$Z = X + Y \qquad p_Z(z) = p_X(z) * p_Y(z)$

$$p_W(w) = \left. \frac{p_Z(z)}{\frac{dw}{dz}} \right|_{z = w \cdot n} \qquad W = \frac{Z}{n} \qquad w = \frac{z}{n} \qquad \frac{dw}{dz} = \frac{1}{n}$$

(??)

$$p_W(w) = n \cdot p_Z(w \cdot n)$$

$$Z = \frac{X+Y}{n} = \frac{W}{n} \qquad z = \frac{w}{n} \qquad \frac{dz}{dw} = \frac{1}{n} \qquad p_W = p_X(w) * p_Y(w)$$

$$p_Z(z) = \left. \frac{p_W(w)}{\frac{\partial z}{\partial w}} \right|_{w = z \cdot n} = n \cdot p_W(nz) = n \cdot p_X(nz) * p_Y(nz) \quad (*)$$

$$Z = \frac{X+Y}{n} = \frac{X}{n} + \frac{Y}{n}$$

ISTO SE DOBIVA

$$Z = \frac{X}{n} + \frac{Y}{n} \xrightarrow{\text{inv. f}} Y = Z \cdot n - X'$$

$$X' = X \xrightarrow{\text{inv. f}} X = X'$$

$$j = \begin{vmatrix} \frac{\partial z}{\partial x} & \frac{\partial z}{\partial y} \\ \frac{\partial x'}{\partial x} & \frac{\partial x'}{\partial y} \end{vmatrix} = \begin{vmatrix} \frac{1}{n} & \frac{1}{n} \\ 1 & 0 \end{vmatrix} = -\frac{1}{n} \qquad |j| = \frac{1}{n}$$

$$p_{ZX'}(z, x') = \left. \frac{p_{XY}(x,y)}{|j|} \right|_{\substack{y = zn - x \\ x = x'}} = n\, p_{XY}(x', nz - x)$$

$$p_{ZX'}(z, x') \qquad p_Z(z) = \int_0^\infty p_{ZX'}(z, x')\, dx'$$

$$p_Z(z) = \int_0^\infty n\, p_{XY}(x', nz - x')\, dx' = n \int_0^\infty p_{XY}(x, n \cdot z - x)\, dx =$$

$$= n \int_0^\infty p_X(x) \cdot p_Y(nz - x)\, dx = n\, p_X(nz) * p_Y(nz)$$

$$M_Z(s) = \int_0^\infty p_Z(z) \cdot e^{+sz}\, dz = \int_0^\infty n\, p_X(nz) * p_Y(nz)\, e^{sz}\, dz =$$

OVA NE 6 FUNKCIONALNA TRANSFORMACIJA NA RV ZATOA NE TREBA :-)

$$n \cdot z = v \qquad dz = \frac{dv}{n}$$

$$= \int_0^\infty n \cdot \frac{p_X(v)}{1/n} \circledast \frac{p_Y(v)}{1/n} \cdot e^{\frac{s}{n} \cdot v} \frac{dv}{n} = n^2 M_X\left(\frac{s}{n}\right) \cdot M_Y\left(\frac{s}{n}\right)$$

$$\rightarrow M_Z(s) = M_X\left(\frac{s}{n}\right) \cdot M_Y\left(\frac{s}{n}\right)$$

④ PP.1 $\Rightarrow$ $p_Z(z) = n\, p_X(nz) * p_Y(nz) = n \int_0^\infty p_X(x) \cdot p_Y(nz-x)\,dx$

$$M(s) = \int_0^\infty p_Z(z) e^{-sz} dz = n \int_0^\infty p_X(nz) * p_Y(nz) e^{-sz} dz$$

$v = nz$, $dz = \frac{1}{n} dv$ } OVA NE E FUNKCIONALNA TRANSFORMACIJA NA RV TUKU ZAMENA VO INTEGRALOT

$$M(s) = n \int_0^\infty p_X(v) * p_Y(v) e^{-\frac{s}{n} \cdot v} \frac{dv}{n} = M_X\left(\frac{s}{n}\right) \cdot M_Y\left(\frac{s}{n}\right)$$

MMV

- BY USING OF INITIAL VALUE THEOREM

$$\frac{\partial^n}{\partial y_1^n} f_{Y_1}(0) = \lim_{s \to \infty} s^{n+1} M_{Y_1}(s)$$

$$M(s) = \int_0^\infty f(x) e^{xs} \cdot dx \qquad M(-s) = \int_0^\infty f(x) e^{-xs} dx$$

$$\frac{dM(s)}{ds} = \int_0^\infty x f(x) e^{xs} dx \qquad \frac{dM(0)}{ds} = \bar{X} = E(X)$$

- LAPLACE TRANSFORM (INITIAL VALUE THEOREM)

$\mathcal{L}\{f(x)\} = F(s)$ $\quad f(x)$ - n-TIME DIFERENCIABLE FUNCTION [A. JEFFREY, HANDBOOK]

$$f^{(r)}(0) = \lim_{s \to \infty} \left\{ s^{r+1} F(s) - s^r f(0) - s^{r-1} f'(0) - \dots s f^{(r-1)}(0) \right\}$$

$r = 0, 1, 2, \dots$ [*$]

$$\mathcal{L}\{f'(x)\} = s \cdot F(s)$$ — H. HSU Diferenciation property $F(s) = \int_0^\infty f(x) e^{-st} dt$

H. HSU Eq. 3.35 MMV

- INITIAL VALUE THEOREM

$$x(0^+) = \lim_{s \to \infty} s X_L(s)$$

UNILATERAL LAPLACE TRANSFORM

$$X_L(s) = \int_{0^-}^\infty f(t) e^{-st} dt$$

- Differentiation in time (UNILATERAL)

$$\frac{dx(t)}{dt} \leftrightarrow s X_L(s) - x(0^-)$$

$\mathcal{L}^{-1}[s X_L(s)]$

$$x(t) = \frac{1}{2\pi} \int_{\sigma - j\infty}^{\sigma + j\infty} X(s) e^{st} ds \qquad \frac{dx(t)}{dt} = \frac{1}{2\pi} \int_{\sigma - j\infty}^{\sigma + j\infty} s X_L(s) e^{st} dt$$

- ZA UNILATERALNA DIFERENCIRANJETO VO VREME E:

$$\frac{dx(t)}{dt} \leftrightarrow sX_L(s) - x(0^-)$$

$$\frac{d^2x(t)}{dt^2} \leftrightarrow s^2X_L(s) - sx(0^-) - x'(0^-)$$

$$\frac{d^nx(t)}{dt^n} \leftrightarrow s^nX_L(s) - s^{n-1}x(0^-) - s^{n-2}x'(0^-) - \dots x^{(n-1)}(0^-)$$

$$x^{(r)}(0^-) = \left.\frac{d^rx}{dt^r}\right|_{t=0^-}$$

• Proof of initial value theorem:

$$sX_L(s) - x(0^-) = \int_{0^-}^{\infty} \frac{dx(t)}{dt} \cdot e^{-st}\,dt = \int_{0^-}^{0^+} \frac{dx(t)}{dt}\,dt + \int_{0^+}^{\infty} \frac{dx(t)}{dt} e^{-st}\,dt$$

$$= x(0^+) - x(0^-) + \int_{0}^{\infty} \frac{dx(t)}{dt} e^{-st}\,dt$$

$$sX_L(s) = x(0^+) + \int_{0}^{\infty} \frac{dx(t)}{dt} e^{-st}\,dt$$

$$\lim_{s\to\infty} sX_L(s) = x(0^+) + \int_0^{\infty} \frac{dx(t)}{dt} \lim_{s\to\infty} e^{-st}\,dt = x(0^+)$$

(→ 0) DOKAZANO!!!

$$x(0^+) = \lim_{s\to\infty} sX_L(s)$$

⊛$ ⇒

$$f^{(r)}(0) = \lim_{s\to\infty} s^{r+1}F(s)$$

→ INITIAL VALUE THEOREM PER POCETNI USLOVI.

---

$$Y_1 = \frac{x_0 + \sum_1^n x_i}{n+1}$$

$$\frac{\partial^n}{\partial Y_1} f_{Y_1}(0) = \lim_{s\to\infty} s^{n+1} M_{Y_1}(s) = ⊛$$

$$M_{Y_1}(-s) = \int_0^{\infty} f_{Y_1}(y_1) e^{-sy_1}\,dy_1$$

$$M_{Y_1}(-s) = \mathcal{L}[f_{Y_1}(y_1)]$$

USE LIMITS PROPERTIES

$$⊛ = \lim_{s\to\infty} s^{n+1} \prod_{i=0}^{n} M_{x_i}\left(\frac{s}{n+1}\right) = (n+1)^{n+1} \lim_{n\to\infty} \prod_{i=0}^{n} \underbrace{\frac{s}{(n+1)} M_{x_i}\left(\frac{s}{n+1}\right)}_{f_{x_i}(0)}$$

$$= (n+1)^{n+1} \prod_{i=0}^{n} \underbrace{\lim_{s\to\infty} \frac{s}{n+1} M_{x_i}\left(\frac{s}{n+1}\right)}_{f_{x_i}(0)} = (n+1)^{n+1} \prod_{i=0}^{n} f_{x_i}(0)$$

$V = \max x_i$ | $V_1 = \min(x_1, x_2, \dots x_n) \dots V_m = \max(x_1, x_2, \dots x_m)$

$x_i : x_1, x_2, x_3, \dots, x_m$ [redacted]

$V_1 < V_2 < V_3 < V_4 \dots < V_m < x$ ⟸ PODLEDUVANI

$$P(V < x) = P(V_m < x) = P(x_1 < x) \cdot P(x_2 < x) \cdots P(x_m < x)$$

$$F_V(x) = \prod_{i=1}^{m} F_{x_i}(x)$$

070227111

3110406 | AHA 3MAX

$$\boxed{\xi_2 = \frac{x_0 + V}{2}}$$

$V = \max x_i$

$$F_V(\sigma) = \prod_{i=1}^{m} F_{x_i}(\sigma) \qquad \frac{\partial^{m-1} f_\sigma}{\partial \sigma^{m-1}}(0) = \frac{\partial^m F_V(0)}{\partial \sigma^m} =$$

$$= \frac{\partial^m}{\partial \sigma}\left(F_V(0)\right) = \frac{\partial^m}{\partial \sigma^m}\left[\prod_{i=1}^{m} F_{x_i}(0)\right] = \frac{\partial^m}{\partial \sigma^m} F^m_{x_i}(0)$$

$$\frac{\partial^2}{\partial \sigma^2}\left[F^2(\sigma)\right] = \frac{\partial}{\partial \sigma}\left[\frac{\partial}{\partial \sigma} F^2(\sigma)\right] = \frac{\partial}{\partial \sigma}\left[2F(\sigma) \cdot F'(\sigma)\right] =$$

$$= 2 \cdot 1 \cdot F'(\sigma) \cdot F'(\sigma) + 2 \cdot F(\sigma) \cdot F''(\sigma) = 2!\, F'^2(\sigma) + 2F(\sigma) \cdot F''(\sigma)$$

$$\frac{\partial^3}{\partial \sigma^3}\left[f^3(\sigma)\right] = 6\left[f'(x)\right]^3 + \underbrace{18 f(x)\left(\frac{d}{dx} f(x)\right) + 3 f(x)^2 \left(\frac{d^3}{dx^3} f(x)\right)}_{o(0)}$$

$$\frac{\partial^{m-1} f_\sigma}{\partial \sigma^{m-1}}(0) = m! \prod_{i=1}^{m} \frac{\partial F_{x_i}}{\partial \sigma}(0) = m! \prod_{i=1}^{m} f_{x_i}(0) + o(0)$$

$o(\sigma)$ – ALL THE OTHERS TERMS.

$o(\sigma)$ – {EACH TERM CONTAINS AT LEAST ONE $F(\sigma)$

$F(0) = 0 \Rightarrow o(0) = 0 \Rightarrow$

$$\boxed{\frac{\partial^{m-1} f_\sigma}{\partial \sigma^{m-1}}(0) = m! \prod_{i=1}^{m} f_{x_i}(0) = \frac{\partial^m}{\partial \sigma^m} F_V(0)}$$

MMV

$$\frac{\partial^n f_{Y_2}(0)}{\partial Y_2^n} = \lim_{s\to\infty} s^{n+1} M_{Y_2}(s) = \lim_{s\to\infty} s^{n+1} M_{x_0}\left(\frac{s}{2}\right) M_V\left(\frac{s}{2}\right)$$

$$= \Big[\underbrace{\lim_{s\to\infty}\left(\frac{s}{2}\right) M_{x_0}\left(\frac{s}{2}\right)}_{f_{x_0}(0)}\Big]\cdot\Big[\underbrace{\lim_{s\to\infty}\left(\frac{s}{2}\right)^n M_V\left(\frac{s}{2}\right)}_{\frac{\partial f_V}{\partial V^{n-1}}(0)}\Big]\cdot 2^{n+1}$$

$$\boxed{\frac{\partial^n f_{Y_2}(0)}{\partial Y_2^n} = 2^{n+1} f_{x_0}(0)\cdot\frac{\partial f_V}{\partial V^{n-1}}}$$

$$\frac{\partial^n f_{Y_2}(0)}{\partial Y_2^n} = 2^{n+1} f_{x_0}(0)\cdot n!\prod_{i=1}^{n} f_{x_i}(0) = 2^{n+1} n! \prod_{i=0}^{n} f_{x_i}(0)$$

$$\boxed{\frac{\partial^n f_{Y_2}(0)}{\partial Y_2^n} = 2^{n+1} n! \prod_{i=0}^{n} f_{x_i}(0)} \quad \text{S}$$

$$\boxed{\frac{\partial^n}{\partial Y_1} f_{Y_1}(0) = (n+1)^{n+1}\prod_{i=0}^{n} f_{x_i}(0)} \quad \text{AP}$$

$$Pe \rightarrow \frac{\prod_{i=1}^{t+1}(2i-1)}{2(t+1)c^{t+1}t!}\,\frac{\partial^t f_{\gamma_r}}{\partial \gamma_r^t}(0)$$

$$\boxed{\frac{Pe^S}{Pe^{AP}} = \frac{2^{n+1} n!}{(n+1)^{n+1}}} \qquad \boxed{\frac{Pe^S}{Pe^{AP}} = \left(\frac{2}{n+1}\right)^{n+1} n!}$$

- Izvestaj od dizajnot [illegible] na sajt.
- Asymptotic expansion for BesselK function

$$\boxed{K_\nu(z) \sim \sqrt{\frac{\pi}{2z}}\, e^{-z}\left\{1 + \frac{\mu-1}{8z} + \frac{(\mu-1)(\mu-9)}{2!(8z)^2} + \frac{(\mu-1)(\mu-9)(\mu-25)}{3!(8z)^3} + \dots\right\}}$$

→ when $|z|$ is large and $\mu = 4\nu^2$ $\qquad z = \frac{\gamma}{c\rho}\sqrt{\frac{2}{p_1 p_2}}$

- Lee 2x1x2 layer CDF (N 9.42)

$$F_\gamma^{MS}(\gamma) = 1 - \frac{\gamma^3 e^{-\frac{a\gamma}{c\rho}}}{p_1p_2c^3\rho^3}\left[\left(a+\frac{c\rho}{\gamma}\right)K_2\left(\frac{\gamma}{c\rho}\sqrt{\frac{2}{p_1p_2}}\right) + \sqrt{\frac{2}{p_1p_2}}\,K_1\left(\frac{\gamma}{c\rho}\sqrt{\frac{2}{p_1p_2}}\right)\right] =$$

$$= 1 - \frac{\gamma^3}{p_1p_2c^3\rho^3}\left[\left\{\left(a+\frac{c\rho}{\gamma}\right)\left(1+\frac{3}{8z}\right) + \sqrt{\frac{2}{p_1p_2}}\left(1+\frac{15}{8z}\right)\right\}\right]\sqrt{\frac{\pi}{2z}}\,e^{-z}$$

$$F_\gamma^{NS} = 1 - \frac{e^{\dots}\,\gamma^3 \sqrt{\frac{\pi}{2\frac{\gamma}{c\varrho}\sqrt{\frac{2}{\beta_1\beta_2}}}}}{\beta_1\beta_2 c^3\varrho^3}\left[\left(a+\frac{c\varrho}{\gamma}\right)\left(1+\frac{3}{8\cdot\frac{\gamma}{c\varrho}\sqrt{\frac{2}{\beta_1\beta_2}}}\right)+\sqrt{\frac{2}{\beta_1\beta_2}}\left(1+\frac{15}{8\frac{\gamma}{c\varrho}\sqrt{\frac{2}{\beta_1\beta_2}}}\right)\right]$$

$e^{-\frac{2\gamma}{c\varrho}}$

TREBA OBRATNO!!!

$$= 1 - \frac{e^{-\frac{\gamma}{c\varrho}\sqrt{2}}\,\gamma^3\sqrt{\frac{\pi}{\frac{2\gamma\sqrt{2}}{c\varrho}}}}{c^3\varrho^3}\left[\left(a+\frac{c\varrho}{\gamma}\right)\left(1+\frac{3c\varrho}{8\gamma\sqrt{2}}\right)+\sqrt{2}\left(1+\frac{15c\varrho}{8\gamma\sqrt{2}}\right)\right]$$

$$= \left| a = \frac{\beta_1+2\beta_2}{2\beta_1\beta_2} = \frac{3}{2}\right| = 1 - \frac{e^{-\frac{\gamma}{c\varrho}\sqrt{2}}\gamma^3}{c^3\varrho^3}\sqrt{\frac{\pi c\varrho}{2\gamma\sqrt{2}}}\left[\left(\frac{3}{2}+\frac{c\varrho}{\gamma}\right)\left(1+\frac{3c\varrho}{8\gamma\sqrt{2}}\right)+\sqrt{2}\left(1+\frac{15c\varrho}{8\gamma\sqrt{2}}\right)\right]$$

$$F_\gamma^{NS} = 1 - \frac{e^{-\frac{\gamma}{c\varrho}\sqrt{2}}\gamma^2\sqrt{\gamma}}{c^3\varrho^3}\sqrt{\frac{\pi c\varrho}{2\sqrt{2}}}\left[\frac{3}{2}+\frac{c\varrho}{\gamma}+\frac{9c\varrho}{16\gamma\sqrt{2}}+\frac{3c^2\varrho^2}{8\gamma^2\sqrt{2}}+\sqrt{2}+\frac{15c\varrho}{8\gamma}\right]$$

$$= 1 - \frac{e^{-\frac{\gamma}{c\varrho}\sqrt{2}}\gamma^2\sqrt{\gamma}}{c^3\varrho^3}\sqrt{\frac{\pi c\varrho}{2\sqrt{2}}}\left[\left(\frac{3}{2}+\sqrt{2}\right)+\frac{1}{\gamma}\left(c\varrho+\frac{9c\varrho}{16\sqrt{2}}+\frac{15c\varrho}{8}\right)+\frac{1}{\gamma^2}\frac{3c^2\varrho^2}{8\sqrt{2}}\right]$$

$$= 1 - \frac{1}{c^3\varrho^3}\sqrt{\frac{\pi c\varrho}{2\sqrt{2}}}\left[\left(\frac{3}{2}+\sqrt{2}\right)\gamma^2\sqrt{\gamma}\,e^{-\frac{\gamma}{c\varrho}\sqrt{2}}+\left(c\varrho+\frac{9c\varrho}{16\sqrt{2}}+\frac{15c\varrho}{8}\right)\gamma\sqrt{\gamma}\,e^{-\frac{\gamma}{c\varrho}\sqrt{2}}+\frac{3c^2\varrho^2}{8\sqrt{2}}\sqrt{\gamma}\,e^{-\frac{\gamma}{c\varrho}\sqrt{2}}\right]$$

$$F_\gamma^{NS} = 1 - F_{\gamma_1} - F_{\gamma_2} - F_{\gamma_3}$$

$$P_e|\gamma = Q\left(\sqrt{d\cdot\gamma}\right)$$

$$P_e^S = \int_0^\infty F_\gamma^{NS}\left(\frac{x^2}{d}\right)\cdot\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}dx$$

$$F_\gamma^{NS} = 1 - A\cdot\gamma^2\sqrt{\gamma}\,e^{-\frac{\gamma}{c\varrho}\sqrt{2}} - B\gamma\sqrt{\gamma}\,e^{-\frac{\gamma}{c\varrho}\sqrt{2}} - C\sqrt{\gamma}\,e^{-\frac{\gamma}{c\varrho}\sqrt{2}} \Rightarrow$$

$$= 1 - A\frac{x^4}{d^2}\frac{x}{\sqrt{d}}e^{-\frac{x^2}{cd\varrho}\sqrt{2}} - B\frac{x^2}{d}\frac{x}{\sqrt{d}}e^{-\frac{x^2}{cd\varrho}\sqrt{2}} - C\frac{x}{\sqrt{d}}e^{-\frac{x^2}{cd\varrho}\sqrt{2}}$$

$$= 1 - \frac{Ax^5}{d^2\sqrt{d}}e^{-\frac{x^2}{cd\varrho}\sqrt{2}} - \frac{Bx^3}{d\sqrt{d}}e^{-\frac{x^2}{cd\varrho}\sqrt{2}} - \frac{Cx}{\sqrt{d}}e^{-\frac{x^2}{cd\varrho}\sqrt{2}} \quad (*1)$$

OLIVERA MURATOVIĆ

QPSK, OSTBC 222 $\Rightarrow c = \frac{T}{4S\cdot K\cdot ld(M)} = \frac{2}{2\cdot 2\cdot 2} = 0.25$

· UNION BOUND FOR SER OF MSK IN AWGN

$$P_S(e) \leq 2Q\left(\sqrt{\frac{2E_S}{N_0}}\cdot\sin\frac{\pi}{M}\right) = 2Q\left(\sqrt{2\cdot\sin^2\frac{\pi}{M}\cdot\frac{E_S}{N_0}}\right)$$

$M=4 \qquad 2\sin^2\frac{\pi}{M} = 2\cdot\frac{2}{4} = 1 \qquad P_S(e) = 2Q\left(\sqrt{\frac{E_S}{N_0}}\right)$

$Q(z) = \frac{1}{2}\,\mathrm{erfc}\,\frac{z}{\sqrt{2}}$ доказ $P_e \doteq \mathrm{erfc}\left(\sqrt{\frac{E_s}{2N_0}}\right) = 2\,\frac{1}{2}\,\mathrm{erfc}\left(\frac{\sqrt{\frac{E_s}{N_0}}}{\sqrt{2}}\right)$

$$\boxed{P_e \doteq 2\,Q\left(\sqrt{\frac{E_s}{N_0}}\right)}$$

НЕ ОДГОВАРА НА АПРОКСИМАЦИЈАТА $P_e = Q\left(\sqrt{c\,\frac{E_s}{N_0}}\right)$

• За QPSK се зема $\boxed{d = 1}$ но изразот за $F_\gamma^{NS}$ (*1) треба да се помножи со $\kappa^2$ =

<u>QPSK</u>

$$\boxed{F_\gamma^{NS} = 2\left(1 - Ax^5 e^{-\frac{x^2}{c\rho}\sqrt{2}} - Bx^3 e^{-\frac{x^2}{c\rho}\sqrt{2}} - Cx\,e^{-\frac{x^2}{c\rho}\sqrt{2}}\right)}$$

OSTBC222 <u>$c = 0.25$</u>

$$F_\gamma(\rho) = 2\left(1 - Ax^5 e^{-\frac{4x^2}{\rho}\sqrt{2}} - Bx^3 e^{-\frac{4x^2}{\rho}\sqrt{2}} - Cx\,e^{-\frac{4x^2}{c\rho}\sqrt{2}}\right)$$

$$A = \frac{e^{-\frac{3x^2}{2c\rho d}}}{c^3\rho^3}\sqrt{\frac{\pi c\rho}{2\sqrt{2}}}\left(\frac{3}{2} + \sqrt{2}\right) \qquad B = \frac{e^{-\frac{3x^2}{2c\rho d}}}{c^3\rho^3}\sqrt{\frac{\pi c\rho}{2\sqrt{2}}}\left(c\rho + \frac{9c\rho}{16\sqrt{2}} + \frac{15c\rho}{8}\right)$$

$$C = \frac{3\cdot e^{-\frac{3x^2}{2c\rho}}}{c^5\rho^5}\sqrt{\frac{\pi c\rho}{2\sqrt{2}}}\;\frac{c^2\rho^2}{8\sqrt{2}} = \frac{3}{c\rho}\sqrt{\frac{\pi c\rho}{2\sqrt{2}}}\;\frac{1}{8\sqrt{2}}\cdot e^{-\frac{3x^2}{2c\rho}}$$

$$\boxed{C = \frac{3}{8}\sqrt{\frac{\pi}{4c\rho\sqrt{2}}}\,e^{-\frac{3x^2}{2c\rho d}}} \qquad \boxed{K = e^{-\frac{3x^2}{2c\rho d}}}$$

$$F_\gamma^{NS} = 1 - \frac{\gamma^3 e^{-\frac{\gamma}{c\rho}\sqrt{2}}\cdot K}{c^3\rho^3}\sqrt{\frac{\pi c\rho}{2\gamma\sqrt{2}}}\left[\left(\frac{3}{2} + \frac{c\rho}{\gamma}\right)\left(1 + \frac{15c\rho}{8\gamma\sqrt{2}}\right) + \sqrt{2}\left(1 + \frac{3c\rho}{8\gamma\sqrt{2}}\right)\right]$$

$$= 1 - \frac{\gamma^3 e^{-\frac{\gamma}{c\rho}\sqrt{2}}\cdot K}{c^3\rho^3}\sqrt{\frac{\pi c\rho}{2\gamma\sqrt{2}}}\left[\frac{3}{2} + \frac{45c\rho}{16\gamma\sqrt{2}} + \frac{c\rho}{\gamma} + \frac{15c^2\rho^2}{8\gamma^2\sqrt{2}} + \sqrt{2} + \frac{3c\rho}{8\gamma}\right]$$

$$= 1 - \frac{K\cdot e^{-\frac{\gamma\sqrt{2}}{c\rho}}}{c^3\rho^3}\sqrt{\frac{\pi c\rho}{2\sqrt{2}}}\left[\left(\frac{3}{2} + \sqrt{2}\right)\cdot\gamma^2\sqrt{\gamma} + \gamma\sqrt{\gamma}\left(\frac{45c\rho}{16\sqrt{2}} + c\rho + \frac{3c\rho}{8}\right) + \sqrt{\gamma}\,\frac{15c^2\rho^2}{8\sqrt{2}}\right]$$

$$= 1 - \underbrace{\frac{1}{c\rho^3}\sqrt{\frac{\pi c\rho}{2\sqrt{2}}}\left(\frac{3}{2} + \sqrt{2}\right)}_{A}\cdot\gamma^2\sqrt{\gamma}\,e^{-\frac{\gamma\sqrt{2}}{c\rho}}\cdot K - \underbrace{\frac{1}{c^3\rho^3}\sqrt{\frac{\pi c\rho}{2\sqrt{2}}}\left(\frac{45c\rho}{16\sqrt{2}} + c\rho + \frac{3c\rho}{8}\right)}_{B}\gamma\sqrt{\gamma}\,e^{-\frac{\gamma\sqrt{2}}{c\rho}}\cdot K$$

$$- \underbrace{\frac{15}{8}\sqrt{\frac{\pi}{4c\rho\sqrt{2}}}}_{C}\cdot\sqrt{\gamma}\,e^{-\frac{\gamma\sqrt{2}}{c\rho}}\cdot K = 1 - \frac{Ax^5 K}{d^2\sqrt{d}}e^{-\frac{x^2\sqrt{2}}{cd\rho}} - \frac{Bx^3 K}{d\sqrt{d}}e^{-\frac{x^2\sqrt{2}}{cd\rho}} - \frac{C\,x\,K}{\sqrt{d}}e^{-\frac{x^2}{cd\rho}\sqrt{2}}$$

(**)

$$F_\gamma^{NS} \doteq 1 - \frac{\gamma^3 e^{-\frac{3\gamma}{2c\rho}}}{c^3\rho^3}\left[\left(\frac{3}{2}+\frac{c\rho}{\gamma}\right)K_2\left(\frac{\gamma\sqrt{2}}{c\rho}\right) + \sqrt{2}\,K_1\left(\frac{\gamma\sqrt{2}}{c\rho}\right)\right]$$

$$\gamma = \frac{x^2}{d} \qquad F_\gamma^{NS} = 1 - \frac{x^6 e^{-\frac{3x^2}{2c\rho d}}}{c^3\rho^3 d^3}\left[\left(\frac{3}{2}+\frac{dc\rho}{x^2}\right)K_2\left(\frac{x^2\sqrt{2}}{dc\rho}\right) + \sqrt{2}\,K_1\left(\frac{x^2\sqrt{2}}{dc\rho}\right)\right]$$

$$\mathrm{Re}|\delta = \rho(\sqrt{d\cdot\delta})$$

$$K_p(z) \sim \sqrt{\frac{\pi}{2z}}\,e^{-z} = \sqrt{\frac{\pi}{x^2\sqrt{2}}\cdot dc\rho}\;e^{-\frac{x^2\sqrt{2}}{dc\rho}}$$

$$F_\gamma^{NS} = 1 - \frac{x^6 e^{-\frac{3x^2}{2c\rho d}}}{c^3\rho^3 d^3}\left[\frac{3}{2}+\sqrt{2}+\frac{dc\rho}{x^2}\right]e^{-\frac{x^2\sqrt{2}}{dc\rho}}\cdot\sqrt{\frac{\pi dc\rho}{x^2\sqrt{2}}} =$$

$K = e^{-\frac{3x^2}{2c\rho d}}$

$$= 1 - \frac{3+2\sqrt{2}}{2}\cdot\frac{x^5 K\cdot e^{-\frac{x^2\sqrt{2}}{dc\rho}}}{c^2\rho^2\sqrt{c\rho}\cdot d^3}\sqrt{\frac{\pi d}{\sqrt{2}}} + \frac{dc\rho\cdot x^3 K\cdot e^{-\frac{x^2\sqrt{2}}{dc\rho}}}{c^3\rho^3 d^2\sqrt{c\rho}}\sqrt{\frac{\pi d}{\sqrt{2}}}$$

$$= 1 - \frac{3+2\sqrt{2}}{2}\cdot\frac{K\cdot x^5 e^{-\frac{x^2\sqrt{2}}{dc\rho}}}{c^2\rho^2\sqrt{c\rho}\,d^3}\sqrt{\frac{\pi d}{\sqrt{2}}}\;[redacted]\;\frac{K\cdot x^3 e^{-\frac{x^2\sqrt{2}}{dc\rho}}}{c\rho\, d^2\sqrt{c\rho}}\sqrt{\frac{\pi d}{\sqrt{2}}}$$

$$\rho_{dB} = 10\log\rho \qquad \rho = 10^{0.1\,\rho_{dB}}$$

• APPROXIMATION WITH 2 TERMS (#) PR.7

$$F = 1 - \left(\frac{A K\cdot x^5}{d^2\sqrt{d}} + \frac{K\cdot B x^3}{d\sqrt{d}} + \frac{K\cdot C x}{\sqrt{d}}\right)e^{-\frac{x^2}{cd\rho}\sqrt{2}} = \qquad K = e^{-\frac{3x^2}{2c\rho d}}$$

$$= 1 - \left(\frac{A\cdot x^5}{d^2} + \frac{B x^3}{d} + Cx\right)\frac{e^{-\left(\frac{3}{2}+\sqrt{2}\right)\frac{x^2}{cd\rho}}}{\sqrt{d}}$$

$$A = \frac{1}{c^3\rho^3}\sqrt{\frac{\pi c\rho}{2\sqrt{2}}}\;\frac{3+2\sqrt{2}}{2} \qquad B = \frac{1}{c^3\rho^3}\sqrt{\frac{\pi c\rho}{2\sqrt{2}}}\left(c\rho + \frac{45 c\rho}{16\sqrt{2}} + \frac{3c\rho}{8}\right)$$

$$C = \frac{15}{8}\sqrt{\frac{\pi}{4c\rho\sqrt{2}}}$$

NE MOŽAM DA GO DOKAŽAM VO MATLE. (NEGDE IMAM GREŠKA)!!!

$$F = 1 - \sqrt{\frac{\pi c\rho}{2\sqrt{2}\,d}}\left(\frac{(3+2\sqrt{2})x^5}{c^3\rho^3 d^2} + \frac{1}{c^3\rho^3}\left(c\rho + \frac{45 c\rho}{16\sqrt{2}} + \frac{3c\rho}{8}\right)\frac{x^3}{d} + \frac{15x}{8\sqrt{2c\rho}}\right)e^{-\frac{3+2\sqrt{2}}{2}\frac{x^2}{c\rho d}}$$

$$F = 1 - \sqrt{\frac{\pi c\rho}{2\sqrt{2}\,d\,c^4\rho^4}}\left[\frac{(3+2\sqrt{2})x^5}{c\rho\, d^2} + \left(1 + \frac{45}{16\sqrt{2}} + \frac{3}{8}\right)\frac{x^3}{d} + \frac{15\,x c^2\rho^2}{8\sqrt{2c\rho}}\right]e^{-\frac{3+2\sqrt{2}}{2}\frac{x^2}{c\rho d}}$$

$$F = 1 - \sqrt{\frac{\pi}{2\sqrt{2}\,d\,c^3\rho^3}}\left[\frac{(3+2\sqrt{2})x^5}{c\rho\, d^2} + \frac{(16\sqrt{2}+45+6\sqrt{2})\,x^3}{16\sqrt{2}\,d} + \frac{15\,x c^2\rho^2}{8\sqrt{2c\rho}}\right]e^{-\frac{3+2\sqrt{2}}{2}\frac{x^2}{c\rho d}}$$

?

$$F = 1 - \sqrt{\frac{\pi}{2\sqrt{2}\,d\,c^3\rho^3}}\left[\frac{(3+2\sqrt{2})x^5}{c\rho\, d^2} + \frac{(45+22\sqrt{2})\,x^3}{16\sqrt{2}\,d} + \frac{15\,x c^2\rho^2}{8\sqrt{2c\rho}}\right]e^{-\frac{3+2\sqrt{2}}{2}\frac{x^2}{c\rho d}}$$

???

