right|$$

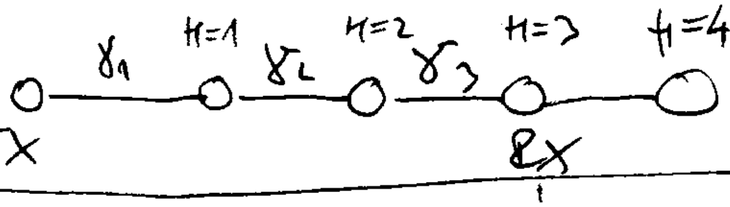
$$\underline{P(\gamma<\gamma_0)} = \int_0^{\frac{\gamma_0}{\bar{\gamma}}} e^{-\mu}\, d\mu = -e^{-\mu}\Big|_0^{\frac{\gamma_0}{\bar{\gamma}}} = 1 - e^{-\frac{\gamma_0}{\bar{\gamma}}}$$

$$\boxed{p(\gamma) = \frac{1}{\bar{\gamma}} e^{-\frac{\gamma}{\bar{\gamma}}}}$$

$$\gamma \to \underline{SNR}$$

$$\bar{\gamma} = \frac{2\sigma^2}{n^2} \qquad \boxed{2\sigma^2 = E(x^2)}$$

$$P(\gamma > \gamma_{th}) = \int_{\gamma_{th}}^{\infty} \frac{e^{-\frac{\gamma}{\bar{\gamma}}}}{\bar{\gamma}}\, d\gamma = -e^{-\mu}\Big|_{\frac{\gamma_{th}}{\bar{\gamma}}}^{\infty} = e^{-\mu}\Big|_{\infty}^{\frac{\gamma_{th}}{\bar{\gamma}}} = e^{-\frac{\gamma_{th}}{\bar{\gamma}}}$$



$$G_n^2 = \frac{1}{\alpha_n^2 + N_{0,N}} \; ; \quad SNR = 10\log \frac{fadedSig^2}{n_0}$$

$$\frac{fadedSig^2}{n_0} = 10^{0.1 \cdot SNR}$$

$$\boxed{n_0 = fadedSig^2 \cdot 10^{-0.1 SNR}}$$

- OUTAGE PROBABILITY GOLDSMITH

$$P_b = \frac{1}{2} erfc\left(\sqrt{\frac{E_b}{N_0}}\right) \qquad P_b = 10^{-3}$$

$$\frac{S}{N} = \frac{E_b/T_{sym}}{N_0 \cdot W} = \frac{E_b}{N_0} \cdot \frac{2T_{sam}}{T_{sym}} = \frac{\log M\, E_b}{N_0} \cdot \frac{2T_{sam}}{T_{sym}}$$

$$M=2 \qquad \frac{S}{N} = \frac{E_b}{N_0} \cdot \frac{2T_{samp}}{T_{sym}} \qquad T_{sym} = 2T_{samp}$$

$$\frac{S}{N}\Big|_{eff} = \frac{E_b}{N_0} \qquad 10^{-3} = \frac{1}{2} erfc\left(\sqrt{SNR}\right) \Rightarrow$$

$$SNR = 4.77477 \qquad SNR_{dB} = 10\log SNR = 6.8 \div \underline{7dB}$$

$$\text{mean}(pow\_y) = \frac{pow\_s}{gamma}$$

$$gamma\_m = \frac{\text{mean}(pow\_s)}{\text{mean}(pow\_y)} \qquad \text{mean}(pow\_y) = \frac{\text{mean}(pow\_s)}{gamma\_m}$$

$$coef = \sqrt{\frac{\text{mean}(pow\_s)}{gamma\_m}}$$

$$\boxed{P_{out}(\gamma < \gamma_0) = 1 - e^{-\frac{\gamma_0}{\gamma_{th}}}}$$

- DIGITAL COMM. OVER FADING CHANNELS (ALOUINI)

$$p_\alpha(\alpha) = \frac{2(1+n^2)e^{-n^2}\alpha}{\Omega} \exp\left(-\frac{(1+n^2)\alpha^2}{\Omega}\right) I_0\left(2n\alpha\sqrt{\frac{1+n^2}{\Omega}}\right)$$

$$\boxed{K = n^2}$$

$$p_\alpha(\alpha) = \frac{2(1+K)e^{-K}\alpha}{\Omega} \exp\left(-\frac{(1+K)\alpha^2}{\Omega}\right) I_0\left(2\sqrt{K}\alpha\sqrt{\frac{1+K}{\Omega}}\right)$$

2.2 MODELING OF FLAT FADING CHANNELS (ALOUINI BOOK)

- α - Random Variable representing fading amplitude

$\Omega = \overline{\alpha^2}$ - mean square value of α I.E AVERAGE FADING POWER

$\gamma = \alpha^2 \cdot \frac{E_s}{N_0}$ - instantaneous SNR PER SYMBOL

$\bar{\gamma} \to \bar{\gamma} = \frac{\Omega E_s}{N_0}$ - AVERAGE SNR PER SYMBOL

MMV

$$p_\gamma(\gamma) = \frac{p_\alpha(\alpha)}{\frac{d\gamma}{d\alpha}}\Bigg|_{\alpha = f(\gamma)} \quad \text{I.e } \alpha = \sqrt{\gamma}\sqrt{\frac{N_0}{E_s}}$$

MMV

$$\boxed{\bar{\gamma} = \frac{\Omega E_s}{N_0} \qquad \frac{E_s}{N_0} = \frac{\bar{\gamma}}{\Omega}} \qquad \boxed{\alpha = \sqrt{\gamma}\sqrt{\frac{\Omega}{\bar{\gamma}}} = \sqrt{\Omega\gamma/\bar{\gamma}}}$$

$$\frac{d\gamma}{d\alpha} = \frac{2\alpha E_s}{N_0} = \frac{2\alpha\bar{\gamma}}{\Omega} = \frac{2\bar{\gamma}}{\Omega}\cdot\sqrt{\frac{\Omega\gamma}{\bar{\gamma}}} = \frac{2\sqrt{\gamma\bar{\gamma}}}{\sqrt{\Omega}}$$

$$\boxed{p_{\bar{\gamma}}(\gamma) = \frac{p_\alpha\left(\sqrt{\frac{\Omega\gamma}{\bar{\gamma}}}\right)}{2\sqrt{\frac{\gamma\bar{\gamma}}{\Omega}}}} \quad (**)$$

$$\boxed{p_\gamma(\gamma) = \frac{p_\alpha\left(\sqrt{\frac{\Omega\gamma}{\bar\gamma}}\right)}{2\sqrt{\frac{\bar\gamma\gamma}{\Omega}}}}$$

MMV

IZRAZ 2.3
Alouini Book

MOMENT GEN. FUNCTION

$$\boxed{M_\gamma(s) = \int_0^\infty p_\gamma(\gamma)\, e^{s\gamma}\, d\gamma}$$

$$\sigma^2 = \overline{(\xi - \bar\xi)^2} = \overline{\xi^2} - \bar\xi^2$$

$$AF = \frac{var(\alpha^2)}{E^2[\alpha^2]} = \frac{E\left((\alpha^2 - \Omega)^2\right)}{\Omega^2} = \frac{E(\gamma^2) - E^2(\gamma)}{[E(\gamma)]^2}$$

AF – AMOUNT OF FADING.

$$p_\gamma(\gamma) = \frac{1}{\frac{d\gamma}{d\alpha}} \frac{2(1+K)e^{-K}\sqrt{\frac{\cancel{\Omega}\gamma}{\bar\gamma}}}{\cancel{\Omega}\sqrt{\Omega}} \cdot \exp\left(-\frac{1+K}{\Omega}\frac{\Omega\gamma}{\bar\gamma}\right) I_0\left(2\sqrt{K}\sqrt{\frac{\Omega\gamma}{\bar\gamma}}\sqrt{\frac{1+K}{\Omega}}\right)$$

$$\boxed{p_\gamma(\gamma) = \frac{1}{\frac{d\gamma}{d\alpha}} \cdot 2(1+K)e^{-K}\sqrt{\frac{\gamma}{\Omega\bar\gamma}}\ \exp\left(-(1+K)\frac{\gamma}{\bar\gamma}\right) I_0\left(2\sqrt{K}\sqrt{(1+K)\frac{\gamma}{\bar\gamma}}\right)}$$

$$\frac{d\gamma}{d\alpha} = 2\sqrt{\frac{\bar\gamma\gamma}{\Omega}}$$

$$p_\gamma(\gamma) = \frac{\cancel{2}(1+K)e^{-K}\sqrt{\frac{\cancel{\gamma}}{\cancel{\Omega}\bar\gamma}}}{\cancel{2}\sqrt{\frac{\bar\gamma\cancel{\gamma}}{\cancel{\Omega}}}}\ \exp\left(-(1+K)\frac{\gamma}{\bar\gamma}\right) I_0\left(2\sqrt{K}\sqrt{(1+K)\frac{\gamma}{\bar\gamma}}\right)$$

$$\boxed{p_\gamma(\gamma) = \frac{(1+K)e^{-K}}{\bar\gamma}\ \exp\left(-(1+K)\frac{\gamma}{\bar\gamma}\right) I_0\left(2\sqrt{K}\sqrt{(1+K)\frac{\gamma}{\bar\gamma}}\right)}$$

MMV

$$p_\alpha(\alpha) = \frac{\alpha}{\sigma^2}\, e^{-\frac{\alpha^2 + A^2}{2\sigma^2}}\, I_0\left(\frac{\alpha \cdot A}{\sigma^2}\right)$$

$$p_\alpha(\alpha) = \frac{2\alpha(1+K)}{\Omega}\ \exp\left(-K - \frac{(1+K)\alpha^2}{\Omega}\right) I_0\left(2\alpha\sqrt{\frac{K(1+K)}{\Omega}}\right)$$

$$p(r) = \frac{r}{\sigma^2} e^{-\frac{r^2+A^2}{2\sigma^2}} I_0\left(\frac{A\cdot r}{\sigma^2}\right)$$

$K = \frac{A^2}{r^2}$ $r = \sqrt{K\cdot A^2} = A\sqrt{K}$ $A^2 = K\cdot r^2$, $A = r\sqrt{K})$

$$p(r) = \frac{\cancel{A\sqrt{K}}\, r}{\Omega/2} e^{-\frac{r^2(1+K)}{2\sigma^2}} I_0\left(\frac{\cancel{A}^2\sqrt{K}}{\sigma^2}\right)$$

$\Omega = 2\sigma^2$

$$p(r) = \frac{2r}{\Omega} e^{-\frac{r^2(1+K)}{2\sigma^2}} I_0\left(\frac{2r^2\sqrt{K}}{\Omega}\right)$$

$\boxed{HMV}$

GOLDSMITH: (EXPRESION FOR RICIAN USING K FACTOR)

$$p_z(z) = \frac{z}{\sigma^2} \exp\left[-\frac{z^2+s^2}{2\sigma^2}\right] I_0\left(\frac{zs}{\sigma^2}\right), \quad x > 0$$

$2\sigma^2 = \sum_{n, n\neq 0} E(\alpha_n^2)$ - NON-LOS multipath comp

$s^2 = \alpha_0^2$ - POWER IN THE LOS comp

$$\boxed{P_r = \int_0^\infty z^2 p_z(z)\,dz = s^2 + 2\sigma^2 = \Omega}$$ → TOTAL POWER

$$K = \frac{s^2}{2\sigma^2}$$

$$\boxed{s^2 = \frac{K\cdot\Omega}{K+1} \qquad 2\sigma^2 = \frac{\Omega}{K+1}}$$

$$p_z(z) = \frac{2z}{\frac{\Omega}{K+1}} \exp\left[-\frac{z^2+\frac{K\Omega}{K+1}}{\frac{\Omega}{K+1}}\right] I_0\left(\frac{z\sqrt{\frac{K\Omega}{K+1}}}{\frac{\Omega}{2\cancel{(K+1)}}}\right)$$

$\sqrt{K+1}$

$$= \frac{2z(K+1)}{\Omega} \exp\left[-\frac{z^2(K+1)+K\Omega}{\cancel{K+1}\,\Omega}\right] I_0\left(\frac{2z\sqrt{K(K+1)}}{\sqrt{\Omega}}\right)$$

$$\boxed{p_z(z) = \frac{2z(K+1)}{\Omega} \exp\left[-\frac{z^2(K+1)}{\Omega} - K\right] I_0\left(\frac{2z\sqrt{K(K+1)}}{\sqrt{\Omega}}\right)}$$

$$\Omega = \int_0^\infty z^2 p_z(z)\,dz$$ – AVERAGE RECEIVED POWER

$$\Omega = s^2 + 2\sigma^2 \qquad K = \frac{s^2}{2\sigma^2}$$

$$\Omega = s^2 + \frac{s^2}{K} = s^2\left(1 + \frac{1}{K}\right) = \frac{K+1}{K}$$

$$\boxed{s^2 = \frac{K\cdot\Omega}{K+1}} \qquad \Omega = K\cdot 2\sigma^2 + 2\sigma^2 \qquad \boxed{2\sigma^2 = \frac{\Omega}{K+1}}$$

$\boxed{\text{MMV}}$

- SNR expression (check Alouini Book) K, N.4.115

MMV $\alpha = \sqrt{\frac{\Omega\gamma}{\bar{\gamma}}}$

$$\frac{d\gamma}{d\alpha} = \frac{d}{d\alpha}\left(\alpha^2 \frac{E_s}{N_0}\right) = \frac{2\alpha E_s}{N_0}$$

$$\bar{\gamma} = \Omega \frac{E_s}{N_0} \qquad \frac{\bar{\gamma}}{\Omega} = \frac{E_s}{N_0} \qquad - -$$

$$\frac{d\gamma}{d\alpha} = 2\alpha \frac{\bar{\gamma}}{\Omega}; \qquad \boxed{\gamma = \alpha^2 \frac{E_s}{N_0}} \Rightarrow$$

$$\alpha = \sqrt{\cancel{\gamma}}\sqrt{\frac{\cancel{N_0}}{\cancel{E_s}}} = \sqrt{\cancel{\gamma}}\sqrt{\frac{\cancel{\Omega}}{\cancel{\bar{\gamma}}}} = \sqrt{\gamma}\sqrt{\frac{\Omega}{\bar{\gamma}}} = \sqrt{\frac{\gamma\Omega}{\bar{\gamma}}}$$

$$p_\gamma(\gamma) = \frac{p_\alpha(\alpha)}{2\alpha\frac{\bar{\gamma}}{\Omega}} = \frac{\Omega}{\cancel{2\alpha}\bar{\gamma}} \cdot \frac{\cancel{2\alpha}(K+1)}{\cancel{\Omega}} \exp\left(-\frac{K+1}{\cancel{\Omega}}\frac{\cancel{\Omega}\gamma}{\bar{\gamma}} - K\right) \cdot I_0\left(2\sqrt{\frac{\cancel{\Omega}\gamma}{\bar{\gamma}}}\,\frac{\sqrt{K(K+1)}}{\cancel{\sqrt{\Omega}}}\right)$$

$$\boxed{p_\gamma(\gamma) = \frac{K+1}{\bar{\gamma}} \exp\left(-(K+1)\frac{\gamma}{\bar{\gamma}} - K\right) \cdot I_0\left(2\sqrt{K(K+1)\frac{\gamma}{\bar{\gamma}}}\right)}$$

SNR DISTRIBUTION FOR RICIAN CHANNEL

$$\bar{\gamma} = \frac{E_s}{N_0}\cdot\Omega \searrow$$

$$\bar{\gamma} = \frac{\text{mean}(pow\_s)}{\boxed{\text{mean}(pow\_n)}}$$

$\bar{\gamma} = SNR \cdot \Omega$ // $SNR = 0:40\ dB$

$\Omega = \text{mean}(|fadedSig|^2)$

$\gamma = \alpha^2 \frac{E_s}{N_0}$

$SNR = \frac{P_s}{P_N} = \frac{E_s/T_{sym}}{N_0 \cdot W} = \frac{E_s \, 1/T_{sym}}{N_0 \cdot \frac{f_s}{2}} = \frac{E_s}{N_0} \cdot \frac{1/T_{sym}}{\frac{1}{2T_{samp}}}$

$SNR = \frac{E_s}{N_0} \cdot \frac{2T_{samp}}{T_{sym}}$ $\frac{E_s}{N_0} = SNR \cdot \frac{0.5T_{sym}}{T_{samp}}$

$T_{sym} = T_{samp} \Rightarrow \frac{E_s}{N_0} = 0.5\,SNR$

$\frac{E_s}{N_0} \Leftrightarrow \frac{J}{W/Hz} \Leftrightarrow$ ~~[illegible]~~ $\frac{J}{W \cdot sec} \Leftrightarrow \frac{\cancel{J}}{\cancel{J}} \Leftrightarrow 1$

$\boxed{\bar{\gamma} = \Omega \frac{E_s}{N_0}} = \underline{0.5\,SNR \cdot \Omega}$ [~~[illegible]~~)

$K = 2 = \frac{a^2}{2\sigma^2}$

$a^2 + 2\sigma^2 = 1$ $2\sigma^2 = 1 - a^2$

$\frac{a^2}{1-a^2} = 2$ $a^2 = 2 - 2a^2$ $a^2 = 2$

$\boxed{a = \pm\sqrt{2}}$ $2\sigma^2 = 1 - a^2 = 1 - 2 = -1$

$2\sigma^2 = 0.1 \Rightarrow a^2 = 1 - 0.1 = 0.9$

$\boxed{a = \sqrt{0.9} = 0.95}$ (0.94868)

$\sigma^2 = \frac{0.1}{2} = 0.05$ $\sigma = 0.2236$

$\boxed{K = \frac{0.9}{0.1} = 9}$

⊙ COOPERATIVE COMMUNICATIONS WITH OUTAGE-OPTIMAL OPPORTUNISTIC RELAYING

$y_D = \alpha_{AB} x_A + n_B$ x_A – TRANSMITTED SIGNAL

$\alpha_{AB} \sim \mathcal{CN}(0, \Omega_{AB})$ → CHANNEL GAIN BETWEEN A-B

$n_B \sim \mathcal{CN}(0, N_0)$ → ADDITIVE GAUSSIAN (AWGN)

$\boxed{\gamma_{AB} \triangleq |\alpha_{AB}|^2}$ $\gamma_{AB} \sim \mathcal{E}\left(\frac{1}{\Omega_{AB}}\right)$

↳ SQUARED CHANNEL STRENGTH

$$\boxed{\overline{\left(\xi - \bar{\xi}\right)^2} = \zeta^2}$$

PDF OF γ_{AD}:

$$\boxed{p_{\gamma_{AD}}(x) = \frac{1}{\Omega_{AD}} \exp\left(-\frac{x}{\Omega_{AD}}\right)}$$

K

$P_{source} = \xi P_{tot}$ $P_{relay} = \sum_{k=1}^{K} P_k = (1-\xi) P_{tot}$

AGGREGATE POWER CONSTRAINT

AVERAGE SIGNAL TO NOISE RATIO

$$\boxed{\eta_{SK} \triangleq \Omega_{SK} \frac{P_{source}}{N_0} \qquad \eta_{KD} \triangleq \Omega_{KD} \frac{P_K}{N_0}}$$

SOURCE K-TH RELAY

$\mathcal{CN}(\mu, \sigma^2)$ – COMPLEX CIRCULARLY SYMMETRIC GAUSS. D.

$\tilde{N}_m(\mu, \Sigma)$ – m-VARIATE GAUSSIAN DISTR.

Σ – COVARIANCE MATRIX

$$\boxed{E\{\gamma_{AD}\} = \Omega_{AD}}$$

$$D \triangleq \left\{k \in S_{relay} : \frac{1}{2} \log_2\left(1 + \xi \gamma_{Sk} \frac{P_{tot}}{N_0}\right) \geq R\right\}$$

$$\Pr\{D_e\} = \prod_{i \in D_e} \Pr\left\{\gamma_{si} \underset{\infty}{\geq} K_1\right\} \prod_{j \notin D_e} \Pr\{\gamma_{sj} \leq K_1\}$$

$$\Pr(\gamma_{si} \geq K_1) = \int_{K_1}^{\infty} \frac{1}{\Omega_{AD}} e^{-x/\Omega_{AD}} dx = -e^{-\frac{x}{\Omega_{AD}}}\Big|_{K_1}^{\infty} = e^{-\frac{K}{\Omega_{AD}}}$$

$$\Pr(\gamma_{ji} \leq K_1) = \int_{\cancel{0}}^{K_1} \frac{1}{\Omega_{AD}} e^{-x/\Omega_{AD}} dx = 1 - e^{-K/\Omega_{AD}}$$

$$\Pr\{D_e\} = \prod_{i \in D_e} \left(1 - e^{-K_1/\Omega_{si}}\right) \prod_{j \notin D_e} \left(1 - e^{-K_1/\Omega_{sj}}\right)$$

$$\boxed{K_1 = \frac{2^{2R}-1}{\xi P_{tot}/N_0}}$$

$$P^{(react)}_{MR-DaF}(\text{outage}) = \sum_{l=0}^{K} \sum_{D_l} \Pr\{\text{outage}|D_l\} \Pr\{D_l\}$$

$$\Pr\{\text{outage}/D_l\} = \Pr\left\{\frac{1}{2} \log_2\left(1 + \sum_{k \in D_l} \gamma_{kD}\right) < R\right\}$$

$$\sum_{k \in D_l} P_k = P_{relay}$$

LET: $\{\varphi_i(D_l)\}_{i=1}^{l} = \{h_{kD}\}_{k \in D_l}$ AND

$$A(D_l) = \text{diag}(\varphi_1(D_l), \varphi_2(D_l), \dots \varphi_l(D_l))$$

LOWER BOUND OF CONDITIONAL PROBABILITY

$$\Pr\{\text{outage}/D_l\} \geq \prod_{k \in D_l} \Pr\{\gamma_{kD} < K_2\}$$

$$K_2 = \frac{2^R - 1}{(1-\xi)\frac{P_{tot}}{N_0}}$$

$$\boxed{P^{(react)}_{OPT-DaF} = \prod_{k=1}^{K} \left[1 - \exp\left\{-\frac{2^{2R}-1}{P_{tot}/N_0}\left(\frac{1}{\xi \Omega_{sk}} + \frac{1}{(1-\xi)\Omega_{kd}}\right)\right\}\right.}$$

$$b^*_{react-DaF} = \arg\max E\{\gamma_{kD}\} = \arg\max_{k \in D_l} \Omega_{kD}$$

$$b^*_{proact-DaF} = \arg\max_{k \in S_{relay}} W_k^{(DaF)} \qquad \gamma_{AB} = |\alpha_{AB}|^2$$

$$\Omega_{AB} = E[\gamma_{AB}]$$

$$W_k^{(DaF)} = \min\{\xi \gamma_{sk}, (1-\xi)\gamma_{kD}\}$$

$$W_k^{(DaF)} = \gamma\left(\frac{1}{\xi \Omega_{sk}} + \frac{1}{(1-\xi)\Omega_{kD}}\right)$$

$$P^{react}_{OPT-DAF} = \prod_{k=1}^{K} \Pr\left\{W_k^{(DaF)} < \frac{2^{2R}-1}{P_{tot}/N_0}\right\} =$$

$$= \prod_{k=1}^{K}\left[1 - \exp\left\{-\frac{2^{2R}-1}{P_{tot}/N_0}\left(\frac{1}{\xi \Omega_{sk}} + \frac{1}{(1-\xi)\Omega_{kD}}\right)\right\}\right]$$

$\frac{1}{\Omega_{AB}}$ - HAZARD RATE $\triangleq \frac{1}{\text{AVERAGE RECEIVED POWER}}$

$\Omega_{AB} = \underline{1}$ ODIČMO ZEMAM VAVA

- AMPLIFY AND FORWARD RELAYING

$$\boxed{x_k = \sqrt{P_k} \frac{y_k}{\sqrt{E\{|y_k|^2\}}}}$$

$$y_D = G x_S + n_D \qquad G = \frac{1}{\sigma} \sum_{k=1}^{K} \sqrt{\frac{P_k}{\gamma_{sk} P_{source} + N_0}} \, a_{sk} a_{kD}$$

$$\sigma^2 = 1 + \sum_{k=1}^{K} \frac{P_k |a_{kD}|^2}{\gamma_{sk} P_{source} + N_0}$$

$$I_{MR-AaF} = \frac{1}{2} \log_2 \left(1 + |G|^2 \frac{P_{source}}{N_0} \right)$$ MUTUAL INFORMATION

$$I(s_i) = ld \frac{1}{P(s_i)}$$ ⇒ INFORMACIJA

$$H(S) = \sum_{i=1}^{N} P(s_i) \cdot ld \frac{1}{P(s_i)}$$ ⇒ ENTROPIJA (PROSJEČNA NEIZVJESNOST ZA POJAVLJIVANJE NA SIMBOL T.E. PROSJEČNO KOL. INFOR.)

$$\boxed{H(S) = \overline{I(s_i)}}$$

$$\frac{297 \times 210}{210 \times 148}$$

1.414 , 1,418

- OPPORTUNISTIC AaF

$$I_{OPP-AaF} = \max_{k \in S_{relay}} \frac{1}{2} ld \left\{ 1 + \frac{\gamma_{sk} \gamma_{kD}}{\frac{\xi}{1-\xi} \gamma_{sk} + \frac{N_0}{P_{relay}} + \gamma_{kD}} \frac{P_{source}}{N_0} \right\}$$

$$b^*_{AaF} = \arg\max_{k \in S_{relay}} W_k^{(AaF)} \qquad W_k^{(AaF)} = \frac{\gamma_{sk} \gamma_{kD}}{\frac{\xi}{1-\xi} \gamma_s \left(1 + \frac{1}{\gamma_{sk}} \right) + \gamma_{kD}}$$

$$p_\gamma(\gamma) = \frac{K+1}{\bar{\gamma}} \exp\left(-(K+1) \frac{\gamma}{\bar{\gamma}} - K \right) I_0 \left(2 \sqrt{K(K+1) \frac{\gamma}{\bar{\gamma}}} \right)$$

$$P_{out} = P(\gamma < \gamma_0) = \int_0^{\gamma_0} p_\gamma(\gamma) \, d\gamma$$

$$\gamma = \alpha^2 \frac{E_s}{N_0} ; \qquad \Omega = E(\gamma) = E[\alpha^2] ;$$

$$\bar{\gamma} = \Omega \cdot \frac{E_s}{N_0}$$

$$\Omega = \int_0^\infty \alpha^2 p(\alpha) \, d\alpha = s^2 + 2\sigma^2 = P_r$$

$$K = \frac{s^2}{2\sigma^2} \qquad \Omega = K \cdot 2\sigma^2 + 2\sigma^2 \qquad 2\sigma^2 = \frac{\Omega}{K+1}$$

$$\Omega = s^2 + \frac{s^2}{K} \qquad s^2 = \frac{K \cdot \Omega}{K+1}$$

$$p(r) = \frac{r}{\sigma^2} e^{-\frac{r^2+A^2}{2\sigma^2}} I_0\left(\frac{A\cdot r}{\sigma^2}\right)$$ $a = \frac{A}{\sigma}$ $z = \frac{r}{\sigma}$

$$\boxed{p(z) = z\, e^{-\frac{z^2+a^2}{2}} I_0(a, z)}$$

$$P(z<z_0) = \int_0^{z_0} p(z)\,dz = 1 - \int_{z_0}^{\infty} z\, e^{-\frac{z^2+a^2}{2}} I_0(a,z)\,dz$$

$$\boxed{P(z<z_0) = 1 - Q(a, z_0)}$$

$\gamma = r^2 \cdot \frac{E_b}{N_0}$ $\Omega = E[\gamma] = E[r^2]$

$\bar{\gamma} = \Omega \cdot \frac{E_b}{N_0}$

$s^2 = \frac{K\Omega}{K+1}$ $\frac{s^2}{\Omega} = \frac{K}{K+1} \Rightarrow \frac{s}{\sqrt{\Omega}} \triangleq \frac{A}{\sigma} = \sqrt{\frac{K}{K+1}}$

$\frac{\gamma}{\bar{\gamma}} = \frac{r^2}{\Omega}$ $\frac{r}{\sqrt{\Omega}} \triangleq \frac{r}{\sigma} = \sqrt{\frac{\gamma}{\bar{\gamma}}}$.

MULTIHOP RALEIGH

$$p_\gamma(\gamma) = \frac{1}{\bar{\gamma}} e^{-\frac{\gamma}{\bar{\gamma}}}$$

$$P(\gamma<\gamma_0) = \int_0^{\gamma_0} \frac{1}{\bar{\gamma}} e^{-\frac{\gamma}{\bar{\gamma}}} d\gamma = \underline{\underline{1-e^{-\frac{\gamma_0}{\bar{\gamma}}}}}$$

$$P(\gamma>\gamma_0) = \int_{\gamma_0}^{\infty} \frac{1}{\bar{\gamma}} e^{-\frac{\gamma}{\bar{\gamma}}} d\gamma = \underline{\underline{e^{-\frac{\gamma_0}{\bar{\gamma}}}}}$$

n-HOP REGENERATIVE

$$P_{out} = 1 - P(\gamma_1>\gamma_0)\,P(\gamma_2>\gamma_0) \cdots P(\gamma_n>\gamma_0)$$

BAREM EDNO DA E : $\gamma_i < \gamma_0 \Rightarrow$ OUTAGE

$$\boxed{P_{out} = 1 - e^{-\frac{\gamma_0}{\bar{\gamma}_1}} \cdot e^{-\frac{\gamma_0}{\bar{\gamma}_2}} \cdots e^{-\frac{\gamma_0}{\bar{\gamma}_n}}}$$

- NON - REGENERATIVE SYSTEM

- END TO END SNR γ_{eq} FALLS BELOW

$$\boxed{p(\gamma)=\frac{m^m\gamma^{m-1}}{\Gamma(m)\,\bar{\gamma}^m}e^{-\frac{m\gamma}{\bar{\gamma}}}}$$

AKO SAKAŠ SO KORISTENJE NA GOTOVATA FORMULA OD N4.pp4 VIDI N4.pp56

NADOLU IZVEDUVANJE ZA NAKAGAMI

$$p(x)=\frac{2m^m\cdot x^{2m-1}}{\Gamma(m)\cdot\Omega^{m}}e^{-\frac{[illegible]}{\Omega}}$$

$$\gamma=x^2\frac{E_s}{N_0}\qquad\bar{\gamma}=E(\gamma)=E(x^2)\frac{E_s}{N_0}=\Omega\frac{E_s}{N_0}$$

$$\boxed{x=\sqrt{\gamma\frac{N_0}{E_s}}}\qquad\boxed{\Omega=\bar{\gamma}\frac{N_0}{E_s}}\qquad x^2=\gamma\frac{N_0}{E_s}$$

$$\frac{d\gamma}{dx}=2x\frac{E_s}{N_0}\qquad p_\gamma(\gamma)=\frac{p_x(x)}{\frac{d\gamma}{dx}}\Bigg|_{x=f(\gamma)}$$

$$p_\gamma(\gamma)=\frac{\cancel{2}m^m\,\gamma^{m-1}\cancel{\left(\frac{N_0}{E_s}\right)^{m-1}}\cdot\cancel{\sqrt{\gamma\frac{N_0}{E_s}}}}{\Gamma(m)\cdot\bar{\gamma}^{m}\cancel{\left(\frac{N_0}{E_s}\right)^{m-1}}\cancel{\left(\frac{N_0}{E_s}\right)}}e^{-\frac{m\gamma\cancel{\frac{N_0}{E_s}}}{\bar{\gamma}\cancel{\frac{N_0}{E_s}}}}\cdot\frac{1}{\cancel{2}\cdot\cancel{\sqrt{\gamma\frac{N_0}{E_s}}}\cdot\cancel{\frac{E_s}{N_0}}}$$

$$\boxed{p_\gamma(\gamma)=\frac{m^m\gamma^{m-1}}{\Gamma(m)\cdot\bar{\gamma}^m}e^{-\frac{m\gamma}{\bar{\gamma}}}}\Rightarrow$$

GAMA DISTRIBUTION

use randg(m)

$$M_{\frac{1}{\gamma}}(s)=E\left[e^{\frac{s}{\gamma}}\right]=\int_0^\infty e^{\frac{s}{\gamma}}p(\gamma)d\gamma=\int_0^\infty e^{\frac{s}{\gamma}}\frac{m^m\gamma^{m-1}}{\Gamma(m)\cdot\bar{\gamma}^m}e^{-\frac{m\gamma}{\bar{\gamma}}}d\gamma$$

$$M_{\frac{1}{\gamma}}(s)=\frac{m^m}{\Gamma(m)\,\bar{\gamma}^m}\int_0^\infty\gamma^{m-1}e^{\frac{s}{\gamma}-\frac{m\gamma}{\bar{\gamma}}}d\gamma$$

GRADSHTEYN :

$$\int_0^\infty x^{\nu-1}e^{-\frac{\beta}{x}-\gamma x}dx=2\left(\frac{\beta}{\gamma}\right)^{\frac{\nu}{2}}K_\nu\left(2\sqrt{\beta\gamma}\right)$$

$\nu\triangleq m$

$x\triangleq\gamma$

$\gamma\triangleq\frac{m}{\bar{\gamma}}$

$\beta\triangleq s$

$$M_{\frac{1}{\gamma}}(s)=\frac{m^m}{\Gamma(m)\,\bar{\gamma}^m}2\cdot\left(\frac{s}{\frac{m}{\bar{\gamma}}}\right)^{m/2}K_m\left(2\sqrt{\frac{sm}{\bar{\gamma}}}\right)=$$

$$=\frac{2}{\Gamma(m)}\frac{m^{m/2}}{\bar{\gamma}^{m/2}}(s)^{m/2}K_m(\ldots)=\frac{2}{\Gamma(m)}\left(\frac{m\cdot s}{\bar{\gamma}}\right)^{m/2}K_m\left(2\sqrt{\frac{sm}{\bar{\gamma}}}\right)$$

• SIMPLE NUMERICAL TECHNIQUE FOR THE INVERSION OF THE LAPLACE TRANSFORM OF CUMULATIVE DIST. FUNCT.

X - POSITIVE (RV) WITH CDF $P_X(x)$

$\hat{P}_X(s)$ - LAPLACE TRANSFORM OF $P_X(x)$

$$\hat{P}_X(s) = \int_{-\infty}^{\infty} P_X(x) \cdot e^{-sx} dx$$

$$P_X(x) = \frac{1}{2\pi j} \int_{a-j\infty}^{a+j\infty} \hat{P}_X(s) e^{sx} ds$$

P_X CAN BE OBTAINED FROM $\hat{P}_X(s)$ BY NUMERIC TECHN.

$\boxed{\text{STEP 1}}$ $s = a + j\mu$ $\qquad ds = j\,d\mu$

$$P_X(x) = \int_{-\infty}^{\infty} \hat{P}_X(x)\, e^{-ax} \cdot e^{-j\mu x} dx$$

$$P_X(x) = \frac{1}{2\pi \cancel{j}} \int_{-\infty}^{\infty} P(a+j\mu)\, e^{+(a+j\mu)x} \cancel{j} d\mu =$$

$$= \frac{e^{ax}}{2\pi} \int_{-\infty}^{\infty} \hat{P}(a+j\mu) \left(\cos \mu x + j \sin(\mu x) \right) d\mu \Rightarrow$$

$$P_X(x) = \frac{e^{ax}}{2\pi} \int_{-\infty}^{\infty} \left\{ Re\left[\hat{P}(a+j\mu)\right] \cos \mu x + Im\left[\hat{P}(a+j\mu)\right] \sin(\mu x) \right\} d\mu$$

$x > 0 \Rightarrow P(-x) = 0$

⊛ $$\int_{-\infty}^{\infty} Re\left[\hat{P}(a+j\mu)\right] \cos \mu x \, d\mu = - \int_{-\infty}^{\infty} Im\left[\hat{P}(a+j\mu)\right] \sin(\mu x) d\mu$$

e.g.: $\hat{P}(a+j\mu) = a + j\mu$

$$\int_{-\infty}^{\infty} (a+j\mu) \cos(\mu x) d\mu = \int_{-\infty}^{\infty} a \cos(\mu x) d\mu + j \int_{-\infty}^{\infty} \mu \cos(\mu x) d\mu$$

$$P_X(x) = \frac{e^{ax}}{\pi} \int_{-\infty}^{\infty} Re\left[\hat{P}(a+j\mu)\right] \cos(\mu x) dx$$

$\boxed{\text{STEP 2}}$ $\boxed{a = \frac{A}{2x}}$

$$P_x(x) = \frac{2e^{A/2}}{\pi}\int_0^\infty \mathrm{Re}\left\{\hat{P}_x\left(\frac{A}{2x}+ju\right)\right\}\underline{\cos(ux)}\,du$$

step: $\boxed{h = \frac{\pi}{2x}}$

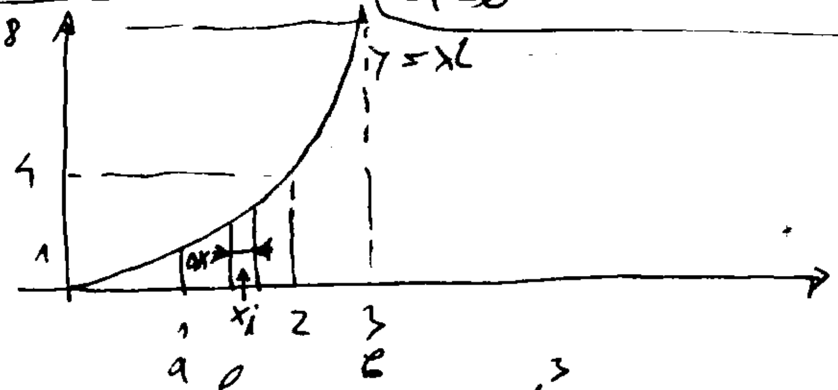
$$P_x(x) = \frac{\cancel{2}e^{A/2}}{\cancel{\pi}}\sum_{n=0}^{\infty}\frac{\cancel{\pi}}{\cancel{2}x}\,\mathrm{Re}\left\{\hat{P}_x\left(\frac{A}{2x}+j\cancel{u}\right)\right\}\cos(nx)+\epsilon(A) \qquad \underline{\frac{n\pi}{\cancel{2}x}}$$

$$P_x(x) = \frac{e^{A/2}}{x}\sum_{n=0}^{\infty}\cancel{\frac{(-1)^n}{d_n}}\,\mathrm{Re}\left\{\hat{P}_x\left(\frac{A}{2x}+j\frac{n\pi}{\cancel{2}x}\right)\right\}(-1)^n+\epsilon(A)$$

$$\cos\left(\frac{n\pi}{2x}\cdot x\right) = \cos\left(\frac{n\pi}{2}\right)$$

$$\underline{u = \frac{n\pi}{x}} \Rightarrow \cos\left(\frac{n\pi}{x}\cdot x\right) = \cos(n\pi) = \underline{(-1)^n}$$

$$P_x(x) = \frac{e^{A/2}}{x}\sum_{n=0}^{\infty}\frac{(-1)^n}{d_n}\,\mathrm{Re}\left\{\hat{P}_x\left(\frac{A+2\pi jn}{2x}\right)\right\}+\epsilon(A)$$



POTSETUVANJE:
„RIEMAN SUM"

$$P = \int_a^b y(x)\,dx = \int_1^3 x^2\,dx = \frac{x^3}{3}\Big|_1^3 = \frac{1}{3}(27-1) = \frac{26}{3}$$

$$P = \sum_{i=1}^{N} f(x_i)\Delta x \qquad \Delta x = \frac{b-a}{N} = \frac{3-1}{N} = \boxed{\frac{2}{N}}$$

$$x_1 = a + \frac{\Delta x}{2}(2-1) = a + \frac{\Delta x}{2} \qquad x_2 = a + \frac{3\Delta x}{2} = a + \frac{3\Delta x}{2}$$

$$x_i = a + \frac{(2i-1)\Delta x}{2} \qquad P = \sum_{i=1}^{N} f(\bar{x}_i)\,\Delta x = \sum_{i=1}^{N} f\left(a+\frac{(2i-1)\Delta x}{2}\right)\Delta x$$

$$p_X(x) = \frac{e^{A/2}}{x} \sum_{n=0}^{\infty} \frac{(-1)^n}{\alpha_n} \mathrm{Re}\left\{ \hat{P}_X\left(\frac{A+2\pi j n}{2x}\right)\right\} + \epsilon(A)$$

$$\alpha_n = \begin{cases} 2 & n=0 \\ 1 & n=1,2,\dots \tilde{N} \end{cases}$$

$$|\epsilon(A)| \leq \frac{e^{-A}}{1-e^{-A}} \doteq e^{-A} \Rightarrow$$ DISCRETIZATION ERROR

STEP 3: TRUNCATING INFINITE SUM

$$\boxed{p_X(x) = \frac{e^{A/2}}{x} \sum_{n=0}^{N} \frac{(-1)^n}{\alpha_n} \mathrm{Re}\left\{ \hat{P}_X\left(\frac{A+2\pi j n}{2x}\right)\right\} + \epsilon(A) + \epsilon(N)}$$

$\epsilon(N)$ - TRUNCATION ERROR

- To ACCELERATE CONVERGENCE USE EULER SUMMATION TECHNIQUE

$$p_X(x) = \sum_{k=0}^{K} 2^{-K} \binom{K}{k} \left[\frac{e^{A/2}}{x} \sum_{n=0}^{N+k} \frac{(-1)^n}{\alpha_n} \mathrm{Re}\left\{ \hat{P}_X\left(\frac{A+2\pi j n}{2x}\right)\right\}\right] + \epsilon(A) + \epsilon(N,K)$$

$-\ \epsilon(N,K)$ OVERALL TRUNCATION ERROR

$$\epsilon(N,K) \doteq \frac{e^{A/2}}{x} \sum_{k=0}^{K} 2^{-K} (-1)^{N+1+k} \binom{K}{k} \mathrm{Re}\left\{ \hat{P}_X\left(\frac{A+2\pi j (N+k+1)}{2x}\right)\right\}$$

$$\binom{n}{k} \binom{10}{1} = \frac{n!}{(n-k)!\,k!} = \frac{10!}{9!\ \cancel{1!}^{\,1}} = \frac{\cancel{9!}\cdot 10}{\cancel{9!}} = 10$$

$\boxed{\text{Alouini Ch. 1.1.2}}$ Outage Probability

$$P_{out,th} = \int_0^{\gamma_{th}} p_\gamma(\gamma)\, d\gamma \qquad p_\gamma(\gamma) = \frac{dP_\gamma(\gamma)}{d\gamma}$$

$$P(\gamma) = 0 \qquad \boxed{\hat{P}_\gamma(s) = \frac{\hat{p}_\gamma(s)}{s}}$$

$$\mathcal{L}\left\{\frac{df(t)}{dt}\right\} = s \cdot F(s)$$

$$\mathcal{L}\left\{\int_{-\infty}^{t} f(\tau)\, d\tau\right\} = \frac{F(s)}{s}$$

$$\mathcal{L}\{P_{out}\} = \mathcal{L}\left\{\int_0^{\gamma_{th}} p_\gamma(\gamma)\,d\gamma\right\} = \frac{\hat{p}_\gamma(s)}{s} = \frac{\mathcal{L}\{p_\gamma(\gamma)\}}{s}$$

$$\hat{P}(s) = \mathcal{L}\{P_{out}\} \Rightarrow \boxed{\hat{P}_\gamma(s) = \frac{\hat{p}_\gamma(s)}{s}}$$

$\mathcal{L}$ TRANSFORM OD INTEGRAL OD FUNKCIJA

$$M_\gamma(s) = \int_{-\infty}^{\infty} e^{+\frac{s}{\gamma}} p_\gamma(\gamma)\,d\gamma \qquad \mathcal{L}\{p_\gamma(\gamma)\} = \hat{p}_\gamma(s) = \int_0^{\infty} p_\gamma(\gamma)e^{-s\gamma}\,d\gamma$$

$$\boxed{M_\gamma(-s) = \int_{-\infty}^{\infty} e^{-\frac{s}{\gamma}} p_\gamma(\gamma)\,d\gamma = \hat{p}_\gamma(s)} \qquad \boxed{\hat{P}_\gamma(s) = \frac{M_\gamma(-s)}{s}}$$

- FROM THE ABOVE STATEMENTS, OUTAGE PROBABILITY CAN BE FOUND FROM THE INVERSE LAPLACE TRANSFORM OF THE RATIO $M_\gamma(-s)/s$; EVALUATED AT: $\gamma = \gamma_{th}$

$$P_{out} = \int_0^{\gamma_{th}} p_\gamma(\gamma)\,d\gamma \Big/_{\mathcal{L}\{\}} \Rightarrow , \quad \hat{P}_{out}(s) = \frac{\hat{p}(s)}{s} = \frac{M_\gamma(-s)}{s}$$

$$P_{out}\Big|_{\gamma=\gamma_{th}} = \mathcal{L}^{-1}\left\{\frac{M_\gamma(-s)}{s}\right\} = \frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty} \frac{M_\gamma(-s)}{s}\exp(s\gamma_{th})\,ds$$

$$P_{out} = \frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty} \frac{M_\gamma(-s)}{s}\, e^{s\gamma_{th}}\,ds$$

MMV

$$\boxed{\bar{\gamma} \triangleq \int_0^{\infty} \gamma\, p_\gamma(\gamma)\,d\gamma}$$

γ - instanteyous SNR (Random variable - RV)

$$M_\gamma(s) = \int_0^{\infty} e^{\gamma s} p_\gamma(\gamma)\,d\gamma \quad \cdot \quad \boxed{\frac{dM_\gamma(s)}{ds}\Big|_{s=0} = \bar{\gamma}}$$

$$\frac{dM_\gamma(s)}{ds}\Big|_{s=0} = \int_0^{\infty} \gamma \cdot \overset{1\,(s=0)}{\cancel{e^{\gamma s}}} \cdot p_\gamma(\gamma)\,d\gamma \Big|_{s=0} = \int_0^{\infty} \gamma \cdot p_\gamma(\gamma)\,d\gamma = \bar{\gamma}$$

MAXIMAL-RATIO COMBINING (MRC)

$\gamma = \sum_{l=1}^{L} \gamma_l$ → OUTPUT SNR IS SUM OF BRANCH SNRs ; L - NUMBER OF CHANNEL COMBINED

If $\gamma_l\big|_{l=1}^{L}$ ARE INDEPENDENT THEN RESULTING MGF IS PRODUCT OF INDIVIDUAL BRANCH SNRs

$$\boxed{M_\gamma(s) = \prod_{l=1}^{L} M_{\gamma_l}(s)}$$

POVTORUVANJE!

$$P_{out} = \int_0^{\gamma_{th}} p_\gamma(\gamma)\,d\gamma \Big/ \mathcal{L}\{\} \Rightarrow \mathcal{L}\{P_{out}\} = \underbrace{\mathcal{L}^{-1}\left\{\int_0^{\gamma_{th}} p_\gamma(\gamma)\,d\gamma\right\}}_{\text{INTEGRATE PROPERTY OF LAPLACE TRANSF.}}$$

$$\boxed{\hat{P}_{out} = \frac{\hat{p}_\gamma(\gamma)}{s}}$$

$$P_{out} = \mathcal{L}^{-1}\left\{\frac{\hat{p}_\gamma(\gamma)}{s}\right\}\Big|_{\gamma=\gamma_{th}} = \frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty} \frac{\hat{p}_\gamma(\gamma)}{s} e^{s\gamma_{th}}\,ds$$

$$M_\gamma(s) = \int_0^\infty e^{\gamma s} p_\gamma(\gamma)\,d\gamma$$

$$M_\gamma(-s) = \int_0^\infty e^{-\gamma s} p_\gamma(\gamma)\,d\gamma = \hat{p}_\gamma(\gamma)$$

$$\boxed{P_{out} = \mathcal{L}\left\{\frac{M_\gamma(-s)}{s}\right\} = \frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty} \frac{M_\gamma(-s)}{s} e^{s\gamma_{th}}\,ds}$$

- PROPERTY OF $\mathcal{L}\{\}$: INTEGRATION IN TIME DOM.

$$\boxed{\int_{-\infty}^{t} x(\tau)\,d\tau = \frac{X(s)}{s}}$$

OVA E NEGDE VO SREDINATA

$x \doteq 10^{-0.5} = 0.31623$

10^{-1} .... 10^{0} 10^{-1}; 0,2026; 0,3004; 0.3971; 0.5066; 0.6018

LOG: 0.6997; 0.7973; 0.9063; 10^0

$10^{-1}\ 10^{-0.7}\ 10^{-0.8}\ 10^{-0.7}$ / $10^{-0.1}\ 10^{0}$ /

SE [illegible] LIMITE NE SE RAMNOM

LIN: 0.1; 0.2; 0.3; 0.4; 0.5; 0.6; 0.7; 0.8; 0.9; 1

OVA NEGDE E NA $\frac{3}{4} = 0.75 \Rightarrow$

$x = 10^{-0.25} = 1.7783$

- LOGARITAMSKA SKALA SE DAJE EKSPONENCIJALNO OD $\boxed{10^x}$ LINEARNO TAKO DA NA OSKA OD $10^0 \div 10^{-1}$ LOGARITAMSKA SU RAVNOMJERNI PORED NA LINIJI ĆE SE POZICIONIRA LINIJE NA:

$$10^0 \; 10^{0.1} \; 10^{-0.2} \; 10^{-0.3} \ldots\ldots 10^{-0.7} \; 10^{-1}$$

$$\boxed{\hat{P}_\gamma(s) = \frac{M_\gamma(-s)}{s}}$$

$$P_\gamma(\gamma) = \frac{e^{\frac{A}{2}}}{\gamma} \sum_{n=0}^{N} \frac{(-1)^n}{\alpha_n} Re\left\{ \hat{P}_\gamma\left(\boxed{\frac{A+2\pi j n}{2\gamma}} \right) \right\} + \epsilon(A) + \epsilon(N)$$

s

$$P_\gamma(\gamma) = \frac{e^{A/2}}{?\,\boxed{\gamma}} \sum_{n=0}^{N} \frac{(-1)^n}{\alpha_n} Re\left\{ \frac{M_\gamma(-s)}{s} \right\} + \epsilon(A) + \epsilon(N)$$

$$P_\gamma(\gamma) = \frac{e^{A/2}}{?\,\boxed{\gamma}} \sum_{n=0}^{N} \frac{(-1)^n}{\alpha_n} Re\left\{ \frac{M_\gamma\left(-\frac{A+2\pi j n}{2\boxed{\gamma}}?\right)}{\frac{A+2\pi j n}{2\boxed{\gamma}}} \right\} + \epsilon(A) + \epsilon(N)$$

$$\boxed{|\epsilon(A)| \leq \frac{e^{-A}}{1-e^{-A}} \doteq e^{-A}}$$

$$\alpha_n = \begin{cases} 2 \; ? & n=0 \\ 1 & n=1,2,\ldots N \end{cases}$$

$$M_{1/\gamma_n}(s) = \frac{2}{\Gamma(m_n)} \left(\frac{m_n s}{\boxed{\gamma_n}} \right)^{\frac{m_n}{2}} K_{m_n}\left(2\sqrt{\frac{m_n s}{\boxed{\gamma_n}}} \right)$$

SEDAK ϵ KAKO ŠTO IMAM GORE NAPISANO

$$P_\gamma(\gamma_{th}) = \frac{e^{A/2}}{\gamma_{th}} \sum_{n=0}^{N} \frac{(-1)^n}{\alpha_n} Re\left\{ \frac{M_\gamma\left(-\frac{A+2\pi j n}{2\gamma_{th}}\right)}{\frac{A+2\pi j n}{2\gamma_{th}}} \right\} + \epsilon(A,N)$$

$$|\epsilon(A,K,N)| \leq \frac{e^{-A}}{1+e^{-A}} + \left| \frac{2^{-K} e^{A/2}}{\gamma_{th}} \sum_{k=0}^{K} (-1)^{N+1+k} \binom{K}{k} Re\left\{ \frac{M_\gamma\left(-\frac{A+2\pi j(N+k+1)}{2\gamma_{th}}\right)}{\frac{A+2\pi j(N+k+1)}{2\gamma_{th}}} \right\} \right|$$

$$P_{out} = P_\gamma(\gamma_{th}) = \frac{2^{-K} e^{A/2}}{\gamma_{th}} \sum_{k=0}^{K} \binom{K}{k} \sum_{n=0}^{N+k} \frac{(-1)^n}{\alpha_n} \cdot Re\left\{ \frac{M_\gamma\left(-\frac{A+2\pi j n}{2\gamma_{th}}\right)}{\frac{A+2\pi j n}{2\gamma_{th}}} \right\} + \epsilon(A,K,N)$$

$$\epsilon = e^{-A} \qquad -A = \ln \epsilon$$

$$\epsilon = 10^{-10} \qquad -A = -10 \ln 10 \qquad \boxed{A = 10 \ln 10}$$

9.5.2 COHERENT EGC

$$\gamma_{EGC} = \frac{\left(\sum_{l=1}^{L} \alpha_l\right)^2 E_s}{L \cdot N_0} \qquad P_{out} \triangleq P_r\{0 \leq \gamma_{EGC} \leq \gamma_{th}\}$$

$$P_{out} = P_r\{0 \leq \alpha_t \leq \alpha_{th}\}$$

$$\boxed{\alpha_t \triangleq \sum_{l=1}^{L} \alpha_l \qquad \alpha_{th} \triangleq \sqrt{\frac{L\gamma_{th}}{E_s/N_0}}}$$

$$M_{\alpha_l}(s) = \frac{\Gamma(2m_l)}{2^{m_l-1}\Gamma(m_l)} \exp\left(\frac{\Omega_l s^2}{8m_l}\right) D_{-2m_l}\left(\frac{-s\sqrt{\Omega_l}}{\sqrt{2m_l}}\right)$$

$D_{-\nu}(\cdot)$ – PARABOLIC CYLINDER FUNCTION

- OR ALTERNATIVELY IN TERMS OF ${}_1F_1[\cdot,\cdot;\cdot]$ – CONFLUENT HYPERGEOMETRIC FUNCTIONS

$$M_{\alpha_l}(s) = \frac{\Gamma(2m_l)}{2^{2m_l-1}\Gamma(m_l)}\left[\frac{\sqrt{\pi}}{\Gamma(m_l+1/2)}\,{}_1F_1\left(m_l, \frac{1}{2}; \frac{\Omega_l s^2}{4m_l}\right) + \frac{\sqrt{\pi\Omega_l}\,s}{\Gamma(m_l)\sqrt{m_l}}\,{}_1F_1\left(m_l+\frac{1}{2}, \frac{3}{2}; \frac{\Omega_l s^2}{4m_l}\right)\right]$$

$$P_{out}(\cdot) = P(\bar{\gamma}_l) \qquad \bar{\gamma}_l = \Omega_l \frac{E_s}{N_0} \qquad l = 1, 2, \ldots L$$

$$\bar{\gamma}_l = \bar{\gamma}_1 e^{-\delta(l-1)} \qquad l = 1, 2, \ldots L$$

FADING DECAY POWER PROFILE

$\bar{\gamma}_1$ – AVERAGE SNR OF THE FIRST (REFERENCE) PROPAGATION PATH

δ – AVERAGE FADING POWER DECAY FACTOR

NORMALIZED AVERAGE SNR : $\boxed{\dfrac{\bar{\gamma}}{\gamma_{th}}}$!!!

$$p_\gamma(\gamma) = \frac{m^m \gamma^{m-1}}{\Gamma(m)\,\bar{\gamma}^m} e^{-\frac{m\gamma}{\bar{\gamma}}}$$

Ова всушност не е накагами туку варијација на гама. Амплитудите на сигналот се дистрибуирани по накагами, а SNR-от по гама.

$$M_{\frac{1}{\gamma}}(s) = \int_0^\infty e^{\frac{s}{\gamma}} p(\gamma)\,d\gamma \ ; \quad M_{\frac{1}{\gamma}}(-s) = \int_0^\infty e^{-\frac{s}{\gamma}} p(\gamma)\,d\gamma$$

$$M_{1/\gamma}(s)=\int_0^\infty e^{-\frac{s}{\gamma}}\frac{m^m\gamma^{m-1}}{\Gamma(m)\bar{\gamma}^m}e^{-\frac{m\gamma}{\bar\gamma}}d\gamma=\frac{m^m}{\Gamma(m)\bar\gamma^m}\int_0^\infty e^{-\frac{s}{\gamma}}\gamma^{m-1}e^{-\frac{m\gamma}{\bar\gamma}}d\gamma$$

$$M_{1/\gamma}(-s)=\frac{m^m}{\Gamma(m)\bar\gamma^m}\underbrace{\int_0^\infty\gamma^{m-1}e^{-\frac{s}{\gamma}-\frac{m\gamma}{\bar\gamma}}d\gamma}_{I}$$

NAJDOBRO IZVEDUVANJE NA MGF ZA NAKAGAMI

Сепак ова случајно се погодило, правилното изведување е на N8.pp82

GRADSHTAJN:

$$\int_0^\infty x^{\nu-1}e^{-\frac{\beta}{x}-\gamma x}dx=\left|\begin{matrix}\nu\triangleq m\\ \beta\triangleq s\\ x\triangleq\gamma\\ \gamma\triangleq m/\bar\gamma\end{matrix}\right|=2\left(\frac{\beta}{\gamma}\right)^{\nu/2}K_\nu\left(2\sqrt{\beta\gamma}\right)$$

$$M_{1/\gamma}(-s)=2\frac{m^m}{\Gamma(m)\bar\gamma^m}\left(\frac{s}{m/\bar\gamma}\right)^{m/2}K_m\left(2\sqrt{s\,\cancel{\gamma}\,\frac{m}{\bar\gamma}}\right)=$$

$$=2\frac{m^{\cancel{m}\;m/2}}{\Gamma(m)\bar\gamma^{\cancel{m}\;m/2}}\frac{s^{m/2}\,\cancel{\bar\gamma^{m/2}}}{\cancel{m^{m/2}}}K_m\left(2\sqrt{s\frac{m}{\bar\gamma}}\right)$$

$$\boxed{M_{1/\gamma}(-s)}=2\frac{m^{m/2}}{\Gamma(m)}\frac{s^{m/2}}{\bar\gamma^{m/2}}K_m\left(2\sqrt{s\frac{m}{\bar\gamma}}\right)=\frac{2}{\Gamma(m)}\left(\frac{ms}{\bar\gamma}\right)^{m/2}K_m\left(2\sqrt{s\frac{m}{\bar\gamma}}\right)$$

$$P_{out}=P_r(\gamma_{eq}<\gamma_{th})=P_r\left(\frac{1}{\gamma_{eq}}>\frac{1}{\gamma_{th}}\right)=1-P_r\left(\frac{1}{\gamma_{eq}}\le\frac{1}{\gamma_{th}}\right)$$

$$=1-\mathcal{L}^{-1}\left(\frac{M_{1/\gamma_{eq}}(-s)}{s}\right)\Bigg|_{1/\gamma_{th}}$$

$$P_r\left(\frac{1}{\gamma_{eq}}\le\frac{1}{\gamma_{th}}\right)=\frac{2^{K}e^{A/2}}{1/\gamma_{th}\,[illegible]}\sum_{k=0}^{K}\binom{K}{k}\sum_{n=0}^{N+k}\frac{(-1)^n}{\alpha_n}\mathrm{Re}\left\{\frac{M_{1/\gamma}\left(-\frac{A+2\pi jn}{2\cdot1/\gamma_{th}}\right)}{\frac{A+2\pi jn}{2\cdot1/\gamma_{th}}}\right\}+E(A,K,N)$$

$$|E(A,K,N)|\le\frac{e^{-A}}{1-e^{-A}}+\left|\frac{e^{-K}e^{A/2}}{1/\gamma_{th}}\sum_{k=0}^{K}(-1)^{N+1+k}\binom{K}{k}\mathrm{Re}\left\{\frac{M_{1/\gamma}\left(-\frac{A+2\pi j(N+k+1)}{2/\gamma_{th}}\right)}{\frac{A+2\pi j(N+k+1)}{2/\gamma_{th}}}\right\}\right|$$

MMV

ZNAČI SOGLASNO (*) NEMA POTREBA VO GORNIVE DVE FORMULI DA GO MENUVAŠ ZNAKOT NA „s"

$$\boxed{s=\frac{A+2\pi(jn)}{2/\gamma_{th}}}$$

ZNAČI BEZ PROMENA NA ZNAK GO VMETNUVAŠ VO (*)

$$P = 10^{-12} = e^{-A} \quad -12 \ln 10 = -A \quad \boxed{A = 12 \ln 10}$$

-HARMONIC MEAN

$$(\gamma_1 = x_1, x_2, \dots x_N)$$

$$\mu_H = \frac{N}{\sum_{i=1}^{N} \frac{1}{x_i}}$$

$$\gamma_{eq2} = \frac{\mu_H}{N}$$

$$\gamma_{eq2} = \left[\left(1 + \frac{1}{\gamma_1}\right)\left(1 + \frac{1}{\gamma_2}\right) - 1\right]^{-1}$$

APPROACH(2)

$$\gamma_{eq3} = \left[\left(1 + \frac{1}{\gamma_1}\right)\left(1 + \frac{1}{\gamma_2}\right)\left(1 + \frac{1}{\gamma_3}\right) - 1\right]^{-1}$$

(*)

APPROACH (1)

$$\gamma_{eq2} = \left(\frac{1}{\gamma_1} + \frac{1}{\gamma_2}\right)^{-1} = \frac{1}{\frac{1}{\gamma_1} + \frac{1}{\gamma_2}} = \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2}$$

$$\gamma_{eq3} = \left(\frac{1}{\gamma_{eq2}} + \frac{1}{\gamma_3}\right)^{-1} = \frac{1}{\left(\frac{1}{\gamma_1} + \frac{1}{\gamma_2}\right) + \left(\frac{1}{\gamma_3}\right)} = \frac{1}{\frac{1}{\gamma_{eq2}} + \frac{1}{\gamma_{eq3}}}$$

IDENTICAL

~~$\gamma_{eq2} =$~~ ~~$\frac{1}{\gamma_{eq2}} = \frac{1}{\frac{1}{\gamma_1} + \frac{1}{\gamma_2}}$~~

$$\frac{1}{\gamma_{eq2}} = \frac{1}{\gamma_1} + \frac{1}{\gamma_2}$$

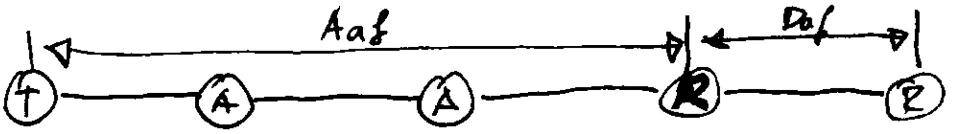
$$\gamma_{eq3} = \frac{1}{\frac{1}{\gamma_1} + \frac{1}{\gamma_2} + \frac{1}{\gamma_3}} = \left[\sum_{i=1} \frac{1}{\gamma_i}\right]^{-1}$$

(*)

$$\gamma_{eq2} = \frac{1}{\left(1 + \frac{1}{\gamma_1}\right)\left(1 + \frac{1}{\gamma_2}\right) - 1} = \frac{1}{\cancel{1} + \frac{1}{\gamma_2} + \frac{1}{\gamma_1} + \frac{1}{\gamma_1 \gamma_2} \cancel{-1}}$$

$$\frac{1}{\gamma_{eq2}} = \left(1 + \frac{1}{\gamma_1}\right)\left(1 + \frac{1}{\gamma_2}\right) - 1 \qquad \left(1 + \frac{1}{\gamma_1}\right)\left(1 + \frac{1}{\gamma_2}\right) = \frac{1}{\gamma_{eq2}} + 1$$

$$\boxed{\frac{1}{\gamma_{eq3}} = \underbrace{\left(1 + \frac{1}{\gamma_1}\right)\left(1 + \frac{1}{\gamma_2}\right)}_{1/\gamma_{eq2} + 1}\left(1 + \frac{1}{\gamma_3}\right) - 1 = \left(\frac{1}{\gamma_{eq2}} + 1\right)\left(1 + \frac{1}{\gamma_3}\right) - 1}$$



OVA MOŽE DA SE SVEDE NA 2 hop Daf SYSTEM

$$\boxed{P_{out} = 1 - P_{Aaf}(\gamma > \gamma_0) \cdot P_{Daf}(\gamma \geq \gamma_0) =}$$

$$= 1 - \left[1 - P_{Aaf}(\gamma < \gamma_0)\right] \cdot P_{Daf}(\gamma \geq \gamma_0) = \underbrace{1 - P_{Daf}(\gamma \geq \gamma_0)}_{P_{Daf}(\gamma \leq \gamma_0)} + P_{Aaf}(\gamma \leq \gamma_0) \cdot P_{Daf}(\gamma \geq \gamma_0)$$

$$P_{out} = P_{Daf}(\gamma < \gamma_0) + P_{Aaf}(\gamma \leq \gamma_0) \cdot P_{Daf}(\gamma \geq \gamma_0)$$

$nSNR\_dB = SNR\_dB - 10\log(g_0) = SNR\_dB - 6.8124$

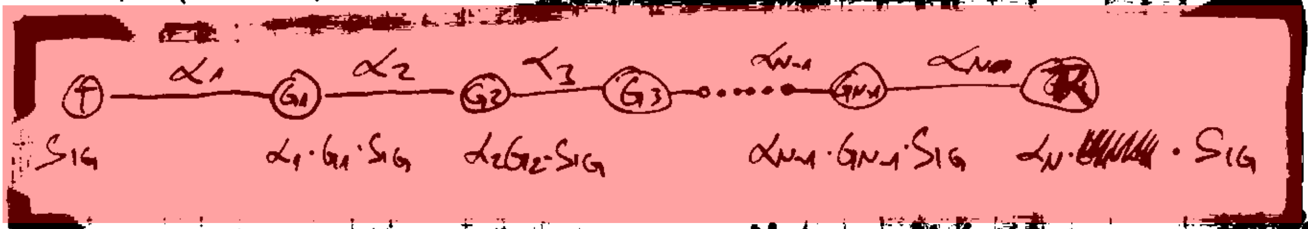
$nSNR\_dB = 25dB \Rightarrow SNR\_dB = 25 + 6.8124 =$

HASNA PAPER APPENDIX

$$\text{SIGNAL POWER} = (\alpha_1^2 \alpha_2^2 \cdots \alpha_N^2)(G_1^2 G_2^2 \cdots G_{N-1}^2)$$

$$\text{NOISE POWER} = N_{01}(G_1^2 G_2^2 \cdots G_{N-1}^2)(\alpha_2^2 \alpha_3^2 \cdots \alpha_N^2) +$$

$$+ N_{0,2}(G_2^2 \cdot G_3^2 \cdots G_{N-1}^2)(\alpha_3^2 \alpha_4^2 \cdots \alpha_N^2) + \cdots + N_{0N}$$



VIDI ZOKI KAKO NA ALTERNATIVEN NAČIN GO IZVEDUVA NA 4.11.69

$$SNR = \gamma_{eq_N} = \frac{\text{SIGNAL POW}}{\text{NOISE POW}} = \frac{\prod_{n=1}^{N} \alpha_n^2 \prod_{n=1}^{N-1} G_n^2}{\sum_{n=1}^{N} N_{0,n} \prod_{t=n+1}^{N} \alpha_t^2 \prod_{t=n}^{N-1} G_t^2} \Bigg/ \prod_{n=1}^{N} N_{0,n} \prod_{n=1}^{N-1} G_n^2$$

$$\text{Noise Power} = \sum_{k=1}^{N} N_{0,k} \prod_{n=k+1}^{N} \alpha_n^2 \prod_{n=k}^{N-1} G_n^2$$

$$\boxed{\gamma_n = \frac{\alpha_n^2}{N_{0N}}}$$

$$\text{Numerator} = \prod_{n=1}^{N} \frac{\alpha_n^2}{N_{0,n}} = \prod_{n=1}^{N} \gamma_n$$

$$\text{Denominator} = \sum_{n=1}^{N} N_{0,n} \frac{\prod_{t=n+1}^{N} \alpha_t^2 \cdot \prod_{t=n}^{N-1} G_t^2}{\prod_{n=1}^{N} N_{0,n} \prod_{n=1}^{N-1} G_n^2} = \sum_{n=1}^{N} N_{0,n} \frac{\prod_{t=n+1}^{N} \gamma_t^2}{\prod_{t=1}^{n} N_{0,n} \prod_{t=1}^{n-1} G_t^2}$$

$n-1$ (AKO GO ZATVIES CE SE NA

$$= \sum_{n=1}^{N} \frac{\prod_{t=n+1}^{N} \gamma_t^2}{\prod_{t=1}^{n-1} N_{0n} \prod_{t=1}^{n-1} G_t^2} \Bigg/\!\!\Bigg/$$

$$\text{Denominator} = \sum_{n=1}^{N} \frac{\prod_{t=n+1}^{N} \gamma_t}{\prod_{t=1}^{n-1} G_t^2 \prod_{t=1}^{n-1} N_{0,t}} = \left| G_t^2 = \frac{1}{\alpha_t^2 + N_{0,t}} \right| =$$

$$= \sum_{n=1}^{N} \frac{\prod_{t=n+1}^{N} \gamma_t}{\prod_{t=1}^{n-1} \cdot \frac{1}{\underbrace{\alpha_t^2 / N_{0t}}_{\gamma_t} + 1}} = \sum_{n=1}^{N} \prod_{t=n+1}^{N} \gamma_t \prod_{t=1}^{n-1} (\gamma_t + 1)$$

$$\gamma_{eq} = \frac{\prod_{n=1}^{N} \gamma_n}{\sum_{n=1}^{N} \prod_{t=n+1}^{N} \gamma_t \prod_{t=1}^{n-1} (\gamma_t + 1)}$$

$$\text{Denominator} = \frac{\sum_{n=1}^{N} \prod_{t=n+1}^{N} \gamma_t \prod_{t=1}^{n-1} (\gamma_t + 1)}{\prod_{n=1}^{N} \gamma_n} = \sum_{n=1}^{N} \frac{\prod_{t=1}^{n-1} (\gamma_t + 1)}{\prod_{t=1}^{n} \gamma_t}$$

$$= \sum_{n=1}^{N} \frac{1}{\gamma_n} \frac{\prod_{t=1}^{n-1} (\gamma_t + 1)}{\prod_{t=1}^{n-1} \gamma_t} = \sum_{n=1}^{N} \frac{1}{\gamma_n} \prod_{t=1}^{n-1} \left(1 + \frac{1}{\gamma_t}\right)$$

$$\gamma_{eq} = \left[\sum_{n=1}^{N} \frac{1}{\gamma_n} \prod_{t=1}^{n-1} \left(1 + \frac{1}{\gamma_t}\right) \right]^{-1}$$

$$\gamma_{eq}^{-1} = \frac{1}{\gamma_1} + \frac{1}{\gamma_2}\left(1 + \frac{1}{\gamma_1}\right) + \frac{1}{\gamma_3}\left(1 + \frac{1}{\gamma_1}\right)\left(1 + \frac{1}{\gamma_2}\right) + \cdots +$$

$$+ \frac{1}{\gamma_N}\left(1 + \frac{1}{\gamma_1}\right)\left(1 + \frac{1}{\gamma_2}\right) \cdots \left(1 + \frac{1}{\gamma_{N-1}}\right)$$

$N=3$

$$\gamma_{eq}^{-1} = \frac{1}{\gamma_1} + \frac{1}{\gamma_2}\left(1 + \frac{1}{\gamma_1}\right) + \frac{1}{\gamma_3}\left(1 + \frac{1}{\gamma_1}\right)\left(1 + \frac{1}{\gamma_2}\right) =$$

$$= \frac{1}{\gamma_1} + \frac{1}{\gamma_2} + \frac{1}{\gamma_1\gamma_2} + \left(\frac{1}{\gamma_3} + \frac{1}{\gamma_1\gamma_3}\right)\left(1 + \frac{1}{\gamma_2}\right) =$$

$$= \frac{1}{\gamma_1} + \frac{1}{\gamma_2} + \frac{1}{\gamma_1\gamma_2} + \frac{1}{\gamma_3} + \frac{1}{\gamma_2\gamma_3} + \frac{1}{\gamma_1\gamma_3} + \frac{1}{\gamma_1\gamma_2\gamma_3}$$

$$\gamma_{eq}^{-1} = 1 + \frac{1}{\gamma_1} + \left(1+\frac{1}{\gamma_1}\right)\left(\frac{1}{\gamma_2} + \frac{1}{\gamma_3}\left(1+\frac{1}{\gamma_2}\right)\right) - 1 =$$

$$= \left(1+\frac{1}{\gamma_1}\right)\left[1 + \frac{1}{\gamma_2} + \frac{1}{\gamma_3}\left(1+\frac{1}{\gamma_2}\right)\right] - 1 =$$

$$= \left(1+\frac{1}{\gamma_1}\right)\left(1+\frac{1}{\gamma_2}\right)\left(1+\frac{1}{\gamma_3}\right) - 1$$

$$\boxed{\gamma_{eq}^{-1}\Big|_{N=3} = \left[\left(1+\frac{1}{\gamma_1}\right)\left(1+\frac{1}{\gamma_2}\right)\left(1+\frac{1}{\gamma_3}\right)\right]^{-1} - 1}$$

DOKAZANO ZA
N=3

$$\boxed{\gamma_{eq} = \prod_{n=1}^{N}\left(1+\frac{1}{\gamma_n}\right) - 1}$$

SE IZVUČE ZATO SE KRATI

$$(T)2\sigma^2 = N_0$$

$$\boxed{SNR = \frac{E_s}{2k\sigma^2}} \longrightarrow \text{SNR PER BIT}$$

$$k = \operatorname{ld} M$$

$$\sigma = \operatorname{sqrt}\left(\frac{E_s}{2k\,SNR}\right)$$

ALTERNAT

$$\boxed{SNR_{bit} = \frac{E_b}{2\sigma^2}}$$

SO KONST. ZA T

$$\boxed{P_B = Q\left(\sqrt{\frac{E_d}{2\boxed{N_0}}}\right)} \qquad E_d = \int [s_1(t) - s_2(t)]^2\,dt$$

UNIPOLAR
0÷A

$$E_d = \int_0^T [A^2 - 0]\,dt = \underline{\underline{A^2T}}$$

$$Q(x) = \frac{1}{2}\operatorname{erfc}\left(\frac{x}{\sqrt{2}}\right) \qquad P_B = Q\left(\sqrt{\frac{A^2T}{2\boxed{N_0}}}\right)$$

$$T\cdot\sigma_N^2 = \frac{N_0}{2}. \qquad N_0 (=) \frac{W}{Hz} (=) \underline{\underline{W\cdot s}}$$

$$\boxed{N_0 = 2\cdot T\cdot\sigma_N^2}$$

$$P_B = Q\left(\sqrt{\frac{A^2\cdot \cancel{T}}{4\cdot\cancel{T}\cdot\sigma_N^2}}\right) = Q\left(\frac{A}{2\sigma_N}\right) = \frac{1}{2}\operatorname{erfc}\left(\frac{A}{2\sqrt{2}\sigma_N}\right)$$

$$E_b = \frac{A^2T + 0}{2} = \frac{A^2T}{2} \qquad A = \sqrt{\frac{2E_b}{T}}$$

$$\sigma_N = \sqrt{\frac{N_0}{2T}} \qquad P_b = \frac{1}{2}\operatorname{erfc}\left(\frac{\sqrt{\frac{\cancel{2}E_b}{\cancel{T}}}}{\cancel{2}\sqrt{2}\sqrt{\frac{N_0}{2\cancel{T}}}}\right) = \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{E_b}{2N_0}}\right)$$

$$SNR_{gm} = \frac{E_b}{N_0} = \frac{E_b}{2\sigma_N^2}$$

$$SNR_{g\_dB} = 10\log\left(\frac{E_b}{N_0}\right) \qquad E_b N_0 = 10^{0.1\,SNR_{g\_dB}}$$

$$\frac{E_b}{N_0} = 10^{0.1\,SNR_{g\_dB}} \qquad \boxed{N_0 = E_b\cdot 10^{-0.1\,SNR_{g\_dB}}}$$

DPSK-2 $E_d = \int_0^T \left(A^{\cancel{2}}+A^{\cancel{2}}\right)^2 dT = 4A^2\cancel{T}$

$$E_b = \frac{A^2+A^2}{2}T = \frac{2A^2}{2}T = A^2T \qquad A=1 \quad \boxed{E_b = 1}$$

DPSK -A:A $\quad E_d = 4A^2T$

$$P_B = Q\left(\sqrt{\frac{E_d}{2N_0}}\right) = Q\left(\sqrt{\frac{4A^2\cancel{T}}{\cancel{2}\cdot 2\cdot\cancel{T}\cdot\sigma_N^2}}\right) = Q\left(\frac{A}{\sigma_N}\right)$$

$$T\cdot\sigma_N^2 = \frac{N_0}{2} \qquad N_0 = 2\cdot T\cdot\sigma_N^2$$

$$P_B = Q\left(\sqrt{\frac{4A^2T}{2N_0}}\right) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

$$\boxed{P_B = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)}$$

$SNR_{g\,dB} = 20$ dB $\qquad SNR_g = 10^{+0,1\cdot 20} = 100$

$$P_B = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{100}\right) = \frac{1}{2}\,\mathrm{erfc}(10) = 1\cdot 10^{-45}$$

MGF OF RAYLEIGH FADING

$$u = \left(-s+\frac{1}{\bar\gamma}\right)\gamma \qquad du = \left(-s+\frac{1}{\bar\gamma}\right)d\gamma$$

$$p_\gamma(\gamma) = \frac{1}{\bar\gamma}e^{-\gamma/\bar\gamma}$$

$$M_\gamma(s) = \int_0^\infty e^{\gamma s}\cdot\frac{1}{\bar\gamma}e^{-\frac{\gamma}{\bar\gamma}}d\gamma = \frac{1}{\bar\gamma}\int_0^\infty e^{-\gamma\left(-s+\frac{1}{\bar\gamma}\right)}d\gamma$$

$$= \frac{1}{\bar\gamma}\left(-s+\frac{1}{\bar\gamma}\right)^{-1}\int_0^\infty e^{-u}\,du = -\frac{1}{\bar\gamma}\left(s-\frac{1}{\bar\gamma}\right)^{-1}\cancel{e^{-u}}\Big|_\infty^0 \rightarrow (1-0)$$

$$= -\frac{1}{\bar\gamma}\,\frac{1}{s-\frac{1}{\bar\gamma}} \quad s-\frac{1}{\cancel{\bar\gamma}}\;\frac{\cancel{\bar\gamma}}{s\bar\gamma-1} = -\frac{1}{s\bar\gamma-1} = \left(1-s\bar\gamma\right)^{-1}$$

$$P_\gamma(\gamma \le \gamma_0) = \int_0^{\gamma_0} p_\gamma(\gamma)\, d\gamma \quad / \mathcal{L}$$

$$\boxed{\hat{P}_\gamma(\gamma) = \frac{\hat{p}_\gamma(\gamma)}{s}\Big|_{\gamma=\gamma_0}}$$

$$\hat{p}_\gamma(\gamma) = \int_0^\infty p_\gamma(\gamma)\, e^{-s\gamma}\, d\gamma$$

$$M_\gamma(s) = \int_0^\infty p_\gamma(\gamma)\, e^{+s\gamma}\, d\gamma \qquad M_\gamma(-s) = \int_0^\infty p_\gamma(\gamma)\, e^{-s\gamma}\, d\gamma = \hat{p}_\gamma(\gamma)$$

$$\hat{P}_\gamma(\gamma) = \frac{M_\gamma(-s)}{s}\Big|_{\gamma=\gamma_0}$$

$$P_\gamma(\gamma \le \gamma_0) = \mathcal{L}^{-1}\left\{\frac{M_\gamma(-s)}{s}\right\}\Big|_{\gamma=\gamma_0}$$

AaF:

$$P(\gamma < \gamma_{th}) = P\left(\frac{1}{\gamma} > \frac{1}{\gamma_{th}}\right) = 1 - P\left(\frac{1}{\gamma} < \frac{1}{\gamma_{th}}\right) =$$
$$= 1 - \mathcal{L}^{-1}\left\{\frac{M_\gamma(-s)}{s}\right\}\Big|_{\frac{1}{\gamma} = \frac{1}{\gamma_{th}}}$$

$$p_R(x) = \frac{1}{\sigma^2} e^{\frac{(x^2-m)^2}{2\sigma^2}}$$

MATLAB GAMMA DISTRIBUTION

$$\boxed{y = f(x, a, b) = \frac{1}{b^a \Gamma(a)} x^{a-1} e^{-\frac{x}{b}}}$$

GENERATE WITH
gamrnd(A,B,m,n)
(ili)
randg(A) → $A = m$, $b = [illegible]$

ODNOSNO NA DISTRIBUCIJATA NA SNR VO NAKAGAMI,

$$\boxed{p_\gamma(\gamma) = \frac{\gamma^{m-1}}{\Gamma(m)\bar{\gamma}^m} e^{-\frac{\gamma}{\bar{\gamma}}}}$$

$$p_r(x) = \frac{m^m x^{2m-1}}{\Gamma(m)\Omega^m} e^{-\frac{mx^2}{\Omega}}$$

$$\gamma = \alpha^2 \frac{E_b}{N_0}, \qquad \boxed{\Omega = E(\alpha^2)}$$

$$\bar{\gamma} = \Omega \frac{E_b}{N_0}$$

$$\boxed{\frac{S}{N} = \frac{\epsilon/T}{N_0 \cdot W} = \frac{\epsilon/T}{N_0 \cdot 1/T} = \frac{\epsilon}{N_0}}$$

- POUT FOR MULTIHOP AAF SYSTEM IN RAYLEIGH CHANNEL

$$P_{out} = P(\gamma < \gamma_0) \qquad p_\gamma(\gamma) = \frac{1}{\bar\gamma} e^{-\gamma/\bar\gamma}$$

$$P(\gamma<\gamma_0) = \int_0^{\gamma_0} \frac{1}{\bar\gamma} e^{-\gamma/\bar\gamma} d\gamma = -e^{-\gamma/\bar\gamma}\Big|_0^{\gamma_0} = +\left(\cancel{\frac{1}{1}} - \cancel{\frac{1}{e^{\gamma/\bar\gamma}}}\right)$$

$$\boxed{P(\gamma<\gamma_0) = P_{out} = 1 - e^{\gamma/\bar\gamma}}$$

RAYLEIGH DISTRIBUTION POUT OF MULTIHOP SYSTEM

$$p_\gamma(\gamma) = \frac{K+1}{\bar\gamma} \exp\left(-(K+1)\frac{\gamma}{\bar\gamma} - K\right) I_0\left(2\sqrt{K(K+1)\frac{\gamma}{\bar\gamma}}\right)$$

$$\boxed{p_x(x) = \frac{2x(K+1)}{\Omega} \exp\left[-\frac{x^2(K+1)}{\Omega} - K\right] I_0\left(\frac{2x\sqrt{K(K+1)}}{\sqrt{\Omega}}\right)}$$

$$p_x(x) = \frac{x}{\sigma^2} \exp\left[-\frac{x^2+s^2}{2\sigma^2}\right] I_0\left(\frac{xs}{\sigma^2}\right)$$

$$K = \frac{s^2}{2\sigma^2} \qquad \Omega = s^2 + 2\sigma^2 = P_r = E[x^2]$$

$$P_{out} = P(\gamma<\gamma_0) = P(x^2 < x_0^2) = \int_0^{x_0} \frac{x}{\sigma^2} e^{-\frac{x^2+s^2}{2\sigma^2}} I_0\left(\frac{xs}{\sigma^2}\right) dx$$

$$= \cancel{\int_0^\infty \frac{x}{\sigma^2} e^{-\frac{x^2+s^2}{2\sigma^2}} I_0\left(\frac{xs}{\sigma^2}\right) dx} \xrightarrow{} 1 - \int_{x_0}^{\infty} \frac{x}{\sigma^2} e^{-\frac{x^2+s^2}{2\sigma^2}} I_0\left(\frac{xs}{\sigma^2}\right) dx =$$

$$= 1 - \mathrm{marcumq}\left(\frac{s}{\sigma}, \frac{x}{\sigma}\right) = 1 - \mathrm{marcumq}\left(\sqrt{2K}, \frac{x}{\sigma}\right)$$

$$2\sigma^2 = \Omega - s^2; \qquad s^2 = 2K\sigma^2$$

$$2\sigma^2 = \Omega - 2K\sigma^2 \qquad 2\sigma^2(1+K) = \Omega \qquad \boxed{\sigma^2 = \frac{\Omega}{2(1+K)}}$$

$$P_{out} = 1 - \mathrm{marcumq}\left(\sqrt{2K}, \sqrt{\frac{2(1+K)}{\Omega}}\, x\right)$$

$$\boxed{P_{out} = 1 - \mathrm{marcumq}\left(\sqrt{2K}, x\sqrt{\frac{2(1+K)}{\Omega}}\right)}$$

$$p_x(x) = \frac{x}{\sigma^2} e^{-\frac{x^2+s^2}{2\sigma^2}} I_0\left(\frac{x\cdot s}{\sigma^2}\right) \qquad \boxed{z = \frac{x}{\sigma} \quad a = \frac{s}{\sigma}}$$

$$\frac{dz}{dx} = \frac{1}{\sigma} \quad x = \sigma\cdot z$$

$$p_z(z) = \frac{p_x(x)}{\frac{dz}{dx}}\Bigg|_{x=f(z)} = \frac{1}{\frac{1}{\sigma}} \frac{\cancel{\sigma}\cdot z}{\cancel{\sigma^2}} e^{-\frac{z^2+a^2}{2}} I_0(z\cdot a)$$

$$p_z(z) = z\, e^{-\frac{z^2+a}{2}} I_0(z\cdot a)$$

$$P(z \le z_0) = 1 - \int_{z_0}^{\infty} z\, e^{-\frac{z^2+a^2}{2}} I_0(z\cdot a)\, dz = \underline{\underline{1 - \text{marq}(a, z_0)}}$$

$$K = \frac{s^2}{2\sigma^2} = \frac{a^2}{2} \Rightarrow \boxed{a = \sqrt{2K}}$$

$$z = \frac{x}{\sigma} \quad z^2 = \frac{x^2}{\sigma^2} \qquad \Omega = s^2 + 2\sigma^2$$

$$\sigma^2 = \frac{\Omega}{2(1+K)}$$

$$z = \frac{x}{\sqrt{\frac{\Omega}{2(1+K)}}} = \boxed{x\sqrt{\frac{2(1+K)}{\Omega}}} \qquad \boxed{\gamma_0 = x_0^2}$$

$$P(z < z_0) = 1 - \text{marq}\left(\sqrt{2K},\ x_0\sqrt{\frac{2(1+K)}{\Omega}}\right.$$

$$P(x < x_0) = 1 - \text{marq}\left(\sqrt{2K},\ \sqrt{\frac{2\gamma_0(1+K)}{\Omega}}\right)$$

$$= 1 - \text{marq}\left(\sqrt{2K},\ \sqrt{\frac{2\gamma_0(1+K)}{\gamma}}\right)$$

MGF OF RAYLEIGH CHANNE (i.e. Pout AaF FOR RAYLEIGH)

$$p_\gamma(\gamma) = \frac{1}{\bar\gamma} e^{-\gamma/\bar\gamma}$$

$$M(-s) = \int_0^\infty \frac{1}{\bar\gamma} e^{-\gamma/\bar\gamma} \cdot e^{-\gamma s}\, d\gamma = \frac{1}{\bar\gamma}\int_0^\infty e^{-\gamma(\bar\gamma^{-1}+s)}\, d\gamma =$$

$$= \frac{1}{\bar\gamma}\int_0^\infty e^{-\gamma(\bar\gamma^{-1}+s)} \frac{d\gamma(\bar\gamma^{-1}+s)}{(\bar\gamma^{-1}+s)} = -\frac{1}{\bar\gamma(\bar\gamma^{-1}+s)} e^{-\gamma(\bar\gamma^{-1}+s)}\Bigg|_0^\infty$$

$$= \frac{1}{\bar\gamma(\frac{1}{\bar\gamma}+s)} = \frac{1}{(1+s\bar\gamma)}$$

$P_{out} = P(\gamma < \gamma_{th})$ $\qquad \hat{P}_{out} = \frac{\hat{P}_\gamma(s)}{s} = \frac{M(-s)}{s}$

$$P_{out} = \mathcal{L}^{-1}\left\{\frac{M(-s)}{s}\right\}\Bigg|_{\gamma=\gamma_{th}}$$

$$M_{1/\bar\gamma}(-s) = \int_0^\infty \frac{1}{\bar\gamma} e^{-\frac{\gamma}{\bar\gamma}} \cdot e^{-\frac{s}{\gamma}}\, d\gamma = \frac{1}{\bar\gamma}\int_0^\infty e^{-\left(\frac{\gamma}{\bar\gamma}+\frac{s}{\gamma}\right)} d\gamma$$

$$= \frac{2\sqrt{g_m\cdot s}}{\cancel{\bar\gamma_m}\ \sqrt{\bar\gamma}}\ \text{BesselK}\left(1, 2\sqrt{\frac{s}{g_m}}\right) \Rightarrow$$ FROM MAPLE!!!

$g_m \triangleq \bar\gamma$

$$\boxed{M_{1/\bar\gamma}(-s) = 2\sqrt{\frac{s}{\bar\gamma}}\ \text{BesselK}\left(1, 2\sqrt{\frac{s}{\bar\gamma}}\right)}$$ RAYLEIGH

- Pout for AaF SYSTEM FOR RICIAN CHANNEL

$$p_\gamma(\gamma) = \frac{K+1}{\bar\gamma}\exp\left(-(K+1)\frac{\gamma}{\bar\gamma} - K\right) I_0\left(2\sqrt{K(K+1)\frac{\gamma}{\bar\gamma}}\right)$$

$$M_{1/\gamma}(-s) = \int_0^\infty e^{-\frac{s}{\gamma}}\ \frac{K+1}{\bar\gamma} e^{-K}\ e^{-(K+1)\frac{\gamma}{\bar\gamma}} I_0\left(2\sqrt{K(K+1)\frac{\gamma}{\bar\gamma}}\right) d\gamma =$$

$$= \frac{K+1}{\bar\gamma} e^{-K}\int_0^\infty e^{-\left[\frac{s}{\gamma}+(K+1)\frac{\gamma}{\bar\gamma}\right]} I_0\left(2\sqrt{K(K+1)\frac{\gamma}{\bar\gamma}}\right) d\gamma = ?$$

- [illegible] NEGO [illegible]
- MATEMATIKA NEGO LOGIKA

$$M_\gamma(-s) = \int_0^\infty \frac{K+1}{\bar\gamma} e^{-K}\ e^{-\left[s\cdot\gamma + (K+1)\frac{\gamma}{\bar\gamma}\right]} I_0\left(2\sqrt{K(K+1)\frac{\gamma}{\bar\gamma}}\right) d\gamma \Rightarrow$$

$$\boxed{M_\gamma(-s) = \frac{\bar\gamma\cdot e^{\frac{K(K+1)}{s\bar\gamma+K+1}}}{s\cdot\bar\gamma+K+1}\ \frac{K+1}{\bar\gamma} e^{-K}} \qquad \boxed{M_\gamma(-s) = \frac{(K+1)\, e^{\frac{K(K+1)}{s\bar\gamma+K+1}-K}}{s\bar\gamma+K+1}}$$

Hasna-Alouini-Paper.mw

$$M_\gamma(s) = \frac{(K+1)\, e^{-\frac{s\bar\gamma K}{s\bar\gamma+K+1}}}{s\bar\gamma+K+1}$$

$$\frac{\cancel{K^2+K} - s\bar\gamma K - \cancel{K^2} - \cancel{K}}{s\bar\gamma+K+1}$$

$$\gamma_{eq} = \left[\sum_{n=1}^{N} \frac{1}{\gamma_n} \right]^{-1} = \Bigg/_{N=3} \frac{1}{\frac{1}{\gamma_1} + \frac{1}{\gamma_2} + \frac{1}{\gamma_3}} = \frac{\gamma_1 \gamma_2 \gamma_3}{\gamma_2\gamma_3 + \gamma_1\gamma_3 + \gamma_1 \gamma_2}$$

$N=2$ $$\gamma_{eq} = \frac{1}{\frac{1}{\gamma_1} + \frac{1}{\gamma_2}} = \frac{\gamma_1\gamma_2}{\gamma_1 + \gamma_2}$$

$$MGF_{eq} = \frac{MGF_1\, MGF_2}{MGF_1 + MGF_2} = \Big/ MGF_1 = MGF_2 \Big/ = \frac{MGF^2}{2\cancel{MGF}}$$

$N=3$ $\gamma_1 = \gamma_2 = \gamma_3$ $$\frac{\gamma^{\cancel{3}}}{3\gamma^{2}} = \boxed{\frac{\gamma}{3}}$$

$$\boxed{M_{\frac{\gamma}{2}}(s) = \frac{2(K+1)\, e^{-\frac{Ks\bar{\gamma}}{s\bar{\gamma}+2K+2}}}{s\bar{\gamma} + 2K + 2}}$$

$$p_\gamma(\gamma) = \frac{K+1}{\bar{\gamma}} \exp\left(-(K+1)\frac{\gamma}{\bar{\gamma}}, -K\right) I_0\left(2\sqrt{K(K+1)\frac{\gamma}{\bar{\gamma}}}\right)$$

$$\boxed{z = \frac{\gamma}{2}} \qquad \frac{dz}{d\gamma} = \frac{1}{2}$$

$$p_z(z) = \frac{p_\gamma(\gamma)}{\frac{dz}{d\gamma}}\Bigg|_{\gamma = 2z} = 2\, \frac{K+1}{\bar{\gamma}} \exp\left(-(K+1)\frac{2z}{\bar{\gamma}} - K\right) I_0\left(2\sqrt{K(K+1)\frac{2z}{\bar{\gamma}}}\right)$$

$$\boxed{M_{\frac{\gamma}{H}}(s) = \frac{H(K+1)\, e^{-\frac{Ks\bar{\gamma}}{s\bar{\gamma}+H\cdot K+H}}}{s\bar{\gamma} + H\cdot K + H}} \circledast$$

FOR N-HOPS

$\circledast$ DANA IST REZULTAT KAKO ,

pout_th = 1 - marcumq(sqrt(2K), sqrt($\frac{2\cdot H\cdot g0(1+K)}{gm}$))

i.e

[redacted]

$$\boxed{P_{out} = 1 - MQ\left(\sqrt{2K}, \sqrt{\frac{2H\gamma_0(1+K)}{\bar{\gamma}}}\right)}$$

$$p_x = \frac{x}{\sigma^2} e^{-\frac{x^2+s^2}{2\sigma^2}} I_0\left(\frac{x\cdot s}{\sigma^2}\right)$$

$$\gamma_{eq} = \frac{\gamma_1\cdot\gamma_2}{\gamma_1+\gamma_2} = \left|\gamma_1=\gamma_2\right| = \frac{\gamma^2}{2\gamma} = \frac{\gamma}{2} \qquad \gamma = x^2$$

$$x_{eq}^2 = \frac{\gamma}{2} = \gamma_{eq} = \sqrt{\frac{\gamma}{2}} = \sqrt{\frac{x^2}{2}} = \frac{x}{\sqrt{2}} = z$$

$$\boxed{z = \frac{x}{\sqrt{2}}} \qquad \boxed{\frac{dz}{dx} = \frac{1}{\sqrt{2}}} \qquad \boxed{x = \sqrt{2}z}$$

$$p_z(z) = \frac{\sqrt{2}z}{\sigma^2}\cdot\sqrt{2}\cdot e^{-\frac{2z^2+s^2}{2\sigma^2}} I_0\left(\frac{\sqrt{2}z\cdot s}{\sigma^2}\right)$$

$$P_{out} = 1 - \int_{z_0}^{\infty} \frac{2z}{\sigma^2}\cdot e^{-\frac{2z^2+s^2}{2\sigma^2}} I_0\left(\frac{\sqrt{2}z\cdot s}{\sigma^2}\right) dz$$

$$\left| w = \frac{\sqrt{2}z}{\sigma} \qquad a = \frac{s}{\sigma} \qquad \frac{dw}{dz} = \frac{\sqrt{2}}{\sigma} \qquad z = \frac{\sigma w}{\sqrt{2}} \right| \quad dz = \frac{\sigma}{\sqrt{2}} dw$$

$$= 1 - \cancel{\frac{2}{\sigma^2}}\,\cancel{\frac{\sigma}{\sqrt{2}}} \int_{\sqrt{2}z_0/\sigma}^{\infty} \frac{\cancel{\sigma} w}{\cancel{\sqrt{2}}} e^{-\frac{w^2+a^2}{2}} I_0(w\cdot a)\, dw$$

$$P_{out} = 1 - \int_{\boxed{\sqrt{2}z_0/\sigma}}^{\infty} e^{-\frac{w^2+a^2}{2}} I_0(w\cdot a)\, dw =$$

$$\boxed{z_0 = \sqrt{\gamma_0}}$$

$$= 1 - \text{marcumq}\left(a, \frac{\sqrt{2}z_0}{\sigma}\right) = 1 - \text{marcumq}\left(a, \frac{\sqrt{\gamma_0}}{\sigma}\right)$$

$$P_{out} = 1 - \text{marcumq}\left(\sqrt{2K}, z_0\sqrt{2}\sqrt{\frac{2(1+K)}{\Omega}}\right)$$

ZA „H"
MORA STAVI
„H"

[illegible]

$$\frac{z_0}{\sigma} = \frac{z_0}{\sqrt{\frac{2(1+K)}{\Omega}}^{-1}} = z_0\sqrt{\frac{2(1+K)}{\Omega}}$$

$$\boxed{\gamma = \sqrt{z_0}}$$

- USTE EDNAŠ Pout ZA RAYLEIGH CH - MULTIHOP SYSTEM

$$p_\gamma(\gamma) = \frac{1}{\bar\gamma} e^{-\gamma/\bar\gamma}$$

$$\gamma_{eq} = \frac{\gamma_1 \cdot \gamma_2}{\gamma_1 + \gamma_2} = \Big/ \gamma_1 = \gamma_2 \Big/ = \frac{\gamma}{2} \qquad \boxed{\gamma_{eq} = \frac{\gamma}{2}} \quad (\Delta\Delta)$$

$$p_{\gamma/2}(\gamma) = \frac{p_\gamma(\gamma)}{\frac{d\gamma_{eq}}{d\gamma}} \Bigg| \gamma = f(\gamma_{eq}) \qquad \frac{d\gamma_{eq}}{d\gamma} = \frac{1}{2} \quad \gamma = 2\gamma_{eq}$$

$$p_{\gamma_{eq}}(\gamma) = p_{\gamma/2}(\gamma) = \frac{\frac{1}{\bar\gamma} e^{-\frac{2\gamma_{eq}}{\bar\gamma}}}{\frac{1}{2}} = 2 \frac{1}{\bar\gamma} e^{-2\gamma_{eq}/\bar\gamma} = \frac{2}{\bar\gamma} e^{-\gamma/\bar\gamma}$$

$$P_{out} = P(\gamma_{eq} < \gamma_0) = \frac{2}{\bar\gamma} \int_0^{\gamma_0} e^{-2\gamma_{eq}/\bar\gamma} d\gamma_{eq} = + e^{-2\gamma_{eq}/\bar\gamma} \Big|_{\gamma_0}^{0}$$

$$\boxed{P_{out} = 1 - e^{-2\gamma_0/\bar\gamma}} \Big|_0$$

N-HOP SYSTEM $\boxed{P_{out} = 1 - e^{-N\gamma_0/\bar\gamma}}$ (**)

SEKAK (**) <u>OTSTAPUVA</u> OD: pout_aaf_rayleigh KOJ E IDENTIČEN SO EMPIRISKITE REZULTATI. "NE MNOGU"

OČIGLEDNO ~~POSTAVUVANJETO~~ (ΔΔ) VODI KON NAŠA GREŠKA. TREBA DA „SE BA

$$\gamma = \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} \qquad p(\gamma_1, \gamma_2) = p(\gamma_1)\, p(\gamma_2)$$

$$\gamma_1' = \gamma_1 \qquad (x^{-1})' = 1x^{-2} = -\frac{1}{x^2}$$

$$j = \begin{vmatrix} \frac{\partial\gamma}{\partial\gamma_1} & \frac{\partial\gamma}{\partial\gamma_2} \\ \frac{\partial\gamma_1'}{\partial\gamma_1} & \frac{\partial\gamma_1}{\partial\gamma_2} \end{vmatrix} = \begin{vmatrix} \left(\frac{\gamma_2}{\gamma_1+\gamma_2}\right)^2 & \left(\frac{\gamma_1}{\gamma_1+\gamma_2}\right)^2 \\ 1 & 0 \end{vmatrix} = -\frac{\gamma_1^2}{(\gamma_1+\gamma_2)^2}$$

$$\frac{\partial}{\partial\gamma_1}\left(\frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2}\right) = \frac{\gamma_2}{\gamma_1+\gamma_2} - \frac{\gamma_1\gamma_2}{(\gamma_1+\gamma_2)^2} = \frac{\cancel{\gamma_2\gamma_1} + \gamma_2^2 - \cancel{\gamma_1\gamma_2}}{(\gamma_1+\gamma_2)^2} = \left(\frac{\gamma_2}{\gamma_1+\gamma_2}\right)^2$$

$$\frac{\partial}{\partial\gamma_2}\left(\frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2}\right) = \left(\frac{\gamma_1}{\gamma_1+\gamma_2}\right)^2$$

$$p_{\gamma\gamma_1'}(\gamma_1', \gamma) = \frac{p(\gamma_1, \gamma_2)}{|j|} = \frac{p(\gamma_1)\,p(\gamma_2)}{\gamma_1^2 / (\gamma_1+\gamma_2)^2} \Bigg| \begin{array}{l} \gamma_1 = \gamma_1' \\ \gamma_2 = \frac{\gamma\gamma_1'}{\gamma_1'-\gamma} \end{array}$$

$$\gamma(\gamma_1 + \gamma_2) = \gamma_1\gamma_2$$
$$\gamma\gamma_1 + \gamma\gamma_2 = \gamma_1\gamma_2 \qquad \gamma_2 = \frac{\gamma\gamma_1}{\gamma_1 - \gamma}$$

$$p(\gamma_1', \gamma) = \frac{p(\gamma_1, \gamma_2)}{\gamma_1^2/(\gamma_1+\gamma_2)^2} \Bigg|_{\substack{\gamma_1 = \gamma_1' \\ \gamma_2 = \frac{\gamma\gamma_1'}{\gamma_1' - \gamma}}}$$

$$p(\gamma) = \int_{-\infty}^{\infty} p(\gamma_1', \gamma)\, d\gamma_1' = \int_{-\infty}^{\infty} \frac{(\gamma_1'+\gamma_2)^2}{\gamma_1'^2}\, p(\gamma_1)\, p(\gamma_2)\, d\gamma_1' \Big/_{\gamma_1'=\gamma_1}$$

$$p(\gamma) = \int_{-\infty}^{\infty} \frac{(\gamma_1+\gamma_2)^2}{\gamma_1^2}\, p(\gamma_1)\, p(\gamma_2)\, d\gamma_1$$

$$p_{\gamma_1}(\gamma_1) = \frac{1}{\bar{\gamma}_1} e^{-\frac{\gamma_1}{\bar{\gamma}_1}} \qquad p_{\gamma_2}(\gamma_2) = \frac{1}{\bar{\gamma}_2} e^{-\frac{\gamma_2}{\bar{\gamma}_2}}$$

$$p(\gamma) = \int_{0}^{\infty} \frac{1}{\bar{\gamma}_2} e^{-\frac{\gamma_2}{\bar{\gamma}_2}} \frac{1}{\bar{\gamma}_1} e^{-\frac{\gamma_1}{\bar{\gamma}_1}} \cdot \frac{(\gamma_1+\gamma_2)^2}{\gamma_1^2}\, d\gamma_1 =$$

$$= \frac{1}{\bar{\gamma}_1 \bar{\gamma}_2} \int_{0}^{\infty} \frac{(\gamma_1+\gamma_2)^2}{\gamma_1^2} e^{-\frac{1}{\bar{\gamma}_2}\frac{\gamma\gamma_1}{\gamma_1-\gamma} - \frac{\gamma_1}{\bar{\gamma}_1}}\, d\gamma_1$$

$$\frac{(\gamma_1+\gamma_2)^2}{\gamma_1^2} = \frac{\left(\gamma_1 + \frac{\gamma\gamma_1}{\gamma_1-\gamma}\right)^2}{\gamma_1^2} = \left(\frac{\frac{\overset{+m}{\cancel{\gamma_1^2}} \cancel{- \gamma_1\gamma + \gamma_1\gamma}}{\gamma_1-\gamma}}{\underset{+m}{\cancel{\gamma_1^{2m}}}}\right)^2 = \frac{\gamma_1^4}{\cancel{\gamma_1^2}(\gamma_1-\gamma)^2}$$

$$\frac{(\gamma_1+\gamma_2)^2}{\gamma_1} = \frac{\gamma_1^2}{(\gamma_1-\gamma)^2}$$

NE GO KORISTAM
NE MOŽE U MATHCAD.

$$\boxed{p(\gamma) = \frac{1}{\bar{\gamma}_1 \bar{\gamma}_2} \int_{0}^{\infty} \frac{\gamma_1^2}{(\gamma_1-\gamma)^2} e^{-\frac{1}{\bar{\gamma}_2}\frac{\gamma\gamma_1}{\gamma_1-\gamma} - \frac{\gamma_1}{\bar{\gamma}_1}}\, d\gamma_1}$$

LOG - NORMAL

$$f(x; \mu, \sigma) = \frac{1}{x\sigma\sqrt{2\pi}} e^{-\frac{(\ln(x)-\mu)^2}{2\sigma^2}}$$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$y = e^x$$
$$x = \ln(y)$$

$$f(y) = \frac{f(x)}{\frac{\partial y}{\partial x}}\Bigg|_{x=\ln(y)} = \frac{e^{-\frac{(\ln y-\mu)^2}{2\sigma^2}}}{\underbrace{\left(e^{\ln(y)}\right)}_{y}\sigma\sqrt{2\pi}}$$

$$\boxed{f(y) = \frac{1}{y\sigma\sqrt{2\pi}} e^{-\frac{(\ln y-\mu)^2}{2\sigma^2}}}$$ LOG-NORMAL DISTRIBUTION

$$\boxed{K = \frac{\sqrt{m^2-m}}{m-\sqrt{m^2-m}}}$$

⊙ DCOFC 1.1.3 AVERAGE BIT PROBABILITY

$$P_b(\epsilon/\gamma) = c_1 e^{-a_1\gamma}$$ → CONDITIONAL BEP

$$P_b(\epsilon) \triangleq \int_0^\infty P_b(\epsilon/\gamma)\, p_\gamma(\gamma)\, d\gamma = \int_0^\infty c_1 e^{-a_1\gamma} p_\gamma(\gamma)\, d\gamma =$$
$$= c_1 \cdot M_\gamma(-a_1)$$

$$\boxed{P_b(\epsilon/\gamma) = \int_{\xi_1}^{\xi_2} c_2 h(\xi)\, e^{-a_2 g(\xi)\gamma}\, d\xi}$$ → DESIRED FORMS OF CONDITIONAL ERROR PROB.

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt = \frac{2}{\sqrt{\pi}} \left[\cancel{\int_0^\infty e^{-t^2} dt} - \int_x^\infty e^{-t^2} dt \right] =$$
$$\xrightarrow{} \frac{\sqrt{\pi}}{2}$$

$$= 1 - \frac{2}{\sqrt{\pi}} \int^\infty e^{-t^2} dt = 1 - \operatorname{erfc}(x)$$

$$P_b(\epsilon) = \int_0^\infty \int_{\xi_1}^{\xi_2} c_2 h(\xi) e^{-a_2 g(\xi)\gamma} d\xi\, p_\gamma(\gamma)\, d\gamma =$$

$$= c_2 \int_{\xi_1}^{\xi_2} h(\xi) \underbrace{\int_0^\infty e^{-a_2 g(\xi)\gamma} p_\gamma(\gamma)\, d\gamma}\, d\xi = c_2 \int_{\xi_1}^{\xi_2} h(\xi)\, M(-a_2 g(\xi))\, d\xi$$

$$P(\varepsilon) = c_2 \int_{\xi_1}^{\xi_2} h_1(\xi) M_\gamma[-a_2 g(\xi)] d\xi$$

GENERAL FORM FOR BEP (BER)

UNIFIED MGF-BASED APPROACH FOR EVALUATING AVERAGE ERROR PROBABILITY

$$P_B(E/\gamma) = P_r\{D/\gamma < 0\} = \int_{-\infty}^{0} \underbrace{p_{D/\gamma}(D)}_{PDF(D/\gamma)} dD = \underbrace{P_{D/\gamma}(0)}_{CDF(D/\gamma)}$$

- ANALOGUOUS TO P_{out} MGF APPROACH

$$\boxed{P_E(E/\gamma) = \frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty} \frac{M_{D/\gamma}(-s)}{s} ds}$$

APPROACH FOR SYMETRIC BINARY MODULATIONS

$$\boxed{M_{D/\gamma}(s) = f_1(s) e^{\gamma f_2(s)}}$$

(D) - DECISION VARIABLE

$$M_{D/\gamma}(s) = f_1(s) e^{\gamma f_2(s)} \qquad \gamma = \sum_{l=1}^{L} \gamma_l$$

$$P_E(E/\gamma) = \frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty} \frac{f_1(s) e^{\gamma f_2(s)}}{s} ds$$

$$P_E(E) = \cancel{\frac{1}{2\pi j}} \int_0^\infty P_E(E/\gamma) p_\gamma(\gamma) d\gamma = \frac{1}{2\pi j}\int_0^\infty \int_{\sigma-j\infty}^{\sigma+j\infty} \frac{f_1(s) e^{\gamma f_2(s)}}{s} p_\gamma(\gamma) ds d\gamma$$

$$= \frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty} \frac{f_1(s)}{s} \underbrace{\int_0^\infty e^{\gamma f_2(s)} p_\gamma(\gamma) d\gamma}_{M_\gamma(f_2(s))} ds = \frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty} \frac{f_1(s)}{s} M(f_2(s)) ds$$

(γ) - FADING VARIABLE

$$M_D(s) = \int_0^\infty \underbrace{f_1(s) e^{\gamma f_2(s)}}_{M_{D/\gamma}(s)} p_\gamma(\gamma) d\gamma = f_1(s) \int_0^\infty e^{\gamma f_2(s)} p_\gamma(\gamma) d\gamma = f_1(s) M_\gamma(f_2(s))$$

$$P_E(E) = \frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty} \frac{M_D(s)}{s} ds$$

$$\boxed{M_D(s) = f_1(s) \prod_{l=1}^{L} M_{\gamma_l}(f_2(s))}$$

$$M_D(s) = f_1(s)\, M_\gamma(f_2(s))$$!!

DECISSION VARIABLE — FADING VARIABLE

- M-ARY NONCOHERENT ORTHOGONAL SYSTEM OVER L-PATH DIVERSITY CHANNEL

$U_{1/\gamma}$ CORRECT $\qquad U_m,\ m = 2,3,\dots,M$ DECISION

$$P_s(C|\gamma; u_1) = P_r\{U_2 < u_1, U_3 < u_1, \dots U_M < u_1 \,|\, \underline{U_1|\gamma = u_1}\}$$

$$= \left[P_r\{U_2 < u_1 \,|\, U_1|\gamma = u_1\}\right]^{M-1} = \left[\int_0^{u_1} p_{U_2}(u_2)\, du_2\right] =$$

$$= \left[1 - [1 - P_{U_2}(u_1)]\right]^{M-1}$$

$$\binom{2}{1} = \frac{2!}{1!\,1!} = 2 \qquad \binom{3}{1} = \frac{3!}{1!\,2!} = 3$$

$$(1+x)^n = \sum_{k=0}^{n} \binom{n}{k} \underline{x^k}$$

$$(1+x)^2 = 1 + 2x + x^2 \ \Big|\ \binom{2}{0}\cancel{x^0} + \binom{2}{1}x^1 + \binom{2}{2}x^2 = 1 + 2x + x^2$$

$$(1+x)^3 = \binom{3}{0}x^0 + \binom{3}{1}x^1 + \binom{3}{2}x^2 + \binom{3}{3}x^3 = 1 + 3x + 3x^2 + x^3$$

$$(1-x)^3 = \sum_{k=0}^{n} \binom{n}{k}(-1)^k x^k = \underline{1 - 3x + 3x^2 - x^3}$$

$$\boxed{P_s(e|\gamma; u_1) = \sum_{k=1}^{M-1} (-1)^{k+1} \binom{M-1}{k} \left(1 - P_{U_2}(u_1)\right)^k = g(u_1)} \qquad \binom{3}{1} = 3$$

$$(1-x)^{M-1} = |M=4| = (1-x)^3 = \sum_{i=1}^{M-1} (-1)^{i+1} x^i =$$

$$= \underline{3x - 3x^2 + x^3} \quad ??$$

$$P_s(e|\gamma) = \int_0^\infty g(u_1)\, p_{U_1|\gamma}(u_1)\, du_1$$

$$P_s(e|\gamma; u_1) = 1 - P_s(c|\gamma; u_1) = 1 - \sum_{k=0}^{M-1} (-1)^k \binom{M-1}{k} \left(1 - P_{U_2}(u_1)\right)^k$$

$$P_s(\epsilon|\gamma) = \int_0^\infty g(\mu_1)\, p_{v_1|\gamma}(\mu_1)\, d\mu_1$$

$$p_{v_1|\gamma}(\mu_1) = \frac{1}{2\pi} \int_{-\infty}^{\infty} M_{v_1|\gamma}(j\omega)\, e^{j\omega\mu_1} d\omega$$

$$\boxed{F(j\omega) = \int_{-\infty}^{\infty} f(t) \cdot e^{-j\omega t} dt}$$

- CHI-SQUARE DISTRIBUTION (WIKIPEDIA)

X_i - K INDEPENDENT NORMALLY DISTRIBUTED RV WITH MEAN 0 AND VARIANCE 1

$Q = \sum_{i=1}^{K} X_i^2$ Q - RV DISTRIBUTED ACCORDING CHI-SQUARE DISTR WITH K-DEGREES OF FREEDOM

PDF: $$f(x;k) = \begin{cases} \frac{1}{2^{k/2}\Gamma(k/2)} x^{(k/2)-1} e^{-x/2} & x > 0 \\ 0 & x \leq 0 \end{cases}$$

$$\boxed{\Gamma(\alpha) = \int_0^\infty t^{\alpha-1} e^{-t} dt}$$

$$\hat{p}_\gamma(s) = \int_{-\infty}^{\infty} p_\gamma(\gamma)\, e^{-s\gamma} d\gamma = M(-s)$$

FOR.

$$\mathcal{F}\{p_\gamma(\gamma)\} = \tilde{P}_\gamma(j\omega) = \int_{-\infty}^{\infty} p_\gamma(\gamma)\, e^{-j\omega\gamma} d\gamma = \underline{M_\gamma(-j\omega)}$$

$$p_\gamma(\gamma) = \mathcal{F}^{-1}\{M(j\omega)\} = \frac{1}{2\pi} \int_{-\infty}^{\infty} M(j\omega) \cdot e^{j\omega\gamma} d\omega$$

- $$M_\gamma(j\omega) = \int_{-\infty}^{\infty} p_\gamma(\gamma) \cdot e^{j\omega\gamma} d\gamma = \tilde{p}^{*}_\gamma(j\omega),$$

[illegible]

$$\left[\mathcal{F}\left\{p_\gamma(\gamma)\right\}\right]^* = M_\gamma(jw)$$

$$P_s(\epsilon/\gamma) = \int_0^\infty g(\mu_1)\, p_{v_1/\gamma}(\mu_1)\, d\mu_1 = \int_0^\infty \frac{1}{2\pi} \int_{-\infty}^{\infty} M_{v_1/\gamma}(jw)\, e^{\ominus jw\mu_1} g(\mu_1)\, d\mu_1\, dw$$

- NON-CENTRAL CHI-SQUARE

$$Q = \sum_{i=1}^{k} \left(\frac{x_i}{\sigma_i}\right)^2$$

x_i – NORMAL VARIABLES WITH MEANS μ_i AND VARIANCES σ_i^2

$$f_x(x;k,\lambda) = \frac{1}{2} e^{-(x+\lambda)/2} \left(\frac{x}{\lambda}\right)^{k/4-1/2} I_{k/2-1}\left(\sqrt{\lambda x}\right)$$

$$MGF(x) = \frac{e^{\lambda t/1-2t}}{(1-2t)^{k/2}} \qquad 2t<1$$

$$P_s(\epsilon/\gamma) = \int_0^\infty \frac{1}{2\pi} \int_{-\infty}^{\infty} M_{v_1/\gamma}(jw)\, e^{-jw\mu_1} g(\mu_1)\, dw\, d\mu_1$$

$$M_{v_1} \triangleq \int_0^\infty M_{v_1/\gamma}(s)\, p_\gamma(\gamma)\, d\gamma = f_1(s)\, M_\gamma(f_2(s))$$

$$\boxed{P(A,B) = P(B)\cdot P(A/B)}$$

- AVERAGING $P_s(\epsilon/\gamma)$ OVER γ

$$P(\epsilon) = \int_0^\infty \frac{1}{2\pi} \int_{-\infty}^{\infty} \Big[\underbrace{\int_0^\infty M_{v_1/\gamma}(jw)\, p_\gamma(\gamma)\, d\gamma}_{M_{v_1}(jw)} \Big] e^{-jw\mu_1} g(\mu_1)\, dw\, d\mu_1$$

$$P(\epsilon) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_0^\infty \underline{M_{v_1}(jw)}\, e^{-jw\mu_1} g(\mu_1)\, dw\, d\mu_1 =$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} f_1(jw)\, M_\gamma(f_2(jw)) \left[\int_0^\infty e^{-jw\mu_1} g(\mu_1)\, d\mu_1\right] dw$$

· FOR GGC

$$\boxed{\gamma = \left[\frac{1}{\sqrt{L}} \sum_{l=1}^{L} \sqrt{\gamma_l}\right]^2}$$

$$x \triangleq \sqrt{\gamma} = \frac{1}{\sqrt{L}} \sum_{l=1}^{L} \sqrt{\gamma_l} = \frac{1}{\sqrt{L}} \sum_{l=1}^{L} x_l$$

$$M_x(s) = \prod_{l=1}^{L} M_{x_l}\left(\frac{s}{\sqrt{L}}\right)$$

$$\boxed{P_b(\epsilon) = \int_0^{\infty} P_b(\epsilon/x)\, p_x(x)\, dx}$$

$/ \mathcal{F}\{\ \}$

$$\boxed{P_b(\epsilon/x) = \int_{\xi_1}^{\xi_2} c_2 h(\xi)\, e^{-a_2 g(\xi) x^2} d\xi}$$

PARSEVALOVA TEOREMA

$$[\text{eff.} f(t)]^2 = \frac{1}{T} \int_{-T/2}^{T/2} f^2(t)\, dt = \sum_{n=-\infty}^{\infty} |F_n|^2$$

$$\mathcal{F}\{P_b(\epsilon)\} = \int_0^{\infty} \int_{-\infty}^{\infty} P_b(\epsilon/x)\, p_x(x)\, e^{-j\omega t}\, dt\, dx$$

$$\mathcal{F}\{P_b(\epsilon/x)\} = G(j\omega)$$

$$\boxed{\int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |X(f)|^2 df}$$

PARSEVAL'S THEOREM

$\bigcirc\!\!\!\!?$

$$P_b(\epsilon) = \frac{1}{2\pi} \int_{-\infty}^{\infty} G(j\omega) \cdot \cancel{M_x} M_x(j\omega)\, d\omega = \frac{1}{2\pi} \int_0 G(j\omega) M_x(j\omega)\, d\omega$$

$$P_B(\varepsilon) = \int_0^\infty P_B(\varepsilon|x)\, p_x(x)\, dx$$

$$P_B(\varepsilon|x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} G(j\omega)\, e^{+j\omega x}\, d\omega$$

$$P_B(\varepsilon) = \frac{1}{2\pi}\int_0^\infty \left[\int_{-\infty}^{\infty} G(j\omega)e^{j\omega x}\,d\omega\right] p_x(x)\,dx = \frac{1}{2\pi}\int_{-\infty}^{\infty} G(j\omega)\,d\omega \underbrace{\int_{-\infty}^{\infty} e^{j\omega x} p_x(x)\,dx}_{M_x(j\omega)}$$

$$\boxed{P_B(\varepsilon) = \frac{1}{2\pi}\int_{-\infty}^{\infty} G(j\omega)M_x(j\omega)\,d\omega}$$

POKAZANO !!!

$$\boxed{\left(\frac{f(x)}{g(x)}\right)' = \frac{f'g - fg'}{g^2} \qquad \frac{\cos\cdot\cos + \sin\sin}{\cos^2} = \frac{1}{\cos}}$$

$$P_B(\varepsilon) = \frac{1}{\pi}\int_0^\infty \mathrm{Re}[G(j\omega)M_x(j\omega)]\,d\omega$$

CHANGE OF VARIABLES:

$\theta = \mathrm{arctg}(\omega) \qquad tg\,\theta = \omega \qquad \left(\frac{\sin\theta}{\cos\theta}\right)' d\theta = d\omega$

$$\sin\theta\cdot\frac{\sin\theta}{\cos^2\theta} + \frac{\cos\theta}{\cos\theta} = 1 + \frac{\sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$\boxed{\frac{d\theta}{\cos^2\theta} = d\omega}$$

$\omega = 0 \Rightarrow \theta = 0$

$\omega = \infty \Rightarrow \theta = \pi/2$

$$P_B(\varepsilon) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \mathrm{Re}[G(j\,tg\theta)M_x(j\,tg\theta)]\,\frac{d\theta}{\cos^2\theta}$$

$$\cos^2\theta = \cos\theta\cdot\cos\theta = \frac{1}{2}[\cos(\theta+\theta) + \overset{1}{\cancel{\cos(\theta-\theta)}}] = \frac{1}{2}[1+\cos 2\theta]$$

$\cos(\alpha+\beta) = \cos\alpha\cdot\cos\beta - \sin\alpha\cdot\sin\beta$

$\cos(\alpha-\beta) = \cos\alpha\cdot\cos\beta + \sin\alpha\cdot\sin\beta$

$\cos^2 2\theta = 1 - \sin^2 2\theta$

$\beta = \alpha$

$1 + \cos(2\alpha) = 2\cos\alpha\cdot\cos\alpha \Rightarrow \cos^2\alpha = \frac{1}{2}[1+\cos 2\alpha]$

~~$\cos^2\theta = \frac{1}{2}[1+$~~ $\quad 2\cos^2\alpha = 1 + \cos 2\alpha$

$1 = \sin^2\alpha + \cos^2\alpha \qquad \cancel{2}\cos^2\alpha = \sin^2 + \cancel{\cos^2} + \cos 2\alpha$

$\cos^2\alpha = \sin^2 + \cos 2\alpha = 1 - \cos 2\alpha$

$\sin^2\alpha = \frac{1}{2}[1 - \cos 2\alpha]$

$\cos^2\alpha = 1 - \sin^2\alpha = 1 - \frac{1}{2} - \frac{\cos 2\alpha}{2}$ ~~[illegible]~~

$= \frac{1}{2}[1 - \cos 2\alpha]^2$

40a

$$\sin(\alpha+\alpha) = \sin\alpha\cdot\cos\alpha + \sin\alpha\cdot\cos\alpha$$
$$\sin(\alpha-\alpha) = \sin\alpha\cdot\cos\alpha - \sin\alpha\cdot\cos\alpha$$

$$\boxed{\sin\alpha\cdot\cos\alpha = \frac{1}{2}\sin(2\alpha)} \qquad \cos^2\alpha = \frac{1}{2}\left[1-\cos 2\alpha\right]$$

$$\frac{1}{\cos^2(\theta)} = \frac{2\cdot tg\theta}{\sin 2\theta} = \frac{2\cdot\sin\theta}{\sin 2\theta\cdot\cos\theta}$$

$$\sin 2\theta = 2\cdot tg\theta\cdot\cos^2\theta$$

$$\cancel{2}\sin\theta\cdot\cos\theta = \cancel{2}\, tg\theta\cdot\cos^2\theta \qquad \sin\theta\cdot\cos\theta = \frac{\sin\theta}{\cancel{\cos\theta}}\cdot\cos^2\theta$$

$$\cos^2\theta = \frac{\sin\theta\cdot\cos\theta}{tg\theta} = \frac{\sin\theta\cdot\cos\theta}{\frac{\sin\theta}{\cos\theta}}$$

$$\cos^2\theta = \frac{\sin\theta}{\sin\theta}\cdot\cos^2\theta = \frac{\sin\theta\cdot\cos\theta}{\sin\theta}\cdot\cos\theta =$$
$$= \frac{\sin\theta\cdot\cos\theta}{\frac{\sin\theta}{\cos\theta}} = \frac{\sin\theta\cdot\cos\theta}{tg\theta} = \frac{1}{2}\,\frac{\sin 2\theta}{tg\theta}$$

$$\boxed{\cos^2\theta = \frac{\sin 2\theta}{2 tg\theta}}$$

$$P_b(\varepsilon) = \frac{2}{\pi}\int_0^{\pi/2}\frac{1}{\sin(2\theta)}\,Re\left[tg\theta\cdot G(j\,tg\theta)\cdot M_x(j\,tg\theta)\right]d\theta$$

$$P_b(\varepsilon|x) = \int_{\xi_1}^{\xi_2} C_2\,h(\xi)\exp\left[-a_2 g(\xi)x^2\right]d\xi$$

VO KNIGATA
ϵ ,, x^2

$$G(j\omega) = \mathcal{F}\{P_b(\varepsilon|x)\} = \int_{\xi_1}^{\xi_2} C_2\,h(\xi)\int_0^{\infty} e^{-a_2 g(\xi)x^2 \ominus j\omega x}\,dx\,d\xi$$

$$\int_0^{\infty} e^{-a_2 g(\xi)x^2 + j\omega x}\,dx = \frac{1}{2a_2 g(\xi)}\left\{\sqrt{\pi a_2 g(\xi)}\;e^{\frac{(j\omega)^2}{4a_2 g(\xi)}} + j\omega\,{}_1F_1\left[1, \frac{3}{2}; \frac{(j\omega)^2}{4a_2 g(\xi)}\right]\right\}$$

${}_1F_1(a,b;c)$ – CONFLUENT HYPERGEOMETRIC FUNCTION

MAXIMAL RATIO COMBINING

259 ÷ 432 KDKFGN
268 – 441 KGAFDP
446 – 4.80 KBCJHB

CH.7 OPTIMUM RECEIVERS FOR FADING CHANNELS

$$s_k(t) = Re\{\tilde{s}_k(t)\} = Re\{\tilde{S}_k(t) \cdot e^{j2\pi f_c t}\}$$

$\tilde{s}_k(t)$ – KOMPLEX BANDPASS SIGNAL

$\tilde{S}_k(t)$ – KOMPLEX BASEBAND – " –

$$r_l(t) = Re\{\alpha_l \tilde{s}_k(t-\tau_l) e^{j\theta_l} + \tilde{n}_l(t)\} =$$

$$= Re\{\alpha_l \tilde{S}_k(t-\tau_l) e^{j2\pi f_c t + \theta_l} + \tilde{N}_l(t) e^{j2\pi f_c t}\} \quad \underline{l = 1, 2, \dots L_p}$$

JEDUCHIM: (KONSEQUANZE)

$$\boxed{y(k) = \sum_{n=1}^{N} h(n) \cdot s(k-n)}$$

$$y(t) = \sum_{k=1}^{N} a_k \cdot s(t - k \cdot \tau) = \sum_{k=1}^{N} a_k\, Re\{\tilde{s}(t-k\tau) \cdot e^{j2\pi f_c (t-k\tau)}\}$$

$$\underline{s(t) = Re\{\tilde{s}(t) \cdot e^{j2\pi f_c t}\}}$$

$$y(t) = Re\left\{\sum_{k=1}^{N} a_k\, \tilde{s}(t-k\tau)\, e^{-j2\pi f_c k\tau}\, e^{j2\pi f_c t}\right\}$$

$$\tilde{y}(t) = \sum_{k=1}^{N} a_k e^{-j2\pi f_c k\tau}\, \tilde{s}(t-k\tau) = \sum_{k=1}^{N} \tilde{a}_k\, \tilde{s}(t-k\tau)$$

$c(t, \tau_k)$

$$\boxed{c(t, \tau_k) = \sum_{k=1}^{N} \underbrace{a_k e^{-j2\pi f_c k\tau}}_{\tilde{a}_k(t, \tau_k)} \delta(\tau - \tau_k)} \qquad \underline{\tau_k = \tau_k(t)}$$

$\{\tilde{N}_l(t)\}_{l=1}^{L_p}$ – STATISTICALLY INDEPENDENT AWGN EACH WITH PSD $\underline{\underline{2N_l}}$ W/Hz

$\{\alpha_l\}_{l=1}^{L_p}$ $\{\theta_l\}_{l=1}^{L_p}$ $\{\tau_l\}_{l=1}^{L_p}$ RANDOM CHANNEL AMPLITUDES, PHASES AND DELAYS

OPTIMUM RECEIVER

$$p\left(s_k(t) \,\Big|\, \{r_l(t)\}\Big|_{l=1}^{L}\right) \qquad k=1,2,\dots M$$

$\bar{s}_k(t)$ – K-TH COMPLEX BASEBAND SIGNAL CHOSEN FROM M EQUIPROBABLE MESSAGE WAVEFORMS

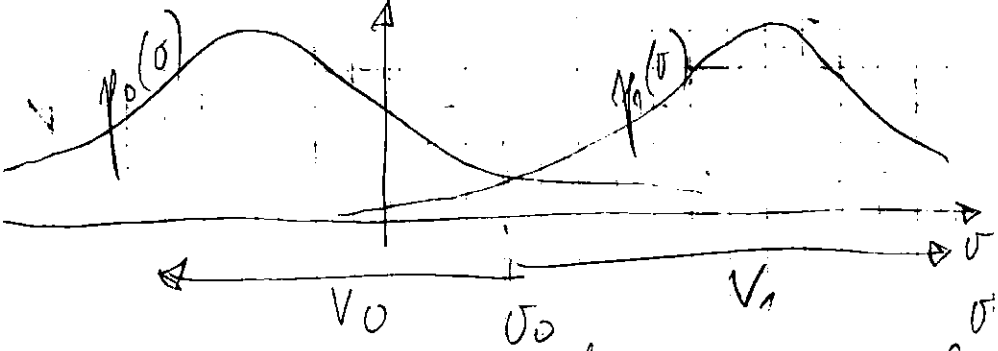
- BAJESOV KRITERIUM, ODLUČUVANJE

$P(H_0)$ $P(H_1)$

$p_0(\sigma)$ $p_1(\sigma)$

$$K = \begin{bmatrix} K_{0,0} & K_{0,1} \\ K_{1,0} & K_{1,1} \end{bmatrix}$$

$$P(H_1/H_0) = Q_{1,0} = \int_{V_1} p_0(\sigma)\,d\sigma = \int_{\sigma_0}^{\infty} p_0(\sigma)\,d\sigma$$



$$P(H_0/H_1) = Q_{0,1} = \int_{V_0} p_1(\sigma)\,d\sigma = \int_{-\infty}^{\sigma_0} p_1(\sigma)\,d\sigma$$

$$P(\varepsilon) = P(H_0)\cdot P(H_1/H_0) + P(H_1)\cdot P(H_0/H_1)$$

- USVOEN SREDEN RIZIK

$$\bar{V}_0 = P(H_0)\left(K_{1,0}\,Q_{1,0} + K_{0,0}\,Q_{0,0}\right)$$

$$\bar{V}_1 = P(H_1)\left(K_{0,1}\,Q_{0,1} + K_{1,1}\,Q_{1,1}\right)$$

$$\boxed{\begin{aligned} K = \bar{V}(\sigma) = {} & P(H_0)\left(K_{1,0}\int_{\sigma_0}^{\infty} p_0(\sigma)\,d\sigma + K_{0,0}\int_{-\infty}^{\sigma_0} p_0(\sigma)\,d\sigma\right) + \\ & P(H_1)\cdot\left(K_{0,1}\int_{-\infty}^{\sigma_0} p_1(\sigma)\,d\sigma + K_{1,1}\int_{\sigma_0}^{\infty} p_1(\sigma)\,d\sigma\right) \end{aligned}}$$

$$\frac{\partial K}{\partial \sigma_0} = P(H_0)\left[-K_{1,0}\,p_0(\sigma_0) + K_{0,0}\,p_0(\sigma_0)\right] + P(H_1)\left[K_{0,1}\,p_1(\sigma_0) + K_{1,1}\,p_1(\sigma)\right]$$

$$\int_a^{\infty} x\,dx = \frac{x^2}{2}\Big|_{a^2}^{\infty}$$

$$\frac{d}{da}\left[\int_0^a x\,dx\right] = \frac{d}{da}\left[\frac{x^2}{2}\Big|_0^a\right] = \frac{d}{da}\left[\frac{a^2}{2}\right] = \frac{2a}{2} = a$$

$$\frac{d}{da}\left[\int_a^0 x\,dx\right] = \frac{d}{da}\left[\frac{x^2}{2}\Big|_a^0\right] = -\frac{d}{da}\left[\frac{a^2}{2}\right] = -a$$

$$\boxed{\frac{\partial K}{\partial v_0} = 0} \qquad P(H_0)\left[K_{1,0}\,\varphi_0(v_0) - K_{0,0}\,\varphi_0(v_0)\right] = P(H_1)\left[K_{0,1}\,\varphi_1(v_0) - K_{1,1}\,\varphi_1(v_0)\right]$$

→ SREDNIOT BAJASOV RIZIK SAKAME DA BIDE MINIMIZIRAN

$$\boxed{\frac{P(H_0)}{P(H_1)} = \frac{K_{0,1}\,\varphi_1(v_0) - K_{1,1}\,\varphi_1(v_0)}{K_{1,0}\,\varphi_0(v_0) - K_{0,0}\,\varphi_0(v_0)}}$$

$$\frac{P(H_0)}{P(H_1)} = \frac{\varphi_1(v_0)}{\varphi_0(v_0)}\,\frac{K_{0,1} - K_{1,1}}{K_{1,0} - K_{0,0}} \qquad \frac{\varphi_0(v_0)}{\varphi_1(v_0)} = \frac{P(H_1)}{P(H_0)}\,\frac{K_{0,1} - K_{1,1}}{K_{1,0} - K_{0,0}}$$

$$\boxed{\frac{\varphi_1(v_0)}{\varphi_0(v_0)} = \frac{P(H_0)}{P(H_1)}\,\frac{K_{1,0} - K_{0,0}}{K_{0,1} - K_{1,1}}} = \lambda_0$$

$$\boxed{\frac{\varphi_1(v_0)}{\varphi_0(v_0)} \gtrless \lambda_0}$$

BAJASOV KRITERIUM

$$\frac{\varphi_1(v)}{\varphi_0(v)} > \lambda_0$$ REALIZIRANA E H_1 (OBLAST V_1)

$$\frac{\varphi_1(v)}{\varphi_0(v)} < \lambda_0$$ REALIZIRANA E H_0 (OBLAST V_0)

K - SIMETRIČNA $\quad K_{0,1} = K_{1,0} \quad K_{1,1} = K_{0,0} \Rightarrow$

$$\frac{\varphi_1(v_0)}{\varphi_0(v_0)} \gtrless \frac{P(H_0)}{P(H_1)} = \lambda_0$$

AKO $P(H_0) = P(H_1) \Rightarrow$

$$\boxed{\frac{\varphi_1(v_0)}{\varphi(v_0)} \gtrless 1}$$

$>$ ZA H_1

$<$ ZA H_0

$P(H_0) = P(H_1) \Rightarrow$ OPTIMALNIOT PRAG E VO PRESEKOT NA $\varphi_1(v)$ SO $\varphi_0(v)$

$$\boxed{\frac{\varphi_1(v)}{\varphi_0(v)} \gtrless 1}$$

$>$ ZA H_1

$<$ ZA H_0

$P(H_0) > P(H_1)$ $\quad v_0$ ODI NANAD V_1

MAKSIMIZ P(E)

TI : MINMAX KRITERIJUM ZA ODLUČIVANJE

$\boxed{P(H_0) = \xi}$ $\boxed{P(H_1) = 1 - \xi}$ $0 \le \xi \le 1$

$$K = \xi\left(K_{1,0}Q_{1,0} + K_{0,0}Q_{0,0}\right) + (1-\xi)\left(K_{0,1}Q_{0,1} + K_{1,1}Q_{1,1}\right)$$

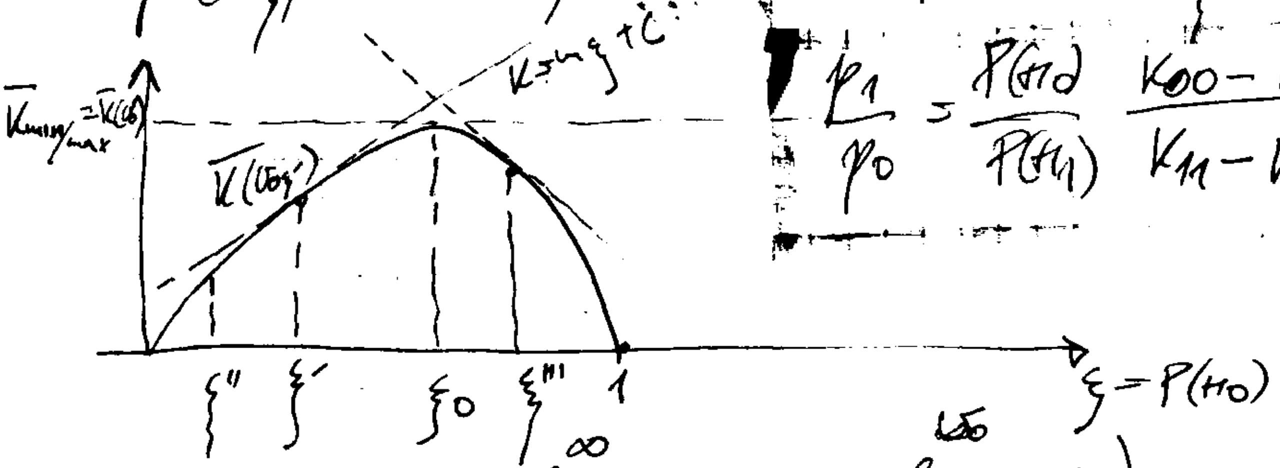
$$= \xi\underbrace{\left(K_{10}Q_{10} + K_{00}Q_{00} - K_{01}Q_{01} - K_{11}Q_{11}\right)}_{m} + \underbrace{K_{01}Q_{01} + K_{11}Q_{11}}_{C}$$

$\boxed{K = m \cdot \xi + C}$ VKUREN SKEPEN KIZIK

PRIMER: $\xi = \xi', \xi'', \xi''' \dots \xi^{(n)}$

$v_0' = ?$ $\boxed{\overline{K}(v_0') = K_{min}}$

$$\frac{p_1(v_0')}{p_0(v_0')} = \frac{P(H_0)}{P(H_1)}\left(\frac{K_{01} - K_{11}}{K_{10} - K_{00}}\right)^{-1} = \lambda_0 \Rightarrow v_0' = \dots$$



$$\frac{p_1}{p_0} = \frac{P(H_0)}{P(H_1)}\,\frac{K_{00} - K_{10}}{K_{11} - K_{01}}$$

$$\overline{K}(v) = P(H_0)\left(K_{10}\int_{v_0}^{\infty} p_0(v)dv + K_{00}\int_{-\infty}^{v_0} p_0(v)dv\right) +$$
$$P(H_1)\left(K_{01}\int_{-\infty}^{v_0} p_1(v)dv + K_{11}\int_{v_0}^{\infty} p_1(v)dv\right)$$

$K_{00} = K_{11} = 1$ $K_{10} = K_{01} = 0$

$$\overline{K}(v) = \xi\int_{-\infty}^{v_0} p_0(v)dv + (1-\xi)\int_{v_0}^{\infty} p_1(v)dv$$

$$p_0(v) = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x+A)^2}{2\sigma^2}}$$

$$p_1(v) = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-A)^2}{2\sigma^2}}$$

$$\frac{p_1(v_{0\xi'})}{p_0(v_{0\xi'})} = \frac{\xi'}{1-\xi'} = \lambda_0$$

$\nearrow$ FOR $K_{00}=K_{11}=1$ ŠTO NE6 LOGIČNO!!

$$\boxed{\frac{\xi'}{1-\xi} \frac{(K_{00}-K_{10})}{(K_{11}-K_{01})}}$$

VAKA E POLOGIČNO

$$\left[e^{-\frac{(x+A)^2}{2\sigma^2}} \cdot e^{+\frac{(x-A)^2}{2\sigma^2}}\right]^{-1} = \frac{\xi'}{1-\xi'}$$

$$\left[e^{\frac{-x^2 - 2Ax - A^2 + x^2 - 2Ax + A^2}{2\sigma^2}}\right]^{-1} = e^{-\frac{4Ax}{2\sigma^2}} = e^{-\frac{2Ax}{\sigma^2}} \quad x = v_{0\xi'}$$

$$e^{+\frac{2A v_{0\xi'}}{\sigma^2}} = \frac{\xi}{1-\xi'} \qquad \frac{2A v_{0\xi'}}{\sigma^2} = \ln \frac{\xi'}{1-\xi'}$$

$$\boxed{v_{0\xi'} = \frac{\sigma^2}{2A} \ln \frac{\xi'}{1-\xi'}}$$

$$\bar{K}(v_{0\xi'}) = \xi' \int_{-\infty}^{v_{0\xi'}} p_0(v)\,dv + (1-\xi') \int_{v_{0\xi'}}^{\infty} p_1(v)\,dv$$

$$\bar{K}(v_{0\xi_0}) = \bar{K}_{min/max} \qquad v_{0\xi_0} = v_0$$

$$\bar{K}(v_0) = \bar{K}_{min/max} \Rightarrow m = 0$$

$$\boxed{\bar{K} = \underset{\to 0}{m} \cdot \xi + c = c}$$

RIZIKOT NE ZAVISI OD ξ ⇒ TOJ E OPTIMIZEN PRAG.

$$m = 0 \Rightarrow$$

$$K_{10} \int_{v_0}^{\infty} p_0(v)\,dv + K_{00} \int^{v_0} p_0(v)\,dv = K_{01} \int_{-\infty}^{v_0} p_1(v)\,dv + K_{11} \int_{v_0}^{\infty} p_1(v)\,dv$$

$$\frac{p_1(v)}{p_0(v)} \gtrless \frac{P(H_0)_m (K_{10}-K_{00})}{(1-P(H_0)_m)(K_{01}-K_{11})} = \lambda_0$$

$P(H_0)_m$ ⇒ APRIORNA VEROJATNOST NA HIPOTEZATA H_0 = KOJA GO ~~MAKSIMIZIRA~~ DADODIOT ČITEMUM

$$K_{00} = K_{11} = 1 \qquad K_{10} = K_{01} = 0 \Rightarrow$$

$$\int_{-\infty}^{v_0} p_0(v)\,dv = \int_{v_0}^{\infty} p_1(v)\,dv$$

$$p_0(v) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{\frac{v-\cancel{(\mu}\,\mu_0)^2}{2\sigma^2}} \qquad p_1(v) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{(v-\mu_1)^2}{2\sigma^2}}$$

$$\frac{p_1(v)}{p_0(v)} = \frac{x\cdot B}{1-x} \qquad e^{+\frac{(v-\mu_0)^2}{2\sigma^2}} \cdot e^{-\frac{(v-\mu_1)^2}{2\sigma^2}} = \frac{x\cdot B}{1-x}$$

$$e^{\frac{\cancel{v^2}-2v\mu_0+\mu_0^2-\cancel{v^2}+2\mu_1 v-\mu_1^2}{\cancel{2\sigma^2}}} = \frac{x\cdot B}{1-x}$$

$$\exp\left[\mu_0^2-\mu_1^2-2v\mu_0+2v\mu_1\right] = \frac{x\cdot B}{1-x}$$

$$e^{\mu_0^2-\mu_1^2}\cdot e^{2v(\mu_1-\mu_0)} = \frac{x\cdot B}{1-x} \qquad e^{2v(\mu_1-\mu_0)} = \frac{x\cdot B}{1-x}\, e^{\mu_1^2-\mu_0^2}$$

$$\cancel{\ln} = \quad 2v\cdot(\mu_1-\mu_0) = \ln\frac{x\cdot B}{1-x} + \mu_1^2-\mu_0^2$$

$$\boxed{v = \frac{1}{2(\mu_1-\mu_0)}\ln\frac{x\cdot B}{1-x} + \frac{\mu_1+\mu_0}{2}} \qquad \boxed{B = \frac{K_{00}-K_{10}}{K_{11}-K_{01}}}$$

ZA SIMETRIČNA MATRICA NA CENI $K_{00}=K_{11}$ $K_{10}=K_{01}$

$$\boxed{B=1}$$

$$\iiint p\left(\{r_l(t)\}_{l=1}^{L} \,\middle|\, s_k(t), \{a_l\}_{l=1}^{L}, \{\theta_l\}_{l=1}^{L}, \{\tau_l\}_{l=1}^{L}\right)\cdot \text{(**)}$$

$$r_l(t) = \mathrm{Re}\left\{\alpha_l\cdot s_k(t-\tau_l)\cdot e^{j\theta_l} + \tilde{n}_l(t)\right\}$$

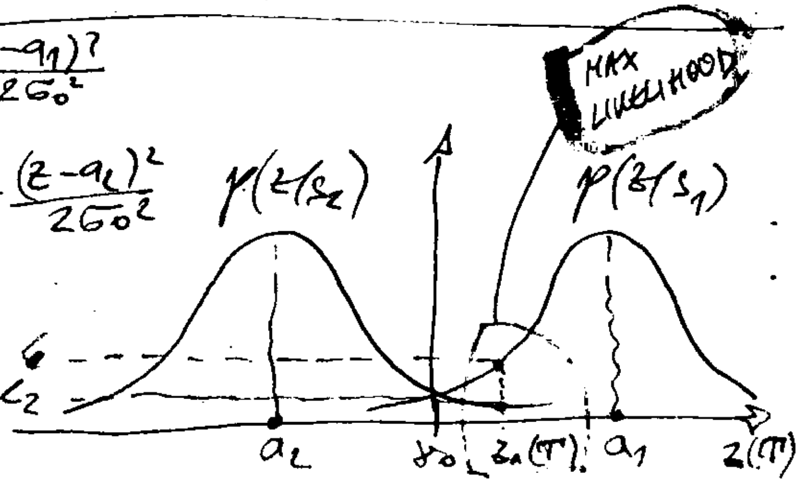
$$r_l(t) = \mathrm{Re}\left\{\alpha_l\cdot \tilde{S}_k(t-\tau_l)\cdot e^{j2\pi f_c t+\alpha} + \tilde{N}_l(t)\, e^{j2\pi f_c t+\theta_l}\right\}$$

$$\text{(**)} = p\left(\{a_l\}_{l=1}^{L}, \{\theta_l\}_{l=1}^{L}, \{\tau_l\}_{l=1}^{L}\right) d\{\alpha_l\}_{l=1}^{L}\, d\{\theta_l\}_{l=1}^{L}\, d\{\tau_l\}_{l=1}^{L}$$

SKLAR (MAX-LI~~KE~~HOOD)

$$p(z|s_1) = \frac{1}{\sigma_0\sqrt{2\pi}}\, e^{-\frac{(z-a_1)^2}{2\sigma_0^2}}$$

$$p(z|s_2) = \frac{1}{\sigma_0\sqrt{2\pi}}\, e^{-\frac{(z-a_2)^2}{2\sigma_0^2}}$$



$$z(T) \underset{H_2}{\overset{H_1}{\gtrless}} \gamma \qquad \frac{p(z/s_1)}{p(z/s_2)} \underset{H_2}{\overset{H_1}{\gtrless}} \frac{P(s_2)}{P(s_1)}$$

$P(s_1), P(s_2)$ - APRIORI PROBABILITIES THAT s_1 AND s_2, RESPECTIVELY, ARE TRANSMITTED

$$z(T) \underset{H_2}{\overset{H_1}{\gtrless}} \frac{a_1+a_2}{2} = \gamma_0$$

check figure on previous page!

$$\cdot z_a(T) \qquad \left[p(z_a|s_1) \overset{H_1}{>} p(z_a|s_2) \qquad P(s_1) = P(s_2) \right]$$

- DETECTOR that minimizes error probability (for equaly probable signal classes) is KNOWN AS MAXIMUM LIKELIHOOD detector.