IMA GREŠKA!!! → (!!! $c\rho$ NE E POD KOREN

$$F_2 = 1 - \sqrt{\frac{\pi c\rho d}{2x^2\sqrt2}}\,\frac{x^6 e^{-\frac{3x^2}{2c\rho d}}}{c^3\rho^3 d^3}\left[\left(\frac32+\frac{dc\rho}{x^2}\right)\left(1+\frac{15c\rho d}{8\cdot x^2\sqrt2}\right)+\sqrt2\left(1+\frac{3c\rho d}{8x^2\sqrt2}\right)\right]\cdot e^{-\frac{x^2\sqrt2}{c\rho d}}$$

$$= 1 - \sqrt{\frac{\pi c\rho d}{2\sqrt2}}\,\frac{x^5}{c^3\rho^3 d^3}\left[\frac32+\frac{45c\rho d}{16x^2\sqrt2}+\frac{dc\rho}{x^2}+\frac{15c^2\rho^2d^2}{8x^4\sqrt2}+\sqrt2+\frac{3\sqrt2 c\rho d}{8x^2\sqrt2}\right]e^{-\frac{(3+\sqrt2)x^2}{2c\rho d}}$$

$$= 1 - \sqrt{\frac{\pi d}{2c\rho}}\,\frac{x^5}{c^2\rho^2 d^3}\left[\frac{3+2\sqrt2}{2}+\frac{c\rho d}{x^2}\left(\frac{45}{16\sqrt2}+\frac11+\frac38\right)+\frac{15c^2\rho^2d^2}{8x^4\sqrt2}\right]e^{-\frac{(3+2\sqrt2)x^2}{2c\rho d}}$$

$\frac{x^5}{c\rho d}$

$$= 1 - \sqrt{\frac{\pi d}{2c\rho\sqrt2}}\,\frac{1}{c\rho d^2}\left[\frac{(3+2\sqrt2)x^5}{2c\rho d}+\left(\frac{45+16\sqrt2+6\sqrt2}{16\sqrt2}\right)x^3+\frac{15c\rho d}{8\sqrt2}x\right]e^{-\frac{(3+2\sqrt2)x^2}{2c\rho d}}$$

$$\boxed{F_2 = 1 - \sqrt{\frac{\pi}{2c^3\rho^3d^3\sqrt2}}\left[\frac{3+2\sqrt2}{2c\rho d}x^5+\frac{45+22\sqrt2}{16\sqrt2}x^3+\frac{15c\rho d}{8\sqrt2}x\right]e^{-\frac{(3+2\sqrt2)x^2}{2c\rho d}}}$$

→ POTVRDENO VO MAPLE

ISTO E SO (II) SAMO ŠTO TAMU VO TRETIOT SOBIROK „$c\rho$" NE E POD KOREN. JAS TAKA GO ZEMAV I ZATOA GREŠEŠE. (VIDI MAPLE MultiloopMIMO.mw (3.12.3))

- SERIES REPRESENTATION OF K BESSEL FUNCTION

$$K_\nu(z) = \frac12\left[\Gamma(\nu)\left(\frac z2\right)^{-\nu}\left(1+\frac{z^2}{4(1-\nu)}+\frac{z^4}{32(1-\nu)(2-\nu)}+\cdots\right)+\Gamma(-\nu)\left(\frac z2\right)^{\nu}\left(1+\frac{z^2}{4(\nu+1)}+\frac{z^4}{32(\nu+1)(\nu+2)}+\cdots\right)\right]$$

$z \to 0 \wedge \nu \in Z$

- SINGLE TERM APPROXIMATION:

$$\boxed{K_\nu(z) \doteq \frac12\Gamma(\nu)\left(\frac2z\right)^{\nu}}$$

→ VO VAKVA FORMA E NA WIKIPEDIA

$$\frac{2}{z^2} = \frac{2c^2\rho^2d^2}{x^4\cdot 2} = \frac{c^2\rho^2d^2}{x^4} \qquad z = \frac{x^2\sqrt2}{c\rho d}$$

$$F_\gamma^{NS} = 1 - \frac{x^6 e^{-\frac{3x^2}{2c\rho d}}}{c^3\rho^3d^3}\left[\left(\frac32+\frac{dc\rho}{x^2}\right)\frac{\Gamma(2)\,2^2}{2z^2}+\sqrt2\,\frac{\Gamma(1)\,2}{2z}\right] =$$

$$= 1 - \frac{x^6 e^{-\frac{3x^2}{2c\rho d}}}{c^3\rho^3d^3}\left[\left(\frac32+\frac{dc\rho}{x^2}\right)\frac{c^2\rho^2d^2}{x^4}+\frac{c\rho d\sqrt2}{x^2\sqrt2}\right] = 1 - \frac{x^6 e^{-\frac{3x^2}{2c\rho d}}}{c^3\rho^3d^3}\left[\frac{3c^2\rho^2d^2}{2x^4}+\frac{c^3\rho^3d^3}{x^6}+\frac{c\rho d}{x^2}\right] = 1 - \frac{x^6 e^{-\frac{3x^2}{2c\rho d}}}{c^3\rho^3d^3}\left[\frac{3c^2\rho^2d^2}{2x^4}+\frac{c^3\rho^3d^3}{x^6}+\frac{c\rho d}{x^2}\right]$$

$$\boxed{F_\gamma^{NS} = 1 - e^{-\frac{3x^2}{2c\rho d}}\left(\frac{3x^2}{2c\rho d}+\frac11+\frac{x^4}{c^2\rho^2d^2}\right)} = 1 - e^{-\frac{3x^2}{2c\rho d}}\left(1+\frac{3x^2}{2c\rho d}+\frac{x^4}{c^2\rho^2d^2}\right)$$

PROOFED MAPLE !!!

$$P_{ea} = \int_0^\infty F_\gamma^{NS}(\rho) \cdot \frac{e^{-x^2/2}}{\sqrt{2\pi}} dx = \frac{1}{2} \frac{(\rho+12)^{5/2} - 30\rho^{3/2} - 264\sqrt{\rho} - \rho^{5/2}}{(\rho+12)^{5/2}}$$

$c = 1/4;\ d = 1$

• VO GENERALEN SLUCAJ

$$P_{eA} = \frac{1}{2}\left(1 - \text{[redacted]}\ \frac{16.5 \cdot \sqrt{c\rho d} + (c\rho d)^{5/2} + 12.5\,(c\rho d)^{3/2}}{(c\rho d+3)^{5/2}}\right)$$

$$P_{eA} = \frac{1}{2}\left(1 - \frac{33\sqrt{c\rho d} + 2c^2\rho^2 d^2\sqrt{c\rho d} + 15\,\rho d\sqrt{c\rho d}}{2\,(c\rho d+3)^{5/2}}\right)$$

$$P_{ea} = \frac{1}{2}\left(1 - \frac{\sqrt{c\rho d}\,(33 + 2\ \text{[redacted]}\ c^2\rho^2 d^2 + 15\,c\rho d)}{2\,(c\rho d+3)^{5/2}}\right)$$

CHECKED W MARKE!!

1 2 3 4 5 6 7 8 9 10
1 2 3 4 5 6 7 8 9

• PSI - DIGAMA FUNCTION (I.E POLIGAMA FUNCTION)

$$\psi(z) = \frac{d}{dz}(\ln\Gamma(z)) = \frac{\Gamma'(z)}{\Gamma(z)}$$

$$\Gamma(z) = \int_0^\infty t^{z-1} e^{-t}\,dt$$

[illegible]

$$\frac{d}{dz} t^{(z-1)} = \frac{d}{dz} e^{(z-1)\ln t} = \ln(t) \cdot t^{z-1}$$

~~$\Gamma(z) = (z-1)\int t^{z-2} e^{-t} dt = (z-1)\cdot\Gamma(z-1)$~~

$$\frac{d}{dz}(\Gamma(z)) = \int_0^\infty \frac{d}{dz}(t^{z-1})\, e^{-t}\,dt = \int_0^\infty \ln(t) \cdot t^{(z-1)} e^{-t}\,dt$$

$$\frac{d(a^x)}{dx} = ?$$

$$a^x = e^{x\ln(a)}$$

$$\frac{d}{dx}(a^x) = \ln(a) \cdot e^{x\ln(a)} = a^x$$

$y = a^x \quad \ln y = x\ln a \quad \rightarrow \quad y = e^{x\ln a}$

$$(a^x)' = \ln a\ a^x$$

$$\frac{d^n}{dz}\left[\Gamma(z)\right] = \int_0^\infty \ln^n(t)\cdot t^{(z-1)} e^{-t}\, dt$$

FROM wolfram.com

- APPROXIMATION OF BesselK FOR $n \in N$

$$K_n(z) = (-1)^{n-1} \log\left(\frac{z}{2}\right)\left(\frac{z}{2}\right)^n \sum_{k=0}^{\infty} \frac{\left(\frac{z}{2}\right)^{2k}}{k!\,(k+n)!} + \frac{1}{2}\left(\frac{z}{2}\right)^{-n} \sum_{k=0}^{n-1} \frac{(-1)^k (n-k-1)!}{k!}\left(\frac{z}{2}\right)^{2k} + \frac{(-1)^n}{2}\left(\frac{z}{2}\right)^n \sum_{k=0}^{\infty} \frac{\psi(k+1)+\psi(k+n+1)}{k!\,(k+n)!}\left(\frac{z}{2}\right)^{2k}$$

- Only FIRST MEMBER OF THE SUMS

$$K_n(z) = (-1)^{n-1} \log\left(\frac{z}{2}\right)\left(\frac{z}{2}\right)^n \cdot \frac{1}{n!} + \frac{1}{2}\left(\frac{z}{2}\right)^{-n}\cdot (n-1)! + \frac{(-1)^n}{2}\left(\frac{z}{2}\right)^n \frac{\psi(1)+\psi(n+1)}{n!}$$

$\psi(1) = -\gamma$

- Euler-Mascheroni constant

$$\gamma = \lim_{n\to\infty}\left(\sum_{k=1}^{n}\frac{1}{k} - \ln(n)\right) = \int_1^\infty \left(\frac{1}{\lfloor x \rfloor} - \frac{1}{x}\right) dx$$

$$\gamma = 0.5772$$

$$K_n(z) = \left[\frac{(-1)^{n-1}}{n!}\log\left(\frac{z}{2}\right) + \frac{(-1)^n}{2}\,\frac{-\gamma+\psi(n+1)}{n!}\right]\left(\frac{z}{2}\right)^n + \frac{2^n}{2\cdot z^n}(n-1)!$$

$$K_n(z) = \left[\frac{(-1)^{n-1}}{n!}\log\left(\frac{z}{2}\right) + \frac{(-1)^n(-\gamma+\psi(n+1))}{2\cdot n!}\right]\left(\frac{z}{2}\right)^n + \frac{2^{n-1}}{z^n}(n-1)!$$

$$K_n(z) = \frac{(-1)^{n-1}}{n!}\left[\log\left(\frac{z}{2}\right) + \frac{\gamma\,\text{[redacted]}\,\psi(n+1)}{2\,\text{[redacted]}}\right]\left(\frac{z}{2}\right)^n + \frac{2^{n-1}}{z^n}(n-1)!$$

OVOJ E DOMINANTEN ČLEN (VIDI N10.9)

- BesselK IS SOLUTION OF

$$z^2\frac{d^2\omega}{dz^2} + z\frac{d\omega}{dz} - (z^2+\nu^2)\omega = 0$$

VANE SIMON 070255851

SIMON STOJANOVSKI, DUŠKO KOSTADINOV

- LITTLE-O NOTATION

$f(x) \in o(g(x))$ — $g(x)$ GROWS MUCH FASTER THAN $f(x)$

$$\lim_{x \to \infty} \frac{f(x)}{g(x)} = 0$$

$f(x) = o(g(x))$ AS $x \to \infty$ IF AND ONLY IF, FOR EVERY $M > 0$, there exist constant $x_0$ SUCH

$$|f(x)| \leq M|g(x)| \text{ for all } x > x_0$$

EXAMPLES:

$2x \in o(x^2)$ ; $2x^2 \notin o(x^2)$ ; $\frac{1}{x} \in o(1)$

- DEFFINITION OF ASYMPTOTIC EXPANSION

$$F(z) = \sum_{k=0}^{\infty} a_k z^k \quad |z| < r \qquad S_n(z) = \sum_{k=0}^{n-1} a_k z^k$$

$$\lim_{n \to \infty} \left( F(z) - S_n(z) \right) = 0$$

$$F(z) - S_n(z) = O(z^n) \qquad |z| \leq r - \varepsilon \quad \varepsilon > 0$$

$$F(z) - S_n(z) = o(z^{n-1}) \qquad z \to 0$$

- ASYMPTOTIC SERIES OF DESCENDING TYPE

$$F(z) \sim \sum_{k=0}^{\infty} a_k z^{-k} \qquad z \to \infty \text{ IN } R$$

$$S_n(z) = \sum_{k=0}^{n-1} a_k z^{-k}$$

IF FOR FIXED "n"

$$\lim z^{n-1} R_n(z) = \lim z^{n-1} \left( F(z) - S_n(z) \right) = 0 \text{ AS } z \to \infty \text{ IN } R$$

— FOR EACH FIXED "n" WE CAN APROXIMATE $F(z)$ AS AS CLOSELY AS WE DESIRE BY TAKING SUFFICIENTLY LARGE "z" IN "R".

$\left|z^{n-1} R_n(z)\right| < \varepsilon$ WITH „ε" ARBITRARY SMALL

$F(z) = S_n(z) + O(|z^{-n}|)$ AS $z \to \infty$

$F(z) = S_n(z) + O(|z^{1-n}|)$

A. G.L (2.138) — OVA E ISTATA FORMULA OD OVAA VO ABRAMOWITZ

$$K_\nu(z) \sim \sqrt{\frac{\pi}{2z}} \frac{e^{-z}}{\Gamma(\nu+\frac{1}{2})} \sum_{k=0}^{\infty} \binom{\nu-\frac{1}{2}}{k} \frac{\Gamma(k+\nu+\frac{1}{2})}{(2z)^k} \quad z \to \infty$$

$\nu = 1$

$$K_1(z) \sim \sqrt{\frac{\pi}{2z}} \frac{e^{-z}}{\Gamma(3/2)} \binom{\frac{1}{2}}{0} \frac{\Gamma(0+1+\frac{1}{2})}{(2z)^0} \to \frac{\Gamma(3/2)}{1}$$

$\Gamma(3/2) \to \frac{\sqrt{\pi}}{2}$

$$K_1(z) \sim \sqrt{\frac{\pi}{2z}} e^{-z}$$

$\Gamma(5/2) = \frac{3\sqrt{\pi}}{4}$ — TWO TERMS APPROXIMATION

$$K_1(z) = \sqrt{\frac{\pi}{2z}} \frac{e^{-z}}{\frac{\sqrt{\pi}}{2}} \left( \frac{\sqrt{\pi}}{2} + \frac{1}{2} \cdot \frac{3\sqrt{\pi}}{4} \frac{1}{2z} \right)$$

$$K_1(z) = \sqrt{\frac{\pi}{2z}} e^{-z} \left(1 + \frac{3}{4} \cdot \frac{1}{2z}\right) = \sqrt{\frac{\pi}{2z}} e^{-z} \left(1 + \frac{3}{8z}\right)$$

ISTO SE DOBIVA SO FORMULATA OD ABRAMOWITZ

$$K_2(z) = \sqrt{\frac{\pi}{2z}} \frac{e^{-z}}{\frac{3\sqrt{\pi}}{4}} \left( \frac{3\sqrt{\pi}}{4} + \frac{3}{2} \cdot \frac{\Gamma(7/2)}{2z} \right) \qquad \Gamma(7/2) = \frac{15}{8}\sqrt{\pi}$$

$$K_2(z) = \sqrt{\frac{\pi}{2z}} e^{-z} \left( 1 + \frac{4}{3\sqrt{\pi}} \cdot \frac{3}{2} \cdot \frac{15\sqrt{\pi}}{16 z} \right) = \sqrt{\frac{\pi}{2z}} e^{-z} \left(1 + \frac{30}{16z}\right)$$

$$K_2(z) = \sqrt{\frac{\pi}{2z}} e^{-z} \left(1 + \frac{15}{8z}\right)$$

ISTO SE DOBIVA SO IZRAZOT OD ABRAMOWITZ

$$K_\nu(x) = \sqrt{\pi} (2x)^\nu e^{-x} z_0(x)$$

$$z_n(x) = U\left(\nu + \frac{1}{2} + n, 2\nu + 1, 2x\right)$$

$$z_0(x) = U\left(\nu + \frac{1}{2}, 2\nu + 1, 2x\right)$$

$$K_\nu(x) = \sqrt{\pi} (2x)^\nu e^{-x} U\left(\nu + \frac{1}{2}, 2\nu + 1, 2x\right)$$

$$K_\nu(z) = \sqrt{\frac{\pi}{2z}}\,\frac{e^{-z}}{\Gamma(\nu+\frac{1}{2})}\sum_{k=0}^{\infty}\binom{\nu-\frac{1}{2}}{k}\frac{\Gamma(k+\nu+\frac{1}{2})}{(2z)^k}$$

$$\Gamma\left(\tfrac{5}{2}\right)=\tfrac{3}{4}\sqrt{\pi} \qquad \Gamma\left(\tfrac{3}{2}\right)=\frac{\sqrt{\pi}}{2}$$

$$K_1(z) = \sqrt{\frac{\pi}{2z}}\,\frac{e^{-z}}{\frac{\sqrt{\pi}}{2\sqrt{2}}}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\frac{\Gamma(k+\frac{3}{2})}{(2z)^k} = \sqrt{\frac{2}{z}}\,e^{-z}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\frac{\Gamma(k+\frac{3}{2})}{(2z)^k}$$

$$K_2(z) = \sqrt{\frac{\pi}{2z}}\,\frac{e^{-z}}{\frac{3}{4}\frac{\sqrt{\pi}}{2\sqrt{2}}}\sum_{k=0}^{\infty}\binom{\frac{3}{2}}{k}\frac{\Gamma(k+\frac{5}{2})}{(2z)^k} = 2\sqrt{\frac{2}{z}}\,e^{-z}\sum_{k=0}^{\infty}\binom{\frac{3}{2}}{k}\frac{\Gamma(k+\frac{5}{2})}{(2z)^k}$$

$$F_\delta^{NS} = 1 - \frac{\delta^3 e^{-\frac{3\delta}{2c\rho}}}{c^3\rho^3}\left[\left(\frac{3}{2}+\frac{c\rho}{\delta}\right)K_2\left(\frac{\delta\sqrt{2}}{c\rho}\right)+\sqrt{2}\,K_1\left(\frac{\delta\sqrt{2}}{c\rho}\right)\right]$$

$$F_\delta^{NS} = 1 - \frac{x^6 e^{-\frac{3x^2}{2c\rho d}}}{c^3\rho^3 d^3}\left[\left(\frac{3}{2}+\frac{c\rho d}{x^2}\right)K_2\left(\frac{x^2\sqrt{2}}{c\rho d}\right)+\sqrt{2}\,K_1\left(\frac{x^2\sqrt{2}}{c\rho d}\right)\right]$$

PART 2 PART 3 PART 4

$$= 1 - \frac{3x^6 e^{-\frac{3x^2}{2c\rho d}}}{2c^3\rho^3 d^3}K_2\left(\frac{x^2\sqrt{2}}{c\rho d}\right) - \frac{x^4 e^{-\frac{3x^2}{2c\rho d}}}{c^2\rho^2 d^2}K_2\left(\frac{x^2\sqrt{2}}{c\rho d}\right) - \frac{\sqrt{2}x^6 e^{-\frac{3x^2}{2c\rho d}}}{c^3\rho^3 d^3}K_1\left(\frac{x^2\sqrt{2}}{c\rho d}\right)$$

$$Pe_4 = \int_0^\infty \frac{\sqrt{2}x^6 e^{-\frac{3x^2}{2c\rho d}}}{c^3\rho^3 d^3}\cdot\sqrt{\frac{2}{\frac{x^2\sqrt{2}}{c\rho d}}}\cdot e^{-\frac{x^2\sqrt{2}}{c\rho d}}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\frac{\Gamma(k+\frac{3}{2})}{\left(\frac{2x^2\sqrt{2}}{c\rho d}\right)^k}\frac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}}\,dx$$

$$= \frac{\sqrt{2}}{c^3\rho^3 d^3}\frac{\sqrt{2c\rho d}}{\sqrt{\sqrt{2}}}\frac{1}{\sqrt{2\pi}}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\Gamma(k+\tfrac{3}{2})\int_0^\infty e^{-\left(\frac{3}{2c\rho d}+\frac{\sqrt{2}}{c\rho d}+\frac{1}{2}\right)x^2}x^5\cdot\frac{1}{\left(\frac{2\sqrt{2}}{c\rho d}\right)^k\cdot x^{2k}}\,dx$$

$$a = \left(\frac{3}{2c\rho d}+\frac{\sqrt{2}}{c\rho d}+\frac{1}{2}\right) \qquad a = \frac{3+2\sqrt{2}+c\rho d}{2c\rho d}$$

3.461.3 GRADSHTEYN

$$\int_0^\infty x^{2n+1}e^{-px^2}\,dx = \frac{n!}{2p^{n+1}}$$

$$Pe_4 = \frac{1}{c^3\rho^3 d^3}\sqrt{\frac{\sqrt{2}c\rho d}{\pi}}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\frac{\Gamma(k+\frac{3}{2})(c\rho d)^k}{(2\sqrt{2})^k}\int_0^\infty e^{-ax^2}x^{5-2k}\,dx$$

$$5-2k = 4-2k+1 = 2(2-k)+1$$

$$\int_0^\infty e^{-ax^2}x^{2(2-k)+1}\,dx = \frac{(2-k)!}{2a^{2-k+1}} = \frac{(2-k)!}{2a^{3-k}}$$

$$Pe_4 = \frac{1}{c^3\rho^3 d^3}\sqrt{\frac{\sqrt{2}c\rho d}{\pi}}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\frac{\Gamma(k+\frac{3}{2})(c\rho d)^k}{(2\sqrt{2})^k}\frac{(2-k)!}{2a^{2-k+1}} =$$

$$= \frac{1}{c^3\rho^3 d^3}\sqrt{\frac{\sqrt{2}c\rho d}{\pi}}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\frac{\Gamma(k+\frac{3}{2})(c\rho d)^k}{(2\sqrt{2})^k}\frac{(2-k)!\,(2c\rho d)^{2-k+1}}{2\left(3+2\sqrt{2}+c\rho d\right)^{2-k+1}}$$

$$P_{e4} = \frac{1}{c'g^{3/2}} \sqrt{\frac{\sqrt{2}\,cg d}{\pi}} \cdot \sum_{k=0}^{\infty} \binom{\frac{1}{2}}{k} \frac{\Gamma(k+\frac{3}{2})\,[\text{illegible}]\,(2-k)!\; 2^{2-k+1} \cdot c g^{3/2}}{(2\sqrt{2})^{k} \cdot 2\,(3+2\sqrt{2}+cgd)^{3-k}}$$

$$P_{e4} = [\text{illegible}] \sqrt{\frac{\sqrt{2}\,cgd}{\pi}} \sum_{k=0}^{\infty} \binom{\frac{1}{2}}{k} \frac{\Gamma(k+\frac{3}{2})\,(2-k)!\; [\text{illegible}]\; 2^{3-k-k-1}}{(\sqrt{2})^{k}\,(3+2\sqrt{2}+cgd)^{3-k}}$$

$$P_{e4} = [\text{illegible}] \sqrt{\frac{\sqrt{2}\,cgd}{\pi}} \sum_{k=0}^{\infty} \binom{\frac{1}{2}}{k} \frac{\Gamma(k+\frac{3}{2})\,(2-k)!\; 2^{2-2k-\frac{k}{2}}}{(3+2\sqrt{2}+cgd)^{3-k}}$$

3065229

$$P_{e4} = \sqrt{\frac{\sqrt{2}\,cgd}{\pi}} \sum_{k=0}^{\infty} \binom{\frac{1}{2}}{k} \frac{\Gamma(k+\frac{3}{2})\,(2-k)!\; 2^{2-\frac{5k}{2}}}{(3+2\sqrt{2}+cgd)^{3-k}}$$

$$\boxed{P_{e4} = 4 \sqrt{\frac{\sqrt{2}\,cgd}{\pi}} \sum_{k=0}^{\infty} \binom{\frac{1}{2}}{k} \frac{\Gamma(k+\frac{3}{2})\,(2-k)!}{2^{\frac{5k}{2}}\,(\underbrace{3+2\sqrt{2}+cgd}_{a})^{3-k}}}$$

$$\Gamma(z-n) = \frac{(-1)^n\,\Gamma(z)}{(1-z)_n}$$

$$\Gamma(k) = (k-1)! \qquad (2-k)! = \Gamma(2-k+1) = \Gamma(3-k)$$

$$\binom{\frac{1}{2}}{k} = \frac{\frac{1}{2}!}{k!\,(\frac{1}{2}-k)!} \qquad \Gamma\left(k+\frac{3}{2}\right) = \left(k+\frac{3}{2}-1\right)! = \left(k+\frac{1}{2}\right)!$$

$$\boxed{S = \sum_{k=0}^{\infty} \frac{\frac{1}{2}!\;(k+\frac{1}{2})!\,(2-k)!}{k!\,(\frac{1}{2}-k)!\;a^{3-k} \cdot 2^{5k/2}}}$$

$$\Gamma(3-k) = \frac{(-1)^k\,\Gamma(3)}{(1-3)_k} = \frac{(-1)^k\,\Gamma(3)}{(-2)_k}$$

$$(-2)_3 = (-2)\cdot(-2+1)(-2+3-1) = (-2)(-1)(0) = 0$$

$$(2)_3 = 2\cdot(2+1)\cdot(2+3-1) = 2\cdot 3\cdot 4 = 24$$

$$(-3)_3 = (-3)\cdot(-3+1)(-3+2) = (-3)\cdot(-2)\cdot(-1) = -6$$

$$\left(\frac{1}{2}-k\right)! = \Gamma\left(\frac{1}{2}-k+1\right) = \Gamma\left(\frac{3}{2}-k\right) = \frac{(-1)^k\cdot\Gamma(\frac{3}{2})}{(1-\frac{3}{2})_k} = \frac{(-1)^k\,\Gamma(\frac{3}{2})}{(-\frac{1}{2})_k}$$

$$S = \sum_{k=0}^{\infty} \frac{\frac{1}{2}!\,(k+\frac{1}{2})!\;\frac{(-1)^k\,\Gamma(3)}{(-2)_k}}{2^{5k/2}\,k!\;\frac{(-1)^k\,\Gamma(\frac{3}{2})}{(-\frac{1}{2})_k}\cdot a^{3-k}} = 2\sum_{k=0}^{\infty} \frac{(k+\frac{1}{2})!\,(-\frac{1}{2})_k}{(-2)_k\,k!\;a^{3-k}\cdot 2^{5k/2}}$$

$$\left(\frac{1}{2}\right)! = \Gamma\left(\frac{3}{2}\right) \qquad \left(-\frac{1}{2}\right)_k = \left(-\frac{1}{2}\right)\cdot\left(-\frac{1}{2}+1\right)\cdot\left(-\frac{1}{2}+2\right)\left(-\frac{1}{2}+3\right) = \left(-\frac{1}{2}\right)\left(+\frac{1}{2}\right)\left(\frac{3}{2}\right)\frac{5}{2} = \frac{15}{16} = \frac{(-1)^k}{2^k}\,[\text{illegible}]\;\frac{(2k-3)!}{[\text{illegible}]}$$

$$\left(-\frac{1}{2}\right)_k = -\frac{(2k-3)!!}{2^k} \quad k\geq 0$$

$$S = 2\sum_{k=0}^{\infty} \frac{\left(k+\frac{1}{2}\right)! \, (-1) \, \frac{(2k-3)!!}{2^k}}{(-2)_k \cdot k! \, b^{3-k} \cdot 2^{5k/2}}$$

$$Pe_4 = -8\sqrt{\frac{\sqrt{2}c\rho d}{\pi}} \sum_{k=0}^{\infty} \frac{\left(k+\frac{1}{2}\right)! \, (2k-3)!!}{(-2)_k \, k! \, (3+2\sqrt{2}+c\rho d)^{3-k}} = 4\sqrt{\frac{\sqrt{2}c\rho d}{\pi}} \sum_{k=0}^{\infty} \binom{\frac{1}{2}}{k} \frac{\Gamma\left(k+\frac{3}{2}\right)(2-k)!}{2^{\frac{5k}{2}} b^{3-k}}$$

- $$Pe_1 = \int_0^{\infty} \frac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}} dx = \frac{1}{2}$$