SIMON CONTINUE...

$$p\left(\{r_l(t)\}_{l=1}^{L_p} \middle| s_k(t)\right) \qquad k=1,2,\dots M$$

CHOSE $s_k(t)$ CORRESPONDING TO THE LARGEST OF THE CONDITIONAL PROBABILITIES (LIKELIHOODS)

$$\underbrace{\text{(ML) maximum likelihood decision rule}}$$

$$\boxed{P(A,B) = P(A)\cdot P(B/A)}$$

$$\boxed{p(x) = \int_{-\infty}^{\infty} p(x,y)\,dy} \qquad p(y) = \int_{-\infty}^{\infty} p(x,y)\,dx$$

$$\sum_{i=1}^{m} P(A_i, B_j) = P(B_j) \qquad \sum_{i=1}^{m} P(A_i, B_j) = \sum_{i=1}^{m} P(B_j)\cdot P(A_i/B_j) = *$$

stat nezavisni $P(A_i/B_j) = P(A_i)$

$$* = P(B_j)\sum_{i=1}^{m} P(A_i) = P(B_j)$$

⑦.1 CASE OF KNOWN AMPLITUDES, PHASES AND DELAYS: COHERENT DETECTION

CONDITIONAL PROBABILITY $p\left(\{r_l(t)\}_{l=1}^{L} \middle| s_k(t), \{a_l\}_{l=1}^{L}, \{\theta_l\}_{l=1}^{L}, \{\tau_l\}_{l=1}^{L}\right)$

IS JOINT GAUSSIAN P.D.F

$$p = \prod_{l=1}^{L_p} K_l \exp\left[\frac{1}{-2N_l}\int_{\tau_l}^{T_s+\tau_l} \left|\tilde{r}_l(t) - a_l \tilde{s}_k(t-\tau_l)e^{j\theta_l}\right|^2 dt\right.$$

$$p = \prod_{l=1}^{4} k_l \exp\left[-\frac{1}{2N_l}\int_{\tau_l}^{T_s+\tau_l} \left|\tilde{r}_1(t) - a_l\,\tilde{s}_l(t-\tau_l)\,e^{i\theta_l}\right|^2 dt\right]$$

- NAKAGAMI-q (HOYT) MODEL

$$p_\alpha(\alpha) = \frac{(1+q^2)\alpha}{q\Omega}\exp\left[-\frac{(1+q^2)^2\alpha^2}{4q^2\Omega}\right] I_0\left(\frac{(1-q^4)\alpha^2}{4q^2\Omega}\right)$$

$$\boxed{\gamma = \alpha^2\cdot\frac{E_s}{N_0}} \qquad \bar{\gamma} = \Omega\frac{E_s}{N_0} \qquad \boxed{\frac{E_s}{N_0} = \frac{\bar{\gamma}}{\Omega}}$$

$$\gamma = \alpha^2\frac{\bar{\gamma}}{\Omega} \qquad p(\gamma) = \frac{p(\alpha)}{\frac{d\gamma}{d\alpha}}\Bigg/ \alpha = \sqrt{\frac{\Omega\gamma}{\bar{\gamma}}}$$

$$\frac{d\gamma}{d\alpha} = 2\alpha\frac{\bar{\gamma}}{\Omega}$$

$$\boxed{\alpha = \sqrt{\frac{\Omega\gamma}{\bar{\gamma}}}}$$

$$\boxed{p_\gamma(\gamma) = \frac{p_\alpha(\alpha)}{2\cdot\sqrt{\frac{\Omega\gamma}{\bar{\gamma}}}\cdot\frac{\bar{\gamma}}{\Omega}} = \frac{p(\alpha)}{2\sqrt{\cdot\frac{\gamma\bar{\gamma}}{\Omega}}}}$$

$$p_\gamma(\gamma) = \frac{1}{2\sqrt{\frac{\gamma\bar{\gamma}}{\Omega}}}\cdot\frac{(1+q^2)\sqrt{\frac{\Omega\gamma}{\bar{\gamma}}}}{q\Omega}\exp\left[-\frac{(1+q^2)^2\frac{\Omega\gamma}{\bar{\gamma}}}{4q^2\Omega}\right] I_0\left(\frac{(1-q^4)\frac{\Omega\gamma}{\bar{\gamma}}}{4q^2\Omega}\right)$$

$$\boxed{p_\gamma(\gamma) = \frac{1+q^2}{2q\cdot\bar{\gamma}}\exp\left[-\frac{(1+q^2)^2\gamma}{4q^2\bar{\gamma}}\right] I_0\left[\frac{(1-q^4)\gamma}{4q^2\bar{\gamma}}\right]}$$

$$MGF = M_\gamma(\gamma) = \int_0^\infty e^{s\gamma}\cdot p_\gamma(\gamma)\,d\gamma$$

MAPLE:

$$M_\gamma(\gamma) = \frac{\cancel{j}(1+q^2)}{\sqrt{-2q^2 - 1 - 2s\bar{\gamma} - q^4 - 4sq^2\bar{\gamma} - 2q^4 s\cdot\bar{\gamma} - 4s^2q^2\bar{\gamma}^2}}$$

$$= \frac{1+q^2}{\sqrt{1+2q^2+q^4+2s\bar{\gamma}+4sq^2\bar{\gamma}+2s\bar{\gamma}q^4+4s^2q^2\bar{\gamma}^2}}$$

- SIMON:

$\boxed{3163971}$

$$M_\gamma(s) = \left[1 - 2s\bar\gamma + \frac{(2s\bar\gamma)^2 q^2}{(1+q^2)^2}\right]^{-1/2} =$$

$$= \left[\frac{(1-2s\bar\gamma)(1+2q^2+q^4) + 4 s^2 \gamma^2 q^2}{(1+q^2)^2}\right]^{-1/2} =$$

$$= \frac{1+q^2}{\sqrt{1+2q^2+q^4-2s\gamma-\underline{4s\bar\gamma q^2-2s\bar\gamma q^4}+4s^2\bar\gamma^2 q^2}}$$

$$\boxed{M_\gamma(-s) = \frac{1+q^2}{\sqrt{1+2q^2+q^4-2s\gamma-4s\bar\gamma q^2-2s\bar\gamma q^4+4s^2\bar\gamma^2 q^2}}}$$

$$p_{\cancel{x}}(\cancel{x}) = \frac{2m^m \cdot x^{2m-1}}{\Gamma(m)\cdot\Omega^{m-1}}\, e^{-\frac{\cancel{m}\cdot x^2}{\Omega}}$$

$$p_\gamma(\gamma) = \frac{m^m\cdot\gamma^{m-1}}{\Gamma(m)\cdot\bar\gamma^m}\, e^{-\frac{m\gamma}{\bar\gamma}}$$

$$\gamma_k = \sum_{i=1}^{k} x_i^2 \quad \rightarrow \quad \gamma_k \rightarrow \text{FOLOWS GAMA DISTRIBUTION}$$

$$\boxed{z^2 = x^2 + y^2}$$

$$z = x\cos\varphi$$
$$z = y\sin\varphi$$

$$z^2 = x^2\cos^2\varphi + y^2 + \sin^2\varphi = x^2 + y^2$$

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{x^2}{2\sigma^2}} \qquad p(z) = ? \qquad \times$$

$$\cancel{zz}\quad p(x,y) = p(x)\cdot p(y) = \frac{1}{\sigma^2 2\pi}\, e^{-\frac{x^2+y^2}{2\sigma^2}}$$

$$\boxed{p(z,\varphi) = |J|\ p(x,y)} \qquad J = \begin{vmatrix} \frac{\partial x}{\partial z} & \frac{\partial x}{\partial \varphi} \\ \frac{\partial y}{\partial z} & \frac{\partial y}{\partial \varphi} \end{vmatrix}$$

$\boxed{\text{RFA-MI}}$

$$p_{\gamma}(\gamma) = \frac{m^{m}\,\gamma^{m-1}}{\Gamma(m)\,\bar{\gamma}^{m}}\, e^{-\frac{m\gamma}{\bar{\gamma}}}$$

$$\gamma = x + jy$$

MMV

- SUM OF GAMMA DISTRIBUTION (WolframMathWorld)

x_1, x_2 – INDEPENDENT RANDOM VARIABLES WITH GAMMA DISTRIBUTIONS WITH PARAMETERS (α_1, θ) (α_2, θ)

$\frac{x_1}{x_1+x_2}$ – IS BETTA DISTRIBUTION VARIATE WITH (α_1, α_2) PARAMETERS

$$\boxed{p(x_1,x_2) = \frac{1}{\Gamma(\alpha_1)\Gamma(\alpha_2)}\, e^{-(x_1+x_2)} \cdot x_1^{\alpha_1-1} \cdot x_2^{\alpha_2-1}} = p(x_1)\cdot p(x_2)$$

ANALOGIJA

$$p(x_1) = \frac{x_1^{\alpha_1-1}}{\Gamma(\alpha_1)\cdot\theta^{\alpha_1}} \cdot e^{-\frac{x_1}{\theta}} \qquad p(x_2) = \frac{x_2^{\alpha_2-1}}{\Gamma(\alpha_2)\theta^{\alpha_2}}\, e^{-\frac{x_2}{\theta}}$$

$$\boxed{y_1 = \frac{x_1}{\theta}} \qquad p(y_1) = \frac{p(x_1)}{\frac{dy_1}{dx_1}}\Bigg/_{x_1 = f(y_1)} \qquad x_1 = \theta\cdot y_1$$

$$p(y_1) = \frac{(\theta y_1)^{\alpha_1-1}}{\Gamma(\alpha_1)\cdot\theta^{\cancel{\alpha_1}\,\alpha_1-1}}\, e^{-y_1}\cdot\cancel{\theta} = \frac{y_1^{\alpha_1-1}\,\cancel{\theta^{\alpha_1-1}}}{\Gamma(\alpha_1)\,\cancel{\theta^{\alpha_1-1}}}\, e^{-y_1}$$

$$\boxed{p(y_1) = \frac{y_1^{\alpha_1-1}}{\Gamma(\alpha_1)}\cdot e^{-y_1}} \Longrightarrow$$

ALTERNATIVE FORM OF GAMMA DISTRIBUTION

$$u = x_1 + x_2 \qquad x_1 = u\cdot v$$

$$v = \frac{x_1}{x_1+x_2} \qquad x_2 = u - u\cdot v$$

$$x_2 = u(1-v)$$

$$\begin{vmatrix} \frac{\partial x_1}{\partial u} & \frac{\partial x_1}{\partial v} \\ \frac{\partial x_2}{\partial u} & \frac{\partial x_2}{\partial v} \end{vmatrix} = \begin{vmatrix} v & u \\ (1-v) & -u \end{vmatrix} = -u\cdot v - u(1-v) =$$

$$= \cancel{-uv} - u + \cancel{uv} = -u$$

$$g(u,v)\,du\,dv = f(x,y)\,\underline{\underline{dx\,dy}} = |J|\,f(x,y)\,du\,dv$$

$$dx\,dy = u\cdot du\,dv = |J|\,du\,dv$$

$$g(u,v) = |J|\cdot f(x,y) = u\cdot\frac{1}{\Gamma(\alpha_1)\Gamma(\alpha_2)}\, e^{-(\cancel{uv}+u-\cancel{uv})}\cdot (u\cdot v)^{\alpha_1-1}\cdot(u-u\cdot v)^{\alpha_2-1}$$

$$g(u,v) = \frac{u}{\Gamma(\alpha_1)\cdot\Gamma(\alpha_2)}\cdot e^{-u}\cdot u^{\alpha_1-1}\, v^{\alpha_1-1}\cdot u^{\alpha_2-1}(1-v)^{\alpha_2-1}$$

$$g(u,v) = \frac{1}{\Gamma(\alpha_1)\cdot\Gamma(\alpha_2)}\, e^{-u}\cdot u^{\alpha_1+\alpha_2-1}\, v^{\alpha_1-1}(1-v)^{\alpha_2-1}$$

$$p(u) = \int_{-\infty}^{\infty} g(u,v)\,dv = \int_0^1 g(u,v)\,dv$$

$$p(u) = \frac{1}{\Gamma(\alpha_1)\Gamma(\alpha_2)} e^{-u} \cdot u^{\alpha_1+\alpha_2-1} \underbrace{\int_0^1 v^{\alpha_1-1}(1-v)^{\alpha_2-1}\,dv}_{I}$$

$$I = \int_0^1 \left(v^{\alpha_1-1} - v^{\alpha_1+\alpha_2-2}\right) dv = \frac{v^{\alpha_1-1+1}}{\alpha_1-1+1}\Big|_0^1 - \frac{v^{\alpha_1+\alpha_2-1}}{\alpha_1+\alpha_2-1}\Big|_0^1$$

$$\boxed{I = \frac{1}{\alpha_1} - \frac{1}{\alpha_1+\alpha_2-1}} \qquad I = \frac{\cancel{\alpha_1}+\alpha_2-1-\cancel{\alpha_1}}{\alpha_1(\alpha_1+\alpha_2-1)}$$

$$\boxed{I = \frac{\alpha_2-1}{\alpha_1(\alpha_1+\alpha_2-1)}} \qquad \boxed{I = B(\alpha_1,\alpha_2)}$$

VIDI Schaum's Advanced Calculus Pg. 379

$$p(u) = \frac{1}{\Gamma(\alpha_1)\Gamma(\alpha_2)} e^{-u} \frac{u^{\alpha_1+\alpha_2-1}(\alpha_2-1)}{\alpha_1(\alpha_1+\alpha_2-1)}$$

(**)

$$\Gamma(x+y) = \int_0^{\infty} e^{-t} \cdot t^{x+y-1}\,dt$$

CONTINUE Pg. 53

$$\Gamma(\alpha+1) = \int_0^{\infty} e^{-t} \cdot t^{\alpha+1-1}\,dt = \underline{\alpha \cdot \Gamma(\alpha)}$$

$$\Gamma(\alpha+1) = \alpha \cdot \Gamma(\alpha)$$

~~$\Gamma(\alpha) = \alpha!$~~ ~~$\Gamma(\alpha+1) = (\alpha+1)! = (\alpha+1)\cdot\alpha! = (\alpha+1)\Gamma(\alpha)$~~

$$\Gamma(\alpha) = \int_0^{\infty} e^{-t} \cdot t^{\alpha-1}\,dt \qquad u = t^{\alpha-1} \quad \underline{du = (\alpha-1)t^{\alpha-2}dt}$$

$$v = \int e^{-t}dt = -e^{-t}$$

$$\Gamma(\alpha) = \underbrace{-t^{\alpha-1} \cdot e^{-t}\Big|_0^{\infty}}_{I_1} + \underbrace{\int_0^{\infty} e^{-t}(\alpha-1)t^{\alpha-2}\,dt}_{I_2}$$

$$I_1 = \frac{t^{\alpha-1}}{e^{+t}}\Big|_0^{\infty} = \cancel{\frac{t^{\infty-1}}{e^{\infty}}} \; \emptyset - \frac{0}{1} = 0 - 0 = 0$$

$$\Gamma(\alpha) = (\alpha-1) \int_0^{\infty} e^{-t} t^{\alpha-2}\,dt = \underline{\underline{(\alpha-1)\,\Gamma(\alpha-1)}}$$

$$\Gamma(x+y) = \int_0^\infty e^{-t}\cdot t^{x+y-1}\,dt \qquad v = \int e^{-t}dt = -e^{-t}$$

$$du = (x+y-1)\,t^{x+y-2}\,dt \qquad u = t^{x+y-1}$$

$$\Gamma(x+y) = \underbrace{-e^{-t}\cdot t^{x+y-1}\Big|_0^\infty}_{\to 0} + \int_0^\infty e^{-t}\cdot(x+y-1)\cdot t^{x+y-2}\,dt$$

$$\Gamma(x+y) = (x+y-1)\underbrace{\int_0^\infty e^{-t}\,t^{x+y-2}\,dt}_{\Gamma(x+y-1)} = (x+y-1)\,\Gamma(x+y-1)$$

$$\Gamma(\alpha) = (\alpha-1)! \qquad \underline{\Gamma(\alpha+1) = \alpha!}$$

$$\frac{a(a+b-1)}{(b-1)}\,\Gamma(a)\cdot\Gamma(b) = \underline{(a+b-1)\,\Gamma(a+1)\,\Gamma(b-1)}$$

(*)

$$\Gamma(a+1) = a\cdot\Gamma(a)$$

$$\Gamma(a+2) = (a+1)\,\Gamma(a+1) = (a+1)\cdot a\cdot\Gamma(a)$$

$$\Gamma(a+3) = (a+2)\,\Gamma(a+2) = (a+2)(a+1)\cdot a\cdot\Gamma(a)$$

$$\Gamma(a+b) = \underbrace{(a+b-1)\cdots(a+2)(a+1)\cdot a}_{b \text{ products}}\;\Gamma(a)$$

$$(a-1)! = \Gamma(a)$$

$$(a+b-1)! = \Gamma(a+b)$$

$$\Gamma(b+a) = \underbrace{(b+a-1)\cdots(b+2)(b+1)\,b}_{a \text{ PRODUCTS}}\cdot\Gamma(b)$$

$$\Gamma(a+b) = (a+b-1)(a+b-2)\cdots(a+2)(a+1)\,a\,\Gamma(a)$$

$$\Gamma(a+b) = \frac{(a+b)}{a+b}\,(a+b-1)(a+b-2)\cdots(a+2)(a+1)\cdot a\,\Gamma(a)$$

$$(*) > (a+b-1)\,\Gamma(a+1)\,\Gamma(b-1) = a\,\Gamma(a+1)\Gamma(b-1) + \underbrace{(b-1)}\,\Gamma(a+1)\underbrace{\Gamma(b-1)} \to \Gamma(b)$$

$$(*) = a\,\Gamma(a+1)\,\Gamma(b-1) + \Gamma(a+1)\,\Gamma(b) =$$

$$= [a\,\Gamma(b-1) + \Gamma(b)]\,\Gamma(a+1) = [a\,\Gamma(b-1) + (b-1)\,\Gamma(b-1)]\,\Gamma(a+1)$$

↑52 $=$ $= (a+b-1)\,\Gamma(a+1)\,\Gamma(b-1)$

~~44~~

$$p(u) = \frac{1}{\Gamma(\alpha_1)\cdot\Gamma(\alpha_2)} e^{-u} \frac{u^{\alpha_1+\alpha_2-1}(\alpha_2-1)}{\alpha_1(\alpha_1+\alpha_1-1)}$$

$$\boxed{p(u) = \frac{e^{-u} u^{\alpha_1+\alpha_2-1}}{\Gamma(\alpha_1+\alpha_1)}}$$

$$\boxed{p(X_1+X_2) = \frac{e^{-x_1-x_2}(x_1+x_2)^{\alpha_1+\alpha_2-1}}{\Gamma(\alpha_1+\alpha_2)}}$$

PDF OF THE SUM OF GAMA RV IS GAMA PDF WITH $\alpha = \alpha_1+\alpha_2$

$$\Gamma(\alpha_1+\alpha_2) = \frac{\alpha_1(\alpha_1+\alpha_2-1)\cdot\Gamma(\alpha_1)\Gamma(\alpha_2)}{(\alpha_2-1)}$$

$$\Gamma(\alpha_1+\alpha_2) = (\alpha_1+\alpha_2-1)\Gamma(\alpha_1+1)\Gamma(\alpha_2-1)$$

$$p(v) = \int_0^\infty g(u,v)\,du$$

$$g(u,v) = \frac{e^{-u}}{\Gamma(\alpha_1)\Gamma(\alpha_2)} u^{\alpha_1+\alpha_2-1} \underline{v^{\alpha_1-1}(1-v)^{\alpha_2-1}}$$

$$p(v) = \frac{v^{\alpha_1-1}(1-v)^{\alpha_2-1}}{\Gamma(\alpha_1)\Gamma(\alpha_2)} \underbrace{\int_0^\infty e^{-u} u^{\alpha_1+\alpha_2-1}\,du}_{\Gamma(\alpha_1+\alpha_2)}$$

$$\boxed{p(v) = \frac{v^{\alpha_1-1}(1-v)^{\alpha_2-1}}{B(\alpha_1,\alpha_2)} \quad \boxed{v = \frac{x_1}{x_1+x_2}}}$$

$$\boxed{B(\alpha_1,\alpha_2) = \frac{\Gamma(\alpha_1)\Gamma(\alpha_2)}{\Gamma(\alpha_1+\alpha_2)}}$$

① MMV

$$\boxed{B(x,y) = \int_0^1 t^{x-1}(1-t)^{y-1}\,dt}$$

MAPLE DIRECTO① GO PAGE

MMV → BETA FUNCTION

Schaum's Outlines – Advanced calculus

$$p(u) = \frac{1}{\Gamma(\alpha_1)\Gamma(\alpha_2)} u^{\alpha_1+\alpha_2-1} e^{-u} \underbrace{\int_0^1 v^{\alpha_1-1}(1-v)^{\alpha_2-1}\,dv}_{B(x,y)}$$

$$p(u) = \frac{u^{\alpha_1+\alpha_2-1}e^{-u}}{\Gamma(\alpha_1)\Gamma(\alpha_2)}\cdot B(x,y) = \frac{u^{\alpha_1+\alpha_2-1}}{\cancel{\Gamma(\alpha_1)\Gamma(\alpha_2)}} e^{-u}\cdot\frac{\cancel{\Gamma(\alpha_1)\Gamma(\alpha_2)}}{\Gamma(\alpha_1+\alpha_2)}$$

$$p(u) = \frac{u^{\alpha_1+\alpha_2-1}e^{-u}}{\Gamma(\alpha_1+\alpha_2)} = \text{GAMMA PDF}(\alpha_1+\alpha_2)$$

$p(\gamma_1) = \frac{\gamma_1^{m_1-1}}{\Gamma(m_1)} e^{-\gamma_1}$ $p(\gamma_2) = \frac{\gamma_2^{m_2-1}}{\Gamma(m_2)} e^{-\gamma_2}$

$\gamma_1 = \frac{\gamma_1'}{\bar{\gamma}}$, $\gamma_2 = \frac{\gamma_2'}{\bar{\gamma}}$ → NORMALIZED SNR

$\gamma = \gamma_1 + \cancel{j}\,\gamma_2$ $\gamma = \gamma_1 + j\gamma_2$

$p(\gamma) = ?$

$\boxed{x = j\gamma}$

$\boxed{\frac{dx}{d\gamma} = j}$

$p(x) = \frac{p(\gamma)}{\frac{\partial x}{\partial \gamma}}$ | $\gamma = f(x) = -jx$

$p(\gamma) = \frac{\gamma^{m-1}}{\Gamma(m)} \cdot e^{-\gamma}$

$p(x) = \frac{(-jx)^{m-1}}{\Gamma(m)} e^{+jx} \cdot \frac{1}{j} = \frac{(-j)^{m-1} \cdot (-j)\, x^{m-1}}{\Gamma(m)} e^{jx}$

$$\boxed{p(x) = \frac{(-j)^{m} \cdot x^{m-1}}{\Gamma(m)} \cdot e^{jx}}$$

$\gamma = \gamma_1 + x$ $v = \gamma_1/(x+\gamma_1)$ $\gamma_1 = v \cdot \gamma$ $x = \gamma - \gamma_1 = \gamma(1-v)$

$p(\gamma_1, x) = \frac{\gamma_1^{m_1-1}}{\Gamma(m_1)} e^{-\gamma_1} \cdot \frac{(-j)^{m_2} x^{m_2-1} e^{jx}}{\Gamma(m_2)}$

$|J| = \begin{vmatrix} \frac{\partial \gamma_1}{\partial \gamma} & \frac{\partial \gamma_1}{\partial v} \\ \frac{\partial x}{\partial \gamma} & \frac{\partial x}{\partial v} \end{vmatrix} = \begin{vmatrix} v & \gamma \\ 1-v & -\gamma \end{vmatrix} = -\cancel{v\gamma} - \gamma + \cancel{\gamma v} = -\gamma$

$p(\gamma, v) = |J|\; p(\gamma_1, x) = +\gamma \cdot \frac{(v \cdot \gamma)^{m_1-1}}{\Gamma(m_1)} \cdot e^{-v \cdot \gamma} \cdot \frac{(-j)^{m_2} \cdot \gamma^{m_2-1} (1-v)^{m_2-1} e^{j(\gamma(1}}{\Gamma(m_2)}$

$p(\gamma, v) = \frac{v^{m_1-1} \cdot \gamma^{m_1} (-j)^{m_2}\, \gamma^{m_2-1} (1-v)^{m_2-1}}{\Gamma(m_1)\,\Gamma(m_2)} e^{-v\cdot\gamma + j\gamma(1-v)}$

$-v\gamma + j\gamma - j\gamma v$

$p(\gamma, v) = (-j)^{m_2} \frac{\gamma^{m_1+m_2-1} \cdot v^{m_1-1} (1-v)^{m_2-1}}{\Gamma(m_1)\,\Gamma(m_2)} e^{j\gamma - \gamma v(1+j)}$

$$p(\gamma, v) = (-j)^{m_2} \frac{\gamma^{m_1+m_2-1} \cdot v^{m_1-1}(1-v)^{m_2-1}}{\Gamma(m_1)\,\Gamma(m_2)} e^{j\gamma - \gamma v(1+j)}$$

$$p(\gamma) = \int_0^1 \cancel{(-j)^{m_2}}\, \gamma(\gamma, v)\, dv = (j)^{m_2} \frac{\gamma^{m_1+m_2-1} e^{j\gamma}}{\Gamma(m_1)\Gamma(m_2)} \underbrace{\int_0^1 v^{m_1-1}(1-v)^{m_2-1} \cancel{e}^{-\gamma v(1+j)}\, dv}_{I\ =?}$$

α-μ -DISTRIBUTION

A SIMPLE, ACCURATE APPROX ... A.C. Moraes

$$f_R(r) = \frac{\alpha \mu^\mu r^{\alpha\mu-1}}{\hat{r}^{\alpha\mu}\,\Gamma(\mu)} e^{-\mu \frac{r^\alpha}{\hat{r}^\alpha}}$$

$$\hat{r} = \sqrt[\alpha]{E[R^\alpha]}$$

α - RELATED TO THE NONLINEARITY OF THE MEDIUM

μ - NUMBER OF MULTIPATH CLASTERS

NAKAGAMI:

$$p_\gamma(\gamma) = \frac{\gamma^{m-1}}{\Gamma(m)\,\bar{\gamma}^{m}} e^{-\frac{m\gamma}{\bar{\gamma}}} = \frac{\gamma^{m-1}}{\Gamma(m)\bar{\gamma}^{m}} \cdot e^{-\frac{m\gamma}{\bar{\gamma}}}$$

$$p_\alpha(\alpha) = \frac{2m^m \alpha^{2m-1}}{\Gamma(m)\cdot\Omega^m} e^{-\frac{m\alpha^2}{\Omega}}$$

$\alpha = 2$ $\quad$ α-μ

$$f_R(r) = \frac{2\mu^\mu r^{2\mu-1}}{\hat{r}^{2\mu}\cdot\Gamma(\mu)} e^{-\mu\frac{r^2}{\hat{r}^2}}$$

$\hat{r}^2 \triangleq \Omega$ $\quad$ $\hat{r} \triangleq \sqrt{\Omega}$

$\mu \triangleq m$ $\quad$ $\alpha \triangleq r$

$\alpha = 2$ $\quad$ $\hat{r}^2 = E[r^2]$ $\qquad$ $\Omega = E[\alpha^2]$

$$\boxed{\gamma = \alpha^2 \cdot \frac{E_b}{N_0} \qquad \bar{\gamma} = E(\alpha^2)\cdot\frac{E_b}{N_0} = \Omega\cdot\frac{E_b}{N_0}}$$

$$f_\alpha(\alpha) = \frac{2\mu^\mu \cdot \alpha^{2\mu-1}}{\Gamma(\mu)\cdot\Omega^\mu} \cdot e^{-\frac{\mu\cdot\alpha^2}{\Omega}}$$

SIR: $\quad z = \left(\frac{x}{y}\right)^2 \qquad x = \sum_{i=1}^{M} X_i$ } SUM OF THE DESIRED SIGNALS AT THE DIVERSITY BRANCHES

$$Y^2 = \sum_{j=1}^{N}\sum_{i=1}^{M} \xi_{i,j}^2$$

- OUTAGE PROBABILITY

$$P_{out} = P_r[z < z_{th}]$$

- UNIFIED APPROACH OF CALCULATING P_{out}

$$F_Z(z_{th}) = \int_0^\infty F_X(y\sqrt{z_{th}})\, f_Y(y)\, dy$$

$$f_R(r) = \frac{\alpha\, \mu^\mu\, r^{\alpha\mu-1}}{\hat{r}^{\alpha\mu}\,\Gamma(\mu)} \exp\left(-\mu\frac{r^\alpha}{\hat{r}^\alpha}\right)$$ α-μ DISTRIB

$$F_R(r) = \frac{\Gamma(\mu, \mu r^\alpha/\hat{r}^\alpha)}{\Gamma(\mu)}$$

$$\boxed{\hat{r} = \sqrt[\alpha]{E(R^\alpha)}}$$

$$F_X(y\sqrt{z_{th}}) = \frac{\Gamma\left(\mu_s, \mu_s\, (y\sqrt{z_{th}})^{\alpha_s}/\hat{x}_s^{\alpha_s}\right)}{\Gamma(\mu_s)}$$

$$\boxed{\hat{x}_s = \sqrt[\alpha_s]{E(X^{\alpha_s})}}$$

$$\cancel{\hat{r}^\alpha = \Omega}$$
$$\cancel{\hat{r} = \sqrt[\alpha]{\Omega}}$$
$$\cancel{\hat{r} = \Omega^{1/\alpha}}$$

NAKAGAMI:

$$p(\gamma) = \frac{m^m\, \gamma^{m-1}}{\Gamma(m)\cdot\bar{\gamma}^m}\cdot e^{-\frac{m\gamma}{\bar{\gamma}}}$$

$$MGF_{1/\gamma}(s) = \frac{2}{\Gamma(m)}\left(\frac{m_n s}{\bar{\gamma}_n}\right)^{m_n/2} K_{m_n}\left(2\sqrt{\frac{m_n s}{\bar{\gamma}_n}}\right) \quad (*1)$$

$$\gamma = \alpha^2\frac{E_s}{N_0} \qquad \Omega = E(\alpha^2) \qquad \bar{\gamma} = \Omega\cdot\frac{E_s}{N_0} \qquad \frac{N_0}{E_s} = \frac{\Omega}{\bar{\gamma}}$$

$$\alpha = \sqrt{\gamma\frac{N_0}{E_s}} = \sqrt{\frac{\gamma\cdot\Omega}{\bar{\gamma}}}$$

$$p_\alpha(\alpha) = 2\frac{m^m\cdot\alpha^{2m-1}}{\Gamma(m)\,\Omega^m}\cdot e^{-\frac{m\alpha^2}{\Omega}}$$

$$p_\gamma(\gamma) = \frac{p_\alpha\left(\sqrt{\frac{\Omega\cdot\gamma}{\bar{\gamma}}}\right)}{2\sqrt{\frac{\bar{\gamma}\gamma}{\Omega}}} = \frac{\cancel{2}\cdot m^m\cdot\alpha^{2m}\cdot\alpha^{-1}}{\cancel{2}\cdot\Gamma(m)\cdot\Omega^m}\cdot\sqrt{\frac{\Omega}{\bar{\gamma}\cdot\gamma}}\cdot e^{-\frac{m\gamma}{\bar{\gamma}}} =$$

$$= \frac{m^m\cdot\left(\frac{\gamma\cdot\cancel{\Omega}}{\bar{\gamma}}\right)^m}{\Gamma(m)\cdot\cancel{\Omega^m}}\cdot\sqrt{\frac{\bar{\gamma}}{\gamma\cdot\cancel{\Omega}}}\cdot\sqrt{\frac{\cancel{\Omega}}{\cancel{\bar{\gamma}}\cdot\gamma}}\, e^{-\frac{m\gamma}{\bar{\gamma}}} = \frac{m^m\cdot\gamma^{m-1}}{\Gamma(m)\cdot\bar{\gamma}^m}\, e^{-\frac{m\gamma}{\bar{\gamma}}}$$

$$f_R(r) = \frac{\alpha \cdot \mu^{\mu} r^{\alpha\mu-1}}{\Gamma(\mu)\cdot \hat{r}^{\alpha\mu}} \exp\left(-\mu \frac{r^{\alpha}}{\hat{r}^{\alpha}}\right)$$

~~$\Omega = E[r^{\alpha}]$~~

$$\boxed{\Omega = E[r^2]}$$

$\alpha = 2 \Rightarrow$ NAKAGAMI

$$r = \sqrt{\frac{\gamma\cdot\Omega}{\bar{\gamma}}} \Rightarrow f_\gamma(\gamma) = \frac{1}{2\sqrt{\frac{\gamma\bar{\gamma}}{\Omega}}} \cdot \frac{\alpha\cdot\mu^{\mu}\cdot r^{\alpha\mu-1}}{\Gamma(\mu)\cdot\hat{r}^{\alpha\mu}} \exp\left(-\mu\frac{r^{\alpha}}{\hat{r}^{\alpha}}\right)$$

$r = \sqrt{\frac{\Omega}{\bar{\gamma}}}$

$$f_\gamma(\gamma) = \frac{1}{2}\sqrt{\frac{\Omega}{\gamma\bar{\gamma}}} \cdot \frac{\alpha\cdot\mu^{\mu}}{\Gamma(\mu)\,\hat{r}^{\alpha\mu}} \left(\sqrt{\frac{\gamma\cdot\Omega}{\bar{\gamma}}}\right)^{\alpha\mu-1} \exp\left(-\frac{\mu}{\hat{r}^{\alpha}}\left(\sqrt{\frac{\gamma\cdot\Omega}{\bar{\gamma}}}\right)^{\alpha}\right)$$

$$f_\gamma(\gamma) = \frac{1}{2}\sqrt{\frac{\Omega}{\gamma\bar{\gamma}}}\, \frac{\alpha\cdot\mu^{\mu}}{\Gamma(\mu)\,\hat{r}^{\alpha\mu}} \cdot \left(\frac{\gamma\Omega}{\bar{\gamma}}\right)^{\frac{\alpha\mu-1}{2}} \exp\left(-\frac{\mu}{\hat{r}^{\alpha}}\left(\frac{\gamma\cdot\Omega}{\bar{\gamma}}\right)^{\alpha/2}\right)$$

$$\boxed{\hat{r}^{\alpha} = E\left(r^{\alpha}\right)}$$

$$\boxed{\frac{\sqrt{\frac{\Omega}{\gamma\bar{\gamma}}}}{\sqrt{\frac{\gamma\cdot\Omega}{\bar{\gamma}}}} = \frac{1}{\gamma}}$$

NAJVOOPŠTENA FORMA NA α-μ DISTRIBUCIJA NA SNAGITE NA SIGNALOT

$\gamma \sim r^2$

$$f_\gamma(\gamma) = \frac{\alpha\cdot\mu^{\mu}}{2\Gamma(\mu)\cdot\hat{r}^{\alpha\mu}\cdot\gamma} \cdot \left(\frac{\gamma\Omega}{\bar{\gamma}}\right)^{\frac{\alpha\mu}{2}} \exp\left(-\frac{\mu}{\hat{r}^{2}}\left(\frac{\gamma\cdot\Omega}{\bar{\gamma}}\right)^{\alpha/2}\right)$$

$\alpha = 2$ $\boxed{\hat{r}^2 = \Omega}$

$$f_\gamma(\gamma) = \frac{\mu^{\mu}}{\Gamma(\mu)\cdot\cancel{\Omega^{\mu}}\cdot\gamma}\left(\frac{\gamma}{\bar{\gamma}}\right)^{\mu}\cdot\cancel{\Omega^{\mu}} \exp\left(-\frac{\mu}{\cancel{\Omega}}\frac{\gamma\cdot\cancel{\Omega}}{\bar{\gamma}}\right)$$

$$\boxed{f_\gamma(\gamma)\Big|_{\alpha=2} = \frac{\mu^{\mu}\,\gamma^{\mu-1}}{\Gamma(\mu)\cdot\bar{\gamma}^{\mu}} \exp\left(-\mu\frac{\gamma}{\bar{\gamma}}\right)}$$ ⇐ NAKAGAMI

$$MGF_{1/\gamma}(s) = \int^{\infty} e^{-\frac{s}{\gamma}} f_\gamma(\gamma)\,d\gamma = \left(\frac{\sqrt{\Omega}}{\hat{r}}\right)^{\alpha\mu}\frac{\alpha}{\Gamma(\mu)}\left(\frac{s\cdot\mu}{\bar{\gamma}}\right)^{\frac{\alpha\mu}{4}} K_{\frac{\alpha\mu}{2}}\left(2\sqrt{\frac{\mu\cdot s}{\bar{\gamma}}}\right)$$

$$\boxed{MGF_{1/\gamma}(s) = \left(\frac{\sqrt{\Omega}}{\hat{r}}\right)^{\alpha\mu}\frac{\alpha}{\Gamma(\mu)}\left(\frac{s\cdot\mu}{\bar{\gamma}}\right)^{\frac{\alpha\mu}{4}} K_{\frac{\alpha\mu}{2}}\left(2\sqrt{\frac{\mu s}{\bar{\gamma}}}\right)}$$

FOR $\alpha = 2$
YOU WILL GET
(*1) PP46

$$f(\gamma) = \frac{\alpha \cdot \mu^{\mu}}{2 \cdot \Gamma(\mu)\, \hat{r}^{\alpha\mu}\, \gamma} \left(\frac{\gamma\Omega}{\bar{\gamma}}\right)^{\alpha\mu/2} e^{-\frac{\mu}{\hat{r}^{2}}\left(\frac{\gamma\Omega}{\bar{\gamma}}\right)^{\alpha/2}} \quad \alpha-\mu \quad \Big\}\ \alpha-\mu \text{ FORMULE}$$

$$p_\gamma(\gamma) = \frac{m^{m} \cdot \gamma^{m-1}}{\Gamma(\mu)\,\bar{\gamma}} e^{-m\frac{\gamma}{\bar{\gamma}}}$$

$\gamma = \mathrm{randg}(m)$

$$f_1(\gamma) = \frac{\alpha \cdot \mu^{\mu} \cdot \gamma^{\alpha\mu/2 - 1}}{2\,\Gamma(\mu) \cdot \bar{\gamma}^{\alpha\mu/2}} \cdot e^{-\mu\frac{\gamma}{\bar{\gamma}}} \cdot \frac{\Omega^{\alpha\mu/2}}{\hat{r}^{\alpha\mu}}$$

$$p_\gamma(\gamma) = \frac{m^{m}\gamma^{m-1}}{\Gamma(m)\,\bar{\gamma}} e^{-\frac{m\gamma}{\bar{\gamma}}}$$

DOBIVANJE NA GAMMA FUNKCIJA SOGLASNO FORMATA VO MATLAB

$x = m\gamma$ $\quad \frac{dx}{d\gamma} = m$ $\quad p(x) = \frac{p(\gamma)}{\frac{\partial x}{\partial \gamma}}\Big|_{\gamma = f(x)}$

$\gamma = \left(\frac{x}{m}\right)$

$$p(x) = \frac{1}{m} \cdot \frac{\cancel{m^{m}} \cdot \frac{x^{m-1}}{\cancel{m^{m-1}}}}{\Gamma(m)\,\bar{\gamma}^{m}} e^{-\frac{x}{\bar{\gamma}}} = \frac{x^{m-1}}{\Gamma(m)\,\bar{\gamma}^{m}} \cdot e^{-\frac{x}{\bar{\gamma}}}$$

$$\boxed{p(x) = \frac{x^{m-1}}{\Gamma(m) \cdot \bar{\gamma}^{m}} e^{-\frac{x}{\bar{\gamma}}}}$$

$$\boxed{x = m \cdot \gamma}$$

DIMENZIJA NA SKALA

$x = \mathrm{randg}(m)$ $\quad m \cdot \gamma = \mathrm{randg}(m); \gamma = \frac{1}{m}\,\mathrm{randg}(m)$

$$\boxed{r = \sqrt{\gamma} = \sqrt{\frac{1}{m}\,\mathrm{randg}(m)}} \Rightarrow$$

VAKA KOGA PRODAV VO NOVATA MI RANDOM GEN (js_GenNakagamiRand) PODOBRO SE FITUVA SO TEORETSKATA KRIVA

- Weibull

$$\alpha^{2} = \frac{\gamma\Omega}{\bar{\gamma}}$$

$$p_\alpha(\alpha) = c\left(\frac{\Gamma\left(1+\frac{2}{c}\right)}{\Omega}\right)^{c/2} \alpha^{c-1} \exp\left[-\left(\frac{\alpha^{2}}{\Omega}\Gamma\left(1+\frac{2}{c}\right)\right)^{c/2}\right] \quad \alpha > 0$$

WEIBULL IZSO

$$\boxed{p_\gamma(\gamma) = \frac{c}{2}\left(\frac{\Gamma\left(1+\frac{2}{c}\right)}{\bar{\gamma}}\right)^{c/2} \gamma^{\frac{c}{2}-1} \exp\left[-\left(\frac{\gamma}{\bar{\gamma}}\Gamma\left(1+\frac{2}{c}\right)\right)^{c/2}\right] \quad \gamma \geq 0}$$

$$P(x < x_0) = \int_0^{x_0} p(x)\,dx \qquad \mathcal{Z}\left[P(x < x_0)\right] = \frac{\hat{p}(s)}{s}$$

$$\hat{p}(s) = \int_{-\infty}^{\infty} p(x)\cdot e^{-sx}\,dx = \underline{\underline{\mathcal{L}[p(x)]}} = MGF(-s)$$

LM 17.06.2009
DL 29.07.2009

$$\boxed{P(x<x_0) = \mathcal{L}^{-1}\left[\frac{MGF(-s)}{s}\right]}$$

$$P_\gamma(\gamma) = 1-\exp\left[-\left(\frac{\gamma}{\bar\gamma}\Gamma\left(1+\frac{2}{c}\right)\right)^{c/2}\right.$$ } CDF FOR WEIBULL DISTRO

RAYLEIGH

$$p(\gamma)=\frac{1}{\bar\gamma}e^{-\frac{\gamma}{\bar\gamma}} \qquad P(\gamma<x)=1-e^{-\frac{\gamma}{\bar\gamma}}$$

MAPLE

$$CDF(z) = 1 - e^{-2\left(\frac{z}{g_m}\right)^{\frac{c}{2}}\cdot\left(\frac{1}{c}\right)^{\frac{c}{2}}\pi^{-\frac{c}{4}}\cdot\Gamma\left(\frac{1}{c}\right)^{\frac{c}{2}}\cdot\Gamma\left(\frac{1}{2}\left(1+\frac{2}{c}\right)\right)^{\frac{c}{2}}}$$

MATHEMATICA

$$MGF_{1/\gamma}(s) = \cdot\, 2^{\frac{c}{4}}\cdot c\ \mathrm{BesselK}\left[-\frac{c}{2}, \sqrt{2}\sqrt{\frac{cs\,\mathrm{Gamma}\left[\frac{2+c}{c}\right]}{\bar\gamma}}\right]\left(\frac{s\Gamma\left(1+\frac{2}{c}\right)}{c\cdot\bar\gamma}\right)^{c/4}$$

Full Simplify

$$\boxed{MGF_{1/\gamma}(s) = 2^{\frac{c}{2}}\cdot c\cdot K_{\frac{c}{2}}\left[2\sqrt{\frac{s\,\Gamma\left(\frac{2}{c}\right)}{\bar\gamma}}\right]\cdot\left(\frac{s\,\Gamma\left(\frac{2}{c}\right)}{c^2\,\bar\gamma}\right)^{c/4}}$$

GO RESI!!! NO RESEN GRETNO

MATLAB WEIBULL DISTRO DEFFINITION

$$y = f(x|a,b) = b\,a^{-b}x^{b-1}e^{-\left(\frac{x}{a}\right)^b}I_{(0,\infty)}(x)$$

$$p_\gamma(\gamma) = \frac{p_\alpha(\alpha)}{2\sqrt{\frac{\gamma\bar\gamma}{\Omega}}}$$

$$\gamma = \alpha^2\cdot\frac{E_b}{N_0} \qquad \bar\gamma = \Omega\frac{E_s}{N_0} \qquad \frac{E_s}{N_0} = \frac{\bar\gamma}{\Omega}$$

$$\gamma = \alpha^2\frac{\bar\gamma}{\Omega} \qquad \boxed{\alpha = \sqrt{\frac{\gamma\Omega}{\bar\gamma}}} \qquad \alpha^2 = \frac{\gamma\Omega}{\bar\gamma}$$

$$p_\gamma(\gamma) = \frac{1}{2}\underbrace{\sqrt{\frac{\Omega}{\gamma\bar\gamma}}\ c\ \frac{\Gamma\left(1+\frac{2}{c}\right)^{c/2}}{\Omega^{c/2}}\cdot\left(\frac{\gamma\Omega}{\bar\gamma}\right)^{\frac{c-1}{2}}}_{①}\exp\left[-\left(\frac{\gamma}{\bar\gamma}\Gamma\left(1+\frac{2}{c}\right)\right)^{c/2}\right]$$

$$① = \left(\frac{\Omega}{\gamma\bar\gamma}\right)^{1/2}\cdot\frac{1}{\Omega^{c/2}}\cdot\frac{\gamma^{\frac{c-1}{2}}\Omega^{\frac{c-1}{2}}}{\bar\gamma^{\frac{c-1}{2}}} = \frac{1}{\cancel{\Omega^{\frac{c-1}{2}}}}\ \frac{\gamma^{\frac{c-1}{2}-\frac{1}{2}}\cancel{\Omega^{\frac{c-1}{2}}}}{\bar\gamma^{\frac{c-1}{2}+\frac{1}{2}}}$$

$$① = \frac{\gamma^{\frac{c}{2}-1}}{\bar\gamma^{c/2}}$$

$$\boxed{p_\gamma(\gamma) = \frac{1}{2}\cdot c\cdot\left(\frac{\Gamma\left(1+\frac{2}{c}\right)}{\bar\gamma}\right)^{c/2}\cdot\gamma^{\frac{c}{2}-1}\exp\left[-\left(\frac{\gamma}{\bar\gamma}\Gamma\left(1+\frac{2}{c}\right)\right)^{\frac{c}{2}}\right]}$$

PROOFED!!!

$\boxed{\text{Theorem 2.1}}$ R is sample of weibul distro with param c then R^β is also a sample of a Weibull distro with parameter c/β

- NORMALIZED FORM OF WEIBUL DISTRO.

$$p_R(r) = c\,r^{c-1}\exp(-R^c)$$

$c=3$ (MATEMATICA)

$$MGF(-s) = \frac{e^{-\sqrt{3/g_m}\,6\cdot\Gamma(5/3)}\left(\sqrt{g_m}\sqrt{6\pi}+6\sqrt{5}\sqrt{\pi}\,\Gamma(5/2)\right)}{6\sqrt{g_m}}$$

MATLAB DEF WEIBULL

$$f(x;a,b) = b\,a^{-b}\,x^{b-1}\,e^{-\left(\frac{x}{a}\right)^b}$$

$$R=\frac{x}{a} \qquad dR = \frac{1}{a}\,dx \qquad x = a\cdot R$$

$$f(R;a,b) = \left(\frac{1}{a}\right)^{-1} b\cdot a^{-b}\,x^{b-1}\,e^{-R^b} = b\cdot\frac{x^{b-1}}{a^{b-1}}\,e^{-R^b} = \underline{b\cdot R^{b-1}e^{-R^b}}$$

$$\boxed{f(R;b) = b\,R^{b-1}e^{-R^b} = p_R(R)}$$

$$p_\alpha(\alpha) = c\left(\frac{\Gamma(1+\frac{2}{c})}{\Omega}\right)^{c/2}\alpha^{c-1}\exp\left(-\frac{\alpha^2}{\Omega}\Gamma\left(1+\frac{2}{c}\right)\right)^{\frac{c}{2}}$$

$$\boxed{b \triangleq \frac{c}{2}} \quad \boxed{a \triangleq \left(\frac{\Omega}{\Gamma(1+\frac{2}{c})}\right)} \quad \boxed{\cancel{A = \frac{1}{\Gamma(1+\frac{2}{c})}}} \quad \boxed{A \triangleq \frac{a\cdot\bar{\gamma}}{\Omega}}$$

$$p_\alpha(\alpha) = c\cdot a^{-\frac{c}{2}}\,\alpha^{2b-1}\exp\left(-\frac{\alpha^{2\cdot\frac{c}{2}}}{a^{\frac{c}{2}}}\right) = 2b\cdot a^{-b}\,\alpha^{2b-1}\cdot e^{-\left(\frac{\alpha^2}{a}\right)^b}$$

$$p_\gamma(\gamma) = \frac{p_\alpha(\alpha)}{2\sqrt{\frac{\gamma\bar{\gamma}}{\Omega}}}\Bigg|_{\alpha=\sqrt{\frac{\gamma\Omega}{\bar{\gamma}}}} = \cancel{2}b\cdot a^{-b}\left(\frac{\gamma\Omega}{\bar{\gamma}}\right)^{\frac{2b-1}{2}}\cdot e^{-\left(\frac{\gamma}{\bar{\gamma}}\frac{\Omega}{a}\right)^b}\cdot\frac{1}{\cancel{2}}\left(\frac{\Omega}{\gamma\bar{\gamma}}\right)^{1/2}$$

$$p_\gamma(\gamma) = b\cdot a^{-b}\,\frac{\gamma^{b-\frac{1}{2}}\cdot\Omega^{b-\cancel{\frac{1}{2}}}}{\bar{\gamma}^{\,b-\cancel{\frac{1}{2}}}}\cdot e^{-\left(\frac{\gamma\Omega}{a\cdot\bar{\gamma}}\right)^b}\cdot\frac{\cancel{\Omega^{1/2}}}{\gamma^{1/2}\,\cancel{\bar{\gamma}^{1/2}}}$$

$$p_\gamma(\gamma) = b\cdot a^{-b}\cdot\left(\frac{\Omega}{\bar{\gamma}}\right)^b\cdot\gamma^{b-1}\,e^{-\left(\frac{\gamma}{a}\right)^b\cdot\left(\frac{\Omega}{\bar{\gamma}}\right)^b} \qquad A = \left(\frac{\Omega}{a\cdot\bar{\gamma}}\right)^{-1} = \frac{a\cdot\bar{\gamma}}{\Omega}$$

$$p_\gamma(\gamma) = b\cdot\left(\frac{\Omega}{a\cdot\bar{\gamma}}\right)^b\gamma^{b-1}\cdot e^{-\left(\frac{\gamma}{A}\right)^b} = b\cdot A^{-b}\cdot\gamma^{b-1}\cdot e^{-\left(\frac{\gamma}{A}\right)^b}$$

$$\boxed{B = b \triangleq \frac{c}{2} \qquad A \triangleq \frac{a \cdot \bar{\gamma}}{\Omega} = \frac{\bar{\gamma}}{\Gamma(1+\frac{2}{c})}}$$

$$\boxed{a = \frac{\Omega}{\Gamma(1+\frac{2}{c})} = \frac{\Omega}{\Gamma(1+\frac{1}{b})} \qquad A = \bar{\gamma}/\Gamma(1+1/b)}$$

$$\boxed{p_\gamma(\gamma) = b A^{-b} \gamma^{b-1} e^{-\left(\frac{\gamma}{A}\right)^b}} \Longrightarrow$$

VAKA GO GENERIRA MATLAB

$$\overline{(\xi-\bar{\xi})^2} = \overline{\xi^2} - \bar{\xi}^2$$

$$\boxed{p_\alpha(\alpha) = 2b \cdot a^{-b} \cdot \alpha^{2b-1} e^{-\left(\frac{\alpha^2}{a}\right)^b}}$$

Theorem 2.1 R sample of a Waibull ITEM(c)

R^β also sample of a Waibull $\left(\frac{c}{\beta}\right)$

$$p_R(R) = c R^{c-1} \exp(-R^c) \qquad R \geq 0$$

$$Y = R^\beta \qquad p_Y(Y) = \left.\frac{p_R(R)}{\frac{dY}{dR}}\right|_{R=f(Y)} = \left.\frac{c R^{c-1} e^{-R^c}}{\beta \cdot R^{\beta-1}}\right|_{R=Y^{\frac{1}{\beta}}}$$

$$p_Y(Y) = \frac{c}{\beta} \cdot \cancel{R^{c-1}} \frac{Y^{\frac{c-1}{\beta}}}{Y^{\frac{\beta-1}{\beta}}} e^{-Y^{\frac{c}{\beta}}} = \frac{c}{\beta} Y^{\frac{c}{\beta} - \cancel{\frac{1}{\beta}} - 1 + \cancel{\frac{1}{\beta}}} e^{-Y^{\frac{c}{\beta}}}$$

$$p_Y(Y) = \frac{c}{\beta} Y^{\frac{c}{\beta}-1} e^{-Y^{\frac{c}{\beta}}} = c_1 \cdot Y^{c_1-1} e^{-Y^{c_1}} \qquad \boxed{c_1 = \frac{c}{\beta}}$$

- MOMENTS OF WEIBULL DISTRO

$$G(l) = \Gamma\left(1+\frac{1}{c}\right) \Rightarrow \boxed{E(R^\beta) = \Gamma\left(1+\frac{\beta}{c}\right)}$$

$$\boxed{\begin{aligned} M_\alpha(s) = c \left(\frac{\Gamma(1+\frac{2}{c})}{\Omega}\right)^{\frac{c}{2}} (2\pi)^{\frac{1-c}{2}} \frac{1}{\sqrt{2}} \left(-\frac{s}{c}\right)^{-c} \cdot \\ G^{c,1}_{1,c}\left(\left(\frac{\Gamma(1+\frac{2}{c})}{\Omega}\right)^{-\Omega} \left(-\frac{s}{c}\right)^{c} \,\middle|\, \begin{matrix} 1 \\ 1, 1+1/c, \ldots, 1+(c-1)/c \end{matrix}\right) \end{aligned}}$$

$G^{c,1}_{1,c}(\cdot)$ - Meijer's G - FUNCTION

$$AF = \frac{\operatorname{var}(\alpha^2)}{(E[\alpha^2])^2} = \frac{\operatorname{var}(R^2)}{[E[R^2]]^2} = \frac{\Gamma(1+\frac{4}{c}) - \Gamma^2(1+\frac{2}{c})}{\Gamma^2(1+\frac{2}{c})}$$

$$\operatorname{var}(R^2) = E[R^4] - E^2[R^2]$$

$$AF = \frac{var(\alpha^2)}{[E[\alpha^2]]^2} = \frac{E[(\alpha^2-\Omega)^2]}{[E[\alpha^2]]^2} = \frac{E[\gamma^2]-E[\gamma]^2}{E^2[\gamma]}$$

$$E(\alpha^2)=\Omega$$

$$E[(\alpha^2-\Omega)^2] = E[\alpha^4 - 2\alpha^2\Omega + \Omega^2] = E[\alpha^4] - 2\Omega E[\alpha^2] + \Omega^2$$

$$= E[\alpha^4] - 2\Omega\cdot\Omega + \Omega^2 = E[\alpha^4] - \cancel{2}\Omega^2 + \cancel{\Omega^2} = \underline{E[\gamma^2]-\Omega^2}$$

$$= \underline{E[\gamma^2] - E[\gamma]^2}$$

1GR MGF 10 sec
2GR MGFe 1 sec

ΛΕΓΟΨΠΕΔ
023172744

$N=20 \quad K=11$

$$N\cdot K(10+1) = N\cdot K \cdot 220\cdot 11 sec = 2420 sec \doteq 40 min$$

A PERFORMANCE STUDY OF DUAL-HOP FG-GAIN

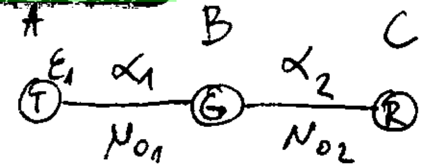
- SYSTEM & CHANNEL MODELS

$$r_B(t) = \alpha_1 s(t) + n_1(t)$$

$$\boxed{r_C = \alpha_2 G(\alpha_1 s(t) + n_1(t)) + n_2(t)}$$

OVA E KLUČOT GLAVNATA FORMULA

$$\gamma_{eq} = \frac{\frac{E_1\alpha_1^2}{N_{01}}\cdot\frac{\alpha_2^2}{N_{02}}}{\frac{\alpha_2^2}{N_{02}} + \frac{1}{G^2 N_{01}}}$$



E_1 – POWER OF TRANSMITTED SIG

E_2 – VLEGUVA TUKA

$$SIGPOWER = E_1\alpha_1^2\cdot\alpha_2^2\cdot G^2$$

$$NOISEPOWER = \alpha_2^2 N_{01}\cdot G^2 + N_{02}$$

$$\frac{S}{N} = \frac{E_1\alpha_1^2\cdot\alpha_2^2\cdot G^2}{\alpha_2^2\cdot N_{01}\cdot G^2 + N_{02}}$$

$$\frac{S}{N} = \frac{\frac{E_1\alpha_1^2\alpha_2^2 G^2}{N_{01}N_{02}}}{\frac{\alpha_2^2 G^2}{N_{02}} + \frac{1}{N_{01}}} = \frac{\frac{E_1\alpha_1^2\alpha_2^2\cancel{G^2}}{N_{01}N_{02}}}{\frac{\alpha_2^2}{N_{02}} + \frac{1}{N_{01}G^2}}$$

VIDI KAKO ZOKI ALTER-NATIVNO GO IZVEDUVA NA N.4.PPGS

$$\boxed{G^2 = \frac{\mathcal{E}_2}{\mathcal{E}_1\alpha_1^2 + N_{01}}}$$

$\mathcal{E}_2$ - POWER OF THE TRANSMITTED SIGNAL AT THE OUTPUT OF THE RELAY

↳ PREPORACANO VO CLANAKOT NA LANEMAN

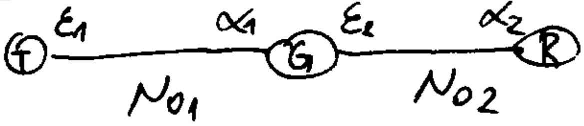
$$\gamma_{eq} = \frac{S}{N} = \frac{\frac{\mathcal{E}_1\alpha_1^2\alpha_2^2}{N_{01}N_{02}}}{\frac{\alpha_2^2}{N_{02}} + \frac{1}{N_{01}G^2}} = \frac{\frac{\mathcal{E}_1\alpha_1^2\alpha_2^2}{N_{01}N_{02}}}{\frac{\alpha_2^2}{N_{02}} + \frac{\mathcal{E}_1\alpha_1^2 + N_{01}}{N_{01}\cdot\mathcal{E}_2}} =$$

$$= \frac{\frac{\mathcal{E}_1\alpha_1^2\alpha_2^2}{N_{01}N_{02}}}{\frac{\alpha_2^2}{N_{02}} + \frac{\mathcal{E}_1\alpha_1^2}{\mathcal{E}_2 N_{01}} + \frac{1}{\mathcal{E}_2}} = \frac{\frac{\mathcal{E}_1\alpha_1^2\alpha_2^2}{N_{01}N_{02}}}{\frac{1}{\mathcal{E}_2}\left[\underbrace{\frac{\mathcal{E}_2\alpha_2^2}{N_{02}}}_{\gamma_2} + \underbrace{\frac{\mathcal{E}_1\alpha_1^2}{N_{01}}}_{\gamma_1} + 1\right]}$$

$$= \frac{\frac{\mathcal{E}_1\alpha_1^2}{N_{01}} \cdot \frac{\alpha_2^2\mathcal{E}_2}{N_{02}}}{[\gamma_1 + \gamma_2 + 1]} = \frac{\gamma_1\cdot\gamma_2}{\gamma_1+\gamma_2+1}$$

$$\boxed{\gamma_{eq} = \frac{\gamma_1\cdot\gamma_2}{\gamma_1+\gamma_2+1}} \qquad \boxed{\gamma_i = \frac{\mathcal{E}_i\alpha_i^2}{N_{0i}} \quad i = 1,2}$$

- BLIND RELAY



$$\boxed{G^2 = \frac{\mathcal{E}_2}{C\cdot N_{01}}}$$

$$S = \mathcal{E}_1\cdot\alpha_1^2\cdot\alpha_2^2 G^2 \qquad \cancel{N_k} = \alpha_2^2\cdot G^2\cdot N_{01} + N_{02}$$

$$\gamma_{eq} = \frac{\frac{\mathcal{E}_1\alpha_1^2\alpha_2^2}{N_{01}N_{02}}}{\frac{\alpha_2^2}{N_{02}} + \frac{1}{N_{01}G^2}} = \frac{\frac{\mathcal{E}_1\alpha_1^2\alpha_2^2}{N_{01}N_{02}}}{\frac{\alpha_2^2}{N_{02}} + \frac{C}{\mathcal{E}_2}} = \frac{\frac{\mathcal{E}_1\alpha_1^2}{N_{01}}\frac{\alpha_2^2\mathcal{E}_2}{N_{02}}}{\frac{\alpha_2^2\mathcal{E}_2}{N_{02}} + C}$$

$$\boxed{\gamma_{eq} = \frac{\gamma_1\gamma_2}{\gamma_2 + C}}$$

FIXED GAIN

$$\bar{\gamma}_1 = \Omega_1\frac{\mathcal{E}_1}{N_{01}}, \quad \bar{\gamma}_2 = \Omega_2\frac{\mathcal{E}_2}{N_{02}}$$

$$\Omega_i = \overline{\alpha_i^2} \qquad i = 1,2$$

↳ AVERAGE FADING POWER.

$$P_{out} = P[\gamma_{eq} < \gamma_{th}] = P\left[\frac{\gamma_1\gamma_2}{\gamma_2 + C} < \gamma_{th}\right]$$

$$P_{out} = P\left[\frac{\gamma_1\gamma_2}{\gamma_2+C} < \gamma_{th}\right] = \int_0^\infty P\left(\frac{\gamma_1\gamma_2}{\gamma_2+C} < \gamma_{th} / \gamma_2\right) p_{\gamma_2}(\gamma_2)\, d\gamma_2$$

$$p_\gamma(\gamma) = \frac{1}{\bar\gamma} \cdot e^{-\gamma/\bar\gamma} \qquad P(\gamma < \gamma_{th}) = \int_0^{\gamma_{th}} \frac{1}{\bar\gamma} e^{-\gamma/\bar\gamma}\, d\gamma$$

$$P(\gamma < \gamma_{th}) = \cancel{\tfrac{1}{\gamma}} - e^{-\gamma/\bar\gamma}\Big|_0^{\gamma_{th}} = - e^{-\gamma_{th}/\bar\gamma} + 1 = 1 - e^{-\gamma_{th}/\bar\gamma}$$

$$P(\gamma_{eq} < \gamma_{th} / \gamma_2) = 1 - e^{-\gamma_{th}/\bar\gamma_1} = 1 - e^{-\frac{\gamma_1\gamma_2}{\gamma_2+C} \cdot \frac{1}{\bar\gamma_1}}$$

$$\boxed{P(A,B) = P(B/A)\, P(A)}$$

$$e^{-(\gamma_{th}/\bar\gamma_1)(1+C/\gamma_2)} \quad \exp\left[-\frac{\gamma_{th}}{\bar\gamma_1}\cdot\left(1+\frac{C}{\gamma_2}\right)\right] = \exp\left[\frac{\gamma_{th}\gamma_2 + \gamma_{th}C}{\bar\gamma_1\gamma_2}\right]$$

$$e^{-\frac{\gamma_{th}}{\bar\gamma_1}} \cdot e^{-\frac{\gamma_{th} C}{\bar\gamma_1\gamma_2}} \qquad \frac{\gamma_1\gamma_2}{C+\gamma_2} = \gamma_{eq} \qquad \boxed{\gamma_1 = \frac{\gamma_{eq}\cdot(C+\gamma_2)}{\gamma_2}}$$

$$P\left[\frac{\gamma_1\gamma_2}{\gamma_2+C} < \gamma_{th}\right] = P\left[\gamma_1 < \frac{\gamma_{th}(\gamma_2+C)}{\gamma_2}\right] = \underline{\underline{1 - e^{-\frac{1}{\bar\gamma_1}\cdot\frac{\gamma_{th}(\gamma_2+C)}{\gamma_2}}}}$$

$$P\left[\frac{\gamma_1\gamma_2}{\gamma_2+C} < \gamma_{th}\right] = \int_0^\infty P\left(\frac{\gamma_1\gamma_2}{\gamma_2+C} < \gamma_{th} / \gamma_2\right) p_{\gamma_2}(\gamma_2)\, d\gamma_2 =$$

$$= \int_0^\infty \left(1 - e^{-\frac{1}{\bar\gamma_1}\frac{\gamma_{th}(\gamma_2+C)}{\gamma_2}}\right) \frac{e^{-\frac{\gamma_2}{\bar\gamma_2}}}{\bar\gamma_2}\, d\gamma_2 =$$

$$= \frac{1}{\bar\gamma_2}\int_0^\infty e^{-\frac{\gamma_2}{\bar\gamma_2}}\, d\gamma_2 - \frac{1}{\bar\gamma_2}\int_0^\infty e^{-\frac{1}{\bar\gamma_1}\gamma_{th}\left(1+\frac{C}{\gamma_2}\right) - \frac{\gamma_2}{\bar\gamma_2}}\, d\gamma_2 =$$

$$= \left(-e^{-\frac{\gamma_2}{\bar\gamma_2}}\right)\Big|_0^\infty - \frac{1}{\bar\gamma_2} e^{-\frac{\gamma_{th}}{\bar\gamma_1}} \int_0^\infty e^{-\frac{\gamma_{th}}{\bar\gamma_1}\cdot\frac{C}{\gamma_2} - \frac{\gamma_2}{\bar\gamma_2}}\, d\gamma_2$$

$$= 1 - \frac{1}{\cancel{\bar\gamma_2}} e^{-\frac{\gamma_{th}}{\bar\gamma_1}} 2\sqrt{\frac{\gamma_{th}\bar\gamma_2\cdot C}{\bar\gamma_1}}\, K_1\left(2\sqrt{\frac{\gamma_{th} C}{\bar\gamma_2\bar\gamma_1}}\right) \rightarrow \text{FROM MAPLE} \rightarrow ⊕$$

RAYLEIGH

$$p_\alpha(\alpha) = \frac{\alpha}{\sigma^2} e^{-\frac{\alpha^2}{2\sigma^2}}.$$

$$\gamma = \alpha^2 \cdot \frac{E_s}{N_0} \qquad \Omega = E[\alpha^2] = 2\sigma^2$$

$$\bar{\gamma} = \Omega \frac{E_s}{N_0} \qquad \frac{E_s}{N_0} = \frac{\bar{\gamma}}{\Omega}$$

MILKA

$$\boxed{\frac{077625}{503}}$$

$$\alpha^2 = \frac{\Omega\gamma}{\bar{\gamma}} \qquad \alpha = \sqrt{\frac{\Omega\gamma}{\bar{\gamma}}} \qquad \boxed{\frac{d\gamma}{d\alpha} = 2\alpha \cdot \frac{\bar{\gamma}}{\Omega}}$$

$$\gamma = \alpha^2 \frac{\bar{\gamma}}{\Omega}.$$

$$p_\gamma(\gamma) = \frac{p_\alpha(\alpha)}{2\sqrt{\frac{\Omega\gamma}{\bar{\gamma}}} \cdot \frac{\bar{\gamma}}{\Omega}} = \frac{p_\alpha(\alpha)}{2\sqrt{\frac{\bar{\gamma}\gamma}{\Omega}}} \Bigg/ \alpha = \sqrt{\frac{\Omega\gamma}{\bar{\gamma}}}$$

$$p_\gamma(\gamma) = \frac{1}{2\sigma^2}\sqrt{\frac{\Omega}{\bar{\gamma}\gamma}} \cdot \sqrt{\frac{\Omega\gamma}{\bar{\gamma}}} \cdot e^{-\frac{\Omega\gamma}{\bar{\gamma}} \cdot \frac{1}{2\sigma^2}} = \frac{1}{\bar{\gamma}} \cdot e^{-\gamma/\bar{\gamma}}$$

$$(*)\ \boxed{P_{out} = 1 - 2\sqrt{\frac{\gamma_{th} C}{\bar{\gamma}_1 \bar{\gamma}_2}}\, e^{-\frac{\gamma_{th}}{\bar{\gamma}_1}} \cdot K_1\left(2\sqrt{\frac{\gamma_{th} C}{\bar{\gamma}_1 \bar{\gamma}_2}}\right)}$$

JOVAN STOSIĆ

DUAL-HOP FIXED GAIN

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$$\text{Denominator} = \sum_{n=1}^{N} \frac{\prod_{t=n+1}^{N} \gamma_t}{\prod_{t=1}^{n-1} G_t^2 \prod_{t=1}^{n-1} N_{0,t}} = \left| G_t^2 = \frac{1}{C \cdot N_{0,t}} \right|$$

$$\text{Denominator} = \sum_{n=1}^{N} \frac{\prod_{t=n+1}^{N} \gamma_t}{\prod_{t=1}^{n-1} \frac{1}{C \cancel{N_{0,t}}} \prod_{t=1}^{n-1} \cancel{N_{0,t}}} = \sum_{n=1}^{N} C^{n-1} \prod_{t=n+1}^{N} \gamma_t$$

$$SNR = \frac{\prod_{n=1}^{N} \gamma_n}{\sum_{n=1}^{N} C^{n-1} \prod_{t=n+1}^{N} \gamma_t} = |N=2| = \frac{\gamma_1 \cdot \gamma_2}{C^0 \cdot \gamma_2 + C \cdot 1} = \frac{\gamma_1 \gamma_2}{\gamma_2 + C}$$

$$\boxed{\frac{1}{\gamma_{eq}} = \left(\frac{\gamma_1 \gamma_2}{\gamma_2 + C}\right)^{-1} = \frac{\gamma_2 + C}{\gamma_1 \gamma_2} = \frac{1}{\gamma_1} + \frac{C}{\gamma_1 \gamma_2}}$$

$N = 3$

$$SNR = \frac{\gamma_1 \gamma_2 \gamma_3}{C^0 \cdot \gamma_2 \cdot \gamma_3 + C^1 \gamma_3 + C^2} = \frac{1}{\frac{1}{\gamma_1} + \frac{C}{\gamma_1 \gamma_2} + \frac{C^2}{\gamma_1 \gamma_2 \gamma_3}}$$

$\gamma_{eq} = \frac{\gamma_1 \gamma_2}{\gamma_1 + C}$ $p(\gamma_1, \gamma_2) = \frac{1}{\bar{\gamma}_1} e^{-\frac{\gamma_1}{\bar{\gamma}_1}} \cdot \frac{1}{\bar{\gamma}_2} e^{-\frac{\gamma_2}{\bar{\gamma}_2}}$

$p(\gamma_1, \gamma_2) = \frac{1}{\bar{\gamma}_1 \bar{\gamma}_2} e^{-\frac{\gamma_1 + \gamma_2}{\bar{\gamma}_1 \bar{\gamma}_2}}$

$\mu = \gamma_1 + c$

$\upsilon = \frac{\gamma_1 \gamma_2}{\gamma_1 + c}$ $\boxed{J = 0}$ $\frac{1}{\upsilon} = \frac{\gamma_1 + c}{\gamma_1 + \gamma_2} = \frac{1}{\gamma_2} + \frac{c}{\gamma_1 \gamma_2}$

$\left.\begin{aligned} \mu &= \gamma_1 + \gamma_2 \\ \upsilon &= \frac{\gamma_1 \cdot \gamma_2}{\gamma_1 + C} \end{aligned}\right\}$ $J = -\frac{\gamma_2 C}{\gamma_1 + C}$

$p(x) \cdot p(y)\, dx\, dy = p(\mu)\, p(\upsilon)\, d\mu\, d\upsilon$

$p(x) \cdot p(y) = |J| \cdot p(\mu) \cdot p(\upsilon)$

$p(\mu) \cdot p(\upsilon) = \frac{1}{|J|}\, p(x)\, p(y)$?

$p(\mu, \upsilon) = \frac{1}{|J|}\, p(x, y)$ $\Big/$ $x = f(\mu, \upsilon)$

$y = f(\mu, \upsilon)$

~~$\frac{1}{\gamma_2} = \frac{1}{\upsilon} + \frac{c}{\gamma_1 \gamma_2}$~~

$\gamma_2 = \upsilon \left(\frac{\gamma_1 + C}{\gamma_1} \right) = \upsilon \left(1 + \frac{C}{\gamma_1} \right)$ $\gamma_1 = \mu + \gamma_2$

$\gamma_2 = \upsilon \left(1 + \frac{c}{\mu + \gamma_2} \right)$ $\frac{\gamma_2}{\upsilon} = 1 + \frac{c}{\mu + \gamma_2}$

$\frac{\gamma_2}{\upsilon} = \frac{\mu + \gamma_2 + c}{\mu + \gamma_2}$ $\mu \gamma_2 + \gamma_2^2 = \mu \upsilon + \gamma_2 \upsilon + \upsilon c$

$\gamma_2^2 + \mu \gamma_2 - \mu \upsilon - \underline{\gamma_2 \upsilon} - \upsilon C = 0$

$\mu = \gamma_2$ $\boxed{\gamma_2 = \mu}$

$\upsilon = \frac{\gamma_1 \gamma_2}{\gamma_1 + c}$ $\upsilon = \frac{\gamma_1 \mu}{\gamma_1 + C}$ $\upsilon \cdot \gamma_1 + c\upsilon = \gamma_1 \mu$

$\gamma_1 (\upsilon - \mu) = -c \cdot \upsilon$

$\boxed{\gamma_1 = \frac{c \cdot \upsilon}{\mu - \upsilon}}$ $J = -\frac{\gamma_2 c}{(\gamma_1 + c)^2}$

$$p(\mu,\upsilon) = \frac{1}{|J|} \cdot p(\gamma_1, \gamma_2) = \frac{(\gamma_1 + c)^2}{\gamma_2 c} \cdot \frac{1}{\gamma_1 \gamma_2} \cdot e^{-\frac{\gamma_1 + \gamma_2}{\gamma_1 \gamma_2}}$$

$$\cancel{p(\mu,\upsilon) = \frac{(\gamma_1 + c)^2}{\gamma_1 \cdot \gamma_2 \cdot c} \cdot \gamma_1}$$

$$\boxed{\begin{aligned} \gamma_2 &= \mu \\ \gamma_1 &= \frac{c\upsilon}{\mu - \upsilon} \end{aligned}}$$

$$p(\mu,\upsilon) = \frac{\left(\frac{c\upsilon}{\mu-\upsilon} + c\right)^2}{\mu \cdot c} \cdot \frac{1}{\gamma_1 \gamma_2} e^{-\frac{\frac{c\upsilon}{\mu-\upsilon} + \mu}{\frac{c\mu\cdot\upsilon}{\mu-\upsilon}}}$$

$$= \frac{c\upsilon + \cancel{c\mu} - \upsilon c}{(\mu-\upsilon)^2 \cdot \cancel{\mu c}} \frac{1}{\gamma_1 \gamma_2} e^{-\frac{c\upsilon + \mu^2 - \upsilon\mu}{\cancel{(\mu-\upsilon)} \frac{c\mu\cdot\upsilon}{\cancel{(\mu-\upsilon)}}}}$$

$$= \frac{1}{(\mu-\upsilon)^2} \cdot \frac{1}{\gamma_1 \gamma_2} \exp\left[- \frac{\mu^2 - \upsilon\mu + c\upsilon}{c\mu \cdot \upsilon}\right]$$

$$p(\mu,\upsilon) = \frac{1}{\gamma_1 \gamma_2 (\mu-\upsilon)^2} \exp\left[- \frac{\mu}{c\upsilon} + \frac{1}{c} - \frac{1}{\mu}\right]$$

$$\boxed{p(\mu,\upsilon) = \frac{e^{+1/c}}{\gamma_1 \gamma_2 (\mu-\upsilon)^2} e^{-\frac{\mu}{c\upsilon} - \frac{1}{\mu}}}$$

$$p(\upsilon) = \int_{-\infty}^{\infty} p(\mu,\upsilon)\, d\mu$$

$f_{X_1X_2}(x_1, x_2)$

~~$y = g(x_1, x_2)$~~ $\qquad Y = g(x_1, x_2)$

$f_Y(y) = \int g(x_1, x_2) \cdot \frac{1}{|J|}$

$F_Y(y) = P_r\{g(\cancel{x_1, x_2}) \le y\} =$

$= \int_0 g(x_1, x_2) \cdot f_{X_1}(x_1) \cdot f_{X_2}(x_2)$

$Y = g(x_1, x_2)$, $Z = X_1$

~~$f_Y(y) = \int f_{X_1}(x_1) f_{X_2}(x_2) \frac{1}{|J|}$~~ $\Big|\, x_1 = \quad dx_1$

x

$f_{YZ}(y, z) = f_{X_1}(x_1) f_{X_2}(x_2) \frac{1}{|J|}$ $\Big|\, X_1 =$

$X_2 =$

$f_Y(y) = \int_0^\infty f_{X_1}(x_1) f_{X_2}(h(x_1, y)) \cdot \frac{1}{|J|} dx_1$

$P\{Y \le y\} = \int_0^y f_Y(t)\, dt = \int_0^\infty \left[\int_0^y f_{X_1}(x_1) f_{X_2}(h(x_1, t)) \cdot \frac{1}{|J|} dt\right] dx_1$

$\boxed{\int_0^y f_{X_2}(h(x_1, t)) \frac{1}{|J|} \cancel{dt}}$

$u = h(x_1, t)$

$du = h'(x_1, t) \cdot dt$

$F_{X_2}(y)$

$$p(y) = \int p(x,y)\cdot \cancel{p(x)}\, dx = \int p(y|x)\, p(x)\, dx$$

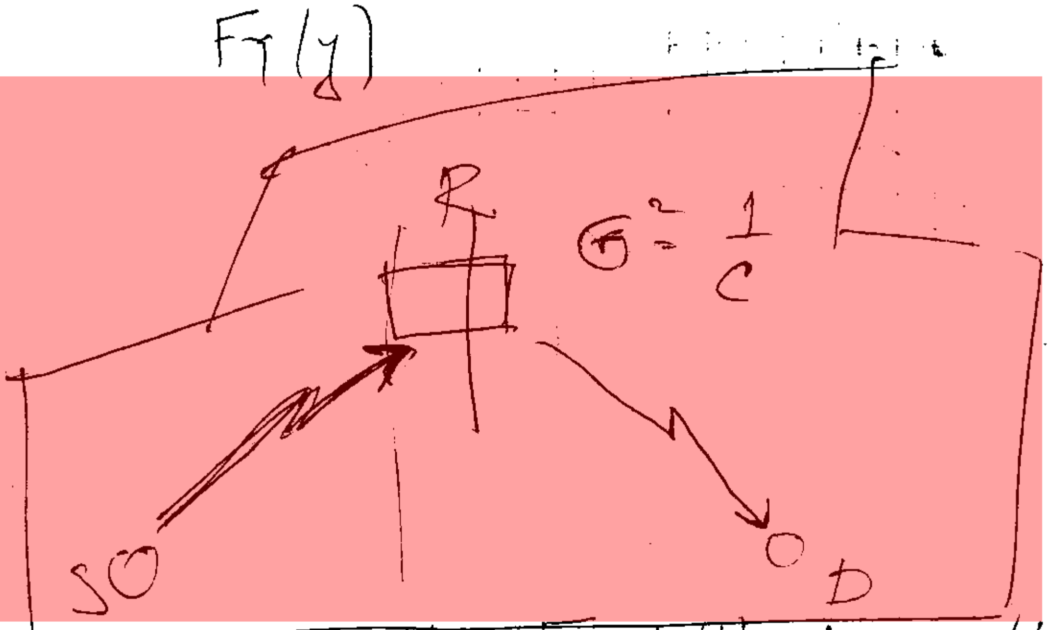
$$f_Y(y) = f_{Y|X}(y,x)\, f_X(x)$$

$$\int_0^y f_Y(t)\, dt =$$

$$F_Y(y) = F_{Y|X}(y,x)\cdot f_X(x)$$

$$f_Y(y) = \int_{-\infty}^{\infty} f_{YX}(x,y)\, dx$$

$F_Y(y)$



$$z_1(t) = \sqrt{E_1}\cdot \alpha_1(t)\cdot s(t) + n_1(t)$$

$$\cancel{z_2(t) = \sqrt{E_1}\cdot \sigma \cdot \sqrt{E_2}}$$

$$z_2(t) = \sqrt{E_2}\cdot \overset{\alpha_2}{\sigma}\, z_1(t) + n_2(t)$$

$$z(t) = \sqrt{E_1}\sqrt{E_2}\cdot \alpha_1(t)\, \alpha_2(t)\cdot \sigma\cdot s(t) + \sqrt{E_2}\cdot \sigma\cdot \alpha_2(t)\cdot \underline{n_1(t)} + n_2(t)$$

$$\gamma = \frac{E_1 E_2 \sigma^2 \alpha_1^2 \alpha_2^2 \cdot 1}{N_1 E_2 \sigma^2 \alpha_2^2 + N_2}$$

$$z_1(t) = \sqrt{E_1}\,\alpha_1(t)\,s(t) + n_1(t)$$

Siml. signal

$$\boxed{\alpha_1 =} \sqrt{n'^2(t) + n''^2(t)}$$

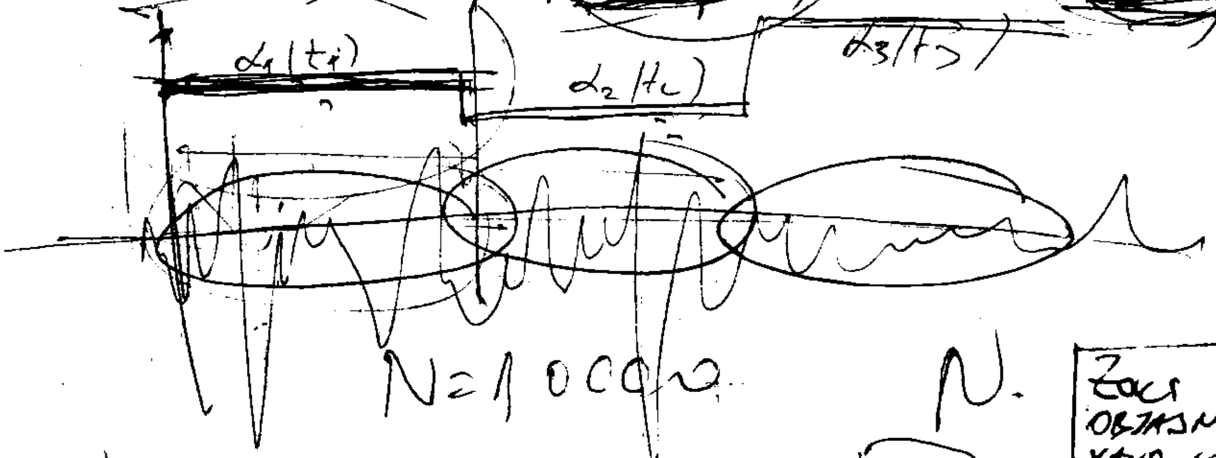
$s(t) \rightarrow$ nizata $N = 10000$ bitovi

α_1 α_1

1 1 0 1 1 0 0 1

$$\bar{\gamma} = 2 \cdot \frac{E_B}{N_0} = \frac{SNR}{} = 2 \cdot SNR$$

$$z_R(t) = \sqrt{E_1}\,\alpha_1(t_0)\,s(t_0) + n(t)$$



N = 10000

N.

Zošto
OBJASNUVA
KAKO SE IZVE-
DUVA T.E.
PRESMETUVA
MOMENTALEN SNR

$z_1(t)$

$$\text{momen. } SNR = \frac{\text{mom. signal}}{\text{signal šum}} = \frac{B}{I}$$

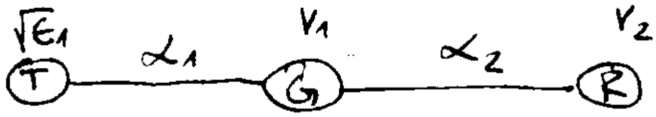
$$B = E_1 \alpha_1^2 s^2(t) = E_1 \alpha_1^2(t)$$

$$I = n^2(t)$$

$$SNR(t) = \frac{E_1 \alpha_1^2(t)}{N_0} \qquad \overline{SNR}$$

- OVA GO IMA VO CLANAK NA BRENAN ZA DIVERSITY (LINEAR DIVERSITY COMBINING TECHNIQUE)
- CLANCITE OD GEORGE I BRENAN

- α_2 NE GO SODRZI α_1 W (IMPLEMENTIRANO VO DualHopAaF-fixgain)
- SUMOT DOZO DA SE MENUVA.
- FIXED GAINS SO ARBITRARY No of HOPS. (KONCEPT ZA CLANAKOT)



$$r_1 = \sqrt{G_1}\cdot\alpha_1\cdot s(t) + n_1(t)$$

$$r_2 = \sqrt{E_2}\cdot\alpha_2 G r_1(t) + n_2 = \overbrace{\sqrt{E_2}\,\alpha_2\cdot\sqrt{E_1}\,\alpha_1 G s(t)}^{SIGNAL} +$$

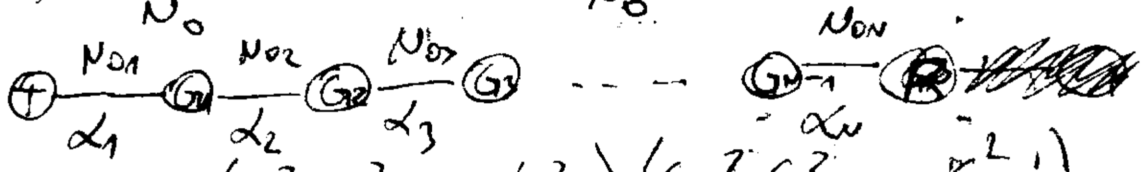
$$+\underbrace{\sqrt{E_2}\cdot\alpha_2 G\cdot n_1(t) + n_2}_{NOISE}$$

$$\boxed{HMV}$$

$$\boxed{\frac{S}{N} = \frac{G_1 E_2 \alpha_2\cdot\alpha_1^2\cdot G^2\cdot 1}{E_2\cdot\alpha_2^2 G^2 N_{01} + N_{02}}}$$

SE PRAŠUVAM ZOŠTO SUMATA NE SE KREVA NA KVADRAT TUKU SOBIROCITE POSEBNO !??
SE RABOTI ZA DVA RAZL. SIG!!!

$$\gamma = \alpha^2\cdot\frac{E_s}{N_0} \qquad \bar{\gamma} = \Omega\,\frac{E_s}{N_0} \qquad \Omega = E(\alpha^2)$$



$$SIG\_POW = (\alpha_1^2\cdot\alpha_2^2\dots\alpha_N^2)(G_1^2 G_2^2\dots G_{N-1}^2)$$

$$NOISE\_POW = N_{01}(\alpha_2^2\cdot\alpha_3^2\dots\alpha_N^2)(G_1^2 G_2^2\dots G_{N-1}^2) +$$

$$+ N_{02}(\alpha_3^2\cdot\alpha_4^2\dots\alpha_N^2)(G_2^2\cdot G_3^2\cdot G_{N-1}^2) + \dots + N_{0N}$$

$$\frac{S}{N} = \frac{\prod_{n=1}^{N}\alpha_n^2\prod_{n=1}^{N-1}G_n^2}{\sum_{n=1}^{N} N_{0,n}\prod_{t=n+1}^{N}\alpha_t^2\prod_{t=n}^{N-1}G_t^2}$$

$$\gamma_{eq} = \frac{\epsilon_1 \epsilon_2 \alpha_1^2 \cdot \alpha_2^2 \cdot G^2}{\epsilon_2 \alpha_2^2 \cdot G^2 \cdot N_{01} + N_{02}} = \frac{\boxed{\frac{\epsilon_1 \alpha_1^2}{N_{01}}} \cdot \frac{\alpha_2^2 \epsilon_2}{N_{02}} \cdot G^2 \cdot \cancel{N_{01} N_{02}}}{\cancel{G^2 \cdot N_{01} N_{02}} \left(\frac{\epsilon_2 \alpha_2^2}{N_{02}} + \frac{1}{G^2 N_{01}} \right)}$$

$$\boxed{\gamma_{eq} = \frac{\gamma_1 \cdot \gamma_2}{\gamma_2 + \frac{1}{G^2 N_{01}}}}$$

KN

KLASIČNA DEFINICIJA NA γ: $\gamma = \alpha^2 \frac{E}{N_0}$ VIDI N4.ppt

$\bar{\gamma} = \Omega \frac{E}{N_0}$

IF: $G^2 = \frac{1}{\epsilon_1 \cdot \alpha_1^2}$

$$\boxed{\gamma_{eq} = \frac{\gamma_1 \cdot \gamma_2}{\gamma_2 + \gamma_1}}$$

$$\boxed{\gamma_{eq} = \frac{\gamma_1 \cdot \gamma_2}{\gamma_2 + \frac{1}{G^2 N_{01}}}}$$

DUAL HOP

$$\boxed{\gamma_{eq} = \frac{1}{\frac{1}{\gamma_1} + \frac{1}{G^2 N_{01} \gamma_1 \gamma_2}}} \quad (*)$$

N-HOPS

$$SNR = \frac{\prod_{u=1}^{N} \alpha_u^2 \cdot \prod_{u=1}^{N-1} G_u^2}{\sum_{u=1}^{N} N_{0,u} \prod_{t=u+1}^{N} \alpha_t^2 \prod_{t=u}^{N-1} G_t^2} = \frac{\frac{\prod_{u=1}^{N} \alpha_u^2 \prod_{u=1}^{N-1} G_u^2}{\prod_{u=1}^{N} N_{0,u}}}{\sum_{u=1}^{N} N_{0,u} \frac{\prod_{t=u+1}^{N} \alpha_t^2 \prod_{t=u}^{N-1} G_t^2}{\prod_{t=1}^{N} N_{0,t}}}$$

$$\text{Denominator} = \sum_{u=1}^{N} \frac{1}{\prod_{t=1}^{u-1} N_{0t}} \cdot \prod_{t=u+1}^{N} \gamma_t \cdot \prod_{t=u}^{N-1} G_t^2$$

$$SNR = \frac{\prod_{u=1}^{N} \gamma_u \prod_{u=1}^{N-1} G_u^2}{\sum_{u=1}^{N} \frac{1}{\prod_{t=1}^{u-1} N_{0t}} \prod_{t=u+1}^{N} \gamma_t \cdot \prod_{t=u}^{N-1} G_t^2}$$

$$SNR = \frac{\prod_{u=1}^{N} \gamma_u}{\sum_{u=1}^{N} \frac{1}{\prod_{t=1}^{u-1} N_{0t} \prod_{t=1}^{u-1} G_t^2} \cdot \prod_{t=u+1}^{N} \gamma_t} = \frac{1}{\sum_{u=1}^{N} \frac{1}{\prod_{t=1}^{u-1} N_{0t} \prod_{t=1}^{u-1} G_t^2 \prod_{t=1}^{u} \gamma_t}}$$

$$\boxed{\gamma_{eq}^{-1} = \sum_{n=1} \frac{1}{\prod_{t=1}^{n-1} N_0 + \prod_{t=1}^{n-1} G_t^2 \prod_{t=1}^{n} \gamma_t \ldots}}$$ ⇒ PO OVAA FORMULA GO
IMPLEMENTIRAV VO
MATLAB.

IF HOPS = 2 $\quad \gamma_{eq}^{-1} = \sum_{n=1}^{2} \frac{1}{\prod_{1}^{n-1} N_0 + \prod_{1}^{n-1} G_t^2 \prod_{1}^{n} \gamma_t} = \frac{1}{\gamma_1} + \frac{1}{N_{01} G_1^2 \gamma_1 \gamma_2}$

$$\boxed{\gamma_{eq} = \frac{1}{\frac{1}{\gamma_1} + \frac{1}{N_{01} G_1^2 \gamma_1 \gamma_2}}}$$ $\boxed{\text{VI DI } ⊛}$

$snr = \frac{E_s}{N_0} \qquad N_0 = 2\sigma^2 \qquad snr = \frac{E_b}{2\sigma^2}; \quad E_s = k \cdot E_b; \quad E_b = \frac{E_s}{k}$

$snr = \frac{E_s}{2k\sigma^2} \qquad \sigma^2 = \frac{E_s}{2k\, snr} \qquad - M = 4 \quad k = ld M = 2$

$\sigma^2 = \frac{E_s}{4 \cdot snr} \qquad \sigma = \frac{1}{2}\sqrt{\frac{E_s}{snr}}$

$M = 2 \Rightarrow \quad \sigma^2 = \frac{E_s}{2 snr} \qquad \sigma = \cancel{\sqrt{\frac{E_s}{2snr}}}$

$E_d = \int_0^T [s_1(t) * s_2(t)]^2 dt = \int_0^T [A + A]^2 dt = \underline{4A^2 \cdot T}$

$s_1(t) = \frac{A}{2} \qquad s_2(t) = -\frac{A}{2} \qquad E = \int_0^T \left[\frac{A^2}{2} + \frac{A^2}{2}\right]^2 dt = A^2 \cdot T$

$E_{b1} = \int_0^T s_1^2(t)\, dt = A^2 T \qquad E_{b2} = \int_0^T A^2 dt = A^2 T$

$E_b = \frac{E_{b1} + E_{b2}}{2} = \frac{A^2T + A^2T}{2} = A^2 T$ $\Big\}$ $\underline{\text{BIPOLAR}}$

$E_s = \overset{1}{k} E_b = E_d \qquad \boxed{[-A, A]}$

$SNRq\text{-}dB = 20dB \qquad E_s = 1 \qquad \boxed{N_0 = 0.01}$

$SNR = 10 \log \frac{\alpha_i^2}{N_0}$

CONTINUATION (PERFORMANCE STUDY)

$P_{out} = 1 - 2\sqrt{\frac{c\gamma_{th}}{\gamma_1 \gamma_2}}\, e^{-\frac{\gamma_{th}}{\gamma_1}} K_1\left(2\sqrt{\frac{c\gamma_{th}}{\gamma_1 \gamma_2}}\right)$

$p_\gamma(\gamma) = \frac{dP_{out}}{d\gamma_{th}} = \frac{2}{\gamma_1} e^{-\frac{\gamma}{\gamma_1}} \left[\sqrt{\frac{c\gamma}{\gamma_1\gamma_2}}\, K_1\left(2\sqrt{\frac{c\gamma}{\gamma_1\gamma_2}}\right) + \frac{c}{\gamma_2} K_0\left(2\sqrt{\frac{c\gamma}{\gamma_1\gamma_2}}\right)\right]$

MAPLE:

$$p_\gamma(\gamma) = 2 e^{-\frac{\gamma}{\bar\gamma_1}} \left(\frac{\sqrt{c\gamma\bar\gamma_1\bar\gamma_2}}{\cancel{\bar\gamma_1}\bar\gamma_2\sqrt{\bar\gamma_2}} K_1\left(2\sqrt{\frac{c\gamma}{\bar\gamma_1\bar\gamma_2}}\right) + \frac{c\cancel{\bar\gamma_1}}{\bar\gamma_1\bar\gamma_2} K_0\left(2\sqrt{\frac{c\gamma}{\bar\gamma_1\bar\gamma_2}}\right)\right)$$

$$\bar\gamma_1\sqrt{\bar\gamma_1}$$

$$p_\gamma(\gamma) = \frac{2 e^{-\frac{\gamma}{\bar\gamma_1}}}{\bar\gamma_1} \left(\sqrt{\frac{c\gamma}{\bar\gamma_1\bar\gamma_2}} K_1\left(2\sqrt{\frac{c\gamma}{\bar\gamma_1\bar\gamma_2}}\right) + \frac{c}{\bar\gamma_2} K_0\left(2\sqrt{\frac{c\gamma}{\bar\gamma_1\bar\gamma_2}}\right)\right)$$

DERIVATE OF BESSEL FUNCTION

$$\boxed{\frac{d}{dx}\left(K_1(x)\right) = -\mathrm{BesselK}(0,x) - \frac{\mathrm{BesselK}(1,x)}{x}}$$

$$K_0(x) = \int_0^\infty \cos(x \sinh t)\, dt = \int_0^\infty \frac{\cos(xt)}{\sqrt{t^2+1}}\, dt$$

- MODIFIED FUNCTION OF SECOND KIND, 0 ORDER

$$z \frac{d}{dz} K_\nu(z) + \nu K_\nu(z) = -z K_{\nu-1}(z)$$

$$\frac{d}{dz} K_\nu(z) = -\frac{\nu}{z} K_\nu(z) - K_{\nu-1}(z)$$

$$\boxed{\frac{d}{dx} K_1(z) = -\frac{1}{x} K_1(x) - K_0(x)}$$

GRADSHTEYN RYZHIK

$$P_{out} = 1 - 2 \underbrace{\sqrt{\frac{c\gamma_{th}}{\bar\gamma_1\bar\gamma_2}}\, e^{-\frac{\gamma_{th}}{\bar\gamma_1}}}_{D_1} \underbrace{K_1\left(2\sqrt{\frac{c\gamma_{th}}{\bar\gamma_1\bar\gamma_2}}\right)}_{D_2}$$

$$\frac{d}{d\gamma_{th}}\left(P_{out}\right) = -2 D_1' \cdot D_2 + 2\sqrt{\frac{c\gamma_{th}}{\bar\gamma_1\bar\gamma_2}}\, e^{-\frac{\gamma_{th}}{\bar\gamma_1}} \cdot \left[\frac{1}{2\sqrt{\frac{c\gamma_{th}}{\bar\gamma_1\bar\gamma_2}}} \cdot K_1\left(2\sqrt{\frac{c\gamma_{th}}{\bar\gamma_1\bar\gamma_2}}\right) + K_0\left(2\sqrt{\frac{c\gamma_{th}}{\bar\gamma_1\bar\gamma_2}}\right)\right]$$

$$= -\cancel{2}\left[\sqrt{\frac{c}{\bar\gamma_1\bar\gamma_2}} \cdot \frac{1}{\cancel{2}} \frac{1}{\sqrt{\gamma_{th}}} \cdot e^{-\frac{\gamma_{th}}{\bar\gamma_1}} - 2\sqrt{\frac{c\gamma_{th}}{\bar\gamma_1\bar\gamma_2}} \cdot \frac{1}{\bar\gamma_1} e^{-\frac{\gamma_{th}}{\bar\gamma_1}}\right] K_1\left(2\sqrt{\frac{c\gamma_{th}}{\bar\gamma_1\bar\gamma_2}}\right) - 2D_1 D_2'$$

$$= -\sqrt{\frac{c}{\bar\gamma_1\bar\gamma_2\gamma_{th}}}\, e^{-\frac{\gamma_{th}}{\bar\gamma_1}} K_1\left(2\sqrt{\frac{c\gamma_{th}}{\bar\gamma_1\bar\gamma_2}}\right) + 2\sqrt{\frac{c\gamma_{th}}{\bar\gamma_1^2\bar\gamma_2}}\, e^{-\gamma_{th}/\bar\gamma_1} K_1\left(2\sqrt{\frac{c\gamma_{th}}{\bar\gamma_1\bar\gamma_2}}\right) +$$

$$+\ e^{-\gamma_{th}/\bar\gamma_1} K_1\left(2\sqrt{\frac{c\gamma_{th}}{\bar\gamma_1\bar\gamma_2}}\right) + 2\sqrt{\frac{c\gamma_{th}}{\bar\gamma_1\bar\gamma_2}}\, e^{-\frac{\gamma_{th}}{\bar\gamma_1}} K_0\left(2\sqrt{\frac{c\gamma_{th}}{\bar\gamma_1\bar\gamma_2}}\right)$$

$$\boxed{M_{\gamma_{eq}} = \frac{1}{\bar\gamma_1 s+1} + \frac{C\bar\gamma_1 s\, e^{\overline{\bar\gamma_2(\bar\gamma_1 s+1)}}}{\bar\gamma_2(\bar\gamma_1 s+1)^2} E_1\left(\frac{C}{\bar\gamma_2(\gamma_1 s+1)}\right)}$$

MGF FOR DUAL-HOP FIXED GAIN SYSTEM

RAYLEIGH

$$E_i(a,z) = \int_1 e^{-ll\cdot z}\, ll^{-a}\, dll$$

$$\boxed{E_1(z) \doteq \int_z^{\infty} \frac{e^{-t}}{t}dt \qquad |\arg z|<\pi}$$

**

OVOJ GO KORISTI MAPLE

Cauchy's PRINCIPAL VALUE OF INTEGRAL

$$E_i(x) = -⨍_{-x}^{\infty} \frac{e^{-t}}{t}dt = ⨍_{-\infty}^{x} \frac{e^{t}}{t}dt$$

$\gamma_{SNR} = \gamma \cdot$ in data rate

$$\gamma_1^{①*} = K\cdot\gamma_1$$

$$\gamma_2 = K\cdot\gamma_2$$

$$\gamma'_{eq} = \left(\frac{1}{\gamma'_1}+\frac{1}{\gamma'_2}\right)^{-1}$$

$$\gamma'_{eq} = \left(\frac{1}{K\gamma_1}+\frac{1}{K\gamma_2}\right)^{-1} = K\left(\frac{1}{\gamma_1}+\frac{1}{\gamma_2}\right)^{-1}$$

$$\boxed{\gamma'_{eq} = K\left(\frac{1}{\gamma_1}+\frac{1}{\gamma_2}\right) \doteq K\cdot\gamma_{eq}}$$

ISTOTO VAŽI ZA BILO KOJ ODNOS NA HOPOVI.

$$SNR = \frac{E_s/T}{N_0\cdot\frac{1}{T}} = \frac{E_s}{N_0}$$

$$\boxed{P_b(E) = \frac{1}{2}M_{\gamma_{eq}}(1)} \Rightarrow$$ BER OF BINARY DPSK

$$G^2 = E\left[\frac{\varepsilon_2}{\varepsilon_1\alpha_1^2+N_0}\right]$$ ⇒ GAIN FOR SEMI-BLIND RELAYS

FOR RAYLEIGH FADING

$$G^2 = E\left[\frac{\varepsilon_2}{\varepsilon_1\alpha_1^2+N_0}\right] = \int_0^{\infty}\frac{\varepsilon_2}{N_0(\gamma+1)}\cdot\frac{1}{\bar\gamma}e^{-\frac{\gamma}{\bar\gamma}}d\gamma =$$

$$= \frac{\varepsilon_2}{N_0\bar\gamma}\int_0^{\infty}\frac{e^{-\gamma/\bar\gamma}}{\gamma+1}d\gamma = \left|\begin{matrix}\mu=\gamma+1 & d\gamma=d\mu\\ \gamma=\mu-1 & \gamma=0\,/\,\mu=1\\ & \gamma=\infty\,/\,\mu=\infty\end{matrix}\right| =$$

$$= \frac{\varepsilon_2}{N_0\bar\gamma}\int_1^{\infty}\frac{e^{\frac{-\mu+1}{\bar\gamma}}}{\mu}d\mu = \frac{\varepsilon_2 e^{+\frac{1}{\bar\gamma}}}{N_0\bar\gamma}\int_1^{\infty}\frac{e^{-\frac{\mu}{\bar\gamma}}}{\mu}d\mu = \frac{\varepsilon_2 e^{\frac{1}{\bar\gamma}}}{N_0\bar\gamma}E_1(1/\bar\gamma)$$

$$G^2 = \int_0^\infty \frac{\varepsilon_2}{\varepsilon_1 \alpha_1^2 + N_0} \cdot \frac{1}{\bar\gamma_1} e^{-\frac{\gamma_1}{\bar\gamma_1}} d\gamma = \int_0^\infty \frac{\varepsilon_2}{N_0(\gamma_1 + 1)} \frac{1}{\bar\gamma_1} e^{-\frac{\gamma_1}{\bar\gamma_1}} d\gamma_1 =$$

$$= \frac{\varepsilon_2}{N_0 \bar\gamma_1^2} \int_0^\infty \frac{e^{-\gamma/\bar\gamma_1}}{\frac{\gamma_1+1}{\bar\gamma_1}} d\gamma = \left| \begin{array}{ll} \mu = \frac{\gamma_1+1}{\bar\gamma_1} & d\mu = \frac{d\gamma_1}{\bar\gamma} \\ \gamma_1 = \bar\gamma\mu - 1 & \gamma_1 = 0 \quad \mu = \frac{1}{\bar\gamma} \\ & \gamma_1 = \infty \quad \mu = \infty \end{array} \right|$$

$$= \frac{\varepsilon_2}{N_0} \int_{1/\bar\gamma_1}^\infty \frac{e^{-\bar\gamma_1 \mu}}{\mu} e^{\frac{1}{\bar\gamma_1}} d\mu = \frac{\varepsilon_2 e^{\frac{1}{\bar\gamma_1}}}{N_0 \bar\gamma_1^2} \int_{1/\bar\gamma_1}^\infty \frac{e^{-\bar\gamma_1 \mu}}{\mu} d\mu \dots$$

$$\downarrow \quad \varepsilon_1(1, \bar\gamma)$$

$$\boxed{\varepsilon_1(a, z) = \int_1^\infty \frac{e^{-x \cdot z}}{x^a} dx}$$

$$G^2 = \left| \begin{array}{ll} \mu = \gamma_1 + 1 & d\mu = d\gamma_1 \\ \gamma_1 = 0 & \mu = 1 \\ \gamma_1 = \mu - 1 & \end{array} \right| = \frac{\varepsilon_2 e^{\frac{1}{\bar\gamma_1}}}{\frac{\varepsilon_1 N_0 \bar\gamma_1}{\varepsilon_1}} \int^\infty \frac{e^{-\frac{\mu}{\bar\gamma_1}}}{\mu} \cdot d\mu = ⊛$$

$$\Omega = E(\alpha^2) \qquad \bar\gamma_1 = \Omega \cdot \frac{\varepsilon_1}{N_0} \qquad \Omega = \frac{\bar\gamma_1 \cdot N_0}{\varepsilon_1}$$

$$⊛ = \frac{\varepsilon_2 e^{1/\bar\gamma_1}}{\varepsilon_1 \Omega_1} \underbrace{\int_1^\infty \frac{e^{-\frac{\mu}{\bar\gamma_1}}}{\mu} d\mu}_{I} = \frac{\varepsilon_2 e^{1/\bar\gamma}}{\varepsilon_1 \Omega_1} \cdot \varepsilon_1\left(\frac{1}{\bar\gamma_1}\right)$$

$$\Gamma(x) = \int_0^\infty e^{-t} \cdot t^{x-1} dt \qquad \boxed{G^2 = \frac{\varepsilon_2 e^{1/\bar\gamma_1}}{\varepsilon_1 \Omega_1} \varepsilon_1\left(\frac{1}{\bar\gamma_1}\right)}$$

Ambrowitz: $\varepsilon_1(z) = \int_z^\infty \frac{e^{-t}}{t} dt$ } I VO MATLAB E VAKA DEFINIRAN !!!

$$I = \int_1^\infty \frac{e^{-\frac{\mu}{\bar\gamma_1}}}{\mu} d\mu = \left| \begin{array}{ll} \frac{\mu}{\bar\gamma_1} = x & d\mu = \bar\gamma_1 dx \\ \mu = \bar\gamma_1 x & \mu = 1 \quad x = \frac{1}{\bar\gamma_1} \end{array} \right| =$$

$$= \frac{1}{\bar\gamma_1} \cancel{\bar\gamma_1} \int_{1/\bar\gamma_1}^\infty \frac{e^{-x}}{x} dx = \underline{\frac{1}{\bar\gamma_1} \cdot \varepsilon_1\left(\frac{1}{\bar\gamma_1}\right)}$$

$$G^2 = \frac{\varepsilon_2}{N_0 \bar{\gamma}_1} \int_0^\infty \frac{e^{-\gamma_1/\bar{\gamma}_1}}{(\gamma_1+1)} d\gamma_1 = \left| \begin{array}{l} \gamma_1 + 1 = \mu \\ \gamma_1 = \mu - 1 \\ \gamma_1 = 0 \quad \mu = 1 \\ d\gamma_1 = d\mu \end{array} \right| = \frac{\varepsilon_2}{N_0 \bar{\gamma}_1} \int \frac{e^{-\mu/\bar{\gamma}_1} e^{+\frac{1}{\bar{\gamma}_1}}}{\mu} d\mu$$

$$G^2 = \frac{\varepsilon_2 e^{1/\bar{\gamma}_1}}{N_0 \bar{\gamma}_1} \int_1^\infty \frac{e^{-\mu/\bar{\gamma}_1}}{\mu} d\mu = \left| \begin{array}{l} \frac{\mu}{\bar{\gamma}_1} = x \quad d\mu = \bar{\gamma}_1 dx \\ \mu = 1 \quad x = 1/\bar{\gamma}_1 \end{array} \right|$$

$$G^2 = \frac{\varepsilon_2 e^{1/\bar{\gamma}_1}}{N_0 \bar{\gamma}_1} \int_{1/\bar{\gamma}_1}^\infty \frac{e^{-x}}{\mu} \cdot \bar{\gamma}_1 dx = \frac{\varepsilon_2 e^{1/\bar{\gamma}_1}}{\varepsilon_1 \left(\frac{N_0 \bar{\gamma}_1}{\varepsilon_1}\right)_2} \boxed{\underbrace{\int_{1/\bar{\gamma}_1}^\infty \frac{e^{-x}}{x} dx}_{E_1\left(\frac{1}{\bar{\gamma}_1}\right)}}$$

$$\boxed{G^2 = \frac{\varepsilon_2 e^{1/\bar{\gamma}_1}}{\varepsilon_1 \Omega} E_1\left(\frac{1}{\bar{\gamma}_1}\right)}$$

DOKAZANO!!!

~~$\frac{1}{C} = G^2 = \frac{\varepsilon_2 e^{1/\bar{\gamma}_1}}{\varepsilon_2 \Omega} E_1\left(\frac{1}{\bar{\gamma}_1}\right) \quad C = \frac{\varepsilon_1 \Omega}{\varepsilon_2 e^{1/\bar{\gamma}_1} E_1\left(\frac{1}{\bar{\gamma}_1}\right)}$~~

$$C = \frac{\varepsilon_2}{G^2 \cdot N_{01}} = \frac{\varepsilon_2}{N_{01} \frac{\varepsilon_2 e^{1/\bar{\gamma}_1}}{\varepsilon_1 \Omega} \cdot E_1\left(\frac{1}{\bar{\gamma}_1}\right)} = \frac{\left(\frac{\varepsilon_1 \Omega}{N_{01}}\right) \bar{\gamma}_1}{e^{1/\bar{\gamma}_1} E_1\left(\frac{1}{\bar{\gamma}_1}\right)}$$

$$\boxed{C = \frac{\bar{\gamma}_1}{e^{1/\bar{\gamma}_1} E_1\left(\frac{1}{\bar{\gamma}_1}\right)}}$$

$$\gamma_{eq} = \frac{\gamma_1 \gamma_2}{\gamma_2 + C}$$

$$\boxed{\gamma_{eq} = \frac{\gamma_1 \gamma_2}{\gamma_2 + \frac{\bar{\gamma}_1}{e^{1/\bar{\gamma}_1} E_1\left(\frac{1}{\bar{\gamma}_1}\right)}}}$$

FIXED GAIN

$$G^2 = \frac{\varepsilon_2}{C \cdot N_{01}}$$

VARIABLE GAIN:

$$G^2 = \frac{\varepsilon_2}{\varepsilon_1 \cdot \alpha_1^2 + N_{01}}$$

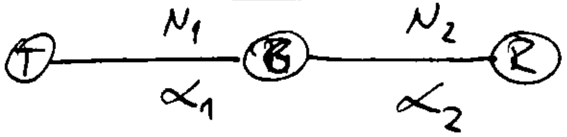
$$\gamma_{eq_2}^{-1} = \sum_{n=1}^{N} \frac{1}{\prod_{t=1}^{n-1} N_{0t} \prod_{t=1}^{n-1} G_t^2 \prod_{t=1}^{n} \gamma_t}$$

$$\boxed{\begin{array}{l} N=2 \\ \gamma_{eq_2}^{-1} = \frac{1}{\gamma_1} + \frac{1}{N_{01} G_1^2 \cdot \gamma_1 \cdot \gamma_2} \end{array}}$$

go_dB = [0, 5, 10]

$go = 10^{0.1 \cdot go\_dB} = [1, 3.1623\ 10]$

$$G_1^2 = \frac{1}{\alpha_1^2} \qquad \gamma_{eq_2}^{-1} = \frac{1}{\gamma_1} + \frac{1}{\boxed{\frac{N_0}{\alpha_1^2}} \cdot \underset{1/\gamma_1}{\cancel{\gamma_1}} \cdot \gamma_2} = \frac{1}{\gamma_1} + \frac{1}{\gamma_2}$$



$$r_1 = \alpha_1 \cdot Sig + \sqrt{N_1/2} = \alpha_1 \cdot Sig + n_1(t)$$
$$\gamma_1 = \alpha_1^2 Sig + N_1$$
$$r_2 = \alpha_2 \circ r_1 + n_2(t) = \alpha_1 \cdot \alpha_2 \cdot Sig + \alpha_2 n_1(t) + n_2(t)$$

- Riemann Sum

$$A = \sum_{i=1}^{N} f(x_i) \cdot \Delta x$$

$$G^2 = \mathcal{E}\left[\frac{\varepsilon_2}{\varepsilon_1 \alpha_1^2 + N_{01}}\right] \qquad \boxed{G^2(\varepsilon_1 \alpha_1^2 + N_{01}) \leq K \varepsilon_2} \quad ⊛$$

- MODIFIED RELAY GAIN

$$G_s^2 = \begin{cases} \frac{\varepsilon_2}{\varepsilon_1 \bar{\alpha}_1} e^{1/\bar{\gamma}_1} E_1\left(\frac{1}{\bar{\gamma}_1}\right) & \gamma_1 < T \\ \frac{\varepsilon_2}{\varepsilon_1 \alpha_1^2 + N_{01}}, & \gamma_1 > T \end{cases}$$

$$T = \frac{K \bar{\gamma}_1}{e^{1/\bar{\gamma}_1} E_1\left(\frac{1}{\bar{\gamma}_1}\right)} - 1$$

$$G^2 = \frac{\varepsilon_2}{\varepsilon_1 \Omega_1} e^{1/\bar\gamma_1} E_1\left(\frac{1}{\bar\gamma_1}\right) \longleftarrow \textcircled{x} \longrightarrow \frac{G^2}{} \cdot N_{01}\left(\overset{T}{\cancel{\gamma_1}} + 1\right) = K \varepsilon_2$$

$$\left(\gamma_1 + 1\right) = \frac{K}{G^2 \cdot N_{01}} \qquad \gamma_1 = \frac{K}{G^2 \cdot N_{01}} - 1$$

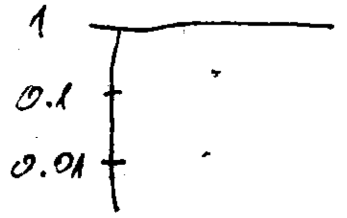
$$\gamma_1 = \frac{K \cancel{\varepsilon_2}}{\cancel{N_{01}} \frac{\cancel{\varepsilon_2}}{\cancel{\varepsilon_1}\ \bar\gamma_1 \cdot \frac{\cancel{N_{01}}}{\cancel{\varepsilon_1}}} \cdot e^{1/\bar\gamma_1} E_1\left(\frac{1}{\bar\gamma_1}\right)} - 1$$

~~$\bar\gamma_1 = E[\alpha_1^2]$~~ ~~$\boxed{\Omega = \bar\gamma_1 \frac{\varepsilon_1}{N_{01}}}$~~

$$\Omega = E[\alpha_1^2]$$

$$\bar\gamma_1 = \Omega \cdot \frac{\varepsilon_1}{N_{01}} \qquad \Omega = \bar\gamma_1 \frac{N_{01}}{\varepsilon_1}$$

$$\boxed{\gamma_1 = \frac{K \cdot \bar\gamma_1}{e^{1/\bar\gamma_1} E_1\left(\frac{1}{\bar\gamma_1}\right)} - 1 = T}$$



$$\gamma_{eq} = \begin{cases} \dfrac{\gamma_1 \gamma_2}{\gamma_2 + \dfrac{\bar\gamma_1}{e^{1/\bar\gamma_1} E_1(1/\bar\gamma_1)}}, & \gamma_1 < T \\ \dfrac{K \gamma_1 \gamma_2}{\gamma_1 + K\gamma_2 + 1}, & \gamma_1 > T \end{cases}$$

- DUAL-HOP NAKAGAMI THEORETICAL

$$p(\gamma_1, \gamma_2) = p(\gamma_1 / \gamma_2) \cdot p(\gamma_2) \qquad \int\int^{\gamma_{th}}$$

$$P_{out} = \int_0^{\gamma_{th}} \int_0^{\gamma_{th}} p(\gamma_1, \gamma_2)\, d\gamma_1\, d\gamma_2 = \int_0^{\gamma_{th}} \int_0^{\gamma_{th}} p(\gamma_1 / \gamma_2)\, p(\gamma_2)\, d\gamma_1\, d\gamma_2$$

$$P_{out} = \int_0^{\infty} P\left[\frac{\gamma_1 \gamma_2}{\gamma_1 + C} < \gamma_{th} / \gamma_2\right] p_{\gamma_2}(\gamma_2)\, d\gamma_2$$

$$P\left[\frac{\gamma_1 \gamma_2}{\gamma_2 + C} < \gamma_{th}\right] = P\left[\gamma_1 < \frac{\gamma_{th}(\gamma_2 + C)}{\gamma_2}\right]$$

$$p_\gamma(\gamma) = \frac{m^m \gamma^{m-1}}{\bar\gamma^m \Gamma(m)} \exp\left(-\frac{m\gamma}{\bar\gamma}\right)$$

$$P_{out} = \int_0^{\gamma_{th}} p_{\gamma_1}(\gamma_1)\, d\gamma_1 = \int_0^{\frac{\gamma_{th}(\gamma_2 + C)}{\gamma_2}} p_{\gamma_1}(\gamma_1)\, d\gamma_1 = P\left[\gamma_1 < \frac{\gamma_{th}(\gamma_2 + C)}{\gamma_2}\right.$$

$$\gamma_{eq} = \frac{\gamma_1 \cdot \gamma_2}{\gamma_2 + C} \qquad \frac{1}{\gamma_{eq}} = \frac{\gamma_2 + C}{\gamma_1 \cdot \gamma_2} = \frac{\cancel{\gamma_2}}{\gamma_1} + \frac{C}{\gamma_1 \cdot \gamma_2}$$

$$\boxed{\gamma_{eq} = \left(\frac{1}{\gamma_1} + \frac{C}{\gamma_1 \cdot \gamma_2}\right)^{-1}}$$

JOBAH CTOVUK
DALI E CRNO ILI
TEMNO SINO?

1.) Equal Power ~~Power~~ Transmission of sources with same average SNR

2) $Y_K = \epsilon_K \cdot \alpha_K Y_{K-1} + n_K$, -- — ✓

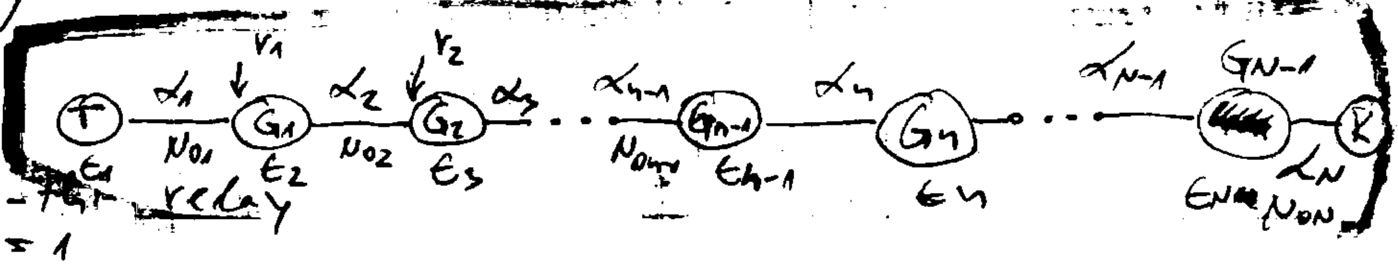
3.) Fig 3 AaF → DaF ✓ -

4.) $n = 2.285$ same shape for K (maching shapes) ✓

5.) C OD PRETHODNOT HOT ✓

6.) ISPRAVI SLUCAJ 1 ✓

7.) Rice + Naka ✓



n - [illegible] relay

$n = 1$

$$v_1 = \sqrt{\epsilon_1}\,\alpha_1 \cdot s + n_1$$

$n = 2$

$$v_2 = \alpha_2 \sqrt{\epsilon_2} \cdot \overset{G_1}{v_1} + n_2 = \alpha_2 \cdot \epsilon_2 \overset{G_1}{\sqrt{}} (\epsilon_1 \alpha_1 s + n_1) + n_2$$

$$= \sqrt{\epsilon_1 \epsilon_2}\,\alpha_1 \cdot \alpha_2 G_1 s + G_1 \sqrt{\epsilon_2}\,\alpha_2 \cdot n_1 + n_2$$

$$\left(\frac{S}{N}\right)_2 = \frac{G_1^2 \epsilon_1 \epsilon_2 \alpha_1^2 \alpha_2^2}{G_1^2 \epsilon_2 \alpha_2^2 \cdot N_{01} + N_{02}} = \frac{G_1 \epsilon_1 \epsilon_2 \alpha_1^2 \alpha_2^2}{\cancel{G_1} N_{01} N_{02} \left(\frac{\epsilon_2 \alpha_2^2}{N_{02}} + \frac{1}{G_1 N_{01}}\right)}$$

$$= \frac{\frac{\epsilon_1 \alpha_1^2}{N_{01}} \frac{\epsilon_2 \alpha_2^2}{N_{02}}}{\frac{\epsilon_2 \alpha_2^2}{N_{02}} + \frac{1}{G_1 N_{01}}} = \frac{\gamma_1 \gamma_2}{\gamma_2 + \frac{1}{G_1 N_{01}}}$$

$\boxed{n=2}$

$$r_2 = G_1 \alpha_2 \sqrt{\epsilon_2}\, r_1 + n_2 = G_1 \sqrt{\epsilon_1}\alpha_1\sqrt{\epsilon_2}\alpha_2 \cdot s(t) + G_1\sqrt{\epsilon_2}\alpha_2 n_1 + n_2$$

$\boxed{n=K}$ → AT THE RECEIVER OF THE K-th RELAY

$$\boxed{r_K = G_{K-1}\,\alpha_K \sqrt{\epsilon_K}\cdot r_{K-1} + n_K}$$

$$\int_0^{+\infty} e^{-u^2} dx = \frac{\sqrt{\pi}}{2}$$

$n=2$

SIG POWER = $G_1^2 \cdot \epsilon_1 \cdot \alpha_1^2 \cdot \epsilon_2 \cdot \alpha_2^2$

NOISE POWER = $G_1^2 \cdot \epsilon_2 \alpha_2^2 \cdot N_{01} + N_{02}$

$n=N$

SIG POWER = $(G_1^2 G_2^2 \cdots G_{N-1}^2)(\epsilon_1\alpha_1^2\epsilon_2\alpha_2^2 \ldots \epsilon_N\alpha_N^2)$

NOISE-POWER = $(G_1^2 G_2^2 \cdots G_{N-1}^2)(\epsilon_2\alpha_2^2\epsilon_3\alpha_3^2 \cdots \epsilon_N\alpha_N^2)N_{01} + (G_2^2 G_3^2 \cdots G_{N-1}^2)(\epsilon_3\alpha_3^2\epsilon_4\alpha_4^2 \cdots \epsilon_N\alpha_N^2)N_{02} + \ldots N_{0N}$

TOP : 5.2 BOTTOM : 5.2
LEFT : 4.4 RIGHT : 4.4

ČLANAKOT OD ERKIP

- ZORAN NEXT STEPS

1.) PRVO MUČEŠE DA PRESMETAM KAPACITET NA MULTIHOP SYSTEM (SO KORISTENJE NA TURBO CODER OD MATLAB) → POTREBNO DA ~~[illegible]~~ PRESMETAM BER ZA FADING.

2.) POTOA MU TREBA DEKA ODOBRENO DA NAPRAVIME ANALIZA NA MULTI-HOP SYSTEM SO POVEĆE ANTENI. ZA TAA CEL PRVO DA IMPLEMENTIRAM POVEĆE ANTENSKI SYSTEM.

... $(H \cdot H^T)$ DECOMPOSITION - - -

IMAM ČITANO VAŽNOV ČLANAK. NAJDI GO!!

- OPTIMAL POWER ALLOCATION (Hasna & Alouini)

$$P_{out} = P_r[\gamma_{min} = \min\{\gamma_1, \gamma_2, \ldots \gamma_N\} \le \gamma_{th}] \quad DaF$$

$$P_{out} = P_r[\gamma_{eq} \le \gamma_{th}]$$

$$\boxed{\gamma_n = G_n \cdot p_n}$$

$$P_{max} = K P_T, \quad 1/N < K < 1$$

---

min P_{out}

subject to $\begin{cases} \sum_{n=1}^{N} p_n = P_T \\ p_n \le P_{max}, \quad n = 1, \ldots N \end{cases}$

• REGENERATIVE SYSTEM WITH NO DIVERSITY:

$$P_{out} = 1 - P_r(\gamma_1 > \gamma_{th}) \cdot P_r(\gamma_2 > \gamma_{th}) =$$

$$= 1 - \int_{\gamma_{th}}^{\infty} \frac{1}{\bar{\gamma}_1} \cdot e^{-\gamma/\bar{\gamma}_1} d\gamma \int_{\gamma_{th}}^{\infty} \frac{1}{\bar{\gamma}_2} e^{-\gamma/\bar{\gamma}_2} d\gamma =$$

$$= 1 - e^{-\frac{\gamma}{\bar{\gamma}_1}} \Big|_{\gamma_{th}}^{\infty} \cdot e^{-\frac{\gamma}{\bar{\gamma}_2}} \Big|_{\gamma_{th}}^{\infty} = 1 - \left(0 - e^{-\frac{\gamma_{th}}{\bar{\gamma}_1}}\right)\left(0 - e^{-\frac{\gamma_{th}}{\bar{\gamma}_2}}\right)$$

$$\boxed{P_{out} = 1 - e^{-\gamma_{th}/\bar{\gamma}_1} e^{-\gamma_{th}/\bar{\gamma}_2} = 1 - e^{-\gamma_{th}\left(\frac{1}{\bar{\gamma}_1} + \frac{1}{\bar{\gamma}_2}\right)}}$$

$$\min \quad P_{out} = 1 - e^{-\gamma_{th}\left(\frac{1}{\bar{\gamma}_1} + \frac{1}{\bar{\gamma}_2}\right)}$$

$$\text{s.t} \quad \begin{cases} p_1 + p_2 = P_T \\ p_1 \leq P_{max} \quad n = 1, 2 \end{cases}$$

$$\boxed{\begin{array}{ll} \max & -\gamma_{th}\left(\frac{1}{\bar{\gamma}_1} + \frac{1}{\bar{\gamma}_2}\right) \\ \text{s.t} & \begin{cases} p_1 + p_2 = P_T \\ p_n \leq P_{max} \quad n = 1, 2 \end{cases} \end{array}}$$

$$\bar{\gamma}_n = G_n p_n \qquad G_n = \left.\frac{\varepsilon_n G_t G_r \lambda^2}{(4\pi)^2 d^\alpha L N_{0n}}\right\} \text{friis formula}$$

$$\boxed{\begin{array}{ll} \max & -\gamma_{th}\left(\frac{1}{G_1 \cdot p_1} + \frac{1}{G_2 p_2}\right) \\ \text{s.t.} & \begin{cases} p_1 + p_2 = P_T \\ p_n \leq P_{max} \end{cases} \end{array}}$$

Lagrange multiplier maximization method

$$J = -\gamma_{th}\left(\frac{1}{G_1 p_1} + \frac{1}{G_2 p_2}\right) - \eta\left(p_1 + p_2 - P_T\right)$$

$$\frac{dJ}{dp_1} = -\gamma_{th} \frac{(-1)}{G_1 p_1^2} - \eta = 0$$

$$\frac{dJ}{dp_2} = -\gamma_{th} \frac{(-1)}{G_2 p_2^2} - \eta = 0$$

$$\boxed{\begin{aligned} p_1^* &= P_T\left[1+\sqrt{\frac{G_1}{G_2}}\right]^{-1} \\ p_2^* &= P_T\left[1+\sqrt{\frac{G_2}{G_1}}\right]^{-1} \end{aligned}}$$

- IN PRACTICE $P_T < 2P_{max}$ $\quad P_{max} = K \cdot P_T \Rightarrow$
  $0.5 < K < 1$

$$J = \gamma_{th}\left(\frac{1}{G_1\gamma_1} + \frac{1}{G_2\gamma_2}\right) + \nu(\gamma_1+\gamma_2-P_T) + \mu_1(\gamma_1 - P_{max}) + \mu_2(\gamma_2 - P_{max})$$

NO CLIPPING : $\mu_1 = \mu_2 = 0$

$$p_1^* = \begin{cases} P_{max} & \frac{G_1}{G_2} < K \\ P_T\left[1+\sqrt{\frac{G_1}{G_2}}\right]^{-1} & K < \frac{G_1}{G_2} < \frac{1}{K} \\ P_T\, P_{max} & \frac{G_1}{G_2} > \frac{1}{K} \end{cases}$$

$$\gamma_2 = P_T - \gamma_1$$

- **DIVERSITY** (REGENERATIVE WITH DIVERSITY)

$$\boxed{P_{out} = \left(1 - e^{-\gamma_{th}\left(\frac{1}{\gamma_1}+\frac{1}{\gamma_2}\right)}\right)\left(1 - e^{-\frac{\gamma_{th}}{\gamma_3}}\right)}$$ DIVERSITY UNIFORM

$$\gamma_3 = G_3 \cdot \gamma_1$$

$$\boxed{\gamma_1^* = P_T\left[1 + \left(\frac{G_2}{G_3}\frac{1-e^{\gamma_{th}\left(\frac{1}{\gamma_1}+\frac{1}{\gamma_2}\right)}}{e^{\gamma_{th}/\gamma_3}-1} + \frac{G_2}{G_1}\right)^{-1/2}\right]^{-1}}$$ DIVERSITY OPTIMAL

$$P_{out} = 1 - \frac{2c}{\sqrt{\gamma_1\gamma_2}} K_1\left(\frac{2c}{\sqrt{\gamma_1\gamma_2}}\right) e^{-\gamma_{th}\left(\frac{1}{\gamma_1}+\frac{1}{\gamma_2}\right)}$$

- PROBLEM FORMULATION

$$\min P_{out} = 1 - \frac{2c}{\sqrt{\gamma_1\gamma_2}} K_1\left(\frac{2c}{\sqrt{\gamma_1\gamma_2}}\right) e^{-\gamma_{th}\left(\frac{1}{\gamma_1}+\frac{1}{\gamma_2}\right)}$$

$$\text{s.t.} \quad \begin{cases} \gamma_1 + \gamma_2 = P_T \\ \gamma_n \leq P_{max} \end{cases} \quad n = 1, 2$$

- DERIVATE EXPRESSION OF Bessel FUNCTION

$$z \frac{d}{dz} K_\nu(z) + \nu K_\nu(z) = -z K_{\nu-1}(z)$$

$$z \frac{d}{dz} K_\nu(z) = -\nu K_\nu(z) - z K_{\nu-1}(z)$$

example:

$$\frac{d}{dz} K_\nu(z) = -\frac{\nu}{z} K_\nu(z) - K_{\nu-1}(z)$$

$$\boxed{K_2'(z) = -\frac{2}{z} K_2(z) - K_1(z)}$$

$$p_1^* = \left[\frac{G_1}{G_2} \frac{1}{(P_T - p_1^*)^2} + \frac{c}{\gamma_{th}} \sqrt{\frac{G_1}{G_2}} \frac{K_0\left(\frac{2c}{\sqrt{\gamma_1 \gamma_2}}\right)}{K_1\left(\frac{2c}{\sqrt{\gamma_1 \gamma_2}}\right)} \cdot \left(\frac{1}{\sqrt{p_1^*(P_T - p_1^*)^3}} - \frac{1}{\sqrt{p_1^{*3}(P_T - p_1^*)}} \right) \right]^{-1/2}$$

$$\boxed{\begin{array}{l} c = \gamma_{th} \text{ IN OPTION } (2) \\ c = \sqrt{\gamma_{th}^2 + \gamma_{th}} \end{array}}$$

$\alpha = 3$

$$G_3 = \frac{G_2}{\left(1 + \left(\frac{G_2}{G_1}\right)^{-1/3}\right)^3}$$

Friis PROPAGATION FORMULA

$$P_r(d) = \frac{P_t G_t G_r \lambda^2}{(4\pi)^2 d^2 L}$$

$$\bar{\gamma}_n = G_n \cdot p_n$$

NUMBER OF HOPS

$$\sum_{n=1}^{N} p_n = P_T$$

lognormal shadowing

$$G_n = \frac{\varepsilon_n G_t G_r \lambda^2}{(4\pi)^2 d^{\alpha} L \, N_{on}}$$

$$\bar{\gamma}_n = \frac{\varepsilon_n p_n G_t G_r \lambda^2}{(4\pi)^2 d^{\alpha} L \, N_{on}}$$

$3 < \alpha < 4$ ⇒ URBAN ENVIRONMENT

$\alpha = 2$ ⇒ FREE SPACE

⊙ EXTENSION TO MULTIHOP TRANSMISSION

· Pout PdF Weibull

$$p(\gamma) = b \cdot A^{-b} g^{b-1} e^{-\left(\frac{g}{A}\right)^b} = b \cdot A^{-b} \gamma^{b-1} e^{-\left(\frac{\gamma}{A}\right)^b}$$

$$A = \frac{\bar{\gamma}}{\Gamma\left(1 + \frac{2}{c}\right)} \qquad b = \frac{c}{2} \qquad A = \frac{\bar{\gamma}}{\Gamma\left(1 + \frac{1}{b}\right)}$$

$$\int_{\gamma_{th}}^{\infty} p(\gamma) d\gamma = \left| \text{MAPLE} \right| = \underline{\underline{e^{-\gamma_{th}^{b} \cdot A^{-b}}}}$$

$$P_{out}\Big|_{DaF\,Wei} = 1 - e^{-\gamma_{th}^{b} A^{-b}} = 1 - e^{-(\gamma_{th}/A)^{b}} = \underline{\underline{1 - e^{-\left(\frac{\gamma_{th}}{A}\right)^{b}}}}$$

$$P_{out} = 1 - \prod_{n=1}^{N} e^{-\gamma_{th}/\bar{\gamma}_n}$$

$\underline{\underline{N=2}}$ $$P_{out_2} = 1 - e^{-\gamma_{th}/\bar{\gamma}_1} \cdot e^{-\gamma_{th}/\bar{\gamma}_2}$$

$N=1$ $$P_{out_1} = 1 - e^{-\gamma_{th}/\bar{\gamma}_1}$$

$$P_{out_2} = 1 - (1 - P_{out_1}) \cdot e^{-\gamma_{th}/\bar{\gamma}_2} = \underline{\underline{1 - e^{\gamma_{th}/\bar{\gamma}_1} \cdot e^{\gamma_{th}/\bar{\gamma}_2}}}$$

VO GENERALEN SLUČAJ:

$N=K$ $$\boxed{P_{out_K} = 1 - (1 - P_{out_{K-1}})\, e^{-\gamma_{th}/\bar{\gamma}_K}}$$

OPTIMIZATION PROBLEM FORMULATION:

$$\min \; P_{out} = 1 - \prod_{n=1}^{N} e^{-\gamma_{th}/\bar{\gamma}_n}$$

$$\text{s.t.} \; \begin{cases} \sum_{n=1}^{N} p_n = P_T \\ p_n \le P_{max} \quad n = 1, 2, \dots, N \end{cases}$$

$$\boxed{p_n^{*} = P_T \left[1 + \sqrt{G_n} \sum_{n=1,\, k \ne n}^{N} \frac{1}{\sqrt{G_k}} \right]^{-1}}$$

⇒ ANALOGNO E NA OVA
ZA DUAL-HOP

$G_1 = 2$ $\quad G_n = 2, 3, \dots 6$ $\quad \boxed{G_n = 2 \cdot G_{n-1}}$

$$G_3 = \frac{G_2}{\left(1 + \left(\frac{G_2}{G_1}\right)^{-1/3}\right)^3}$$

$G_1 = 1$
$G_2 = 10$ $\quad G_3 = ?$

$$G_3 = \frac{10}{\left(1 + \frac{1}{\sqrt[3]{10^3}}\right)^3} = \cancel{\frac{10}{(1+0{,}001)^3}} = \cancel{\frac{10}{1.003}} = \cancel{\underline{\underline{9.9701}}}$$

$$G = \frac{10}{1 + \frac{1}{\sqrt[3]{10}}} = 3.2 \,//$$

- DIRECT LINK

$$\gamma_u = \frac{\varepsilon_u p_u G_t G_r \lambda^2}{(4\pi)^2 d^\alpha \cdot L N_{0,u}}$$

$\boxed{\alpha = 3}$

$$\gamma_d = \frac{\varepsilon_u p_u G_t G_r \lambda^2}{(4\pi)^2 (2d)^\alpha \cdot L N_{0,u}} = \frac{1}{2^\alpha} \overbrace{\left(\frac{\varepsilon_u p_u G_t G_r \lambda^2}{(4\pi)^2 d^\alpha \cdot L N_{0,u}} \right)}^{\bar{\gamma}_1}$$

$$\boxed{\gamma_d = \frac{\bar{\gamma}_1}{2^\alpha}}$$

$\bar{\gamma}_1 = G_1 \cdot p_1 \qquad \bar{\gamma}_2 = G_2 \cdot p_2$

$G_1 = 1 \qquad G_2 = 10$

$\gamma_d = ?$

$$G_1 = \frac{\varepsilon_1 \cancel{p_1} G_{t_1} G_{r_1} \lambda^2}{(4\pi)^2 d^\alpha \cdot L \cdot N_{0,u}} \qquad G_2 = \frac{\varepsilon_1 G_{t_2} G_{r_2} \lambda^2}{(4\pi)^2 (\cancel{2}d)^\alpha \cdot L \cdot N_{0,u}}$$

$G_{t_2} G_{r_2} = 10 \cdot G_{t_1} G_{r_1} \qquad \boxed{\frac{G_{t_2} G_{r_2}}{G_{t_1} G_{t_1}} = 10}$

$p_1 + p_2 = P_t \qquad \underline{\bar{\gamma}_1 = G_1 \cdot p_1} \qquad \underline{\bar{\gamma}_2 = G_2 \cdot p_2}$

$$J = -\gamma_{th} \left(\frac{1}{G_1 p_1} + \frac{1}{G_2 p_2} \right) - \eta (p_1 + p_2 - P_T)$$

$$\frac{\partial J}{\partial p_1} = \frac{\gamma_{th}}{G_1 p_1^2} - \eta = 0$$

LAGRANGE MULTIPLIER METHOD

$$\frac{\partial J}{\partial p_2} = \frac{\gamma_{th}}{G_2 p_2^2} - \eta = 0 \Rightarrow \eta = \frac{\gamma_{th}}{G_2 p_2^2}$$

$$\frac{\partial J}{\partial \eta} = p_1 + p_2 - P_T = 0 \Rightarrow p_1 = P_T - p_2 \qquad p_2 = P_T - p_1$$

$$\frac{\cancel{\gamma_{th}}}{G_1 p_1^2} = \frac{\cancel{\gamma_{th}}}{G_2 p_2^2} \qquad G_1 p_1^2 = G_2 p_2^2 = G_2 (P_T^2 - 2P_T p_1 + p_1^2)$$

$$G_1 p_1^2 = G_2 P_T^2 - 2 G_2 P_T p_1 + G_2 p_1^2$$

$$(G_2-G_1)p_1^2 - 2G_2P_Tp_1 + G_2P_T^2 = 0$$

$$p_{1,2} = \frac{\cancel{2}G_2P_T \pm \sqrt{\cancel{4}G_2^2P_T^2 - \cancel{4}(G_2-G_1)G_2P_T^2}}{\cancel{2}(G_2-G_1)} =$$

$$= \frac{1}{G_2-G_1}\left(G_2P_T \pm \sqrt{\cancel{G_2^2P_T^2} - \cancel{G_2^2P_T^2} + G_1G_2P_T^2}\right) =$$

$$= \frac{1}{G_2-G_1}\left(G_2P_T \pm P_T\sqrt{G_1G_2}\right) = \frac{P_T\left(G_2 \pm \sqrt{G_1G_2}\right)}{G_2-G_1}\cdot\frac{\sqrt{G_1G_2}}{\sqrt{G_1G_2}}$$

$$= \frac{P_T\left(G_2\sqrt{G_1G_2} \pm G_1G_2\right)}{(G_2-G_1)\sqrt{G_1G_2}}$$

~~$G_2 \pm \sqrt{G_1G_2}) =$~~

$$\boxed{p_{1,2} = \frac{P_T\left(G_2 \pm \sqrt{G_1G_2}\right)}{G_2-G_1}} \quad ⊛$$

$$p_1 = \frac{P_T}{1+\frac{\sqrt{G_1}}{\sqrt{G_2}}}$$ [illegible]

⊛ $$p_{1,2} = \frac{P_T\cancel{G_2}\left(1 \overset{1}{\underset{2}{\pm}} \sqrt{\frac{G_1}{G_2}}\right)}{\cancel{G_2}\left(1-\frac{G_1}{G_2}\right)} = \frac{P_T\left(1 \overset{1}{\underset{2}{\pm}} \sqrt{\frac{G_1}{G_2}}\right)}{\left(1-\frac{G_1}{G_2}\right)}$$

$$\boxed{p_1^1 = \frac{P_T}{1-\sqrt{\frac{G_1}{G_2}}} \qquad p_1^2 = \frac{P_T}{1+\sqrt{\frac{G_1}{G_2}}}} \longrightarrow p < P_T$$

$$\boxed{p_1 = \frac{P_T}{\sqrt{\frac{G_1}{G_2}}+1}}$$ ✓ DOKAZANO

AKO SE REŠI PO p_2

$$\boxed{p_2 = \frac{P_T}{1+\sqrt{\frac{G_2}{G_1}}}}$$

020145234

$$P_{out} = 1 - 2\sqrt{\frac{C\gamma_{th}}{\bar\gamma_1\bar\gamma_2}}\, e^{-\frac{\gamma_{th}}{\bar\gamma_1}} K_1\left(2\sqrt{\frac{C\gamma_{th}}{\bar\gamma_1\bar\gamma_2}}\right)$$

(*)

$$P_{out} = 1 - \frac{2c_1}{\sqrt{\bar\gamma_1\bar\gamma_2}} K_1\left(\frac{2c_1}{\sqrt{\bar\gamma_1\bar\gamma_2}}\right) e^{-\gamma_{th}\left(\frac{1}{\bar\gamma_1}+\frac{1}{\bar\gamma_2}\right)}$$

(**)

$c_1 = \sqrt{\gamma_{th}^2+\gamma_{th}}$ [8]
$c_1 = \gamma_{th}$ [9]

(*) - IZRAZOT OD „PERFORMANCE STUDY ---"

(**) - ČISTIOT IZRAZ OD „OPTIMAL POWER ALL..."

$$C = \frac{\bar\gamma_1}{e^{1/\bar\gamma_1} E_1\left(\frac{1}{\bar\gamma_1}\right)}$$

(*) →

$$P_{out} = 1 - 2\sqrt{\frac{\cancel{\bar\gamma_1}\gamma_{th}}{e^{1/\bar\gamma_1}E_1\left(\frac{1}{\bar\gamma_1}\right)\cancel{\bar\gamma_1}\bar\gamma_2}}\, e^{-\frac{\gamma_{th}}{\bar\gamma_1}} K_1\left(2\sqrt{\frac{\gamma_{th}}{\bar\gamma_2E_1\left(\frac{1}{\bar\gamma_1}\right)}}\, e^{-\frac{1}{2\bar\gamma_1}}\right)$$

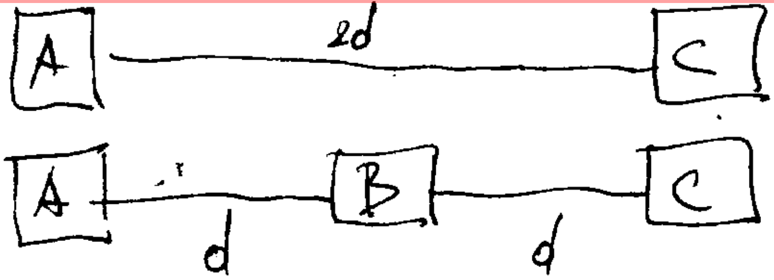
$$= 1 - 2\sqrt{\frac{\gamma_{th}}{\bar\gamma_2E_1\left(\frac{1}{\bar\gamma_1}\right)}}\, e^{-\frac{\gamma_{th}}{\bar\gamma_1}}\cdot e^{-\frac{1}{2\bar\gamma_1}} K_1\left(2e^{-\frac{1}{2\bar\gamma_1}}\sqrt{\frac{\gamma_{th}}{\bar\gamma_2E_1\left(\frac{1}{\bar\gamma_1}\right)}}\right)$$

$G_1 = l_i \cdot G_2$ $\boxed{l_i = 1:5:100}$

- EQUIVALENT COVERAGE / POWER CONSUMPTION
 book: Wireless Communications Systems A. Molisch.

(*)

(**)



$\boxed{P_r = \frac{P_T}{D^\alpha}}$

$\alpha = 2$ FREE SPACE

(*) $P_T = 2P$

$$\bar\gamma_D = \frac{2P_T}{\sigma^2(2d)^2} = \frac{2P}{\sigma^2 4\cdot d^2} = \frac{P}{\sigma^2 2d^2}$$

OPŠT SLUČAJ

$$\boxed{\bar\gamma_D = \frac{2P}{\sigma^2(2d)^\alpha}}$$

(**) $\bar\gamma_R = \frac{P}{\sigma^2 d^2}$ FREE SPACE; OPŠT SLUČAJ $\boxed{\bar\gamma_R = \frac{P}{\sigma^2 d^\alpha}}$

- $\alpha = 2$

$$P_{out_D} = 1 - e^{-\gamma_{th}/\bar{\gamma}_D} = 1 - e^{-\frac{\sigma^2 \cdot 2d^2}{P}}$$

$$P_{out_{R\_eq}} = 1 - e^{-\frac{\gamma_{th}}{\gamma_R}} \cdot e^{-\frac{\gamma_{th}}{\gamma_R}} = 1 - e^{-\frac{2}{\gamma_R}\gamma_{th}} = 1 - e^{-2\frac{\sigma^2 d^2}{P}\gamma_{th}}$$

$$\boxed{P_{out_D} = P_{out_{R\_eq}}}$$

$\boxed{S_{2}DO}$

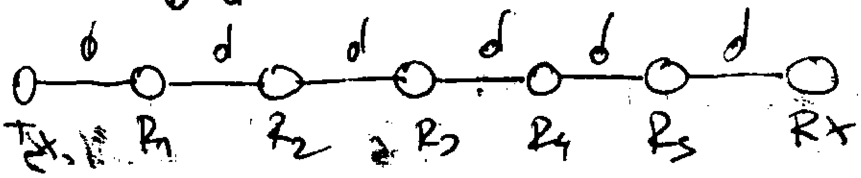
- $\boxed{\alpha = 3}$

$$P_{out_D} = 1 - e^{-\gamma_{th}/\bar{\gamma}_D} = 1 - e^{-\cancel{\gamma}\gamma_{th}/\bar{\gamma}_R} \quad -2\gamma_{th}/\bar{\gamma}_R$$

$$P_{out_{R-eq}} = 1 - e^{-2\gamma_{th}/\bar{\gamma}_R} = 1 - e$$

TUKA MI TEKNA ZOŠTO ZOŠ [illegible] $\bar{\gamma}_i$ DA E ISTO ZA SEKOJ HOP!

$$\bar{\gamma}_R = \frac{P}{\sigma^2 d^\alpha} = \frac{P}{\sigma^2 d^3} \qquad \bar{\gamma}_D = \frac{2P}{\sigma^2 (2d)^3} = \frac{2P}{\sigma^2 8 \cdot d^3} = \frac{\bar{\gamma}_R}{\cancel{4}}$$



- NEWTON METHOD

$$\boxed{x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}} \qquad \boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$$

$$p_1^* = P_T\left[1 + \left(\frac{G_2}{G_3}\,\frac{1 - e^{-\gamma_{th}(1/\bar{\gamma}_1 + 1/\bar{\gamma}_2)}}{e^{\gamma_{th}/\bar{\gamma}_3} - 1} + \frac{G_2}{G_1}\right)^{-1/2}\right]^{-1} =$$

$$\left[1 + \left(\frac{G_2}{G_3}\,\frac{1 - e^{-\gamma_{th}(1/\bar{\gamma}_1 + 1/\bar{\gamma}_2)}}{e^{\gamma_{th}/\bar{\gamma}_3} - 1} + \frac{G_2}{G_1}\right)^{-1/2}\right] - \frac{P_T}{P_1} = 0$$

- METHOD 2: EQUIVALENT ~~AVERAGE~~ SNR

$$\gamma_{eq} = \frac{1}{\prod_{n=1}^{N}\left(1 + \frac{1}{\gamma_n}\right) - 1}$$

$N = 2$

$$\gamma_{eq} = \frac{1}{\left(1 + \frac{1}{\gamma_1}\right)\left(1 + \frac{1}{\gamma_2}\right) - 1} = \frac{1}{\cancel{1+}\frac{1}{\gamma_2} + \frac{1}{\gamma_1} + \frac{1}{\gamma_1\gamma_2}\cancel{-1}}$$

$$= \frac{\gamma_1\gamma_2}{\gamma_1 + \gamma_2 + 1} = \left|\begin{matrix}\gamma_1 = \gamma_2 \\ = \bar{\gamma}_R\end{matrix}\right| = \frac{\bar{\gamma}_R^2}{2\bar{\gamma}_R + 1}$$

$$\gamma_{eq}^{-1} = \frac{1}{\gamma_1} + \frac{1}{\gamma_2} + \dots \frac{1}{\gamma_6} = \left|\gamma_1 = \gamma_2 = \dots \gamma_6 = \gamma\right|$$

$$\gamma_{eq}^{-1} = \frac{6}{\gamma}$$

• APPLICATION OF HARMONIC MEAN STATISTICS

$$\gamma_{eq} = \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2 + 1} \qquad G^2 = \frac{1}{a_1^2 + N_0}$$

$$\gamma_{eq_{21}} = \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} \qquad G^2 = \frac{1}{a_1^2}$$

$$P_t = \underbrace{\left(2 + 2^2 + 2^3 + 2^4 + \dots + 2^6\right)}_{(*)} \frac{P_T}{\frac{2(1-2^6)}{1-2}} = (*) \frac{P_T}{2 \cdot 63} = (*) \cdot \frac{P_T}{126}$$

$$S = \frac{1-2^N}{1-2}$$

$$S = 2^1 + 2^2 + \dots + 2^N$$

$$2S = 2^2 + \dots + 2^N + 2^{N+1}$$

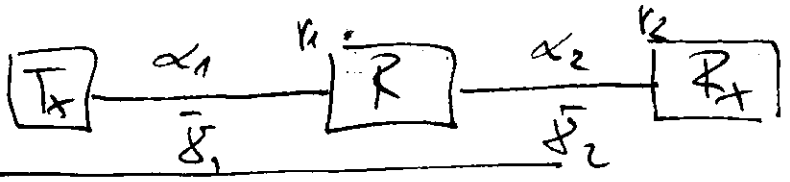
$$S - 2S = 2 - 2^{N+1}$$

$$\boxed{S = \frac{2(1-2^N)}{1-2}}$$

• BER FOR DUAL-HOP SYSTEM

$$M_{\gamma_{eq}}(s) = \frac{1}{(\bar{\gamma}_1 s + 1)} + \frac{C \bar{\gamma}_1 s \cdot e^{(C/\bar{\gamma}_2(\bar{\gamma}_1 s + 1))}}{\bar{\gamma}_2 (\bar{\gamma}_1 s + 1)^2} E_1\left(\frac{C}{\bar{\gamma}_2(\bar{\gamma}_1 + 1)}\right)$$

$$P_b(E) = \frac{1}{2} M_{\gamma_{eq}}(1)$$



1592

1580
1591
1594
1595

$$\boxed{\bar{\gamma}_1 = \bar{\gamma}_2 = 0 : 1 : 30}$$

$$r_1 = \alpha_1 \cdot s_1 + n_1$$

$$r_2 = \alpha_2 \cdot s_2 + n_2 \; ; \quad s_2 = G \cdot r_1 = G \cdot \alpha_1 \cdot s_1 + G n_1$$

$$r_2 = \alpha_2 (G \cdot \alpha_1 \cdot s_1 + G n_1) + n_2 = \underline{\alpha_1 \cdot \alpha_2 \cdot G} \cdot s_1 + \underline{\underline{\alpha_2 G n_1 + n_2}}$$

$$\gamma_{eq} = \frac{S}{N} = \frac{\alpha_1^2 \cdot \alpha_2^2 \cdot G^2}{\alpha_2^2 G^2 N_{01} + N_{02}} = \frac{\alpha_1^2 \alpha_2^2 \cdot G^2}{N_{01} N_{02} \left(\frac{\alpha_2^2 G^2}{N_{02}} + \frac{1}{N_{01}} \right)}$$

$$\gamma_{eq} \frac{S}{N} = \frac{\frac{\alpha_1^2}{N_{01}} \cdot \frac{\alpha_2^2}{N_{02}} \cancel{G^2}}{\cancel{G^2} \left(\frac{\alpha_2^2}{N_{02}} + \frac{1}{G^2 N_{01}} \right)} = \frac{\frac{\alpha_1^2}{N_{01}} \cdot \frac{\alpha_2^2}{N_{02}}}{\frac{\alpha_2^2}{N_{02}} + \frac{1}{G^2 N_{01}}} = \frac{\gamma_1 \gamma_2}{\gamma_2 + \frac{1}{G^2 N_{01}}}$$

IF: $\boxed{G^2 = \frac{1}{\alpha_1^2}}$ $\boxed{\frac{S}{N} = \frac{\gamma_1 \gamma_2}{\gamma_2 + \gamma_1}}$

IF: $G^2 = \frac{1}{\alpha_1^2 + N_{01}}$; $\frac{S}{N} = \frac{\gamma_1 \gamma_2}{\gamma_2 + \frac{\alpha_1^2 + N_{01}}{N_{01}}} = \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2 + 1}$

$\boxed{C = \frac{\varepsilon_2}{G^2 N_{01}}}$ $\boxed{G^2 = \frac{\varepsilon_2}{C N_{01}}}$

$\boxed{\varepsilon_1 = \varepsilon_2 = \text{mData.SNR}}$

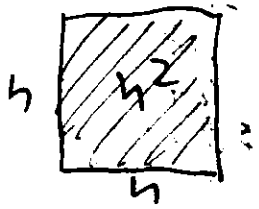
$\boxed{\gamma_1 = \frac{\varepsilon_1 \cdot \alpha_1^2}{N_{01}}}$ → NAMESTO mData.SNR NAZLOGICNO AI ECCO DA SDVAM $\boxed{\text{mData.e}}$!!!

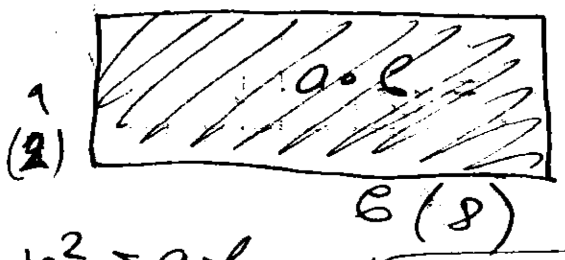
12TB ~ 24.990$ 1TB = 2082$ ≈ 2100$

$$\boxed{\gamma_{eq}^{-1} = \frac{\mu_H}{N} = \frac{1}{\frac{1}{\gamma_1} + \frac{1}{\gamma_2} + \dots \frac{1}{\gamma_N}}}$$

$$\mu_H(x_1, x_2) = \frac{2}{\frac{1}{x_1} + \frac{1}{x_2}} = \frac{2 \cdot x_1 x_2}{x_1 + x_2}$$

GEOMETRIC MEAN: 4, 8 $g_m = \sqrt{2 \cdot 8} = 4$





$$\boxed{h_2 = 2 \cdot 12 = 24}$$

$h^2 = a \cdot b$ $\boxed{h = \sqrt{ab} = 4}$

ZNAČI POENTATA E DA IMA ISTA POVRŠINA

$4^2 = 2 \cdot 8$ $\underline{\underline{16 = 16}}$

- GAMMA RV

$$\boxed{p_X(x) = \frac{x^{\alpha-1} e^{-x/\beta}}{\beta^\alpha \Gamma(\alpha)} u(x)}$$

$u(x) \rightarrow$ UNIT STEP FUNCTION

$X = G(\alpha, \beta) \Rightarrow$ NOTATION THAT X IS GAMMA DISTRIBUTED

- **Theorem 1** (PDF OF THE HARMONIC MEAN OF TWO GAMMA RVs)

$X_i \sim G(\alpha, \beta)$, $i = 1, 2$ $X = \mu_H(X_1, X_2)$

$$p_X(x) = \frac{\sqrt{\pi}\beta^{-\alpha}}{\Gamma^2(\alpha)} \left(\frac{x}{2}\right)^{\alpha-1} e^{-\frac{2x}{\beta}} \Psi\left(\frac{1}{2}-\alpha, 1-\alpha, \frac{2x}{\beta}\right) u(x)$$

$\Psi(\cdot, \cdot, \cdot) \Rightarrow$ CONFLUENT HYPERGEOMETRIC FUNCTION

$$\Gamma(a) U(a, b, z) = \int_0^\infty e^{-zt} t^{\alpha-1} (1+t)^{b-a-1} dt$$

- KUMMER'S EQUATION

$$\boxed{z \frac{d^2 w}{dz^2} + (b-z) \frac{dw}{dz} - aw = 0}$$

Corollary 1: (CDF OF THE HARMONIC MEAN OF TWO GAMMA RVs)

$$P_X(x) = \frac{\sqrt{\pi} x}{2^{2\alpha-2} \Gamma^2(\alpha) \beta} G_{23}^{21}\left(\frac{2x}{\beta} \middle| \begin{matrix} 0, \alpha - \frac{1}{2} \\ \alpha-1, 2\alpha-1, -1 \end{matrix}\right)$$

$G_{pq}^{mn}(\cdot)$ - Meijer's G - function

Corollary 2: (MGF OF THE HARMONIC MEAN OF TWO GAMMA RVs)

$X_i \sim \mathcal{G}(\alpha,\beta) \quad i=1,2 \quad X=\mu_H(x_1,x_2)$

$M_X(s) = \mathcal{E}_X(e^{sx})$

$M_x(s) = {}_2F_1(\alpha, 2\alpha; \alpha+\frac{1}{2}; \frac{\beta}{2}s)$

${}_2F_1(\cdot,\cdot;\cdot;\cdot) \Rightarrow$ GAUSS HYPERGEOMETRIC FUNCTION

$$\mathcal{E}(X^n) = \frac{d^n}{ds^n}(M_x(s))\Big|_{s=0} = \frac{\beta(\alpha)_n(2\alpha)_n}{2(\alpha+\frac{1}{2})_n}$$

$(x)_n = \frac{\Gamma(x+n)}{\Gamma(x)}$ $\Rightarrow$ Pochhammer's SYMBOL

$$M_\gamma(s) = \int_0^\infty e^{\gamma s}\cdot p(\gamma)\,d\gamma$$

$$\frac{d M_\gamma(s)}{ds} = \int_0^\infty \gamma\cdot e^{\gamma s}\cdot p(\gamma)\,d\gamma\Big|_{s=0} = \underbrace{\int_0^\infty \gamma\cdot p(\gamma)\,d\gamma}_{\bar{\gamma}}$$

$$\boxed{{}_2F_1(a,b;c;z) = \sum_{n=0}^\infty \frac{(a)_n(b)_n}{(c)_n}\frac{z^n}{n!} = \frac{\Gamma(b)}{\Gamma(a)\Gamma(b)}\sum_{n=0}^\infty \frac{\Gamma(a+n)\Gamma(b+n)}{\Gamma(c+n)}\frac{z^n}{n!}}$$

GAUSS HYPERGEOM. FUNCTION!!!