- $Pe_2 = ?$ $\quad K_2(z) = 2\sqrt{\frac{2}{z}} e^{-z} \sum_{k=0}^{\infty} \binom{\frac{3}{2}}{k} \frac{\Gamma\left(k+\frac{5}{2}\right)}{(2z)^k}$

$$K_2\left(\frac{x^2\sqrt{2}}{c\rho d}\right) = 2\sqrt{\frac{2c\rho d}{x^2\sqrt{2}}} \, e^{-\frac{x^2\sqrt{2}}{c\rho d}} \sum_{k=0}^{\infty} \binom{\frac{3}{2}}{k} \frac{\Gamma\left(k+\frac{5}{2}\right)}{(2\cdot x^2\sqrt{2})^k} \cdot c^k\rho^k d^k$$

$$Pe_2 = \frac{1}{\sqrt{2\pi}} \frac{3\cdot 2}{2c^3\rho^3 d^3} \int_0^{\infty} e^{-\underbrace{\left(\frac{3}{2c\rho d}+\frac{\sqrt{2}}{c\rho d}+\frac{1}{2}\right)}_{a} x^2} \sqrt{\frac{2c\rho d}{\sqrt{2}}} \cdot x^5 \sum_{k=0}^{\infty} \binom{\frac{3}{2}}{k} \frac{\Gamma\left(k+\frac{5}{2}\right) \cdot c^k\rho^k d^k}{2^{3k/2} \cdot x^{2k}} dx$$

$$= \frac{6}{2\sqrt{2\pi} c^3\rho^3 d^3} \cdot \sqrt{\sqrt{2}c\rho d} \sum_{k=0}^{\infty} \binom{\frac{3}{2}}{k} \frac{\Gamma\left(k+\frac{5}{2}\right) c^k\rho^k d^k}{2^{3k/2}} \underbrace{\int_0^{\infty} x^{5-2k} e^{-ax^2} dx}_{=\frac{(2-k)!}{2a^{3-k}}}$$

$$Pe_2 = \frac{3}{c^3\rho^3 d^3}\sqrt{\frac{c\rho d}{\pi\sqrt{2}}} \sum_{k=0}^{\infty} \binom{\frac{3}{2}}{k} \frac{\Gamma\left(k+\frac{5}{2}\right) c^k\rho^k d^k}{2^{3k/2}} \cdot \frac{(2-k)! \, 2^{3-k} (c\rho d)^{3-k}}{2\cdot b^{3-k}}$$

$$Pe_2 = 3\sqrt{\frac{c\rho d}{\pi\sqrt{2}}} \sum_{k=0}^{\infty} \binom{\frac{3}{2}}{k} \frac{\Gamma\left(k+\frac{5}{2}\right)(2-k)!}{2^{\frac{3k}{2}-3+k+1} \, b^{3-k}} = 12\sqrt{\frac{c\rho d}{\pi\sqrt{2}}} \sum_{k=0}^{\infty} \binom{\frac{3}{2}}{k} \frac{\Gamma\left(k+\frac{5}{2}\right)(2-k)!}{2^{5k/2} b^{3-k}}$$

- $Pe_3 = ?$ $\quad Pe_3 = \frac{2\sqrt{\sqrt{2}c\rho d}}{c^2\rho^2 d^2\sqrt{2\pi}} \int_0^{\infty} e^{-ax^2} \cdot x^3 \sum_{k=0}^{\infty} \binom{\frac{3}{2}}{k} \frac{\Gamma\left(k+\frac{5}{2}\right) c^k\rho^k d^k}{2^{3k/2} x^{2k}} dx$

$$3-2k = \overbrace{2(1-k)}^{n=} + 1$$

$$= \frac{2\sqrt{\sqrt{2}c\rho d}}{c^2\rho^2 d^2\sqrt{2\pi}} \sum_{k=0}^{\infty} \binom{\frac{3}{2}}{k} \frac{\Gamma\left(k+\frac{5}{2}\right) c^k\rho^k d^k}{2^{3k/2}} \underbrace{\int_0^{\infty} e^{-ax^2} x^{3-2k} dx}_{\frac{(1-k)!}{2a^{2-k}}}$$

$$Pe_3 = \frac{2}{c^2\rho^2 d^2}\sqrt{\frac{c\rho d}{\pi\sqrt{2}}} \sum_{k=0}^{\infty} \binom{\frac{3}{2}}{k} \frac{\Gamma\left(k+\frac{5}{2}\right) c^k\rho^k d^k \cdot 2^{2-k} (c\rho d)^{2-k} (1-k)!}{2^{3k/2} \cdot 2 \cdot b^{2-k}} = 4\sqrt{\frac{c\rho d}{\pi\sqrt{2}}} \sum_{k=0}^{\infty} \binom{\frac{3}{2}}{k} \frac{\Gamma\left(k+\frac{5}{2}\right)(1-k)!}{2^{5k/2} b^{2-k}}$$

$$Pe = \frac{1}{2} - \sqrt{\frac{c\rho d}{\pi\sqrt{2}}} \left[ 12\sum_{k=0}^{\infty} \binom{\frac{3}{2}}{k} \frac{\Gamma\left(k+\frac{5}{2}\right)(2-k)!}{2^{5k/2} b^{3-k}} + 4\sum_{k=0}^{\infty} \binom{\frac{3}{2}}{k} \frac{\Gamma\left(k+\frac{5}{2}\right)(1-k)!}{2^{5k/2} b^{2-k}} + 4\sqrt{2}\sum_{k=0}^{\infty} \binom{\frac{1}{2}}{k} \frac{\Gamma\left(k+\frac{3}{2}\right)(2-k)!}{2^{5k/2} b^{3-k}} \right.$$

VASIL VASILEVSKI — VESTAK PREPORAKA OD DRAGI

- DOMI DIZAJN (LILJANA KOVACEVA 070 219 971)

KEMOFARM 3090600 KASTE ISIROV

BILJANA OD DDK

OSNOVEN SUD SKOPJE 2

$$K_\nu(z) = \frac{\Gamma(\nu)}{2}\left(\frac{z}{2}\right)^{\nu}\left(1 + \frac{z^2}{4(1-\nu)} + \frac{z^4}{32(1-\nu)(2-\nu)} + \dots + \frac{z^{2k}}{2\cdot 2^{2k}(1-\nu)(2-\nu)\cdots(k-\nu)}\right)$$

- Ako se zeme samo sledniot clen od izrazot za n ed od wolfram.com

$$K_n(z) = \frac{1}{2}\left(\frac{z}{2}\right)^{-n}\sum_{k=0}^{n-1}\frac{(-1)^k(n-k-1)!\left(\frac{z}{2}\right)^{2k}}{k!} \quad (\#*)$$

NAJDOBRA APROXIMACIJA ZA $z \to 0$.

$$P_{ea} = \frac{1}{2} - \frac{135}{8}\frac{\sqrt{\rho}}{(\rho+3)^{7/2}} - \frac{21}{4}\frac{\rho^{5/2}}{(\rho+3)^{7/2}} - \frac{75}{4}\frac{\rho^{3/2}}{(\rho+3)^{7/2}} - \frac{1}{2}\frac{\rho^{7/2}}{(\rho+3)^{7/2}}$$

NAJDOBAR IZRAZ ZA 2x1+2

$$P_{ea} = \frac{1}{2} - \frac{135}{8}\frac{\sqrt{c\rho d}}{(c\rho d+3)^{7/2}} - \frac{21}{4}\frac{(c\rho d)^{5/2}}{(c\rho d+3)^{7/2}} - \frac{75}{4}\frac{(c\rho d)^{3/2}}{(c\rho d+3)^{7/2}} - \frac{1}{2}\frac{(c\rho d)^{7/2}}{(c\rho d+3)^{7/2}}$$

MMV

$$P_b(E) \doteq \frac{2}{\log_2(M)} Q\left(\sqrt{\frac{2E_b \log_2 M}{N_0}}\sin\frac{\pi}{M}\right) = \frac{2}{ldM} Q\left(\sqrt{2\sin^2\frac{\pi}{M} E_s N_0}\right)$$

$$P_b \sim \frac{P_s}{ldM} \rightarrow P_s \sim 2Q\left(\sqrt{2\sin^2\frac{\pi}{M}E_s N_0}\right)$$

- Sakam da proveram kako ce odi za 4+1+4 system

$$F_{\gamma_{MS}}(\gamma) = 1 - \frac{d^3}{dw^3}\mathcal{L}^{-1}\left\{\frac{M_w(s)}{s^N}\right\}\Big|_{w=\frac{c\rho}{\gamma}}$$

$$\mathcal{L}^{-1}\left\{\frac{M_w(s)}{s^N}\right\} = \frac{2e^{-\frac{5}{4w}}}{(4)^2(\Gamma(4))^2 w}\cdot K_4\left(\frac{2}{\sqrt{N}w}\right)$$

$16\cdot 36 = 576$

$$z K_\nu'(z) = -\nu K_\nu(z) - z K_{\nu-1}(z) \qquad K_\nu'(z) = -\frac{\nu}{z} K_\nu(z) - K_{\nu-1}(z)$$

$$K_2'(z) = -\frac{2}{z} K_2(z) - K_1(z)$$

$$\mathcal{L}^{-1}\left\{\frac{M_w(s)}{s^N}\right\} = \frac{2 e^{-\frac{5}{4w}} K_4\left(\frac{1}{w}\right)}{g \cdot 2^{5} \cdot w} \quad \boxed{N=4} \qquad \boxed{\mathcal{L}^{-1}\left\{\frac{M_w(s)}{s^N}\right\} = \frac{e^{-\frac{5}{4w}} \cdot K_4\left(\frac{1}{w}\right)}{g \cdot 2^5 \cdot w}}$$

$$F_{\gamma_{NS}}(\gamma) = 1 - \left[\frac{d^3}{dw^3} \mathcal{L}^{-1}\left\{\frac{M_w(s)}{s^N}\right\}\right] \Big/ w = \frac{c\rho}{\gamma}$$

$$F_{\gamma_{NS}}(\gamma) = 1 - \frac{g^7 \cdot e^{-\frac{5g}{c\rho}}}{g \cdot 2^{11} c^7 \rho^7}\left[\left(\frac{480c^2\rho^2}{g^2} + \frac{300c\rho}{g} + 125 + \frac{384c^3\rho^3}{g^3}\right) K_4\left(\frac{g}{c\rho}\right)\right.$$
$$\left. + \left(300 + \frac{240c\rho}{g} + \frac{192c^2\rho^2}{g^2}\right) \cdot K_3\left(\frac{g}{c\rho}\right) + 64 K_1\left(\frac{g}{c\rho}\right) + 240 K_2\left(\frac{g}{c\rho}\right)\right]$$

- FOR $c = \frac{1}{3}$ $d = 2$

$$(a+b)^4 = \binom{4}{0} a^4 + \binom{4}{1} a^{4-1} b^1 + \ldots$$

$$\rho^{11/2} = \sqrt{\rho^{11}} = \sqrt{\rho^8 \cdot \rho^3} = \sqrt{\rho^{10} \cdot \rho} = \rho^5 \cdot \sqrt{\rho}$$

- SO UPOTREBA NA (#*) OD PR. 17 I SO [illegible] OTFRLA-NJE NA MALITE CLENOVI ZA GOLEMO „$\rho$" ASIMPTOTSKIOT IZRAZ E:

$$\boxed{P_{ea}(\rho) = \frac{1}{2} - \left(2^{10}\rho^7 + 13634122 5\rho^4 + 285 \cdot 2^7 \rho^8 + 5450625\rho^6 + 22675 \cdot 2^3 \rho^7 + 66383955\,\rho^3/2\right) \frac{4^7 \sqrt{\rho}}{2^{10}(4\rho + 15)^{19/2}}}$$

4×1×4

- VNIMAVAJ!!! TOČNIOT IZRAZ E VO MultilopMIMO2.nb

## ☐ APROXIMATE BER EXPRESSION OF DISTRIBUTED ALAMOUTI'S CODE (Z. Yi, I.-M. Kim)

$$P_{ea} = \frac{1}{2} - \frac{135 + 42\rho^2 + 150\rho + 4\rho^3}{8\rho^3} \cdot \frac{\rho^3\sqrt{\rho}}{(\rho+3)^{7/2}}$$

NAVRAĆANJE NA MOITE KALK.

$$4\rho^3 + 150\rho + 42\rho^2 + 135 \qquad 4\left(\rho^2 + \frac{75}{2}\right) + 42\left(\rho^2 + \frac{45}{14}\right)$$

$$4\rho^3 + 42\rho^2 + 150\rho + 135 \qquad 4\rho^2\left(\rho + \frac{21}{2}\right) + 150\left(\rho + \frac{9}{10}\right)$$

$$P_{ea} = \int_{-\infty}^{\infty} \delta(1,x)\, f(x)\, dx = -\frac{d}{dx}\left[f(x)\right]\Big|_{x=0}$$

VIDI MAKE HELP ZA Delta(...)

$$x^2 = \frac{cgd}{u}$$

$$-\frac{1}{2}\int_{-\infty}^{\infty} \delta\left(1, \frac{cgd}{x^2}\right) \frac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}}\, dx = \Big| \; u = \frac{cgd}{x^2}$$

$$du = -2\frac{cgd}{x^3}\, dx$$

$x \to +\infty \quad u \to +0$

$x \to -\infty \quad u \to -0$

$$x = \sqrt{\frac{cgd}{u}}$$

$$du = + \frac{2\,cgd}{\frac{cgd}{u}\sqrt{\frac{cgd}{u}}}\, dx = + 2u\sqrt{\frac{u}{cgd}}\, dx$$

$$dx = + \frac{du}{2u\sqrt{\frac{u}{cgd}}}$$

$$I = -\frac{1}{2\sqrt{2\pi}}\int_{-0}^{0} \delta(1,u)\; e^{-\frac{cgd}{2u}} \frac{du}{2u\sqrt{\frac{u}{cgd}}}$$

10-274/4

$$P_{ea} = \frac{\sqrt{cgd}}{4\sqrt{\pi}}\frac{d}{du}\left[\frac{e^{-\frac{cgd}{2u}}}{u\sqrt{u}}\right]\Big|_{u=0}$$

$$P_{ea} = \lim_{u\to 0} \frac{\sqrt{2cgd}\; e^{-\frac{cgd}{2u}}(cgd-3u)}{16\sqrt{\pi}\, u^{7/2}}$$

$$P_{ea} = \frac{\sqrt{2cgd}}{16\sqrt{\pi}} \lim_{u\to 0}(cgd-3u) \lim_{u\to 0} \frac{1}{u^{7/2}\, e^{+\frac{cgd}{2u}}}$$

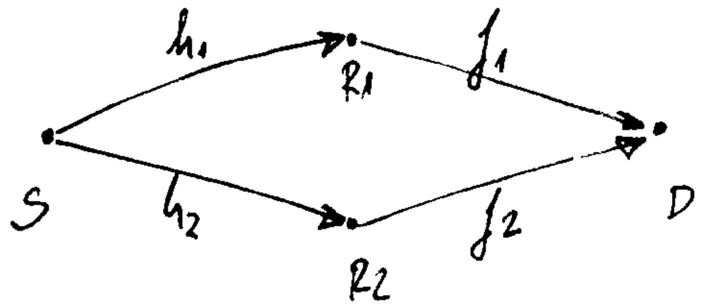
($\lim (cgd-3u) \to cgd$; $e^{+\frac{cgd}{2u}} \to \infty$; 2)

$$\lim_{u\to 0} \frac{\frac{1}{u^{7/2}}}{e^{\frac{cgd}{2u}}}$$

$$\mathcal{L}^{-1}\left[\frac{(1-s)(2-s)}{2s^2}\right] = \frac{1}{2}\mathcal{L}^{-1}\left[\frac{(1-s)(2-s)}{s^2}\right]$$

$$= \frac{1}{2}\mathcal{L}^{-1}\left[\frac{2}{s^2} - \frac{3}{s} + 1\right] = \frac{1}{2}\left[2w - 3 + \delta(t)\right]$$

■ CONTINUE... (Z. Zi PAPER)




$$h_k = \bar{h}_k \sqrt{d_{s,k}^{-\beta_{s,k}}}$$

$\bar{h}_k = CN(0,1)$

$\beta_{s,k}$ – PATH LOSS EXPONENT

$d_{s,k}$ – NORMALIZED DISTANCE TO "k"-TH RELAY

$$f_k = \bar{f}_k \sqrt{d_{k,d}^{-\beta_{k,d}}} \qquad \bar{f}_k \sim CN(0,1)$$

$$\Omega_{h_k} = E[h_k^2] = d_{s,k}^{-\beta_{s,k}} \qquad \Omega_{f_k} = E[f_k^2] = d_{k,d}^{-\beta_{k,d}}$$

■ $E_s$ – TRANSMISSION POWER

k – RELAY
t – TIME SLOT

$$Y_{k,t} = \sqrt{E_s}\, h_k x_t + n_{k,t} \qquad n_{k,t} \sim CN(0, \sigma_n^2)$$

– AMPLIFICATION COEFFICIENT

$$\rho_k = \sqrt{\frac{G_{rk}}{E_s \Omega_{h_k} + \sigma_n^2}} \rightarrow$$

INTEGER IZRAZ ZA BLIND RELAY

$$y_1 = \rho_1 f_1 Y_{1,1} - \rho_2 f_2 Y_{2,2}^* + n_1 \qquad n_1 \sim CN(0, \sigma_n^2)$$

$$y_2 = \rho_1 f_1 Y_{1,2} + \rho_2 f_2 Y_{2,1}^* + n_2$$

$$\hat{x}_1 = \rho_1 f_1^* h_1^* y_1 + \rho_2 f_2 h_2^* y_2^*$$

$$\gamma = \frac{E_s\left(\rho_1^2 |f_1 h_1|^2 + \rho_2^2 |f_2 h_2|^2\right)}{\sigma_n^2\left(\rho_1^2 |f_1|^2 + \rho_2^2 |f_2|^2 + 1\right)}$$

→ VD REFERENCA [15]

$$\hat{x}_1 = \rho_1 f_1^* h_1^* \rho_1 f_1 Y_{1,1} - \rho_1 f_1^* h_1^* \rho_2 f_2 Y_{2,2}^* + \rho_1 f_1^* h_1^* n_1 +$$

$$+ \rho_2 f_2 h_2^* \rho_1^* f_1^* Y_{1,2}^* + \rho_2 f_2 h_2^* \rho_2^* f_2^* Y_{2,1} + \rho_2 f_2 h_2^* n_2^* =$$

($Y_{1,1} = \sqrt{E_s} h_1 x_1 + n_{1,1}$, $Y_{2,2} = \sqrt{E_s} h_2 x_2 + n_{2,2}$, $Y_{1,2} = \sqrt{E_s} h_1 x_2 + n_{1,2}$, $Y_{2,1} = \sqrt{E_s} h_2 x_1 + n_{2,1}$)

$$= \rho_1^2 |f_1|^2 h_1^* Y_{1,1} - \rho_1 \rho_2 f_1^* f_2 h_1^* Y_{2,2}^* + \rho_1 \rho_2 f_1^* f_2 h_2^* Y_{1,2}^* +$$

$$+ |\rho_2|^2 |f_2|^2 h_2^* Y_{2,1} + \rho_1 h_1^* h_1 n_1 + \rho_2 f_2 h_2^* n_2^*$$

$$= \rho_1^2 |f_1|^2 |h_1|^2 \sqrt{E_s} x_1 - \rho_1 \rho_2 f_1^* f_2 h_1^* h_2^* \sqrt{E_s} x_2^* + \rho_1^* \rho_2 f_1^* f_2 h_1^* h_2^* \sqrt{E_s} x_2^* +$$

$$+ |\rho_2|^2 |f_2|^2 |h_2|^2 x_1 + \mu(n)$$

$$= \left(\rho_1^2 |f_1|^2 |h_1|^2 + |\rho_2|^2 |f_2|^2 |h_2|^2\right) x_1 + (-\rho_1 + \rho_1^*) \rho_2 f_1 f_2 h_1^* h_2^* \sqrt{E_s} x_2 + \mu(n)$$

$$= /\rho_1 = \rho_1^*/ = \sqrt{E_s}\left(\rho_1^2 |f_1 h_1|^2 + \rho_2^2 |f_2 h_2|^2\right) x_1 + \mu(n)$$

MAPLE: MULTIHOP MIMO2.mw

$$\mu(n) = \rho_2 f_2 \bar{h}_2 \rho_1 f_1 \bar{h}_{12} + \rho_1^2 f_1^* h_1^* f_1 n_{11} + \rho_1 f_1 h_1 \rho_2 f_2 n_{22} + \rho_1 f_1^* h_1^* n_1 + \rho_2^2 f_2 h_2^* f_2^* n_{21} + \rho_2 f_2 \bar{h}_2 \bar{n}_2$$

$$E[\mu^2(n)] = N_0 \left( \rho_2^2 f_2^2 \bar{h}_2^2 \rho_1^2 \bar{f}_1^2 + \rho_1^4 |f_1|^4 h_1^{*2} - \rho_1^2 \rho_2^2 f_1^2 f_2^2 h_1^2 + \rho_1 f_1 h_1 + \rho_2 f_2 h_2 \right)$$

$$= N_0 \left[ \rho_1^2 \rho_2^2 \bar{f}_1^2 f_2^2 (-\bar{h}_1^2 + \bar{h}_2^2) + \rho_1^4 |f_1|^4 \bar{h}_1^2 + \rho_2^4 |f_2|^4 \bar{h}_2^2 + \rho_1 f_1 h_1 \right.$$

$$= N_0 \left[ \rho_1^2 \rho_2^2 \bar{f}_1^2 f_2^2 (-\bar{h}_1^2 + h_2^2) + \rho_1^4 |f_1|^4 \bar{h}_1^2 + \rho_2^4 |f_2|^4 \bar{h}_2^2 + \rho_1^2 \bar{f}_1 \bar{h}_1 \right]$$

$$E[\mu^2(n)] = N_0 \left[ \rho_1^2 \rho_2^2 \bar{f}_1^2 f_2^2 (-\bar{h}_1^2 + \bar{h}_2^2) + \rho_1^2 |f_1|^2 \bar{h}_1^2 (1 + |f_1|^2 \rho_1^2) + \rho_2^4 |f_2|^4 \bar{h}_{12}^2 + \rho_2^2 |f_2|^2 \bar{h}_{12}^2 \right)$$

$$= N_0 \left[ -\rho_1^2 \rho_2^2 \bar{f}_1^2 f_2^2 \bar{h}_1^2 + \rho_1^2 \rho_2^2 \bar{f}_1^2 f_2^2 \bar{h}_2^2 + \rho_1^2 \bar{f}_1^2 \bar{h}_1^2 (1 + |f_1|^2 \rho_1^2) + \rho_2^4 f_2^4 \bar{h}_{12}^2 + \rho_2^2 |f_2|^2 \bar{h}_{12}^2 \right]$$

TREBA DO APSOLUTNA VREDNOST!!

$$E[|\mu(n)|^2] = \sigma_N^2 \left[ +|\rho_1|^2 |\rho_2|^2 |f_1|^2 |f_2|^4 |h_1|^2 + |\rho_1^2 \rho_2|^2 |f_1|^2 |f_2|^4 |h_{12}|^2 + |\rho_1|^2 |f_1|^2 |h_1|^2 + |\rho_1|^4 |f_1|^4 |h_1|^2 + |\rho_2|^4 |f_2|^4 |h_2|^2 + |\rho_2|^2 |f_2|^2 |h_2|^2 \right.$$

(2), (4), (3), (1), (5), (6)

$$= \sigma_N^2 \left[ \rho_1^2 \rho_2^2 f_1^2 f_2^2 (|h_1|^2 + |h_2|^2) + \rho_1^2 f_1^2 h_1^2 (1 + f_1^2 \rho_1^2) + \rho_2^2 f_2^2 h_2^2 (1 + \rho_2^2 f_2^2) \right.$$

PROVERENO VO MAPLE

IDENTICNO!!!

$$(\rho_1^2 f_1^2 h_1^2 + \rho_2^2 f_2^2 h_2^2)(\rho_1^2 f_1^2 + \rho_2^2 f_2^2 + 1) =$$

$$\rho_1^4 f_1^4 h_1^2 + \rho_1^2 f_1^2 h_1^2 \rho_2^2 f_2^2 + \rho_1^2 f_1^2 h_1^2 + \rho_1^2 \rho_2^2 f_1^2 f_2^2 h_2^2 + \rho_2^4 f_2^4 h_2^2 + \rho_2^2 f_2^2 h_2^2$$

(1), (2), (3), (4), (5), (6)

$$P_N = E[|\mu(n)|^2] = \sigma_N^2 \left( \rho_1^2 |f_1 h_1|^2 + \rho_2^2 |f_2 h_2|^2 \right) \left( \rho_1^2 |f_1|^2 + \rho_2 |f_2|^2 + 1 \right)$$

NOISE POWER

$$P_S = \left( \rho_1^2 |f_1 h_1|^2 + \rho_2^2 |f_2 h_2|^2 \right)^2$$

$$\gamma = \frac{P_S}{P_N} = \frac{P_S}{E[|\mu(n)|^2]} = \frac{\sigma_S (\rho_1 |f_1 h_1|^2 + \rho_2^2 |f_2 h_2|^2)^2}{\sigma_N^2 (\rho_1 |f_1 h_1|^2 + \rho_2 |f_2 h_2|^2)(\rho_1^2 |f_1|^2 + \rho_2 |f_2|^2 + 1)} = (\$)$$

• M-QAM MODULACIJA. CONDITIONAL BER

$$P_b(\gamma) = \frac{1}{\log_2 M} \sum_{j=1}^{\log_2\sqrt{M}} P_{b,j}(\gamma)$$ → IZRAZ OD REFERENCA [17]

$$P_{b,j}(\gamma) = \frac{2}{\sqrt{M}} \sum_{i=0}^{(1-2^{-j})\sqrt{M}-1} \left\{ (-1)^{\lfloor 2^{j-1} \cdot i/\sqrt{M} \rfloor} \left( 2^{j-1} - \left\lfloor \frac{2^{j-1} i}{\sqrt{M}} + \frac{1}{2} \right\rfloor \right) \underbrace{Q\left[(2i+1)\sqrt{\frac{3\gamma}{M-1}}\right]}_{(*)} \right\}$$

• Exact BER is:

$$P_b = E[P_b(\gamma)] \qquad P_b = \frac{1}{\log_2 M} \sum_{j=1}^{\log_2\sqrt{M}} P_{b,j}$$

$$(*) = Q\left[(2i+1)\sqrt{\frac{3\gamma}{M-1}}\right] = Q\left(\sqrt{\frac{3(2i+1)^2}{M-1} \cdot \gamma}\right) = Q\left(\sqrt{g 2\gamma}\right)$$

$$g = \frac{3(2i+1)^2}{2(M-1)} \qquad A = \frac{g}{\sin^2\theta} \qquad g = \frac{3(2i+1)^2}{2(M-1)}$$

$$P_s = \frac{1}{\pi} \int_0^{(M-1)\pi/M} M_\gamma\left(-\frac{g}{\sin^2\theta}\right) d\theta \qquad (*)$$

OVA VAŽI ZA CONDITIONAL SER: DCFC 8.23

$$P_s(E) = \frac{1}{\pi} \int_0^{(M-1)\pi/M} \exp\left(\frac{-g}{\sin^2\theta} E_s N_0\right) d\theta \qquad (\#)$$

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-\frac{t^2}{2}} dt \qquad Q(z) = \frac{1}{2} \operatorname{erfc}\left(\frac{z}{\sqrt{2}}\right)$$

$$\operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-t^2} dt \qquad \operatorname{erfc}\left(\frac{z}{\sqrt{2}}\right) = \frac{2}{\sqrt{\pi}} \int_{z/\sqrt{2}}^{\infty} e^{-t^2} dt =$$

$$u = t \cdot \sqrt{2} \qquad t = \frac{u}{\sqrt{2}}$$

$$t = \frac{z}{\sqrt{2}} \rightarrow u = z \qquad dt = \frac{du}{\sqrt{2}}$$

$$= \frac{2}{\sqrt{\pi}} \int_z^{\infty} e^{-\frac{u^2}{2}} \frac{du}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{\pi}} \int_z^{\infty} e^{-u^2/2} du = 2Q(z)$$

• CRAIG FORMULA:

$$Q(x) = \int_0^{\pi/2} \exp\left(\frac{-x^2}{2\sin^2\theta}\right) d\theta$$

$$Q\left(\sqrt{a\cdot 2\cdot \gamma}\right) = \int_0^{\pi/2} \exp\left(\frac{-g\cdot\gamma}{\sin^2\theta}\right) d\theta \qquad g = \frac{3(2i+1)^2}{2(M-1)}$$

OVA ODGOVARA NA (#) (OSVEN GORNATA GRANICA NA INTEGRALOT)
PA ZATOA VAŽI (*)

$$P_s = \frac{1}{\pi} \int_0^{(M-1)\pi/M} M_\gamma\left(-\frac{3(2i+1)^2}{2(M-1)\cdot\sin^2(\theta)}\right) d\theta$$

$$P'_{b,j} = \frac{1}{\pi} \int_0^{\pi/2} M_\gamma\left(-\frac{3(2i+1)^2}{2(M-1)\sin^2(\theta)}\right) d\theta$$

→ SAMO DELOT SO Q FUNKCIJA

- Celiot $P_{b,j}$ e:

$$P_{b,j} = \frac{2}{\sqrt{M}} \sum_{i=0}^{(1-2^{-j})\sqrt{M}-1} \left\{ (-1)^{\lfloor 2^{j-1} i/\sqrt{M} \rfloor} \left(2^{j-1} - \left\lfloor \frac{2^{j-1} i}{\sqrt{M}} + \frac{1}{2} \right\rfloor\right) \frac{1}{\pi} \int_0^{\pi/2} M_\gamma\left(-\frac{3(2i+1)^2}{2(M-1)\sin^2(\theta)}\right) d\theta \right\}$$

→ OVOJ IZRAZ MOŽE DA GO KORISTIŠ ZA PRESMETKA NA BER ZA BILO KOJ SISTEM SO MGF – $M_\gamma(\dots)$, KOJ KORISTI QAM-M.

$$c = \frac{T}{n_t^s \cdot K \cdot \mathrm{ld} M}$$

448

$$\frac{T}{K} = \frac{8}{4} = 2$$

$n_t^s = 4$

$$c = 2/4 = 1/2$$

$$c = \frac{\text{rate}}{n_t^s}$$

• P.A. Anghel, On The Performance of Distributed Space-Time Coding Systems

$$\mu_r[i+1] = \sqrt{\alpha_R}\, c_{sr}[i]\left[r^*[2i+1], -r^*[2i]\right]^T$$ } "R" uses band "B" to transmit this signal

$$\mu_s[i+1] = \sqrt{\alpha_B}\left[s[2i], s[2i+1]\right]^T$$ } source uses band B to transmitt this signal