- Pochammer's symbol

$(z)_n = z(z+1)(z+2)\cdots(z+n-1) = \frac{\Gamma(z+n)}{\Gamma(z)}$

$\Gamma(z) = (z-1)!$

- $X = \mu_H(X_1,x_2) \qquad \mathcal{E}(X) = \frac{2\beta\alpha^2}{2\alpha+1}$

$$M_X(s) = {}_2F_1\left(\alpha, 2\alpha, \alpha+\frac{1}{2}; \frac{\beta}{2}s\right)$$

$$\left.\frac{d^n}{ds^n} M_X(s)\right|_{s=0} ; \qquad \frac{d^n}{dz^n} F(a,b;c;z) = \frac{(a)_n (b)_n}{(c)_n} F(a+n, b+n; c+n; z)$$

$$\frac{d^n}{ds^n} M_X(s) = \boxed{\frac{\beta}{2}} \frac{(\alpha)_n \cdot (2\alpha)_n}{\left(\alpha+\frac{1}{2}\right)_n} F\left(\alpha+n, 2\alpha+n; \alpha+\frac{1}{2}+n; 0\right) \quad 1$$

MATLE VELJ DOKLA OVA FRSZA DA SE KLONE $M_n y^n$

$$x(x-1)y'' + ((a+b+1)x - c)y' + aby = 0$$

$\boxed{x=1}$ $\quad 0 \cdot y'' + ((a+b+1) - c)y' + aby = 0$

$\boxed{x=0}$ $\quad -cy' + aby = 0 \qquad y = C \cdot e^{\frac{abx}{c}}$

$$\boxed{x=0 \quad y = C = 1}$$

$$E(x,1) = \frac{1}{2} \frac{a \cdot b \cdot d F([a+1, b+1], [c+1], \frac{1}{2}dx)}{c}$$

$\boxed{x=0}$ $\quad E(0,1) = \dfrac{1}{2} \dfrac{\alpha \cdot 2\alpha \cdot \beta}{\left(\alpha+\frac{1}{2}\right)} = \dfrac{2\alpha^2 \cdot \beta}{2\alpha+1}$

- HARMONIC MEAN OF F VARIATES

DEFINITION: „Y" FOLLOWS CENTRAL F DISTRO IF:

BSVEB ZITO

$$\boxed{f_Y(y) = \frac{\left(\frac{v}{m}\right)^{\frac{v}{2}} y^{\frac{v}{2}-1}}{B\left(\frac{v}{2}, \frac{m}{2}\right)\left(1+\frac{v}{m}y\right)^{\frac{v+m}{2}}} U(y)}$$

$$\Gamma(\alpha) = \int_0^\infty t^{\alpha-1} \cdot e^{-t} dt \qquad \beta(x,y) = \int_0^1 t^{x-1}(1-t)^{y-1} dt$$

$$r_B(t) = \alpha_1 s(t) + \boxed{n_1(t)}$$ — one sided spectral density PSD - N_0

$$r_C(t) = \alpha_2 G(\alpha_1 s(t) + n_1(t)) + n_2(t)$$

$$r_c(t) = \alpha_2 G \cdot \alpha_1 s(t) + \alpha_2 G n_1(t) + n_2(t)$$

$$P_S = \alpha_1^2 \cdot \alpha_2^2 \cdot G^2$$

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$$P_N = \alpha_2^2 \cdot N_{01} + N_{02}$$

$$\gamma_{eq} = \frac{P_S}{P_N} = \frac{\alpha_1^2 \alpha_2^2 G^2}{G^2 \alpha_2^2 N_{01} + N_{02}} = \frac{\alpha_1^2 \alpha_2^2 \cancel{G^2}}{\cancel{G^2} N_{01} N_{02} \left(\frac{\alpha_2^2}{N_{02}} + \frac{1}{G^2 N_{01}}\right)}$$

$$\boxed{\gamma_{eq_2} = \frac{\frac{\alpha_1^2}{N_{01}} \cdot \frac{\alpha_2^2}{N_{02}}}{\frac{\alpha_2^2}{N_{02}} + \frac{1}{G^2 N_{01}}}} \qquad \boxed{G^2 = \frac{1}{\alpha_1^2 + N_{01}}}$$

$$\gamma_{eq_1} = \frac{\gamma_1 + \gamma_2}{\gamma_2 + \frac{\alpha_1^2 + N_{01}}{N_{01}}} = \frac{\gamma_1 + \gamma_2}{\gamma_1 + \gamma_2 + 1}$$

$$G^2 = \frac{1}{\alpha_1^2} \Rightarrow \boxed{\gamma_{eq_2} = \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2}}$$

$$\gamma_{eq2} = \frac{\mu_H(\gamma_1, \gamma_2)}{2} \qquad \gamma \sim G\overset{\alpha,\beta}{\left(m, \frac{\bar\gamma}{m}\right)}$$

$$p_\gamma(\gamma) = \frac{\gamma^{\alpha-1} e^{-\frac{\gamma}{\beta}}}{\beta^\alpha \Gamma(\alpha)} = \frac{\gamma^{m-1}}{\Gamma(\alpha)\left(\frac{\bar\gamma}{m}\right)^m} e^{-\frac{m\gamma}{\bar\gamma}}$$

$$\boxed{p_\gamma(\gamma) = \frac{m^m \gamma^{m-1}}{\Gamma(\alpha) \cdot \bar\gamma^m} e^{-\frac{m\gamma}{\bar\gamma}}} \qquad \boxed{\begin{aligned}\alpha &= m \\ \beta &= \frac{\bar\gamma}{m}\end{aligned}}$$

OD TUKA SLEDUVA

$$p_H(\gamma) = 2\, p_x(2\gamma) \qquad \boxed{x = 2\gamma}$$

$$p_x(x) = \frac{\sqrt{\pi}\, \bar\gamma^{-m}}{\Gamma^2(m)\, m^m} \left(\frac{x}{2}\right)^{m-1} e^{-\frac{2x \cdot m}{\bar\gamma}} \psi\left(\frac{1}{2} - m, 1 - m, \frac{2xm}{\bar\gamma}\right) U(x)$$

$$\boxed{\begin{aligned} &y = 2x \qquad x = \frac{y}{2} \\ &p(y) = \left.\frac{p(x)}{\frac{\partial y}{\partial x}}\right|_{x = f(y)} \end{aligned}}$$

$$\gamma = \frac{x}{2} \qquad p(\gamma) = \left.\frac{p(x)}{\frac{\partial \gamma}{\partial x}}\right|_{x = 2\gamma}$$

$$\frac{\partial \gamma}{\partial x} = \frac{1}{2} \qquad p(\gamma) = 2 \cdot p(2\gamma)$$

$$p_H(\gamma) = 2 \frac{\sqrt{\pi}\, \bar\gamma^{-m}}{m^m \Gamma^2(m)} \cdot \gamma^{m-1} e^{-\frac{4\gamma \cdot m}{\bar\gamma}} \psi\left(\frac{1}{2} - m, 1 - m, \frac{4\gamma m}{\bar\gamma}\right) U(\gamma)$$

JS

$$p_\Gamma(\gamma) = \frac{2\sqrt{\pi}}{\Gamma^2(m)}\left(\frac{m}{\bar\gamma}\right)^m \cdot \gamma^{m-1} e^{-\frac{4\gamma m}{\gamma}} \psi\left(\frac{1}{2}-m, 1-m, \frac{4\gamma m}{\bar\gamma}\right) U(\gamma)$$

$$E(x) = \frac{2\beta\alpha^2}{2\alpha+1} \qquad \alpha = m \quad \beta = \frac{\bar\gamma}{m}$$

$$E(x) = \frac{2\bar\gamma\, m^{\cancel{2}}}{\cancel{m}(2m+1)} = \frac{2\bar\gamma m}{2m+1} \qquad \boxed{x = 2\gamma}$$

$$E(2\gamma) = \frac{2\bar\gamma m}{2m+1} \qquad \boxed{\cancel{2}E(\bar\gamma_{eq}) = \frac{\cancel{2}\bar\gamma m}{2m+1}}$$

OVA E GLAVNIOT MOMENT

***

$$\boxed{E(\bar\gamma_{eq}) = \frac{\bar\gamma m}{2m+1}} \qquad E(2\gamma) = 2\bar\gamma_{eq} \qquad \boxed{\bar\gamma_{eq} = \frac{\bar\gamma m}{2m+1}}$$

- Outage PROBABILITY

$$P_{out} = P[\gamma \leq \gamma_{th}] = \int_0^{\gamma_{th}} p_\Gamma(\gamma)\, d\gamma$$

$$P_\Gamma(\gamma) = \cancel{2}\, \frac{\sqrt{\pi}\, 2\gamma}{2^{2m-2}\Gamma^2(m)} \frac{m}{\bar\gamma}\, G^{21}_{23}\left(\frac{4\gamma m}{\bar\gamma}\,\middle|\, \begin{matrix} 0, m-\frac{1}{2} \\ m-1, 2m-1, -1 \end{matrix}\right).$$

$$P_{out} = P_\Gamma(\gamma) = \frac{\sqrt{\pi}\left(\frac{\gamma m}{\bar\gamma}\right)}{2^{2m-3}\Gamma^2(m)}\, G^{21}_{23}\left(4\frac{\gamma m}{\gamma}\,\middle|\, \begin{matrix} 0, m-\frac{1}{2} \\ m-1, 2m-1, -1 \end{matrix}\right)$$

- INTERFERENCE LIMITED SYSTEMS

SIR λ_i, $i = 1, 2$; N_I – NUMBER OF INTERFERERS

$$\boxed{p_\Lambda(\lambda) = B(m_D, N_I m_I)\, C^{m_D} \lambda^{m_D-1} (C\lambda+1)^{-m_D-N_I m_I} U(\lambda)}$$

$$C = \frac{m_D \Omega_I}{m_I \Omega_D}$$

$p_\Lambda(\lambda)$ - FOLLOWS CENTRAL F DISTRIBUTION

$$X = \frac{U_1/d_1}{U_2/d_2}$$

U_1, U_2 chi-square distributions with d_1 & d_2 degrees of freedom

$$f(x) = \sqrt{\frac{(d_1x)^{d_1} d_2^{d_2}}{(d_1x+d_2)^{d_1+d_2}}} \Big/ xB\left(\frac{d_1}{2}, \frac{d_2}{2}\right)$$

$x, d_1, d_2 > 0$

B – betta funct

200850 77.28.15.169

Ovenazh SIR

$$\boxed{\lambda_{eq} = \frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2}} \qquad P_{out} = P(\lambda < \lambda_{th}) = \int_0^{\lambda_{th}} p_\lambda(\lambda)\, d\lambda \qquad \boxed{C = \frac{m_D \Omega_I}{m_I \Omega_D}} \quad \boxed{\frac{m_I}{m_D} = \frac{\Omega_I}{C\Omega_D}}$$

$$p_\Lambda(\lambda) = \frac{B(N_I m_I, \frac{1}{2})\, C^{m_D - N_I m_I}}{2^{2N_I m_I - 1} B^2(m_D, N_I m_I)} \frac{\lambda^{m_D - N_I m_I - 1}}{(1 + C\lambda)^{m_D + N_I m_I}} \cdot$$

$$_2F_1\left(m_D + N_I m_I, N_I m_I; N_I m_I + \frac{1}{2}; -\frac{C^{-1}}{4\lambda(C\lambda+1)}\right) U(\lambda)$$

GAUSS HYPERGEOM

$a = m_D$ $\quad u = N_I m_I \Rightarrow$ ZAMENA VO (10)

$$p_Y(y) = \frac{2\left(\frac{m_D}{m_I N_I}\right)^{m_D N_I} B\left(\frac{m_I N_I}{2}, \frac{1}{2}\right)}{B^2\left(\frac{m_D}{2}, \frac{N_I m_I}{2}\right) \cancel{\frac{N_I m_I + 1}{2}; -\frac{\left(\frac{N_I m_I}{m_D}\right)^2}{y\left(y + 2\frac{N_I m_I}{m_D}\right)}\Big) U(\lambda)}} y^{\frac{m_D - N_I m_I - 2}{2}} \left(y + 2\frac{N_I m_I}{m_D}\right)^{-\frac{m_D + N_I m_I}{2}}$$

$$\cdot\ _2F_1\left(\frac{m_D + N_I m_I}{2}, \frac{N_I m_I}{2}, \frac{N_I m_I + 1}{2}; \underbrace{-\frac{\left(\frac{N_I m_I}{m_D}\right)^2}{y\left(y + 2\frac{N_I m_I}{m_D}\right)}}_{(*)}\right)$$

$$(*) = \frac{C^{-1}}{4\lambda(C\lambda+1)}; \quad ① = \frac{\left(\frac{N_I m_I}{m_D}\right)^2}{2\lambda\left(2\lambda + 2\frac{N_I m_I}{m_D}\right)} = \frac{\left(\frac{1}{C}\right)^2}{2\lambda\left(2\lambda + \frac{2}{C}\right)}$$

$$\Omega_I = \Omega_D = \Omega \Rightarrow \boxed{C = \frac{m_D}{m_I}} \qquad \text{ZNAČI: } \boxed{C = \frac{m_D}{m_I N_I}}$$

$$① = \frac{1}{4\lambda C^2\left(\lambda + \frac{1}{C}\right)} = \frac{1}{4\lambda C(C\lambda+1)} = \frac{C^{-1}}{4\lambda(C\lambda+1)}$$

~~40~~ ISPUŠTA DECENDETO SO ,2"

$$p_Y(2\lambda) = p_\Lambda(\lambda) = \frac{2\, C^{m_I N_I} B\left(m_I N_I, \frac{1}{2}\right) \cdot 2^{m_D - N_I m_I - 1}}{B^2(m_D, N_I m_I)} \frac{\lambda^{m_D - N_I m_I - 1}}{\left(2\left(\lambda + \frac{1}{C}\right)\right)^{m_D + N_I m_I}}$$

$$\cdot\ _2F_1\left(m_D + N_I m_I, N_I m_I; N_I m_I + \frac{1}{2}; \frac{C^{-1}}{4\lambda(C\lambda+1)}\right) =$$

$$p_\Lambda(\lambda) = \frac{② \, C^{m_I N_I + m_D + N_I m_I} B\left(m_I N_I, \frac{1}{2}\right) \lambda^{m_D - N_I m_I + 1}}{2^{2N_I m_I \, \textcircled{-1}} B^2(m_D, N_I m_I) (\lambda C + 1)^{m_D + N_I m_I}} \cdot (\dots)$$

(?)

$$C = \frac{m_D \Omega_I}{m_I \Omega_D} \qquad m_D = m_I \Rightarrow C = \frac{\Omega_I}{\Omega_D}$$

$$C \cdot \lambda = \underbrace{\left(\frac{\Omega_I}{\Omega_D}\right)}_{SIR^{-1}} \lambda = \frac{\lambda}{SIR} \qquad \frac{SIR}{\lambda} = \frac{1}{C\lambda} \qquad \boxed{C\lambda = \left(\frac{SIR}{\lambda}\right)^{-1}}$$

- AVERAGE BER

$$M_\gamma(s) = {}_2F_1\left(\alpha, 2\alpha; \alpha + \frac{1}{2}; \frac{\beta}{2} s\right) \qquad \boxed{\alpha = m \quad \beta = \frac{\bar{\gamma}}{m}}$$

$$M_\gamma(s) = {}_2F_1\left(m, 2m; m + \frac{1}{2}; \frac{\bar{\gamma}}{4m} \cdot s\right)$$

OVA E ZGODI [illegible] PRETPOSTAVKA $\bar{\gamma} \equiv \bar{\gamma}_{eq}$

CLOSED FORM FOR MGF FOR DUALHOP SYSTEM

$$\boxed{P_e(\epsilon) \stackrel{\Delta}{=} \int_0^\infty P_e(\epsilon/\gamma)\, p_\gamma(\gamma)\, d\gamma}$$

- MGF of ~~[illegible]~~ SNR for MRC (MAXIMAL RATIO COMBINER)

$$\gamma_t = \gamma_0 + \sum_{l=1}^{L} \gamma_t$$

$$M_{\gamma_t}(s) = M_{\gamma_0}(s) \prod_{i=1}^{L} M_{\gamma_t}(s)$$

$$M_\gamma(s) = \int_{-\infty}^{\infty} e^{\gamma s} p(\gamma)\, d\gamma$$

$$\gamma = \gamma_1 + \gamma_2 \qquad M_\gamma(s) = \int_{-\infty}^{\infty} e^{\gamma_1 s} \cdot e^{\gamma_2 s} p(\gamma)\, d\gamma$$

$$\boxed{M_\gamma(s) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{\gamma_1 s} e^{\gamma_2 s} p(\gamma_1)\, p(\gamma_2)\, d\gamma_1\, d\gamma_2}$$

$$M_\gamma(s) = \int_{-\infty}^{\infty} e^{\gamma_1 s} p(\gamma_1)\, d\gamma_1 \int_{-\infty}^{\infty} e^{\gamma_2 s} p(\gamma_2)\, d\gamma_2 = \underline{M(\gamma_1) M(\gamma_2)}$$

$$P_e(\epsilon/\gamma_1, \gamma_2) = P_e(\epsilon/\gamma_1) + P_e(\epsilon/\gamma_2) - 2P_e(\epsilon/\gamma_1) P_e(\epsilon/\gamma_2)$$

i.i.d $\quad P_e = 2P_e(\epsilon) - 2P_e^2(\epsilon)$

- DPSK over i.i.d in NAKAGAMI

$$P_b(e) = P(0)\,P(1/0) + P(1)\,P(0/1)$$

$$p_\gamma(\gamma) = \frac{m^m \gamma^{m-1}}{\Gamma(m)\,\bar{\gamma}^m}\, e^{-\frac{m\gamma}{\bar{\gamma}}}$$

$$p_\alpha(\alpha) = \frac{m^m \alpha^{2m-1}}{\Gamma(m)\,\Omega^m}\, e^{-\frac{m\alpha^2}{\Omega}}$$

- RAYLEIGH: $\boxed{p_\alpha(\alpha) = \frac{\alpha}{\sigma^2} \cdot e^{-\frac{\alpha^2}{2\sigma^2}}} = \frac{2\alpha}{\Omega}\, e^{-\frac{\alpha^2}{\Omega}}$

$$\boxed{p_\alpha(\alpha) = \frac{2\alpha}{\Omega}\, e^{-\frac{\alpha^2}{\Omega}}} \qquad \boxed{2\sigma^2 = \Omega} \qquad \sigma = \sqrt{\frac{\Omega}{2}}$$

ISTA RELACIJA KAKO σ_N vs N_0

$\sigma_N = \sqrt{\frac{N_0}{2}}$

$$p_{0\alpha}(\alpha) = \frac{m^m (\alpha+1)^{2m-1}}{\Gamma(m)\,\Omega_0^m}\, e^{-\frac{m(\alpha+1)^2}{\Omega_0}}$$

LOGICAL 0
" $-A$ " $A = 1$

$$p_{1\alpha}(\alpha) = \frac{m^m (\alpha-1)^{2m-1}}{\Gamma(m)\,\Omega_1^m}\, e^{-\frac{m(\alpha-1)^2}{\Omega_1}}$$

LOGICAL 1
" A " $A = 1$

- RAYLEIGH

$$P_b = \cancel{\frac{1}{2}}\left(\frac{1}{2}\sqrt{\pi} - \frac{1}{2}\sqrt{\pi}\,\mathrm{erf}(1)\right) \cdot \cancel{2} = \frac{\sqrt{\pi}}{2}\,\mathrm{erfc}(1)$$

$$\boxed{Q(z) = \frac{1}{2}\,\mathrm{erf}\,\frac{z}{\sqrt{2}}}$$

SVAKAK: $P_b = Q\left(\sqrt{\frac{E_d}{2N_0}}\right)$ $\qquad E_d = \int_0^T [s_1(t) - s_2(t)]^2\,dt = 4A^2T$

$$P_b = Q\left(\sqrt{\frac{4A^2T}{2N_0}}\right) \qquad E_b = \int_0^T A^2\,dt = A^2T$$

$$P_b = Q\left(\sqrt{\frac{4E_b}{2N_0}}\right) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$$

$E_b = 1;\ N_0 = 1$ $\qquad \boxed{P_b = \frac{1}{2}\,\mathrm{erfc}(1)}$

- MODULATION PERFORMANCE IN FADING AND MULTIPATH CHANNEL (RAPPAPORT)

$$r(t) = \alpha(t) e^{-j\theta(t)} s(t) + n(t) \qquad 0 \le t \le T$$

$$\boxed{P_e = \int_0^\infty P_e(x)\, p(x)\, dx}$$

MNOGU VAZNA DEFINICIJA ZA BER VO FLAT FEDING KANAL

KAKO SE DOAGJA DO NEA, VIDI NONCOHERENT BSK NA pp.101!!!

MMV

AVERAGING THE ERROR IN AWGN ~~CH~~ CHANNELS OVER THE FADING PROBABILITY DENSITY FUNCTION!!!

$P_e(x)$ – PROBABILITY OF ERROR FOR AN ARBITRARY SNR – x, WHERE

$$\boxed{x = \frac{\alpha^2 E_b}{N_0}}$$

α – GAIN OF THE CHANNEL

$p(x)$ – PROBABILITY DENSITY FUNCTION OF x DUE TO THE FADING CHANNEL

$$p(x) = \frac{1}{\bar\gamma} e^{-\frac{x}{\bar\gamma}} = \frac{1}{\Gamma} e^{-\frac{x}{\Gamma}}$$

– RAPPAPORT NOTATION

$$\boxed{\Gamma = \overline{\alpha^2} \frac{E_b}{N_0}}$$

$$\boxed{P_e(x) = \frac{1}{2} \operatorname{erfc}\left(\sqrt{\gamma}\right)_{x \equiv \gamma}}$$

BER ZA BPSK

$$\boxed{P_e(\gamma) = \frac{1}{2} \operatorname{erfc}\left(\sqrt{\gamma}\right)}$$

$$P_e = \int_0^\infty P_e(\gamma) \cdot \underline{p(\gamma)}\, d\gamma = \int_0^\infty \frac{1}{2} \operatorname{erfc}\left(\sqrt{\gamma}\right) \cdot \frac{1}{\bar\gamma} e^{-\frac{\gamma}{\bar\gamma}}\, d\gamma$$

$$P_e = \frac{1}{2\bar\gamma} \int_0^\infty e^{-\frac{\gamma}{\bar\gamma}} \operatorname{erfc}\left(\sqrt{\gamma}\right) d\gamma$$

$$\boxed{P_e = \frac{1}{2} \frac{\sqrt{\bar\gamma + 1} - \sqrt{\bar\gamma}}{\sqrt{\bar\gamma + 1}} = \frac{1}{2}\left(1 - \sqrt{\frac{\bar\gamma}{\bar\gamma + 1}}\right)}$$

AVERAGE BER RAYLEIGH IN FADING CHANNEL WITH USING PSK

© DTK
$$2\sigma_N^2 = \frac{N_0}{T}$$

$$\boxed{\sigma_N = \sqrt{\frac{N_0}{2T}}}$$

PSK
$$P(\varepsilon) = \frac{1}{2}\,\mathrm{erfc}\,\frac{\hat{\rho}}{\sqrt{2}} \qquad \hat{\rho} = \frac{A}{\sigma_N}$$

$$\gamma = \frac{E_B}{N_0} = \frac{A^2 \cancel{T}}{2\sigma_N^2 \cancel{T}} = \frac{A^2}{2\sigma_N^2} = \frac{\hat{\rho}^2}{2} \qquad \boxed{\hat{\rho} = \sqrt{2\gamma}}$$

KONVERZIJA OD DTK VO SKALAR MMV!!!

$$P(\varepsilon) = \frac{1}{2}\,\mathrm{erfc}\,\frac{\sqrt{\cancel{2}\gamma}}{\sqrt{\cancel{2}}} = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\gamma}\right)$$

$\boxed{\text{FSK COHERENT}}$
$$P(\varepsilon) = \frac{1}{2}\,\mathrm{erfc}\,\frac{\hat{\rho}}{2} = \frac{1}{2}\,\mathrm{erfc}\left(\frac{\sqrt{2\gamma}}{2}\right) = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{\gamma}{2}}\right)$$

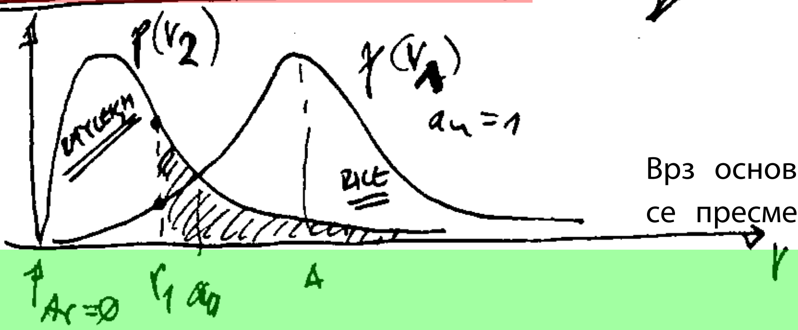
- AVERAGE BER IN RAYLEIGH BY USING $\boxed{\text{FSK COHERENT}}$:

$$P_e = \int_0^\infty P_{FSK}(\gamma)\cdot p_{RAY}(\gamma)\,d\gamma = \int_0^\infty \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{\gamma}{2}}\right)\frac{1}{\bar{\gamma}}e^{-\frac{\gamma}{\bar{\gamma}}}\,d\gamma$$

$$\boxed{P_e = \frac{1}{2}\left(1 - \sqrt{\frac{\bar{\gamma}}{\bar{\gamma}+2}}\right)}$$

$\boxed{\text{DTK SLIKATA}}$

- NONCOHERENT FSK



$$v(t) = r_1(t) - r_2(t)$$

$$\boxed{r_1(t) - r_2(t) \gtrless 0} \quad \begin{matrix} > \text{„1"} \\ < \text{„0"} \end{matrix}$$

Врз основ на оваа пресметка
се пресметува и BER во даден фединг канал

$\boxed{\text{MMV}}$

$P(r_2 > r_1)$, KOGA $a_n = 1$
$P(r_1 > r_2)$, KOGA $a_n = -1$
$r_1(t_0) = r_1$

$$P(r_2 > r_1) = P(0/1)_{r_1} = \int_{r_1}^{\infty} p(r_2)\,dr_2 = \int_{r_1}^{\infty} \frac{r_2}{\sigma_N^2} e^{-\frac{r_2^2}{2\sigma_N^2}}\,dr_2 = e^{-\frac{r_1^2}{2\sigma_N^2}}$$

$$P(0/1) = \overline{P(0/1)_{r_1}} = \int_0^\infty P(0/1)_{r_1}\,p(r_1)\,dr_1 = \int_0^\infty e^{-\frac{r_1^2}{2\sigma_N^2}}\cdot\frac{r_1}{\sigma_N^2}\,e^{-\frac{r_1^2+A^2}{2\sigma_N^2}}\,I_0\left(\frac{Ar_1}{\sigma_N^2}\right)dr_1$$

$$P(0/1) = \frac{1}{2}e^{-\frac{A^2}{4\sigma_N^2}}$$

101

$$P(0|1) = \frac{1}{2} e^{-\frac{A^2}{4\sigma_N^2}} = \frac{1}{2} e^{-\frac{A^2}{2\cdot N_0}} = \frac{1}{2} e^{-\frac{\mathcal{E}_b}{2N_0}} = \frac{1}{2} e^{-\frac{\gamma}{2}}$$

NONCOHERENT FSK BER: $\boxed{P(\mathcal{E}) = \frac{1}{2} e^{-\gamma/2}}$

- AVERAGE BER IN RAYLEIGH FADING BY USING NONCOHERENT FSK

$$Pe = \int_0^\infty P_{BKN}(\mathcal{E})\cdot p(\gamma)\,d\gamma = \int_0^\infty \frac{1}{2} e^{-\gamma/2}\cdot \frac{1}{\bar\gamma}\cdot e^{-\frac{\gamma}{\bar\gamma}}\,d\gamma$$

$\boxed{Pe = \frac{1}{2+\bar\gamma}}$ (USING MAPLE)

Ste87
Yao 92

- DPSK $\boxed{Pe_{DPSK} = \frac{1}{2} e^{-\gamma}}$

$\boxed{Pe = \int_0^\infty Pe_{DPSK}(\mathcal{E})\cdot p(\gamma)\,d\gamma = \frac{1}{2(\bar\gamma+1)}}$ → AVERAGE BER FOR DPSK IN RAYLEIGH CHANNEL

- GMSK (GAUSSIAN MINIMUM SHIFT KEYING)

$$Pe = Q\left(\sqrt{\frac{2C\cdot \mathcal{E}_b}{N_0}}\right) = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{C\,\mathcal{E}_b}{N_0}}\right) = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{C\gamma}\right)$$

$$C \doteq \begin{cases} 0.68 & BT = 0.25 \\ 0.85 & BT = \infty \end{cases}$$

BT – BANDWIDTH – BIT DURATION PRODUCT

$B = \frac{f_s}{2} = \frac{1}{2T_s}$ $\boxed{B\cdot T_s = 0.5}$ IDEAL NYQUIST

$\boxed{B = (1+r)\frac{f_s}{2} \qquad r = 0 \div 1}$

$$\boxed{Pe = \frac{1}{2}\,\frac{\sqrt{\frac{1+C\bar\gamma}{C\bar\gamma}} - 1}{\sqrt{\frac{1+C\bar\gamma}{C\bar\gamma}}} = \frac{1}{2}\left(1 - \sqrt{\frac{C\bar\gamma}{1+C\bar\gamma}}\right)}$$

$\boxed{\text{Ex. 5.12}}$ $P_e = ?$ IN RICIAN FADING (DPSK & NONCOH FSK)

$$P_{b\,DPSK} = \frac{1}{2} e^{-\gamma} \qquad P_{e\,NFSK} = \frac{1}{2} e^{-\gamma/2}$$

$$P_e = \int_0^\infty P_b(\gamma)\, p(\gamma)\, d\gamma \qquad p_\alpha(\alpha) = \frac{\alpha}{\sigma^2} e^{-\frac{\alpha^2-A^2}{2\sigma^2}} I_0\left(\frac{\alpha \cdot A}{\sigma^2}\right)$$

$$p_\gamma(\gamma) = \frac{(K+1)}{\bar\gamma} \exp\left(-(K+1)\frac{\gamma}{\bar\gamma} - K\right) I_0\left(2\sqrt{K(K+1)\frac{\gamma}{\bar\gamma}}\right) \qquad K = \frac{A^2}{2\sigma^2}$$

$$P_e = \frac{1}{2}\int_0^\infty e^{-\gamma} \frac{K+1}{\bar\gamma} e^{-(K+1)\frac{\gamma}{\bar\gamma}} \cdot e^{-K} I_0\left(2\sqrt{K(K+1)\frac{\gamma}{\bar\gamma}}\right) d\gamma$$

$$P_e = \frac{1}{2} \frac{K+1}{\bar\gamma} e^{-K} \int_0^\infty e^{-(K+1)\frac{\gamma}{\bar\gamma} - \gamma} I_0\left(2\sqrt{K(K+1)\frac{\gamma}{\bar\gamma}}\right) d\gamma$$

SOLVED BY MAPLE

$$\boxed{\begin{aligned} P_{e\,DPSK} &= \frac{1}{2} \frac{(K+1)}{\bar\gamma+K+1} \exp\left(-\frac{K\bar\gamma}{\bar\gamma+K+1}\right) \\ P_{e\,NFSK} &= \frac{K+1}{\bar\gamma+2K+2} \exp\left(-\frac{K\bar\gamma}{\bar\gamma+2K+2}\right) \end{aligned}}$$

⊙ BER FOR NAKAGAMI FADING

- DPSK $\boxed{P_b = \frac{1}{2} e^{-\gamma}}$

$$P_e = \int_0^\infty P_e(\gamma) \cdot p_\gamma(\gamma)\, d\gamma = \frac{1}{2}\int_0^\infty e^{-\gamma} \cdot \frac{m^m \gamma^{m-1}}{\Gamma(m) \cdot \bar\gamma^m} \cdot e^{-\frac{m\gamma}{\bar\gamma}} d\gamma$$

$$P_e = \frac{1}{2} \frac{m^m}{\Gamma(m) \cdot \bar\gamma^m} \int_0^\infty \gamma^{m-1} e^{-\frac{m\gamma}{\bar\gamma} - \gamma} d\gamma = \frac{1}{2}\left(\frac{\bar\gamma+m}{m}\right)^{-m}$$

$$\cancel{P_e = \frac{1}{2}\left(\frac{\bar\gamma+m}{m}\right)^{-m} \bar\gamma^m = \frac{1}{2}\left(\frac{m}{\bar\gamma+m}\right)^m \bar\gamma^m \cdot \bar\gamma^{-1}} = \frac{1}{2\cancel{\bar\gamma}} \cancel{\left(\frac{m \cdot \bar\gamma}{\bar\gamma+m}\right)^m}$$

$$\boxed{P_{e,DPSK} = \frac{1}{2}\left(\frac{m}{\bar\gamma+m}\right)^m}$$

BER FOR NAKAGAMI FADING IN SINGLEHOP DPSK SYSTEM

- DUALHOP REGENERATIVE SYSTEM IN NAKAGAMI FADING

$$\boxed{P_b = \left(\frac{m}{m+\bar\gamma}\right)^m - \frac{1}{2}\left(\frac{m}{m+\bar\gamma}\right)^{2m}}$$

⊙ ALTERNATIVE REPRESENTATION OF CLASSICAL FUNCTIONS (ALOUINI & SIMON BOOK)

- GAUSSIAN Q-FUNCTION

$$Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} dy \quad \Big\}$$

COMPLEMENT OF CDF OF GAUSSIAN PDF WITH $\mu = 1$ $\sigma = 1$

$$Q(x) = \frac{1}{\pi} \int_0^{\pi/2} \exp\left(-\frac{x^2}{2\sin^2\theta}\right) d\theta$$

$$Q(x) = \frac{1}{2} \operatorname{erfc} \frac{x}{\sqrt{2}}$$

Gradshteyn

$$\Phi(x) = \operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$$

$$I = \int_u^\infty \frac{\sqrt{x-u}}{x} e^{-\mu x} dx = \sqrt{\frac{\pi}{\mu}} e^{-\mu u} - \pi\sqrt{u}\left(1 - \Phi(\sqrt{\mu u})\right)$$

$$I = \int_u^\infty \frac{\sqrt{x-u}}{x} e^{-\mu x} dx = \sqrt{\frac{\pi}{\mu}} e^{-\mu u} - \pi\sqrt{u}\left(\underbrace{1 - \operatorname{erf}(\sqrt{\mu u})}_{\operatorname{erfc}(\sqrt{\mu u})}\right)$$

$$I = \sqrt{\frac{\pi}{\mu}} e^{-\mu u} - 2\pi \boxed{\frac{\sqrt{u}}{2} \operatorname{erfc}\left(\frac{\sqrt{2\mu u}}{\sqrt{2}}\right)} = \sqrt{\frac{\pi}{\mu}} e^{-\mu u} - 2\pi\sqrt{u}\, Q(\sqrt{2\mu u})$$

$$\sqrt{u} \cdot Q(\sqrt{2\mu u})$$

$$Q(\sqrt{2\mu u}) = \frac{1}{2\pi\sqrt{u}}\left(-\int_u^\infty \frac{\sqrt{x-u}}{x} e^{-\mu x} + \sqrt{\frac{\pi}{\mu}} e^{-\mu u}\right)$$

$$Q(\sqrt{2\mu u}) = \frac{1}{2\sqrt{\pi\mu u}} e^{-\mu u} - \frac{1}{2\pi\sqrt{u}} \underbrace{\int_u^\infty \frac{\sqrt{x-u}}{x} e^{-\mu x} dx}_{I_1}$$

$$I_1 = \int_u^\infty \frac{\sqrt{x-u}}{x} e^{-\mu x} dx = \left| \begin{matrix} x = y^2 \\ dx = 2y\, dy \\ x = u \quad y = \sqrt{u} \end{matrix} \right| = \int_{\sqrt{u}}^\infty \frac{\sqrt{y^2-u}}{\sqrt{y}} e^{-\mu y^2} 2y\, dy$$

$$= \int_{\sqrt{u}}^\infty \sqrt{y^2-u}\, e^{-\mu y^2} 2\sqrt{y}\, dy$$

$\boxed{070201005}$ vozče ↑ [redacted]

$\boxed{\begin{matrix}\text{Azokanoa}\\ 075848966\end{matrix}}$

$$I_1 = \int_{\mu}^{\infty} \frac{\sqrt{x-\mu}}{x} e^{-\mu x} dx$$

$x = tg\, y \qquad y = arctg\, x \qquad \begin{matrix} x = \mu & y = arctg(\mu) \\ x = \infty & y = \pi/2 \end{matrix}$

$$dx = \left(\frac{\sin(y)}{\cos(y)}\right)' dy = \frac{+\cos(y)\cdot\cos(y) + \sin(y)\sin(y)}{\cos^2(y)} dy = \frac{1}{\cos^2(y)} dy$$

$$dx = (1 + tg^2(y))\, dy = (1+x^2)\, dy \qquad \boxed{dy = \frac{dx}{1+x^2}}$$

$$\int \frac{dx}{1+x^2} = \left| \begin{matrix} x = tg\, y \\ y = arctg\, x \end{matrix} \right| = \int dy = y = arctg(x)$$

$$I_1 = \int_{arctg(\mu)}^{\pi/2} \frac{\sqrt{tg\,y-\mu}}{tg\,y} e^{-\mu\, tg\,y} \frac{dy}{\cos^2 y} = \int_{arctg(\mu)}^{\pi/2} \frac{\sqrt{tg\,y-\mu}}{tg\,y} (1+tg^2 y) e^{-\mu\, tg\,y} dy$$

$$\frac{1}{2}\left(1 - erf\, \frac{\sqrt{2}x}{2}\right) = \frac{1}{2}\, erfc \frac{\cancel{x}}{\sqrt{2}}$$

$$I_2 = \frac{1}{\pi}\int_0^{\pi/2} e^{-x^2/(2\sin^2\theta)} d\theta = \left| \mu = \frac{x^2}{2} \right| = \frac{1}{\pi}\int_0^{\pi/2} e^{-\mu/(\sin^2\theta)} d\theta$$

$$x = \frac{1}{\sin^2\theta} \qquad dx = \sin^{(-2+1)}(\theta)\cos\theta\, d\theta = \frac{\cos\theta}{\sin^3\theta} d\theta = \frac{d\theta}{tg\theta \sin^2\theta}$$

$$d\theta = tg\theta\cdot\sin^2\theta\, dx = tg\theta \frac{dx}{x}$$

$$I_2 = \frac{1}{\pi}\int_a^{\cancel{\pi/2}} e^{-\mu x} \left(-\frac{2dx}{x^3}\right)$$

$$\rightarrow x^2 = \frac{1}{\sin\theta} \qquad 2x\,dx = -\frac{1}{\sin^2\theta} d\theta = -x^4 d\theta \qquad \boxed{d\theta = -\frac{2dx}{x^3}}$$

$$\sin\theta = \frac{1}{x^2} \qquad \sin^2\theta = \frac{1}{x^4} \rightarrow$$

$$I_2 = -\frac{2}{\pi}\int_a^b \frac{e^{-x\mu}}{x^3} dx \quad \left| \begin{matrix} \theta = 0 \Rightarrow x = \infty \\ \theta = \frac{\pi}{2} \Rightarrow x = 1 \end{matrix} \right| = \frac{2}{\pi}\int_1^{\infty} \frac{e^{-x\mu} dx}{x^3}$$

$$I_2 = \frac{1}{\pi}\int_0^{\pi/2} \exp\left(-\frac{x^2}{2\sin^2\theta}\right) d\theta = \frac{2}{\pi}\int_1^{\infty} \frac{e^{-x\mu}}{x^3}\, dx$$

~~$= \mu^2\left(-\frac{1}{2}\frac{e^{-\mu x}}{\mu^2 x^2} + \frac{1}{2}\frac{e^{-\mu x}}{\mu x}\right)\Big|_1^{\infty} - \frac{1}{2}\int_1^{\infty} e^{-x\mu}$~~

~~$-Ei(1, \mu x) = \int \frac{e^{-x\mu}}{x}\, dx$~~

$$I_2 = \frac{2}{\pi}\left(\frac{1}{2}e^{-\mu} - \frac{1}{2}\mu\cdot e^{-\mu} + \frac{1}{2}\mu^2 \overbrace{\int_1^{\infty}\frac{e^{-x\mu}}{x}\, dx}^{\boxed{E_1(\mu)}}\right)$$

$$I = \int_{-\infty}^{\infty} e^{-x^2} dx = \left(\int_{-\infty}^{\infty} e^{-x^2} dx \int_{-\infty}^{\infty} e^{-x^2} dx\right)^{1/2} = \left(\int_{-\infty}^{\infty} e^{-x^2} dx \int_{-\infty}^{\infty} e^{-y^2} dy\right)^{1/2} =$$

$$I^2 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-(x^2+y^2)}\, dx\, dy = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-(x^2+y^2)}\, r\cdot dr\, d\theta$$

$x = r\cdot\cos\theta$
$y = r\cdot\sin\theta$
$x^2+y^2 = r^2$

$$J = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} \end{vmatrix} = \begin{vmatrix} \cos\theta & -r\cdot\sin\theta \\ \sin\theta & r\cdot\cos\theta \end{vmatrix}$$

$J = r\cos^2\theta + r\sin^2\theta = r(\sin^2\theta + \cos^2\theta) = r$

$dx\,dy = J\, dr\cdot d\theta = r\, dr\, d\theta$

$p(x,y)\, dx\, dy = p(r,\theta)\cdot J\cdot dr\cdot d\theta \qquad p(r,\theta) = p(x,y)\,\frac{dx\,dy}{dr\,d\theta} = \frac{p(x,y)}{\frac{dr\,d\theta}{dx\,dy}}$

$$I^2 = \left| \begin{matrix} x = -\infty & \cos\theta = \frac{x}{r} = -1 \Rightarrow \theta = \pi \\ r = 0..\infty & \theta = 0..2\pi \end{matrix} \right| = \int_0^{2\pi}\int_0^{\infty} e^{-r^2}\, r\, dr\, d\theta$$

$$I^2 = 2\pi\cdot\int_0^{\infty} e^{-r^2} r\, dr = \Big|\ \cancel{e^{-r^2}}\ \Big|^{0} = \pi\int_0^{\infty} e^{-r^2} d(r^2) = \pi\int_0^{\infty} e^{-\mu} d\mu$$

$$I^2 = \pi(-e^{-\mu})\Big|_0^{\infty} = -\pi\left(e^{-\infty}\xrightarrow{}0 - e^0\right) = \pi \qquad \boxed{I = \sqrt{\pi}}$$

$$Q(x)=\int_x^\infty \frac{1}{\sqrt{2\pi}}e^{-y^2/2}\,dy = \left(\int_x^\infty \frac{1}{\sqrt{2\pi}}\overset{(*)}{e^{-y^2/2}}\,dy \int_x^\infty \frac{1}{\sqrt{2\pi}}e^{-y^2/2}\,dy\right)^{1/2}$$

$$Q^2 = \left(\frac{1}{2\pi}\right)\left(\int_\mu^\infty e^{-y^2/2}\,dy \int_\mu^\infty e^{-x^2/2}\,dx\right) = \frac{1}{2\pi}\left(\iint_{\mu\ \mu}^{\infty} e^{-\frac{x^2+y^2}{2}}\,dx\,dy\right)$$

$x = r\cdot\cos\theta$
$y = r\cdot\sin\theta$
$x^2+y^2 = r^2$

$|J| = r$

$\theta = 0 \div \pi$
$r = \frac{\mu}{\sin\theta} \div \infty$

$$\boxed{r = \frac{\mu}{\sin\theta}}$$

~~$Q^2 = \frac{1}{4\pi}\int_\mu^\infty\int_0^\pi e^{-\frac{r^2}{2}}\,r\cdot dr\,d\theta = \frac{1}{4\pi}\int_0^\pi d\theta\int_{\frac{\mu}{\sin\theta}}^{\infty} e^{-\frac{r^2}{2}}\,r\,dr$~~

~~$Q^2 = \frac{1}{2\pi}\int_{\frac{\mu}{\sin\theta}}^{\infty} e^{-r^2/2}\,r\,dr$~~

$$\frac{1}{2\pi}\int_\mu^\infty\int_0^\infty e^{-\frac{x^2+y^2}{2}}\,dx\,dy$$

$x = \mu \quad r = \frac{\mu}{\cos\theta}$
$x = \infty \quad r = \infty$
$y = 0 \quad \theta = \operatorname{arctg}\left(\frac{0}{r}\right) \rightarrow 0$
$y = \infty \quad \theta = \frac{\pi}{2}$

$$Q^2 = \frac{1}{2\pi}\int_{\frac{\mu}{\cos\theta}}^{\infty}\int_0^{\pi/2} e^{-r^2/2}\,r\,dr\,d\theta = \frac{1}{2\pi}\int_0^{\pi/2}\underbrace{\left[\int_{\frac{\mu}{\cos\theta}}^{\infty} e^{-\frac{r^2}{2}}\,r\,dr\right]}_{I}d\theta$$

$$I = \int_{\frac{\mu}{\cos\theta}}^{\infty} e^{-\frac{r^2}{2}}\,r\,dr = \frac{2}{2}\int_{\frac{\mu}{\cos\theta}}^{\infty} e^{-\frac{r^2}{2}}\,d\frac{r^2}{2} = -e^{-\frac{r^2}{2}}\Bigg|_{\frac{\mu}{\cos\theta}}^{\infty}$$

$$I = e^{-\frac{\mu^2}{2\cos^2\theta}} - 0$$

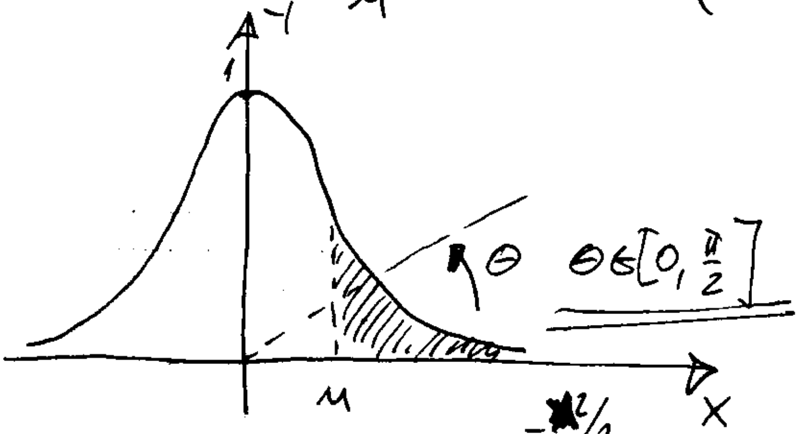
$$\boxed{Q = \frac{1}{2\pi}\int_0^{\pi/2} e^{-\frac{\mu^2}{2\cos^2\theta}}\,d\theta}$$

SUM BIL MNOGU
BLISKU !!! VIDI SIMON
APPENDIX 4A
SAMO SUM TREBAL
DA INTEGRIRAM

$$\int_0^\infty \frac{1}{2\pi}e^{-y^2/2}\,dy = 1$$

(*)

$$Q(\cancel{\mu}) = \frac{1}{\sqrt{2\pi}} \int_{\mu}^{\infty} e^{-\frac{x^2}{2}} dx = \left(\frac{1}{2\pi} \cdot \int_{\mu}^{\infty} e^{-\frac{x^2}{2}} dx \int_{\mu}^{\infty} e^{-\frac{y^2}{2}} dy \right)$$



$$Q(\mu) = \int_{\mu}^{\infty} dx \int_{0}^{e^{-x^2/2}} e^{-\frac{x^2}{2}} dy$$

$x = r \cdot \cos\theta$

$y = r \cdot \sin\theta$

$\dot{J} = r$

$$Q(\mu) = \int_{\mu}^{\infty} \int_{0}^{e^{-x^2/2}} e^{-x^2/2}\, dx\, dy$$

$x = \mu \quad r = \frac{\mu}{\cos\theta}$

$x = \infty \quad r = \infty$

$y = 0 \quad \theta = \arcsin\frac{y}{r} = 0$

$y = e^{-\frac{x^2}{2}} \quad \theta = \arcsin\left(\frac{e^{-\frac{x^2}{2}}}{r}\right)$

$$Q(\mu) = \int_{\frac{\mu}{\cos\theta}}^{\infty} \int_{0}^{\arcsin\left(\frac{e^{-x^2/2}}{r}\right)} e^{-\frac{r^2}{2\cos^2\theta}}\, r\, dr\, d\theta$$

$2 \times (\;)$

~~ZATO~~ ŠTO ZBYAK

$\theta = 0 \div \frac{\pi}{2}$ IMAKO

TKON $\theta = 0 \div \pi$

$$Q(\mu) = \left(\frac{1}{2\pi} \int_{\mu}^{\infty} e^{-\frac{x^2}{2}} dx \int_{\mu}^{\infty} e^{-\frac{y^2}{2}} dy \right)^{\frac{1}{2}} = \frac{1}{\sqrt{2\pi}} \left(\int_{\mu}^{\infty} \int_{\mu}^{\infty} e^{-\frac{x^2+y^2}{2}} dx\, dy \right)^{1/2}$$

$\boxed{\dot{J} = r}$

$x = r \cdot \cos\theta$

$y = r \cdot \sin\theta$

$\boxed{x^2 + y^2 = r^2}$

$x = \mu \rightarrow r = \frac{\mu}{\cos\theta} \quad x = \infty \rightarrow r = \infty$

$y = \mu \rightarrow \theta = \arcsin\left(\frac{\mu}{r}\right) \quad y = \infty \rightarrow \theta = \frac{\pi}{2}$

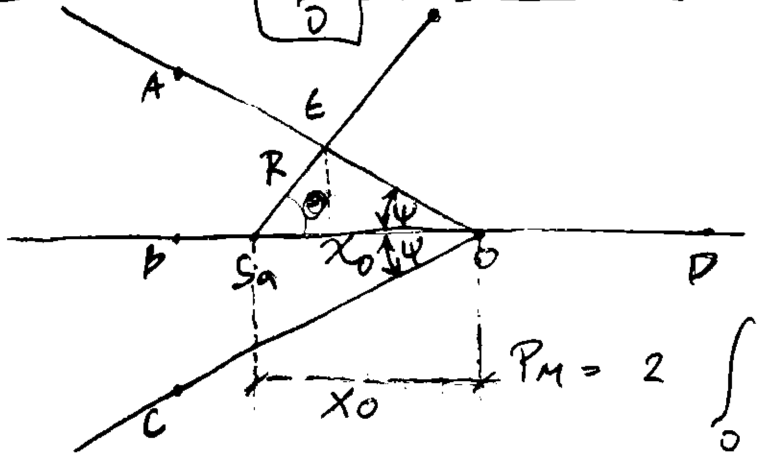
$\theta = 0 \ldots \pi/2 \Rightarrow$ VIDI SLIKA

$$Q^2(\mu) = \frac{2}{2\pi} \int_{0}^{\pi/2} \int_{\frac{\mu}{\cos\theta}}^{\infty} e^{-\frac{r^2}{2}} r \cdot dr\, d\theta = \frac{1}{\pi} \int_{0}^{\pi/2} \int_{\mu/\cos\theta}^{\infty} e^{-\frac{r^2}{2}} d\left(\frac{r^2}{2}\right) = \int_{0}^{\pi/2} -e^{-\frac{r^2}{2}} \Big|_{\mu/\cos\theta}^{\infty} d\theta$$

$$Q^2(\mu) = \frac{1}{\pi} \int_{0}^{\pi/2} e^{-\frac{r^2}{2}} \Big|_{0}^{\mu/\cos\theta} d\theta = \frac{1}{\pi} \int_{0}^{\pi/2} e^{-\frac{\mu^2}{2\cos^2\theta}} d\theta$$

$$Q^2(\mu) = \frac{1}{\pi}\int_0^{\pi/2} e^{-\frac{\mu^2}{2\cos^2\theta}}\,d\theta$$

CRAIG GEOMETRIC APPROACH



Sa - SIGNAL POINT

ABCO - DECISION REGION

MMV

$$P_M = 2\int_0^{\pi-\psi} d\theta \int_R^{\infty} p(r,\theta)\,dr$$

$$p(r,\theta) = \frac{r}{2\pi\sigma^2}\exp\left(-\frac{r^2}{2\sigma^2}\right)$$

FOR ADDITIVE NARROWBAND WHITE GAUSSIAN NOISE HAVING INDEPENDENT IN-PHASE AND QUADRATURE COMPONENTS

$$P_M = \cancel{2}\int_0^{\pi-\psi} d\theta \underbrace{\int_R^{\infty} \frac{r}{\cancel{2}\pi\sigma^2} e^{-r^2/2\sigma^2}\,dr}_{I}$$

$$\mu = \frac{r^2}{2\sigma^2} \qquad d\mu = \frac{r\,dr}{\sigma^2}$$

$$\cancel{r} = R \Rightarrow \mu = \frac{R^2}{2\sigma^2}$$

$$I = \frac{1}{\pi}\int_R^{\infty} e^{-r^2/2\sigma^2}\frac{r\,dr}{\sigma^2} = \frac{1}{\pi}\int_{\frac{R^2}{2\sigma^2}}^{\infty} e^{-\mu}\,d\mu = \frac{1}{\pi}e^{-\mu}\Big|_{\infty}^{R^2/2\sigma^2} = \frac{1}{\pi}e^{-\frac{R^2}{2\sigma^2}}$$

$$\boxed{P_M = \int_0^{\pi-\psi}\frac{1}{\pi}e^{-\frac{R^2}{2\sigma^2}}\,d\theta}$$

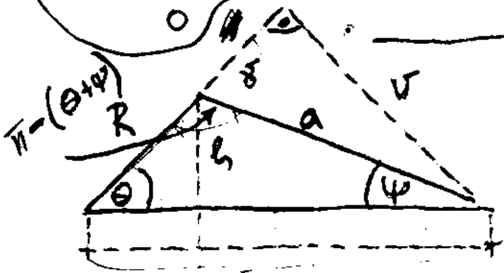
$$R = \frac{x_0 \sin\psi}{\sin(\theta+\psi)}$$ LOW OF SINES

$$\frac{h}{a} = \sin\psi \qquad h = a\cdot\sin\psi$$

$$\frac{h}{R} = \sin\theta \qquad h = R\cdot\sin\theta$$

$$a\sin\psi = R\sin\theta \qquad \frac{\sin\psi}{R} = \frac{\sin\theta}{a}$$

$$\gamma = 2\pi - [\pi-(\theta+\psi)] = 2\pi - \pi + \theta+\psi = \pi+\theta+\psi$$



LAW OF THE SINES

$$\underbrace{\frac{\sin\theta}{a} = \frac{\sin\psi}{R} = \frac{\sin(\pi-(\theta+\psi))}{x_0}}$$

$$\frac{v}{x_0} = \sin\theta$$

$$R = \frac{x_0\cdot\sin\psi}{\sin(\pi-(\theta+\psi))} = \frac{x_0\cdot\sin\psi}{\sin(\theta+\psi)}$$

$$\sin(\pi-(\theta+\psi)) = \sin(\theta+\psi) = \frac{v}{a} = \frac{x_0\cdot\sin\theta}{a}$$

$$\sin\theta = \frac{a}{x_0}\cdot\sin(\theta+\psi)$$

$$\sin\theta = \frac{a}{R}\cdot\sin\psi \qquad \frac{\cancel{a}}{R}\cdot\sin\psi = \frac{\cancel{a}}{x_0}\cdot\sin(\theta+\psi) \rightarrow \frac{\sin\psi}{R} = \frac{\sin(\theta+\psi)}{x_0}$$

$$\Rightarrow \boxed{\frac{\sin\psi}{R} = \frac{\sin\theta}{a} = \frac{\sin(\theta+\psi)}{x_0}}$$ LAW OF SINES

$$P_M = \frac{1}{\pi} \int_0^{\pi-\psi} e^{-\frac{R^2}{2\sigma^2}} d\theta \qquad R = \frac{x_0 \sin\psi}{\sin(\theta+\psi)}$$

$$P_M = \frac{1}{\pi} \int_0^{\pi-\psi} \exp\left(-\frac{x_0^2 \sin^2\psi}{2\sigma^2 \sin^2(\theta+\psi)}\right) d\theta \quad \xrightarrow{} \quad \boxed{\theta+\psi = \pi - \phi}$$

$$\boxed{\frac{x_0^2}{2\sigma^2} = \frac{E_s}{N_0} = \gamma_s}$$

$$\phi = \pi - (\theta+\psi) \qquad d\phi = -d\theta$$

$$\theta = 0 \qquad \phi = \pi - \psi$$

$$\theta = \pi - \psi \qquad \phi = \cancel{\pi} - \cancel{\pi} + \cancel{\psi} - \cancel{\psi} = 0$$

[illegible]

$$P_M = \frac{1}{\pi} \int_{\pi-\psi}^{0} -\exp\left(-\frac{\gamma_s \sin^2\psi}{\sin^2(\pi-\phi)}\right) d\phi = \frac{1}{\pi} \int_0^{\pi-\psi} \exp\left(-\frac{\gamma_s \sin^2\psi}{\sin^2\phi}\right) d\phi$$

$$\boxed{P_M = \frac{1}{\pi} \int_0^{\pi-\psi} \exp\left(-\frac{\gamma_s \sin^2\psi}{\sin^2\phi}\right) d\phi}$$

§ ERROR PROBABILITY OF MPSK (CRAIG)

⓪ PROAKIS:

$$P_M = 1 - \int_{-\pi/M}^{\pi/M} p_{\Theta_r}(\Theta_r) d\Theta_r$$

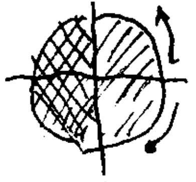
$$s_m(t) = g(t) \cos\left[2\pi f_c t + \frac{2\pi}{M}(m-1)\right] \qquad 1 \leq m \leq M \quad 0 \leq t \leq T$$

BPSK $M=2$

$$s_m(t) = g(t) \cos(2\pi f_c t) \cdot \cos\left[\frac{2\pi}{M}(m-1)\right] + g(t) \sin(2\pi f_c t) \sin\left[\frac{2\pi}{M}(m-1)\right]$$

$$\boxed{\left\{\cos\left[\frac{2\pi}{M}(m-1)\right], \sin\left[\frac{2\pi}{M}(m-1)\right]\right\}}$$

$$\boxed{\begin{array}{ll} & M=2 \\ m=1 & \{1,0\} \\ m=2 & \{-1,0\} \end{array}}$$



DECISION REGION

$\boxed{M=2}$ DPSK

$$m=1 \qquad \{\cos(0), \sin(0)\} = \{1, 0\}$$

$$m=2 \qquad \{\cos(\pi), \sin(\pi)\} = \{-1, 0\}$$

VECTOR REPRESENTATION:

$$s_m = \left[\sqrt{E_s} \cos\frac{2\pi}{M}(m-1), \sqrt{E_s} \sin\frac{2\pi}{M}(m-1)\right] \qquad E_s = \frac{E_g}{2}$$

$g(t)$ - PULSE SHAPE OF TRANSMITTED SIGNAL

- CORRELATION METRICS: $\boxed{C(r, s_m) = r \cdot s_m}$ $m = 1, 2, \ldots M$
(FOR EQUAL ENERGY SIGNALS) ↳ CORRELATION DETECTOR

- CORRELATION ~~MATRIX~~ METRIX COULD BE ALTERNATIVELY PRESENTED:

$$C(r, s_m) = 2\int_0^T r(t)\, s_m(t)\, dt - \underbrace{\int_0^T s_m^2(t)\, dt}_{E_m}$$ (ENERGY OF SIGNAL)

PROOF: $C(r, s_m) = 2\sum_{n=1}^{N} r_n s_{mn} - \sum_{n=1}^{N} s_{mn}^2$ $m = 1, 2, \dots M$

$$r_n = \int_0^T r_n(t) f_n(t)\, dt$$

$$s_{mn} = \int_0^T s_m(t) f_n(t)\, dt$$

$\{f_n(t)\}$ ~~FORMS~~ REPRESENTS ORTHOGONAL BASIS FUNTIONS OF SIGNAL SPACE

$$r(t) = \sum_{n=1}^{N} r_n f_n(t) \qquad s_m(t) = \sum_{n=1}^{N} s_{mn} f_n(t)$$

$$C(r, s_m) = 2\int_0^T \left(\sum_{n=1}^{N} r_n f_n(t)\right) \left(\sum_{l=1}^{N} s_{ml} f_l(t)\right) dt -$$

$$- \int_0^T \left(\sum_{n=1}^{N} s_{mn} f_n(t) \sum_{l=1}^{N} s_{ml} f_l(t)\right) dt =$$

$$= 2\sum_{n=1}^{N} r_n \sum_{l=1}^{N} s_{ml} \underbrace{\int_0^T f_n(t) \cdot f_l(t)\, dt}_{\delta_{nl}} - \sum_{n=1}^{N} s_{mn} \sum_{l=1}^{N} s_{ml} \underbrace{\int_0^T f_n(t) f_l(t)\, dt}_{\delta_{nl}}$$

$$\delta_{nl} = \begin{cases} 1 & n = l \\ 0 & n \neq l \end{cases}$$

$$\boxed{C(r, s_m) = 2 \cdot \sum_{n=1}^{N} r_n \cdot s_{mn} - \sum_{n=1}^{N} s_{mn}^2}$$ DOKAZANO!!

- PHASE DETECTOR

$$\theta_r = tg^{-1} \frac{r_2}{r_1} = \mathrm{arctg}\, \frac{r_2}{r_1}$$

- $\theta_r = 0$ $\qquad s_0 = [\sqrt{E_s}, 0]$

$r_1 = \sqrt{E_s} + n_1$ } components of the

$r_2 = n_2$ } received signal

u_1, u_2 - JOINTLY GAUSSIAN

$E(r_1)=\sqrt{E_s}$; $E(r_2)=0$

$$p_r(r_1,r_2)=\frac{1}{2\pi\sigma_r^2}\cdot e^{-\frac{(r_1-\sqrt{E_s})^2+r_2^2}{2\sigma_r^2}}$$

$$\sigma_{r_1}^2=\sigma_{r_2}^2=\frac{N_0}{2}=\sigma_r^2$$

- PDF from θ:

$V=\sqrt{r_1^2+r_2^2}$ $\qquad \theta_r=\operatorname{arctg}\frac{r_2}{r_1}$

$$\boxed{\begin{aligned}r_1&=V\cdot\cos\theta_r\\ r_2&=V\cdot\sin\theta_r\end{aligned}}$$

$$J=\begin{vmatrix}\frac{\partial r_1}{\partial V} & \frac{\partial r_1}{\partial\theta_r}\\ \frac{\partial r_2}{\partial V} & \frac{\partial r_2}{\partial\theta_r}\end{vmatrix}=\begin{vmatrix}\cos\theta_r & -V\cdot\sin\theta_r\\ \sin\theta_r & V\cdot\cos\theta_r\end{vmatrix}=V\underbrace{(\cos^2\theta_r+\sin^2\theta_r)}_{1}=V$$

$$p_r(V,\theta_r)=|J|\cdot p(r_1,r_2)=\frac{V}{2\pi\sigma_r^2}\cdot\exp\left(-\frac{(V\cos\theta_r-\sqrt{E_s})^2+V^2\sin^2\theta_r}{2\sigma_r^2}\right)$$

$$p_r(V,\theta_r)=\frac{V}{2\pi\sigma_r^2}\exp\left(-\frac{V^2\cos^2(\theta_r)-2V\sqrt{E_s}\cos\theta_r+E_s+V^2\sin^2\theta_r}{2\sigma_r^2}\right)$$

$$\boxed{p_{V,\theta_r}(V,\theta_r)=\frac{V}{2\pi\sigma_r^2}\exp\left(-\frac{V^2-2V\sqrt{E_s}\cos\theta_r+E_s}{2\sigma_r^2}\right)}$$ MMV

~~ZDRUŽENA~~ GUSTINA NA VEROJATNOST NA AMPLITUDATA I FAZATA NA SIGNALOT (PP signal) POSLE KANALNIOT (NARROWBAND) FILTER

$$p_{\theta_r}(\theta_r)=\int_0^\infty p_{V,\theta_r}(V,\theta_r)\,dV$$

$$\boxed{\gamma_s=\frac{E_s}{2\sigma_r^2}}\qquad 2\sigma_r^2=N_0 \qquad 2\sigma_r^2=\frac{E_s}{\gamma_s}$$

$$p_{V,\theta_r}(V,\theta_r)=\frac{V\gamma_s}{\pi E_s}\exp\Big(\underbrace{-\frac{V^2}{E_s}\cdot\gamma_s+\frac{2V\sqrt{E_s}\cos\theta_r}{E_s}\cdot\gamma_s-\gamma_s}_{*}\Big)$$

$$\boxed{*=?}\ -\frac{1}{2}\left(V-\sqrt{4\gamma_s}\cos\theta_r\right)^2=-\frac{1}{2}\left(V^2-2V\sqrt{4\gamma_s}\cos\theta_r+4\gamma_s\cos^2\theta_r\right)=①$$

$$①=-\frac{V^2}{2}+V\sqrt{4\gamma_s}\cos\theta_r-2\gamma_s\cos^2\theta_r$$

$$①-2\gamma_s\sin^2\theta_r=-\frac{V^2}{2}+V\sqrt{4\gamma_s}\cos\theta_r\underbrace{-2\gamma_s\cos^2\theta_r-2\gamma_s\sin^2\theta_r}_{2\gamma_s}=$$

$$=-\frac{V^2}{2}+V\sqrt{4\gamma_s}\cos\theta_r-2\gamma_s=-\frac{V^2}{2}+2V\sqrt{\gamma_s}\cos\theta_r+2\gamma_s$$

IF: $\boxed{\sigma_r^2=1}$ $E_s=\gamma_s$

$$p_{V,\theta_r}(V,\theta_r)=\frac{V}{2\pi}\exp\left(-\frac{V^2}{2}+\frac{2V\cdot\sqrt{E_s}\cdot\cos\theta_r}{2}-\frac{E_s}{2}\right)$$

$$p_{V,\theta_r}(V,\theta_r)=\frac{V}{2\pi}\exp\frac{1}{2}\left(-V^2+2V\sqrt{E_s}\cos\theta_r-E_s\sin^2\theta_r-E_s\cos^2\theta_r\right)$$

$$p_{r,\theta_r}(r,\theta_r) = \frac{r}{2\pi}\, \exp\left(-\frac{\gamma_s}{2}\sin^2\theta_r\right) \cdot \exp\left(-\left(r^2 - 2r\sqrt{\mathcal{E}_s}\cos\theta_r + \gamma_s\cos^2\theta_r\right)/2\right)$$

$$p_{r,\theta_r}(r,\theta_r) = \frac{r}{2\pi}\, e^{-\frac{\gamma_s}{2}\sin^2\theta_r} \cdot e^{-(r-\sqrt{\gamma_s}\cdot\cos\theta_r)^2/2}$$

IF: $\boxed{\mathcal{E}_s = 4\gamma_s}$ $\boxed{p_{r,\theta_r}(r,\theta_r) = \frac{r}{2\pi}\, e^{-2\gamma_s\sin^2\theta_r} \cdot e^{-(r-\sqrt{4\gamma_s}\cos\theta_r)^2/2}}$

ACCORDING PROAKIS

SOGLASNO PROAKIS 5-2-55

$$\boxed{p_{\theta_r}(\theta_r) = \int_0^{\infty} p_{r,\theta_r}(r,\theta_r)\,dr = \frac{e^{-2\gamma_s\sin^2\theta_r}}{2\pi} \int_0^{\infty} r\,e^{-(r-\sqrt{4\gamma_s}\cos\theta_r)^2/2}\,dr}$$

HMV

IF: $\mathcal{E}_s = \gamma_s$ (ACCORDING ME)

$$\boxed{p_{\theta_r}(\theta_r) = \frac{e^{-\frac{\gamma_s}{2}\sin^2\theta_r}}{2\pi} \int_0^{\infty} r\cdot e^{-(r-\sqrt{\gamma_s}\cos\theta)^2/2}\,dr}$$

- WHEN $s_1(t)$ IS TRANSMITTED DECISION ERROR IS MADE IF THE NOISE CAUSES THE PHASE TO FALL OUTSIDE OF RANGE $-\frac{\pi}{M} \leq \theta_r \leq \frac{\pi}{M}$, HENCE PROBABILITY OF SYMBOL ERROR IS:

$$\boxed{P_M = 1 - \int_{-\pi/M}^{\pi/M} p_{\theta_r}(\theta_r)\,d\theta_r}$$

ERROR PROBABILITY OF M-ARY PSK

$M=2$ $$P_2 = Q\left(\sqrt{\frac{2\mathcal{E}_b}{N_0}}\right) = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{\mathcal{E}_b}{N_0}}\right) = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\gamma_b}\right)$$

$M=4$ $$P_{4c} = (1-P_2)^2 = \left[1 - Q\left(\sqrt{\frac{2\mathcal{E}_b}{N_0}}\right)\right]^2$$ ⇒ PROBABILITY OF CORRECT DECISION

$$P_4 = 1 - P_{4c} = 1 - \left(1 - Q\left(\sqrt{\frac{2\mathcal{E}_b}{N_0}}\right)\right)^2 =$$

$$= \cancel{1} - \cancel{1} + 2Q\left(\sqrt{\frac{2\mathcal{E}_b}{N_0}}\right) - Q^2\left(\sqrt{\frac{2\mathcal{E}_b}{N_0}}\right) = \underline{2Q\left(\sqrt{\frac{2\mathcal{E}_b}{N_0}}\right)\left(1-\frac{1}{2}Q\left(\sqrt{\frac{2\mathcal{E}_b}{N_0}}\right)\right)}$$

- APPROXIMATION $\frac{\mathcal{E}_s}{N_0} \gg 1$ $|\theta_r| \leq \frac{\pi}{2}$

$$\boxed{p_{\theta_r}(\theta_r) \doteq \sqrt{\frac{2\gamma_s}{\pi}}\,\cos\theta_r\, e^{-\gamma_s\sin^2\theta_r}}$$

$Q(z) = \frac{1}{2}\mathrm{erfc}\,\frac{z}{\sqrt{2}}$

$$P_M = 1 - \mathrm{erf}\left(\sqrt{2g}\,\sin\left(\frac{\pi}{M}\right)\right) = \left(\frac{1}{2}\mathrm{erfc}\left(\frac{\sqrt{2g}\,\sin\left(\frac{\pi}{M}\right)}{\sqrt{2}}\cdot\sqrt{2}\right)\cdot 2\right)$$

$$\boxed{P_M = 2 \cdot Q\left(2\sqrt{\gamma}\sin\frac{\pi}{M}\right)}$$

$$P_M = 1-\operatorname{erf}\left(\sqrt{\gamma}\cdot\sin\frac{\pi}{M}\right)\cdot\sqrt{2} = \sqrt{2}\left(1-\operatorname{erf}\left(\sqrt{\gamma}\cdot\sin\frac{\pi}{M}\right)\right) + \underline{1-\sqrt{2}}$$

$$P_M = \sqrt{2}\ \operatorname{erfc}\left(\sqrt{\gamma}\cdot\sin\frac{\pi}{M}\right)+1-\sqrt{2} = 2\cdot\sqrt{2}\cdot\underbrace{\frac{1}{2}\operatorname{erfc}\left(\frac{\sqrt{2\gamma}\sin\pi/M}{\sqrt{2}}\right)}_{Q\left(\sqrt{2\gamma_s}\sin\pi/M\right)}+1-\sqrt{2}$$

$$\boxed{P_M = 2\sqrt{2}\ Q\left(\sqrt{2\gamma_s}\sin\frac{\pi}{M}\right)+1-\sqrt{2}}\qquad ???$$

$$P_\Theta(\Theta) = \sqrt{\frac{2\gamma}{\pi}}\cos\Theta\, e^{-2\gamma\sin^2\Theta} \qquad = 2Q\left(2\sqrt{\gamma}\sin\frac{\pi}{M}\right)$$

MAPLE : $P_M(\gamma) = \operatorname{erfc}\left(\sqrt{2\gamma}\sin\left(\frac{\pi}{M}\right)\right) = 2\,\frac{1}{2}\operatorname{erfc}\left(\frac{2\sqrt{\gamma}\sin\frac{\pi}{M}}{\sqrt{2}}\right)$

$$I = \int_{-\pi/M}^{\pi/M}\sqrt{\frac{2\gamma_s}{\pi}}\cos\Theta_r\, e^{-2\gamma_s\sin^2\Theta_r}\,d\Theta_r$$

$u = \sin\Theta$

$du = \cos\Theta\, d\Theta$

$\Theta = -\pi/M \quad u = -\sin\frac{\pi}{M}$

$\Theta = \pi/M \quad u = \sin\pi/M$

$$\cancel{I = \frac{1}{\frac{2\gamma_s}{\sqrt{2}\sqrt{\gamma}}}\sqrt{\frac{2\gamma}{\pi}}\int_{-\sin\left(\frac{\pi}{M}\right)}^{\sin\frac{\pi}{M}} e^{-2\gamma_s\cdot u^2}\,d(2\gamma_s\cdot u) = \frac{(-1)}{\sqrt{2\gamma\pi}}\cdot e^{-2\gamma_s\cdot u^2}\Big|_{-\sin\frac{\pi}{M}}^{\sin\frac{\pi}{M}}}$$

$u = +\sqrt{2\gamma_s}\cdot\sin\Theta \qquad du = \sqrt{2\gamma_s}\cos\Theta\cdot d\Theta$

$\Theta = -\pi/M \qquad u = -\sqrt{2\gamma_s}\sin\left(\frac{\pi}{M}\right)$

$\Theta = \pi/M \qquad u = \sqrt{2\gamma_s}\sin\left(\frac{\pi}{M}\right)$

KAKO DOAGA DO OVA NE ZNAM!!

$$I = \frac{2}{\sqrt{\pi}}\int_{-\sqrt{2\gamma_s}\sin\left(\frac{\pi}{M}\right)}^{\sqrt{2\gamma_s}\sin\left(\frac{\pi}{M}\right)} e^{-u^2/2}\cdot d\left(\frac{u}{2}\right) \doteq \frac{2}{\sqrt{\pi}}\int_{\sqrt{2\gamma_s}\sin\left(\frac{\pi}{M}\right)}^{\infty} e^{-u^2/2}\,du = \text{①}$$

$$= \cancel{\frac{2}{\sqrt{\pi}}\operatorname{erfc}\left(\frac{\sqrt{2\gamma_s}\sin\frac{\pi}{M}\cdot\sqrt{2}}{\sqrt{2}}\right)}$$

$\frac{u}{\sqrt{2}} = x \qquad du = dx\cdot\sqrt{2}$

$u = \sqrt{2\gamma_s}\sin\frac{\pi}{M} \qquad x = \sqrt{\gamma_s}\sin\frac{\pi}{M}$

$$\circledast = \frac{2}{\sqrt{\pi}}\int_{\sqrt{\gamma_s}\sin\frac{\pi}{M}}^{\infty}\sqrt{2}\,e^{-x^2}dx = \cancel{2\frac{2\sqrt{2}}{\sqrt{\pi}}\frac{1}{2}\operatorname{erfc}\left(\frac{\sqrt{\gamma_s}\sin\frac{\pi}{M}\cdot\sqrt{2}}{\sqrt{2}}\right)}$$

$$\circledast = \sqrt{2}\ \underbrace{\frac{2}{\sqrt{\pi}} \int_{\sqrt{\gamma_s}\sin\frac{\pi}{M}}^{\infty} e^{-t^2} dt}_{\mathrm{erfc}} = \underbrace{\sqrt{2}\cdot 2\cdot \frac{1}{2}\,\mathrm{erfc}\,\frac{\sqrt{2\gamma_s}\sin\frac{\pi}{M}}{\sqrt{2}}}_{\boxed{Q\left(\sqrt{2\gamma_s}\sin\frac{\pi}{M}\right)}} = 2\sqrt{2}\, Q\left(\sqrt{2\gamma_s}\sin\frac{\pi}{M}\right)$$

$$\circledast = \sqrt{2}\ \mathrm{erfc}\left(\sqrt{\gamma}\cdot\sin\frac{\pi}{M}\right)$$

$$\boxed{Q(x) = \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} e^{-t^2/2} dt} \Rightarrow$$

SIMON & ALOUINI

$$\circledast = \frac{2}{\sqrt{\pi}} \int_{\sqrt{\gamma_s}\sin\frac{\pi}{M}}^{\infty} e^{-u^2/2} du = 2\sqrt{2}\ \underbrace{\frac{1}{\sqrt{2\pi}} \int_{\sqrt{2\gamma}\sin\frac{\pi}{M}}^{\infty} e^{-u^2/2} du}_{Q(x)} = 2\sqrt{2}\, Q\left(\sqrt{2\gamma}\sin\frac{\pi}{M}\right)$$

FINALNA VREDNOST ZA APROKSIMACIJATA E:

$$\boxed{P_M = \mathrm{erfc}\left(\sqrt{\gamma_s}\sin\left(\frac{\pi}{M}\right)\right) = 2\,Q\left(\sqrt{2\gamma_s}\sin\left(\frac{\pi}{M}\right)\right) \quad \gamma_s = \gamma_b \cdot \mathrm{ld} M = \gamma_b \cdot k}$$

SYMBOL ERROR PROBABILITY

- IF GRAY CODE IS USED FOR MAPPING:

$$\boxed{P_b \doteq \frac{1}{k}\cdot P_M}$$

BIT ERROR PROBABILITY

MMV

$$P_{\theta_r}(\theta_r) = \frac{e^{-2\gamma_s \sin^2\theta_r}}{2\pi} \int_0^{\infty} v\, e^{-(v-\sqrt{4\gamma_s}\cos\theta_r)^2} dv$$

$$P_M = 1 - \int_{-\pi/M}^{\pi/M} p_{\theta_r}(\theta_r)\, d\theta_r$$

MPSK BER PROAKIS

$$P_M = \frac{1}{\pi} \int_0^{\pi-\psi} \exp\left(-\frac{\gamma_s \sin^2\theta}{\sin^2(\phi)}\right) d\phi$$

MPSK BER CRAIG

NE VAŽI ~~ZA~~ $M=2$ (IZLEGUVA 2x POGOLEMO OD VISTIN)

VISTINSKOTO P_M ZA $M=2$ E:

$$P_2 = Q\left(\sqrt{2\gamma_b}\right) = \underline{\frac{1}{2}\mathrm{erfc}\left(\sqrt{\gamma_b}\right)}$$

$$P_2 = \frac{1}{\pi}\int_0^{\pi-\psi} \exp\left(-\frac{\gamma_s \sin^2(\psi)}{\sin^2(\phi)}\right) d\phi$$

$$\boxed{\psi = \frac{\pi}{2}}$$

CHECK PICTURE

$$P_2 = \frac{1}{\pi}\int_0^{\pi/2} e^{-\frac{\gamma_s \sin^2(\psi)}{\sin^2\phi}} d\phi$$

OD DRUGA STRANA

$$P_2 = \frac{1}{2}\,\mathrm{erfc}\sqrt{\gamma_s} = Q\left(\sqrt{2\gamma_s}\right)$$

$$Q\left(\sqrt{2\gamma_s}\right) = \frac{1}{\pi}\int_0^{\pi/2} e^{-\frac{\gamma_s \sin^2\frac{\pi}{2}}{\sin^2\phi}} d\phi$$

$\sin^2\frac{\pi}{2} \to 1$

- $x = \sqrt{2\gamma_s} \qquad 2\gamma_s = x^2 \qquad \boxed{\gamma_s = \frac{x^2}{2}}$

$$\boxed{Q(x) = \frac{1}{\pi}\int_0^{\pi/2} e^{-\frac{x^2}{2\sin^2\phi}} d\phi}$$

KONEČNO DOKAZ ZA ALTERNATIVNATA FORMA NA GAUSOVA Q FUNKCIJA !!!

$$\mathrm{erfc}\left(\sqrt{\gamma_s}\right) = \frac{1}{\pi}\int_0^{\pi/2} e^{-\frac{\gamma_s}{\sin^2\phi}} d\phi$$

$x = \sqrt{\gamma_s} . \ \gamma_s = x^2$

$$\boxed{\mathrm{erfc}(x) = \frac{1}{\pi}\int_0^{\pi/2} e^{-\frac{x^2}{\sin^2\phi}} d\phi}$$

$$\boxed{\begin{aligned} Q(-x) &= 1 - Q(x) \\ \mathrm{erfc}(-x) &= 2 - \mathrm{erfc}(x) \end{aligned}}$$

$$\boxed{Q(x) \leq \frac{1}{2} e^{-x^2/2}}$$

- TWO-DIMENSIONAL CASE

$$\boxed{Q(x_1, y_1; 0) = Q(x_1)\cdot Q(x_2)}$$

$$Q(x_1, y_1; \rho) = \frac{1}{2\pi\sqrt{1-\rho^2}}\int_{x_1}^{\infty}\int_{y_1}^{\infty} \exp\left[-\frac{x^2+y^2-2\rho xy}{2(1-\rho^2)}\right] dx\,dy$$

BIVARIATE NORMAL DISTRO:

$$p(x_1, x_2) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}} \exp\left[-\frac{z}{2(1-\rho^2)}\right]$$

$$z = \frac{(x_1-\mu_1)^2}{\sigma_1^2} - \frac{2\rho(x_1-\mu_1)(x_2-\mu_2)}{\sigma_1\sigma_2} + \frac{(x_2-\mu_2)^2}{\sigma_2^2}$$

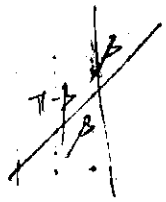
$\rho = \mathrm{cov}(x_1,x_2) = \frac{V_{12}}{\sigma_1\sigma_2}$ → CORRELATION of x_1 & x_2

V_{12} – COVARIANCE

$$V_{12} = E\left[(x_1-\mu_1)(x_2-\mu_2)\right]$$

$\mu_1, \mu_2, = 0 \quad \sigma_1, \sigma_2 = 1 \Rightarrow$

$$\boxed{f(x,y) = \frac{1}{2\pi\sqrt{1-\rho^2}}\, e^{-\frac{x^2-2\rho xy+y^2}{\sigma^2}}}$$



$\mu = x - x_1 \quad d\mu = dx \quad x = x_1 \Rightarrow \mu = x_1 - x_1 = 0$

$\upsilon = y - y_1 \quad d\upsilon = dy \quad y = y_1 \Rightarrow \upsilon = y_1 - y_1 = 0$

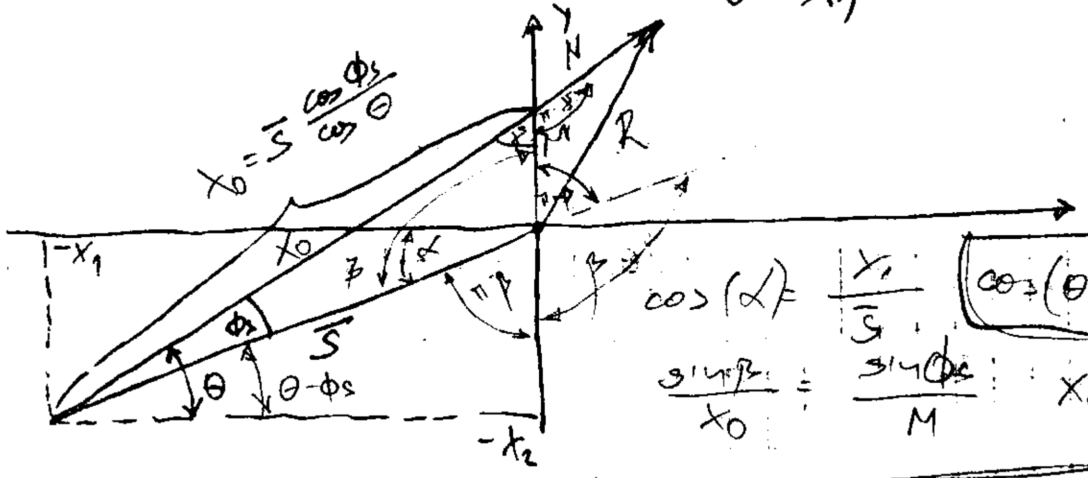
$x = \mu + x_1 \quad y = \upsilon + y_1$

$$Q(x_1, y_1; \rho) = \frac{1}{2\pi\sqrt{1-\rho^2}} \int_0^\infty\int_0^\infty \exp\left[\frac{(\mu+x_1)^2+(\upsilon+y_1)^2-2\rho(\mu+x_1)(\upsilon+x_1)}{2(1-\rho^2)}\right] d\mu\, d\upsilon$$

$$= \left|\begin{matrix}\mu = x\\ \upsilon = y\end{matrix}\right| = \frac{1}{2\pi\sqrt{1-\rho^2}} \int_0^\infty\int_0^\infty \exp\left[\frac{(x+x_1)^2+(y+y_1)^2-2\rho(x+x_1)(y+y_1)}{2(1-\rho^2)}\right] dx\, dy$$

SIGNAL VECTOR $s = (-x_1, -y_1)$ falls IN UPPER RIGHT QUADRANT OF (x,y) PLANE.

$$\bar{S} = \sqrt{x_1^2+y_1^2} \qquad \phi_s = \mathrm{arctg}\,\frac{y_1}{x_1}$$



$$\boxed{x_2 = \bar{S}\cdot\cos(\theta-\phi_s)}$$

!!! GREŠKA TREBA x_1

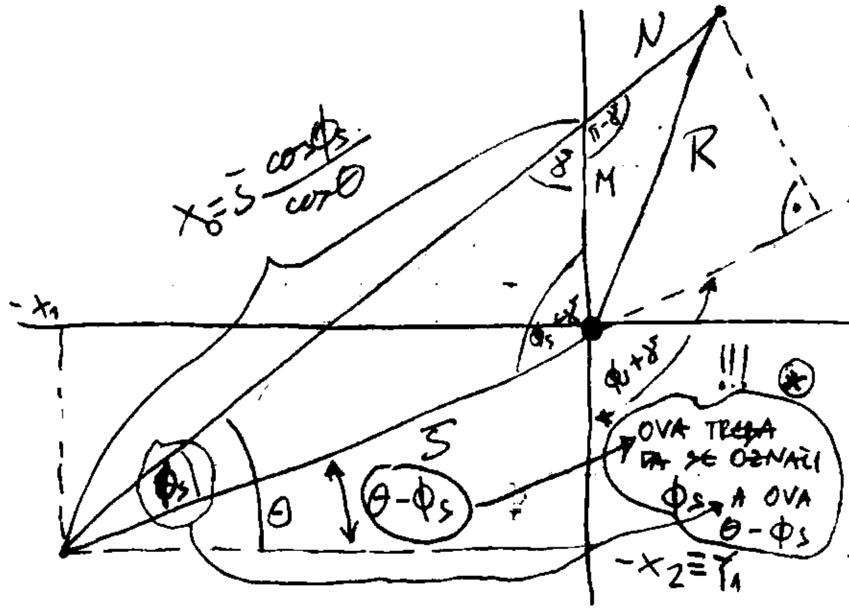
$$\cos(\alpha) = \frac{x_1}{\bar{S}} \qquad \boxed{\cos(\theta-\phi_s) = \frac{x_2}{\bar{S}}}$$

$$\frac{\sin\beta}{x_0} = \frac{\sin\phi_s}{M} \qquad x_0 = \frac{M\cdot\sin\beta}{\sin\phi_s}$$

$$\sin\theta = \frac{x_2+M}{x_0} \qquad \boxed{M = x_0\cdot\sin\theta - x_2 = x_0\cdot\sin\theta - \bar{S}(\theta-\phi_s)}$$

$$\frac{\sin\gamma}{\bar{S}} = \frac{\sin\phi_s}{M} \qquad \cos\gamma = \frac{M+x_2}{x_0} \qquad \gamma = \frac{\pi}{2}-\theta$$

$$\sin\theta = \frac{M+x_2}{x_0}$$



⊛ OČIGLEDNO SLIKATA VO KNIGATA E POGREŠNA ZOŠTO VEZAT !

$$\phi_s = \operatorname{arctg}\frac{Y_1}{X_1} = \operatorname{arctg}\frac{x_2}{x_1}$$

TOGAŠ VSUŠNOST: $\theta-\phi_s \triangleq \phi_s$

$$\cos(\theta-\phi_s)=\frac{x_2}{\bar{S}} \qquad \left.\begin{aligned} x_2 &= \bar{S}\cdot\cos(\theta-\phi_s)^{\#} \\ x_1 &= \bar{S}\ \sin(\theta-\phi_s)^{\#}\end{aligned}\right\} \text{GREŠKA!!}$$

$$\left.\begin{aligned}\frac{\sin\phi_s}{M} &= \frac{\sin\delta}{\bar{S}} = \frac{\sin(\phi_s+\delta)}{x_0}\cdot\checkmark \\ \sin\theta &= \frac{M+x_2}{x_0} = \frac{M+\bar{S}\cos(\theta-\phi_s)}{x_0},^{\#}\end{aligned}\right\} \quad \text{2 RAVENKI} \quad \text{3 NEPOZNATI } (x_0, \delta, M)$$

$$\to \underline{M = x_0\sin\theta-\bar{S}\cos(\theta-\phi_s)}$$

$$x_0\cdot\sin\phi_s = M\cdot\sin(\phi_s+\delta) \qquad \delta=?$$

$$x_0^2=X_1^2+(M+x_2)^2 = x_1^2+M^2+2Mx_2+x_2^2=$$

$$= \cancel{\bar{S}^2\cdot\sin^2(\theta-\phi_s)} + (x_0\sin\theta-\bar{S}\cos(\theta-\phi_s))^2+2(x_0\sin\theta-\bar{S}\cos(\theta-\phi_s))\cdot$$
$$\cdot\ \bar{S}\cdot\cos(\theta-\phi_s) + \underline{\bar{S}^2\cdot\cos^2(\theta-\phi_s)} =$$

$$= \bar{S}^2 + x_0^2\sin^2\theta - \cancel{2x_0\bar{S}\sin\theta}\cdot\cos(\theta-\phi_s) + \underline{\bar{S}^2\cos^2(\theta-\phi_s)} +$$
$$+ \ 2\bar{S}\cancel{x_0\sin\theta}\cdot\cos(\theta-\phi_s) - \underline{2\bar{S}^2\cdot\cos^2(\theta-\phi_s)}$$

$$x_0^2 = \bar{S}^2 - S^2\cos^2(\theta-\phi_s) + x_0^2\sin^2\theta$$

$$x_0^2(1-\sin^2\theta) = \bar{S}^2(1-\cos^2(\theta-\phi_s)) \qquad x_0^2 = \bar{S}^2\frac{\sin^2(\theta-\phi_s)}{\cos^2\theta}$$

$$\boxed{x_0=\frac{\bar{S}\sin(\theta-\phi_s)}{\cos\theta},} \qquad \sin(\theta-\phi_s)=\sin\theta\cdot\cos\phi_s-\sin\phi_s\cdot\cos(\theta)$$

$$\cos(\theta-\phi_s)=\frac{x_1}{\bar{S}} \qquad \boxed{\begin{aligned} x_1 &= \bar{S}\cdot\cos(\theta-\phi_s) \\ x_2 &= \bar{S}\cdot\sin(\theta-\phi_s)\end{aligned}}$$

$$M = x_0\cdot\sin\theta - x_2 = x_0\cdot\sin\theta - \bar{S}\cdot\sin(\theta-\phi_s)$$

$$x_0^2 = \cancel{\bar{S}^2\cdot\cos^2(\theta-\phi_s)} + (x_0\cdot\sin\theta-\bar{S}\sin(\theta-\phi_s))^2+2(x_0\cdot\sin\theta-\bar{S}\sin(\theta-\phi_s))\cdot$$
$$\bar{S}\cdot\sin(\theta-\phi_s) + \underline{\bar{S}^2\cdot\sin^2(\phi-\phi_s)} =$$

$$= \bar{S}^2 + x_0^2\sin^2\theta - 2x_0\cancel{\bar{S}\sin\theta}\sin(\theta-\phi_s) + \cancel{\bar{S}^2\sin^2}(\phi-\phi_s) + \cancel{2\bar{S}x_0\sin\theta}\cdot\sin(\theta-\phi_s) -$$
$$- \cancel{2}\bar{S}^2\sin^2(\theta-\phi_s) = \bar{S}^2+x_0^2\sin^2\theta - \bar{S}^2\sin^2(\theta-\phi_s)$$

$$x_0^2 - x_0^2\sin^2\theta = \bar{S}^2 - \bar{S}^2\sin^2(\theta-\phi_s) \Rightarrow x_0^2\cdot\cos^2\theta = \bar{S}^2\cdot\cos^2(\theta-\phi_s)$$

$$\boxed{x_0 = \frac{\bar{S}\cdot\cos(\theta-\phi_s)}{\cos(\theta)}}$$

$$\boxed{x_0 = \frac{\bar{S}\,\cos(\theta-\phi_s)}{\cos(\theta)}\quad \text{MMV}}$$

VO SIMON, ALOUINI VEZAT: $x_0 = \frac{\bar{S}\cos\phi_s}{\cos\theta}$ SE SOVPAĐA AKO SE ZEME ⊛

$$Q(x_1,y_1;\rho) = \frac{1}{2\pi\sqrt{1-\rho^2}}\int_0^\infty\int_0^\infty \exp\left[-\frac{(x+x_1)^2+(y+y_1)^2-2\rho(x+x_1)(y+y_1)}{2(1-\rho^2)}\right]dx\,dy$$

$$\boxed{\bar{S} = \sqrt{x_1^2+y_1^2} \qquad \phi_s = \text{arctg}\frac{y_1}{x_1}}$$

TOČNA SLIKA:

$(x+x_1) = N\cdot\cos\theta$

$x = -x_1 + N\cdot\cos\theta$

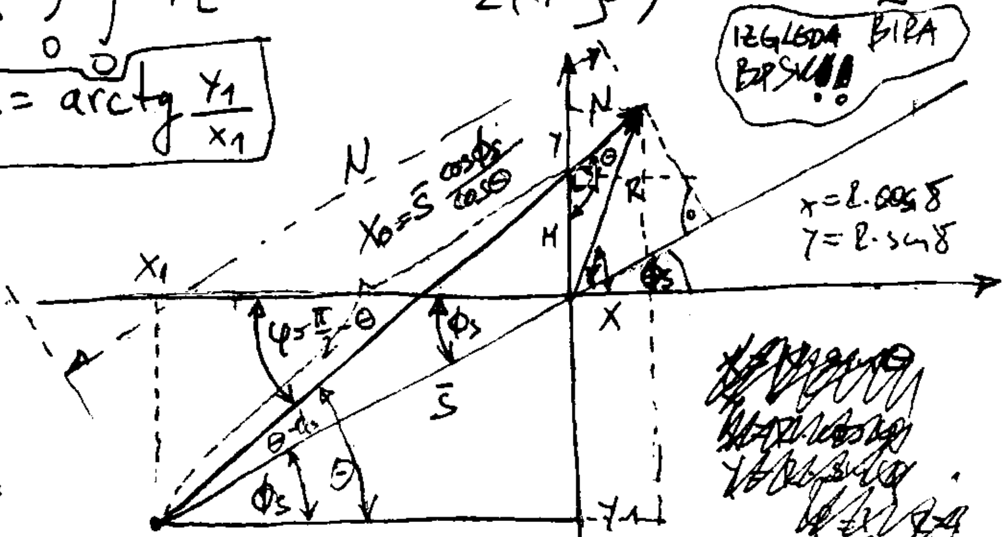
$(y+y_1) = N\cdot\sin\theta$

$y = -y_1 + N\cdot\sin\theta$

$\jmath = \begin{vmatrix} \frac{\partial x}{\partial N} & \frac{\partial x}{\partial\theta} \\ \frac{\partial y}{\partial N} & \frac{\partial y}{\partial\theta}\end{vmatrix} = N$ //

$x_1 = \bar{S}\cdot\cos\phi_s$

$y_1 = \bar{S}\cdot\sin\phi_s$



$$Ne^{j\theta} = (x+x_1) + j(y+y_1)$$

$$(x+x_1)^2 + (y+y_1)^2 = N^2(\cos^2\theta+\sin^2\theta) = N^2$$

$$2\rho(x+x_1)(y+y_1) = 2\rho\, N\cdot\cos\theta\; N\cdot\sin\theta = 2\rho N^2\cdot\sin\theta\cdot\cos\theta$$

$$\sin 2\theta = \sin(\theta+\theta) = \sin\theta\cdot\cos\theta + \sin\theta\cdot\cos\theta = \underline{2\sin\theta\cdot\cos\theta}$$

$$\boxed{2\rho(x+x_1)(y+y_1) = \rho N^2\sin 2\theta}$$

SOGLASNO PRISTAPOT VO J.W. CRAIG

PRETP. SE ZBOROT ZA BPSK

$$(x+x_1)^2 + (y+y_1)^2 - 2\rho(x+x_1)(y+y_1) = N^2 - \rho N^2\sin(2\theta)$$
$$= N^2(1-\rho\sin(2\theta))$$

$$Q(x_1,y_1,\rho) = \frac{1}{2\pi\sqrt{1-\rho^2}}\int_0^\infty\int_0^\infty \exp\left[\frac{N^2(1-\rho\sin(2\theta))}{2(1-\rho^2)}\right]dx\,dy =$$

$$\frac{1}{2\pi\sqrt{1-\rho^2}}\int_{\theta_1}^{\theta_2} d\theta \int_{x_0}^{\infty} N\cdot e^{-\underbrace{\frac{N^2(1-\rho\sin(2\theta))}{2(1-\rho^2)}}_{K}}\,dN = \left| \begin{matrix} N^2\cdot K = M \\ dM = 2N\cdot K\cdot dN \\ N\cdot dN = \frac{dM}{2K} \\ N = x_0 \quad M = K\cdot x_0^2 \end{matrix} \right|$$

ŠTO JACOBIAN

I

$$I = \int_{K\cdot x_0^2}^{\infty} e^{-M}\frac{dM}{2K} = \frac{1}{2K}e^{-M}\Big|_{\infty}^{K\cdot x_0^2} = \frac{1}{2K}e^{-Kx_0}$$ //

$$Q(x_1,y_1,\rho) = \frac{1}{2\pi\sqrt{1-\rho^2}}\int_{\theta_1}^{\theta_2} \frac{e^{-kx_0^2}}{2k}\cdot d\theta \quad \Big/ \; k = \frac{1-\rho\sin 2\theta}{2(1-\rho^2)} \Big/ =$$

$$= \frac{1}{2\pi\sqrt{1-\rho^2}}\int_{\theta_1}^{\theta_2} \frac{\sqrt{1-\rho^2}\,\cancel{1-\rho^2}}{1-\rho\sin 2\theta}\, \exp\left(-\frac{1-\rho\sin 2\theta}{1-\rho^2}\cdot\frac{\bar{s}^2\cos^2\phi_s}{2\cdot\cos^2\theta}\right)d\theta$$

$$\boxed{\begin{aligned}\sin 2\theta &= \sin(\pi-2\varphi)\\ &= \sin 2\varphi\end{aligned}}$$

$$\cos\theta = \cos\left(\tfrac{\pi}{2}-\varphi\right) = \sin\varphi$$

$$\varphi = \frac{\pi}{2}-\theta \qquad \boxed{\theta = \frac{\pi}{2}-\varphi} \qquad d\varphi = -d\theta$$

$$Q(x_1,y_1,\rho) = \frac{1}{2\pi}\int_{\theta_1}^{\theta_2} \frac{\sqrt{1-\rho^2}}{1-\rho\sin 2\theta}\, \exp\left(-\frac{1-\rho\sin 2\theta}{1-\rho^2}\,\frac{\bar{s}^2\cos^2\phi_s}{2\cos^2\theta}\right)d\theta$$

$$= \frac{1}{2\pi}\int_{\varphi_2}^{\varphi_1} \frac{\sqrt{1-\rho^2}}{1-\rho\sin 2\varphi}\, \exp\left(-\frac{1-\rho\sin 2\varphi}{1-\rho^2}\,\frac{\bar{s}^2\cos^2\phi_s}{2\cdot\sin^2\varphi}\right)d\varphi$$

$$\left.\begin{aligned} x+x_1 &= N\cos\theta\\ y+y_1 &= N\cdot\sin\theta\end{aligned}\right\}$$

PROBABILITY THAT SIGNAV VECTOROT $s = (-x, -y_1)$ SE NAJDE VO GORNIOT DESEN KVADRANT.

$$\theta = \underset{\theta_1}{\phi_s} \div \underset{\theta_2}{\tfrac{\pi}{2}}, \qquad \varphi = \frac{\pi}{2}-\theta \qquad \varphi = \underset{\varphi_1}{\frac{\pi}{2}-\phi_s} : \underset{\varphi_2}{0}$$

$$\theta = \underset{\theta_1}{0} \div \underset{\theta_2}{\phi_s} \qquad \varphi = \frac{\pi}{2}-\theta \qquad \varphi = \underset{\varphi_1}{\frac{\pi}{2}} \div \underset{\varphi_2}{\frac{\pi}{2}-\phi_s}$$

ZABELEŠKA NA SLIKATA ZA DA GO POKRIE CEL KVADRANT I θ TREBA DA ODI: $0 \div \phi_s$ I POTOA OD $\phi_s \div \frac{\pi}{2}$

$$Q(x_1,y_1,\rho) = \frac{1}{2\pi}\int_0^{\phi_s} \frac{\sqrt{1-\rho^2}}{1-\rho\sin 2\theta}\, \exp\left(-\frac{1-\rho\sin 2\theta}{1-\rho^2}\,\frac{\bar{s}^2\sin^2\phi_s}{2\cdot\sin^2\theta}\right)d\theta \;\text{(\#)} +$$

$$+ \frac{1}{2\pi}\int_{\phi_s}^{\frac{\pi}{2}} \frac{\sqrt{1-\rho^2}}{1-\rho\sin 2\theta}\, \exp\left(-\frac{1-\rho\sin 2\theta}{1-\rho^2}\,\frac{\bar{s}^2\cos^2\phi_s}{\cos^2\theta}\right)d\theta \;\text{(\&)}$$

PREKU φ

GRANICITE TREBA DA BIDAT: $0 \div \frac{\pi}{2}-\phi_s$; $\frac{\pi}{2}-\phi_s \div \frac{\pi}{2}$

$$Q(x_1,y_1,\rho) = \frac{1}{2\pi}\int_0^{\phi_s} \frac{\sqrt{1-\rho^2}}{1-\rho\sin 2\varphi}\, \exp\left(-\frac{1-\rho\sin 2\varphi}{1-\rho^2}\,\frac{\bar{s}^2\sin^2\phi_s}{2\cdot\sin^2\varphi}\right)d\varphi \;\text{(\#)} +$$

$$+ \frac{1}{2\pi}\int_0^{\frac{\pi}{2}-\phi_s} \frac{\sqrt{1-\rho^2}}{1-\rho\sin 2\theta}\, \exp\left(-\frac{1-\rho\sin 2\varphi}{1-\rho^2}\,\frac{\bar{s}^2\cos^2\phi_s}{2\cdot\sin^2\varphi}\right)d\varphi \;\text{(\&)}$$

$$\Theta(x_1, y_1, \rho) = \frac{1}{2\pi}\int_0^{\operatorname{arctg}\frac{y_1}{x_1}} \frac{\sqrt{1-\rho^2}}{1-\rho\sin 2\varphi}\exp\left(-\frac{1-\rho\sin 2\varphi}{1-\rho^2}\frac{x_1^2}{2\cdot\sin^2\varphi}\right)d\varphi \; + \quad (\#)$$

$$+ \frac{1}{2\pi}\int_0^{\frac{\pi}{2}-\operatorname{arctg}\frac{y_1}{x_1}} \frac{\sqrt{1-\rho^2}}{1-\rho\sin 2\varphi}\exp\left(-\frac{1-\rho\sin 2\varphi}{1-\rho^2}\frac{y_1^2}{2\cdot\sin^2\varphi}\right)d\varphi \quad (\#\#)$$

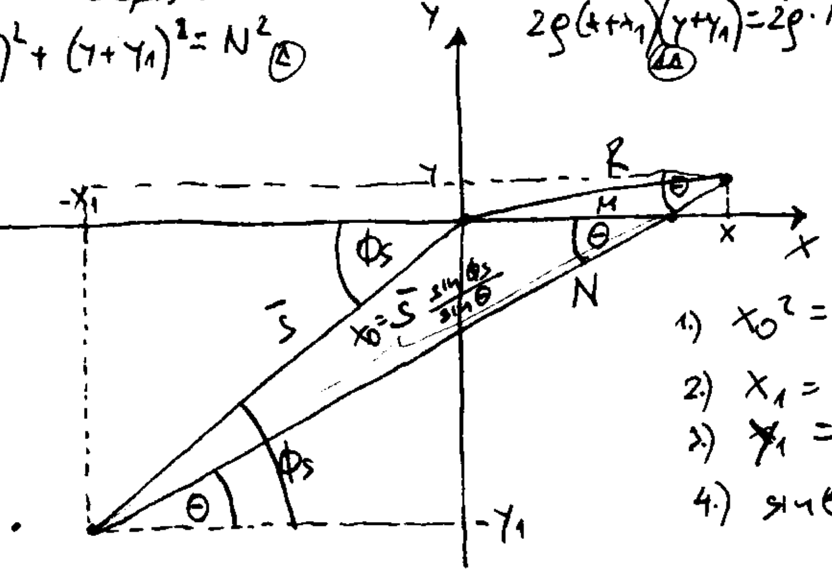
ZA OVOJ INTEGRAL NEMAŠE POTREBA OD VOVEDUVANJE NA POMOŠNA PROMENLIVA φ' ZATOA ŠTO e VO FORMA $\int_0^x \ldots dz$

- ~~[illegible]~~
- ZA INTEGRALOT (#) TREBA DA SE GLEDA VTORATA SLIKA

$$(x+x_1)^2 + (y+y_1)^2 = N^2 \quad (1)$$

$$2\rho(x+x_1)(y+y_1) = 2\rho\cdot N\cdot\cos\Theta\cdot N\cdot\sin\Theta = 2\rho N^2\sin\Theta\cos\Theta = \rho N^2\sin(2\Theta) \quad (1a)$$

SISTEM RAVENKI SO 4 NEPOZNATI



1.) $x_0^2 = (x_1+M)^2 + y_1^2$

2.) $x_1 = \bar S\cdot\cos\Phi_s$

3.) $y_1 = \bar S\cdot\sin\Phi_s$

4.) $\sin\Theta = \dfrac{x_1+M}{\bar S\cos\Phi_s}$ $\quad M = \bar S\sin\Theta - x_1$

$x_1 + M = \bar S\sin\Theta$

~~$x_0^2 = x_1^2 + 2M\cdot x_1 + M^2 + y_1^2 = \bar S^2\cdot\cos^2\Phi_s + 2(\bar S\cdot\sin\Theta - x_1)\bar S\cos\Phi_s + \bar S^2\cdot\sin^2\Phi_s + M^2$~~

~~$= \bar S^2 + 2\bar S^2\sin\Theta\cdot\cos\Phi_s - 2\bar S^2\cos^2\Phi_s + \bar S^2\sin^2\Theta - 2\bar S^2\sin\Theta\cdot\cos\Phi_s + \bar S^2\cdot\cos^2\Phi$~~

~~$x_0^2 = 2\bar S^2 - 2\bar S^2\cos^2\Phi_s = 2\bar S^2(1-\cos^2\Phi_s)$~~

~~$x_0^2 = \bar S^2 - \bar S^2\cos^2\Phi_s + \bar S^2\sin^2\Theta$~~

$$x_0^2 = (x_1+M)^2 + y_1^2 = \bar S^2\sin^2\Theta + \bar S^2\sin^2\Phi_s \qquad x_0(1-\sin^2\Theta) = \bar S^2\sin^2\Phi$$

$$\boxed{x_0^2 = \bar S^2\frac{\sin^2\Phi_s}{\cos^2\Theta}} \qquad \boxed{x_0 = \frac{\bar S\sin\Phi_s}{\sin\Theta}}$$

DOKAZANO!!!

Φ - (1a)

$$(\#) = \frac{1}{2\pi\sqrt{1-\rho^2}}\int_{\Theta_1}^{\Theta_2} d\Theta\int_{x_0}^{\infty} N\cdot\exp\left[-\frac{N^2(1-\rho\sin 2\Theta)}{2(1-\rho^2)}\right]dN = \left| x_0 = \bar S\frac{\sin\Phi_s}{\sin\Theta}\right| =$$

$$= \frac{1}{2\pi\sqrt{1-\rho^2}}\int_{\Theta_1=0}^{\Theta_2=\Phi_s}\frac{\cancel{1-\rho^2}\ \sqrt{1-\rho^2}}{1-\rho\sin 2\Theta}\exp\left(-\frac{1-\rho\sin 2\Theta}{1-\rho^2}\cdot\frac{\overbrace{\bar S^2\sin^2\Phi_s}^{y_1^2}}{\sin^2\Theta}\right)d\Theta =$$

$$= \frac{1}{2\pi}\int_0^{\Phi_s}\frac{\sqrt{1-\rho^2}}{1-\rho\sin(2\Theta)}\exp\left(-\frac{1-\rho\sin 2\Theta}{1-\rho^2}\frac{y_1^2}{\sin^2\Theta}\right)d\Theta$$

DOKAZANO e (#) !!!
NEMA POTREBA NA ZAMENA SO φ!!

- Ако ~~x_1~~ дистрибуцијата на x_1 не е корелирана со y_1 $\boxed{\rho=0}$

$$\boxed{Q(x_1,y_1;0)=Q(x_1)\cdot Q(y_1)}=\frac{1}{2\pi}\int_0^{\pi/2-\operatorname{arctg}\frac{y_1}{x_1}}\exp\left[-\frac{x_1^2}{2\sin^2\theta}\right]d\theta+\frac{1}{2\pi}\int_0^{\operatorname{arctg}\frac{y_1}{x_1}}\exp\left[-\frac{y_1^2}{2\sin^2\theta}\right]d\theta$$

$x_1=y_1=x \quad \operatorname{arctg}1=\frac{\pi}{4}$

$$\boxed{Q(x,x;0)=\frac{1}{\pi}\int_0^{\pi/4}\exp\left[-\frac{x^2}{2\sin^2\theta}\right]d\theta \qquad Q(x,x,0)=Q^2(x)}$$

SINGLE INTEGRAL FORM FOR THE SQUARE OF GAUSSIAN Q-FUNCTION

$$\boxed{Q^2(x)=\frac{1}{\pi}\int_0^{\pi/4}e^{-x^2/2\sin^2\theta}\,d\theta}$$

MMV

→ ВНИМАВАЈ!!! ОВА Е ЗА: $\boxed{\rho=0}$

ЗА $\rho\neq0$ ВИДИ N4. pp. 141

Simon & Alouini Appendix 4A

⊙

$$\int_u^\infty\frac{e^{-\mu x}dx}{x\sqrt{x-u}}=\frac{\pi}{\sqrt{u}}\left[1-\Phi(\sqrt{u\mu})\right]=\frac{\pi}{\sqrt{u}}\left[1-\operatorname{erf}(\sqrt{u\mu})\right]=\frac{\pi}{\sqrt{u}}\operatorname{erfc}(\sqrt{u\mu})$$

$$\int_u^\infty\frac{e^{-\mu x}dx}{x\sqrt{x-u}}=\frac{\pi}{\sqrt{u}}\operatorname{erfc}(\sqrt{u\mu})\Big/\frac{1}{2}e^{\mu u} \qquad \int_u^\infty\frac{e^{\mu u-\mu x}}{x\sqrt{x-u}}dx=\frac{\pi}{\sqrt{u}}e^{\mu u}\operatorname{erfc}(\sqrt{u\mu})$$

$\boxed{u=y^2}$

$$\int_{y^2}^\infty\frac{e^{\mu y^2-\mu x}}{x\sqrt{x-y^2}}dx=\frac{\pi}{\sqrt{y^2}}e^{\mu y^2}\operatorname{erfc}(\sqrt{\mu y^2})$$

$u=x-y^2$
$du=dx$
$x=u+y^2$

$x=y^2 \quad u=0$
$x=\infty \quad u=\infty$

$$\int_0^\infty\frac{e^{-\mu u}}{(u+y^2)\sqrt{u}}du=\frac{\pi}{\sqrt{y^2}}e^{\mu y^2}\operatorname{erfc}(y\sqrt{\mu});\quad\int_0^\infty\frac{e^{-\mu u}}{(u+y^2)\sqrt{u}}du=\frac{\pi}{\sqrt{y^2}}e^{\mu y^2}\operatorname{erfc}(y\sqrt{\mu})$$

$u=t^2;\ t=\sqrt{u};\ du=2t\,dt=2\sqrt{u}\,dt$

$$\boxed{\int_0^\infty\frac{e^{-\mu u}}{(u+y^2)\sqrt{u}}du=\frac{\pi}{y}e^{\mu y^2}\operatorname{erfc}(y\sqrt{\mu})}\ \frac{1}{2}\cdot\int_0^\infty\frac{e^{-\mu t^2}\,\cancel{2t}\,dt}{(t^2+y^2)\,\cancel{t}}=\frac{\pi}{2\cdot y}e^{\mu y^2}\operatorname{erfc}(y\sqrt{\mu})$$

- $y=1 \quad \mu=z^2 \Rightarrow$

$$\int_0^\infty\frac{e^{-z^2t^2}dt}{(t^2+1)}=\frac{\pi}{2\cdot1}e^{z^2}\operatorname{erfc}(z)$$

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$$\int_0^\infty \frac{e^{-z^2(1+t^2)}}{(t^2+1)}\,dt = \frac{\pi}{2}\,erfc(z)$$

$$\sin^2\theta = \frac{1}{1+t^2}$$

$$\cos^2\theta = \frac{t^2}{1+t^2}$$

$$dt = \cancel{\frac{d\theta}{1+t^2}}\ \frac{-(t^2+1)\,d\theta}{}$$

$$\boxed{d\theta = -\frac{dt}{1+t^2}}$$

$$\frac{2}{\pi}\int_{\theta_1}^{\theta_2} (-1)\cdot e^{-z^2/\sin^2\theta}\,d\theta = erfc(z) \quad ⊛$$

$$\theta = arctg\left(\frac{1}{1+t^2}\right) \qquad \sin\theta = \sqrt{\frac{1}{1+t^2}}$$

$$\cancel{2}\sin\theta\,\cos\theta\cdot d\theta = -\frac{\cancel{2}t}{(1+t^2)^2}\cdot dt = \frac{1}{\cancel{\sqrt{1+t^2}}}\cdot\frac{\cancel{t}}{\cancel{\sqrt{1+t^2}}}\,d\theta = \frac{\cancel{t}}{(1+t^2)^2}\,dt$$

$$\sin^2\theta + \cos^2\theta = 1 \qquad \frac{1}{1+t^2} + \cos^2\theta = 1 \qquad \cos^2\theta = \frac{-\cancel{1}+\cancel{1}+t^2}{t^2+1}$$

$$\boxed{\cos^2\theta = \frac{t^2}{t^2+1}}$$

$$\boxed{d\theta = \frac{dt}{1+t^2}}$$

$$\sin\theta = \sqrt{\frac{1}{1+t^2}}$$

$$\boxed{\begin{array}{lll} t = 0 & \sin\theta = 1 & \theta_1 = \frac{\pi}{2} \\ t = \infty & \sin\theta = 0 & \theta_2 = 0 \end{array}}$$

$$⊛\ -\frac{2}{\pi}\int_{\pi/2}^{0} e^{-z^2/\sin^2\theta}\,d\theta = erfc(z)$$

$$\boxed{\frac{2}{\pi}\int_0^{\pi/2} e^{-z^2/\sin^2\theta}\,d\theta = erfc(z)}$$

DOKAZAN

$$Q(z) = \frac{1}{2}\,erfc\left(\frac{z}{\sqrt{2}}\right)$$

$$\frac{2}{\pi}\int_0^{\pi/2} e^{-z^2/\sin^2\theta}\,d\theta = 2\cdot\frac{1}{2}\,erfc\left(\frac{\sqrt{2}z}{\sqrt{2}}\right) = 2\,Q(\sqrt{2}z)$$

$$Q(\sqrt{2}z) = \frac{1}{\pi}\int_0^{\pi/2} e^{-z^2/\sin^2\theta}\,d\theta$$

$$\boxed{z = \frac{x}{\sqrt{2}}} \qquad Q(x) = \frac{1}{\pi}\int_0^{\pi/2} e^{-\frac{x^2}{2\sin^2\theta}}\,d\theta$$

$$Q(z) = \frac{1}{2}\,erfc\left(\frac{z}{\sqrt{2}}\right) = \frac{1}{2}\cdot\frac{2}{\sqrt{\pi}}\int_{z/\sqrt{2}}^{\infty} e^{-x^2}\,dx$$

$$Q(z) = \frac{1}{2}\frac{2}{\sqrt{\pi}}\int_{z/\sqrt{2}}^{\infty} e^{-x^2}dx \qquad x = \frac{y}{\sqrt{2}} \qquad dx = \frac{dy}{\sqrt{2}}$$

$$x = \frac{z}{\sqrt{2}} \quad y = z$$

$$x = \infty \quad y = \infty$$

$$Q(z) = \frac{1}{2}\frac{2}{\sqrt{\pi}}\int_z^{\infty} e^{-\frac{y^2}{2}}\frac{dy}{\sqrt{2}} = \frac{1}{\sqrt{2\pi}}\int_z^{\infty} e^{-y^2/2}dy$$

$$\mathrm{erfc}(0) = \frac{2}{\sqrt{\pi}}\int_0^{\infty} e^{-\frac{x^2}{2}}dx = 1$$

$$\mathrm{erfc}(z) = \frac{2}{\sqrt{\pi}}\int_z^{\infty} e^{-x^2}dx = \left| \begin{matrix} x = \frac{y}{\sqrt{2}} & x^2 = \frac{y^2}{2} \\ dx = \frac{dy}{\sqrt{2}} & x = z \quad y = \sqrt{2}z \end{matrix} \right|$$

$$\mathrm{erfc}(z) = \frac{2}{\sqrt{\pi}}\frac{1}{\sqrt{2}}\int_{\sqrt{2}z}^{\infty} e^{-y^2/2}dx = 2\underbrace{\frac{1}{\sqrt{2\pi}}\int_{\sqrt{2}z}^{\infty} e^{-y^2/2}dx}_{Q(\sqrt{2}z)}$$

$$\boxed{\mathrm{erfc}(z) = 2Q(\sqrt{2}z)} \qquad \boxed{Q(z) = \frac{1}{2}\mathrm{erfc}\left(\frac{z}{\sqrt{2}}\right)}$$

MMV

$z = 0 \qquad \mathrm{erfc}(0) = 1 = 2Q(\sqrt{2}z) = 2Q(0)$

$$\boxed{Q(0) = \frac{1}{2}}$$

$$Q(z) = 2\underbrace{\int_0^{\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{y^2}{2}}dy}_{1}\underbrace{\int_z^{\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}dx}$$

MMV

$\equiv$

PRV KVADRANT

MISLIM 6 PODOLO PRVU SLIKA
$x > 0 \quad y > 0$
$\exists$ I KVADRANT
MMV

$$Q(z) = 2\,\frac{1}{2\pi}\int_0^{\infty}\int_z^{\infty} e^{-\frac{x^2+y^2}{2}}dx\,dy$$

$$Q(z) = 2\cdot\frac{1}{2\pi}\int_0^{\pi/2}\underbrace{\int_{z/\cos\varphi}^{\infty} e^{-\frac{r^2}{2}}r\,dr}\,d\varphi$$

$x = r\cos\varphi$

$y = r\cdot\sin\varphi$

$y = 0 \quad \varphi = 0$

$y = \infty \quad \varphi = \frac{\pi}{2}$

$x^2 + y^2 = r^2$

$J = r \qquad dx\,dy = r\cdot dr\,d\varphi$

$x = z \qquad \boxed{r = \frac{z}{\cos\varphi}}$ MMV

$x = \infty \quad r = \infty$

$$I = \int_z^{\infty} r e^{-r^2/2}dr = \cancel{\frac{2}{2}}\int_{z/\cos\varphi}^{\infty} e^{-r^2/2}d\left(\frac{r^2}{2}\right) = -e^{-\frac{r^2}{2}}\Big|_{z/\cos\varphi}^{\infty} = \cancel{e^{-\frac{z^2}{2}}} = e^{-\frac{z^2}{2\cos\varphi}}$$

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$$Q(z)=\frac{1}{\pi}\int_0^{\pi/2} e^{-\frac{z^2}{2\cos\varphi}}d\varphi = \frac{1}{\pi}\int_0^{\pi/2} e^{-\frac{z^2}{2\cos\varphi}}d\varphi$$

$$\boxed{\varphi=\frac{\pi}{2}-\Theta} \quad d\varphi=-d\Theta \quad \begin{matrix}\varphi=0 & \Theta=\frac{\pi}{2}\\ \varphi=\frac{\pi}{2} & \Theta=0\end{matrix} \quad \cos\left(\frac{\pi}{2}-\Theta\right)=\sin\Theta$$

$$Q(z)=-\frac{1}{\pi}\int_{\pi/2}^{0} e^{-\frac{z^2}{2\sin\Theta}}d\Theta = \frac{1}{\pi}\int_0^{\pi} e^{-\frac{z^2}{2\sin\Theta}}d\Theta$$

FORMULA 4.2

- PROOF OF 4.9

$$\boxed{Q^2(x)=\frac{1}{\pi}\int_0^{\pi/4} e^{-\frac{x^2}{2\sin\Theta}}d\Theta}$$

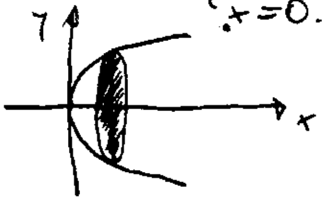
$$Q^2(z)=\left(\frac{1}{\sqrt{2\pi}}\right)^2\int_z^{\infty} e^{-\frac{x^2}{2}}dx\int_z^{\infty} e^{-\frac{y^2}{2}}dy=\left(\frac{1}{\sqrt{2\pi}}\right)^2\int_z^{\infty}\int_z^{\infty} e^{-\frac{x^2+y^2}{2}}dx\,dy$$

$x=r\cdot\cos\varphi$
$y=r\cdot\sin\varphi$
$J=r$

$x=z \Rightarrow r=\frac{z}{\cos\varphi}$
$x=\infty \Rightarrow r=\infty$
$\varphi=0\div\pi/4$

$r=\frac{y}{\sin\varphi}$
$\varphi=\frac{\pi}{4}\div\frac{\pi}{2}$

VOLUMES: ① $y=\sqrt{x}$ $\quad x=y^2$
$x=0\dots1$



$P_1=dx\cdot R^2\pi \qquad R=y^2=(\sqrt{x})^2=x$

$$V=\int_0^1 P_1\,dP=\int_0^1 x\cdot\pi\,dx=\frac{x^2}{2}\pi\Big|_0^1=\frac{\pi}{2}$$

② $y=x^3 \quad y=8\ \&\ x=0 \qquad x=\sqrt[3]{y}$

$P_1=dy\cdot R^2\cdot\pi=dy\cdot x^2\pi=\underline{\underline{dy\sqrt[3]{y^2}}}\,\pi \qquad V=\int_0^8\sqrt[3]{y^2}\,\pi\,dy$

$$Q^2(z)=\frac{1}{2\pi}\int_0^{\pi/4}\int_{\frac{z}{\sin\varphi}}^{\infty} e^{-\frac{r^2}{2}}r\,dr+\frac{1}{2\pi}\int_{\pi/4}^{\pi/2}\int_{z/\cos\varphi}^{\infty} e^{-r^2/2}r\,dr=\frac{1}{2\pi}\int_0^{\pi/4} e^{-\frac{z^2}{2\sin^2\varphi}}d\varphi+$$

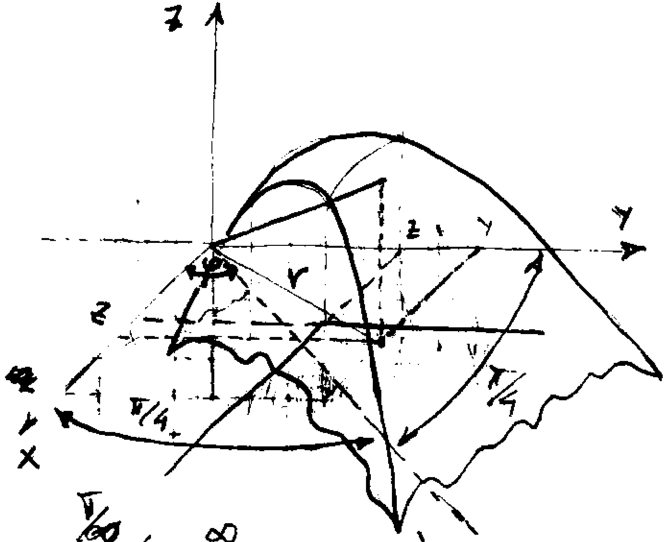
$$+\frac{1}{2\pi}\int_{\pi/4}^{\pi/2} e^{-\frac{z^2}{2\cos^2\varphi}}d\varphi$$

SECOND INTEGRAL

$\Theta=\frac{\pi}{2}-\varphi \qquad \varphi=\frac{\pi}{2}-\Theta \qquad d\varphi=-d\Theta$
$\varphi=\frac{\pi}{4} \quad \underline{\Theta=\frac{\pi}{2}-\frac{\pi}{4}=\frac{\pi}{4}}$
$\varphi=\frac{\pi}{2} \quad \Theta=0$

$$Q^2(z)=\frac{1}{2\pi}\int_0^{\pi/4} e^{-\frac{z^2}{2\sin^2\varphi}}d\varphi+\frac{1}{2\pi}\int_0^{\pi/4} e^{-\frac{z^2}{2\sin^2\Theta}}d\Theta=\underline{\underline{\frac{1}{\pi}\int_0^{\pi/4} e^{-\frac{z^2}{2\sin^2\Theta}}d\Theta}}$$

DOKAZANO



$$Q^2(z) = K \int_z^{\infty} \int_z^{\infty} e^{-\frac{x^2+y^2}{2}} dx\,dy$$

$x = r \cdot \cos\varphi$ $J = r$

$y = r \cdot \sin\varphi$

$$Q^2(z) = K \int_z^{\infty} \left(\int_z^{\infty} e^{-\frac{x^2+y^2}{2}} dy \right) dx =$$

$y = z \Rightarrow r = \frac{z}{\sin\varphi}$

$y = \infty \Rightarrow r = \infty$

$$= K \int_{\pi/4}^{\pi/2} \left(\int_{z/\sin\varphi}^{\infty} e^{-\frac{r^2}{2}} r\,dr \right) d\varphi + \underbrace{K \int_z^{\infty} \left(\int_z^{\infty} e^{-\frac{x^2+y^2}{2}} dx \right) dy}_{I_2} ;$$

$$I_2 = \int_{\boxed{0}}^{\boxed{\pi/4}} \left(\int_{\boxed{\frac{z}{\cos\varphi}}}^{\infty} e^{-\frac{r^2}{2}} r\,dr \right) d\varphi$$

$$Q^2(z) = K \int_{\pi/4}^{\pi/2} e^{-\frac{z^2}{2\sin^2\varphi}} d\varphi + K \int_0^{\pi/4} e^{-\frac{z^2}{2\cos^2\varphi}} d\varphi$$

$\theta = \frac{\pi}{2} - \varphi$

$\varphi = \frac{\pi}{2} - \theta$

$d\varphi = -d\theta$

$\varphi = 0$ $\theta = \frac{\pi}{2}$

$\varphi = \pi/4$ $\theta = \frac{\pi}{4}$

$$Q^2(z) = K \int_{\pi/4}^{\pi/2} e^{-\frac{z^2}{2\sin^2\varphi}} d\varphi + K \int_{\pi/4}^{\pi/2} e^{-\frac{z^2}{2\sin^2\theta}} d\theta = 2K \int_{\pi/4}^{\pi/2} e^{-\frac{z^2}{2\sin^2\theta}} d\theta$$

ZATOA TREBA OBRATNO DA SE ZEMO!!! $\pi/4 \div \pi/2$

ZNACI SO OBRATNI GRANICI:

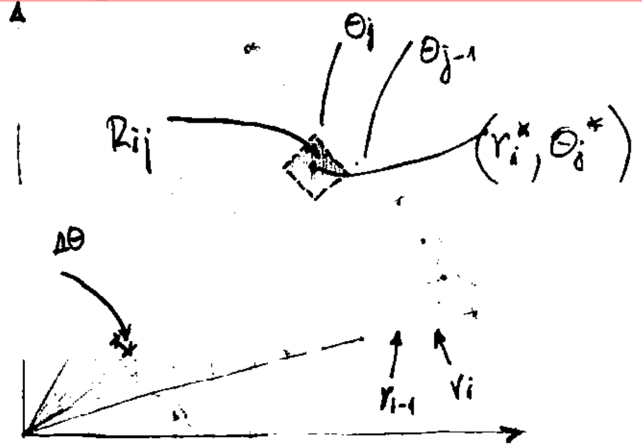
$$Q^2(z) = K \int_0^{\pi/4} \underbrace{\int_{z/\sin\varphi}^{\infty} e^{-r^2/2} dr}_{\text{po „}y\text{"}} d\varphi + K \int_{\pi/4}^{\pi/2} \underbrace{\int_{z/\cos\varphi}^{\infty} e^{-r^2/2} dr}_{\text{po „}x\text{"}} d\varphi = K \int_0^{\pi/4} e^{-\frac{z^2}{2\sin^2\varphi}} du + K \int_{\pi/4}^{\pi/2} e^{-\frac{z^2}{2\cos^2\varphi}} d\varphi$$

vidi ← pr.123

$$\boxed{Q^2(z) = \frac{1}{\pi} \int_0^{\pi/4} e^{-\frac{z^2}{2\sin^2\varphi}} d\varphi}$$

PODOBRO SE GLEDA NA PR.134 I PR.135

⊙ STEWART: DOUBLE INTEGRALS IN POLAR COORDINATES



- AREA OF A SECTOR OF CIRCLE WITH CENTRAL ANGLE θ IS: $\frac{1}{2} r^2 \theta$

$x^2 + y^2 = R^2 \qquad y = \sqrt{R^2 - x^2}$

$x = r \cdot \cos\varphi \qquad J = r$

$y = r \cdot \sin\varphi$

$\int_{\theta_1}^{\theta_2} d\varphi \int$

VIDI PP. 128
FAZI POVRŠINATA NA TRIAGOLNIKOT
MMV!!

$$I = 2\int_{-R}^{R} \sqrt{R^2 - x^2}\, dx = \left| \begin{array}{l} x = R\cdot\cos\varphi \\ x^2 = R^2\cdot\cos\varphi \\ dx = -R\cdot\sin\varphi\, d\varphi \\ x=-R \quad \varphi = +\pi \\ x = R \quad \varphi = 0 \end{array} \right| = -2\int_0^{\pi} R\underbrace{\sqrt{1-\cos^2\varphi}}_{\sin\varphi} R\sin\varphi\, d\varphi$$

$$I = +2R^2 \int_0^{\pi} \sqrt{1-\cos^2\varphi}\; d(\cos\varphi) = -2R^2 \underbrace{\int_0^{\pi} \sin^2\varphi\, d\varphi}\; = 2R^2 \underbrace{\int_0^{\pi} (1 - \cos^2_\varphi)\, d\varphi}_{\circledast \text{ AREA OF CIRCLE}}$$

ZA SECTOR
GENERAL

$$\boxed{I_1 = R^2 \int_{\theta_1}^{\theta_2} \sin^2\varphi\, d\varphi}$$

$\cos(2\varphi) = \cos^2\varphi - \sin^2\varphi =$

~~$\cos^2(\varphi)+$~~ $= \cos^2(\varphi) - 1 + \cos^2\varphi$

$\cos(\varphi - \varphi) = \cos^2(\varphi) + \sin^2\varphi$

$\cos(2\varphi) = 2\cos^2(\varphi) - 1 \qquad \cos^2\varphi = \frac{1}{2}(1 + \cos 2\varphi)$

$$I_1 = R^2 \int_{\theta_1}^{\theta_2} (1 - \cos^2\varphi)\, d\varphi = R^2 \int_{\theta_1}^{\theta_2} \left(1 - \frac{1}{2} - \frac{1}{2}\cos 2\varphi\right) d\varphi$$

$$I_1 = R^2 \int_{\theta_1}^{\theta_2} \frac{1}{2}(1 - \cos 2\varphi)\, d\varphi = \frac{R^2}{2}\left[\varphi\Big|_{\theta_1}^{\theta_2} - \frac{1}{2}\int_{\theta_1}^{\theta_2} \cos 2\varphi\, d(2\varphi) \right] =$$

$$= \frac{R^2}{2}\left((\theta_2 - \theta_1) - \frac{1}{2}\sin(2\varphi)\Big|_{\theta_1}^{\theta_2}\right)$$

$I = \int \sin^2\varphi\, d\varphi = \frac{1}{2}(\varphi - \sin\varphi\cdot\cos\varphi)$

$$\circledast = 2R^2\left[\int_0^{\pi} \sin^2\varphi\, d\varphi\right] = -2R^2 \frac{1}{2}\left(\varphi - \frac{\sin 2\varphi}{2}\right)\Big|_0^{\pi} = \frac{\cancel{2}R^2}{\cancel{2}}[\pi - 0] = R^2\pi$$

MMV

- $\theta_1 = 0 \quad \theta_2 = \theta$ SECTOR!

$$I_1 = \frac{R^2}{2}\left(\varphi - \frac{1}{2}\sin 2\varphi\right)\Big|_0^{\theta} = \frac{R^2}{2}\left(\theta - \frac{1}{2}\sin 2\theta\right)$$

VIDI PP. 128

$\theta = \frac{\pi}{2} \Rightarrow 0$

$\frac{R^2}{2}\left(\frac{\pi}{2} - \frac{\cancel{\sin\pi}}{2}\right) = \frac{R^2\pi}{4}$ W

① **PROOF:**

$\Delta\theta = \theta_j - \theta_{j-1}$

$$A_j = R^2\int_0^{\theta_j}\sin^2\varphi\,d\varphi = \frac{R^2}{2}\left(\theta_j - \frac{1}{2}\sin 2\theta_j\right) \approx \frac{R^2}{2}\theta_j$$

$$A_{j-1} = R^2\int_0^{\theta_{j-1}}\sin^2\varphi\,d\varphi = \frac{R^2}{2}\left(\theta_{j-1} - \frac{1}{2}\sin 2\theta_{j-1}\right) \approx \frac{R^2\theta_{j-1}}{2}$$

$$R_{ij} = \left\{(r,\theta) \mid r_{i-1}\le r\le r_i,\ \theta_{j-1}\le\theta\le\theta_j\right\}$$

$$r_i^* = \frac{1}{2}\ \frac{r_i+r_{i-1}}{2} \qquad \theta_j^* = \frac{\theta_i+\theta_{j+1}}{2}$$

$$\boxed{\theta_j - \theta_{j-1} = \Delta\theta}$$

• AREA OF R_{ij}

$$\Delta A_{ik} = \cancel{R^2}\theta_j\ \frac{r_i^2\cdot\Delta\theta}{2} - \frac{r_{i-1}^2\cdot\Delta\theta}{2} = \frac{1}{2}\left(r_i^2 - r_{i-1}^2\right)\Delta\theta$$

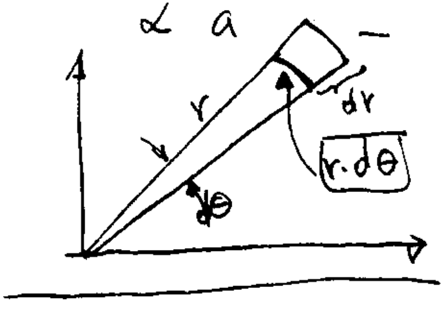
$$\boxed{\Delta A_i = \underbrace{\frac{\Delta\theta}{2}(r_i+r_{i-1})}_{r_i^*}\underbrace{(r_i-r_{i-1})}_{\Delta r} = r_i^*\Delta r\Delta\theta}$$ MMV

$$\boxed{\sum_{i=1}^{m}\sum_{j=1}^{n} f\left(r_i^*\cos\theta_j^*, r_i^*\sin\theta_j^*\right)\Delta A_i = \sum_{i=1}^{m}\sum_{j=1}^{n} f\left(r_i^*\cos\theta_j^*, r_i^*\sin\theta_j^*\right)r_i^*\Delta r\Delta\theta}$$

$$g(r,\theta) = r_i^* f\left(r_i^*\cos\theta_j^*, r_i^*\sin\theta_j^*\right) \qquad \sum_{i=1}^{m}\sum_{j=1}^{n} g(r_i^*,\theta_j^*)\Delta r\Delta\theta$$

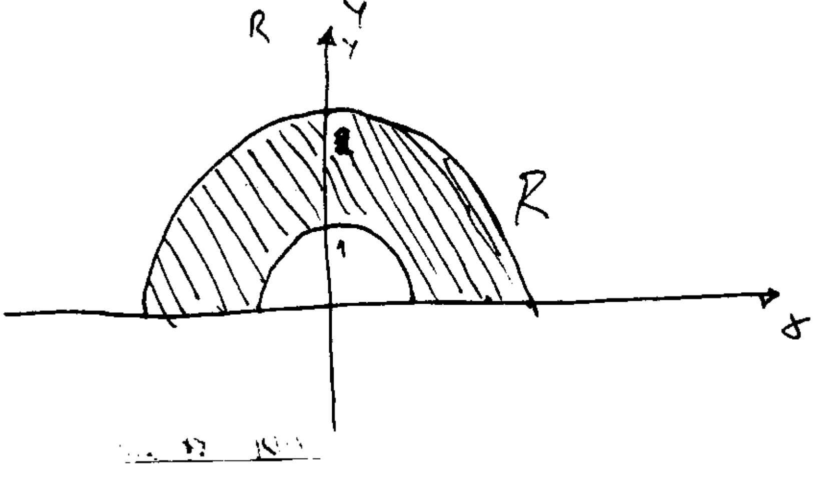
$$\iint_R f(x,y)\,dA = \lim_{\substack{m\to\infty\\ n\to\infty}}\sum_{i=1}^{m}\sum_{j=1}^{n} g(r_i^*,\theta_j^*)\Delta r\Delta\theta = \int_\alpha^\beta\int_a^b g(r,\theta)\,dr\,d\theta =$$

$$= \int_\alpha^\infty\int_a^b f(r\cos\theta, r\sin\theta)\,r\,dr\,d\theta \qquad \boxed{dA = r\,d\theta\cdot dr}$$



EXAMPLE 1

$$\iint_R (3x+4y^2)\,dA$$



$$\int_0^{\pi} d\varphi \int_1^{2} r\cdot\left(3r\cdot\cos\varphi + 4r^2\cdot\sin^2\varphi\right) dr = \int_0^{\pi} 15\sin^2\varphi + 7\cos\varphi \, d\varphi$$

$$= \cancel{-15\cos\varphi\Big|_0^{\pi} + 7\sin\varphi\Big|_0^{\pi} = -15(-1-1) = 30}$$

$$= -\frac{15}{2}\underbrace{\cancel{\frac{\sin(2\varphi)}{2}}}_{\to 0}\Big|_0^{\pi} + \frac{15}{2}\varphi\Big|_0^{\pi} + 7\sin\varphi\Big|_0^{\pi} = \frac{15\pi}{2}$$

$\boxed{\text{EXAMPLE 2}}$ $V = ?$ SOLID BOUNDED BY $z=0$ AND $\boxed{z = 1 - x^2 - y^2}$

$$V = \int_{-1}^{1}\left[\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \left(1 - x^2 - y^2\right) dy\right] dx$$

$x = r\cdot\cos\varphi$
$y = r\sin\varphi$
$J = r$

$$V = \int_0^{2\pi}\int_0^{1} \left(1 - r^2\right) r\,dr\,d\varphi = \frac{\pi}{2}$$

$\boxed{\text{EQUATION 3}}$

$$D = \left\{(r,\theta) \mid \alpha \le \theta < \beta,\ h_1(\theta) \le r \le h_2(\theta)\right\}$$

$$\iint_D f(x,y)\,dA = \int_{\alpha}^{\beta}\int_{h_1(\theta)}^{h_2(\theta)} f(r\cos\theta, r\sin\theta)\, r\,dr\,d\theta$$

$\boxed{\text{MMV}}$

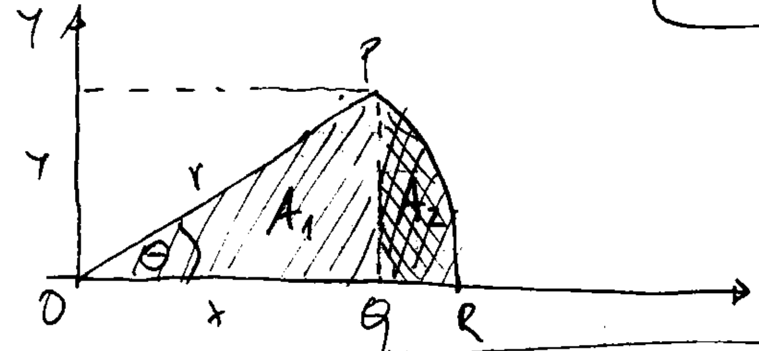
$f(x,y) = 1 \qquad h_1(\theta) = 0 \quad h_2(\theta) = h(\theta)$

$$A(D) = \iint_D 1\,dA = \int_{\alpha}^{\beta}\int_0^{h(\theta)} 1\cdot r\,dr\,d\theta = \int_{\alpha}^{\beta} \frac{r^2}{2}\Big|_0^{h(\theta)} d\theta$$

$$\boxed{A(D) = \int_{\alpha}^{\beta} \frac{h^2(\theta)}{2}\,d\theta}$$

Stewart (Excercises) 7.3

PR. 35 PROVE FORMULA $\boxed{A = \frac{1}{2} r^2\theta}$ FOR AREA OF SECTOR



$x^2+y^2=r^2$

$y=\pm\sqrt{r^2-x^2}$

$A=A_1+A_2$

$A_1=\frac{x\cdot y}{2}$

$x=r\cdot\cos\theta$

$y=r\cdot\sin\theta$

$\boxed{A_1=\frac{r^2\cos\theta\cdot\sin\theta}{2}}$

MMV

AKO INTEGRUJEŠ PO "φ" MORA DA VYMAŇAŠ DOLA
$\varphi=\theta\div 0$

$$A_2=\int_{r\cos\theta}^{r}\sqrt{r^2-x^2}\,dx=\frac{1}{2}x\sqrt{r^2-x^2}+\frac{1}{2}r^2\arcsin\left(\frac{x}{r}\right)$$

$$=-\frac{1}{2}r^2\sin\theta\cdot\cos\theta+\frac{1}{2}r^2\cdot\theta$$ (DOKÁŽ)

$\boxed{A=\frac{1}{2}r^2\theta}$

$$A=A_1+A_2=\frac{r^2\cos\theta\sin\theta}{2}-\frac{r^2\sin\theta\cos\theta}{2}+\frac{1}{2}r^2\theta$$

$y=r\cdot\sin\theta$

$\theta=\arcsin\frac{y}{r}$

$dy=r\cos\theta\,d\theta=r\sqrt{1-\sin\theta}\,d\theta$

$$\boxed{\frac{d\theta}{dy}=\frac{1}{r\sqrt{1-\sin\theta}}=\frac{1}{\sqrt{r^2-y^2}}}$$

$y=\operatorname{arctg}x$ $x=\operatorname{tg}y$ $dx=\frac{\cos y\cos y+\sin y\sin y}{\cos^2 y}dy$

$$dx=\frac{1}{\cos^2 y}dy=(1+\operatorname{tg}^2y)dy=(1+x^2)dy \quad \boxed{\frac{dy}{dx}=\frac{1}{1+x^2}}$$

$$A_2=\int_{r\cos\theta}^{r}\sqrt{r^2-x^2}\,dx=\left|\begin{array}{l}x=r\cos\varphi\\ dx=-r\sin\varphi\,d\varphi\\ \boxed{\varphi=\theta\div 0}\end{array}\right|=-\int_{\theta}^{0}r\sqrt{1-\cos^2\varphi}\cdot r\sin\varphi\,d\varphi$$

$$=-\int_{\theta}^{0}r^2\cdot\sin^2\varphi\,d\varphi=-\frac{r^2}{2}\int_{\theta}^{0}(1-\cos 2\varphi)\,d\varphi=-\left(\underbrace{\frac{r^2}{2}\varphi\Big|_{\theta}^{0}-\frac{r^2}{4}\int_{\theta}^{0}\cos(2\varphi)\,d(2\varphi)}_{①}\right)$$

$\cos 2\theta=\cos^2\theta-\sin^2\theta$
$\cos^2\theta=1-\sin^2\theta$

$\cos 2\theta=1-2\sin^2\theta$
$\sin^2\theta=\frac{1}{2}(1-\cos 2\theta)$

$$① = \frac{r^2}{2}\theta+\frac{r^2}{4}\sin(2\varphi)\Big|_{\theta}^{0}=\underline{\frac{r^2}{2}\theta-\frac{r^2}{4}\sin 2\theta}=\frac{r^2}{2}\theta-\frac{r^2}{2}\sin\theta\cos\theta$$

-128

ALTERNATIVELY: (80, x") ↙①

$$A_2 = -\left(\frac{r^2}{2}\varphi - \frac{r^2}{4}\sin 2\varphi\right)\Bigg|_a^b = -\frac{r^2}{2}\varphi + \frac{r^2}{2}\left([illegible]\right)$$

$\sin\varphi \cdot \cos\varphi$

$$\left| \begin{array}{l} x = r\cdot\cos\varphi \quad y = r\cdot\sin\varphi \qquad \varphi = \arccos\frac{x}{r} \\ y = \sqrt{r^2-x^2} \qquad y^2 = r^2 - x^2 \end{array} \right|$$

$\frac{1}{2}x\cdot y$

$$A_2 = -\frac{r^2}{2}\arccos\frac{x}{r} + [illegible] = -\frac{r^2}{2}\arccos\frac{x}{r} + \frac{x}{2}\sqrt{r^2-x^2}$$

$$A_2 = \frac{1}{2}\left(-r^2\arccos\frac{x}{r} + x\sqrt{r^2-x^2}\right)$$

[illegible]

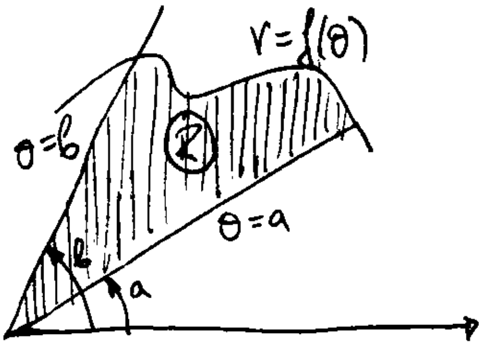
$$A_2 = \frac{1}{2}\left(x\sqrt{r^2-x^2} - r^2\arccos\frac{x}{r}\right)\Bigg|_{r\cos\theta}^{r}$$

$$A_2 = \frac{1}{2}\left(r\cancel{\sqrt{r^2-r^2}}^{\,0} - r^2\cancel{\arccos 1}^{\,0}\right) - \frac{1}{2}\left(r\cos\theta\sqrt{r^2-r^2\cos^2\theta} - r^2\cdot\theta\right)$$

$$\boxed{A_2 = -\frac{r^2}{2}\cos\theta\sin\theta + \frac{r^2}{2}\theta}$$

$$\boxed{A = A_1 + A_2 = -\frac{r^2\theta}{2}}$$

DOKAZANO!!



$0 < b - a < 2\pi$

$[a, b] = [\theta_0, \theta_1, \ldots, \theta_n]$

$\Delta\theta = \theta_1 - \theta_0 \qquad \Delta\theta = \theta_i - \theta_{i-1}$

$\Delta A_i \doteq \frac{1}{2} f^2(\theta_i^*)\,\Delta\theta$

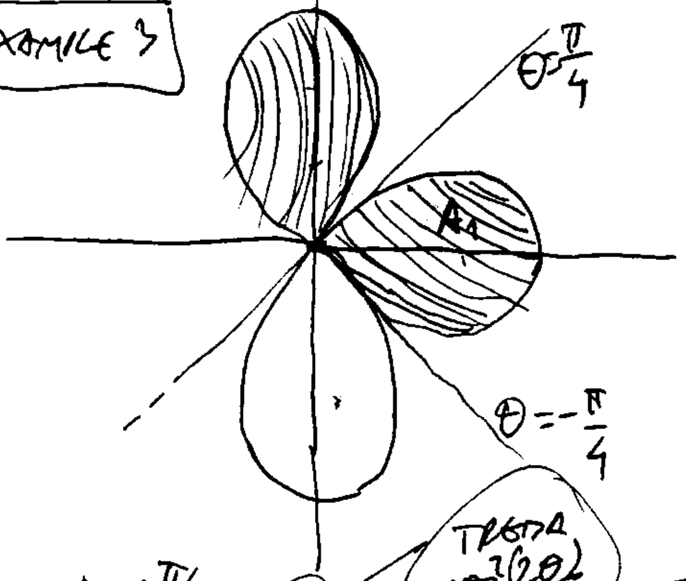
- TOTAL AREA OF R: $A \approx \sum_{i=1}^{n} \frac{1}{2} f^2(\theta_i^*)\Delta\theta$

$$\boxed{\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{2} f^2(\theta_i^*)\Delta\theta = \int_a^b \frac{1}{2}\left[f(\theta)\right]^2 d\theta}$$

$$\boxed{A = \int_a^b \frac{1}{2} f^2(\theta)\,d\theta = \int_a^b \frac{1}{2} r^2\,d\theta}$$

$r = f(\theta)$!

EXAMPLE 3



$\boxed{r = \cos 2\theta}$

$$A = \int_0^{\pi/4} \int_0^{\cos 2\theta} r\,dr\,d\theta$$

$$A = \int_0^{\pi/4} \left(\frac{r^2}{2} \Big|_0^{\cos 2\theta} \right) d\theta$$

TREBA $\frac{\cos^2(2\theta)}{2}$

$$A = \int_0^{\pi/4} \frac{\cos^2(\theta)}{2} d\theta = \frac{1}{2} \int_0^{\pi/4} \frac{1}{2}(1 + \cos 2\theta) d\theta = \frac{1}{4}\left(\theta + \frac{1}{2} \sin 2\theta\right)\Big|_0^{\frac{\pi}{4}}$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta = \cos^2\theta - 1 + \cos^2\theta = 2\cos^2\theta - 1$$

$$\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta)$$

$$\boxed{A = \frac{1}{4}\left(\frac{\pi}{4} + \frac{1}{2} \underset{1}{\sin\frac{\pi}{2}}\right) - \underbrace{\frac{1}{4}\left(0 + \frac{1}{2} \cdot 0\right)}_{0} = \frac{\pi}{16} + \frac{1}{8}}$$

$$A = \int_0^{\pi/4} \frac{\cos^2(2\theta)}{2} d\theta = \left| \begin{array}{l} 2\theta = \alpha \\ d\theta = \frac{d\alpha}{2} \\ \theta = 0 \; \alpha = 0 \\ \theta = \frac{\pi}{4} \; \alpha = \frac{\pi}{2} \end{array} \right| = \int_0^{\pi/2} \cos^2(\alpha) \frac{d\alpha}{4}$$

$$A_1 = \frac{1}{8}\left(\alpha + \frac{1}{2} \sin 2\alpha\right)\Big|_0^{\pi/2} = \frac{1}{8}\left(\frac{\pi}{2} + \underset{0}{\sin \pi}\right) - \frac{1}{8}\left(0 + \frac{1}{2} \underset{0}{\sin 0}\right)$$
$$= \frac{\pi}{16}$$

$$\boxed{A = 2 \cdot A_1 = \frac{\pi}{8}}$$

EXAMPLE 4

VOLUME UNDER $z = x^2 + y^2$ AND INSIDE CILINDER! $x^2 + y^2 = 2x$?