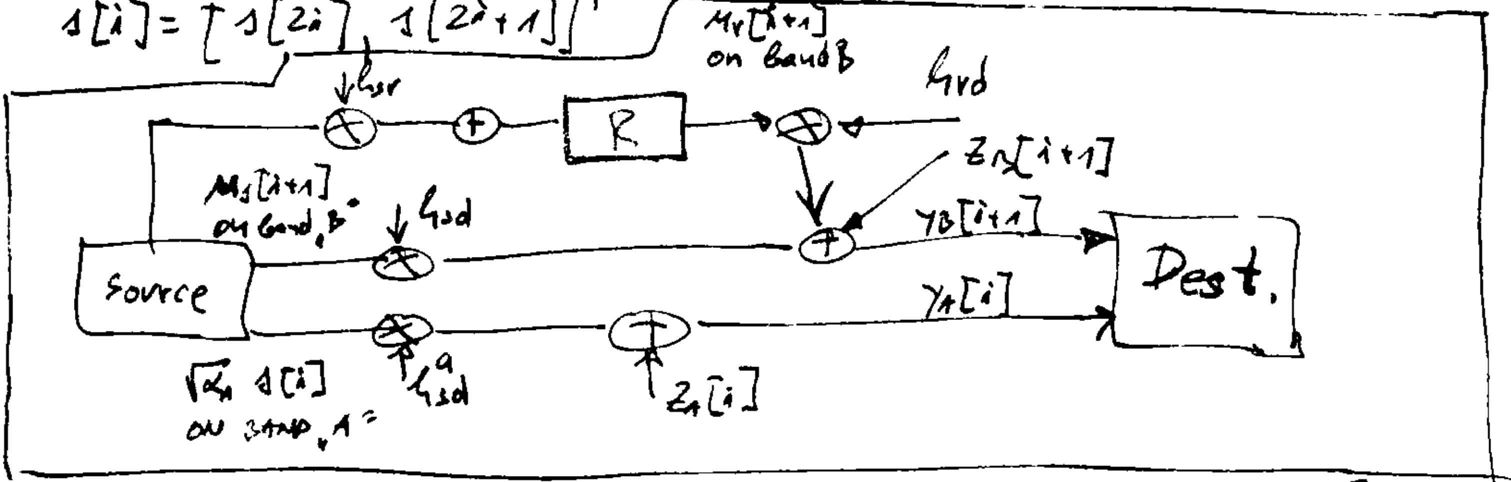
$$r[i] = \left[r[2i], r[2i+1]\right]^T := h_{sr}[i]\sqrt{\alpha_A}\, s[i] + z_r[i]$$

$$z_r[i] = \left[z_r[2i], z_r[2i+1]\right]^T$$ AWGN noise vectors

$$y_A[i] = \left[y_A[2i], y_A[2i+1]\right]^T = h_{sd}^a[i]\sqrt{\alpha_A}\, s[i] + z_A[i] \quad \mu_s[i+1]$$

$$y_B[i+1] = \left[y_B[2i+2]\ \ y_B[2i+3]\right]^T = h_{rd}[i+1]\mu_r[i+1] + h_{sd}[i+1] + z_B[i+1]$$

$$s[i] = \left[s[2i], s[2i+1]\right]^T$$




$$z_A[i] = \left[z_A[2i], z_A[2i+1]\right]^T; \quad z_B[i+1] = \left[z_B[2i+2], z_B[2i+3]\right]^T;$$

• Other assumptions:

$$\alpha_A + \alpha_B \le \varepsilon \quad \text{and} \quad \alpha_R \le \varepsilon_R / N_R$$ (noise of R-D link)

• Interesna rabota e što vo recto izbira

$$c_{sR}[i] = \frac{h_{sr}^*[i]}{|h_{sr}[i]|}$$

$$\epsilon\left[Q\left(\sqrt{k\gamma}\right)\right] \to \frac{\prod_{k=1}^{t}(2i-1)}{2k^t}\ \frac{p_\gamma^{(t-1)}(0)}{t!} \qquad \epsilon[\gamma] \to \infty$$

$$\gamma_{t_1} = \frac{\alpha_A \gamma_{sr} \sigma_R \gamma_{rd} + \alpha_B \gamma_{sd}}{\sigma_R \gamma_{rd} + 1} + \alpha_A \gamma_{sd}^a$$

$$\sigma_R = \alpha_R \cdot N_R$$

$$\gamma_{t_1} < \gamma_{t_1}^u = \alpha_A\left(\gamma_{sr} + \gamma_{sd}\right) + \frac{\alpha_B \gamma_{sd}}{\sigma_R \gamma_{rd} + 1}$$

$$p_\gamma(\mu) = \mu^{t-1}\frac{p_\gamma^{(t-1)}(0)}{(t-1)!} + O(\mu^t)$$

ako pdf-ot može da se napiše vo vakva forma togaš:

• Example (za $2\times1\times2$ sistemot od I.H. Lee

$$t = 2$$

$$p_\gamma^{(1)}(0) = \frac{5}{4c^2 \rho^2}$$

$$P_e = E\left[Q\left(\sqrt{K\gamma}\right)\right] = \frac{1\cdot 3}{2\cdot 2^2}\cdot\frac{1}{2!}\cdot\frac{5}{4c^2\rho^2} = \frac{15}{2^4\cdot 4c^2\rho^2} = \frac{15}{2^6 c^2\rho^2}$$

MSK 2x2x2 $c = \frac{T}{nS\cdot K\cdot \mathrm{ld}M} = \frac{2}{2\cdot 2\cdot 1} = \frac{1}{2}$

- MNOGU ZAUA FINTA E OVA ZOŠTO

$$M(-s) = \int_0^\infty p(\gamma)e^{-s\gamma}d\gamma \qquad P_\Gamma(\gamma) = P_r(\Gamma<\gamma) = \int_0^\gamma p(\gamma)d\gamma$$

$$\boxed{p(\gamma) = \frac{dP_\Gamma(\gamma)}{d\gamma}} \qquad \frac{dM(-s)}{ds} = \int_0^\infty -s\,p(\gamma)e^{-s\gamma}d\gamma$$

$$\boxed{\frac{dM(-s)}{ds} = -s\;M(-s)} \qquad M(0) = \int_0^\infty p(\gamma)d\gamma$$

$$\mathcal{L}\left[p(\gamma)\right] = M(-s) \qquad \mathcal{L}\left[\frac{dP_\Gamma(\gamma)}{d\gamma}\right] = M(-s) \therefore s\cdot\hat{P}(s) = M(-s)$$

$$\hat{P}(s) = \frac{M(-s)}{s} \qquad \boxed{P = \mathcal{L}^{-1}\left[\frac{M(-s)}{s}\right]}$$

$$p(\gamma) = \frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty} M(-s)e^{s\gamma}ds \qquad p(0) = \frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty} M(-s)ds$$

$$\frac{dp(\gamma)}{d\gamma} = \frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty} s\cdot M(-s)e^{s\gamma}ds = \mathcal{L}\left[s\cdot\left[M(-s)\right]\right]$$

$$\boxed{M(s) = \int_0^\infty p(\gamma)e^{s\gamma}d\gamma \qquad \bar{\gamma} = \left.\frac{dM(s)}{ds}\right|_{s=0} = M^{(1)}(0)}$$

$$\frac{dp(0)}{d\gamma} = \frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty} s\cdot M(-s)ds$$

- OD SUŠTINSKO ZNAČENJE SE REFERENCITE [11], [14], [23]
- MNOGU MI IZGLEDA VAŽNO DA SE DOKAŽE KAKO SE DOBIVA MGF-OT VO (25) POAGAJĆI OD (24).

DOKAZANO NA N10.54

- CDF ZA 2x2x2 SISTEM

$$\boxed{M_W(s) = \frac{s^4}{s\,\bar{\gamma}^4}\left[K_4\left(2\sqrt{s/\bar{\gamma}}\right)\right]^2}$$

$$P(W < x) = \mathcal{L}^{-1}\left[\frac{M_W(s)}{s}\right] \leftarrow \text{NIO.25}$$

$$P\left(\frac{1}{\gamma_{eq}} < x\right) = \mathcal{L}^{-1}\left[\frac{M_W(s)}{s}\right] \qquad P\left(\gamma_{eq} > \frac{1}{x}\right) = 1 - P\left(\gamma_{eq} < \frac{1}{x}\right)$$

$$P\left(\gamma_{eq} < \frac{1}{x}\right) = 1 - P\left(\gamma_{eq} > \frac{1}{x}\right) = 1 - P\left(W < \frac{1}{\gamma_{eq}}\right)$$

$$P_{\gamma_{eq}} = 1 - \mathcal{L}^{-1}\left[\frac{M_W(s)}{s}\right]\Bigg|_{w=\frac{1}{\gamma_{eq}}}$$

OVA IZVEDUVANJE E BITNO!!!

$$W = \frac{1}{\gamma_1} + \frac{1}{\gamma_2} \qquad W = \frac{1}{\gamma_{eq}} \qquad W = \frac{1}{\Gamma}$$

$$P_\Gamma(\gamma_{eq}) = P_R(\Gamma < \gamma_{eq}) = P_R\left(\frac{1}{W} < \gamma_{eq}\right) = P_R\left(W > \frac{1}{\gamma_{eq}}\right)$$

$$= 1 - P_R\left(W < \frac{1}{\gamma_{eq}}\right) = 1 - P_W\left(\frac{1}{\gamma_{eq}}\right)$$

MMV

$$P_\Gamma(\gamma_{eq}) = 1 - \mathcal{L}^{-1}\left[\frac{M_W(s)}{s}\right]\Bigg|_{w=\frac{1}{\gamma_{eq}}}$$

$$M_W(s) = \frac{s^4}{9\bar{\gamma}^4} K_4^2\left[2\sqrt{\frac{s}{\bar{\gamma}}}\right]$$

$$\frac{d^{(N-1)}}{ds^{N}}\left(\mathcal{L}^{-1}\left[\frac{M_W(s)}{s^N}\right]\right) = ? \qquad \frac{d}{dw}\left\{\mathcal{L}^{-1}\left[\frac{M_W(s)}{s^N}\right]\right\} =$$

$$= \left[\frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty} \frac{M_W(s)}{s^N} e^{+sw} ds\right]' = \frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty} s\,\frac{M_W(s)}{s^N}\, ds$$

$$\frac{d^{(N-1)}}{dw^{(N-1)}}\mathcal{L}^{-1}\left[\frac{M_W(s)}{s^N}\right] = \mathcal{L}^{-1}\left[\frac{M_W(s)}{s}\right] \Rightarrow \boxed{\text{MMV}}$$

$$P_\Gamma(\gamma_{eq}) = 1 - \mathcal{L}^{-1}\left[\frac{M_W(s)}{s}\right]\Bigg|_{w=\frac{1}{\gamma_{eq}}} = 1 - \frac{d^{(N-1)}}{dw^{(N-1)}}\left[\frac{M_W(s)}{s^N}\right]$$

$$P_\Gamma(\gamma_{eq}) = 1 - \mathcal{L}^{-1}\left[\frac{1}{g\Gamma^4} K_4^2\left[2\sqrt{\frac{1}{\delta}}\right]\right]^{(3)}\Bigg|_{w=\frac{1}{\delta}}$$

$$\boxed{P_\Gamma(\gamma_{eq}) = 1 - \frac{1}{g\Gamma^4}\frac{d^3}{dw^3}\left[\mathcal{L}^{-1}\left\{K_4^2\left[2\sqrt{\frac{1}{\delta}}\right]\right\}\right]\Bigg|_{w=\frac{1}{\delta}}}$$

PRUDNIKOV 3.16.6.6

$$v_\pm = \sqrt{s}\left(\sqrt{b+a} \pm \sqrt{b-a}\right)$$

$$\mathcal{L}\left[\frac{1}{x} e^{-\frac{b}{x}} K_\nu\left(\frac{a}{x}\right)\right] = 2K_\nu(v_-)\cdot K_\nu(v_+)$$

$\boxed{b=a}$ $\quad v_+ = \sqrt{s}\sqrt{2a} \quad v_- = \sqrt{s}\sqrt{2a}$

$$\mathcal{L}\left[\frac{1}{x} e^{-\frac{b}{x}} K_\nu\left(\frac{a}{x}\right)\right] = 2K_\nu^2\left[\sqrt{2s\cdot a}\right]$$

$$= 2K_\nu^2\left[2\sqrt{\frac{s\cdot a}{2}}\right] = 2K_\nu^2\left[2\sqrt{\frac{s}{2/a}}\right] \quad \boxed{\delta = \frac{2}{a}}$$

$$\mathcal{L}^{-1}\left[K_\nu^2\left[2\sqrt{\frac{s}{2/a}}\right]\right] = \frac{1}{2x} e^{-\frac{a}{x}} K_\nu\left(\frac{a}{x}\right)$$

$$a = \frac{2}{\delta} \Rightarrow \mathcal{L}^{-1}\left[K_4^2\left[2\sqrt{\frac{1}{\delta}}\right]\right] = \frac{1}{2x} e^{-\frac{2}{x\delta}} K_4\left(\frac{2}{\delta\cdot x}\right)$$

$$x \triangleq w \quad \boxed{\mathcal{L}^{-1}\left\{K_4^2\left[2\sqrt{\frac{1}{\delta}}\right]\right\} = \frac{1}{2w} e^{-\frac{2}{w\delta}} K_4\left(\frac{2}{\delta w}\right)}$$

- N×N×N SYSTEM

$$\boxed{M_W(-1) = \frac{4}{\Gamma^2(N^2)}\left(\frac{1}{\delta}\right)^{N^2} K_{N^2}^2\left(2\sqrt{\frac{1}{\delta}}\right)}$$

→ OVOJ IZRAZ E VERIFIKUVAN OD MOMENT ČLANAK ZA ICUMT10

T.E. OD WICNC10

T.E. OD I.H.Lee PCF...

$\boxed{N=4}$ $$M_W(-1) = \frac{4}{\Gamma^2(16)}\left(\frac{1}{\delta}\right)^{16} K_{16}^2\left(2\sqrt{\frac{1}{\delta}}\right)$$

$$\mathcal{L}^{-1}\left[K_{16}^2\left[2\sqrt{\frac{1}{\delta}}\right]\right] = \frac{1}{2w} e^{-\frac{2}{w\delta}} K_{16}\left(\frac{2}{\delta\cdot w}\right)$$

} 15-TI IZVOD OD OVA MATRICE NE VAŽI!!!

TREBA DA SE PROBA SO ASIMPTOTSKA APROKSIMACIJA NA Bessel K ZA INTEGER · N

$$\gamma_{t_1} < \gamma_{t_1}^{M} := \alpha_A\left(\gamma_{sd}^{a} + \gamma_{sr}\right) + \frac{\alpha_B \gamma_{sd}}{\sigma_R \gamma_{rd} + 1} \qquad \sigma_R = \alpha_R \cdot N_R$$

$$\xi_d = \frac{\alpha_B \gamma_{sd}}{\sigma_R \gamma_{rd} + 1} \qquad p_{\xi_d}(t) = \int_0^\infty (\sigma_R u + 1)\, p_{\gamma_{sd}}(\sigma_R u t + t)\, p_{\gamma_{rd}}(u)\, du$$

---

☐ ~~POVTORUVANJE OD L. ZANG:~~

$$\gamma_1 = \frac{\bar{\gamma}\, Y Z}{Y + Z} \qquad \boxed{f_Z(z) = \frac{z^{N^2-1}}{\Gamma(N^2)} e^{-z}} \quad \text{GAMMA DISTR.}$$

$$f_{\gamma_1}(\gamma) = \frac{d P_{\gamma_1}(\gamma)}{d\gamma} = \frac{d}{d\gamma} P_r\left(\frac{\bar{\gamma} Y Z}{Y+Z} \le \gamma\right) = \frac{d}{d\gamma} P_r\left(\frac{YZ}{Y+Z} \le \frac{\gamma}{\bar{\gamma}}\right)$$

$$= \frac{d}{d\gamma} \int_0^\infty P_r\left(\frac{Yz}{Y+z} \le \frac{\gamma}{\bar{\gamma}}\right) f_Z(z)\, dz = \frac{d}{d\gamma} \int_0^\infty P_r\left(Yz \le \frac{\gamma}{\bar{\gamma}}(Y+z)\right) f_Z(z)\, dz$$

$$= \frac{d}{d\gamma}\left[\int_0^{\gamma/\bar{\gamma}} P_r\left(Yz \le \frac{\gamma}{\bar{\gamma}}(Y+z)\right) f_Z(z)\, dz + \int_{\gamma/\bar{\gamma}}^\infty P_r\left(Yz \le \frac{\gamma}{\bar{\gamma}}(Y+z)\right) f_Z(z)\, dz\right]$$

$$= \frac{d}{d\gamma}\left[\int_0^{\gamma/\bar{\gamma}} P_r\left[Y\left(z - \frac{\gamma}{\bar{\gamma}}\right) \le \frac{\gamma z}{\bar{\gamma}}\right] f_Z(z)\, dz + \int_{\gamma/\bar{\gamma}}^\infty P_r\left[Y\left(z - \frac{\gamma}{\bar{\gamma}}\right) \le \frac{\gamma z}{\bar{\gamma}}\right] f_Z(z)\, dz\right]$$

$$0 \le z \le \frac{\gamma}{\bar{\gamma}} \Rightarrow P\left[Y\left(z - \frac{\gamma}{\bar{\gamma}}\right) \le \frac{\gamma z}{\bar{\gamma}}\right] = 1$$

$$z = 0 \quad P\left[-\frac{Y\gamma}{\bar{\gamma}} \le 0\right] = 1 \quad \checkmark$$

$$z = \frac{\gamma}{\bar{\gamma}} \quad P\left[0 \le \left(\frac{\gamma}{\bar{\gamma}}\right)^2\right] = 1 \quad \checkmark$$

$$= \frac{1}{\bar{\gamma}} \frac{d}{d\gamma}\left[\int_0^{\gamma/\bar{\gamma}} f_Z(z)\, dz + \int_{\gamma/\bar{\gamma}}^\infty P_r\left[Y\left(z - \frac{\gamma}{\bar{\gamma}}\right) \le \frac{\gamma z}{\bar{\gamma}}\right] f_Z(z)\, dz\right] = \quad \boxed{N1031}$$

USE GENERALIZED LIEBNITZ RULE

$$= \frac{1}{\bar{\gamma}} f_Z\left(\frac{\gamma}{\bar{\gamma}}\right) + \frac{d}{d\gamma}\left[\int_{\gamma/\bar{\gamma}}^\infty P_r\left[Y \le \frac{\gamma z}{\bar{\gamma} z - \gamma}\right] f_Z(z)\, dz\right] =$$

$$= \frac{1}{\bar{\gamma}} f_Z\left(\frac{\gamma}{\bar{\gamma}}\right) + \int_{\gamma/\bar{\gamma}}^\infty \frac{d P_r\left[Y \le \frac{\gamma z}{\bar{\gamma} z - \gamma}\right]}{d\gamma} \frac{\partial\left(\frac{\gamma z}{\bar{\gamma} z - \gamma}\right)}{\partial \gamma} f_Z(z)\, dz - \frac{\partial\left(\frac{\gamma}{\bar{\gamma}}\right)}{\partial \gamma} \cdot f_Z(\ldots) \cdot \lim_{z \to \frac{\gamma}{\bar{\gamma}}} P_r\left[Y \le \frac{\gamma z}{\bar{\gamma} z - \gamma}\right] \quad (\to 1)$$

$$f_{\gamma_1}=\int_{\gamma/\bar\gamma}^{\infty} f_\gamma\left(\frac{\gamma z}{\bar\gamma z-\gamma}\right)\frac{z(\bar\gamma z-\gamma)+\gamma(\bar\gamma z)}{(\bar\gamma z-\gamma)^2}f_Z(z)\,dz=\int_{\gamma/\bar\gamma}^{\infty} f\left(\frac{\gamma z}{\bar\gamma z-\gamma}\right)\frac{\bar\gamma z^2}{(\bar\gamma z-\gamma)^2}f_Z(z)\,dz$$

$$f_{\gamma_1}=\int_{\gamma/\bar\gamma}^{\infty} f_\gamma\left(\frac{\gamma z}{\bar\gamma z-\gamma}\right)\frac{z^2}{\left(z-\frac{\gamma}{\bar\gamma}\right)^2}\cdot\frac{1}{\bar\gamma}f_Z(z)\,dz \qquad f_Y(y)=\frac{y^{N^2-1}}{\Gamma(N^2)}e^{-y}$$

$$f_Z(z)=\frac{z^{N^2-1}}{\Gamma(N^2)}\cdot e^{-z}$$

[illegible]

$$f_{\gamma_1}(\gamma)=\int_{\gamma/\bar\gamma}^{\infty}\frac{\left(\frac{\gamma z}{\bar\gamma z-\gamma}\right)^{N^2-1}}{\Gamma(N^2)}\cdot e^{-\frac{\gamma z}{\bar\gamma z-\gamma}}\cdot\frac{z^2}{\left(z-\frac{\gamma}{\bar\gamma}\right)^2}\;\frac{z^{N^2-1}}{\Gamma(N^2)\,\bar\gamma}e^{-z}\,dz$$

OBIČNA ZAMENA!!!

VO OVOJ SLUČAJ NEMA FUNKCIONALNA TRANSFORMACIJA NA SL. KOM.

$$=\frac{1}{\Gamma^2(N^2)\,\bar\gamma}\int_{\gamma/\bar\gamma}^{\infty}\left(\frac{\gamma z}{\bar\gamma z-\gamma}\right)^{N^2-1}\frac{(\bar\gamma z)^2\cdot z^{N^2-1}}{(\bar\gamma z-\gamma)^2}\,e^{-\frac{\gamma z+\bar\gamma z^2-\gamma z}{\bar\gamma z-\gamma}}\,dz$$

$$f_{\gamma_1}(\gamma)=\frac{1}{\Gamma^2(N^2)\,\bar\gamma}\int_{\gamma/\bar\gamma}^{\infty}\frac{(z)^{2N^2}\cdot\gamma^{N^2-1}\cdot\bar\gamma^2\cdot e^{-\frac{\bar\gamma z^2}{\bar\gamma z-\gamma}}}{(\bar\gamma z-\gamma)^{N^2+1}}\,dz$$

$$f_{\gamma_1}(\gamma)=\frac{\gamma^{N^2-1}\,\bar\gamma^2}{\Gamma^2(N^2)\,\bar\gamma}\int_{\gamma/\bar\gamma}^{\infty}\frac{z^{2N^2}e^{-\frac{\bar\gamma z^2}{\bar\gamma z-\gamma}}}{(\bar\gamma z-\gamma)^{N^2+1}}\,dz=\left|\xi=\frac{\gamma}{\bar\gamma}\right|=$$

$$=\frac{\gamma^{N^2-1}\,\bar\gamma}{\Gamma^2(N^2)}\int_{\xi}^{\infty}\frac{z^{2N^2}e^{-\frac{z^2}{z-\xi}}}{\bar\gamma^{N^2+1}(z-\xi)^{N^2+1}}\,dz=\underbrace{\frac{\gamma^{N^2-1}}{\Gamma^2(N^2)\,\bar\gamma^{N^2}}}_{K}\int_{\xi}^{\infty}\frac{z^{2N^2}e^{-\frac{z^2}{z-\xi}}}{(z-\xi)^{N^2+1}}\,dz$$

VO SPOREDBA SO IZRAZOT NA N9.75 SE RAZLIKUVA VO ZBORO $\bar\gamma$ VO IMENITELOT ZATOA ŠTO TAMU UŠTE NE E SPROVEDENA PROMENATA.

- ZAMENA: $z-\xi=y$ $\quad dz=dy$ $\quad z=\xi \;\; y=0$ $\quad \boxed{z=y+\xi}$

$$f_{\gamma_1}(\gamma)=\underbrace{\frac{\gamma^{N^2-1}}{\Gamma^2(N^2)\,\bar\gamma^{N^2}}}_{K}\int_{0}^{\infty}\frac{(y+\xi)^{2N^2}}{y^{N^2+1}}e^{-\frac{(y+\xi)^2}{y}}\,dy=K\cdot\int_{0}^{\infty}\frac{(y+\xi)^{2N^2}}{y^{N^2+1}}e^{-\left(\frac{y^2+2y\xi+\xi^2}{y}\right)}\,dy$$

$$f_{\gamma_1}(\gamma) = K\int_0^\infty \frac{(y+\xi)^{2N^2}}{y^{N^2+1}} e^{-y-\frac{\xi^2}{y}} \cdot e^{-2\xi}\, dy = K\cdot e^{-2\xi}\int_0^\infty \frac{(y+\xi)^{2N^2}}{y^{N^2+1}} e^{-y-\frac{\xi^2}{y}}\, dy$$

$$= K e^{-2\xi}\int_0^\infty \sum_{k=0}^{2N^2}\binom{2N^2}{k} y^{2N^2-k}\cdot\xi^k \cdot \frac{e^{-y-\frac{\xi^2}{y}}}{y^{N^2+1}}\, dy = K e^{-2\xi}\int_0^\infty \sum_{k=0}^{2N^2}\binom{2N^2}{k} y^{N^2-k-1}\xi^k e^{-y-\frac{\xi^2}{y}}\, dy$$

$$= K e^{-2\xi}\sum_{k=0}^{2N^2}\binom{2N^2}{k}\xi^k\int_0^\infty y^{N^2-k-1} e^{-y-\frac{\xi^2}{y}}\, dy$$

- GRADSHTEYN 3.471.9

$$\int_0^\infty x^{\nu-1} e^{-\frac{\beta}{x}-\gamma x}\, dx = 2\left(\frac{\beta}{\gamma}\right)^{\frac{\nu}{2}} K_\nu\left(2\sqrt{\beta\gamma}\right)$$

$$f_{\gamma_1}(\gamma) = K e^{-2\xi}\sum_{k=0}^{2N^2}\binom{2N^2}{k}\xi^k\cdot 2\left(\frac{\xi^2}{1}\right)^{\frac{N^2-k}{2}} K_{N^2-k}\left(2\sqrt{\xi^2}\right) =$$

$$= \frac{2\gamma^{N^2-1} e^{-2\xi}}{\Gamma^2(N^2)\,\bar\gamma^{N^2}}\cdot\xi^{N^2}\sum_{k=0}^{2N^2}\binom{2N^2}{k}K_{N^2-k}\left(2\sqrt{\xi^2}\right)$$

SAMO ZAMENA ⇓ $\xi = \frac{\gamma}{\bar\gamma}$

$$\boxed{f_{\gamma_1}(\gamma) = \frac{2\gamma^{2N^2-1} e^{-\frac{2\gamma}{\bar\gamma}}}{\Gamma^2(N^2)\,\bar\gamma^{2N^2}}\sum_{k=0}^{2N^2}\binom{2N^2}{k}K_{N^2-k}\left(\frac{2\gamma}{\bar\gamma}\right)}$$

DOKAZANO!!!

⊙ SE VRAĆAM NA ČLANAKOT OD ANGHEL (MMV)

$$\xi_d = \frac{\alpha_B\gamma_{sd}}{\sigma_R\gamma_{rd}+1} = \frac{\alpha_B\Gamma_{sd}}{\sigma_R\Gamma_{rd}+1}$$ ← UNIMANAJ VO ČLANAKOT NA ANGHEL BEZ $\alpha_B$.

$$f_{\Gamma_{sd}}(\gamma_{sd}) = \frac{\gamma^{N^2-1}}{\Gamma(N^2)}e^{-\gamma}$$

$\sigma_R = \alpha_R\cdot N_R$

$$p_{\xi d}(t) = \int_0^\infty (\sigma_R m+1)\, p_{sd}(\sigma_R m t + t)\, p_{rd}(m)\, dm$$

$$p_{\xi d}(t) = \frac{dP_{\xi d}}{dt} = \frac{d}{dt}P_R\left(\frac{\alpha_B\Gamma_{sd}}{\sigma_R\Gamma_{rd}+1}\le t\right) = \frac{d}{dt}\int_0^\infty P_R\left(\frac{\alpha_B\Gamma_{sd}}{\sigma_R X+1}\le t\right)p_{rd}(x)\,dx$$

$$= \frac{d}{dt}\left[\int_0^t P_r\left[\alpha_B\Gamma_{sd}\le(\sigma_R x+1)t\right]p_{rd}(x)\,dx + \int_t^\infty P_r\left[\alpha_B\Gamma_{sd}\le t(\sigma_R x+1)\right]p_{rd}(x)\,dx\right.$$

$$= \frac{d}{dt}\int_0^\infty P_R\left(\frac{\alpha_B x}{\sigma_R\Gamma_{rd}+1}\le t\right)p_{sd}(x)\,dx = \frac{d}{dt}\left[\int_0^t P_R\left(\alpha_B x\le t(\sigma_R\Gamma_{RD}+1)\right)p_{sd}(x)\,dx + \int_t^\infty P_R\left(\alpha_B x\le t(\sigma_R\Gamma_{RD}+1)\right)p_{sd}(x)\,dx\right]$$

$$x=0 \quad P[\alpha_B \cdot 0 \le t(\sigma_R \Gamma_{RD}+1)] = 1$$

$$x=t \quad P[\alpha_B \cdot t \le t(\sigma_R \Gamma_{RD}+1)] = 1$$

**MMV** MNOGU VAZNO IZVEDUVANJE!!!

$$\frac{\xi_d}{\alpha_B} = \frac{\Gamma_{sd}}{\sigma_R \Gamma_{rd}+1} \qquad \boxed{Z = \frac{\xi_d}{\alpha_B}} \qquad Z = \frac{\Gamma_{sd}}{\sigma_R \Gamma_{rd}+1}$$

$$p_Z(z) = \frac{dP_Z(z)}{dz} = \frac{d}{dz} P_R(Z \le z) = \frac{d}{dz} P_R\left(\frac{\Gamma_{sd}}{\sigma_R \Gamma_{rd}+1} \le z\right) =$$

$$= \frac{d}{dz}\int_0^{\infty} P_R\left(\frac{x}{\sigma_R\Gamma_{rd}+1} \le z\right) p_{sd}(x)\,dx = \frac{d}{dz}\int_0^{\infty} P_r\left(x \le z(\sigma_R\Gamma_{rd}+1)\right) p_{sd}(x)\,dx$$

$$= \frac{d}{dz}\int_0^{z} P_r\left(x \le z(\sigma_R\Gamma_{rd}+1)\right) p_{sd}(x)\,dx + \frac{d}{dz}\int_z^{\infty} P_r\left(x \le z(\sigma_R\Gamma_{rd}+1)\right) p_{sd}(x)\,dx$$

$$= \begin{array}{ll} x=0 & P_r(0 \le z(\sigma_r\Gamma_{rd}+1)) = 1 \\ x=z & P_r(z \le z(\sigma_r\Gamma_{rd}+1)) = 1 \end{array} \Bigg/ = p_{sd}(z) + \underbrace{\frac{d}{dz}\int_z^{\infty} P_r\left(x \le z(\sigma_R\Gamma_{rd}+1)\right) p_{sd}(x)\,dx}_{I}$$

- Generalized Liebnitz Rule **MMV**

$$\Phi(\alpha) = \int_{a(\alpha)}^{b(\alpha)} f(x,\alpha)\,dx$$

$$\boxed{\frac{\partial \Phi(\alpha)}{\partial \alpha} = \int_{a(\alpha)}^{b(\alpha)} \frac{\partial f(x,\alpha)}{\partial \alpha}\,dx + \frac{\partial b(\alpha)}{\partial \alpha} f(b,\alpha) - \frac{\partial a(\alpha)}{\partial \alpha} f(a,\alpha)}$$

$$p_Z(z) = p_{sd}(z) + \int_z^{\infty} \frac{\partial P_r\left(\left(\frac{x}{z}-1\right)\frac{1}{\sigma_R} \le \Gamma_{rd}\right)}{\partial\left(\frac{x-z}{z\sigma_R}\right)} \frac{\partial}{\partial z}\left(\frac{x-z}{z\sigma_R}\right) \cdot p_{sd}(x)\,dx +$$

$$+ \underbrace{\frac{\partial \infty}{\partial z}}_{0} P_r\left(x \le z(\sigma_R\Gamma_{rd}+1)\right) - \lim_{x\to z}\left\{\underbrace{P_r\left(x \le z(\sigma_r\Gamma_{rd}+1)\right)}_{1} p_{sd}(z)\right\}$$

$$p_Z(z) = \int_z^{\infty} \frac{\partial}{\partial\left(\frac{x-z}{z\sigma_R}\right)}\left\{1 - P_r\left[\Gamma_{rd} \le \frac{x-z}{z\sigma_r}\right]\right\} \underbrace{\frac{-z-(x-z)}{\sigma_R z^2}}_{-\frac{x}{\sigma_R z^2}} p_{sd}(x)\,dx =$$

$$= \int_z^{\infty} p_{rd}\left(\frac{x-z}{z\sigma_r}\right) \frac{x \cdot p_{sd}(x)}{\sigma_R \cdot z^2}\,dx = \frac{1}{\sigma_R z^2}\int_z^{\infty} p_{rd}\left(\frac{x-z}{z\sigma_R}\right) \cdot x \cdot p_{sd}(x)\,dx$$

$$p_{\mathcal{Z}}(z) = \frac{1}{\sigma_R z^2} \int_z^{\infty} p_{rd}\left(\frac{x-z}{z\sigma_r}\right) \cdot x \cdot p_{sd}(x)\, dx \qquad \left| \begin{array}{l} u = \frac{x-z}{z\sigma_r} \quad du = \frac{dx}{z\sigma_r} \\ x = z \quad u = 0 \\ u z \sigma_r + z = x \end{array} \right|$$

$$p_{\mathcal{Z}}(z) = \frac{1}{\sigma_R z^2} \int_0^{\infty} p_{rd}(u) \cdot (u z \sigma_r + z)\, p_{sd}(u z \sigma_r + z) \cdot z\sigma_r\, du$$

$$p_{\mathcal{Z}}(z) = \frac{1}{z} \int_0^{\infty} p_{rd}(u) \cdot z(u\sigma_r + 1)\, p_{sd}(u z \sigma_r + z)\, du$$

$$p_{\mathcal{Z}}(z) = \int_0^{\infty} p_{rd}(u) \cdot (u\sigma_r + 1) \cdot p_{sd}(u \cdot z\sigma_r + z)\, du$$