PARABOLOID

D: $x^2 - 2x + 1 + y^2 = 1$ $\qquad (x-1)^2 + y^2 = 1$

$$\boxed{V = \iint_D (x^2+y^2)\,dx\,dy}$$

$x = r\cos\varphi$
$y = r\cdot\sin\varphi$
$\mathcal{J} = r$

$x^2+y^2=r^2$

$$V = \int_\alpha^\beta \int_{h_1(\theta)}^{h_2(\theta)} f(r\cos\theta, r\sin\theta)\, r\,dr\,d\theta = \int_\alpha^\beta \int_{h_1(\theta)}^{h_2(\theta)} r^2\cdot r\,dr\,d\theta =$$

VIDI SLIKA VO KNIGA!!!

$$= \int_{-\pi/2}^{\pi/2} \int_{h_1(\theta)}^{h_2(\theta)} r^3\,dr\,d\theta$$

$r^2 = 2r\cos\theta$ $\quad \boxed{r = 2\cos\theta}$

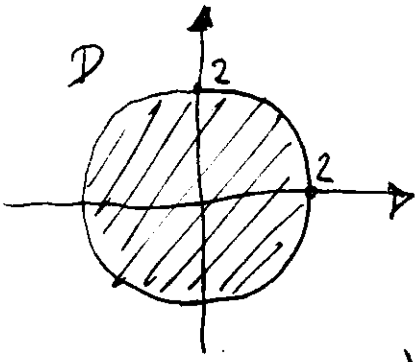
$$V = \int_{-\pi/2}^{\pi/2} \int_0^{2\cos\theta} r^3\,dr\,d\theta = \int_{-\pi/2}^{\pi/2} \frac{r^4}{4}\Bigg|_0^{2\cos\theta} d\theta = \int_{-\pi/2}^{\pi/2} \frac{16\cdot\cos^4\theta}{4}\,d\theta =$$

~~[illegible]~~

$$= 4\cdot\frac{3}{8}\pi = \frac{3\pi}{2}$$

Problem 21

UNDER PARABOLOID $z = x^2+y^2$ $\quad z = r^2$
ABOVE DISK $x^2+y^2 \le 9$



$$V = \iint_D f(x,y)\,dx\,dy = \int_\alpha^\beta \int_{h_1(\theta)}^{h_2(\theta)} f(r\cos\theta, r\sin\theta)\, r\,dr\,d\theta$$

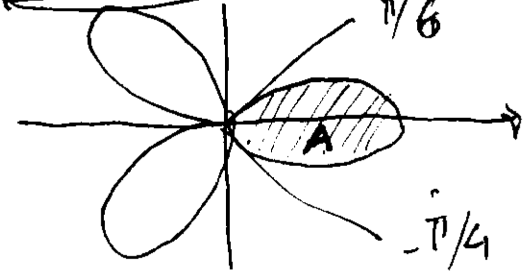
$$V = \int_0^{2\pi} \int_0^{3} r^2\cdot r\,dr\,d\theta = \int_0^{2\pi} \frac{r^4}{4}\Bigg|_0^2 d\theta$$

$$V = \frac{81}{4} \int_0^{2\pi} d\theta = \text{[illegible]} = \frac{81\pi}{2}$$

$\cos(3\theta) = 0$
$3\theta = \frac{\pi}{2}$ $\quad \theta = \frac{\pi}{6}$

Prob. 17

$r(\theta) = \cos(3\theta)$



$$A = 2\int_0^{\pi/6} \int_0^{\cos(3\theta)} r\,dr\,d\theta = 2\int_0^{\pi/6} \frac{\cos^2(3\theta)}{2}\,d\theta$$

$$\boxed{A = \frac{\pi}{12}}$$

MMV

FIND AREA WITH USAGE OF $\iint$

Problem 19 $r_1 = \cos\theta$ $r_2 = \underline{\underline{\sin\theta}}$

$$A = \int_{\alpha}^{\beta}\int_{\sin\theta}^{\cos\theta} r\,dr\,d\theta = \int_0^{\pi/2} \frac{r^2}{2}\Big|_{\sin\theta}^{\cos\theta} d\theta = \frac{1}{2}\int_0^{\pi/2}(\cos^2\theta - \sin^2\theta)\,d\theta$$

$$A = \frac{1}{2}\int_0^{\pi/2} \cos 2\theta\, d\theta = \frac{1}{4}\sin 2\theta\Big|_0^{\pi/2} = \frac{1}{4}(0 - 0) = 0 \;??$$

$$A = A_1 + A_2$$

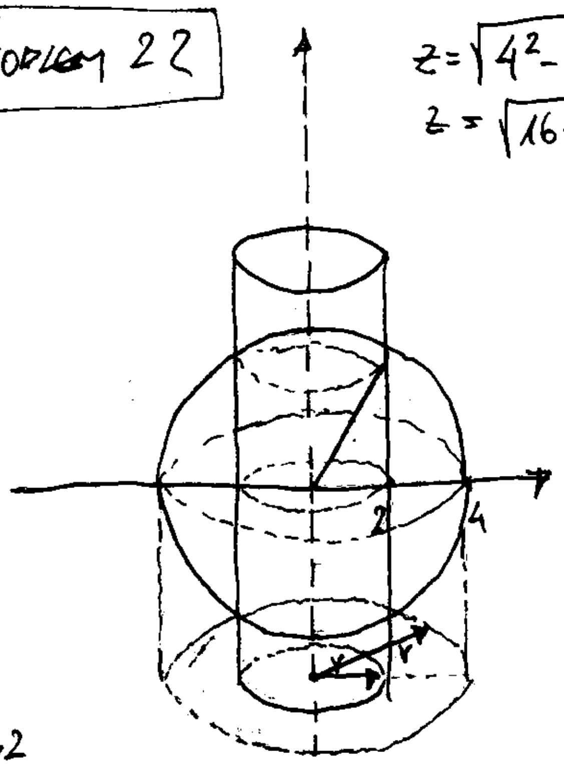
$$A_1 = \int_{\pi/4}^{\pi/2}\int_0^{\cos\theta} r\,dr\,d\theta = \int_{\pi/4}^{\pi/2} \frac{\cos^2\theta}{2}\,d\theta = \left(\frac{1}{2}\theta + \frac{1}{4}\sin 2\theta\right)\Big|_{\pi/4}^{\pi/2}$$

$$\boxed{A_1 = -\frac{1}{8} + \frac{\pi}{16}}$$

$$A_2 = \int_0^{\pi/4}\int_0^{\sin\theta} r\,dr\,d\theta = \int_0^{\pi/4} \frac{\sin^2\theta}{2}\,d\theta = -\frac{1}{8} + \frac{\pi}{16}$$

$$\boxed{A = 2\cdot A_1 = A_1 + A_2 = \frac{\pi}{8} - \frac{1}{4}} \qquad A = \frac{1}{8}(\pi - 2)$$

Problem 22



$z = \sqrt{4^2 - x^2 - y^2}$

$z = \sqrt{16 - r^2}$

$x^2 + y^2 + z^2 = 16$ sphere

cylinder:

$x^2 + y^2 = 4$ $r^2 = 4$ $r = 2$

$x = r\cos\theta$

$y = r\cdot\sin\theta$

$$V = \int_{\alpha}^{\beta}\int_{h_1(\theta)}^{h_2(\theta)} f(r\cos\theta, r\sin\theta)\,r\,dr\,d\theta$$

$$V = 2\int_0^{2\pi}\int_2^4 \sqrt{16 - r^2}\; r\,dr$$

$V =$ [illegible] $32\sqrt{3}\pi$ //

- $\boxed{Pr.27}$ $V = ?$ of sphere with radius „a"

$x^2+y^2+z^2=a^2$. $\boxed{z = \sqrt{a^2-x^2-y^2} = \sqrt{a^2-r^2}}$

D: $\boxed{x^2+y^2=a^2}$ $V = \iint_D f(x,y)\,dx\,dy$

$$V = 2\int_0^{2\pi}\int_0^a \underbrace{\sqrt{a^2-r^2}\,r\,dr}_{I}\,d\theta = \boxed{\frac{4a^3\pi}{3}}$$ MAPLE.!!!

$$I = \frac{1}{2}\int_0^a \sqrt{a^2-r^2}\;d(r^2) = -\frac{1}{2}\int_0^a \sqrt{a^2-r^2}\;d(a^2-r^2) = -\frac{1}{2}\left(a^2-r^2\right)^{3/2}\Big|_0^a$$

$$= -\frac{1}{2}\left[-(a^2)^{3/2}\right] = \frac{a^3}{2}$$

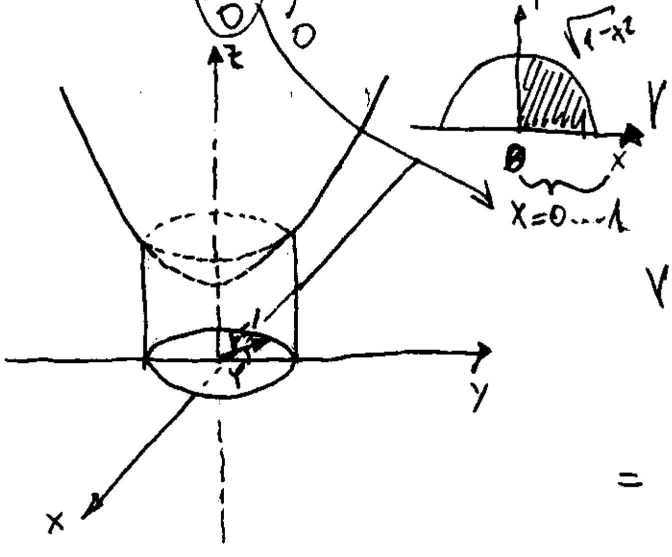
$$V = 2\cdot\int_0^{2\pi} d\theta \cdot I = 4\pi\cdot I = \frac{4\pi a^3}{2} \Leftarrow$$

$\boxed{Pr.29}$ $$V = \int_0^1\int_0^{\sqrt{1-x^2}} e^{x^2+y^2}\,dy\,dx$$

[illegible]

$r^2 = x^2+y^2$

$x = r\cdot\cos\varphi$
$y = r\cdot\sin\varphi$



$$V = \int_0^{\pi/2}\int_0^1 e^{+r^2}\,r\,dr\,d\varphi$$

$$V = \int_0^{\pi/2} \frac{1}{2}\,e^{r^2}\Big|_0^1\,d\varphi =$$

$$= \int_0^{\pi/2}\frac{1}{2}\left(e^1-1\right)d\varphi = \frac{\pi}{2}\,\frac{1}{2}(e-1)$$
$$= \underline{\pi/4\,(e-1)}$$

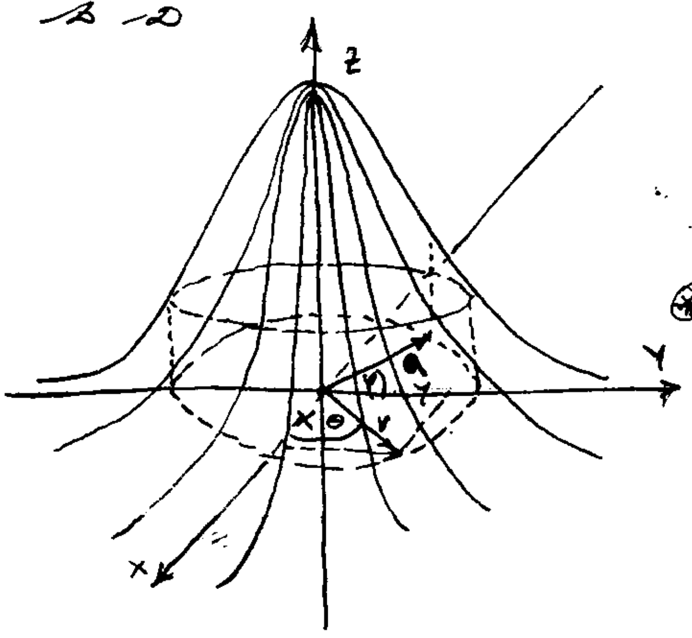
$\boxed{Pr.36}$ $$I = \iint_{R^2} e^{-x^2+y^2}\,dA = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-(x^2+y^2)}\,dx\,dy =$$

$$= \lim_{a\to\infty}\iint_{D_a} e^{-(x^2+y^2)}\,dA$$

$\boxed{\text{Da – DISK WITH RADIUS „a"}}$

ⓐ SHOW THAT:

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-(x^2+y^2)}\,dA=\pi$$



$x = r\cos\varphi \qquad J = r$

$y = r\sin\varphi$

$x^2+y^2=r^2$

[redacted]

$$I=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-x^2+y^2}\,dx\,dy=$$

$$\circledast \rightarrow =2\int_0^{\pi}\int_0^{\infty} e^{-r^2}\,r\,dr\,d\theta=$$

$$=-2\int_0^{\pi}\frac{1}{2}e^{-r^2}\Big|_0^{\infty}d\theta=$$

$$=-\int_0^{\pi}(0-1)\,d\theta=\int_0^{\pi}d\theta=\theta\Big|_0^{\pi}=\pi$$

⊛ МОЖЕ ДА ГО ПРЕТСТАВИШ КАКО:

$$\boxed{\lim_{a\to\infty} 2\int_0^{\pi}\int_0^{a} e^{-r^2}\,r\,dr\,d\theta=\pi}$$

$$\boxed{\int_{-\infty}^{\infty} e^{-x^2}\,dx=\sqrt{\pi}.}$$

MMV

ⓒ

$$I=\underbrace{\int_{-\infty}^{\infty} e^{-x^2}\,dx}_{I_1}\;\underbrace{\int_{-\infty}^{\infty} e^{-y^2}\,dy}_{I_1}=\pi$$

$$I=I_1\cdot I_1=I_1^2=\pi$$

$$\boxed{I_1=\sqrt{\pi}}$$

ⓓ $t=\sqrt{2}\,x$

$dt=\sqrt{2}\,dx$

$x=-\infty \quad t\to-\infty$

$x\to\infty \quad t\to\infty$

$$\boxed{x^2=t^2/2}$$

$$I=\int_{-\infty}^{\infty} e^{-x^2}\,dx=\sqrt{\pi}$$

$$I=\int_{-\infty}^{\infty} e^{-t^2/2}\,\frac{dt}{\sqrt{2}}=\frac{1}{\sqrt{2}}\int_{-\infty}^{\infty} e^{-t^2/2}\,dt$$

$$\boxed{\int_{-\infty}^{\infty} e^{-t^2/2}\,dt=\sqrt{2\pi}}$$

$$\boxed{2\int_0^{\pi}\int_0^{a} e^{-r^2}\,r\,dr\,d\theta=\pi(1-e^{-a^2})}$$

$$Q^2(z)=\frac{1}{2\pi}\int_{z:\,x=f(y)}^{\infty} e^{-\frac{x^2}{2}}dx\int_{z:\,y=f(x)}^{\infty} e^{-\frac{y^2}{2}}dy$$

$x=r\cos\theta$
$y=r\sin\theta$

$\boxed{z>0 \Rightarrow \text{I KVADRANT} \quad \text{VIDI SLIKA PP. 134}}$

$h_1(\theta)=\frac{z}{\sin\theta}$

$g_1(\theta)=\frac{z}{\cos\theta}$

$$Q^2(z)=\frac{1}{2\pi}\int_0^{\pi/2}\int_{h_1(\theta)}^{h_2(\theta)} e^{-\frac{r^2}{2}}r\,dr\,d\theta=$$

$$=\frac{1}{2\pi}\int_0^{\pi/4}\int_{h_1(\theta)}^{\infty} e^{-\frac{r^2}{2}}r\,dr\,d\theta+\frac{1}{2\pi}\int_{\pi/4}^{\pi/2}\int_{g_1(\theta)}^{\infty} e^{-\frac{r^2}{2}}r\,dr\,d\theta=$$

$$=\frac{1}{2\pi}\int_0^{\pi/4}\int_{z/\sin\theta}^{\infty} e^{-\frac{r^2}{2}}d\left(\frac{r^2}{2}\right)d\theta+\frac{1}{2\pi}\int_{\pi/4}^{\pi/2}\int_{z/\cos\theta}^{\infty} e^{-\frac{r^2}{2}}d\left(\frac{r^2}{2}\right)d\theta=$$

$r=\frac{y}{\sin\theta}$

$r=\frac{x}{\cos\theta}$

$$=\frac{1}{2\pi}\int_0^{\pi/4} e^{-\frac{z^2}{2\sin^2\theta}}d\theta+\frac{1}{2\pi}\underbrace{\int_{\pi/4}^{\pi/2} e^{-\frac{z^2}{2\cos^2\theta}}d\theta}_{I}$$

$$I=\int_{\pi/4}^{\pi/2} e^{-\frac{z^2}{2\cos^2\theta}}d\theta=\left|\begin{matrix}\varphi=\frac{\pi}{2}-\theta & d\varphi=-d\theta\\ \theta=\frac{\pi}{4}\ \ \varphi=\frac{\pi}{4} & \theta=\frac{\pi}{2}\ \ \varphi=0\end{matrix}\right|=$$

$$=-\int_{\pi/4}^{0} e^{-\frac{z^2}{2\sin\varphi}}d\varphi=\int_0^{\pi/4} e^{-\frac{z^2}{2\sin\varphi}}d\varphi$$

$$Q^2(z)=\frac{1}{2\pi}\int_0^{\pi/4} e^{-\frac{z^2}{2\sin^2\theta}}d\theta+\frac{1}{2\pi}\int_0^{\pi/4} e^{-\frac{z^2}{2\sin^2\varphi}}d\varphi$$

$$\boxed{Q^2(z)=\frac{1}{\pi}\int_0^{\pi/4} e^{-\frac{z^2}{2\sin^2\theta}}d\theta}$$

• OTHER FORMS FOR ONE- AND TWO-DIMENSIONAL CASES .

$$Q(x_1,y_1;\rho) = \frac{1}{2\pi}\int_0^{\frac{\pi}{2}-\operatorname{arctg}\left(\frac{y_1}{x_1}\right)} \frac{\sqrt{1-\rho^2}}{1-\rho\sin 2\phi}\exp\left[-\frac{x_1^2}{2}\,\frac{1-\rho\sin 2\phi}{(1-\rho^2)\sin^2\phi}\right]d\phi$$

$$+\frac{1}{2\pi}\int_0^{\operatorname{arctg}\left(\frac{y_1}{x_1}\right)} \frac{\sqrt{1-\rho^2}}{1-\rho\sin 2\phi}\exp\left[-\frac{y_1^2}{2}\,\frac{1-\rho\sin 2\phi}{(1-\rho^2)\sin^2\phi}\right]d\phi \qquad Ⓐ$$

CHANGE OF VARIAB.:

$$\boxed{\theta = \operatorname{arctg}\left(\frac{\operatorname{tg}\phi \pm \rho}{\sqrt{1-\rho^2}}\right)}$$

$y = \operatorname{arctg} x$

$dy = \frac{dx}{1+x^2} \qquad \frac{dy}{dx} = \frac{1}{1+x^2}$

$$\operatorname{tg}\theta = \frac{\operatorname{tg}\phi \pm \rho}{\sqrt{1-\rho^2}}$$

~~$(\operatorname{tg} y)' = \left(\frac{\sin y}{\cos y}\right)' = \frac{\cos^2 y + \sin^2 y}{\cos^2 y} = \frac{1}{\cos^2 y} = 1+\operatorname{tg}^2 y$~~

~~$(\operatorname{tg} y)' = 1+x^2 \qquad \frac{dy}{dx} =$~~

$$\boxed{dx = \frac{dy}{1+x^2}}$$

$$\int \frac{dx}{1+x^2} = \operatorname{arctg} x \Rightarrow \boxed{\operatorname{arctg}' x = \frac{1}{1+x^2}}$$

$y = \operatorname{arctg} x \qquad \operatorname{tg} y = x \qquad \left(\frac{\sin y}{\cos y}\right)' dy = dx$

$\frac{\cos^2 y + \sin^2 y}{\cos^2 y}\, dy = dx \qquad (1+\operatorname{tg}^2 y)\, dy = dx \qquad \boxed{dy = \frac{dx}{1+x^2}}$

$$\boxed{\frac{dy}{dx} = \frac{1}{1+x^2}}$$

$$\frac{1}{\cos^2\theta}\, d\theta = \frac{1}{\sqrt{1-\rho^2}}\,\frac{1}{\cos^2\phi}\cdot d\phi$$

$$\frac{d\theta}{d\phi} = \frac{1}{1+\left(\frac{\operatorname{tg}\phi\pm\rho}{\sqrt{1-\rho^2}}\right)^2}\cdot\frac{1}{\sqrt{1-\rho^2}}\cdot\frac{1}{\cos^2\phi} \Rightarrow$$

$$= \frac{\left(\sqrt{1-\rho^2}\right)^{\cancel{2}}}{\cancel{1-\rho^2}+\operatorname{tg}^2\phi \pm 2\rho\operatorname{tg}\phi+\cancel{\rho^2}}\cdot\cancel{\frac{1}{\sqrt{1-\rho^2}}}\;\frac{1}{\cos^2\phi}$$

$$\frac{d\theta}{d\phi} = \frac{\sqrt{1-\rho^2}}{1\pm 2\rho\operatorname{tg}\phi+\operatorname{tg}^2\phi}\cdot\frac{1}{\cos^2\phi} \quad ⊛$$

$$\circledast = \underline{\cos^2\phi} \pm 2\rho\sin\phi\cdot\cos\phi + \underline{\sin^2\phi} = 1 \pm \cancel{2}\rho\frac{1}{\cancel{2}}\cdot\sin 2\phi$$

$$\boxed{\frac{d\theta}{d\phi} = \frac{\sqrt{1-\rho^2}}{1 \pm \rho\sin 2\phi}} \qquad d\theta = \underbrace{\frac{\sqrt{1-\rho^2}}{1\pm\rho\sin\textcircled{2\phi}}\,d\phi}_{\text{SMENI VO } \textcircled{1}}$$

$$\phi = \frac{\pi}{2} - \operatorname{arctg}\left(\frac{y_1}{x_1}\right) \qquad \theta = \operatorname{arctg}\left(\frac{\overbrace{\operatorname{tg}\left(\frac{\pi}{2}-\operatorname{arctg}\textcircled{\frac{y_1}{x_1}}\right)}^{\circledast\circledast} \pm\rho}{\sqrt{1-\rho^2}}\right)$$

$$\circledast\circledast = \operatorname{tg}\left(\frac{\pi}{2} - \operatorname{arctg}\frac{y_1}{x_1}\right) = \operatorname{tg}\left(\frac{\pi}{2}-\alpha\right) = \operatorname{ctg}(\alpha)$$

$$\operatorname{tg}(\alpha-\beta) = \frac{\sin(\alpha-\beta)}{\cos(\alpha-\beta)}$$

$$\operatorname{tg}\left(\frac{\pi}{2}-\alpha\right) = \frac{\sin\left(\frac{\pi}{2}-\alpha\right)}{\cos\left(\frac{\pi}{2}-\alpha\right)} = \frac{\underset{1}{\sin\frac{\pi}{2}}\cdot\cos\alpha - \underset{0}{\cos\frac{\pi}{2}}\cdot\sin\alpha}{\cos\left(\frac{\pi}{2}\right)\cdot\cos\alpha + \underset{1}{\sin\frac{\pi}{2}}\sin\alpha}$$

$$\operatorname{tg}\left(\frac{\pi}{2}-\alpha\right) = \frac{\cos\alpha}{\sin\alpha} = \cot(\alpha) = \underline{\operatorname{ctg}(\alpha)}$$

$$\underline{\operatorname{ctg}(\operatorname{arctg}(\alpha))} = \frac{1}{\operatorname{tg}(\operatorname{arctg}(\alpha))} = \frac{1}{\alpha}$$

$$\boxed{\theta = \operatorname{arctg}\left(\frac{\frac{x_1}{y_1}\pm\rho}{\sqrt{1-\rho^2}}\right) \qquad \text{FOR: } \phi = \frac{\pi}{2} - \operatorname{arctg}\left(\frac{y_1}{x_1}\right)}$$

$$\phi = 0 \Rightarrow \theta = \operatorname{arctg}\left(\frac{\pm\rho}{\sqrt{1-\rho^2}}\right) = -\operatorname{arctg}\frac{\rho}{\sqrt{1-\rho^2}}$$

$$\gamma = \operatorname{arctg}(-\alpha) \qquad \operatorname{tg}\gamma = -\alpha \qquad -\operatorname{tg}\gamma = \alpha \qquad \operatorname{tg}(-\gamma) = \alpha$$

$$-\gamma = \operatorname{arctg}\alpha \qquad \boxed{\gamma = -\operatorname{arctg}\alpha} \qquad \operatorname{tg}(-\alpha) = -\operatorname{tg}(\alpha)$$

$$A = \frac{1\mp\rho\sin 2\phi}{\cancel{(1-\rho^2)}\sin^2\phi} = ?$$

$$\operatorname{tg}\theta = \frac{\operatorname{tg}\phi\pm\rho}{\sqrt{1-\rho^2}} \qquad \sqrt{1-\rho^2}\operatorname{tg}\theta \mp\rho = \operatorname{tg}\phi$$

$$\boxed{\operatorname{tg}\phi = \sqrt{1-\rho^2}\operatorname{tg}\theta \mp\rho} \qquad \phi = \operatorname{arctg}\left(\sqrt{1-\rho^2}\operatorname{tg}\theta\mp\rho\right)$$

$$A = \frac{1\mp\rho\sin 2\phi}{\cancel{(1-\rho^2)}\sin^2\phi}\,\frac{\cos 2\phi}{\cos 2\phi} = \frac{\frac{1}{\cos 2\phi}\mp\rho\operatorname{tg}2\phi}{(1-\rho^2)\,\frac{\sin^2\phi}{\cos 2\phi}}$$

$$A = \frac{1\mp\rho\ 2\sin\phi\cdot\cos\phi}{\cancel{(1-\rho^2)}\sin\phi\cdot\sin\phi} = \frac{1}{\sin^2\phi} \mp\rho\ 2\frac{1}{\operatorname{tg}\phi} = \frac{1}{\sin^2\phi}\mp\frac{2\rho}{\sqrt{1-\rho^2}(\operatorname{tg}\theta\mp\rho)}$$

$$A = \frac{1}{1-\rho^2}\,\frac{1 \mp \rho\sin 2\phi}{\sin^2\phi} = \frac{1-\rho 2\sin\phi\cdot\cos\phi}{(1-\rho^2)\sin^2\phi}$$

$$\mathrm{tg}\,\phi = \sqrt{1-\rho^2}\,\mathrm{tg}\,\theta \mp \rho$$

$$A = \frac{\sin^2\phi + \cos^2\phi - 2\rho\sin\phi\cdot\cos\phi}{(1-\rho^2)\sin^2\phi} = \frac{1+\frac{1}{\mathrm{tg}^2\phi} - 2\rho\frac{1}{\mathrm{tg}\,\phi}}{(1-\rho^2)}$$

$$A = \frac{\frac{\mathrm{tg}^2\phi + 1 - 2\rho\,\mathrm{tg}\,\phi}{\mathrm{tg}^2\phi}}{(1-\rho^2)} = \frac{A_1}{A_2} = \frac{1+\left(\sqrt{1-\rho^2}\,\mathrm{tg}\,\theta \mp \rho\right)^2 - 2\rho\overset{\sqrt{1-\rho^2}\mathrm{tg}\theta\mp\rho}{\cancel{\mathrm{tg}\,\phi}}}{(1-\rho^2)\left(\sqrt{1-\rho^2}\,\mathrm{tg}\,\theta \mp \rho\right)^2}$$

$$A_1 = 1 + (1-\rho^2)\mathrm{tg}^2\theta \textcircled{$\mp$} \cancel{2\rho\sqrt{1-\rho^2}}\,\mathrm{tg}\,\theta + \underline{\underline{\rho^2}} - \cancel{2\rho\sqrt{1-\rho^2}\,\mathrm{tg}\,\theta} \textcircled{$\pm$} \underline{\underline{2\rho^2}}$$

$$A_1 = 1 + (1-\rho^2)\,\mathrm{tg}^2\theta - \rho^2 = (1-\rho^2) + (1-\rho^2)\,\mathrm{tg}^2\theta$$

$$A_1 = (1-\rho^2)(1+\mathrm{tg}^2\theta)$$

$$A = \frac{A_1}{A_2} = \frac{\cancel{(1-\rho^2)}(1+\mathrm{tg}^2\theta)}{\cancel{(1-\rho^2)}\left(\sqrt{1-\rho^2}\,\mathrm{tg}\,\theta \mp \rho\right)^2} = \frac{1+\mathrm{tg}^2\theta}{\left(\sqrt{1-\rho^2}\,\mathrm{tg}\,\theta \mp \rho\right)^2}$$

$$A = \frac{A_1}{A_2} = \frac{\frac{\cos^2\theta + \sin^2\theta}{\cos^2\theta}}{\left(\sqrt{1-\rho^2}\,\mathrm{tg}\,\theta \pm \rho\right)^2} = \frac{1}{\cos^2\theta\cdot\mathrm{tg}^2\theta\left(\sqrt{1-\rho^2} \pm \rho\,\mathrm{ctg}\,\theta\right)^2}$$

$$\boxed{A = \frac{1}{\sin^2\theta\left(\sqrt{1-\rho^2} \pm \rho\,\mathrm{ctg}\,\theta\right)^2}}$$

$$Q(x_1, y_1; \rho) = \frac{1}{2\pi}\int_{-\mathrm{arctg}\left(\frac{\rho}{\sqrt{1-\rho^2}}\right)}^{\mathrm{arctg}\left(\left(\frac{x_1}{y_1} \mp \rho\right)/\sqrt{1-\rho^2}\right)} \exp\left(-\frac{x_1^2}{2}\,\overset{\textcircled{B}}{\frac{1}{\sin^2\theta\left(\sqrt{1-\rho^2} \pm \rho\,\mathrm{ctg}\,\theta\right)^2}}\right)d\theta$$

$$+\frac{1}{2\pi}\int_{-\mathrm{arctg}\left(\frac{\rho}{\sqrt{1-\rho^2}}\right)}^{\mathrm{arctg}\left(\left(\frac{y_1}{x_1} \pm \rho\right)/\sqrt{1-\rho^2}\right)} \exp\left(-\frac{y_1^2}{2\sin^2\theta\left(\sqrt{1-\rho^2} \pm \rho\,\mathrm{ctg}\,\theta\right)^2}\right)d\theta$$

VIDI: ㊵

⑩

2.: $\phi = \operatorname{arctg}\frac{y_1}{x_1}$ $\qquad \theta = \operatorname{arctg}\left(\frac{\operatorname{tg}\phi \pm \varrho}{\sqrt{1-\varrho^2}}\right) = \operatorname{arc}\left(\frac{\frac{y_1}{x_1} \pm \varrho}{\sqrt{1-\varrho^2}}\right)$

$$\boxed{\alpha = \operatorname{arctg}\frac{\varrho}{\sqrt{1-\varrho^2}}} \qquad \operatorname{tg}\alpha = \frac{\varrho}{\sqrt{1-\varrho^2}}$$

$$\beta = \frac{1}{\left(\sqrt{1-\varrho^2} + \varrho \operatorname{ctg}\theta\right)^2} = \frac{1}{\left(\sqrt{1-\varrho^2} + \varrho\frac{\cos\theta}{\sin\theta}\right)^2} = \frac{\sin^2\theta}{\underbrace{\left(\sqrt{1-\varrho^2}\sin\theta + \varrho\cos\theta\right)^2}_{⊛}}$$

$$\sqrt{1-\varrho^2} = \frac{\varrho}{\operatorname{tg}\alpha} \qquad \varrho = \sqrt{1-\varrho^2}\operatorname{tg}\alpha$$

$$⊛ = \left(\frac{\varrho}{\operatorname{tg}\alpha}\sin\theta + \varrho\cos\theta\right)^2 = \varrho^2\left(\frac{\cos\alpha}{\sin\alpha}\cdot\sin\theta + \cos\theta\right)^2$$

$$= \frac{\varrho^2}{\sin^2\alpha}\Big(\underbrace{\cos\alpha\cdot\sin\theta + \cos\theta\cdot\sin\alpha}_{\sin(\theta+\alpha)}\Big)^2 = \frac{(1-\varrho^2)\operatorname{tg}^2\alpha}{\sin^2\alpha}\sin^2(\alpha+\theta)$$

$$\Rightarrow \left| 1-\varrho^2 = \frac{\varrho^2}{\operatorname{tg}^2\alpha} \right| = \frac{\varrho^2}{\sin^2\alpha}\sin^2(\alpha+\theta)$$

$$⊛ = \left(\sqrt{1-\varrho^2}\right)^2\left(\sin\theta + \frac{\varrho}{\sqrt{1-\varrho^2}}\cos\theta\right)^2 = \left(\sqrt{1-\varrho^2}\right)^2\left(\sin\theta + \operatorname{tg}\alpha\cos\theta\right)^2$$

$$= \frac{1-\varrho^2}{\cos^2\alpha}\left(\sin\theta\cos\alpha + \sin\alpha\cdot\cos\theta\right)^2 = \frac{\varrho^2}{\sin^2\alpha}\sin^2(\alpha+\theta)$$

$$\operatorname{tg}^2\alpha = \frac{\varrho^2}{1-\varrho^2} \qquad \frac{\sin^2\alpha}{\cos^2\alpha} = \frac{\varrho^2}{1-\varrho^2}$$

$$1-\cos^2\alpha = \frac{\varrho^2\cos^2\alpha}{1-\varrho^2} \qquad 1-\cos^2\alpha - \varrho^2 + \varrho^2\cancel{\cos^2\alpha} = \varrho^2\cancel{\cos^2\alpha}$$

$$\boxed{\cos^2\alpha = 1-\varrho^2}$$

$$⊛ = \sin^2(\alpha+\theta)$$

$$\beta = \frac{\sin^2\theta}{\left(\sqrt{1-\varrho^2}\sin\theta + \varrho\cos\theta\right)^2} = \frac{\sin^2\theta}{\sin^2(\alpha+\theta)}$$

$$Q_m(x_1, y_1; \rho) = \frac{1}{2\pi} \int_{-\alpha}^{\operatorname{arctg}\left(\frac{x_1/y_1}{\sqrt{1-\rho^2}} - \tan\alpha\right)} \exp\left(-\frac{x_1^2}{2\sin^2(\alpha+\theta)}\right) d\theta +$$

$$+ \frac{1}{2\pi} \int_{-\alpha}^{\overset{\beta}{\boxed{\operatorname{arctg}\left(\frac{y_1/x_1}{\sqrt{1-\rho^2}} - \operatorname{tg}\alpha\right)}}} \exp\left(-\frac{y_1^2}{2\sin^2(\alpha+\theta)}\right) d\theta$$

$\phi = \theta + \alpha \quad \theta = \phi - \alpha \quad d\theta = d\phi \quad \theta = -\alpha \quad \phi = 0; \quad \theta = \beta \quad \phi = \alpha + \beta$

$\operatorname{arctg} x + \operatorname{arctg} y = ?$ $\qquad \underline{\underline{\operatorname{arctg}(x)}} + \operatorname{arctg}(X) = \frac{\pi}{2}$

$y = \operatorname{arctg} x \qquad \operatorname{tg} y = x \qquad \frac{1}{x} = \operatorname{ctg} y$

$$\boxed{y = \operatorname{arcctg}\left(\frac{1}{x}\right)}$$

$$\boxed{\operatorname{arctg}(x) + \operatorname{arctg}\left(\frac{1}{y}\right) = \frac{\pi}{2}}$$

$$Q(x_1, y_1; \rho) = \frac{1}{2\pi} \int_{-0}^{\alpha + \operatorname{arctg}\left(\frac{y_1/x_1}{\sqrt{1-\rho^2}} - \operatorname{tg}\alpha\right)} \exp\left(-\frac{x_1^2}{2\sin^2(\phi)}\right) d\phi' +$$

$$+ \frac{1}{2\pi} \int_{0}^{\boxed{\alpha + \operatorname{arctg}\left(\frac{y_1/x_1}{\sqrt{1-\rho^2}} - \operatorname{tg}\alpha\right)}(*)} \exp\left(-\frac{y_1^2}{2\sin^2(\phi)}\right) d\phi$$

$$\alpha = \operatorname{arctg} \frac{\rho}{\sqrt{1-\rho^2}}$$

$\operatorname{arctg}(x + y) = \delta \qquad x + y = \operatorname{tg}\delta \qquad \operatorname{tg}\alpha = \frac{\rho}{\sqrt{1-\rho^2}}$

$$(*) = \operatorname{arctg} \operatorname{tg}(\alpha) + \operatorname{arctg}\left(\frac{y_1/x_1}{\sqrt{1-\rho^2}} - \operatorname{tg}\alpha\right)$$

$$(*) = \operatorname{arctg}\frac{\rho}{\sqrt{1-\rho^2}} + \operatorname{arctg}\left(\frac{y_1/x_1 - \rho}{\sqrt{1-\rho^2}}\right)$$

$$\operatorname{tg}(\alpha+\beta) = \frac{\sin\alpha \cdot \cos\beta + \sin\beta \cdot \cos\alpha}{\cos\alpha \cos\beta - \sin\alpha \cdot \sin\beta} = \frac{\operatorname{tg}\alpha + \operatorname{tg}\beta}{1 - \operatorname{tg}\alpha \cdot \operatorname{tg}\beta}$$

$$\alpha + \beta = \operatorname{arctg} \frac{\operatorname{tg}\alpha + \operatorname{tg}\beta}{1 - \operatorname{tg}\alpha \cdot \operatorname{tg}\beta}$$

$$\mathrm{arctg}\,\frac{x}{y} = \pi(1-\mathrm{sgn}\,y)/2 + (\mathrm{sgn}(y))\,\mathrm{arctg}\left(\frac{x}{|y|}\right)$$

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$$Q(x_1,y_1;\rho) = \frac{1}{2\pi}\int_0^{\mathrm{arctg}\frac{\sqrt{1-\rho^2}\,x_1/y_1}{1-\rho x_1/y_1}} \exp\left[-\frac{x_1^2}{2\sin^2\phi}\right]d\phi \; +$$

$$+\frac{1}{2\pi}\int_0^{\mathrm{arctg}\left(\frac{\sqrt{1-\rho^2}\,y_1/x_1}{1-\rho\, y_1/x_1}\right)} \exp\left(-\frac{y_1^2}{2\sin^2\phi}\right)d\phi\,.$$

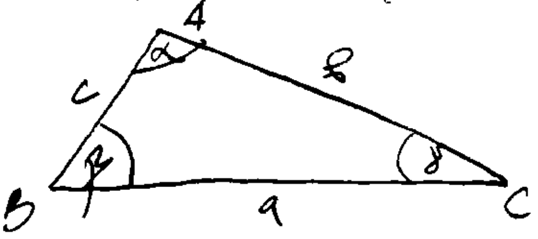
$$\sqrt{1-\rho^2} = \sqrt{(1-\rho)(1+\rho)}$$

$\rho \neq 0 \quad x_1 = y_1 = x$

$$\boxed{Q(x,x;\rho) = \frac{1}{\pi}\int_0^{\mathrm{arctg}\frac{\sqrt{1-\rho^2}}{1-\rho}} \exp\left(\frac{x^2}{2\sin^2\phi}\right)d\phi = \frac{1}{\pi}\int_0^{\mathrm{arctg}\frac{\sqrt{1+\rho}}{\sqrt{1-\rho}}} \exp\left(\frac{x^2}{2\sin^2\phi}\right)d\phi}$$

MMV

LOW OF TANGENTS



$$\boxed{\frac{a-b}{a+b} = \frac{\mathrm{tg}\,\frac{\alpha-\beta}{2}}{\mathrm{tg}\,\frac{\alpha+\beta}{2}}}$$

$$Q(x,x;0) = \frac{1}{\pi}\int_0^{\pi/4} \exp\left(\frac{x^2}{2\sin^2\phi}\right)d\phi = Q^2(x)$$

$$Q(x,x;1) = \frac{1}{\pi}$$

$$\lim_{\rho\to 1}\frac{\sqrt{1-\rho^2}}{1-\rho} = \lim_{\rho\to 1}\frac{-1}{\sqrt{1-\rho^2}}(-2\rho) = \lim_{\rho\to 1}\frac{2\rho}{\sqrt{1-\rho^2}} = \frac{2}{0} = \infty$$

$\mathrm{arctg}\,\infty = \pi/2$

$$Q(x,x,1) = \frac{1}{\pi}\int_0^{\pi/2} \exp\left(\frac{x^2}{2\sin^2\phi}\right)d\phi = Q(x)$$

- ALTERNATIVE REPRESENTATION OF HIGHER POWERS OF THE GAUSSIAN Q-FUNCTION

$$f(x) \overset{\Delta}{=} \frac{1}{\pi^2}\int_0^{\phi_M} \exp\left(-\frac{x^2}{2\sin^2\phi}\right)d\phi \int_0^{\xi_M}\exp\left(-\frac{x^2}{2\sin^2\xi}\right)d\xi \qquad \phi_M,\xi_M \le \frac{\pi}{2}$$

$$\boxed{\chi = \frac{1}{\sin\phi} \qquad \gamma = \frac{1}{\sin\xi}} \qquad R = \sqrt{\chi^2+\gamma^2} \qquad \beta = \operatorname{arctg}\frac{\gamma}{\chi}$$

$$R^2 = \frac{1}{\sin^2\phi} + \frac{1}{\sin^2\xi} = \frac{\sin^2\xi + \sin^2\phi}{\sin^2\phi\cdot\sin^2\xi}$$

~~$\sin\phi = \frac{1}{\chi}$ $\phi = \arcsin\frac{1}{\chi}$~~ $\phi = \phi_M$ ~~$\chi = \frac{1}{\sin\phi_M}$~~

$\phi = 0$ ~~$\chi = \infty$~~

$\chi = R\cdot\cos\beta$

$\gamma = R\cdot\sin\beta$

$R^2 = \sqrt{\chi^2+\gamma^2}$

ZAŠTO NE TREBA TUKA DA SE DODADE $|J|$?

$$f(x) = \frac{1}{\pi^2}\int_0^{\phi_M}\exp\left(\frac{x^2\cdot\chi^2}{2}\right)d\phi\int_0^{\xi_M}\exp\left(\frac{\chi^2\cdot\gamma^2}{2}\right)d\xi$$

$$d\chi = \left(\frac{1}{\sin\phi}\right)'\cdot d\phi = \frac{(-1)\cos\phi}{\sin^2\phi}\,d\phi = -\frac{\cos\phi}{\sin^2\phi}\,d\phi$$

$$d\gamma = \left(\frac{1}{\sin\xi}\right)' d\xi = -\frac{\cos\xi}{\sin^2\xi}\,d\xi$$

$$d\chi = -\sqrt{1-\sin^2\phi}\cdot\chi^2\cdot d\phi = -\chi^2\sqrt{1-\frac{1}{\chi^2}}\,d\phi = -\chi\sqrt{\chi^2-1}\,d\phi$$

$$\boxed{d\chi = -\chi\sqrt{\chi^2-1}\;d\phi} \qquad \boxed{d\gamma = -\gamma\sqrt{\gamma^2-1}\;d\xi}$$

$$f(x) = \frac{1}{\pi^2}\int_{\infty}^{1/\sin\phi_M}\exp\left(-\frac{x^2\chi^2}{2}\right)\frac{d\chi}{\chi\sqrt{\chi^2-1}}\int_{1/\sin\xi_M}^{\infty}\exp\left(-\frac{\chi^2\gamma^2}{2}\right)\frac{d\gamma}{\gamma\sqrt{\gamma^2-1}}$$

$$f(t) = \frac{1}{\pi^2} \int_{\infty}^{1/\sin\phi_M} \int_{\infty}^{1/\sin\xi_M} \exp\left(-\frac{x^2(x^2+y^2)}{2}\right) \frac{dx\,dy}{xy\sqrt{(x^2-1)(y^2-1)}}$$

$$f(t) = \frac{1}{\pi^2} \int_{\infty}^{1/\sin\phi_M} \int_{\infty}^{1/\sin\xi_M} \exp\left(-\frac{x^2R^2}{2}\right) \frac{dx\,dy}{\underbrace{xy\sqrt{x^2y^2-x^2-y^2+1}}_{①}}$$

$x = R\cdot\cos\beta \qquad y = R\cdot\sin\beta \qquad x^2+y^2=R^2$

$$① = R\cos\beta\cdot R\sin\beta\sqrt{R^4\cos^2\beta\cdot\sin^2\beta - R^2+1} =$$

$$= \frac{R^2}{2}\sin 2\beta\sqrt{R^4\frac{\sin^2 2\beta}{4} - R^2+1} = \frac{R^2}{4}\sin 2\beta\cdot$$

$$\sqrt{R^4\sin^2 2\beta - 4(R^2-1)}$$

$x = \infty \qquad R = \frac{x}{\cos\beta} \qquad R=\infty$

$x = \frac{1}{\sin\phi_M} \qquad R = \frac{x}{\cos\beta} = \frac{1}{\cos\beta\cdot\sin\phi_M}$

$y = \infty \qquad \sin\beta = \frac{y}{R} \qquad \beta = \frac{\pi}{2}$

$y = \frac{1}{\sin\xi_M} \qquad \sin\beta = \frac{1}{R\cdot\sin\xi_M} \qquad \beta = \arcsin\frac{1}{R\cdot\sin\xi_M}$

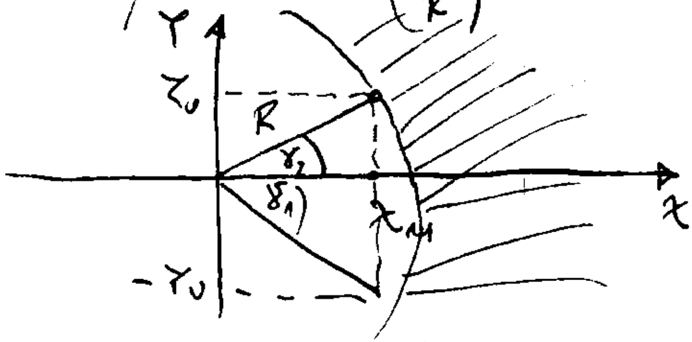
$\sin\beta = \frac{y_u}{R} \qquad \boxed{\beta = \arcsin\frac{y_u}{R}}$

$|\dot{O}|$

$$f(t) = \frac{1}{\pi^2} \int_{1/\sin\phi_M}^{\infty} \int_{1/\sin\xi_M}^{\infty} \exp\left(-\frac{t^2R^2}{2}\right) \frac{\cancel{R}\,d\beta\,dR}{\frac{R^{\cancel{2}}}{4}\cdot\sin 2\beta\sqrt{R^4\sin^2 2\beta - 4(R^2-1)}}$$

VIDI SLKA NAP.144

$$= \frac{1}{\pi^2} \int_{1/\cos\beta\sin\phi_M}^{\infty} \int_{\arcsin\left(\frac{y_u}{R}\right)}^{\infty} \exp\left(-\frac{t^2R^2}{2}\right) \frac{4\,d\beta\,dR}{R\sin 2\beta\sqrt{R^4\sin^2 2\beta-4(R^2-1)}}$$

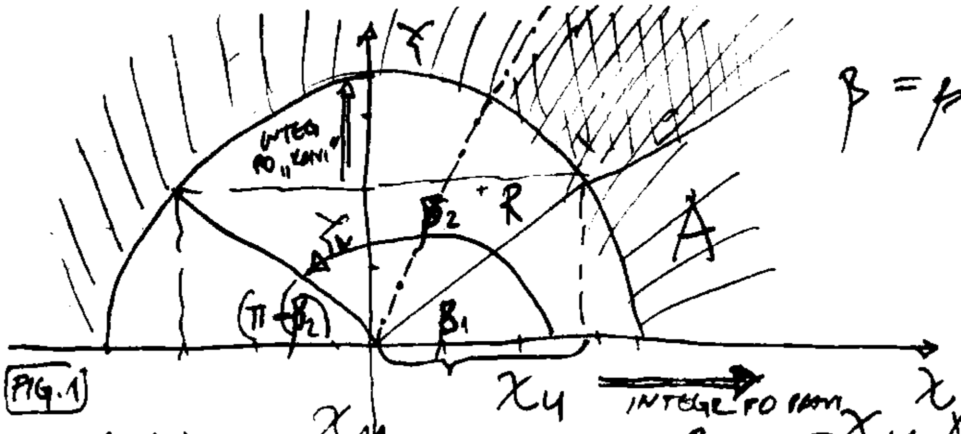


$\sin\gamma_1 = \frac{\cancel{-y_u}}{R}$

$\gamma = \arcsin\frac{y_u}{R}$

$\cos\gamma_2 = \frac{x_M}{R}$

[FIG.1]



$$\beta = \beta_1 \div \beta_2$$

$$x_u = \frac{1}{\cos\phi_u}$$

$$y_u = \frac{1}{\sin\xi_u}$$

$$\beta_v = \operatorname{arctg}\left(\frac{y_v}{x_v}\right)$$

$$\cos\beta_1 = \frac{x_u}{R}. \qquad \cos\beta_2 = \frac{-x_u}{R}$$

$$\sin(\pi-\beta_2) = \frac{y_u}{R}$$

$$\sin\beta_2 = \frac{y_u}{R}$$

$$\beta_1 = \arccos\frac{x_u}{R}$$

$$\beta_2 = \arcsin\frac{y_u}{R}$$

ALTERNATIVELY:

$$\sin\beta_1 = \frac{y_u}{R} \qquad \beta_1 = \arcsin\frac{y_v}{R}$$

$$\cos\beta_2 = +\frac{x_v}{R} \qquad \beta_2 = \arccos\left(\frac{x_v}{R}\right)$$

β_2 IN FIRST QUADRANT

$$f(x) = \frac{1}{\pi^2}\int_{\sqrt{x_v^2+y_v^2}}^{\infty}\int_{\arcsin\frac{y_v}{R}}^{\arccos\left(\frac{x_v}{R}\right)} \exp\left(-\frac{x^2R^2}{2}\right)\frac{4\,d\beta\,d\varrho}{R\sin 2\beta\sqrt{R^4\sin^2 2\beta-4(R^2-1}}$$

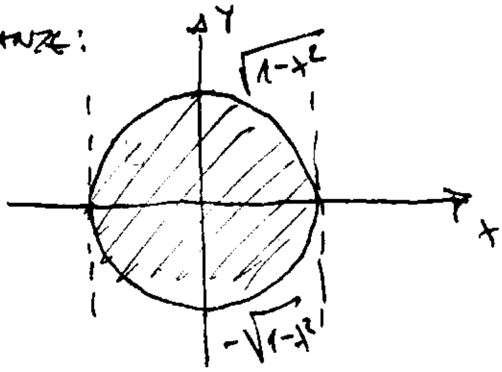
TA OPFAĆA MREŽATA (#) OD FIG.1

OTUKA I OD SKICATA MOŽE ŠI SKONAŠ GRANICITE [illegible], R.

$$f(x) = \frac{1}{\pi^2}\int_{x_v}^{\infty}\int_{y_v}^{\infty} \exp\left(-\frac{x^2R^2}{2}\right)\frac{dx\,dy}{xy\sqrt{(x^2-1)(y^2-1)}}$$

VIDI FIG 1

POTSETUVANJE:



$$z = x^2+y^2$$

$$V = \int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} x^2+y^2\,dx\,dy$$

$$g(\theta,\xi) \doteq -1+2\,\frac{\sin^2\xi-\sin^2\theta-\sin^4\xi\cos^2\theta}{(\sin^2\xi-\sin^2\theta)\cos^2 2\theta}$$

$$f(x) = \frac{1}{\pi^2}\int_0^{\sin^{-1}\left(\sin\frac{\phi_0}{\sqrt{2}}\right)} \cos^{-1} g(\theta,\phi_0)\exp\left(-\frac{x^2}{2\sin^2\theta}\right)d\theta$$

4.2 MARCUM Q-FUNCTION

$$Y = \sum_{k=1}^{2} x_k^2$$ CHI-SQUARE RANDOM VARIABLE

$$Q_1(s, \sqrt{Y}) = \int_{\sqrt{Y}}^{\infty} x \exp\left[-\frac{x^2+s^2}{2}\right] I_0(sx)\,dx$$

$$Q_1(\alpha, \beta) = \int_{\beta}^{\infty} x \exp\left[-\frac{x^2+\alpha^2}{2}\right] I_0(\alpha x)\,dx$$

- SERIES FORM OF FIRST-ORDER MARCUM Q-FUNCTION

$$\boxed{Q_1(\alpha, \beta) = e^{-\frac{\alpha^2+\beta^2}{2}} \sum_{k=0}^{\infty} \left(\frac{\alpha}{\beta}\right)^k I_k(\alpha\beta) = \exp\left[-\frac{\beta^2}{2}(1+\xi^2)\right] \sum_{k=0}^{\infty} \xi^k I_k(\beta^2)} \quad \boxed{\xi \triangleq \frac{\alpha}{\beta}}$$

$$I_0(u) = \frac{1}{2\pi} \int_0^{2\pi} e^{u\cos(x)}\,dx$$

(PROAKIS PP. 44

$$\boxed{I_\alpha(x) = \sum_{k=0}^{\infty} \frac{(x/2)^{\alpha+2k}}{k!\,\Gamma(\alpha+k+1)} \quad \boxed{x \geq 0}}$$

→ α-th ORDER MODIFIED BESSEL FUNCTION
INFINITE SERIES REPRESENTATION

- GENERALIZED MARCUM Q FUNCTION

$$Q_m(\alpha, \beta) = \int_{\beta}^{\infty} x \left(\frac{x}{a}\right)^{m-1} e^{-\frac{x^2+a^2}{2}} I_{m-1}(ax)\,dx =$$

$$= Q_1(a, b) + e^{\frac{a^2+b^2}{2}} \sum_{k=1}^{m-1} \left(\frac{b}{a}\right)^k I_k(ab)$$

$$Q_1(a, b) = e^{-\frac{a^2+b^2}{2}} \sum_{k=0}^{\infty} \left(\frac{a}{b}\right)^k I_k(ab)$$

REPRESENTATION OF MODIFIED BESSEL FUNCTION OF K-TH ORDER

$$\boxed{I_k(z) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \left(-j\,e^{-j\theta}\right)^k e^{-z\sin\theta}\,d\theta \quad \boxed{MM}} \qquad I_0(z) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{-z\sin\theta}\,d\theta$$

- INCOMPLETE TORONTO FUNCTION

$$T_B(m, n, r) = 2r^{n-m+1} e^{-r^2} \int_0^B t^{m-n} e^{-t^2} I_n(2rt)\,dt.$$

$$T_{\beta/\sqrt{2}}\left(1, 0, \frac{\alpha}{\sqrt{2}}\right) = 1 - Q_1(\alpha, \beta)$$

• RELATION BETWEEN MARCUM'S & GAUSSIAN Q FUNCTION BY USING THE ASYMPTOTIC FORM OF $I_0(x)$

DTK:

$$Q(\hat{\rho}, b_0) \doteq \text{erf}^*\left(\frac{\hat{\rho}-b_0}{\sqrt{2}}\right) \qquad \boxed{\text{erfc}(x) = \frac{1}{x\sqrt{\pi}} e^{-x^2} \quad x \gg 1}$$

$$\text{erf}^*(x) = \frac{1}{\sqrt{\pi}}\int_{-\infty}^{x} e^{-x^2} dx = \underbrace{\frac{1}{\sqrt{\pi}}\int_{-\infty}^{0} e^{-x^2} dx}_{\frac{\sqrt{\pi}}{2}} + \underbrace{\frac{1}{\sqrt{\pi}}\int_{0}^{x} e^{-x^2} dx}_{\frac{\text{erf}(x)}{2}}$$

MMV

$$\boxed{\text{erf}^*(x) = \frac{1}{2} + \frac{1}{2}\text{erf}(x) = \frac{1}{2}\left(1 + \underline{\underline{\text{erf}(x)}}\right)} \Rightarrow$$ ~~VORVELITA NA~~ erf-VI !!!

$$Q(\hat{\rho}, b_0) \doteq \frac{1}{2}\left(1 + \text{erf}(x)\right) \doteq \frac{1}{2}\left(1 + 1 - \text{erfc}(x)\right) =$$

$$= \frac{1}{2}\left(2 - \frac{1}{x\sqrt{\pi}} e^{-x^2}\right) \doteq 1 - \frac{1}{2x\sqrt{\pi}} e^{-x^2} \qquad \boxed{x = \frac{\hat{\rho}-b_0}{\sqrt{2}}}$$

$$Q(\hat{\rho}, b_0) = 1 - \frac{1}{\sqrt{2}(\hat{\rho}-b_0)\sqrt{\pi}} e^{-\frac{(\hat{\rho}-b_0)^2}{2}}$$

$$\boxed{Q(\alpha, \beta) = 1 - \frac{1}{(\alpha-\beta)\sqrt{2\pi}} e^{-\frac{(\alpha-\beta)^2}{2}}}$$

APROKSIMACIJA SOGLASNO DTK.

ASYMPTOTIC FORMS OF BESSEL FUNCTIONS:

• FOR LARGE ARGUMENTS: $x \gg |\alpha^2 - \frac{1}{4}|$

$$I_\alpha(x) \doteq \frac{e^x}{\sqrt{2\pi x}}\left(1 + \frac{(1-2\alpha)(1+2\alpha)}{8x} + \dots\right)$$

$$K_\alpha(\alpha) \doteq \sqrt{\frac{\pi}{2x}} e^{-x}$$

MMV

BY: C. BENDER

Advanced Mathematical Methods

EXAMPLE 5 pp.94

$$y(x) = c_1 x^{-1/2} e^{x} w(x)$$

$$w(x) \sim 1 - \frac{(4v^2-1^2)}{1!\,8x} + \frac{(4v^2-1^2)(4v^2-3^2)}{2!\,(8x)^2} - \dots \quad x \to \infty$$

$I_v(x) = ?$ $\quad I_5(x) = ?$ $\quad \boxed{v = 5}$ $\quad \boxed{c_1 = (2\pi)^{-1/2}}$

$$I_5(x) \doteq (2\pi)^{-1/2} e^x x^{-1/2}\left[1 - \frac{(4\cdot 25-1)}{1-8x} + \frac{(4\cdot 25-1)(4\cdot 25-9)}{2!\,(8x)^2} \dots\right]$$

$$I_0(x) = \frac{e^x}{\sqrt{2\pi x}}\left[1 - \frac{1}{1-8x} + \frac{9}{2(8x)^2} - \dots\right]$$

TOA 3TO NEDOSTASUVA E VO DCOPC 11.15 ↑ NEDOSTASUVA IZVEDUVANZE! (VO TETTEK NA RABOTA)

$$Q_1(\alpha,\beta) = Q_1(\beta\xi,\beta) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{1+\xi\sin\theta}{1+2\xi\sin\theta+\xi^2}\exp\left(-\frac{\beta^2}{2}(1+2\xi\sin\theta+\xi^2)\right)d\theta$$

$\boxed{4.39}$

$\beta > \alpha \geq 0 \quad 0 \leq \xi \leq 1$

$\boxed{4.42}$

$$Q_1(\alpha,\beta) = Q_1(\alpha,\alpha\xi) = 1 + \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{\xi^2+\xi\sin\theta}{1+2\xi\sin\theta+\xi^2}\exp\left(-\frac{\alpha^2}{2}(1+2\xi\sin\theta+\xi^2)\right)d\theta$$

$\alpha > \beta \geq 0 \quad 0 \leq \xi < 1$

$$\frac{1}{1+\xi}\exp\left[-\frac{\beta^2(1+\xi)^2}{2}\right] \leq Q_1(\beta\xi,\beta) \leq \frac{1}{1-\xi}\exp\left[-\frac{\beta^2(1-\xi)^2}{2}\right]$$

$$\frac{\beta}{\beta+\alpha}\exp\left[-\frac{(\beta+\alpha)^2}{2}\right] \leq Q(\overset{\alpha}{\cancel{\beta\xi}},\beta) \leq \frac{\beta}{\beta-\alpha}\exp\left[-\frac{(\beta-\alpha)^2}{2}\right]$$

MANLE

$$\frac{\cancel{(1-\xi)}\, e^{-\frac{\beta^2}{2}(1-2\xi+\xi^2)}}{\underbrace{(\xi-1)^{\cancel{2}}}_{\cancel{(\xi-1)(\xi+1)}}} = \frac{e^{-\frac{\beta^2}{2}(\xi-1)^2}}{(1-\xi)\cancel{(\xi+1)}} = \frac{e^{-\frac{\beta^2}{2}(1-\xi)^2}}{1-\xi}$$

$$\boxed{\xi\,\frac{e^{-\frac{1}{2}(1-\xi)^2}}{1-\xi\cancel{(1+\xi)}}}$$

$$1 + \frac{\xi e^{-\frac{\alpha^2}{2}(1-\xi)^2}}{1-\xi} \leq Q_1(\alpha,\alpha\xi)$$

$$1 + \frac{\beta\, e^{-\frac{1}{2}(\alpha-\beta)^2}}{\alpha-\beta} \leq Q_1(\alpha,\overset{\beta}{\alpha\xi})$$

TIGHTER LOWER AND UPPER BOUNDS

$$\exp\left[-\frac{(\beta+\alpha)^2}{2}\right] \leq Q_1(\overset{\alpha}{\cancel{\beta\xi}},\beta) \leq \exp\left[-\frac{(\beta-\alpha)^2}{2}\right]$$

$\beta > \alpha \geq 0$

$$1 - \frac{1}{2}\left\{\exp\left[-\frac{(\alpha-\beta)^2}{2}\right] - \exp\left[-\frac{(\alpha+\beta)^2}{2}\right]\right\} \leq Q_1(\alpha,\beta)$$

$\alpha > \beta \geq 0$

- GENERALIZED (mth-Order) Marcum Q-Function

$$\boxed{Q_m(s, \sqrt{y}) = \frac{1}{s^{m-1}} \int_{\sqrt{y}}^{\infty} x^m e^{-\frac{x^2+s^2}{2}} I_{m-1}(sx)\, dx}$$

$$Q_m(\alpha, \beta) = \frac{1}{\alpha^{m-1}} \int_{\beta}^{\infty} x^m e^{-\frac{x^2+\alpha^2}{2}} I_{m-1}(\cancel{\alpha} x)\, dx$$

- TORONTO FUNCTION

$$T_{\beta/\sqrt{2}}\left(2m-1, m-1, \frac{\alpha}{\sqrt{2}}\right) = 1 - Q_m(\alpha, \beta)$$

ASYMPTOTIC FORM OF BESSEL FUN (MMU)

$$\boxed{I_m(x) = \frac{e^x}{\sqrt{2\pi x}} \left(1 - \frac{4m^2-1^2}{1!\, 8x} + \frac{(4m^2-1^2)(4v^2-3^2)}{2!\, (8x)^2} - \dots\right) \quad x \to \infty}$$

$$\boxed{Q_m(\alpha, \beta) \doteq \int_{\beta}^{\infty} x \left(\frac{x}{\alpha}\right)^{m-1} e^{-\frac{x^2+\alpha^2}{2}} \frac{e^x}{\sqrt{2\pi x}}\, dx} = \circledast$$

- RICIAN FADING

$$p(\alpha) = \alpha \cdot e^{-\frac{\alpha^2+A^2}{2}} I_0(A \cdot \alpha) \qquad p(\gamma) = \dots$$

$$P_{out} = 1 - \int_{\gamma_0}^{\infty} p(\gamma)\, d\gamma = \int_0^{\gamma_0} p(\gamma)\, d\gamma$$

$$p(\gamma) = \underbrace{\frac{(1+K)e^{-K}}{\gamma}}_{C} \exp\left(-(K+1)\frac{\gamma}{\gamma}\right) I_0\left(2 \sqrt{\frac{K(1+K)\gamma}{\gamma}}\right)$$

$$\boxed{P_{out} = C \int_0^{\gamma_0} e^{-(K+1)\frac{\gamma}{\gamma}} I_0\left(2\sqrt{(K+1)K\frac{\gamma}{\gamma}}\right) d\gamma}$$

$$MGF_{\gamma} = \int_0^{\infty} e^{s\gamma} p(\gamma)\, d\gamma \qquad MGF_{1/\gamma} = \int_0^{\infty} e^{s/\gamma} p(\gamma)\, d\gamma$$

$$MGF_{1/\bar\gamma}(-s) = \int_0^\infty e^{-\frac{s}{\gamma}} \frac{(1+K)e^{-K}}{\bar\gamma} e^{-(K+1)\frac{\gamma}{\bar\gamma}} \underbrace{I_0\left(2\sqrt{K(1+K)\frac{\gamma}{\bar\gamma}}\right)}_{\doteq \frac{e^{x}}{\sqrt{2\pi x}}} d\gamma =$$

ASYMPTOTIC APPROXIMATION!

$$= \frac{(1+K)}{\bar\gamma} \int_0^\infty e^{-\left(\frac{s}{\gamma} + (K+1)\frac{\gamma}{\bar\gamma}\right)} \frac{e^{2\sqrt{K(1+K)\frac{\gamma}{\bar\gamma}}}}{\sqrt{2\pi\, 2\sqrt{K(1+K)\frac{\gamma}{\bar\gamma}}}} d\gamma$$

$$= \frac{1+K}{\bar\gamma} \int_0^\infty e^{-\left(\frac{s}{\gamma}+(K+1)\frac{\gamma}{\bar\gamma}\right)} \frac{e^{2\sqrt{K(1+K)\frac{\gamma}{\bar\gamma}}}}{2\sqrt{\pi}\sqrt[4]{K(1+K)}\left(\frac{\gamma}{\bar\gamma}\right)^{1/4}} d\gamma$$

$$= \underbrace{\frac{(1+K)\,(\bar\gamma)^{1/4}}{2\bar\gamma\sqrt{\pi}\,(K(1+K))^{1/4}}}_{C=} \int_0^\infty \frac{e^{-\frac{s}{\gamma}-(K+1)\frac{\gamma}{\bar\gamma}+2\sqrt{K(1+K)\frac{\gamma}{\bar\gamma}}}}{\sqrt[4]{\gamma}} d\gamma$$

WITH ASYMPTOTIC APPROX

RICIAN 4

$$\boxed{MGF_{1/\bar\gamma}(-s) = C \int_0^\infty \gamma^{-1/4} e^{-\frac{s}{\gamma}-(K+1)\frac{\gamma}{\bar\gamma}+2\sqrt{K(1+K)\frac{\gamma}{\bar\gamma}}} d\gamma}$$

PAK NE MOŽE DA GO REŠAT INTEGRALOT NI MAPLE NI MATHEMATICA !!!

$$\circledast \doteq Q_n(\alpha,\beta) = \underbrace{\left[\int_\beta^\infty \exp\left[-\frac{(x-\alpha)^2}{2}\right]\right]}_{Q(\beta-\alpha)} \left(\frac{\beta}{\alpha}\right)^{n-\frac{1}{2}} \frac{1}{\sqrt{2\pi}}$$

$$\boxed{Q_n(\alpha,\beta) = \frac{1}{\sqrt{2\pi}}\left(\frac{\beta}{\alpha}\right)^{n-\frac{1}{2}} Q(\beta-\alpha)}$$

$$\boxed{Q_m(\alpha,\beta)=\frac{1}{\alpha^{m-1}}\int_\beta^\infty x^m e^{-\frac{x^2+\alpha^2}{2}} I_{m-1}(\alpha x)\,dx}$$

→ GENERALIZED m-th ORDER MARCUM'S Q FUNCTION

$$\boxed{dv = x\,e^{-\frac{x^2+\alpha^2}{2}}\,dx}$$

$$\boxed{u = x^{m-1} I_{m-1}(\alpha x)} \qquad v = \int x\,e^{-\frac{x^2+\alpha^2}{2}}\,dx$$

$$\boxed{I_{m-1}(x) - I_{m+1}(x) = \frac{2m}{x} I_m(x)} \quad Ⓐ$$

BESSEL FUNCTION RECURSION RELATION (Abramowitz & Stegun)

(MAPLE GO [illegible] SO SIMPLIFY)

$$Q_m(\alpha,\beta)=\frac{1}{\alpha^{m-1}}\int_\beta^\infty \underbrace{x^{m-1} I_{m-1}(\alpha x)}_{u}\ \underbrace{x\,e^{-\frac{x^2+\alpha^2}{2}}dx}_{dv} = \frac{1}{\alpha^{m-1}}\int_\beta^\infty u\,dv$$

$$Q_m(\alpha,\beta)=\frac{1}{\alpha^{m-1}}\Big[(u\cdot v)\Big|_\beta^\infty - \underbrace{\int_\beta^\infty \underline{v}\,du}_{I}\Big]$$

$$\underline{v} = \int x\,e^{-\frac{x^2+\alpha^2}{2}}\,dx = \int e^{-\frac{x^2+\alpha^2}{2}}\,d\left(\frac{x^2+\alpha^2}{2}\right) = \underline{-e^{-\frac{x^2+\alpha^2}{2}}}$$

$$Q_m(\alpha,\beta)=\frac{1}{\alpha^{m-1}}\Big[-x^{m-1}I_{m-1}(\alpha x)\cdot e^{-\frac{x^2+\alpha^2}{2}}\Big|_\beta^\infty + \int_\beta^\infty e^{-\frac{x^2+\alpha^2}{2}}\,du\Big]$$

(**)

$$u = x^{m-1} I_{m-1}(\alpha x)$$

IZVOD OD MODIFICIRANA BESSELOVA FUNKCIJA

$$\frac{d I_m(x)}{dx} = \underbrace{I_{m+1}(x) + \frac{m I_m(x)}{x}}_{MAPLE} \qquad \frac{d I_{m-1}(x)}{dx} = I_m(x) + \frac{(m-1) I_{m-1}(x)}{x}$$

$$\frac{d I_m(\alpha x)}{dx} = \alpha\cdot I_{m+1}(\alpha x) + \frac{m I_m(\alpha x)}{x} \qquad I_m'(x) = I_{m+1}(x) + \frac{m I_m(x)}{x}$$

$$I_m(x) = \sum_{k=0}^\infty \frac{\left(\frac{x}{2}\right)^{\alpha+2k}}{k!\,\Gamma(\alpha+k+1)} \qquad I_m'(x) = \sum_{k=0}^\infty \frac{(\alpha+2k)\left(\frac{x}{2}\right)^{\alpha+2k-1}}{k!\,\Gamma(\alpha+k+1)}\cdot\frac{1}{2}$$

$$\Gamma(x) = \int_0^\infty t^{x-1} e^{-t}\,dt \quad , \quad \Gamma(x+1) = x\,\Gamma(x)$$

$$\Gamma(x) = (x-1)\,\Gamma(x-1)$$

$$I_\alpha'(x) = \underbrace{\sum_{k=0}^{\infty} \frac{\frac{\alpha}{2}\left(\frac{x}{2}\right)^{\alpha+2k-1}}{k!\,\Gamma(\alpha+k+1)}}_{S1} + \underbrace{\sum_{k=0}^{\infty} \frac{k\left(\frac{x}{2}\right)^{\alpha+2k-1}}{k!\,\Gamma(\alpha+k+1)}}_{S2}$$

$$S2 = \sum_{k=0}^{\infty} \frac{\left(\frac{x}{2}\right)^{\alpha+2k-1}}{(k-1)!\,\Gamma(\alpha+k+1)}$$

$$S_1 = \frac{\alpha}{2}\frac{2}{x}\underbrace{\sum_{k=0}^{\infty} \frac{\left(\frac{x}{2}\right)^{\alpha+2k}}{k!\,\Gamma(\alpha+k+1)}}_{I_\alpha(x)} = \frac{\alpha \cdot I_\alpha(x)}{x}$$

$$I_{\alpha+1}(x) = \sum_{k=0}^{\infty} \frac{\left(\frac{x}{2}\right)^{\alpha+2k+1}}{k!\,\Gamma(\alpha+k+2)}$$

$$\boxed{\begin{gathered}\Gamma(\alpha+k+2) = (\alpha+k+1)\,\Gamma(\alpha+k+1) \\ \Gamma(\alpha+k+1) = \frac{\Gamma(\alpha+k+2)}{\alpha+k+1}\end{gathered}}$$

$$I_{\alpha+1}(x) = \sum_{k=0}^{\infty} \frac{\left(\frac{x}{2}\right)^{\alpha+2k-1}\cdot\left(\frac{x}{2}\right)^{2}}{k!\,(\alpha+k+1)\,\Gamma(\alpha+k+1)}$$

$$S_2 = \sum_{k=0}^{\infty} \frac{(\alpha+k+1)\left(\frac{x}{2}\right)^{\alpha+2k-1}}{(k-1)!\,\Gamma(\alpha+k+2)} \quad \text{?}$$

$$e^{j\theta} = \cos\theta + j\sin\theta$$
$$e^{-j\theta} = \cos\theta - j\sin\theta$$
$$\sin\theta = \frac{e^{j\theta}-e^{-j\theta}}{2j}$$

- INTEGRALNA FORMA (PRAVILEN PRISTAP)

$$I_k(z) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \left(-j e^{j\theta}\right)^k e^{-z\sin\theta}\, d\theta$$

$$\boxed{I_k'(z) = \frac{dI_k(z)}{dz}}$$

$$I_k'(z) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \left(-j e^{j\theta}\right)^k (-\sin\theta)\, e^{-z\sin\theta}\, d\theta$$

TREBA DA IMA -?! SUM GO "PROPUSTIL!!!

SEPAK SE DOBIVA DOBAR REZULTAT!!

$$\sin\theta = \frac{1}{2j}\left(e^{+j\theta} - e^{-j\theta}\right) = \frac{j}{2}\left(e^{-j\theta} - e^{j\theta}\right)$$

$$I_k'(z) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \left(-j e^{j\theta}\right)^k \cdot \frac{j}{2} e^{-j\theta} e^{-z\sin\theta}\, d\theta + \frac{1}{4\pi}\int_{-\pi}^{\pi} \left(-j e^{j\theta}\right)^k \cdot \left(-j e^{j\theta}\right) e^{-z\sin\theta}\, d\theta$$

$$= \frac{1}{4\pi}\int_{-\pi}^{\pi} \left(-j e^{j\theta}\right)^k \frac{1}{\left(-j e^{j\theta}\right)} e^{-z\sin\theta}\, d\theta + \underbrace{\frac{1}{4\pi}\int_{-\pi}^{\pi} \left(-j e^{j\theta}\right)^{k+1} e^{-z\sin\theta}\, d\theta}_{I_{k+1}(z)/2}$$

$$I_k'(z) = \underbrace{\frac{1}{4\pi}\int_{-\pi}^{\pi}\left(-j e^{j\theta}\right)^{k-1} e^{-z\sin\theta}\, d\theta}_{I_{k-1}(z)/2} + \frac{I_{k+1}(z)}{2}$$

$$\boxed{I_k'(z) = \frac{I_{k-1}(z) + I_{k+1}(z)}{2}}$$ – MATHEMATICA GO DAVA ISTOTO !!!

$$\boxed{I_k'(\alpha z) = \frac{\alpha}{2}\left[I_{k+1}(\alpha z) + I_{k-1}(\alpha z)\right]}$$

ZA ∞^2 OVOJ ČLEN E 0 !!!

(**) ⇒

$$Q_m(\alpha,\beta) = \frac{1}{\alpha^{m-1}}\left[-x^{m-1} I_{m-1}(\alpha x)\, e^{-\frac{x^2+\alpha^2}{2}}\Big|_{\beta}^{\textcircled{\infty}} + \underbrace{\int_{\beta}^{\infty} e^{-\frac{x^2+\alpha^2}{2}}\, du}_{I_1}\right]$$

$$u = x^{m-1}\boxed{I_{m-1}(\alpha x)} \qquad I_{m-1}'(\alpha x) = \frac{\alpha}{2}\left[I_m(\alpha x) + \underbrace{I_{m-2}(\alpha x)}_{I_m'(\alpha x)}\right]$$

$$\frac{du}{dx} = (m-1)x^{m-2} I_{m-1}(\alpha x) + x^{m-1}\frac{\alpha}{2}\left[I_m(\alpha x) + I_{m-2}(\alpha x)\right] ⑪$$

---

① $$\boxed{I_{m-1}(x) - I_{m+1}(x) = \frac{2m}{x} I_m(x)} \qquad I_{m-1}(x) = \frac{2m}{x} I_m(x) + I_{m+1}$$

$$I_m'(\alpha z) = \frac{1}{2}\left[I_{m+1} + \frac{2m}{x} I_m(x) + I_{m+1}(x)\right] =$$

$$= \left[I_{m+1}(x) + \frac{m}{x} I_m(x)\right] \qquad \boxed{I_m'(x) = I_{m+1}(x) + \frac{m}{x} I_m(x)}$$

ZNAČI GO DOBIV REŠENIETO ŠTO GO DAVA MAPLE SO KORISTENJE NA REKURZIVNATA FORMULA ①

$$I_{m-1}'(x) = I_m(x) + \frac{(m-1) I_{m-1}(x)}{x} \qquad \left(I_{m-1}'(\alpha x) = \alpha I_m(\alpha x) + \frac{m-1}{x} I_{m-1}(\alpha x)\right)$$

$$\frac{du}{dx} = (m-1)x^{m-2} I_{m-1}(\alpha x) + x^{m-1}\left[\alpha I_m(\alpha x) + \frac{m-1}{x} I_{m-1}(\alpha x)\right] =$$

$$= \underline{(m-1)x^{m-2} I_{m-1}(\alpha x)} + \alpha x^{m-1} I_m(\alpha x) + \underline{(m-1)x^{m-2} I_{m-1}(\alpha x)}$$

$$\frac{du}{dx} = 2(m-1)x^{m-2} I_{m-1}(\alpha x) + \alpha x^{m-1} I_m(\alpha x)$$

$$Q_m(\alpha,\beta) = \frac{1}{\alpha^{m-1}}\left[\beta^{m-1} I_{m-1}(\alpha\cdot\beta)\cdot e^{-\frac{\beta^2+\alpha^2}{2}}\right] + \frac{1}{\alpha^{m-1}}\int_{\beta}^{\infty} e^{-\frac{x^2+\alpha^2}{2}}\cdot 2(m-1)x^{m-2} I_{m-1}(\alpha x)\, dx$$

$$+ \frac{1}{\alpha^{m-1}}\int_{\beta}^{\infty} e^{-\frac{x^2+\alpha^2}{2}}\cdot \alpha\cdot x^{m-1} I_m(\alpha x)\, dx$$

$$Q_m(\alpha,\beta) = \underline{\left(\frac{\beta}{\alpha}\right)^{m-1} I_{m-1}(\alpha\beta)\, e^{-\frac{\alpha^2+\beta^2}{2}}} + \frac{2(m-1)}{\alpha^{m-1}} \int_\beta^\infty x^{m-2} e^{-\frac{x^2+\alpha^2}{2}} I_{m-1}(\alpha x)\,dx$$

$$+ \frac{1}{\alpha^{m-2}} \int_\beta^\infty x^{m-1} e^{-\frac{x^2+\alpha^2}{2}} I_m(\alpha x)\,dx$$

$$Q_{m-1}(\alpha,\beta) = \frac{1}{\alpha^{m-2}} \int_\beta^\infty x^{m-1} e^{-\frac{x^2+\alpha^2}{2}} I_{m-2}(\alpha x)\,dx$$

$$Q_m(\alpha,\beta) = \frac{1}{\alpha^{m-1}} \int_\beta^\infty x^{m} e^{-\frac{x^2+\alpha^2}{2}} I_{m-1}(\alpha x)\,dx$$

- PRISTAP PREKU (A1):

$$I_1 = \int_\beta^\infty e^{-\frac{x^2+\alpha^2}{2}} (m-1)\, x^{m-2} I_{m-1}(\alpha x)\,dx + \frac{\alpha}{2}\int_\beta^\infty e^{-\frac{x^2+\alpha^2}{2}} x^{m-1} I_m(\alpha x)\,dx +$$

$$+ \frac{\alpha}{2} \int_\beta^\infty e^{-\frac{x^2+\alpha^2}{2}} x^{m-1} I_{m-2}(\alpha x)\,dx$$

$I_2 =$

$$\boxed{\frac{I_1}{\alpha^{m-1}}} = \overbrace{\frac{(m-1)}{\alpha^{m-1}} \int_\beta^\infty x^{m-2} e^{-\frac{x^2+\alpha^2}{2}} I_{m-1}(\alpha x)\,dx}^{I_3} + \frac{1}{2\alpha^{m-2}} \int_\beta^\infty x^{m-1} e^{-\frac{x^2+\alpha^2}{2}} I_m(\alpha x)\,dx$$

$$+ \frac{1}{②\alpha^{m-2}} \int_\beta^\infty x^{m-1} e^{-\frac{x^2+\alpha^2}{2}} I_{m-2}(\alpha x)\,dx$$

$Q_{m-1}(\alpha,\beta)$

$$I_{m-1}(\alpha x) - I_{m+1}(\alpha x) = \frac{2m\, I_m(\alpha x)}{\alpha x} \qquad \boxed{I_{m-1} = I_{m+1} + \frac{2m\, I_m}{\alpha x}}$$

$$I_3 = \underbrace{\frac{2m(m-1)}{\alpha^{m-1}} \int_\beta^\infty x^{m-1} e^{-\frac{x^2+\alpha^2}{2}} I_m(x)\,dx}_{=\alpha\cdot Q(\alpha,\beta)} + \underbrace{\frac{m-1}{\alpha^{m-1}} \int_\beta^\infty x^{m-2} e^{-\frac{x^2+\alpha^2}{2}} I_{m+1}(x)\,dx}_{?\ +Q_{m-1}(\alpha,\beta)}$$

$$I_2 = 2m(m-1)\cdot\alpha\cdot Q(\alpha,\beta) + \frac{m-1}{\alpha^{m-1}} \int_\beta^\infty x^{m-2} e^{-\frac{x^2+\alpha^2}{2}} I_{m+1}(x)\,dx + \frac{\alpha}{2\alpha^{m-2}} \int_\beta^\infty x^{m-2} e^{-\frac{x^2+\alpha^2}{2}} \frac{I_m(\alpha x)}{\alpha\cdot x}$$

(?)

- VTORIOT ČLEN ME MORAM DA GO IZVEDAM!!

$$\boxed{Q_m(\alpha,\beta) = \left(\frac{\beta}{\alpha}\right)^{m-1} e^{-\frac{\alpha^2+\beta^2}{2}} I_{m-1}(\alpha\beta) + Q_{m-1}(\alpha,\beta)}$$

GOTOVA FORMULA OD DCoFC !!