VNIMAVAJ VO ANGHEL $\mathcal{Z} \triangleq \xi_d$ MMV @

DOKAZAN IZRAZ: (9a) OD P.A. ANGHEL On The Perfor...

$$\mathcal{Z} = \frac{\xi_d}{\alpha_B}$$

$$\xi_d = \alpha_B \cdot \mathcal{Z}$$

$$x_d = \alpha_B \cdot z \qquad \frac{dx_d}{dz} = \alpha_B$$

$$z = \frac{x_d}{\alpha_B}$$

$$p_{\xi d}(x_d) = \left.\frac{p_{\mathcal{Z}}(z)}{\frac{dx_d}{dz}}\right|_{z = \frac{x_d}{\alpha_B}} = \frac{1}{\alpha_B}\, p_{\mathcal{Z}}\left(\frac{x_d}{\alpha_B}\right)$$

$$\gamma_{t_1}^{M} = \alpha_A(\gamma_{sd}^{a} + \gamma_{sr}) + \frac{\alpha_B \gamma_{sd}}{\sigma_R \gamma_{rd} + 1} \quad (*)$$

$\triangleq \mathcal{Z}$ (VNIMAVAJ VO CLANAKOT NA ANGHEL OVA GO OBELEZUVAAT SO $\xi_d$)

- SEKOGAŠ MORA DA NAJDEŠ pdf ZA NEKOJA SLUČAJNA PROMENLIVA TOGAŠ OBAVEZNO OSLOBODUVAJ SE OD KONSTANTITE ZOŠTO PRI PRESMETKA NA MGF TIE ČISTO MNOŽAT VO ARGUMENTOT OD REZULTANTNATA MGF

$$\gamma_{t_1}^{M} = \alpha_A \gamma_{sd}^{a} + \alpha_A \gamma_{sr} + \alpha_B \cdot \mathcal{Z} = \gamma_1 + \gamma_2 + \xi_d$$

$$MGF = M_1 \cdot M_2 \cdot M_3$$

$$M_3(-s) = \int_0^{\infty} p_{\xi d}(x_d)\, e^{-s x_d}\, dx_d = \int_0^{\infty} \frac{1}{\alpha_B} p_{\mathcal{Z}}\left(\frac{x_d}{\alpha_B}\right) e^{-\alpha_B \cdot z \cdot s}\, dx_d$$

$$= \int_0^{\infty} p_{\mathcal{Z}}(z)\, e^{-z(\alpha_B s)}\, dz = M_{\mathcal{Z}}(-\alpha_B s)$$

MMV

$$P_1^{M} = \frac{1}{\pi} \int_0^{\pi/2} M_{\gamma_{sd}^{a}}\left(-\frac{\alpha_A}{\sin^2\theta}\right) \cdot M_{\gamma_{sr}}\left(-\frac{\alpha_A}{\sin^2\theta}\right) \cdot M_{\mathcal{Z}}\left(-\frac{\alpha_B}{\sin^2\theta}\right) d\theta$$

$\alpha_A + \alpha_D = \varepsilon \qquad \alpha_D = \varepsilon - \alpha_A \qquad (\sigma_R \gamma_{rd} + \ldots$

$$\gamma_{t_1} = \frac{\alpha_A \gamma_{sr} \sigma_R \gamma_{rd} + \alpha_D \gamma_{sd}}{\sigma_R \gamma_{rd} + 1} + \alpha_A \gamma_{sd}^a = \frac{\alpha_A \gamma_{sr} \sigma_R \gamma_{rd} + (\varepsilon - \alpha_A)\gamma_{sd} + \alpha_A \gamma_{sd}^a \ldots}{\sigma_R \gamma_{rd} + 1}$$

$$\gamma_{t_1} = \frac{\alpha_A \gamma_{sr} \sigma_R \gamma_{rd} + \varepsilon \gamma_{sd} - \alpha_A \gamma_{sd} + \alpha_A \gamma_{sd}^a \cdot \sigma_R \gamma_{rd} + \alpha_A \gamma_{sd}^a}{\sigma_R \gamma_{rd} + 1}$$

$$\gamma_{t_1}^{w} = \frac{\alpha_A \sigma_R \gamma_{rd} (\gamma_{sd}^a + \gamma_{sr}) + \alpha_A (\gamma_{sd}^a + \gamma_{sr}) + (\varepsilon - \alpha_A) \cdot \gamma_{sd}}{\sigma_R \gamma_{rd} + 1} =$$

$$\min_{\alpha_A, \sigma_R} \left( \frac{\alpha_A \sigma_R \gamma_{rd} (\gamma_{sd}^a + \gamma_{sr}) + \alpha_A (\gamma_{sd}^a + \gamma_{sr} - \gamma_{sd}) + \varepsilon \gamma_{sd}}{\sigma_R \gamma_{rd} + 1} \right)$$

s.t.: $\alpha_A \leq \varepsilon \qquad \alpha_R \leq \varepsilon_R \qquad \sigma_R = \alpha_R \cdot N_R$

$$p_\gamma(\mu) = \mu^{t-1} \frac{p_\gamma^{(t-1)}(0)}{(t-1)!} + o(\mu^t)$$

$$E[Q(\sqrt{k\gamma})] \to \frac{\prod_{i=1}^{t} (2i-1)\, p_\gamma^{(t-1)}(0)}{2k^t \quad t!} \qquad E[\gamma] \to \infty$$

$$p_{\gamma_{sd}^a}(\mu) = \mu^{t_{sd}^a - 1} \frac{p_{\gamma_{sd}^a}^{(t_{sd}^a - 1)}(0)}{(t_{sd}^a - 1)!} + o(\mu^{t_{sd}^a}); \quad p_{\gamma_{sd}}(\mu) = \mu^{t_{sd}-1} \frac{p_{\gamma_{sd}}^{(t_{sd}-1)}(0)}{(t_{sd}-1)!} + o(\mu^{t_{sd}})$$

$$p_{\gamma_{sr}}(\mu) = \mu^{t_{sr}-1} \frac{p_{\gamma_{sr}}^{(t_{sr}-1)}(0)}{(t_{sr}-1)!} + o(\mu^{t_{sr}})$$

- AKO SE RAZGLEDUVA SAMO $\gamma_{sd}^a + \gamma_{sr}$ OD (*) SLEDI:

$(t_{sd}^a + t_{sr} - 1)$th DERIVATE: ← INITIAL VALUE THEOREM OF LAPLACE TRANSFORM.

$$p_{\gamma_{sd}^a + \gamma_{sr}}^{(t_{sd}^a + t_{sr} - 1)}(0) = \lim_{s \to \infty} s^{t_{sd}^a + t_{sr}} M_{\gamma_{sd}^a + \gamma_{sr}}(s) = \lim_{s \to \infty} s^{t_{sd}^a} M_{\gamma_{sd}^a}(s) \cdot$$

$$\lim_{s \to \infty} s^{t_{sr}} M_{\gamma_{sr}}(s) = p_{\gamma_{sd}^a}^{(t_{sd}^a - 1)}(0)\, p_{\gamma_{sr}}^{(t_{sr}-1)}(0)$$

CONTINUE ... NIO.36

⊙ SHORT JUMP TO A. RIBEIRO, SYMBOL ERROR PROBAB...

$$\bar{P}_e = \int_0^\infty Q(\sqrt{k \bar{\gamma} \hat{\gamma}})\, p_\gamma(\hat{\gamma})\, d\hat{\gamma} \qquad Q(0) = \frac{1}{2}$$

$$Q(z) = \frac{1}{2} \operatorname{erfc} \frac{z}{\sqrt{2}} \qquad Q(0) = \frac{1}{2} \operatorname{erfc}(0) = \frac{1}{2} \cdot 1 = \frac{1}{2}$$

• VO CLANAKOT OD Wang POKAZUVAAT DEKA ZA

$p_\gamma(\bar{\gamma}) = a\,\bar{\gamma}^t + o(\bar{\gamma}) \quad \bar{\gamma} \to \infty$ (TREBA DA SE POMINE CLANAKOT!!!)

$$\bar{P}_e \to \frac{2^t a\,\Gamma(t+3/2)}{\sqrt{\pi}\,(t+1)} (k\bar{\gamma})^{-(t+1)}$$

$$a = \frac{1}{t!} \frac{\partial^t p_\gamma(0)}{\partial \gamma^t}$$

$$\bar{P}_e \to \frac{\prod_{i=1}^{t+1}(2i-1)}{2(t+1)k^{(t+1)}} \cdot \frac{1}{t!} \frac{\partial^t p_\gamma(0)}{\partial \gamma^t}$$

$$\Gamma\left(n+\frac{1}{2}\right) = \frac{\sqrt{\pi}}{2^n} \prod_{k=1}^{n}(2k-1)$$

PROPOSITION 1: CONSIDER THREE NONNEGATIVE INDEPENDENT RANDOM VARIABLES $X, Y, \& Z$

$x_0 = p_X(0) \quad y_0 = p_Y(0) \quad z_0 = p_Z(0)$

ASSUME: $x_0, y_0, z_0 > 0$

$$V = \frac{XY}{X+Y} + Z$$

$$\frac{\partial p_V}{\partial v}(0) = (x_0 + y_0)\,z_0 \quad (*)$$

MOZE TUKA DA SE VMETNE

22,4 GB (24.079.374.180)
22.684 f. 2.006 d.

• STATIKA (HTZ), INSTITUTOT ZA SEIZMOLOGIJA
• ZAKON ZA GRADBA: br. 18 14.02.2011 str. 17.

VIKTOR (070309099)

$\sqrt{\frac{1}{2} + \frac{1}{2}} = \sqrt{1} = 1$

$$g(x,y) = \frac{xy}{x+y}$$

AUXILIARY RANDOM VARIABLE $W = g(X,Y)$

$$p_W(w) = \iint_{\{(x,y):\,g(x,y)=w\}} \frac{p_X(x)\,p_Y(y)}{|\nabla g(x,y)|}\, dx\,dy$$

OVDE TREBA JACOBIAN A NE GRADIENT!!! (PLONKIS 2.1.2) ISTOTO SE DOBIVA!! VIDI/ MATLE: Multinomial...

Norm( )

$\nabla g(x,y)$ – GRADIENT OF $g(x,y)$

$$|\nabla g(x,y)| = \left\| \frac{\partial g(x,y)}{\partial x}\vec{i} + \frac{\partial g(x,y)}{\partial y}\vec{j} \right\| = \sqrt{\left(\frac{\partial g(x,y)}{\partial x}\right)^2 + \left(\frac{\partial g(x,y)}{\partial y}\right)^2}$$

$$|\nabla g(x,y)| = \sqrt{\frac{x^4+y^4}{(x+y)^4}}$$

$$W = \frac{XY}{X+Y}$$

$$w = \frac{xy}{x+y} = 0$$

$$p_W(0) = p_X(0)\underbrace{\int_0^\infty \frac{p_Y(y)\,dy}{\nabla g(0,y)}}_{I_1} + \underbrace{p_Y(0)\int_0^\infty \frac{p_X(x)}{\nabla g(x,0)}\,dx}_{I_2} = I_1 + I_2$$

$$p_W(0) = \iint_{\{(x,y):\, g(x,y)=0\}} \frac{p_X(x)\,p_Y(y)}{|\nabla g(x,y)|}\,dx\,dy$$

VIDI IZVEDOVANJE DOKONČ!!!

---

$$g(x,y) = \frac{x \cdot y}{x+y} \qquad J = \begin{vmatrix} \frac{\partial g}{\partial x} & \frac{\partial g}{\partial y} \\ \frac{\partial h}{\partial y} & \frac{\partial h}{\partial x} \end{vmatrix} = \begin{bmatrix} \frac{\partial g}{\partial x} & \frac{\partial g}{\partial y} \\ 1 & 1 \end{bmatrix}$$

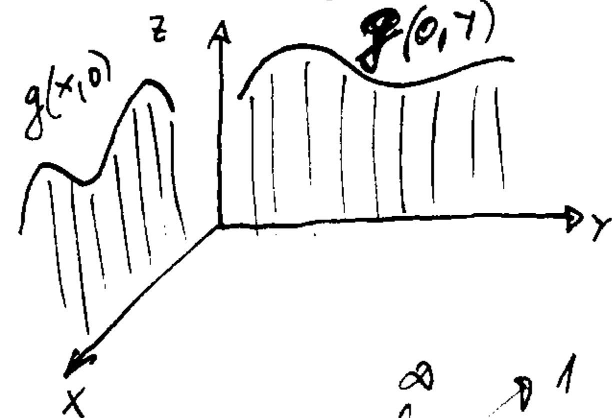
$$h(x,y) = x + y$$

$$= \left[\frac{\partial g}{\partial x} \quad \frac{\partial g}{\partial y}\right] \qquad \boxed{??}$$

---

$\boxed{x=0}$ $$I_1 = \iint_{(0,y):\, g(0,y)=w} \frac{p_X(0)\cdot p_Y(y)}{|\nabla g(x,y)|}\,dy$$

ZNAČI NE INTEGRIRAŠ PO OBLASTI (RAVNI) TEMVEČ PO INTERVAL!




$$I_1 = p_X(0)\int_0^\infty \frac{p_Y(y)}{|\nabla g(0,y)|}\,dy$$

$$|\nabla g(0,y)| = \sqrt{\frac{y^4}{y^4}} = 1$$

$$|\nabla g(x,0)| = \sqrt{\frac{x^4}{x^4}} = 1$$

$$p_W(0) = p_X(0)\int_0^\infty p_Y(y)\,dy + p_Y(0)\int_0^\infty p_X(x)\,dx = p_X(0) + p_Y(0)$$

(the integral $\int_0^\infty p_Y(y)\,dy \to 1$)

$\boxed{w_0 = x_0 + y_0}$ DOKAZANO!!!

$$L_V(s) = L_W(s)\cdot L_Z(s)$$

$$\boxed{\begin{aligned} L_W(s) &= \mathcal{L}[p_W(w)] = M_W(-s) \\ L_Z(s) &= \mathcal{L}[p_Z(z)] = M_Z(-s) \end{aligned}}$$

• INITIAL VALUE THEOREM FOR LAPLACE TRANSFORM: (N10.2-3)

$$f^{(r)}(0) = \lim_{s\to\infty} s^{r+1} F(s)$$

BIDEJĆI $V = W + Z \Rightarrow M_V = M_W \cdot M_Z$ PA AKO VRZ OVA SE PRIMENI INITIAL VALUE THEOREM SE DOBIVA $

N10.34 (*) $\Rightarrow$ $\frac{\partial p_V(0)}{\partial v} = (p_X(0) + p_Y(0)) \cdot p_Z(0) = p_W(0) \cdot p_Z(0)$

OD OVDE SLEDI OVA!!

$$\frac{\partial p_V(0)}{\partial v} = \lim_{s\to\infty} s M_W(-s) \cdot \lim_{s\to\infty} s \cdot M_Z(-s)$$ $

$$\frac{\partial p_V(0)}{\partial v} = \lim_{s\to\infty} s^2 \overbrace{M_W(-s) M_Z(-s)}^{M_V(-s)}$$

---

• RETURN BACK TO P.A. ANGHEL, On the performance... (CONTINUE FROM N10.33) (UPOTREBUVAM SO NOMENKLATURA OD ČLANAKOT)

$$p_{\xi d}^{(tsd-1)}(u) = \int_0^\infty (1+\sigma_R v)^{tsd} p_{\gamma sd}^{(tsd-1)}(u + \sigma_R \cdot u \cdot v) \, p_{\gamma N}(v) \, dv$$ SLEDI!

$$p_{\xi d}(u) = \int_0^\infty (\sigma_R v + 1) \, p_{\gamma sd}(u + \sigma_R u \cdot v) \, p_{\gamma rd}(v) \, dv$$ (@) N10.32 $\xi d \triangleq Z$

$$p'_{\xi d}(u) = \int_0^\infty (\sigma_R v + 1) \cdot (1 + \sigma_R v) \, p'_{\gamma sd}(u + \sigma_R u \cdot v) \, p_{\gamma rd}(v) \, dv$$

$$p''_{\xi d}(u) = \int_0^\infty (\sigma_R v + 1)^3 \, p''_{\gamma sd}(u + \sigma_R u \cdot v) \, p_{\gamma rd}(v) \, dv$$

(&#)

$$p_{\xi d}^{(tsd-1)}(0) = p_{\gamma sd}^{(tsd-1)}(0) \underbrace{\int_0^\infty (1+\sigma_R v)^{tsd} \, p_{\gamma rd}(v) \, dv}_{E\left[(1+\sigma_R v)^{tsd}\right]}$$ $\triangleq \gamma_{rd}$

$$p_{\xi d}^{(tsd-1)}(0) = p_{\gamma sd}^{(tsd-1)}(0) \, E\left[(1+\sigma_R \gamma_{rd})^{tsd}\right]$$ (#)

DOKAZANO!!!

$$\gamma_{t_1}^M = \alpha_A \gamma_{sd}^a + \alpha_A \gamma_{sr} + \alpha_B \xi_d$$

$$M_{\gamma_{t_1}^M}(-s) = M_{\gamma_{sd}^a}(\alpha_A s) \cdot M_{\gamma_{sr}}(\alpha_A \cdot s) \cdot M_{\xi d}(\alpha_B \xi_d)$$ MMV

- BY USING INITIAL VALUE THEOREM

$$p_{\gamma_{t_1}^M}^{(t_{sd}^a + t_{sr} + t_{\xi d} - 1)}(0) = \lim_{s\to\infty} s^{t_{sd}^a + t_{sr} + t_{\xi d}} M_{\gamma_{sd}^a}(\alpha_A s) \cdot M_{\gamma_{sr}}(\alpha_A \cdot s) \cdot M_{\xi d}(\alpha_B \xi_d)$$

$$t = t_{sd}^a + t_{sr} + t_{\xi d}$$

$$= \lim_{s\to\infty} s^{t_{sd}^a} \cdot M_{\gamma_{sd}^a}(\alpha_A s) \cdot \lim_{s\to\infty} s^{t_{sr}} \cdot M_{\gamma_{sr}}(-\alpha_A s) \cdot \lim_{s\to\infty} \left[M_{\xi d}(-\alpha_B \xi_d) \cdot s^{t_{\xi d}}\right]$$

$$p^{(t-1)}_{\gamma^{sm}_{t_1}}(0) = \frac{\lim_{s\to\infty}(\alpha_A s)^{t^a_{sd}} M_{\gamma^a_{sd}}(-\alpha_A s)}{\alpha_A^{t^a_{sd}}} \cdot \frac{\lim_{s\to\infty}(\alpha_A s)^{t_{sr}} M_{\gamma^s_{sr}}(-\alpha_A s)}{\alpha_A^{t_{sr}}} \cdot \frac{\lim_{s\to\infty}(\alpha_B s) M_{\gamma^s_{gd}}}{\alpha_B^{t_{gd}}}$$

(1)

$$p^{(t-1)}_{\gamma^{sm}_{t_1}}(0) = \frac{p^{(t^a_{sd}-1)}_{\gamma^a_{sd}}(0)}{\alpha_A^{t^a_{sd}}} \cdot \frac{p^{(t_{sr}-1)}_{\gamma^s_{sr}}(0)}{\alpha_A^{t_{sr}}} \cdot \frac{p^{(t_{gd}-1)}_{\gamma^s_{gd}}(0)}{\alpha_B^{t_{gd}}}$$ ← (#)

MHV

$$p_{\gamma^{sm}_{t_1}}(0) = E\left[(1+G_R\gamma_{rd})^{t_{sd}}\right] \frac{p^{(t_{sd}-1)}_{\gamma^a_{sd}}(0)}{\alpha_A^{t^a_{sd}}} \cdot \frac{p^{(t_{sr}-1)}_{\gamma_{sr}}(0)}{\alpha_A^{t_{sr}}} \cdot \frac{p^{(t_{gd}-1)}_{\gamma^s_{gd}}(0)}{\alpha_D^{t_{gd}}}$$

(*$)

$$E\left[Q\left(\sqrt{k\gamma}\right)\right] = \frac{\prod_{i=1}^{t}(2i-1)}{2k^t} \frac{p_\gamma^{(t-1)}(0)}{t!}$$

OVA GO DOBIVAŠ ILI SO (*$) ILI SO (1) !!!

• PROPOSITION 1: $Z = \max\{X, Y\}$ X, Y ARE RANDOM VARIABLES THAT ADMITS TAYLOR SERIES EXPANSION ABOUT ZERO. THE PDF OF Z COMPUTED AT ZERO CAN BE WRITEN AS:

$$p_Z^{(t-1)}(0) = C_t^{t_x} p_X^{(t_x-1)}(0)\, q_Y^{(t_y-1)}(0)$$

WHERE $t_x, t_y$ ARE SMALLEST INTEGERS GREATE OR EQUAL TO 1 FOR WHICH:

$$p_X^{(t_x-1)}(0) \neq 0 \qquad p_Y^{(t_y-1)}(0) \neq 0$$ "

PROOF:

$$p_Z(n) = p_X(n)P_Y(n) + P_X(n)p_Y(n)$$

KAKO DOAĐA DO OVA! PRETPOSTAVUVAM OTTAMU ŠTO SE ZNAE DEKA $P_Z(x) = P_X(x)\cdot P_Y(x)$

$$P_Z(x) = P_r(Z \le x) \qquad P_X(x) = P_r(X \le x) \quad P_Y(x) = P_r(Y \le x)$$

$$\frac{dP_Z(x)}{dx} = p_Z(x) \qquad p_Z(x) = P_Y(x)\frac{dP_X(x)}{dx} + P_X(x)\frac{dP_Y(x)}{dx}$$

$$p_Z(x) = p_X(x)\cdot P_Y(x) + p_Y(x)\cdot P_X(x)$$

$t = t_x + t_y$

DOKAZ!!!

EXAMPLE: $t_x = 1 \quad t_y = 1 \qquad t = t_x + t_y = 1 + 1 = 2$

$$\frac{d^2}{dx}p_Z(x) = \frac{d}{dx}\left(p_X'(x)P_Y(x) + p_Y'(x)\cdot P_X(x) + p_Y(x)P_X(x) + p_Y(x)P_X'(x)\right)$$

$$= p_X''P_Y(x) + p_X'(x)P_Y'(x) + p_Y'(x)P_X(x) + p_Y'\cdot p_X' + p_Y''P_X(x) + p_Y'P_X + p_Y'p_X' + p_Y(x)p_X''(x)$$

$P_X(0)=0 \quad P''_X(0)=0 \quad P''_Y(0)=0 \quad P_Y(0)=0$

$$\boxed{P_Z^{(t-1)}(0) = P_X^{(t_x-1)}(0)\cdot P_Y^{(t_y-1)}(0)} \quad \text{FOR: } Z=\max(X,Y)$$

- ZNAČI SE DOBIVA ISTE POLOMAKTEN IZRAZ OD N10.37

$$\gamma_{t_1}^{(1)} = \varepsilon_R\gamma_{rd}\,\varepsilon\gamma_{sr}/(\varepsilon_R\gamma_{rd}+1) + \varepsilon\gamma_{sd}^a \qquad \gamma_{t_1}^{(2)}=\varepsilon\gamma_{sd}$$

IT: BOBAN TODOROVSKI

$$\gamma_{t_1}^0 = \max\{\gamma_{t_1}^{(1)},\gamma_{t_2}^{(2)}\}$$

$$P_{\gamma_{t_1}^0}^{(t-1)}(0) = C_t^{t_{sd}}\,\frac{P_{\gamma_{sd}}^{(t_{sd}-1)}(0)}{\varepsilon^{t_{sd}}}\cdot P_{\gamma_{t_1}^{(1)}}^{(t_{sd}^a+t_{sr}-1)}(0)$$

- MALKU BUNI ŠTO VO ČLANAKOT NA RISERIO SE RABOTI SO $W=XY/(X+Y)$, A OVDE VO ČLANAKOT NA ANGHEL SE RABOTI SO $\boxed{XY/(X+1)}$

$$\xi_r = \frac{\varepsilon_R\gamma_{rd}\gamma_{sr}}{\varepsilon_R\gamma_{rd}+1}\ ; \quad P_{\xi_r}^{(t_{sr}-1)}(0) = P_{\gamma_{sr}}^{(t_{sr}-1)}(0)\int_0^\infty\left(\frac{1+\varepsilon_R\gamma_{rd}}{\varepsilon_R\gamma_{rd}}\right)^{t_{sr}} f_{\gamma_{rd}}(v)\,dv$$

N10.36

$$P_{\xi_r}^{(t_r-1)}(0) = E\left[\left(\frac{1+\varepsilon_r\gamma_{rd}}{\varepsilon_R\gamma_{rd}}\right)^{t_{sr}}\right]\cdot P_{\gamma_{sr}}^{(t_r-1)}(0)$$

$$P_{\gamma_{t_1}^{(1)}}^{(t_{sd}^a+t_{sr}-1)}(0) = \lim_{s\to\infty} s^{t_{sd}^a+t_{sr}}\, M_{\xi_R}(\varepsilon\cdot s)\, M_{\gamma_{sd}^a}(\varepsilon s) =$$

$$= \frac{\lim_{s\to\infty}(\varepsilon s)^{t_{sd}^a}M_{\gamma_{sd}^a}(\varepsilon s)}{\varepsilon^{t_{sd}^a}}\cdot\frac{\lim(\varepsilon s)^{t_{sr}}M_{\xi_R}(\varepsilon\cdot s)}{\varepsilon^{t_{sr}}}$$

$$\boxed{P_{\gamma_{t_1}^{(1)}}^{(t_{rd}^a+t_{sr}-1)}(0) = \frac{P_{\gamma_{sd}^a}^{(t_{sd}^a-1)}(0)\cdot P_{\xi_R}^{(t_{sr}-1)}(0)}{\varepsilon^{t_{sd}^a+t_{sr}}}}$$

ZAMENI!

$$t=t_{sd}+t_{sd}^a+t_{sr}$$

$$P_{\gamma_{t_1}^0}^{(t-1)}(0) = C_t^{t_{sd}}\,\frac{P_{\gamma_{sd}}^{(t_{sd}-1)}(0)\cdot P_{\gamma_{sd}^a}^{(t_{sd}^a-1)}(0)\cdot P_{\gamma_{sr}}^{(t_{sr}-1)}(0)\cdot E\left[\left(\frac{1+\varepsilon_r\gamma_{rd}}{\varepsilon_R\gamma_{rd}}\right)^{t_{sr}}\right]}{\varepsilon^{t_{sd}^a+t_{sr}+t_{sd}}}$$

$$\boxed{P_{\gamma_{t_1}^0}^{(t-1)}(0) = C_t^{t_{sd}}\cdot E\left[\left(\frac{1+\varepsilon_r\gamma_{rd}}{\varepsilon_R\gamma_{rd}}\right)^{t_{sr}}\right]\cdot\frac{P_{\gamma_{sd}}^{(t_{sd}-1)}(0)\,P_{\gamma_{sd}^a}^{(t_{sd}^a-1)}\,P_{\gamma_{sr}}^{(t_{sr}-1)}(0)}{\varepsilon^t}}$$

DOKAZAN IZRAZ (11) OD P.A. ANGHEL

## C. Variable Amplification at the Relay

- Automatic gain controller:

$$\alpha_R = \frac{\beta_R}{e_{sr}(\gamma_{sr}+1)} \qquad \beta_R \leq \frac{E_R}{N_R}$$

$$e_{sr} = E\left[1/(\gamma_{sr}+1)\right] \qquad \gamma_{t_1} = \frac{\alpha_A \gamma_{sr} \sigma_R \gamma_{rd}}{e_{sr}(\sigma_R\gamma_{rd}/e_{sr} + \gamma_{sr}+1)} + \frac{(\gamma_{sr}+1)\alpha_B \gamma_{rd}}{\sigma_R\gamma_{rd}/e_{sr}+\gamma_{sr}+1} + \alpha_A \gamma_{sd}^{a}$$

$$\gamma_{t_1} = \frac{\alpha_A \gamma_{sr}\sigma_R\cdot\gamma_{rd} + \alpha_B\gamma_{sd}}{\sigma_R\gamma_{Rd}+1} + \alpha_A\gamma_{sd}^{a} \qquad \sigma_R = \beta_R \cdot N_R$$

$$\gamma_{t_1} = \frac{\alpha_A \cdot \gamma_{sr} \cdot \frac{\overset{\sigma_R}{(\beta_R N_R)}\gamma_{rd}}{e_{sr}(\gamma_{sr}+1)}}{\sigma_R \frac{(\beta_R N_R)\gamma_{rd}}{e_{sr}(\gamma_{sr}+1)}+1} + \frac{\alpha_B\gamma_{sd}}{\frac{\sigma_R\gamma_{rd}}{e_{sr}(\gamma_{sr}+1)}+1} + \alpha_A\gamma_{sd}$$

$$\gamma_{t_1} = \frac{\alpha_A\gamma_{sr}\cdot\sigma_R\cdot\gamma_{rd}}{\sigma_R\cdot\gamma_{rd} + e_{sr}(\gamma_{sr}+1)} + \frac{\alpha_B\gamma_{sd}\, e_{sr}(\gamma_{sr}+1)}{e_{sr}\left[\sigma_R\gamma_{rd}/e_{sr} + \gamma_{sr}+1\right]} + \alpha_A\gamma_{sd}$$

$$\boxed{\gamma_{t_1} = \frac{\alpha_A\gamma_{sr}\sigma_R\cdot\gamma_{rd}}{e_{sr}(\sigma_R\cdot\gamma_{rd}/e_{sr} + \gamma_{sr}+1)} + \frac{\alpha_B\gamma_{sd}(\gamma_{sr}+1)}{\sigma_R\gamma_{rd}/e_{sr}+\gamma_{sr}+1} + \alpha_A\gamma_{sd}}$$

$$\tilde{\gamma}_{t_1} = \frac{\alpha_A\gamma_{sr}\sigma_R\cdot\gamma_{rd}}{e_{sr}(\sigma_R\gamma_{rd}/e_{sr}+\gamma_{sr}+1)} + \frac{\alpha_B\gamma_{sd}}{\sigma_R\gamma_{rd}+1} + \alpha_A\gamma_{sd}^{a}$$

$$\tilde{\gamma}_{t_1} = \alpha_A\cdot\xi(\sigma_R) + \alpha_B\xi_d + \alpha_A\gamma_{sd}^{a} \qquad \xi(\sigma_R) = \frac{\sigma_R}{e_{sr}}\frac{\gamma_{sr}\gamma_{rd}}{(\sigma_R\frac{\gamma_{rd}}{e_{sr}}+\gamma_{sr}+1)}$$