USE RECURSION

$$Q_m(\alpha,\beta) = \left(\frac{\beta}{\alpha}\right)^{m-1} e^{-\frac{\alpha^2+\beta^2}{2}} I_{m-1}(\alpha,\beta) + \left(\frac{\beta}{\alpha}\right)^{m-2} e^{-\frac{\alpha^2+\beta^2}{2}} \cdot I_{m-2}(\alpha,\beta) + Q_{m-2}(\alpha,\beta)$$

$$Q_m(\alpha,\beta) = \frac{1}{\alpha^{m-1}} \int_{\beta}^{\infty} x^m \cdot e^{-\frac{x^2+\alpha^2}{2}} \cdot I_{m-1}(\alpha \cancel{x})\, dx$$

$$\lim_{m \to \infty} Q_m(\alpha,\beta) = 1 \, ; \quad Q_{\infty}(\alpha,\beta) = 1 \qquad Q_{-\infty}(\alpha,\beta) = 0$$

$$I_m(x) = \sum_{k=0}^{\infty} \frac{\left(\frac{x}{2}\right)^{m+2k}}{k!\, \Gamma(m+k+1)} \qquad \lim_{m \to \infty} \frac{\left(\frac{x}{2}\right)^{m}}{\Gamma(m+k+1)} = 0$$ MATLB

$$Q_m(\alpha,\beta) = \sum_{i=1}^{\infty} \left(\frac{\beta}{\alpha}\right)^{m-i} e^{-\frac{\alpha^2+\beta^2}{2}} I_{m-i}(\alpha,\beta)$$

$$Q_m(\alpha,\beta) = e^{-\frac{\alpha^2+\beta^2}{2}} \sum_{\nu=1-m}^{\infty} \left(\frac{\alpha}{\beta}\right)^{\nu} I_{-\nu}(\alpha\beta)$$

$$\boxed{I_{-\nu}(x) = I_{\nu}(x)}$$

$$\boxed{Q_m(\alpha,\beta) = 1 - e^{-\frac{\alpha^2+\beta^2}{2}} \sum_{\nu=m}^{\infty} \left(\frac{\beta}{\alpha}\right)^{\nu} I_{\nu}(\alpha\beta)}$$

SERIES FORM OF MARCUM'S Q FUNCTION

$$\boxed{Q_m(\alpha,\beta) = e^{-\frac{\alpha^2+\beta^2}{2}} \sum_{\nu=1-m}^{\infty} \left(\frac{\alpha}{\beta}\right)^{\nu} I_{\nu}(\alpha\beta)}$$

$m = 1$

$$Q_1(\alpha,\beta) = 1 \underset{\pi}{-} e^{\frac{\alpha^2+\beta^2}{2}} \sum_{\nu=1}^{\infty} \left(\frac{\beta}{\alpha}\right)^{\nu} I_{\nu}(\alpha\beta)$$

$$I_{\nu}(x) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \left(-je^{\theta}\right)^{\nu} \cdot e^{-x \sin\theta}\, d\theta$$ MODIFIED OF I KIND

ABRAMOWITZ

$$J_0(x) = \frac{1}{\pi} \int_0^{\pi} \cos(x \sin\theta)\, d\theta \qquad J_{\nu}(x) = \frac{1}{\pi} \int_0^{\pi} \cos(x \sin\theta - \nu\cdot\theta)\, d\theta$$

BESSEL OF I KIND

$$Q_m(\xi\beta,\beta) = \exp\left[-\frac{\beta^2}{2}(1+\xi^2)\right] \sum_{\nu=1-m}^{\infty} \xi^{\nu} I_{\nu}(\xi\beta^2) \qquad 0^+ \leq \xi \triangleq \frac{\alpha}{\beta} < 1$$

$$Q_m(\alpha,\beta) = \cancel{\sum_{i=1}^{n}}\; 1 - \exp\left[-\frac{\alpha^2}{2}(1+\xi^2)\right]\sum_{r=m}^{\infty}\xi^r I_r(\alpha^2\xi)$$

$$\boxed{j = e^{i\frac{\pi}{2}} \quad -j = e^{-i\frac{\pi}{2}}} \qquad 0 \leq \xi = \frac{\beta}{\alpha} < 1$$

$$Q_m(\beta\xi,\beta) = \exp\left[-\frac{\beta^2}{2}(1+\xi^2)\right]\frac{1}{2\pi}\int_{-\pi}^{\pi}\left[\sum_{r=1-m}^{\infty}\left(-j e^{j\theta}\right)^{-r}\xi^r e^{-\xi\beta^2\sin\theta}\right]d\theta =$$

$$= \frac{e^{-\frac{\beta^2}{2}(1+\xi^2)}}{2\pi}\int_{-\pi}^{\pi}\left\{\sum_{r=1-m}^{N-m}\left[e^{j\left(\theta+\frac{\pi}{2}\right)}\xi\right]^{+r} + \sum_{r=N-m+1}^{\infty}\left[\xi e^{j\left(\theta+\frac{\pi}{2}\right)}\right]^r\right\}e^{-\xi\beta^2\sin\theta}\,d\theta$$

$$s = \sum_{n=1-m}^{N-m} q^n \qquad s = ?$$

$$n = i+m-1$$

$$i = n+m-1$$

$$\boxed{s = \sum_{i=0}^{N-1} q^{i+m-1}}$$

$$\begin{cases} n = 1-m \\ i = 1-m+m-1 = 0 \end{cases}$$

$$\begin{cases} n = N-m \\ i = \cancel{N}-m+m-1 = N-1 \end{cases}$$

$$s = q^{m-1}\underbrace{\sum_{i=0}^{N-1} q^i}_{S_1}$$

$$S_1 = 1 + q + q^2 + \dots q^{N-1}$$

$$q S_1 = q + q^2 + \dots q^N$$

$$S_1(1-q) = 1 - q^N \qquad \boxed{S_1 = \frac{1-q^N}{1-q}}$$

$$\boxed{s = q^{m-1}\frac{1-q^N}{1-q}} \qquad \boxed{q = \xi e^{j\left(\theta+\frac{\pi}{2}\right)}}$$

$$S_2 = \sum_{r=M}^{\infty} q^r$$

$$i = r-M \qquad r = i+M$$

$$r = M \quad i = 0 \qquad r \to \infty \quad i \to \infty$$

$$S_2 = \sum_{i=0}^{\infty} q^{i+M} = q^M\sum_{i=0}^{\infty} q^i$$

$$\boxed{S_2 = \frac{q^M}{1-q}} \qquad \boxed{M = N-m+1}$$

$$q = \xi e^{j\left(\theta+\frac{\pi}{2}\right)}.$$

$$Q_m(\beta\xi,\beta) = \frac{e^{-\frac{\beta^2}{2}(1+\xi^2)}}{2\pi}\int_{-\pi}^{\pi}\left[\xi^{m-1}e^{j(m-1)\left(\theta+\frac{\pi}{2}\right)}\frac{1-\xi^N e^{jN\left(\theta+\frac{\pi}{2}\right)}}{1-\xi e^{j\left(\theta+\frac{\pi}{2}\right)}} + \right.$$

$$\left. + \frac{\xi^{N-m+1}e^{j(N-m+1)\left(\theta+\frac{\pi}{2}\right)}}{1-\xi e^{j\left(\theta+\frac{\pi}{2}\right)}}\right]e^{-\xi\beta^2\sin\theta}\,d\theta$$

$$\boxed{\begin{aligned} Q_m(\beta\xi,\beta) &= \frac{1}{2\pi}\int_{-\pi}^{\pi}\xi^{-(m-1)}\frac{\left\{\cos\left[(m-1)\left(\theta+\frac{\pi}{2}\right)\right] - \xi\cos\left[m\left(\theta+\frac{\pi}{2}\right)\right]\right\}}{1+2\xi\sin\theta+\xi^2}\,\cdot \\ &\cdot \exp\left[-\frac{\beta^2}{2}\left(1+2\xi\sin\theta+\xi^2\right)\right]d\theta \qquad 0 < \xi \leq \frac{1}{\cancel{\alpha}\beta} < 1 \end{aligned}} \quad \boxed{4.69}$$

- SMALL ARGUMENT FORM FOR MODIFIED BESSEL FUNCTION

$$Q_n(\alpha,\beta)=\frac{1}{\alpha^{n}}\int_\beta^\infty x^{n}e^{-\frac{x^2+\alpha^2}{2}}I_{n-1}(\alpha x)\,dx$$

$$\boxed{I_\nu(z)\doteq\frac{(z/2)^\nu}{\Gamma(\nu+1)}}$$ RAV

$$Q_n(\xi\beta,\beta)=?\quad \text{IF: } \xi\to 0 \qquad \boxed{\xi=\frac{\alpha}{\beta}}$$

$$\left.\begin{matrix}\xi\to 0\\ \alpha\to 0\end{matrix}\right\}\Rightarrow$$ SMALL ARGUMENT $\boxed{\alpha\cdot x}$

$$Q_n(\alpha,\beta)=\frac{1}{\alpha^{n-1}}\int_\beta^\infty x^{n}e^{-\frac{x^2+\alpha^2}{2}}\frac{(\alpha x/2)^{n-1}}{\Gamma(n)}\,dx$$

$$Q_n(\alpha,\beta)=\frac{1}{\cancel{\alpha^{n-1}}}\frac{\cancel{\alpha^{n-1}}}{\Gamma(n)\cdot 2^{n-1}}\int_\beta^\infty x^{2n-1}e^{-\frac{x^2+\alpha^2}{2}}\,dx$$

$$\boxed{Q_n(\alpha,\beta)=\frac{1}{\Gamma(n)\cdot 2^{n-1}}\int_\beta^\infty x^{2n-1}e^{-\frac{x^2+\alpha^2}{2}}\,dx}$$

MAPLE:

$$\boxed{Q(\alpha,\beta)=\cancel{\tfrac{1}{2}}\,\frac{e^{-\frac{\alpha^2}{2}}\Gamma\left(n,\frac{\beta^2}{2}\right)}{\cancel{\left(\frac{1}{2}\right)^n}\Gamma(n)\cdot\cancel{2^{n-1}}}}\boxed{=\frac{e^{-\frac{\alpha^2}{2}}\Gamma\left(n,\frac{\beta^2}{2}\right)}{\Gamma(n)}}$$

$$\alpha\to 0\qquad e^{-\alpha^2/2}\to 1$$

$$\boxed{Q(0,\beta)=\frac{\Gamma\left(n,\frac{\beta^2}{2}\right)}{\Gamma(n)}}$$

$$\boxed{\begin{aligned}\Gamma(\alpha)&=\int_0^\infty t^{\alpha-1}e^{-t}\,dt\\ \Gamma(\alpha,\beta)&=\int_\beta^\infty t^{\alpha-1}e^{-t}\,dt\end{aligned}}$$

$$\int_\beta^\infty x^{2n-1}e^{-\frac{x^2+\alpha^2}{2}}dx=e^{-\frac{\alpha^2}{2}}\int_\beta^\infty x^{2n-1}e^{-\frac{x^2}{2}}\,dx=\circledast$$

dx → dm

$$x^2=2m$$

$$m=\frac{x^2}{2}\qquad dm=x\,dx\qquad \begin{matrix}x=\beta\\ m=\beta^2/2\end{matrix}\qquad x^{2n-1}=x^{2n-2}\cdot x=(x^2)^{n-1}\,x$$

$$\circledast=e^{-\frac{\alpha^2}{2}}\int_{\beta^2/2}^\infty(2m)^{n-1}e^{-m}\,dm=2^{n-1}e^{-\frac{\alpha^2}{2}}\int_{\beta^2/2}^\infty m^{n-1}e^{-m}\,dm$$

$$=\underline{\underline{2^{n-1}e^{-\frac{\alpha^2}{2}}\Gamma\left(n,\frac{\beta^2}{2}\right)}}$$

$$Q(0,\beta) = \frac{\Gamma(m,\frac{\beta^2}{2})}{\Gamma(m)} \Rightarrow Q(0,\beta) = \frac{\beta^{2m}}{2^{m-1}\Gamma(m)} \int_0^{\pi/2} \frac{\cos\theta}{(\sin\theta)^{1+2m}} e^{-\frac{\beta^2}{2\sin^2\theta}} d\theta$$

- SPECIAL CASES! (m - INTEGER

$$\Gamma(1+n,x) = n!\, e^{-x} \sum_{m=0}^{n} \frac{x^m}{m!} \qquad n = 0,1,\dots$$

$$\Gamma(1+n,x) = \boxed{n!}\, e^{-x} \sum_{k=0}^{n} \frac{x^k}{k!} \qquad n = 0,1,\dots$$

$$\Gamma\left(m,\frac{\beta^2}{2}\right) = ? \qquad m = n+1 \qquad x = \frac{\beta^2}{2}$$

$$n = m-1$$

$$\Gamma\left(m,\frac{\beta^2}{2}\right) = (m-1)!\; e^{-\frac{\beta^2}{2}} \sum_{k=0}^{m-1} \frac{\left(\frac{\beta^2}{2}\right)^k}{k!}$$

- ZA m,n = INTEGER VALI : $\Gamma(m) = (m-1)!$

$$Q(0,\beta) = \cancel{(m-1)!}\; e^{-\frac{\beta^2}{2}} \sum_{k=0}^{m-1} \frac{1}{k!}\left(\frac{\beta^2}{2}\right)^k \cdot \frac{1}{\cancel{(m-1)!}}$$

$$\boxed{Q(0,\beta) = \sum_{k=0}^{m-1} \exp\left[-\frac{\beta^2}{2}\right] \frac{(\beta^2/2)^k}{k!}}$$

$$Q(\alpha,\beta) = \int_\beta^\infty x\, e^{-\frac{x^2+\alpha^2}{2}} I_0(\alpha x)\, dx$$

- SPECIAL CASE OF ANOTHER FORM OF MARCUM'S Q FUNCTION

$$\boxed{Q_m(\alpha,\beta) = \sum_{n=0}^{\infty} e^{-\frac{\alpha^2}{2}} \frac{(\alpha^2/2)^n}{n!} \sum_{k=0}^{n+m-1} e^{-\frac{\beta^2}{2}} \frac{(\beta^2/2)^k}{k!}}$$

SIMILARLY AS FOR $Q(\beta\zeta,\beta)$ ON PP. 156

$$\boxed{Q(\alpha,\alpha\zeta) = 1 - \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{\zeta^m\left\{\cos\left[m\left(\theta+\frac{\pi}{2}\right)\right] - \zeta\cos\left[(m-1)\left(\theta+\frac{\pi}{2}\right)\right]\right\}}{1+2\zeta\sin\theta+\zeta^2} \cdot \exp\left[\frac{\alpha^2}{2}(1+2\zeta\sin\theta+\zeta^2)\right] d\theta \qquad 0<\zeta \triangleq \frac{\beta}{\alpha}<1}$$

$\boxed{4.77}$

⊙ UPPER AND LOWER BOUNDS

$$\boxed{Q_n(\alpha,\beta) = e^{-\frac{\alpha^2+\beta^2}{2}} \sum_{n=1}^{m-1} \left(\frac{\beta}{\alpha}\right)^n I_n(\alpha\beta) + Q_1(\alpha,\beta)}$$

① $$I_n(z) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \underbrace{\left(-j e^{-j\theta}\right)^n}_{W} e^{-z\sin\theta}\, d\theta$$

$$\boxed{I_n(z) = \frac{1}{\pi}\int_0^{\pi} e^{z\cos\theta} \cos n\theta \, d\theta}$$

$$W = \left(-j e^{-j\theta}\right)^n = e^{-j\left(\frac{\pi}{2}+\theta\right)\cdot n} = \cos\left(\theta n + n\frac{\pi}{2}\right) + \underbrace{j\sin\left(n\theta + n\frac{\pi}{2}\right)}_{⑪}$$

$$\cos\left(n\theta + n\frac{\pi}{2}\right) = \cos(n\theta)\cdot\cos\left(n\frac{\pi}{2}\right) - \sin(n\theta)\cdot\sin\left(n\frac{\pi}{2}\right)$$

- $I_n(z)$ - REALNA FUNKCIJA $\Rightarrow$ ⑪ $= 0$

$$I_n(z) = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{z\cos\theta} \cos\left(n\theta + n\frac{\pi}{2}\right) d\theta$$

$$\boxed{I_{-n}(z) = I_n(z)}$$

$I_n(z)$ e PARNA FUNKCIJA ZA „n" INTEGER (VIDI ABRAMOWITZ FIG 9.9)

$$I_n(z) = \frac{1}{\pi}\int_0^{\pi} e^{z\sin\theta} \cos\left(n\theta + n\frac{\pi}{2}\right) d\theta$$

$$I_\nu(z) = \frac{1}{\pi}\int_0^{\pi} e^{z\cos\theta}\cos(\nu\theta)\,d\theta - \underbrace{\frac{\sin(\nu\pi)}{\pi}\int_0^{\infty} e^{-z\cosh t - \nu t}\,dt}_{(*)}$$

* If „ν" IS INTEGER i.e. $\nu = n$ $\sin(n\pi) = 0 \Rightarrow (*) = 0$

$$I_n(z) = \frac{1}{\pi}\int_0^{\pi} e^{z\cos\theta}\cos(n\theta)\,d\theta$$

---

① $$I_n(z) = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{-j\left(\theta+\frac{\pi}{2}\right)\cdot n} e^{-z\sin\theta}\, d\theta$$ KAKO SE DOBIVA?

$$I_n(z) = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{-z\sin\theta - j\left(\theta+\frac{\pi}{2}\right)n}\, d\theta = \frac{1}{2\pi}\int_{-\pi/2}^{3\pi/2} e^{-t}(z\cos\theta + j)^{-1}\, dt$$?

$t = z\sin\theta + j\left(\theta + \frac{\pi}{2}\right)$ $\quad dt = (z\cos\theta + j)\,d\theta$ $\quad d\theta = \frac{dt}{z\cos\theta + j}$

$\theta = -\pi$ $\quad t = 0 + \frac{\pi}{2}$ $\quad \theta = \pi$ $\quad t = \frac{3\pi}{2}$

$$\boxed{I_n(z) = (-j)^{-n} J_n(jz)}$$

$$\boxed{\begin{aligned} J_n(z) &= \frac{1}{\pi}\int_0^{\pi} \cos(z\sin\theta - n\theta)\,d\theta \\ &= \frac{j^{-n}}{\pi}\int_0^{\pi} e^{jz\cos\theta}\cos(n\theta)\,d\theta \end{aligned}}$$

Ⓘ

$$\cos(z\sin\theta - n\theta) = \frac{1}{2}\Big[e^{j(z\sin\theta - n\theta)} + \underset{?}{e^{-j(z\sin\theta - n\theta)}}\Big] =$$

$$= \frac{1}{2} e^{-jn\theta}\Big[e^{jz\sin\theta} + e^{-jz\sin\theta}\Big] = e^{-jn\theta}\cdot\cos(z\sin\theta)\,?$$

$$= \frac{1}{2} e^{jz\sin\theta}\cdot e^{-jn\theta} + \frac{1}{2} e^{-jz\sin\theta}\cdot e^{+jn\theta} = (*)$$

$$\cos(z\sin\theta - n\theta) = \cos(z\sin\theta)\cdot\cos(n\theta) + \sin(z\sin\theta)\cdot\sin(n\theta)$$

$$(*) = \frac{1}{2}\Big[\cos(z\sin\theta) + j\sin(z\sin\theta)\Big]e^{-jn\theta} + \Big[\cos(z\sin\theta) - j\sin(z\sin\theta)\Big]e^{jn\theta}$$

$$= \frac{1}{2}\cos(z\sin\theta)\Big[e^{-jn\theta} + e^{jn\theta}\Big] + \frac{1}{2} j\sin(z\sin\theta)\Big[e^{-jn\theta} - e^{jn\theta}\Big]$$

$$= \cos(z\sin\theta)\cdot\cos(n\theta) + \frac{1}{2j}\sin(z\sin\theta)\underbrace{\Big[e^{jn\theta} - e^{-jn\theta}\Big]}_{\sin(n\theta)}$$

$$\frac{1}{2}\Big(e^{jz\sin\theta} + e^{-jz\sin\theta}\Big)\frac{1}{2}\Big(e^{+jn\theta} + e^{-jn\theta}\Big) + \frac{1}{2j}\Big(e^{jz\sin\theta} - e^{-jz\sin\theta}\Big)\frac{1}{2j}\Big(e^{jn\theta} - e^{-jn\theta}\Big)$$

$$= \frac{1}{4}\Big[\Big(\cancel{e^{jz\sin\theta + jn\theta}} + \cancel{e^{jz\sin\theta - jn\theta}} + e^{-jz\sin\theta + jn\theta} + \cancel{e^{-jz\sin\theta - jn\theta}}\Big) -$$

$$\Big(\cancel{e^{jz\sin\theta + jn\theta}} - \cancel{e^{jz\sin\theta - jn\theta}} - e^{-jz\sin\theta + jn\theta} + \cancel{e^{-jz\sin\theta - jn\theta}}\Big)$$

$$= \frac{1}{4}\Big(2e^{-jz\sin\theta + jn\theta} + 2e^{+jz\sin\theta - jn\theta}\Big) = \frac{1}{2}\underbrace{\Big[e^{jz\sin\theta - jn\theta} + e^{-jz\sin\theta + jn\theta}\Big]}_{\text{ISTO NA } (*)}$$

– Ne MOŽAM DA GO DOKAŽAM Ⓘ

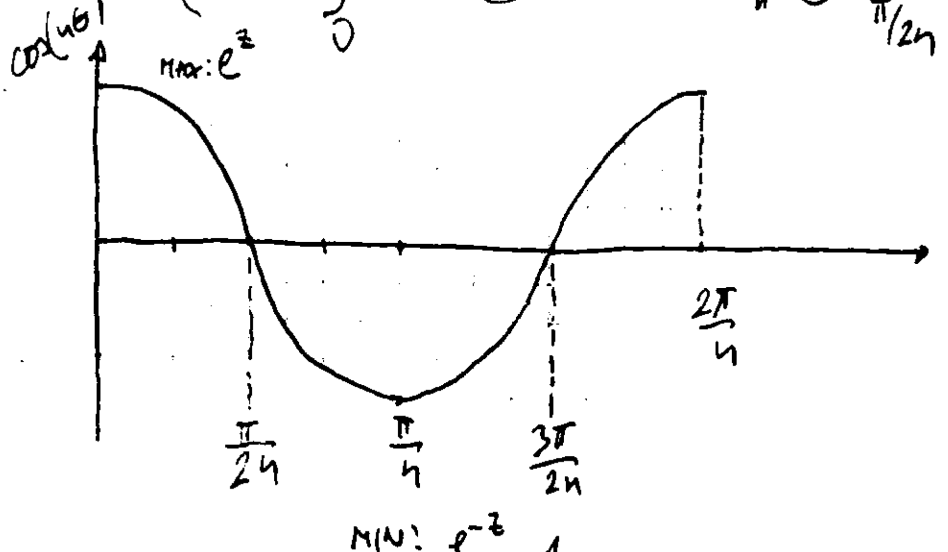
$$Q_m(\alpha,\beta) = e^{-\frac{\alpha^2+\beta^2}{2}} \sum_{n=1}^{m-1} \left(\frac{\beta}{\alpha}\right)^n I_n(\alpha\beta) + Q_1(\alpha,\beta)$$

$$I_n(z) = \frac{1}{\pi}\int_0^\pi \boxed{e^{z\sin\theta}} \cos(n\theta)\,d\theta \longrightarrow \begin{matrix} \text{MAX: } e^{z} \\ \text{MIN: } e^{-z} \end{matrix}$$

$$Q_m(\alpha,\beta) = e^{-\frac{\alpha^2+\beta^2}{2}} \sum_{n=1}^{m-1} \left(\frac{\beta}{\alpha}\right)^n \frac{1}{\pi}\int_0^\pi e^{z\sin\theta}\cos(n\theta)\,d\theta + Q_1(\alpha,\beta)$$

$$I_n(z) \le \frac{n}{2}\left(\frac{1}{\cancel{2}\pi} e^{z}\int_0^\pi \cos(n\theta)\,d\theta\right)$$ $\boxed{\cos(n\theta) \quad n\text{-fold PERIODICITY}}$

$$I_n(z) \le \frac{n}{2}\left(\frac{1}{\pi} e^{z}\int_0^{\pi/2n} \cos(n\theta)\,d\theta + \frac{1}{\pi} e^{-z}\int_{\pi/2n}^{3\pi/2n} \cos(n\theta)\,d\theta + \frac{1}{\pi} e^{z}\int_{3\pi/2n}^{2\pi/n} \cos(n\theta)\,d\theta\right)$$



$$I_n(z) \le \frac{n}{2}\left(\frac{e^{z}}{\pi}\,\frac{1}{n}\,\overset{1}{\sin(n\theta)\Big|_0^{\pi/2n}} + \frac{e^{-z}}{\pi}\,\frac{1}{n}\,\overset{-2}{\sin(n\theta)\Big|_{\pi/2n}^{3\pi/2n}} + \frac{e^{z}}{\pi}\,\frac{1}{n}\,\overset{1}{\sin(n\theta)\Big|_{3\pi/2n}^{2\pi/n}}\right)$$

$$= \frac{n}{2}\left(\frac{2e^{z}}{n\pi} - \frac{2e^{-z}}{n\pi}\right) = \frac{n}{2}\left(\frac{2(e^{z}-e^{-z})}{n\pi}\right) = \frac{e^{z}-e^{-z}}{\pi}$$

$$\boxed{I_n(z) \le \frac{e^{z}-e^{-z}}{\pi}}$$ MMV!!!

$\longrightarrow$ UPPER BOUND OF $I_n(z)$

$$Q_m(\alpha,\beta) \le e^{-\frac{\alpha^2+\beta^2}{2}} \sum_{n=1}^{m-1} \left(\frac{\beta}{\alpha}\right)^n \frac{e^{z}-e^{-z}}{\pi} + Q_1(\alpha,\beta)$$

$$Q_1(\alpha,\beta) \le \exp\left[-\frac{(\beta-\alpha)^2}{2}\right]$$

$$Q_m(\alpha,\beta) \leq e^{-\frac{\alpha^2+\beta^2}{2}} \sum_{k=1}^{m-1} \left(\frac{\beta}{\alpha}\right)^k \frac{e^z - e^{-z}}{\pi} + e^{-\frac{(\beta-\alpha)^2}{2}} =$$

$$= e^{-\frac{\alpha^2+\beta^2}{2}} \cdot \frac{(e^z - e^{-z})}{\pi} \frac{\frac{\beta}{\alpha}\left(1-\left(\frac{\beta}{\alpha}\right)^{m-1}\right)}{1-\frac{\beta}{\alpha}} + e^{-\frac{(\beta-\alpha)^2}{2}} = \qquad \boxed{z=\alpha\cdot\beta}$$

$$= e^{-\frac{\alpha^2+\beta^2}{2}} \frac{(e^{\alpha\beta} - e^{-\alpha\beta})}{\pi} \frac{1-\left(\frac{\beta}{\alpha}\right)^{m-1}}{\alpha/\beta - 1} + e^{-\frac{(\beta-\alpha)^2}{2}}$$

$$S = q + q^2 + \dots + q^{m-1}$$

$$qS = q^2 + q^3 + \dots + q^m$$

$$\boxed{S - qS = q - q^m = \frac{q(1-q^{m-1})}{1-q}}$$

$$= \frac{e^{\frac{\alpha^2-2\alpha\beta+\beta^2}{2}} - e^{-\frac{\alpha^2+2\alpha\beta+\beta^2}{2}}}{\pi} \cdot \frac{1-\left(\frac{\beta}{\alpha}\right)^{m-1}}{\alpha/\beta - 1} + e^{-\frac{(\beta-\alpha)^2}{2}}$$

$$= \frac{e^{-\frac{(\alpha-\beta)^2}{2}} - e^{-\frac{(\alpha+\beta)^2}{2}}}{\pi} \frac{1-\left(\frac{\beta}{\alpha}\right)^{m-1}}{\alpha/\beta - 1} + e^{-\frac{(\beta-\alpha)^2}{2}}$$

$$= \exp\left(-\frac{(\beta-\alpha)^2}{2}\right) + \frac{1}{\pi}\left(\frac{\beta}{\alpha}\right)^{m-1} \frac{\left(\frac{\alpha}{\beta}\right)^{m-1} - 1}{\alpha/\beta - 1} \left[e^{-\frac{(\alpha-\beta)^2}{2}} - e^{-\frac{(\alpha+\beta)^2}{2}}\right]$$

$$\boxed{Q_m(\alpha,\beta) \leq \exp\left(-\frac{\beta^2(1-\zeta)^2}{2}\right) + \frac{1}{\pi} \frac{1}{\zeta^{m-1}} \frac{1-\zeta^{m-1}}{1-\zeta} \left[e^{-\frac{\beta^2(\zeta-1)^2}{2}} - e^{-\frac{\beta^2(\zeta+1)^2}{2}}\right]}$$

UPPER BOUND

- LOWER BOUND

ALTERNATIVE FORM OF $I_m(z)$

$$\boxed{I_m(z) = \frac{(z/2)^m}{\sqrt{\pi}\,\Gamma(m+\frac{1}{2})} \int_0^\pi e^{z\cos\theta} \sin^{2m}\theta\, d\theta}$$

$$I_m(z) = \frac{(z/2)^m}{\sqrt{\pi}\,\Gamma(m+\frac{1}{2})} e^{-z} \int_0^\pi \sin^{2m}\theta\, d\theta$$

MIN VALUE

$$\boxed{\int_0^{\pi/2} \sin^{2m} x\, dx = \int_0^{\pi/2} \cos^{2m} x\, dx = \frac{(2m-1)!!}{(2m)!!} \frac{\pi}{2}}$$

GRADSHTEYN
3.621.3

$$m = 2 \Rightarrow \frac{(4-1)!!}{4!!} \frac{\pi}{2} = \frac{3}{8} \cdot \frac{\pi}{2} = \frac{3\pi}{16} \qquad \int_0^\pi \sin^4 x\, dx = 2 \cdot \frac{3\pi}{16} = \frac{3\pi}{8}$$

$$I_n(z) \geq \frac{(z/2)^n}{\sqrt{\pi}\,\Gamma(n+\frac{1}{2})} e^{-z} \frac{(2n-1)!!}{2n!!}\frac{\pi}{2}$$

$$\boxed{5!! = 5\cdot 3\cdot 1 = 15}$$

$$\boxed{\Gamma\left(n+\frac{1}{2}\right) = \frac{\sqrt{\pi}}{2^n}(2n-1)!!}$$

GRADSHTEYN 8.339.2

$$I_n(z) \geq \frac{(z/\cancel{2})^n}{\cancel{\sqrt{\pi}}\cdot\frac{\cancel{\sqrt{\pi}}}{\cancel{2^n}}\,\cancel{(2n-1)!!}} e^{-z}\frac{\cancel{(2n-1)!!}}{2n!!}\frac{\pi}{2} = \frac{z^n e^{-z}}{2n!!}$$

$$Q_m(\alpha,\beta) \geq e^{-\frac{\alpha^2+\beta^2}{2}}\sum_{n=1}^{m-1}\left(\frac{\beta}{\alpha}\right)^n \frac{(\alpha\beta)^n e^{-\alpha\beta}}{2n!!} + e^{-\frac{(\beta+\alpha)^2}{2}} = \xleftarrow{} 4.61$$

$$= e^{-\frac{(\alpha^2+2\alpha\beta+\beta^2)}{2}}\sum_{n=1}^{m-1}\frac{\beta^{2n}}{2n!!} + e^{-\frac{(\beta+\alpha)^2}{2}} =$$

$$= e^{-\frac{(\alpha+\beta)^2}{2}}\left[\sum_{n=1}^{m-1}\frac{\beta^{2n}}{2n!!}+1\right] = e^{-\frac{(\alpha+\beta)^2}{2}}\sum_{n=0}^{m-1}\frac{\beta^{2n}}{2n!!}$$

$$\boxed{(2n)!! = (2n)!! = 2^n\cdot n!}$$

$n = 4$

$$(2\cdot 8)!! = (16)!! = 16\cdot 14\cdot 12\cdot 10\cdot 8\cdot 6\cdot 4\cdot 2 = 2^8(8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1)$$

$$Q_m(\alpha,\beta) \geq e^{-\frac{(\alpha+\beta)^2}{2}}\sum_{n=0}^{m-1}\frac{\beta^{2n}}{2^n\cdot n!} = e^{-\frac{(\alpha+\beta)^2}{2}}\sum_{n=0}^{m-1}\frac{(\beta^2/2)^n}{n!}$$

$$\boxed{Q_m(\beta\xi,\beta) \geq e^{-\frac{\beta^2(1+\xi)^2}{2}}\sum_{n=0}^{m-1}\frac{(\beta^2/2)^n}{n!} \qquad 0<\xi=\frac{\alpha}{\beta}<1}$$

- $\xi = 0$

$$Q_m(0,\beta) \geq e^{-\beta^2/2}\sum_{n=0}^{m-1}\frac{(\beta^2/2)^n}{n!}$$

- FOR $0\leq\xi=\beta/\alpha<1$

$$Q_1(\alpha,\beta) \geq 1-\frac{1}{2}\left\{\exp\left[-\frac{(\alpha-\beta)^2}{2}\right]-\exp\left[-\frac{(\alpha+\beta)^2}{2}\right]\right\} \quad +4.61$$

$$Q_m(\alpha,\beta) \geq e^{-\frac{(\alpha+\beta)^2}{2}}\sum_{n=1}^{m-1}\frac{(\beta/2)^{2n}}{n!} + 1-\frac{1}{2}\left\{e^{-\frac{(\alpha-\beta)^2}{2}}-e^{-\frac{(\alpha+\beta)^2}{2}}\right\} \quad \Delta$$

⊙ APPROACH OF LOWER & UPPER BOUNDING BY USING CAUSHY & SCHWARTZ INEQUALITY

$$\left|\int_a^b g_1(\theta) g_2(\theta)\, d\theta\right|^2 \leq \int_a^b |g_1(\theta)|^2 d\theta \int_a^b |g_2(\theta)|^2 d\theta .$$

$$Q_m(\rho\xi,\rho) = \frac{1}{2\pi}\int_0^{2\pi} \xi^{-(m-1)} \underbrace{\frac{[\cos(m-1)\theta - \xi\cos(m\theta)]}{1-2\xi\cos\theta+\xi^2}}_{g_2(\theta)}$$

$$\exp\Big[-\underbrace{\frac{\rho^2}{2}(1-2\xi\cos\theta}_{g_1(\theta)}+\xi^2)\Big]\, d\theta \quad 0<\xi<\frac{\alpha}{\rho}<1$$

$$g_1(\theta) = \exp(\rho^2\xi\cos\theta) \qquad g_2(\theta) = \frac{\cos(m-1)\theta - \xi\cos m\theta}{1-2\xi\cos\theta+\xi^2}$$

$$= \frac{1}{2\pi}\int_0^{2\pi} \xi^{-(m-1)} g_2(\theta)\cdot g_1(\theta)\, e^{-\frac{\rho^2}{2}(1+\xi^2)}\, d\theta =$$

$$= \frac{1}{2\pi} \underbrace{\xi^{-(m-1)} e^{-\frac{\rho^2}{2}(1+\xi^2)}}_{C} \int_0^{2\pi} g_1(\theta) g_2(\theta)\, d\theta \leq$$

$$\leq \frac{C}{2\pi}\sqrt{\int_0^{2\pi}|g_1(\theta)|^2 d\theta}\sqrt{\int_0^{2\pi}|g_2(\theta)|^2 d\theta} = C\sqrt{\frac{1}{2\pi}\int_0^{2\pi}|g_1(\theta)|^2 d\theta\,\frac{1}{2\pi}\int_0^{2\pi}|g_2(\theta)|^2 d\theta}$$

$$\frac{1}{2\pi}\int_0^{2\pi}|g_1(\theta)|^2 d\theta = \frac{1}{2\pi}\int_0^{2\pi} e^{2\rho^2\xi\cos\theta}\, d\theta = I_0(2\rho^2\xi)$$

$$\int_0^{2\pi}|g_2(\theta)|^2 d\theta = \frac{1}{2}\left[(2m-1)\xi^{2(m-1)} + \frac{1}{1-\xi^2}\right]$$

$$Q_m(\rho\xi,\rho) = \xi^{-(m-1)} e^{-\frac{\rho^2}{2}(1+\xi^2)}\sqrt{I_0(2\rho^2\xi)}\sqrt{\frac{1}{2}\left[(2m-1)\xi^{2m-1} + \frac{1}{1-\xi^2}\right]}$$

$$= e^{-\frac{\rho^2}{2}(1+\xi^2)}\sqrt{I_0(2\rho^2\xi)}\sqrt{\frac{1}{2}\left[(2m-1) + \xi^{-2(m-1)}/(1-\xi^2)\right]} \quad 0<\frac{\alpha}{\rho}<1 \quad (16)$$

$$\boxed{\sqrt{I_0(x)} \leqslant \cosh\left(\frac{x}{2}\right)} \qquad \cosh\frac{x}{2} = \frac{e^{x/2}+e^{-x/2}}{2}$$

$$Q_m(\beta\zeta,\beta) \leqslant e^{-\frac{\beta^2}{2}(1+\zeta^2)} \frac{e^{\beta^2\zeta}+e^{-\beta^2\zeta}}{2} \sqrt{\frac{1}{2}\left[(2m-1)+\frac{\zeta^{2(m-1)}}{1-\zeta^2}\right.}$$

$$= \frac{1}{2}\left[e^{-\frac{\beta^2 \; \beta^2\zeta^2+2\beta^2\zeta}{2}} + e^{-\frac{\beta^2+\beta^2\zeta^2+2\beta^2\zeta}{2}}\right]\sqrt{\ldots}$$

$$\boxed{Q_m(\beta\zeta,\beta) \leqslant \frac{1}{2}\left[e^{-\frac{\beta^2}{2}(1-\zeta)^2} + e^{-\frac{\beta^2}{2}(1+\zeta)^2}\right]\sqrt{\frac{1}{2}\left[(2m-1)+\frac{\zeta^{2(m-1)}}{1-\zeta^2}\right]}}$$

$$\boxed{0<\zeta=\frac{\alpha}{\beta}<1}$$

$$\boxed{Q_m(\alpha,\alpha\zeta) \geqslant 1-\frac{1}{2}\sqrt{\frac{\zeta^{2m}}{2(1-\zeta^2)}}\left[e^{-\frac{\alpha^2}{2}(1-\zeta)^2} + e^{-\frac{\alpha^2}{2}(1+\zeta)^2}\right]}$$

$$\boxed{0<\zeta=\frac{\beta}{\alpha}<1}$$

- MGF_1/γ̄ FOR RICIAN FADING WITH POLYNOMIAL APROX

$$p(\gamma) = \frac{K+1}{\bar{\gamma}} e^{-\frac{(K+1)\gamma}{\bar{\gamma}}-K} I_0\left(2\cdot\sqrt{\frac{K(K+1)\cancel{\gamma}}{\bar{\gamma}}}\right)$$

$$p(\gamma) \doteq \frac{K+1}{\bar{\gamma}} e^{-\frac{(K+1)\gamma}{\bar{\gamma}}-K}\left(1+3.5156229\left(\frac{4\cdot K(K+1)\cancel{\gamma}}{\bar{\gamma}}\right)^2\frac{1}{C^2}\right)$$

$$\boxed{MGF_{1/\bar{\gamma}} = \frac{4K\cdot e^{-K}\sqrt{s}}{\sqrt{(K+1)\bar{\gamma}}} K_1\left(2\sqrt{\frac{(K+1)s}{\bar{\gamma}}}\right) + \frac{\overbrace{2\cdot 14.0625}^{28.125} K\cdot e^{-K} K(K+1)s}{\bar{\gamma}(K+1)\cdot C^2}\left[2K_0\left(2\sqrt{\frac{(K+1)s}{\bar{\gamma}}}\right) + 2\sqrt{\frac{\bar{\gamma}}{(K+1)s}} K_1\left(2\sqrt{\frac{(K+1)s}{\bar{\gamma}}}\right)\right]}$$

$$\boxed{C=3.75} \qquad \boxed{\frac{28.125}{C^2} \doteq 2}$$

$$\boxed{MGF_{1/\bar{\gamma}} = 4Ke^{-K}\sqrt{\frac{s}{(K+1)\bar{\gamma}}} K_1\left(2\sqrt{\frac{(K+1)s}{\bar{\gamma}}}\right) + \frac{4Ke^{-K}K(K+1)s}{\bar{\gamma}(K+1)}\left[K_0\left(2\sqrt{\frac{(K+1)s}{\bar{\gamma}}}\right) + \sqrt{\frac{\bar{\gamma}}{(K+1)s}} K_1\left(2\sqrt{\frac{(K+1)s}{\bar{\gamma}}}\right)\right.}$$

☐ ELEKTROMAGNETIKA BOOKLET – KRIVOLINISKI INTEGRALI

⊙ KRIVOLINISKI INTEGRAL OD I RED (DOLŽ DADENA KRIVA)

$$\int_\gamma f(x,y,z)\,dl = \int_a^b f(x(t),y(t),z(t))\sqrt{x'^2(t)+y'^2(t)+z'^2(t)}\,dt$$

$\gamma : [a,b] \to R^3$ T.E. $t \to [x(t),y(t),z(t)]$

$t \in [a,b]$ $[x(t),y(t),z(t)] \in R^3$

- RAMNINSKA KRIVA: $dl = \sqrt{1+y'^2}\,dx$ $y=y(x)$

$$\int_\gamma f(x,y)\,dl = \int_a^b \sqrt{1+y'^2(x)}\,dx$$

- PARAMETARSKI OBLIK ZA RAMNINSKA KRIVA

$dl = \sqrt{x'^2(t)+y'^2(t)}\,dt$ $\int_\gamma f(x,y)\,dl = \int_a^b f(x,y)\sqrt{x'^2(t)+y'^2(t)}\,dt$

- KRIVOLINISKI PO KRUŽNICA:

$\int_C f(x,y)\,dl$ $dl = \sqrt{\rho^2+\rho'^2}\,d\varphi$ $\rho = a\cos\varphi$

KRUŽNICA VO POLAREN KOORD SISTEM

⊙ KRIVOLINISKI INTEGRAL OD II RED

$$\int_\gamma f(x,y,z)\,dx,\quad \int_\gamma f(x,y,z)\,dy,\quad \int_\gamma f(x,y,z)\,dz$$

$\gamma : [a,b] \to R^3$ $t \in [a,b] \to [x(t),y(t),z(t)] \in R^3$

$$\int_\gamma f(x,y,z)\,dx = \int_a^b f(x(t),y(t),z(t))\,x'(t)\,dt$$

– SOSTAVENI KI PO KOORDINATI

$$\int_\gamma P(x,y,z)\,dx + Q(x,y,z)\,dy + R(x,y,z)\,dz$$

Ⓑ PRESMETKA NA KI OD II RED SO TOTALEN DIFERENCIJAL

$P(x,y,z)\,dx + Q(x,y,z)\,dy + R(x,y,z)\,dz$

OVOJ IZRAZ E TOTALEN DIFERENCIJAL NA FUNKCIJATA $u(x,y,z)$ AKO I SAMO AKO:

$R\,Q\,Ⓡ$ $Q\,z\,Ⓟ$
$ⓧ\,y\,z$ $ⓩ\,x\,1$

$$\frac{\partial P}{\partial y} = \frac{\partial Q}{\underline{\partial z}} \quad \frac{\partial Q}{\partial z} = \frac{\partial R}{\partial x} \;\text{i}\; \frac{\partial R}{\underline{\partial x}} = \frac{\partial P}{\partial z}$$

✓

TOGAŠ: $dM = P(x,y,z)dx + Q(x,y,z)dy + R(x,y,z)dz$

$$M = \int_\gamma Pdx + Qdy + Rdz = \int_{x_0}^{x} P(x,y,z)dx + \int_{y_0}^{y} Q(x_0,y,z)dy + \int_{z_0}^{z} R(x_0,y_0,z)dz$$

$$\boxed{\int_{x_0,y_0,z_0}^{x_1,y_1,z_1} Pdx + Qdy + Rdz = \int_{x_0}^{x_1} P(x,y_0,z_0)dx + \int_{y_0}^{y_1} Q(x_1,y,z_0)dy + \int_{z_0}^{z_1} R(x_1,y_1,z)dz}$$

- PRESMETUVANJE NA KRIVOLINISKI INTEGRAL SO POMOŠ NA GRINOVA FORMULA

$$\boxed{\iint_D \left[\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right] dxdy = \oint_L Pdx + Qdy}$$

L - ZATVORENA KRIVA
D - ZATVORENA KRIVA I NEJZINA VNATREŠNOST

⑤ POVRŠINSKI INTEGRAL (INTEGRAL NA FUNKCIJA PO DADENA POVRŠINA)

$$\iint_\Gamma f(x,y,z)\,ds = \iint_D f(x(u,v), y(u,v), z(u,v))\sqrt{EG-F^2}\,dudv$$

Γ — POVRŠINA VO PROSTOROT

$$E = \frac{\partial x}{\partial u} + \frac{\partial y}{\partial u} + \frac{\partial z}{\partial u} \qquad G = \frac{\partial x}{\partial v} + \frac{\partial y}{\partial v} + \frac{\partial z}{\partial v}$$

$$F = \frac{\partial x}{\partial u}\frac{\partial x}{\partial v} + \frac{\partial y}{\partial u}\frac{\partial y}{\partial v} + \frac{\partial z}{\partial u}\frac{\partial z}{\partial v}$$

$$(u,v) \in D \to (x(u,v), y(u,v), z(u,v)) \in \Gamma$$

---

⑥ FORMULA GAUS - OSTROGRADSKI

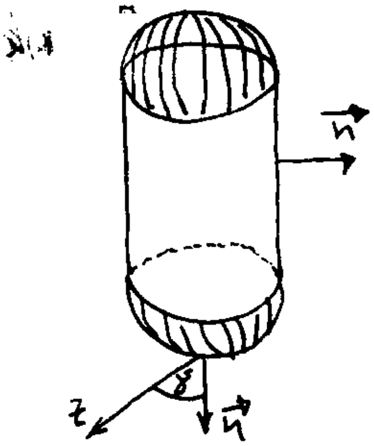
$(x,y,z) \to R^3$ T.E. $R^3 \to R^3$

$$\vec{F}(x,y,z) = P(x,y,z)\vec{i}_x + Q(x,y,z)\vec{k}_y + R(x,y,z)\vec{i}_z$$

PRESLIKUVANJE VO VEKTORSKO POLE!!!

- AKO POSTOI FUNKCIJA: $M(x,y,z)$ T.S.!

$$\left.\frac{\partial M}{\partial x} = P \qquad \frac{\partial M}{\partial y} = Q \qquad \frac{\partial M}{\partial z} = R \right\}$$ POTENCIJALNO VEKTORSKO POLE!!!



$$\iiint_V \underbrace{\left(\frac{\partial P}{\partial x}+\frac{\partial Q}{\partial y}+\frac{\partial R}{\partial z}\right)}_{\operatorname{div}\vec{F}} dx\,dy\,dz =$$

$$= \int_{\Gamma} \left[P\cos(x,\vec{n}) + Q\cos(x,\vec{n}) + R\cos(x,\vec{n})\right] dx\,dy$$

Γ – POVRŠINA SO KOJA E OBLOŽEN DOMENOT „V".

$$\boxed{\iiint_V \operatorname{div}\vec{F}\, dV = \iint_{\Gamma} \vec{F}\,\vec{n}\,ds \qquad d\vec{s} = \vec{n}\,ds}$$

FORMULA GAUSS-OSTROGRADSKI

VECTOR CALCULUS (Stewart)

- POVTORUVANJE ARC LENGTH

$C:\quad y=f(x) \qquad a\leq x\leq b$

$$L=\lim_{n\to\infty}\sum_{i=1}^{n}|P_{i-1}P_i|$$

$$P_{i-1}P_i = \sqrt{(x_i-x_{i-1})^2+(y-y_i)^2} = \sqrt{\Delta x^2+\Delta y^2}$$

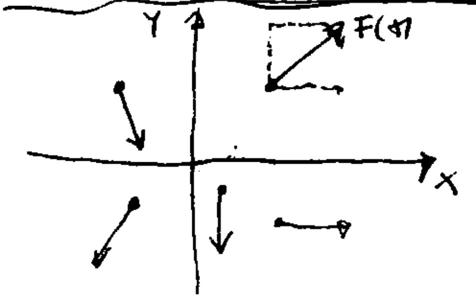
- MEAN VALUE THEOREM

$$f(x_i)-f(x_{i-1}) = f'(x_i^*)\,(x_i-x_{i-1})$$

$$\Delta y = f'(x_i^*)\,\Delta x$$

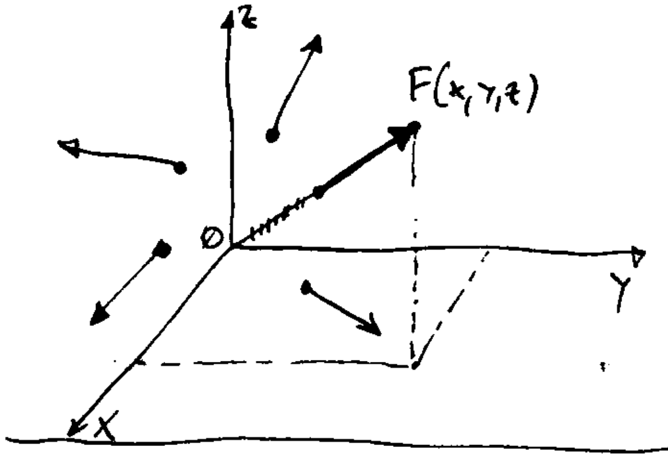
$$P_{i-1}P_i = \sqrt{\Delta x^2(1+f'^2(x_i))} = \Delta x\sqrt{1+f'^2(x)}$$

$$\boxed{L=\lim_{n\to\infty}\sum_{i=1}^{n}\sqrt{1+f'^2(x)}\,\Delta x = \int_a^b\sqrt{1+f'^2(x)}\,dx}$$



$$\boxed{\begin{aligned}F(x,y)&=P(x,y)\,i+Q(x,y)\,j=\langle P(x,y),Q(x,y)\rangle\\ F&=Pi+Qj=P\,\vec{i_x}+Q\,\vec{i_y}\end{aligned}}$$

P, Q – SCALAR FUNCTIONS OF 2 VARIABLES
i.e. SCALAR FIELDS



$F(x,y,z) = P(x,y,z)\,i + Q(x,y,z)\,j + R(x,y,z)\,k$

SECTION 13 (POVTORUVANJE ZA VEKTORSKI FUNKCII)

$r(t) = \langle f(t), g(t), h(t) \rangle = f(t)\,i + g(t)\,j + h(t)\,k$

- t ⇒ INDEPENDENT VARIABLE (e.g. time)

(13.1)

$\boxed{\text{EXAMPLE 1:}}$ $r(t) = \langle t^3, \ln(3-t), \sqrt{t} \rangle$

KOMPONENT FUNCTIONS ARE: $f(t) = t^3$; $g(t) = \ln(3-t)$; $h(t) = \sqrt{t}$

DOMAIN OF „r"

$3 - t > 0$ $\boxed{t < 3}$ $t \geq 0$ $0 \leq t < 3$ $\underline{t \in [0,3)}$

- $\lim_{t\to a} r(t) = L$ ⇒ DOLZINATA / NASOKATA NA VEKTOROT $r(t)$ SE PRIBLIZUVA NA DOLZINATA I NASOKATA NA VEKTOROT „L".

- IF $r(t) = \langle f(t), g(t), h(t) \rangle$

$$\lim_{t\to a} \langle \lim_{t\to a} f(t), \lim_{t\to a} g(t), \lim_{t\to a} h(t) \rangle$$

(13.1)

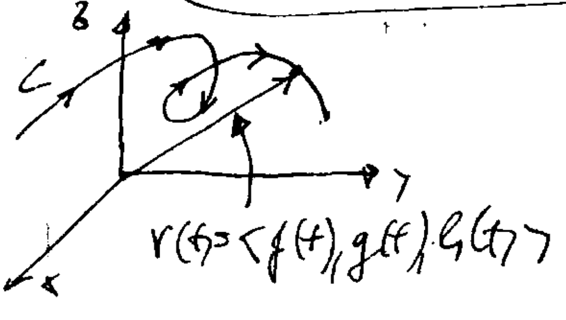
$\boxed{\text{EXAMPLE 2:}}$ $\lim_{t\to 0} r(t)$ $r(t) = \underbrace{(1+t^3)}_{f(t)}\,i + \underbrace{te^{-t}}_{g(t)}\,j + \underbrace{\frac{\sin t}{t}}_{h(t)}\,k$

$\lim_{t\to 0} f(t) = 1$ $\lim_{t\to 0} g(t) = 0$ $\lim_{t\to 0} \frac{\sin t}{t} = 1$

$$\boxed{\lim_{t\to 0} r(t) = i + k}$$

- VECTOR FUNCTION $r(t)$ IS CONTINUOUS AT „a" IF:

$$\boxed{\lim_{t\to a} r(t) = r(a)}$$



set „C" OF ALL POINTS (x,y,z)

$\underbrace{x = f(t) \quad y = g(t) \quad z = h(t)}$

REAL VALUE FUNCTION ON INTERVAL „t"

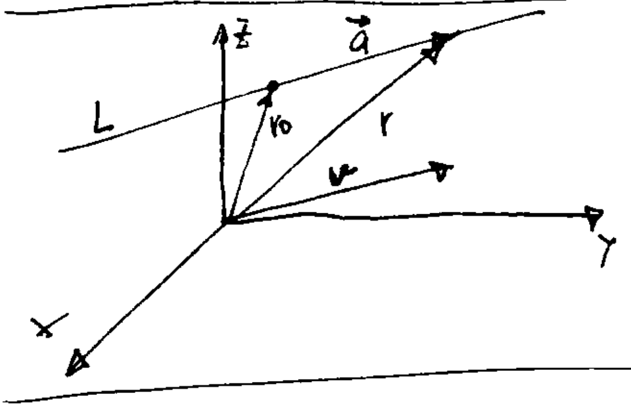
C - IS A SPACE CURVE; , $t \in I$

$r(t) = <f(t), g(t), h(t)>$ POSITION VECTOR OF POINT.

$P(f(t), g(t), h(t))$ ON „C"

$\boxed{41.13}$ $\boxed{EX.3}$ DESCRIBE CURVE DEFINED BY VECTOR FUNCTION

$$\left.\begin{matrix} r(t) = <1+t, 2+5t, -1+6t> \\ x = 1+t \quad y = 2+5t \quad z = -1+6t \end{matrix}\right\} ⊛$$



$r = r_0 + \vec{a}$

$a \parallel v \rightarrow \boxed{r = r_0 + t \cdot v}$

PARAMETER

VECTOR EQUATION OF LINE „L"

COMPONENT FORM OF VECTOR „v":

$v = <a, b, c> \qquad t \cdot v = <ta, tb, tc>$

$r = <x, y, z> \qquad r_0 = <x_0, y_0, z_0>$

$$\boxed{<x, y, z> = <x_0 + ta, y_0 + t \cdot b, z_0 + t \cdot c>}$$

$x = x_0 + ta \quad y = y_0 + tb \quad z = z_0 + tc$

⊛ → $v = <\overset{x}{1}, \overset{y}{5}, \overset{z}{6}>$

$\boxed{<x_0, y_0, z_0> = <1, 2, -1>}$ } LINE „L" PASSING THROUGH <1, 2, -1> AND PARALLEL TO: $v = <1, 5, 6>$

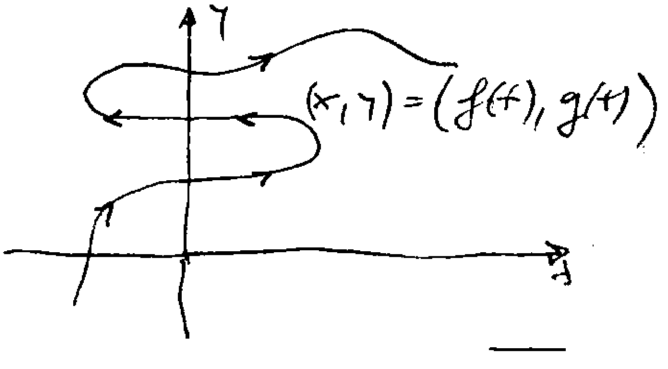
$r = r_0 + t \cdot v \qquad r_0 = <1, 2, -1> \qquad v = <1, 5, 6>$

- PLANE CURVES

$$\boxed{r(t) = <t^2 - 2t, t+1> = (t^2 - 2t)\,\hat{i} + (t+1)\,\hat{j}}$$

$i = <1, 0> \qquad j = <0, 1>$

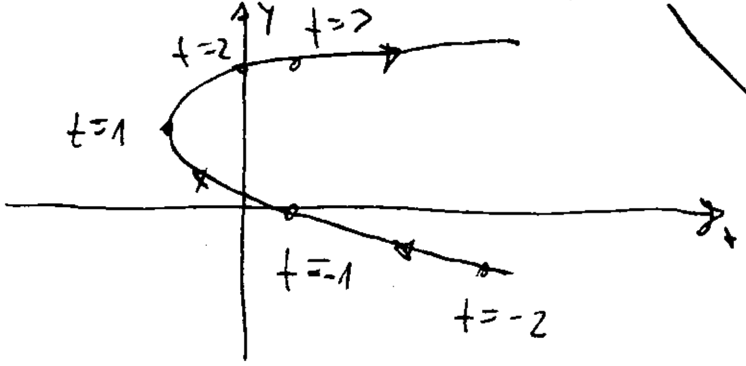
10.1 CURVES DEFINED BY PARAMETRIC EQUATIONS



$x = f(t) \quad y = g(t).$

EXAMPLE 1

$x = t^2 - 2t \qquad y = t + 1$



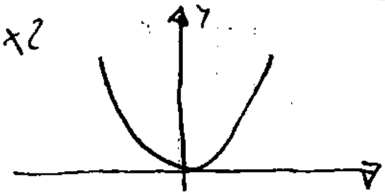
t	x	y
-2	8	-1
-1	3	0
-	-	-

$t = y - 1$

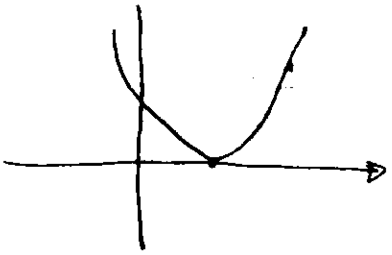
$$\boxed{\begin{aligned} x &= (y-1)^2 - 2(y-1) \\ x &= y^2 - 2y + 1 - 2y + 2 \\ x &= y^2 - 4y + 3 \end{aligned}}$$

MMW

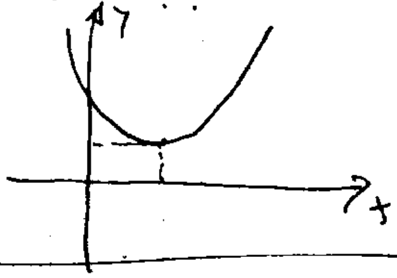
$y = x^2$



$y = (x-1)^2$



$y = (x-1)^2 + 1$



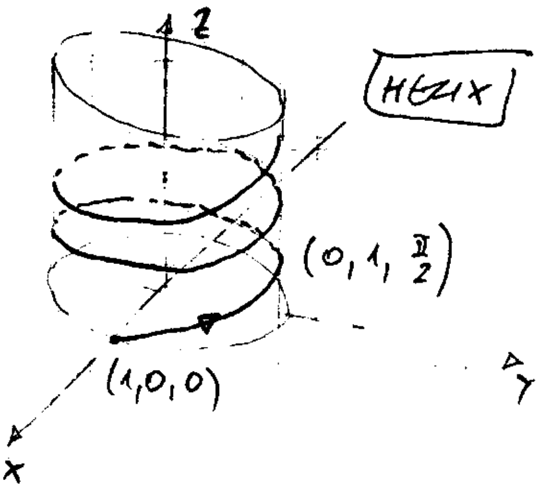
Ch.13 Ex.4 Sketch curve with vector equation

$$\boxed{r(t) = \cos t\,\hat{i} + \sin t\,\hat{j} + t \cdot k}$$

$r(t) = \langle f(t), g(t), h(t) \rangle$

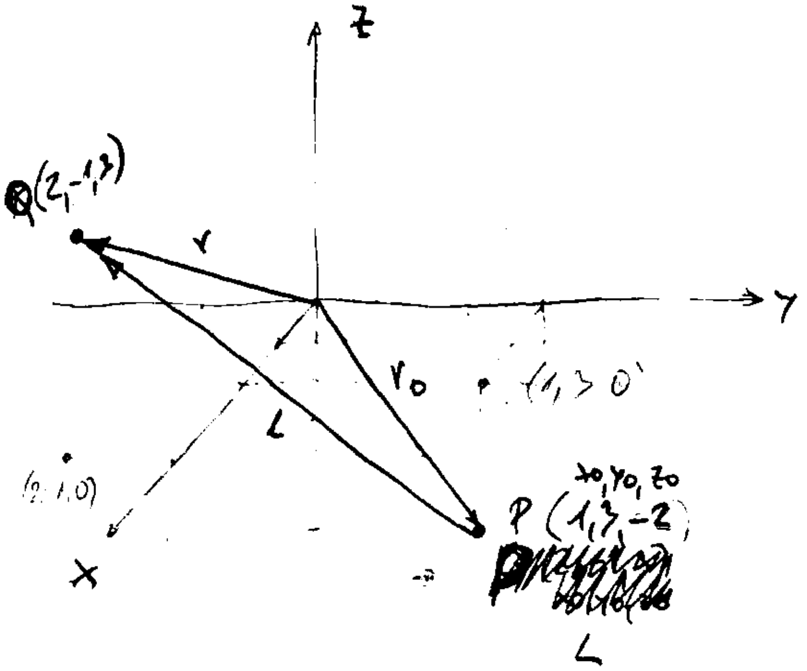
$x = f(t) = \cos t$
$y = g(t) = \sin t$
$z = h(t) = t$

$x^2 + y^2 = \cos^2 t + \sin^2(t) = 1$



Ex.5 FIND VECTOR EQUATION AND PARAMETRIC EQUATIONS FOR THE LINE SEGMENT THAT JOINTS $P(1, 3, -2)$ AND $Q(2, -1, 3)$

$x_1, y_1, z_1 \qquad x_2, y_2, z_2$



$r = r_0 + \ell = r_0 + t \cdot v$

$r_0 = \langle 1, 3, -2 \rangle$

$r = \langle x, y, z \rangle$

$\langle x, y, z \rangle = \langle 1, 3, -2 \rangle + t \langle a, b, c \rangle$

$x = 1 + a \cdot t$
$y = 3 + b \cdot t$
$z = -2 + c \cdot t$

$y - y_1 = \frac{y_2 - x_1}{x_2 - x_1}(x - x_1)$

$\boxed{\text{URAVNA } 2}$

- MY APPROACH

$PQ = \langle 2-1, -1-3, 3+2 \rangle = \langle 1, -4, 5 \rangle$

$\boxed{x = 1 + t \;;\; y = 3 - 4t \;;\; z = -2 + 5t}$

$\boxed{\langle x, y, z \rangle = \langle 1, 3, -2 \rangle + t \langle 1, -4, 5 \rangle = r_0 + t \cdot v}$

- STEWART APPROACH

$\boxed{\text{LINE SEGMENT}}$ THAT JOINTS THE TIP OF VECTOR r_0 TO THE TIP OF VECTOR r_1

$\boxed{r(t) = (1-t) r_0 + t \cdot r_1}$ $t = 0..1$ $\boxed{0 \le t \le 1}$

$r_0 = \langle 1, 3, -2 \rangle$ $r_1 = \langle 2, -1, 3 \rangle$

$r(t) = (1-t) \langle 1, 3, -2 \rangle + t \langle 2, -1, 3 \rangle =$

$= \underline{t \langle 2-1, -1-3, 3+2 \rangle} + \langle 1, 3, -2 \rangle = \underline{\underline{t \langle 1, -4, 5 \rangle + \langle 1, 3, -2 \rangle}}$

$\boxed{12.5}$ $\boxed{\text{ex. } 1}$ $r_0 = \langle 5, 1, 3 \rangle$ paralel to $i + 4j - 2k$

$r = r_0 + t \cdot v = \langle 5, 1, 3 \rangle + t \langle 1, 4, -2 \rangle$

$\boxed{x = 5 + t \quad y = 1 + 4t \quad z = 3 - 2t}$

ALTERN!

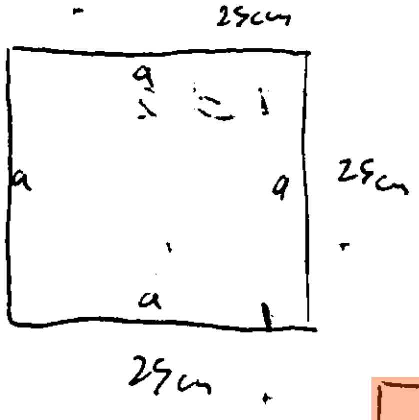
$r = 5i + j + 3k + t(i + 4j - 2k) = (5+t)i + (1+4t)j + (3-2t)k$

$t = 1$ $x = 6$ $y = 5$ $z = 1$ $\langle 6, 5, 1 \rangle$
$t = -1$ $x = 4$ $y = -3$ $z = 5$ $\langle 4, -3, 5 \rangle$

$r_0 = \langle 6, 5, 1 \rangle$

$r = \langle 6, 5, 1 \rangle + t \langle 1, 4, -2 \rangle = \underline{\underline{(6+t)i + (5+4t)j + (1-2t)k}}$

$x = 6 + t$ $y = 5 + 4t$ $z = 1 - 2t$



1 KUTIJA 3a 25cm

$3a \cdot 0{,}25 = 1$ $0{,}75a = 1$

10 STRANA : $1 / 3 \cdot 0{,}25$

$3a \cdot 0{,}10 = 0.3a$

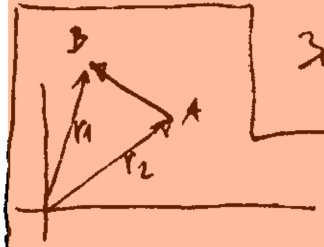
OSTATOK 0.45a

$3a \cdot 0{,}15 = 0{,}45a$

$3 \cdot 25 = 75$

$3 \cdot 10 = 30$

$75 - 30 = 45$



$\overrightarrow{AB} + r_2 = r_1$

$\overrightarrow{AB} = r_1 - r_2$

$r_0 = \langle 5, 1, 3 \rangle$ $v = \langle 2, 8, -4 \rangle = 2i + 8j - 4k = \langle a, b, c \rangle$

$r = r_0 + t \cdot v = 5i + j + 3k + t(2i + 8j - 4k) =$

$= (5 + 2t)i + (1 + 8t)j + (3 - 4t)k$

$x = 5 + 2t$ $y = 1 + 8t$ $z = 3 - 4t$

$x = x_0 + at$ $y = y_0 + bt$ $z = z_0 + ct$

$\frac{x - x_0}{a} = t$ $\frac{y - y_0}{b} = t$ $\frac{z - z_0}{c} = t$

$$\boxed{\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}}$$

Симетрична форма на равенка на права!

$a = 0 \Rightarrow \boxed{x = x_0 \quad \frac{y - y_0}{b} = \frac{z - z_0}{c}} \Rightarrow$ L LEZI VO VERTIKALNA RAMNINA

$c = 0 \Rightarrow \frac{x - x_0}{a} = \frac{y - y_0}{b}$ $\boxed{y - y_0 = \frac{b}{a}(x - x_0)}$

$\boxed{12.5}$ $\boxed{\text{Exp. 2}}$ $A = (\overset{x_0}{2}, \overset{y_0}{4}, \overset{z_0}{-3})$ $B(\overset{x_1}{3}, \overset{y_1}{-1}, \overset{z_1}{1})$

$r = r_0 + t \cdot v$ $AB = \langle 3-2, \underbrace{-1-4}_{(b)}, \underbrace{1+3}_{(c)} \rangle = \langle 1, -5, 4 \rangle$

$r = \langle 2, 4, -3 \rangle + t\langle 1, -5, 4 \rangle = (2 + t)i + (4 - 5t)j + (-3 + 4t)k$

$x = 2 + t$ $y = 4 - 5t$ $z = -3 + 4t$ $\boxed{\frac{x - 2}{1} = \frac{-y + 4}{5} = \frac{z + 3}{4}}$

XY PLANE $\Rightarrow z = 0 \Rightarrow 4t = 3$ $\boxed{t = \frac{3}{4}}$

$x = 2 + \frac{3}{4} = \frac{11}{4}$ $y = 4 - \frac{5 \cdot 3}{4} = \frac{16 - 15}{4} = \frac{1}{4}$

MOZE I OD TUKA

$\boxed{\frac{x - 2}{1} = -\frac{y + 4}{5} = \frac{z}{4}}$

↓ CONTINUE ON NOTEBOOK 5

09 MESEC SMETKI

KOZLE	1249,50
NEREZI	102,00 MKD
OHRID	695,00 MKD

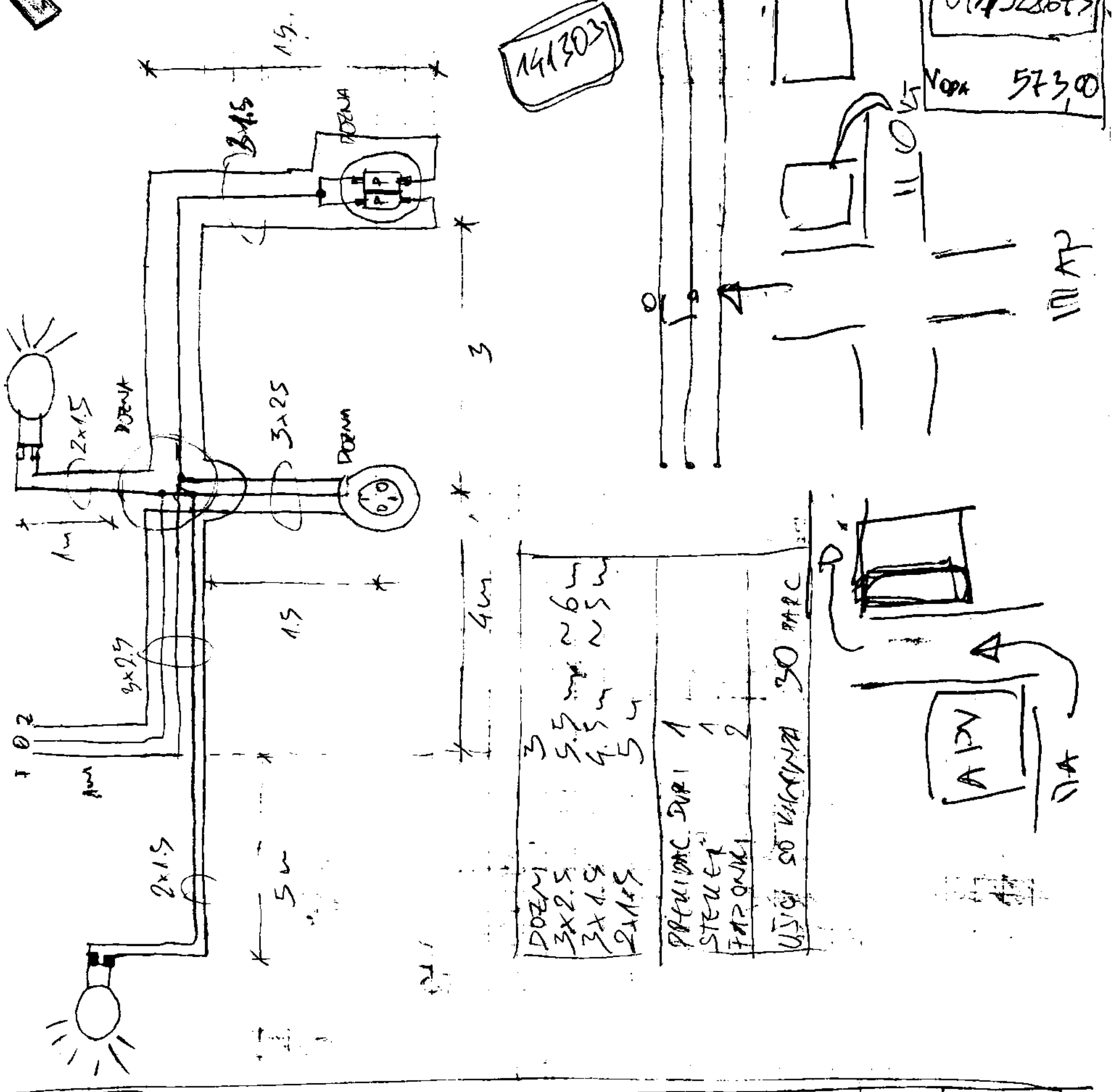
10 MESEC SMETKI
EVN

KOZLE	2077,0
NEREZI	8325,00
OHRID	1133,90

IZLAZ
07~~1~~928673

VODA 573,00

141303



OHRID

16980 - PATOT MEDU GJURCITE I TASEVCI
16981 - KATASTARSKA PARCELA NA TETKA VERE
IZVOD OD KATASTARSKI PLAN 79 I 1995
NAJPOCNA SECISTE AKO VAKA

NEREZI TROŠOCI :

PES: 2x 1400,00 = 2800
CEM: 6x 350,00 = 2100
TRAN: 1x 1000,00 = 1000

5900

CEM: 10x 350,00 = 3500
RAKA: 4 x 1200,00 = 4800

CEM	3x 350,00	1050
PES	2x1300,00	2600
TR		1000

4700

TOTAL 18900

VRATA		5000
CIGLI	65x20	1300
USJEMAL		1000

~~2~~6000,00 MKD SUBTOTAL

RAKA (2x1200)	2400,00
CEMENT (6x350)	2100,00
ZICA ZA OGRADA	1125,00

[redacted]625,00

2x STIROPOR
2x LEPAK (CEM+DEZ) ⑦
~~MREZA~~ 15m2 + DIPZI

3[redacted],00

		KOLIČINA	CENA	TOTAL
~~[illegible]~~	STIROPOR	50	12 540	6270
LEPAK	DEZ	10	327,75	3278
ADRIB		8	441,75	3534
RAKA				6000

(381,64 PO m² = 6€/m²) [redacted]

TOTAL - TOTAL 59723

VRATA	+ 5000
PLASTIČNA MASA ~~ZA~~ COKLE (x2)	+ 7200
CEMENT (x2)	700

VKUPNO 72623

STIROPOR	16	125	2031
⊗ POFIX LEPAK	5	327	1638
ADRIB	5	441	2208

[redacted]
5278

⊗ + } 30 m²

LEPAK	7	30/7 ≈ 5 [illegible]
ADRIB	5	30/5 = 6 [illegible]

LEPAK	1 VREĆA	5 KVADRAT
ADRIB	1 VREĆA	6 KVADRATI

VLATKO (070 379 379)

+1 732 981 0060

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BILJANA KRTOLICA – ПРИРОДА

VIOLETA MITOVSKA

GLUMA

- INFORMATIKA, TVOREŠTVO (ĆE IM KAŽE NA DECATA)
- RABOTNI LISTOVI ZA MATEMATIKA MOŽAT DA SE KUPAT!!?
- OSIGURUVANJE 100,00 MKD

  DA SE IZJAVAT KOJ SAKA PA POSLE ĆE DEFINIRA KAKO ĆE SE PLATI
- OBEZBEDUVANJE 2x 250MKD

  ĆE DOBIJAT UPLATNICI
- RAZVIGOR 40,00 MKD

- SLEDNIOT MESEC DA SE DONESAT PARITE ZA OSIGURUVANJE (100,00 MKD)

08.09

- EVN [illegible]	~~360,00~~	102,00	22610 SOSTOJBA
- EVN OHRID	1764,50		
- EVN KOZLE	360,00		

2.099	NOVECENTO
3.017	APOCALYPSE NOW
3.002	CHINATOWN
2.194	Good, Bad & Ugly

075·212·787 VLADO

IRENA

06.2009	smetki
05.2009	1142,50
06.2009	1075,50
07.2009	1082,50

VODA 06. 429,00
07. 729,00

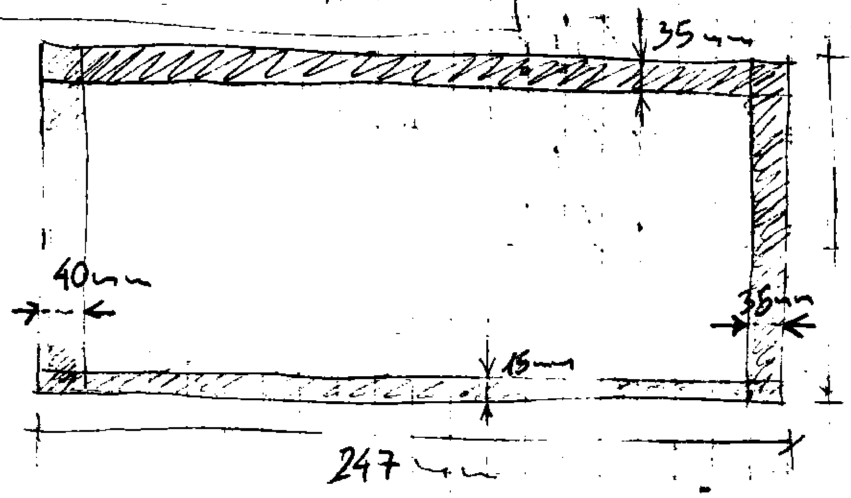
DV2+K

x 9.4 3 326 KBS ~92 mm

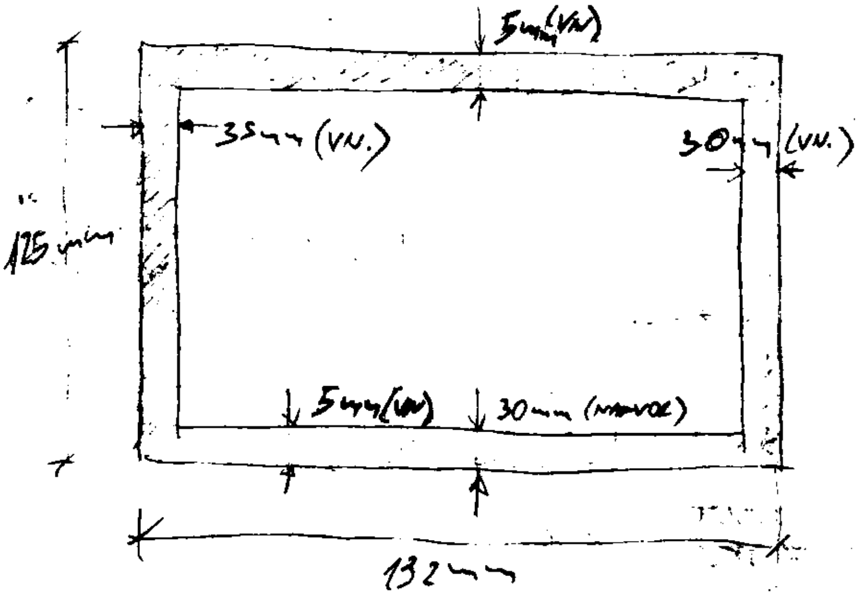
ŠIROKI	TESNI
533	247
434	264
382	108
1369 mm	619 mm

5.07 x 2.29 x
390 290
1950 MKD 667 MKD 2620 MKD

• SOBA DOCO

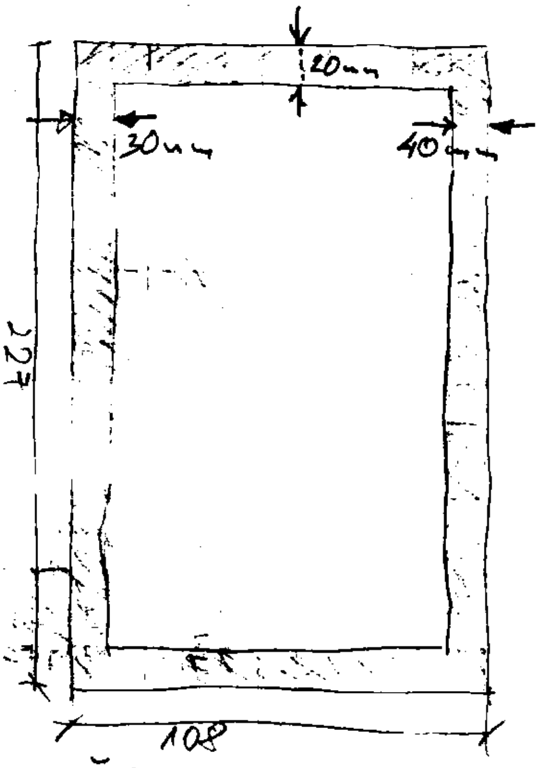


ŠIROKI!
2x143+247
= 533 mm
TESNI!
1 x 247 = 247



ŠIROKI: 2x125 = 250 mm + 132 = 382 mm
TESNI: 2x132 = 264 mm

0800 ÷ 0807



ŠIROKI:
2 x 227 = 454 mm
TESNI
1 x 108 = 108 mm

176