MGF of $\xi(\sigma_R)$ is only unknown

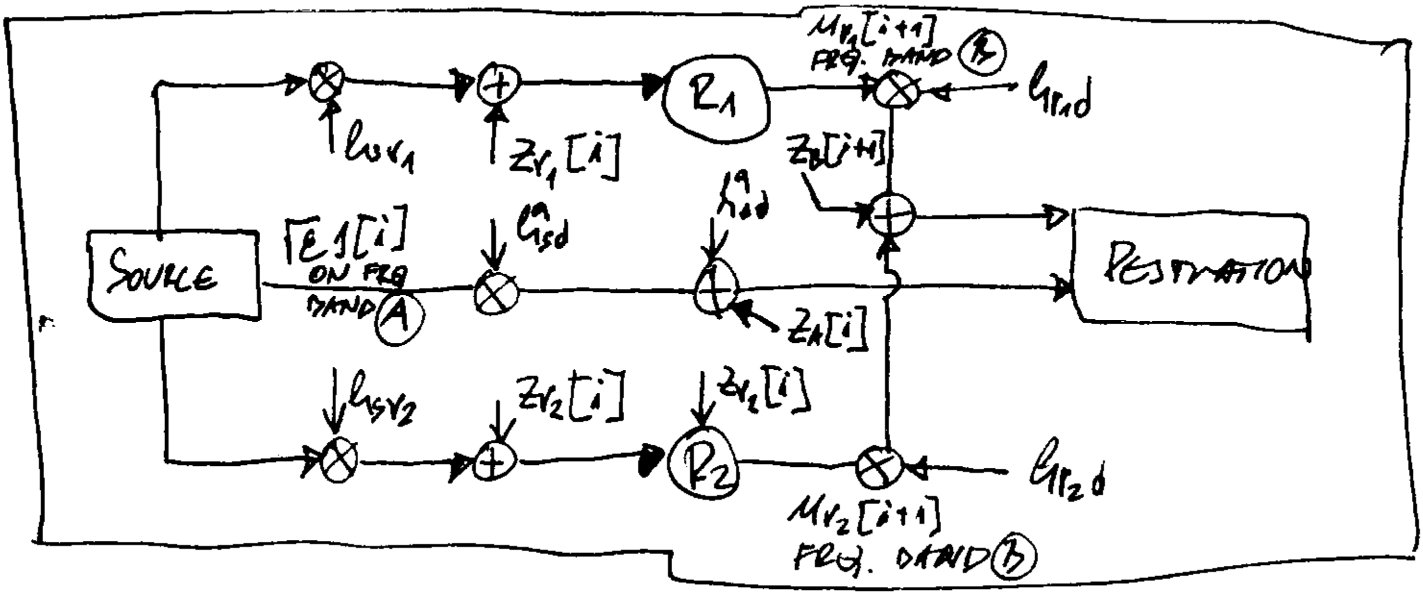
## IV. Extension to a two relay system

$$u_{r_1}[i+1] = \sqrt{\alpha_{R_1}}\frac{h_{sr_1}[i]}{|h_{sr_1}[i]|}\left[r_1^*[2i+1], -r_1^*[2i]\right]^T$$

$$u_{r_2}[i+1] = \sqrt{\alpha_{R_2}}\frac{h_{sr_2}[i]}{|h_{sr_2}|} r_2[i]$$

$$y_D[i+1] = h_{r_1d}[i+1]\,u_{r_1}[i+1] + h_{r_2d}[i+1]\,u_{r_2}[i+1] + z_B[i+1]$$

A2.3 $\alpha_{R_l} \in [|h_{r_l d}|^2] \geq N_0/N_{R_l} \quad l \in \{1,2\}$




$$r_q[i] = [r_q[2i], r_q[2i+1]]^T = h_{sr_q}[i]\sqrt{\alpha_A}\,1[i] + z_{r_q}[i] \quad q \in \{1,2\}$$

$$\alpha_A = \varepsilon \qquad \alpha_{R_q} \le \varepsilon_{R_q}/N_{R_q} \quad q \in [1,2]$$

- THE RECEIVER SHOULD USE FOLOWING MATRIX FOR ELIMINATING ISI:

$$H_2 = \begin{bmatrix} \sqrt{\alpha_{R_2}}\,|h_{sr_2}[i]|\,h_{r_2d}[i+1], & \sqrt{\alpha_{R_1}}\,|h_{sr_1}[i]|\,h_{r_1d}[i+1] \\ -\sqrt{\alpha_{R_1}}\,|h_{sr_1}[i]|\,h^*_{r_1d}[i+1], & \sqrt{\alpha_{R_2}}\,|h_{sr_2}[i]|\,h^*_{r_2d}[i+1] \end{bmatrix}$$

$$\gamma_{t_2}(\sigma) = \frac{\sigma_{R_1}\gamma_{r_1d}\,\varepsilon\,\gamma_{sr_1} + \sigma_{R_2}\gamma_{r_2d}\,\varepsilon\,\gamma_{sr_2}}{\sigma_{R_1}\gamma_{R_1d} + \sigma_{R_2}\gamma_{r_2d} + 1} + \varepsilon\gamma^a_{sd}$$

→ ZA IMPLEMENTACIJA NA PRIEMNIKOT (MRC)

$$\sigma = [\sigma_{R_1}, \sigma_{R_2}]^T \qquad \sigma_{R_q} = \alpha_{R_q} N_{R_q} \quad q \in [1,2]$$

## A. ERROR PERFORMANCE

- UPPER BOUND OF $\gamma_{t_2}(\sigma)$

$$\gamma^u_{t_2} = \varepsilon\left[\rho_d\gamma_{sr_1} + (1-\rho_d)\gamma_{sr_2} + \gamma^a_{sd}\right]$$

$$\rho_d = \frac{\sigma_{R_1}\gamma_{r_1d}}{\sigma_{R_1}\gamma_{r_1d} + \sigma_{R_2}\gamma_{r_2d}} \le 1 \quad (*)$$

- RANDOM VARIABLES $\rho_d\gamma_{sr_1}$ AND $(1-\rho_d)\gamma_{sr_2}$ CONDITIONED ON $\rho_d$ ARE INDEPENDENT THEREFORE:

$$\xi_s = \rho_d\gamma_{sr_1} + (1-\rho_d)\gamma_{sr_2} = \xi_{s1} + \xi_{s2}$$

$$M_{\xi_s}(s) = \int_0^1 M_{\gamma_{sr_1}}(\rho s)\, M_{\gamma_{sr_2}}((1-\rho)s)\, p_{\rho_d}(\rho)\, d\rho$$

OVA E MNOGU INTERESEN IZRAZ !!!

$$p_{\xi_S}(\gamma) = \frac{d P_R(\xi_S \leq \gamma)}{d\gamma} = \frac{d}{d\gamma} P_R\left(\rho_d \gamma_{sr_1} + (1-\rho_d)\gamma_{sr_2} \leq \gamma\right)$$

$$= \frac{d}{d\gamma} \int_0^\infty P_R\left(\rho\, \gamma_{sr_1} + (1-\rho)\gamma_{sr_2} \leq \gamma\right) p_{\rho_d}(\rho)\, d\rho =$$

$$= \frac{d}{d\gamma} \int_0^\infty P_R\left(\rho(\gamma_{sr_1} - \gamma_{sr_2}) + \gamma_{sr_2} \leq \gamma\right) p_{\rho_d}(\rho)\, d\rho$$

$\rho \in [0, \gamma]$ $\quad \rho = 0 \quad 0(\gamma_{sr} - \gamma_{sr}) + \gamma_{sr_2} \leq \gamma$

$\rho = \gamma \quad \gamma\gamma_{sr_1} - \gamma\gamma_{sr_2} + \gamma_{sr_2} \leq \gamma$

$$\gamma\gamma_{sr_1} + \gamma_{sr_2}(1-\gamma) \leq \gamma$$

---

$\xi_{S_1} = \rho_d \gamma_{sr_1}$

$$p_{\xi_{S_1}}(\gamma) = \frac{d P_R(\xi_{S_1} \leq \gamma)}{d\gamma} = \frac{d}{d\gamma} P_R\left(\rho_d \gamma_{sr} \leq \gamma\right)$$

$$= \frac{d}{d\gamma} \int_0^\infty P_R(x\gamma_{sr} \leq \gamma)\, p_{\rho_d}(x)\, dx$$

$\boxed{\gamma_{sr} > 1}$

$x = 0: \; P_R(0 \leq \gamma) = 1$

$x = \gamma: \; P_R(\underline{\gamma \cdot \gamma_{sr}} \leq \gamma) = 0$

$x = \infty \; P_R(\infty \cdot \gamma_{sr} \leq \gamma) = 0$

---

- Vidi N10.32:

$$M_{\xi_S}(s) = M_{\xi_{S_1}}(s) \cdot M_{\xi_{S_2}}(s) = M_{\gamma_{sr_1}}(\rho_d s) \cdot M_{\gamma_{sr_2}}((1-\rho_d)s)$$

za $\underline{\rho_d = \text{const}}$

$$M_{\xi_S}(s) = \int_0^\infty M_{\gamma_{sr_1}}(\rho s) \cdot M_{\gamma_{sr_2}}(s - \rho s) \cdot p_{\rho_d}(\rho)\, d\rho =$$

(*) N.10.40

$$= \int_0^\infty M_{\gamma_{sr_1}}(\rho s)\, M_{\gamma_{sr_2}}((1-\rho)s)\, p_{\rho_d}(\rho)\, d\rho = \underline{\int_0^1 M_{\gamma_{sr_1}}(\rho s)\, M_{\gamma_{sr_2}}(s-\rho s)\, p_{\rho_d}(\rho)\, d\rho}$$

$$(1-\rho_d)\gamma_{sr_2} = \gamma_{sr_2}\frac{\sigma_{R_1}\gamma_{r_1d} + \sigma_{R_2}\gamma_{r_2d} - \sigma_{R_1}\gamma_{r_1d}}{\sigma_{R_1}\gamma_{r_1d} + \sigma_{R_2}\gamma_{r_2d}} = \frac{\sigma_{R_2}\gamma_{r_2d}}{\sigma_{R_1}\gamma_{r_1d} + \sigma_{R_2}\gamma_{r_2d}}$$

$$\xi_s = \rho_d\gamma_{sr_1} + (1-\rho_d)\gamma_{sr_2}$$

$$\boxed{p_{\xi_s}(x) = ?}$$

$$M_{\xi_s}(-1) = \int_0^\infty p_{\xi_s}(x)\, e^{-x1}\, dx$$

$U = XY$ — $f(x)$ PDF OF $X$, $g(y)$ PDF OF $Y$

$$\boxed{f(u) = \int_0^\infty f(y)\cdot g(u/y)\, dy/y}$$

- PAPOULIS: 6-2 ONE FUNCTION OF TWO VARIABLES

300010000131325 MO-DA DESIGN

155€ ZA VLADA VO OHRIA

---

$Z = g(x,y)$

- WITH $z$ A GIVEN NUMBER WITH $D_z$ WE DENOTE REGION OF XY PLANE SUCH THAT $g(x,y) \le z$

$$\{Z < z\} = \{g(x,y) \le z\} = \{(x,y) \in D_z\}$$

$$\boxed{F_Z(z) = P\{Z \le z\} = P\{(x,y) \in D_z\} = \iint_{D_z} f(x,y)\,dx\,dy}$$

- $z < g(x,y) < z + dz$

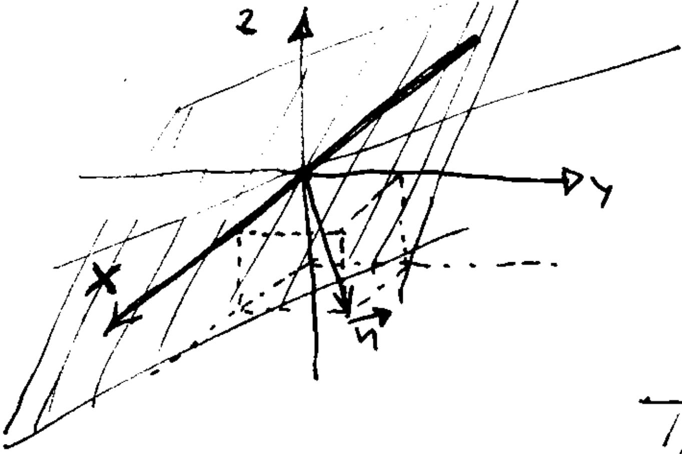
$$\{z < Z \le z + dz\} = \{(x,y) \in \Delta D_z\}$$

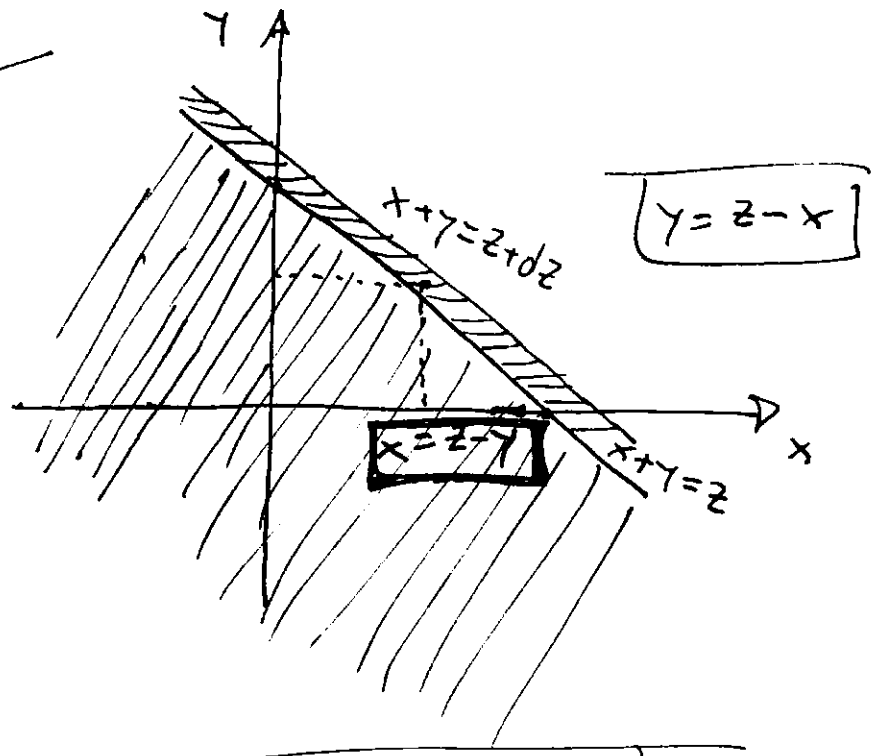
$$\boxed{f_Z(z)\,dz = P\{z < Z \le z + dz\} = \iint_{\Delta D_z} f(x,y)\,dx\,dy}$$

- ILLUSTRATIONS:

$\boxed{Z = X + Y}$ $\quad x + y - z = 0 \quad \vec{n}(\vec{x} - \vec{x_0})$

$$\vec{n} = \langle 1, 1, -1\rangle$$




$$F_Z(z) = \int_{-\infty}^{\infty} \int_{-\infty}^{z-y} f(x,y)\,dx\,dy$$

$$f_Z(z) = \frac{d}{dz} F_Z(z)$$

$$f_Z(z) = \int_{-\infty}^{\infty} \left\{ \frac{d}{dz} \int_{0}^{z-y} f(x,y)\,dx \right\} dy = \int_{-\infty}^{\infty} f(z-y,y)\,dy$$

- INDEPENDENCE AND CONVOLUTION

$$f(x,y) = f_X(x) \cdot f_Y(y)$$

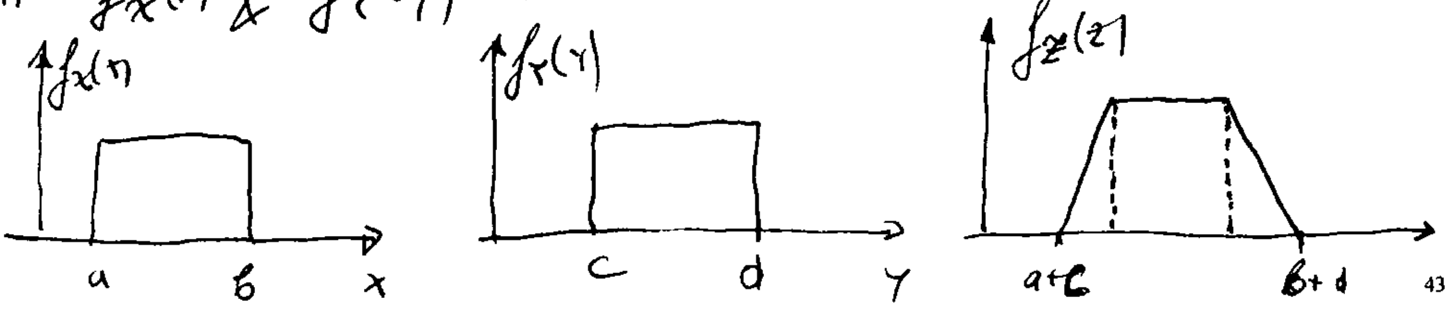
$$f_Z(z) = \int_{-\infty}^{\infty} f_X(z-y) \cdot f_Y(y)\,dy$$

$f_X(x) = 0 \quad x < 0 \qquad f_Y(y) = 0 \quad y < 0 \Rightarrow$

VIDI: MULTIMEDIA MIMO2.wk CH. 12

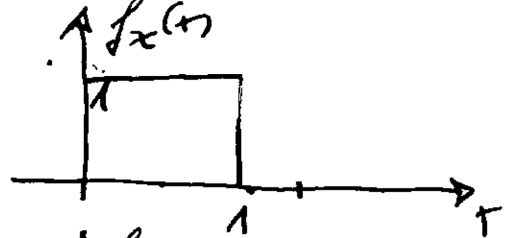
$$f_Z(z) = \int_{0}^{\infty} f(z-y)\,f(y)\,dy$$

- IF $f_X(x)$ & $f_Y(y)$ ARE UNIFOR IN GIVEN INTERVAL

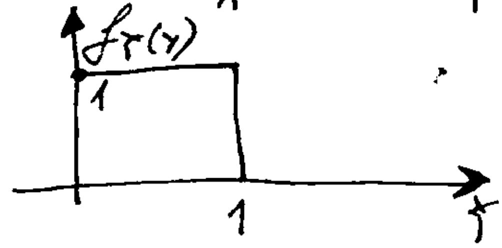


**EXAMPLE:**

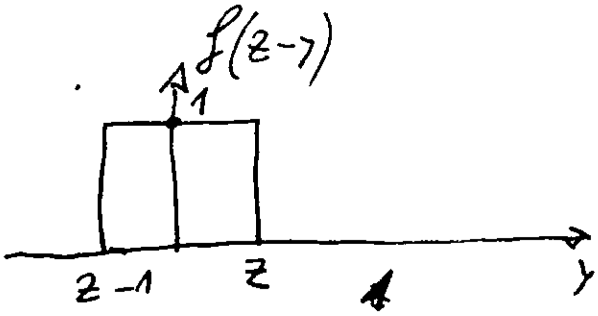
$$f_X(x) = \begin{cases} 0 & x < 0 \\ 0.5 & x \leq 1 \\ 0 & x > 1 \end{cases}$$




$$f_Y(y) = \begin{cases} 0 & y < 0 \\ 0.5 & y \leq 1 \\ 0 & y > 1 \end{cases}$$




$$f_Z(z) = \int_0^{\infty} f_X(z-y) \cdot f_Y(y)\, dy$$

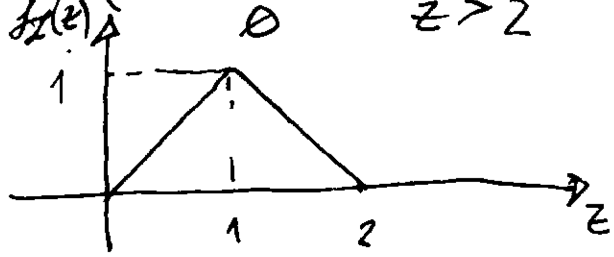



$$f_X(z-y) = \begin{cases} 0 & z-y < 0 \\ 0.5 & z-y \leq 1 \\ 0 & y > 1 \end{cases}$$

$$f_{Z_1}(z) = \int_0^z dy = z \qquad f_{Z_2}(z) = \int_{z-1}^{1} dy$$

$$f_{Z_2}(z) = [1 - z + 1] = [2 - z] = -z + 2 = -(z-2)$$

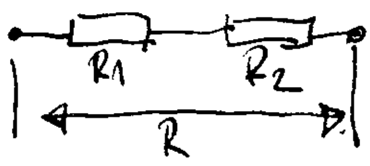
$$f_Z(z) = \begin{cases} 0 & z < 0 \\ z & 0 < z < 1 \\ -z+2 & z < 2 \\ 0 & z > 2 \end{cases}$$



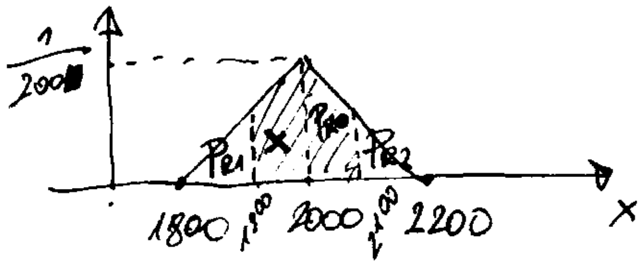

$$P = 2 \cdot 1 \cdot \frac{1}{2} = 1$$

- ISTOVO OVA MNOGU LESNO SE DOBIVA SO KORISTENJE NA STATISTICS PAKETOT VO MultiSim MIMO... SECTION (12).
- PRIMEROT SO DVA OTPORNIKA (OTPORNOSTA VARIRA PO UNIFORMNA RASPREDELBA)

$$R_1 = 900 \div 1100\,\Omega$$
$$R_2 = 900 \div 1100\,\Omega$$
$$R = R_1 + R_2$$



• VELOTATMOSFA DEMA $R = 1800 \div 2000\,\Omega$




$P_R = 400[redacted] \cdot \frac{1}{200[redacted]} \cdot \frac{1}{2} = 1$

$\frac{100[redacted]}{200[redacted]} = \frac{x}{\frac{1}{200[redacted]}}$ $200x = \frac{1}{2}$ $\boxed{x = \frac{1}{400}}$

$P_{R1} = \frac{1}{400} \cdot 100 \cdot \frac{1}{2} = \frac{1}{8}$

$P_{R0} = P_R - P_{R1} - P_{R2} = 1 - \frac{1}{8} - \frac{1}{8}$

$\boxed{P_{R0} = 1 - \frac{1}{4} = \frac{3}{4} = 0.75}$

Example 6-7.

$f_x(x) = \alpha e^{-\alpha x} U(x) \quad f_y(y) = \beta e^{-\beta y} U(y)$;

$z > 0$

$$f_z(z) = \int_0^\infty f(z-y) f(y)\, dy = \int_0^z \alpha \cdot \beta\, e^{-\alpha(z-y)} e^{-\beta y} dy$$

$$= \alpha \cdot \beta \cdot e^{-\alpha z} \int_0^z e^{+\alpha y} \cdot e^{-\beta y} dy = \alpha \cdot \beta\, e^{-\alpha z} \int_0^z e^{(\alpha-\beta)y} dy$$

$$= \alpha \cdot \beta\, e^{-\alpha z} \left.\frac{e^{(\alpha-\beta)y}}{\alpha-\beta}\right|_0^z = \frac{\alpha \cdot \beta\, e^{-\alpha z}}{\alpha-\beta}\left(e^{(\alpha-\beta)z} - 1\right)$$

$$f_z(z) = \frac{\alpha \cdot \beta}{\alpha - \beta}\left[e^{-\beta z} - e^{-\alpha z}\right] = \frac{\alpha \cdot \beta}{\beta - \alpha}\left[e^{-\alpha z} - e^{-\beta z}\right]$$

$$\lim_{\beta \to \alpha} \frac{\alpha \cdot \beta}{\beta - \alpha}\left(e^{-\alpha z} - e^{-\beta z}\right) = \lim_{\beta \to \alpha} \frac{\left[\alpha \cdot \beta\left(e^{-\alpha z} - e^{-\beta z}\right)\right]'}{\left[\beta - \alpha\right]'}$$

$$= \lim_{\beta \to \alpha} \alpha\left(e^{-\alpha z} - e^{-\beta z}\right) + \alpha \cdot \beta\left(+ z e^{-\beta z}\right) =$$

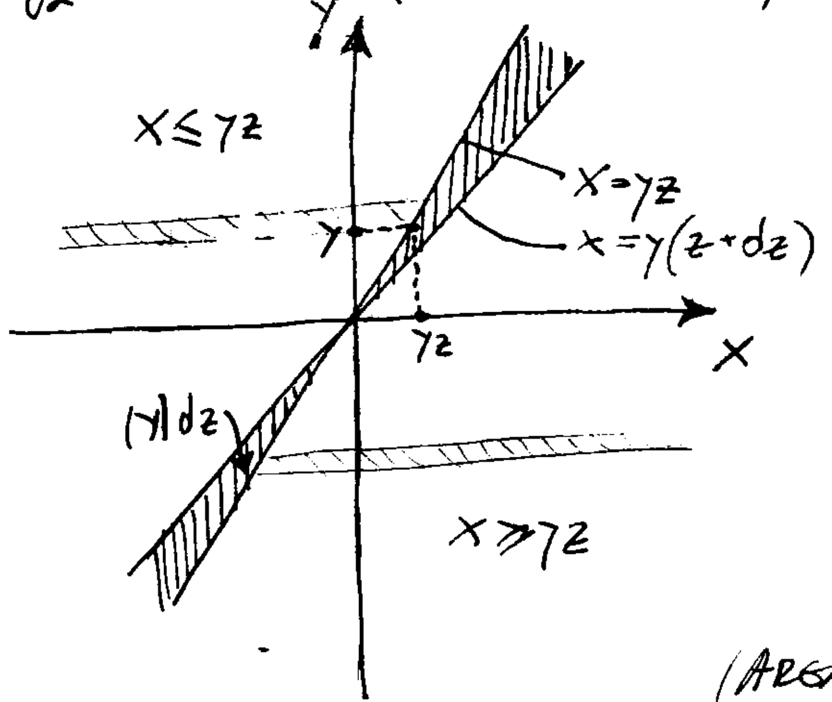
$$= \alpha\left(e^{-\alpha z} - e^{-\alpha z}\right) + z \cdot \alpha \cdot \beta\, e^{-\beta z}$$

$$\boxed{f_z(z) = z \cdot \alpha \cdot \beta\, e^{-\beta z} = z \cdot \alpha^2 e^{-\alpha z}}$$

▪ ILLUSTRATION 2: $\boxed{z = x/y}$

$$F_z(z) = \int_0^\infty \int_{-\infty}^{yz} f(x,y)\, dx\, dy + \int_{-\infty}^0 \int_{yz}^\infty f(x,y)\, dx\, dy$$

**6-36** $f_{z}(z)\,dz = P\left(z < \mathbf{z} < z+dz\right) = \iint_{\Delta D_z} f(x,y)\,dx\,dy$




$\begin{pmatrix}\text{AREA OF THE} \\ \text{TRIANGULAR AREA}\end{pmatrix} = |y|\,dy\,dz$

$\Delta D_z: \quad z < \frac{x}{y} < z + dz$

$$f_z(z) = \int_{-\infty}^{\infty} |y|\, f(zy, y)\, dy$$ ;

**NORMAL DENSITIES:** (x & y ARE JOINTLY NORMAL)

$$f(x,y) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-r^2}} \exp\left[-\frac{1}{2(1-r^2)}\left(\frac{x^2}{\sigma_1^2} - 2r\frac{xy}{\sigma_1\sigma_2} + \frac{y^2}{\sigma_2^2}\right)\right]$$

THEN THEIR RATIO HAS CAUCHY DENSITY CENTERED AT $r\sigma_1/\sigma_2$:

$$f_z(z) = \frac{\sigma_1\sigma_2\sqrt{1-r^2}/\pi}{\sigma_2^2\left(z - r\sigma_1/\sigma_2\right)^2 + \sigma_1^2(1-r^2)}$$

$f(x,y) = f(-x,-y)$

$$f_z(z) = \int_{0}^{\infty} y\, \frac{2}{2\pi\sigma_1\sigma_2\sqrt{1-r^2}} \exp\left[-\frac{y^2}{2(1-r^2)}\left(\frac{z^2}{\sigma_1^2} - 2r\frac{z}{\sigma_1\sigma_2} + \frac{1}{\sigma_2^2}\right)\right] dy$$

$$= \frac{K^2}{2\pi\sigma_1\sigma_2\sqrt{1-r^2}} \int_0^\infty e^{-\frac{y^2}{K^2}}\, d\left(\frac{y}{K}\right)^2 ; \quad K = \frac{2(1-r^2)}{\frac{z^2}{\sigma_1^2} - 2r\frac{z}{\sigma_1\sigma_2} + \frac{1}{\sigma_2^2}}$$

$$f_Z(z) = \frac{\sqrt{1-r^2}}{\pi\sigma_1\sigma_2\left(\frac{z^2}{\sigma_1^2} - 2r\frac{z}{\sigma_1\sigma_2} + \frac{1}{\sigma_2^2}\right)}\left(e^{-\infty} - e^0\right)\cdot(-1)$$

$$\boxed{f_Z(z) = \frac{\sqrt{1-r^2}}{\pi\sigma_1\sigma_2\left(\frac{z^2}{\sigma_1^2} - \frac{2rz}{\sigma_1\sigma_2} + \frac{1}{\sigma_2^2}\right)}} \qquad f_Z(z) = \frac{\sigma_1^2\sigma_2^2\sqrt{1-r^2}}{\pi\sigma_1\sigma_2\left(z^2\sigma_2^2 - 2r\sigma_1\sigma_2 z + \sigma_1^2\right)}$$

$$f_Z(z) = \frac{\sigma_1\sigma_2\sqrt{1-r^2}}{\pi\left(z^2\sigma_2^2 - 2r\sigma_1\sigma_2 z + \sigma_1^2\right)}$$

~~770~~ 720€    ~~670~~

$$\sigma_2^2\left(z - r\sigma_1/\sigma_2\right)^2 + \sigma_1^2(1-r^2) = \sigma_2^2\left(z^2 - 2\frac{r\sigma_1 z}{\sigma_2} + \frac{r^2\sigma_1^2}{\sigma_2^2}\right) + \sigma_1^2 - \sigma_1^2 r^2$$

$$= \sigma_2^2 z^2 - 2r\sigma_1\sigma_2 z + r^2\sigma_1^2 + \sigma_1^2 - \sigma_1^2 r^2 = \sigma_2^2 z^2 - 2r\sigma_1\sigma_2 z + \sigma_1^2$$

$$\boxed{f_Z(z) = \frac{\sigma_1\sigma_2\sqrt{1-r^2}/\pi}{\sigma_2^2\left(z - r\sigma_1/\sigma_2\right)^2 + \sigma_1^2(1-r^2)}}$$

PDF FOR $Z = \frac{X}{Y}$

$$\boxed{F_Z(z) = \int_0^z f_Z(z)\,dz = \frac{1}{2} + \frac{1}{\pi}\operatorname{arctg}\frac{\sigma_2 z - r\sigma_1}{\sigma_1\sqrt{1-r^2}}} \quad (*)$$

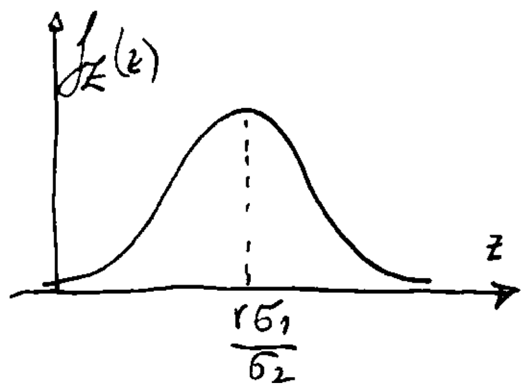
CDF FOR $Z = X/Y$

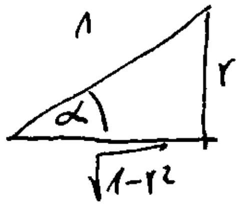
- QUADRANT MASSES. SO KORISTENJE NA (*) MOŽE DA SE POKAŽE DEKA MASITE NA VEROJATNOST VO ČETIRITE KVADRANTI OD XY RAMNINATA SE:

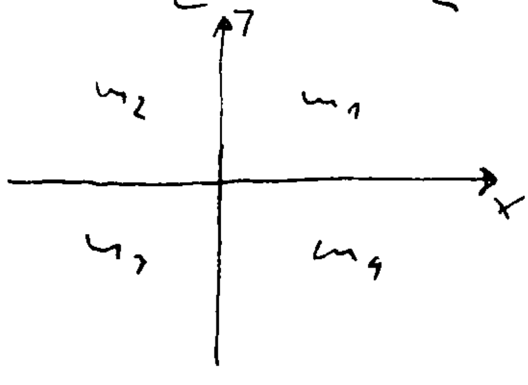
$$m_1 = m_3 = \frac{1}{4} + \frac{\alpha}{2\pi} \qquad m_2 = m_4 = \frac{1}{4} - \frac{\alpha}{2\pi}$$

KADE ŠTO:

$$\alpha = \arcsin(r) = \operatorname{arctg}\frac{r}{\sqrt{1-r^2}} \qquad -\frac{\pi}{2} \le \alpha \le \frac{\pi}{2}$$



- SECOND & FORTH QUADRANT

$$\boxed{\frac{x}{y} \le 0} \qquad m_2 + m_4 = P(z \le 0) = F_z(0) = \frac{1}{2} - \frac{1}{\pi}\operatorname{arctg}\frac{r}{\sqrt{1-r^2}}$$

$$m_2 = m_4 \quad m_1 = m_3 \qquad 2m_2 = \frac{1}{2} - \frac{1}{\pi}\operatorname{arctg}\frac{r}{\sqrt{1-r^2}}$$

$$m_2 = \frac{1}{4} - \frac{1}{2\pi}\operatorname{arctg}\frac{r}{\sqrt{1-r^2}} = \frac{1}{4} - \frac{\alpha}{2\pi} = m_4$$

$$2\left(\frac{1}{4} - \frac{\alpha}{2\pi}\right) + 2m_3 = 1 \qquad m_3 = \frac{1}{2} - \frac{1}{4} + \frac{\alpha}{2\pi} = \frac{1}{4} + \frac{\alpha}{2\pi} = m_1$$

**NORMAL DENSITIES FOR $z = \sqrt{x^2+y^2}$**

(a) $$f(x,y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

THE REGION $D_z$ IS CIRCLE $x^2+y^2 \le z^2$ AND $F_z(z)$ EQUALS PROBABILITY MASSES IN THE CIRCLE. IF $f(x,y) = g(r)$ IS CIRCULARY SIMETRICAL THEN:

$$F_z(z) = 2\pi \int_0^z r g(r)\,dr \quad z > 0 \quad (*)$$

→ VIDI NAO.49

$$x = z\cos\varphi \qquad x^2+y^2 = z^2(\cos^2\varphi + \sin^2\varphi) = z^2$$
$$y = z\sin\varphi$$

$$f(z,\varphi) = \frac{f(x,y)}{|\hat{J}|^{-1}} \Bigg/ \begin{matrix} x = f(z,\varphi) \\ y = f(z,\varphi) \end{matrix}$$

$$f_z(z) = \int_0^{2\pi} f_z(z,\varphi)\,d\varphi = 2\pi f(z,\varphi)$$

$$|\hat{J}| = \begin{vmatrix} \frac{\partial x}{\partial z} & \frac{\partial y}{\partial z} \\ \frac{\partial x}{\partial \varphi} & \frac{\partial y}{\partial \varphi} \end{vmatrix} = \begin{vmatrix} \cos\varphi & \sin\varphi \\ -z\sin\varphi & z\cos\varphi \end{vmatrix} = z\cos^2\varphi + z\sin^2\varphi = z$$

$$f(z,\varphi) = z \cdot f(x,y) = z \cdot \frac{1}{2\pi\sigma^2} e^{-\frac{z^2}{2\sigma^2}}$$

ZNAČI SE POJAVLJA RAYLEIGH RASPODJELA ZA: $z = \sqrt{x^2+y^2}$

$$F_z(z) = \int_0^z \frac{r}{\sigma^2} e^{-\frac{r^2}{2\sigma^2}}\,dr = \int_0^z e^{-\frac{r^2}{2\sigma^2}}\,d\left(\frac{r^2}{2\sigma^2}\right)$$

$$F_z(z) = -\left(e^{-\frac{z^2}{2\sigma^2}} - e^0\right) = \left(1 - e^{-\frac{z^2}{2\sigma^2}}\right) \quad z > 0 \quad (*)(\$)$$

(b) $$f(x,y) = \frac{1}{2\pi\sigma^2} e^{-[(x-\eta)^2 + y^2]/2\sigma^2}$$

$$\Delta D_z \qquad z < \sqrt{x^2+y^2} < z + dz$$

$$x = z\cos\theta \quad y = z\sin\theta \qquad dx\,dy = z\,dz\,d\theta$$

$$f_Z(z)\,dz = \iint_{D_z} f(x,y)\,dx\,dy = \frac{1}{2\pi\sigma^2}\int_0^{2\pi} e^{-[(z\cos\theta - y)^2 + (z\sin\theta)^2]/2\sigma^2}\, z\,dz\,d\theta$$

$$= \frac{1}{2\pi\sigma^2}\int_0^{2\pi} e^{-[z^2\cos^2\theta - 2zy\cos\theta + y^2 + z^2\sin^2\theta]/2\sigma^2}\, z\,dz\,d\theta =$$

$$= \frac{z}{2\pi\sigma^2}\, e^{-\frac{(z^2+y^2)}{2\sigma^2}} \left[\int_0^{2\pi} e^{\frac{2zy\cos\theta}{2\sigma^2}}\, d\theta\right] dz$$

→ RAJSOVA RASPREDELBA $z = \sqrt{x^2+y^2}$

$$f_Z(z) = \frac{z}{\sigma^2}\, e^{-\frac{z^2+y^2}{2\sigma^2}} \cdot I_0\left(\frac{zy}{\sigma^2}\right) \qquad z > 0$$

$$I_0\left(\frac{zy}{\sigma^2}\right) = \frac{1}{2\pi}\int_0^{2\pi} e^{\frac{zy}{\sigma^2}\cos\theta}\, d\theta$$

- OD ⊛ MOŽE DA SE DOBIE $F_Z(z)$

$$f(x,y) = \frac{1}{2\pi\sigma^2}\, e^{-\frac{r^2}{2\sigma^2}}$$

$$F_Z(z) = 2\pi \cdot \frac{1}{2\pi\sigma^2}\int_0^z r\, e^{-\frac{r^2}{2\sigma^2}}\, dr = \left(1 - e^{-\frac{z^2}{2\sigma^2}}\right)$$

ZNAČI NAMESTO SO JACOBIAN ODI SO ⊛ !!!

- OD ⊛⊛ (T.E.) SO IZVOD SE DOBIVA $f_Z(z)$

$$f_Z(z) = \frac{z}{\sigma^2}\, e^{-\frac{z^2}{2\sigma^2}}$$

→ ZNAČI, $z = \sqrt{x^2+y^2}$ E RASPREDELENO PO RAYLEIGH RASPREDELBA !!!

- DEFINITIVNO MISLAM DEKA IZVODOT SO JACOBIAN E POKROFESIONALEN.

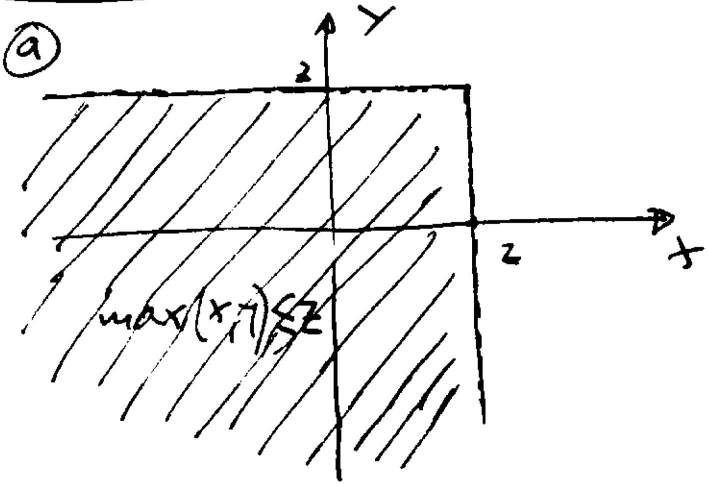
EXAMPLE 6.8: CONSIDER SINE WAVE:

$$x\cos\omega t + y\sin\omega t = r\cos(\omega t + \theta)$$

$r = \sqrt{x^2+y^2}$ IF $x, y$ ARE NORMAL RVs THEN „r" FOLOWS RAYLEIGH DISTRIBUTION

## 4. $Z = \max(X,Y)$ $W = \min(X,Y)$

(a)

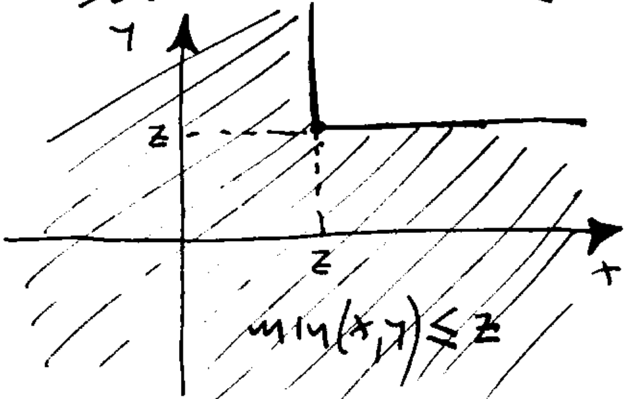



$D_Z$ IS REGION OF XY PLANE SUCH THAT
$\max(x,y) \le z$
IS DEFINING SET OF POINTS SUCH THAT: AND
$x \le z$ & $y \le z$
HENCE:
$$F_Z(z) = P_r(Z < z) = F_{XY}(z,z)$$
$$= P_r(X \le z, Y \le z)$$

- IF X & Y ARE INDEPENDENT

$$F_Z(z) = F_X(z)\, F_Y(z) \qquad f_Z(z) = f_X(z)\cdot F_Y(z) + f_Y(z)\cdot F_X(z)$$

(b) $D_W$ SUCH THAT $\min(x,y) \le w$ IS SET OF POINTS SUCH AS: $X \le W$ OR $Y \le W$




- OD TEORIJA NA INFORMACII

$$P(A \text{ ili } B) = P(A) + P(B) - P(A,B)$$

ILI ↑ — NO NE VO ISTO VREME „A" I „B"!!!

$$F_W(w) = F_X(x) + F_Y(y) - F_{XY}(x,y)$$

- IF RVs X & Y ARE INDEPENDENT IT IS SIMPLER TO REPREZENT THE RESULT IN TERMS OF RELIABILITY FUNCTION.

$$R_X(x) = P\{X > x\} = 1 - F_X(x)$$
$$R_W(w) = R_X(w)\, R_Y(w)$$
$$-f_X(x) = -f_X(w)\cdot R_Y(w) - f_Y(w)\cdot R_X(w)$$

$$\boxed{f_X(x) = f_X(w)\cdot R_Y(w) + f_Y(w)\cdot R_X(w)}$$

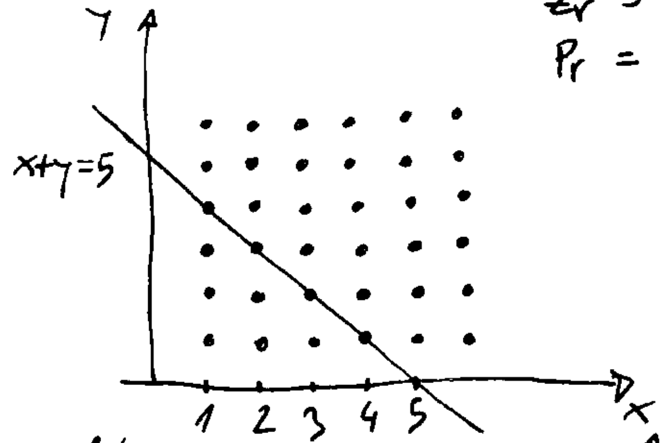
- Discrete type $X \in \{x_i : i = 1,2 \ldots\}$
$Y \in \{y_k : k = 1,2,\ldots\}$

$$z_r = g(x_i, y_k)$$

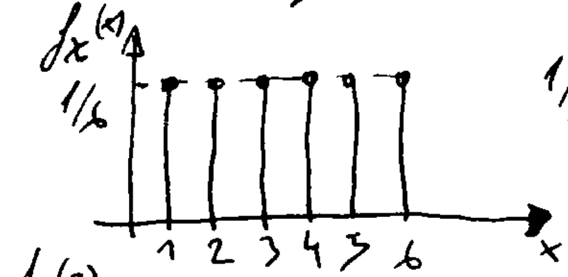
$X(f_i f_k) = i$ $\quad Y(f_i f_k) = k$ $\quad z_r = x_i + y_k$

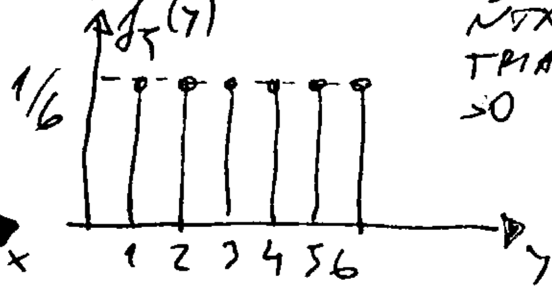
$P_r = \frac{(n)}{36}$ — NUMBER OF POINTS ON THE LINE: $x+y=z_r$

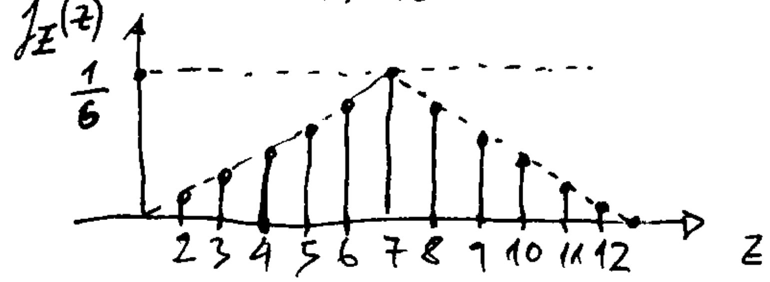
| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $z_r$ = | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| $P_r$ = | $\frac{1}{36}$ | $\frac{2}{36}$ | $\frac{3}{36}$ | $\frac{4}{36}$ | $\frac{5}{36}$ | $\frac{6}{36}$ | $\frac{5}{36}$ | $\frac{4}{36}$ | $\frac{3}{36}$ | $\frac{2}{36}$ | $\frac{1}{36}$ |




MENE OVA MI IZGLEDA KAKO PRIMER ZA IZVEDUVANJETO NA N10.44. Pr ODGOVARA NA PDF, A NE NA CDF. KAKO ŠTO SE GLEDA OD N10.44 REZULTANTNATA PDF E TRIAGOLNIK I SE DOBIVA SO KONVOLUCIJA.




---

• SE VRAĆAM NAZAD NA ČLANAKOT OD ANGHEL.

$$\xi_s := \rho_d \gamma_{sr_1} + (1-\rho_d)\gamma_{sr_2} \qquad \rho_d = \frac{\sigma_{R_1}\bar{\gamma}_{r_1 d}}{\sigma_{R_1}\bar{\gamma}_{r_1 d} + \sigma_{R_2}\bar{\gamma}_{r_2 d}}$$

$$M_{\xi_s}(s) = \int_0^1 M_{\gamma_{sr_1}}(\rho s)\, M_{\gamma_{sr_2}}(s-\rho s)\, f_{\rho_d}(\rho)\, d\rho$$

---

- PRODUCT OF TWO INDEPENDENT RANDOM GAMMA VARIABLES (M. J. MALIK PAPER ON: DISTRIBUTION OF ...)

$$V = X \cdot Y \qquad f_V(v) = f_{XY}(x,y)$$

$$F_V(v) = \iint_{D_v} f_{X,Y}(x,y)\,dx\,dy \qquad (x,y) \in D_v$$

$$P[V \le v] = P[(x,y) \in D_v]$$

$$f_V(v) = \frac{dF_V(v)}{dv} = \frac{d}{dv}\int_{-\infty}^{\infty}\int_{-\infty}^{v/y} f_{X,Y}(x,y)\,dx\,dy$$

$$f_U(u) = \int_{-\infty}^{\infty} \frac{d}{du}\left[\int_{-\infty}^{u/y} f(x,y)\,dx\right] dy = \int_{-\infty}^{\infty} f\left(\frac{u}{y}, y\right) \frac{d}{du}\left(\frac{u}{y}\right) dy$$

$$f_U(u) = \int_{-\infty}^{\infty} f\left(\frac{u}{y}, y\right)\frac{dy}{y} = \left| \begin{matrix}\text{IF} \\ X \,\&\, Y \\ \text{ARE INDEPENDENT}\end{matrix} \right| = \int_{-\infty}^{\infty} f\left(\frac{u}{y}\right)\cdot g(y)\,\frac{dy}{y}$$

$$= \left| \begin{matrix}\text{IF} \\ X \,\&\, Y \\ \text{ARE} \\ \text{IDENTICALLY} \\ \text{DISTRIBUTED}\end{matrix} \right| = \int_{-\infty}^{\infty} f(y)\cdot g\left(\frac{u}{y}\right)\frac{dy}{y}$$

} ↗ DOKAZAN IZRAZ (2.1) OD H. J. MALIK DISTRIBUTION OF PRODUCT

$$f(x; k, \theta) = \frac{x^{k-1}}{\Gamma(k)\,\theta^k}\, e^{-\frac{x}{\theta}}$$

} GAMMA DISTRIBUTION WIKIPEDIA DEFFINITION

GENERALIZED FORM OF DISTRIBUTION OF PRODUCT OF TWO I.I.D RANDOM VARIABLES

$$f_U(u) = \int_{-\infty}^{\infty} f(y)\cdot g\left(\frac{u}{y}\right)\frac{dy}{y} \qquad U = X\cdot Y$$

MMV

$f(x; a_1, d_1, \gamma)$ $\qquad g(y; a_2, d_2, \gamma)$

$$f(x; a_1, d_1, \gamma) = \frac{\gamma\, x^{d_1-1}}{\Gamma\left(\frac{d_1}{\gamma}\right) a_1^{d_1}}\, e^{-\left(\frac{y}{a_1}\right)^{\gamma}}$$

$$g(y; a_2, d_2, \gamma) = \frac{\gamma\, y^{d_2-1}}{\Gamma\left(\frac{d_2}{\gamma}\right) a_2^{d_2}}\, e^{-\left(\frac{y}{d_2}\right)^{\gamma}}$$

$$f(u) = \int_0^{\infty} \frac{\gamma\, y^{d_1-1}}{\Gamma\left(\frac{d_1}{\gamma}\right) a_1^{d_1}}\, e^{-\left(\frac{y}{a_1}\right)^{\gamma}} \cdot \frac{\gamma \left(\frac{u}{y}\right)^{d_2-1}}{\Gamma\left(\frac{d_2}{\gamma}\right) a_2^{d_2}}\, e^{-\left(\frac{u}{a_2 y}\right)^{\gamma}}\, \frac{dy}{y}$$

$C$ =

$$= \frac{\gamma^2\, u^{d_2-1}}{\Gamma\left(\frac{d_1}{\gamma}\right)\Gamma\left(\frac{d_2}{\gamma}\right) a_1^{d_1} a_2^{d_2}} \int_0^{\infty} y^{d_1-d_2-1}\, e^{-\left(\frac{y}{a_1}\right)^{\gamma} - \left(\frac{u}{a_2 y}\right)^{\gamma}}\, dy$$

$$\left(\frac{y}{a_1}\right)^{\gamma} = t \qquad \frac{\gamma\, y^{\gamma-1}}{a_1^{\gamma}}\, dy = dt \qquad \left| \begin{matrix} y = 0 & t = 0 \\ y = \infty & t = \infty \end{matrix} \right| \qquad \boxed{y = a_1 t^{1/\gamma}}$$

$$f(M) = c \int_0^\infty a_1^{d_1-d_2} \cdot t^{(d_1-d_2)/\gamma} \cdot \exp\left(-t - \frac{M^\gamma}{a_1^\gamma a_2^\gamma t}\right) \frac{dt \cdot a_1^\gamma \cdot \gamma^{-1}}{\Gamma \, \gamma^{\gamma-1}} \quad (= \bar{t})$$

$$= \frac{c \cdot a_1^{d_1-d_2}}{\gamma} \int_0^\infty t^{(d_1-d_2)/\gamma - 1} \cdot \exp\left(-t - \frac{M^\gamma}{a_1^\gamma a_2^\gamma t}\right) dt \qquad \left(\frac{c \cdot a_1^{d_1-d_2}}{\gamma} = C_1\right)$$

$$C_1 = \frac{\gamma^2 M^{d_2-1} \cdot a_1^{d_1-d_2}}{\Gamma\left(\frac{d_1}{\gamma}\right)\Gamma\left(\frac{d_2}{\gamma}\right) a_1^{d_1} a_2^{d_2} \cdot \gamma} = \frac{\gamma M^{d_2-1}}{\Gamma\left(\frac{d_1}{\gamma}\right)\Gamma\left(\frac{d_2}{\gamma}\right) \cdot a_1^{d_2} \cdot a_2^{d_2}}$$

$$f(M) = \frac{\gamma M^{d_2-1}}{\Gamma\left(\frac{d_1}{\gamma}\right)\Gamma\left(\frac{d_2}{\gamma}\right)(a_1 a_2)^{d_2}} \int_0^\infty t^{d_1/\gamma - d_2/\gamma - 1} e^{-t - \frac{M^\gamma}{a_1^\gamma a_2^\gamma t}} dt$$

$$\boxed{\int_0^\infty e^{-t - \frac{z^2}{4t}} t^{-(\upsilon+1)} dt = 2\left(\frac{z}{2}\right)^{-\upsilon} \cdot K_\upsilon(z)}$$

VARIJANTA NA GRADSHTEYN 3.471.9

$$\frac{z^2}{4} = \frac{M^\gamma}{a_1^\gamma a_2^\gamma}$$
$$z = \frac{2 M^{\gamma/2}}{a_1^{\gamma/2} a_2^{\gamma/2}}$$
$$\nu = \frac{d_2 - d_1}{\gamma}$$

- GRADSHTEYN 3.471.9

$$\int_0^\infty x^{\nu-1} e^{-\frac{\beta}{x} - \gamma x} dx = 2\left(\frac{\beta}{\gamma}\right)^{\frac{\nu}{2}} K_\nu\left(2\sqrt{\beta\gamma}\right)$$

- IF: $\gamma = 1$, $\beta = \frac{z^2}{4}$, $x = t$

$$\int_0^\infty t^{\nu-1} e^{-\frac{z^2}{t} - t} dt = 2\left(\frac{z^2}{4}\right)^{\frac{\nu}{2}} K_\nu\left(2\sqrt{\frac{z^2}{4}}\right)$$

$$= 2 \cdot \left(\frac{z}{2}\right)^\nu \cdot K_\nu\left(2 \cdot \frac{z}{2}\right) = 2\left(\frac{z}{2}\right)^\nu \cdot K_\nu(z)$$

- IF $\boxed{\nu = -\upsilon}$

$$\int_0^\infty t^{-\upsilon-1} e^{-\frac{z^2}{t} - t} dt = 2\left(\frac{z}{2}\right)^{-\upsilon} \cdot K_{-\upsilon}(z) = 2\left(\frac{z}{2}\right)^{-\upsilon} K_\upsilon(z)$$

$$\left(K_{-\upsilon}(z) \triangleq K_\upsilon(z)\right)$$

DOKAZAN

---

$$f(M) = C_1 \int_0^\infty e^{-t - \left(\frac{2 M^{\gamma/2}}{a_1^{\gamma/2} a_2^{\gamma/2}}\right)^2 / 4t} t^{-\frac{(d_2-d_1)}{\gamma} - 1} dt$$

$$= C_1 \cdot 2 \left(\frac{M^{\gamma/2}}{a_1^{\gamma/2} a_2^{\gamma/2}}\right)^{-\frac{d_2-d_1}{\gamma}} \cdot K_{\frac{d_2-d_1}{\gamma}}\left(\frac{2 M^{\gamma/2}}{a_1^{\gamma/2} a_2^{\gamma/2}}\right)$$

$$f_U(u) = \frac{2p\, u^{d_2-1}}{\Gamma\left(\frac{d_1}{\gamma}\right)\Gamma\left(\frac{d_2}{\gamma}\right)(a_1a_2)^{d_2}} \left(\frac{u}{a_1a_2}\right)^{-\frac{d_2-d_1}{2}} K_{\frac{d_2-d_1}{2}}\left(2\left(\frac{u}{a_1a_2}\right)^{\frac{\gamma}{2}}\right)$$

→ PDF OF PRODUCT OF TWO GAMMA VARIABLES

- FOR $\gamma = 1$ (NORMALNO ZAS IMAOTAM SO TAKVI PDF)

$$p_U(u) = \frac{2\, u^{d_2-1}}{\Gamma(d_1)\cdot\Gamma(d_2)(a_1a_2)^{d_2}} \cdot \left(\frac{u}{a_1a_2}\right)^{-\frac{d_2-d_1}{2}} K_{\frac{d_2-d_1}{2}}\left(2\sqrt{\frac{u}{a_1a_2}}\right)$$

DISTRIBUTION OF PRODUCT OF GAMMA VARIABLES !!! MMV

- $Z = X\cdot Y$ $(z = x\cdot y)$ $\qquad F_Z(z) = P(Z \leq z) = \iint_{D_z} f(x,y)\,dx\,dy$

$$f_Z(z) = \int_{-\infty}^{\infty} f(y)\cdot g\left(\frac{z}{y}\right)\frac{dy}{y}$$ (VIDI NA0.52)

$$f_{Y/X}(y/x) = \frac{f_{XY}(x,y)}{f_X(x)} \qquad f_{XY}(x,y) = f_{Y/X}(y/x)\cdot f_X(x)$$

$$MGF_Z(s) = M_Z(s) = \int_{-\infty}^{\infty} f_Z(z)\cdot e^{zs}\,dz = \int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty} f(y)\,g\left(\frac{z}{y}\right)\frac{dy}{y}\right]e^{zs}\,dz \quad (*)$$

• AKO SE TRGNE OD REZULTATOT

$$M_Z(s) = \int_{-\infty}^{\infty} M_Y(xs)\, p_X(x)\,dx = \int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty} p_Y(y)\cdot e^{xs\cdot y}\,dy\right]p_X(x)\,dx$$

$$= \int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty} p_X(x)\, e^{xsy}\,dx\right] p_Y(y)\,dy = \int_{-\infty}^{\infty} M(ys)\, p_Y(y)\,dy$$

$(*) \Rightarrow$ $|dz = y\,dx|$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(y)\cdot g(x)\frac{dy}{y}\, e^{xys}\, y\cdot dx \qquad \Big/ \; f(y) = p_Y(y), \; g(x) = p_X(x)$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(y)\, e^{xys}\,dy\; g(x)\,dx = \int_{-\infty}^{\infty} M_Y(xs)\, p_X(x)\,dx$$

TELEKOM: 6.527,00 $\boxed{N=2}$

$$P_{ea}\Big|_{2\times1\times2} = \frac{2^{N^2}-1}{2^{2(N^2-1)}c^2\rho^2} = \frac{16-1}{2^{2\times3}c^2\rho^2} = \frac{15}{64\cdot c^2\rho^2}$$

$\boxed{N=4}$

$$P_{ea}\Big|_{4\times1\times4} = \frac{8995}{65536c^4\rho^4} = \frac{8995}{2^{16}c^4\rho^4} = \frac{8995}{2^{N^2}c^N\rho^N}$$

$2^x = 64$

$2^y = 65536$

$$P_{ea}\Big|_{2\times1\times2} = \frac{15\cdot2^2}{2^8 c^N\rho^N}$$

$$N^{\frac{N^2}{2}} = 4^{\frac{16}{2}} = 4^8 = \boxed{65536}$$

$$2^{\frac{4}{2}} = 2^2 = 4$$

$$\frac{2}{N^{\frac{N}{2}}\cdot\Gamma^2(N)} = \Big|N=2\Big| = \frac{2}{2^1\cdot\Gamma^2(N)} = \frac{2}{2} = 1$$

$$(+)\ 2^2\cdot(2\cdot N)^{N} = 4(2\cdot2)^2 = 4^3 = 16\cdot4 = 64 \qquad \underline{(4\cdot2)^2 = 64}$$

$$(2\cdot N)^{N} = (4\cdot4)^4 = (2^4)^4 = 2^{16}$$

| | | |
|---|---|---|
| $N=2$ | $C=64=2^6$ | $2\cdot2^4=(4\cdot8)$ |
| $N=4$ | $C=65536=2^{16}$ | |
| $N=8$ | $C=2^{40}$ | $(8\cdot8)^8 = (2^6)^8 = 2^{48}$ $(4\cdot8)^8 = 2^{40}$ |
| $N=16$ | $C=2^{96}$ | $(16\cdot16)^{16} = (2^8)^{16} = 2^{128}$ |
| | | $(4\cdot16)^{16} = (2^6)^{16} = 2^{96}$ |

ZNAČI GENERALNA FORMULA ZA IMENITELOT E:

$$C_2 = (4\cdot N)^N \qquad P_{ea}\Big|_{N\times1\times N} = \frac{C_1}{(4\cdot N)^N c^N\rho^N} = \frac{C_1}{C_2\, c^N\rho^N}$$

$\boxed{C_1 = ?}$

$$\lim_{g\to0}\frac{d^2F(g,2)}{dg^2} = \frac{5}{4\rho^2c^2}$$

$$\lim_{g\to 0}\left(\frac{d^2}{dg^2}F(g,2)\right)=\frac{5}{4g^2c^2} \qquad \lim_{g\to 0}\left(\frac{d^4}{dg^4}F(g,4)\right)=\frac{257}{256c^4g^4}$$

$$\lim_{g\to 0}\left(\frac{d^8}{dg^8}F(g,8)\right)=\frac{16777217}{16777216\,c^8g^8}$$

- GENERALNA FORMA:

$$\lim_{g\to 0}\left(\frac{d^N}{dg^N}F(g,N)\right)=\frac{N^N+1}{N^N\cdot c^N g^N}$$

- TOGAŠ GENERALNATA FORMA NA ASIMPTOTSKIOT DEL ĆE BIDE:

$$P_{ea}(g,c,N)=\frac{N^N+1}{2\cdot 2^N\cdot N!\cdot N^N c^N\cdot g^N}\cdot\prod_{i=1}^{N}(2i-1) \quad (*)$$

$N=2$

$$\prod_{i=1}^{2}(2i-1)=1\cdot 3$$

$N=3$

$$\prod_{i=1}^{3}(2i-1)=1\cdot 3\cdot 5$$

$N=4$

$$\prod_{i=1}^{4}(2i-1)=1\cdot 3\cdot 5\cdot 7$$

MOŽE DA SE NAJDE VRSKA SO DVOEN FAKTORIEL

$$\prod_{i=1}^{N}(2i-1)=2^N\cdot\prod_{i=1}^{N}\left(i-\frac{1}{2}\right)=2^N\cdot\underbrace{\left(1-\frac{1}{2}\right)\left(2-\frac{1}{2}\right)\cdots\left(N-1-\frac{1}{2}\right)}_{\Pi\left(N-\frac{1}{2}\right)}\left(N-\frac{1}{2}\right)$$

$$\Pi\left(N-\frac{1}{2}\right)=\left(1-\frac{1}{2}\right)\left(2-\frac{1}{2}\right)\cdots\left(N-\frac{1}{2}-1\right)$$

$N=2$ $\quad \Pi\left(N-\frac{1}{2}\right)=\left(-\frac{1}{2}\right)$

$$\prod_{i=1}^{N}(2i-1)=2^N\cdot\prod_{i=1}^{N}\left(i-\frac{1}{2}\right)=2^N\cdot\left(N-\frac{1}{2}\right)!$$

$$P_{ea}(g,c,N)=\frac{(N^N+1)\,2^N\cdot\left(N-\frac{1}{2}\right)!}{2\cdot 2^N\cdot N!\,N^N c^N\cdot g^N\sqrt{\pi}}$$

MARCE VIKA VAKA NO NE ZNAM OD KADE MU IZLEZE!!!

$$\Gamma(\alpha) = \int_0^\infty t^{\alpha-1} e^{-t}\, dt \qquad \Gamma\left(\tfrac{1}{2}\right) = \int_0^\infty \frac{e^{-t}}{\sqrt{t}}\, dt$$

$$\Gamma\left(\tfrac{1}{2}\right) = \int_0^\infty \frac{1}{e^{-t}\sqrt{t}}\, dt \quad \Bigg| \; u = \sqrt{t} \quad t = u^2 \quad 2u\,du = dt \quad t=0 \; u=0 \quad t=\infty \; u=\infty \Bigg|$$

$$\Gamma\left(\tfrac{1}{2}\right) = \int_0^\infty \frac{e^{-u^2}}{u} 2u\,du = 2\int_0^\infty e^{-u^2} du = 2 \cdot \frac{\sqrt{\pi}}{2} = \sqrt{\pi}$$

($\int_0^\infty e^{-u^2} du \to \frac{\sqrt{\pi}}{2}$)

DOKAZANO: $\boxed{\Gamma\left(\tfrac{1}{2}\right) = \sqrt{\pi}}$

$$\Gamma(\alpha) = (\alpha - 1)!$$

$$\Gamma\left(\tfrac{1}{2}\right) = \left(\tfrac{1}{2} - 1\right)! = \left(-\tfrac{1}{2}\right)!$$

$$\Gamma\left(n + \tfrac{1}{4}\right) = \frac{1 \cdot 5 \cdot 9 \cdot 13 \cdots (4n-3)}{4^n}$$

Ambrowitz 6.1.12 INTERESEN DOKAZ! DOKAZI GO I $\Gamma(n+\frac{1}{4})$

$n = 0$

$$\Gamma\left(n+\tfrac{1}{2}\right) = \left(n - \tfrac{1}{2}\right)! = \Gamma\left(\tfrac{1}{2}\right)\underbrace{\tfrac{1}{2}}_{n=1}\underbrace{\left(2 - \tfrac{1}{2}\right)}_{n=2}\underbrace{\left(3 - \tfrac{1}{2}\right)}_{n=3} \cdots \underbrace{\left(n - \tfrac{1}{2}\right)}_{n=n}$$

$$= \left(\tfrac{1}{2}\right)^n (4-1)(6-1)\cdots(2n-1) \cdot \Gamma\left(\tfrac{1}{2}\right) \stackrel{?}{=} (2n-1)!!$$

$$= \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{2^n} \cdot \Gamma\left(\tfrac{1}{2}\right) = \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{2^n}\sqrt{\pi}$$

($\Gamma(\frac{1}{2}) \to \sqrt{\pi}$)

POKAZANO!!!

- Wolfram.com (SAVE IN WIKI FOLDER)

$$\Gamma\left(n + \frac{p}{2}\right) = \frac{1}{2^n}\Gamma\left(\frac{p}{2}\right)\prod_{k=1}^{n}(p + k2 - 2) \qquad n \in N, \; p \in N^+$$

$p = 1 \quad 2 = 2$

$$\Gamma\left(n + \tfrac{1}{2}\right) = \frac{1}{2^n}\Gamma\left(\tfrac{1}{2}\right)\prod_{k=1}^{n}(1 + 2k - 2) = \frac{\Gamma(\frac{1}{2})}{2^n}\prod_{k=1}^{n}(2k-1)$$

ZNAČI OD 6.1.12 SLEDI DELWS (DOKAZ GORE)

$$\prod_{k=1}^{n}(2k-1) = (2n-1)!! = \frac{\Gamma(n+\frac{1}{2})2^n}{\Gamma(\frac{1}{2})} = \frac{2^n\Gamma(n+\frac{1}{2})}{\sqrt{\pi}}$$

GRADSHTEYN 8.339.2, AMBROWITZ 6.1.12

- ZATOA (*) (N10.56) ~~MOŽE DA SE~~ ~~ZAPIŠE~~ ZAPIŠE KAKO

$$\boxed{P_{ea}(\rho, c, N) = \frac{(N^N+1)\, 2^N \cdot \Gamma(N+\frac{1}{2})}{2 \cdot 2^N \cdot N! \cdot N^N \cdot c^N \rho^N \sqrt{\pi}} = \frac{(N^N+1)\,\Gamma(N+\frac{1}{2})}{2 \cdot N! \cdot N^N \cdot c^N \rho^N \sqrt{\pi}}}$$

MMV

$$P(g, g_m) = \frac{e^{-\frac{e_2}{g_m}}}{3 g_m^7}\Big[ -g^4\left(g_m^3 + 2 g_m^2 g + 2 g^2 g_m + \frac{4}{3} g^3\right) K_4(\cdot) - g^5\left(2 g g_m + g_m^2 + g^2\right)$$

$$\cdot K_3(\ldots) + 3 g_m^7 e^{\frac{e_2}{g_m}} - 4 g^7 K_2(\ldots) - \frac{4}{3} g^7 K_1(\ldots)\Big] =$$

$$= 1 - \frac{e^{-\frac{2z}{g_m}} g^7}{3 g_m^7}\Big[\left(\frac{g_m^3}{g^3} + \frac{2 g_m^2}{g^2} + \frac{2 g_m}{g} + \frac{4}{3}\right) K_4(\cdots) + \left(2\frac{g_m}{g} + \frac{g_m^2}{g^2} + 1\right)$$

$$\cdot K_3(\ldots) + 4 K_2(\ldots) + \frac{4}{3} K_1(\ldots)\Big]$$

☐ Za N+N+N (VIDI MultiHopMIMO.nw d. 12.17)

$$L_m(\rho, c, N) = \lim_{g \to 0} \frac{d^{N^2}}{d\rho^{N^2}}\left[F(g, \rho, c, N)\right]$$

$$L_m(\rho, c, 2) = \frac{2}{\rho^4 c^4} \qquad L_m(\rho, c, 3) = \frac{2}{\rho^9 c^9}$$

$$L_m(\rho, c, 4) = \frac{2}{\rho^{16} c^{16}}$$

$$\boxed{P_{ea} = \underbrace{\left[\frac{1}{2 \cdot 2^{N^2} (N^2)!} \prod_{i=1}^{N^2} (2i-1)\right]}_{C_m} \cdot \frac{2}{\rho^{N^2} c^{N^2}}}$$

$$C_m = \frac{1}{2 \cdot 2^{N^2} (N^2)!} \cdot \prod_{i=1}^{N^2} (2i-1) \qquad \prod_{i=1}^{N^2} (2i-1) = \frac{\Gamma\left(N^2 + \frac{1}{2}\right) \cdot 2^{N^2}}{\sqrt{\pi}}$$

$$C_m = \frac{1}{2 \cdot 2^{N^2} (N^2)!} \cdot \frac{\Gamma\left(N^2 + \frac{1}{2}\right) \cdot 2^{N^2}}{\sqrt{\pi}}$$

$$\boxed{P_{ea} = \frac{\Gamma\left(N^2 + \frac{1}{2}\right)}{2 \cdot (N^2)! \sqrt{\pi}} \frac{2}{\rho^{N^2} c^{N^2}}} \qquad \boxed{P_{ea} = \frac{\Gamma\left(N^2 + \frac{1}{2}\right)}{(N^2)! \sqrt{\pi}\, \gamma^{N^2}}}$$

- GRADSHTEYN 8.446

$$K_n(z) = \frac{1}{2}\left(\frac{2}{z}\right)^n \sum_{k=0}^{n-1} (-1)^k \frac{(n-k-1)!}{k!} \cdot \left(\frac{z}{2}\right)^{2k} +$$

$$+ (-1)^{n+1} \cdot \left(\frac{z}{2}\right)^n \sum_{k=0}^{\infty} \frac{\left(\frac{z}{2}\right)^{2k}}{k!(n+k)!}\left[\ln\frac{z}{2} - \frac{1}{2}\psi(k+1) - \frac{1}{2}\psi(n+k+1)\right]$$

$$P_{ea4} = \frac{1}{2} - \frac{\rho^{19/2}}{2\left(\rho + \frac{15}{4}\right)^{9/2}} - \frac{64283}{2^{22}} \cdot \frac{\rho^{1/2}}{\left(\rho + \frac{15}{4}\right)^{9/2}} - \frac{285}{2^4} \frac{\rho^{15/2}}{\left(\rho + \frac{15}{4}\right)^{9/2}}$$

$$-\frac{676885521375}{2^{21}}\frac{\rho^{5/2}}{\left(\rho+\frac{15}{4}\right)^{9/2}}-\frac{1443773584375}{2^{22}}\frac{\rho^{3/2}}{\left(\rho+\frac{15}{4}\right)^{9/2}}$$

$$-\frac{72675}{2^{7}}\frac{\rho^{15/2}}{\left(\rho+\frac{15}{4}\right)^{9/2}}-\frac{66387935}{2^{12}}\frac{\rho^{11/2}}{\left(\rho+\frac{15}{4}\right)^{9/2}}-\frac{48243780225}{2^{18}}\frac{\rho^{7/2}}{\left(\rho+\frac{15}{4}\right)^{9/2}}$$

$$-\frac{136341225}{2^{21}}\frac{\rho^{9/2}}{\left(\rho+\frac{15}{4}\right)^{9/2}}-\frac{5450625}{2^{11}}\frac{\rho^{13/2}}{\left(\rho+\frac{15}{4}\right)^{9/2}}$$

$$P_{ea2}=\frac{1}{2}-\frac{75}{4}\frac{\rho^{3/2}}{(\rho+3)^{7/2}}-\frac{135}{8}\frac{\rho^{1/2}}{(\rho+3)^{7/2}}-\frac{21}{4}\frac{\rho^{5/2}}{(\rho+3)^{7/2}}-\frac{1}{2}\frac{\rho^{7/2}}{(\rho+3)^{7/2}}$$

$$\boxed{P_{ea2}=\frac{1}{2}-\frac{135}{8}\frac{\rho^{1/2}}{(\rho+3)^{7/2}}-\frac{75}{4}\frac{\rho^{3/2}}{(\rho+3)^{7/2}}-\frac{21}{4}\frac{\rho^{5/2}}{(\rho+3)^{7/2}}-\frac{1}{2}\frac{\rho^{7/2}}{(\rho+3)^{7/2}}}$$

- AKO OVAJ SQ , $c\cdot\rho$ I SE KORISTI FACTOR SE DOBIVA:

$$\boxed{P_{ea2}=\frac{1}{2}-\frac{\sqrt{2c\rho}\,(135+300c\rho+168c^2\rho^2+32c^3\rho^3)}{8\cdot(2\cdot c\cdot\rho+3)^{7/2}}}$$

↳ OVA VAŽI ZA 2+1+2 ZA BPSK , DICO KAKVO „c"

$$P_{ea4}=\frac{1}{2}-\frac{\sqrt{c\rho}\,(32659375+220684375c\rho+613008250c^2\rho^2+1058848400c^3\rho^3+1149081600c^4\rho^4+837224320c^5\rho^5+413440000c^6\rho^6+132300800c^7\rho^7+24903680c^8\rho^8+2097152c^9\rho^9)}{8\,(4c\rho+5)^{19/2}}$$

$$P_{ea2}=\frac{1}{2}-\frac{\sqrt{2c\rho}\,\left[135+150\cdot(2c\rho)+42(2c\rho)^2+4\cdot(2c\rho)^3\right]}{8\,(2c\rho+3)^{7/2}}$$

$$=\frac{1}{2}-\frac{\sqrt{N\cdot c\rho}\,\left[135+150\cdot(Nc\rho)+42(Nc\rho)^2+4(Nc\rho)^3\right]}{8\,(N\cdot c\rho+N-1)^{\frac{2N+1}{2}}}$$

$$\frac{19}{2}=\frac{N^2+4-1}{2}=\frac{16+3}{2}=\frac{19}{2}\qquad\frac{N^2+2-1}{2}=\frac{4+1}{2}=\frac{5}{2}$$

$N(N+1)-1 = /N=4/ = 4\cdot 5-1 = 19$

$N(N+1)-1 = /N=2/ = 2\cdot 3-1 = 5$

---

$N(N-1)-1 = (4\cdot 3)+$

---

$N^2-(N+1) = 16-5 = 9$

$N^2-(N+1) = /N=2/ = 4-3 = 1$

$(N+1)^2 - N^2 = 9-4 = 5$ $\quad +N-1$

$N^{\frac{N}{2}} = (\sqrt{N})^N = / \; 2^4 = 16$

$N=2 \qquad N^{N/2} = 2^1 + N-1 = 2+2-1 = 3$

$N=4 \qquad N^{N/2} = 4^2 + 3 =$

---

$N_1^a + N_1 - b = 9$

$N_2^a + N_2 - b = 3$

[illegible]

$b = N^a + N - 3$

$4^a + 4 - b = 9$

$2^a + 2 - b = 3$

$b = 2^a + 2 - 3 = 2^a - 1$

$4^a + 4 - 2^a + 1 = 9$

$4^a - 2^a = 9-5 = 4$

$4^a - 2^a = 4$

$4^a - 4 = 2^a \qquad 2^{2a} - 2^a = 4 \qquad 2^a(2^a - 1) = 4$

---

$a\cdot 4 + 4 - b = 9$

$a\cdot 2 + 2 - b = 3$

$aN + N - b = x$

$4a + 4 - 2a + 1 = 9$

$b = 2a - 1$

$2a + 5 = 9$

$2a = 4 \qquad a = 2$

$b = 2\cdot a - 1 = 3$

NMV $\quad 2N + N - 3 = 3N - 3 = 3(N-1)$ } ЕКСПОНЕНТ НА СФ ВО БРОИТЕЛОТ

---

$4^a + 4 - b = 19$

$2^a + 2 - b = 7$

$b = 2^a - 5$

$4^a + 4 - 2^a + 5 = 19$

$4^a - 2^a = 10$

---

$4a + 4 - b = 19$

$2a + 2 - b = 7$

$b = 2a - 5$

$4a + 4 - 2a + 5 = 19$

$2a = 10 \qquad a = 5$

$b = 5$

$5N + N - 5 = 6N - 5$ ⟹ EKSPONENT NA IMENITELOT

- GENERALNA FORMA NA APROKSIMIRANIOT BER E:

$$P_{ea} = \frac{1}{2} - \frac{\sqrt{N \cdot c\rho}}{(Nc\rho + N + 1)^{\frac{6N-5}{2}}} \cdot \sum_{k=0}^{3(N-1)} c(k) \cdot (N \cdot c \cdot \rho)^k$$

MMV

$c\rho = \gamma$

$$\lim_{\gamma \to \infty} P_{ea} = \frac{(N^N + 1)\,\Gamma(N + \frac{1}{2})}{2\,N!\,N^N\,\gamma^N\,\sqrt{\pi}}$$

$C(k) = ?$

$$P_{ea} = \frac{1}{2} - \frac{\sqrt{N \cdot \gamma}}{(N \cdot \gamma + N + 1)^{\frac{6N-5}{2}}} \cdot \sum_{k=0}^{3(N-1)} c(k) \cdot (N \cdot \gamma)^k$$

$$\lim_{\gamma \to \infty} P_{ea} = \frac{1}{2} - \frac{\sqrt{N\gamma}\,(N \cdot \gamma)^{3(N-1)}}{(N \cdot \gamma + N + 1)^{\frac{6N-5}{2}}} \cdot C(3(N-1)) \quad ?$$

---

$$P_{ea3} = \frac{1}{2} - \frac{3\sqrt{3}\sqrt{cr}\,(161770 + 1112937c^2r^2 + 983376c^3r^3 + 621250cr + 494208c^4r^4 + 134784c^5r^5 + 15552c^6r^6)}{128\,(3cr + 4)^{13/2}}$$

– 11 –

- So Lim vo Maple se dobiva

$$P_{eas} = \frac{1}{2} - \cdot C \qquad C = \frac{\left(\frac{1}{2} - P_{eas}\right)}{C(3(N-1))}$$

$$P_{eas} = \frac{(N^N + 1)\,\Gamma(N + \frac{1}{2})}{2\,N!\,N^N\,c^N\,\rho^N\,\sqrt{\pi}}$$

$168 : 4 = 42$

$$C(3(N-1)) = \frac{1}{2} - \frac{(N^N + 1)\,\Gamma(N + \frac{1}{2})}{2\,N!\,N^N\,c^N\,\rho^N\,\sqrt{\pi}}$$

$$P_{ea2} = \frac{1}{2} - \frac{\sqrt{2g}\,(135 + 300g + 168g^2 + 32g^3)}{8\,(2g - 3)^{7/2}}$$

$$= \frac{1}{2} - \frac{\sqrt{2g}\,(135 + 150(2g) + 42(2g)^2 + 4 \cdot (2g)^3)}{8\,(2g - 3)^{7/2}}$$

$C(0) = \frac{135}{8}$ | $C(1) = \frac{150}{8} = \frac{75}{4}$ | $C(2) = \frac{42}{8} = \frac{21}{4}$ | $C(3) = \frac{4}{8} = \frac{1}{2}$

$$\prod_{i=1}^{t}(2i-1) = \frac{2^t\,\Gamma(t+\frac{1}{2})}{\sqrt{\pi}}$$

$$\frac{\prod_{i=1}^{t}(2i-1)}{2\cdot 2^t\cdot t!} = \frac{2^t\,\Gamma(t+\frac{1}{2})}{2\cdot 2^t\cdot t!\sqrt{\pi}} = \frac{\Gamma(t+\frac{1}{2})}{2\cdot t!\cdot\sqrt{\pi}}$$

$C20 = \frac{135}{8}$ $2^3$ $\quad C30 = \frac{242655}{64}$ $2^6$ $\quad C40 = \frac{32659375}{128}$ $2^7$

$2^{N+1}$ - $2^N\cdot N! = 2^2\cdot 2! = 8$

$3^2\cdot 6$.

$2^{2N-1} = 2^3 = 8$

$2^5 = 32$

$$C50 = \frac{6890312994796375}{2^{18}}$$

---

$aN_1 + N_1 - b = 3 \qquad b = aN_1 + N_1 - 3 = 2a + 2 - 3 = 2a - 1$

$aN_2 + N_2 - b = 18 \qquad 5a + 5 - b = 18$

~~aN2+N2 aN ... = 18~~ $\qquad 5a + 5 - 2a + 1 = 18$

$3a = 12 \quad \boxed{a = 4} \quad \boxed{b = 7}$

$\boxed{4N + N - 7} \qquad \boxed{5N - 7}$

$15 - 7 = 8$

$20 - 7 = 13$

$$\frac{6N-5}{2} = \frac{12-5}{2} = \frac{7}{2}$$

---

$a\cdot 2 + 2 - b = 3 \qquad b = 2a - 1 \qquad \boxed{b = 2 - 1 = 1}$

$a\cdot 4 + 4 - b = 7 \qquad 4a + 4 - 2a + 1 = 7 \qquad 2a = 2 \quad \boxed{a = 1}$

$N + N - 1 = \boxed{2N - 1}$

$$C[3(N-1)] = \frac{1}{2} - \frac{(N^N+1)\,\Gamma(N+\frac{1}{2})}{2N!\,N^N c^N \rho^N\sqrt{\pi}}$$

---

$$\frac{1}{2\sqrt{\pi}}\left(\sqrt{\pi} + \frac{2\cdot 2^{\frac{M-2}{2}}\sqrt{8}\,\Gamma(\frac{5}{2}-M)\,M^{\frac{9-3M}{2}}}{\underbrace{(M\cdot 8 + M + 1)^{\frac{7-2M}{2}}}_{A}}\right. - 3 \qquad \frac{2^{\frac{M-2}{2}}\sqrt{8}\,\Gamma(\frac{5}{2}-M)\,M^{\frac{7-3M}{2}}}{A}$$

$$-5\,\frac{2^{\frac{M-2}{2}}\sqrt{\bar\delta}\,\Gamma\left(\frac{5}{2}-M\right)M^{\frac{5-3M}{2}}}{\left(M\bar\delta+M+1\right)^{\frac{7-2M}{2}}}+\frac{2^{\frac{M}{2}}\sqrt{\bar\delta}\,\Gamma\left(\frac{5}{2}-M\right)M^{\frac{5-3M}{2}}}{\underbrace{\left(M\cdot\bar\delta+M+1\right)^{\frac{5-2M}{2}}}_{B}}$$

$$\frac{2^{\frac{M}{2}}\sqrt{\bar\delta}\,\Gamma\left(\frac{5}{2}-M\right)M^{\frac{7-3M}{2}}}{\left(M\bar\delta+M+1\right)^{\frac{5-2M}{2}}}$$

3203231

$$P_{ea}(\bar\delta)=\frac{1}{2\sqrt{\pi}}\left[\sqrt{\pi}+\frac{2^{\frac{M-2}{2}}\sqrt{\bar\delta}\,\Gamma\left(\frac{5}{2}-M\right)}{\left(M\cdot\bar\delta+M+1\right)^{\frac{7-2M}{2}}}\left(2M^{\frac{9-3M}{2}}-3M^{\frac{7-3M}{2}}-5M^{\frac{5-3M}{2}}\right)\right.$$

$$\left.+\frac{2\cdot 2^{\frac{M-2}{2}}\sqrt{\bar\delta}\,\Gamma\left(\frac{5}{2}-M\right)}{\left(M\cdot\bar\delta+M+1\right)^{\frac{5-2M}{2}}}\left(2M^{\frac{5-3M}{2}}+M^{\frac{7-3M}{2}}\right)\right]$$

$$\frac{6M-5}{2}-\frac{-2M+7}{2}=\frac{6M-5+2M-7}{2}=\frac{8M-12}{2}=4M-6$$

$$P_{ea}(\bar\delta)=\frac{1}{2}+\frac{2^{\frac{M-2}{2}}\sqrt{\bar\delta}\,\Gamma\left(\frac{5}{2}-M\right)}{\left(M\bar\delta+M+1\right)^{\frac{6M-5}{2}}}\left[\left(2M^{\frac{9-3M}{2}}-3M^{\frac{7-3M}{2}}-5M^{\frac{5-3M}{2}}\right)\cdot\right.$$

$$\left.\left(M\bar\delta+M+1\right)^{2M+1}+2\left(2\cdot 2M^{\frac{5-3M}{2}}+M^{\frac{7-3M}{2}}\right)\cdot\left(M\bar\delta+M+1\right)^{2M}\right]$$

$$\frac{6M-5}{2}-\frac{-2M+5}{2}=\frac{6M-5+2M-5}{2}=\frac{8M-10}{2}=4M-5$$

$$(a+b)^n=\sum_{i=1}^{n}\binom{n}{i}a^i b^{n-i}$$

$$M-\frac{5}{2}-M+\frac{7}{2}=\frac{7-5}{2}=1$$

$$x^{-\frac{7}{2}+M}\cdot x=x^{1-\frac{7}{2}+M}=x^{\frac{2-7}{2}+M}=x^{-\frac{5}{2}+M}$$

$$P_{ea2}=\frac{1}{2}-\frac{\sqrt{\bar\delta}\,\Gamma\left(\frac{5}{2}-M\right)M^{-\frac{3M}{2}}\,2^{\frac{M-2}{2}}\left(M\bar\delta+M+1\right)^{-\frac{7}{2}+M}M^{5/2}\left[3(M+1)+2M(M-1)\bar\delta\right]}{2\sqrt{\pi}}$$

$$P_{ea2}=\frac{1}{2}-\frac{\sqrt{\bar\delta}\,\Gamma\left(\frac{5}{2}-M\right)M^{\frac{-3M+5}{2}}\cdot 2^{\frac{M-2}{2}}\left[3(M+1)+2M(M-1)\bar\delta\right]}{2\sqrt{\pi}\left(M\bar\delta+M+1\right)^{\frac{7-2M}{2}}}$$

ova e za: $\alpha(M,z)=\frac{1}{2}\left(\frac{z}{2}\right)^{-M}$

$$M^{\frac{5-3M}{2}} = M^{\frac{1+4-3M}{2}} = \sqrt{M}\cdot M^{\frac{4-3M}{2}} = \sqrt{M}\cdot M^2 \cdot M^{-\frac{3M}{2}}$$

$$Pea2 = \frac{1}{2}\ \frac{\sqrt{M\cdot\bar{\delta}}\ \Gamma\left(\frac{5}{2}-M\right) 2^{\frac{M-2}{2}}\left(3(M+M)M^2 + 2M(M-1)M^2\bar{\delta}\right)}{M^{\frac{3M}{2}}\cdot 2\sqrt{\pi}\ \left(M\bar{\delta}+M+1\right)^{\frac{6M-5}{2}}}$$

$$\frac{6M-5}{2} - \frac{7-2M}{2} = \frac{8M-12}{2} = 4M-6$$

$$Pe2 = \frac{1}{2} + \sum_{k=0}^{M-1} \frac{(-1)^k M^{k-M+\frac{5}{2}}\bar{\delta}^{-M+\frac{5}{2}}\left(M+1+M\bar{\delta}\right)^{M-\frac{5}{2}}}{\left(1+(1+\bar{\delta})M\right)^{2k}}$$

$$\frac{\Gamma(M-k)\,\Gamma\left(2k-M+\frac{5}{2}\right)}{\sqrt{\pi}\ \Gamma(k)\ \Gamma(M^2)} - \sum_{k=0}^{M-1} P23(\delta, M, k)$$

$$= \frac{1}{2} + \frac{\bar{\delta}^{\frac{5}{2}-M} M^{\frac{5}{2}-M}}{2\sqrt{\pi}\left(M\bar{\delta}+M+1\right)^{-M+7/2}\Gamma^2(M)} \sum_{k=0}^{M-1} \frac{(-1)^k M^k (M\bar{\delta}+M+1)\cdot 2}{\Gamma(k)\left(\bar{\delta}M+M+1\right)^{2k}} - \sum_{k=0}^{M-1} P23(\bar{\delta}, M, k)$$

$$\Gamma(M-k)\,\Gamma\left(2k-M+\frac{5}{2}\right)$$

$$(*) = \frac{\bar{\delta}^{5/2-M} M^{5/2-M}}{2\sqrt{\pi}\cdot C^{7/2-M}\ \Gamma^2(M)} \sum_{k=0}^{M-1} \frac{(-1)^k M^k\left(M-2k-\frac{1}{2}\right)\Gamma(M-k)\,\Gamma\left(2k-M+\frac{1}{2}\right)}{\Gamma(k)\ C^{2k}\cdot k}$$

$$\cdot\left(M-2k-\frac{3}{2}\right)\left(2(M+1)k + \frac{3M}{2} + M^2\bar{\delta} + \frac{3}{2} - M\cdot\bar{\delta}\right)$$

$$\boxed{C = \left(\bar{\delta}M+M+1\right)}$$

$$\Gamma(M-k) = (M-k-1)!$$

$$Pe2 = \frac{1}{2} + \frac{(M\cdot\bar{\delta})^{\frac{5}{2}-M}}{2\sqrt{\pi}\left(M\bar{\delta}+M+1\right)^{\frac{7}{2}-M}\Gamma^2(M)} \sum_{k=0}^{M-1} \frac{(-1)^k M^k\left[\left((2M\bar{\delta}+2M+2)(k)\Gamma(M-k)\,\Gamma\left(2k-M+\frac{5}{2}\right)\right) - \left(M-2k-\frac{1}{2}\right)\Gamma(M-k)\,\Gamma\left(2k-M+\frac{1}{2}\right)\left(M-2k-\frac{3}{2}\right)\left(2(M+1)k + \frac{3M}{2} + M^2\bar{\delta} + \frac{3}{2} - M\bar{\delta}\right)\right]}{k\cdot\Gamma(k)\, C^{2k}}$$

$$\boxed{Pe2 = \frac{1}{2} - \frac{(M\cdot\bar{\delta})^{\frac{5}{2}-M}}{2\sqrt{\pi}\left(M\bar{\delta}+M+1\right)^{\frac{7}{2}-M}\Gamma^2(M)} \sum_{k=0}^{M-1} \frac{(-1)^k M^k\cdot\Gamma(M-k)\,\Gamma\left(2k-M+\frac{1}{2}\right)\left(\frac{3}{2}+M^2\bar{\delta}+\frac{3M}{2}-(k+1)\bar{\delta}M\right)\left(M-2k-\frac{3}{2}\right)\left(M-2k-\frac{1}{2}\right)}{k\,\Gamma(k)\left(M\bar{\delta}+M+1\right)^{2k}}}$$

MKV

$$C_1(k) = (-1)^k M^k\cdot\Gamma(M-k)\,\Gamma\left(2k-M+\frac{1}{2}\right)\left(\frac{3}{2}+M^2\bar{\delta}+\frac{3M}{2}-(k+1)\bar{\delta}M\right)\left(M-2k-\frac{3}{2}\right)\left(M-2k-\frac{1}{2}\right)$$

$$Pea = \frac{1}{2} - \frac{\sqrt{N\delta}}{(N\delta+N+1)^{\frac{6N-5}{2}}} \sum_{k=0}^{3(N-1)} c(k)(N\cdot\delta)^k$$

3090600

07020402S

Keste

$i = 2k \quad k = M-1 \quad i = 2(M-1)$

$i = 3k \quad k = 0 \quad i = 0 \quad k = (M-1) \quad i = 3(M-1)$

$$\Gamma\left(n+\frac{1}{2}\right) = \frac{(2n-1)!!}{2^n}\sqrt{\pi}$$

$$\Gamma(-z) = -\frac{\pi \csc(\pi z)}{z\,\Gamma(z)} = -\frac{\pi}{z\,\Gamma(z)\cdot \sin(z\pi)}$$

$$A = \frac{\Gamma\left(6i - M + \frac{5}{2}\right)\Gamma(M-3i)}{\Gamma(3i+1)}$$

$$\Gamma(z-n) = \frac{(-1)^n\,\Gamma(z)}{(1-z)_n}$$

$$(x)_n = x(x-1)(x-2)\cdots(x-n+1)$$

$$A = \frac{(2(6i-M))!!}{2^{6i-M}}\;\frac{(-1)^{3i}\,\Gamma(M)}{(1-M)_{3i}\;\Gamma(3i+1)}$$

$$\sum_{i=1}^{M} a_i = \sum_{j=0}^{M-1} a_{j+1} \qquad j = i-1$$

$$Pea = \frac{1}{2} - \frac{\sqrt{N\delta}}{(N\delta+N+1)^{\frac{6N-5}{2}}} \sum_{k=0}^{3(N-1)} C(k)(N\cdot\delta)^k$$

$$\frac{7-2M}{2} - \frac{6M-5}{2} = \frac{7-8M+5}{2} = \frac{12-8M}{2} = 6-4M$$

$$\frac{7-2M}{2} - (6-4M) = \frac{6M-5}{2} \qquad \frac{7-2M}{2} + (4M-6) = \frac{6M-5}{2}$$

$$Pe2 = \frac{1}{2} - \frac{(M\cdot\delta)^{\frac{1}{2}+2-M}\cdot(M\delta+M+1)^{4M-6}}{2\sqrt{\pi}\,(M\delta+M+1)^{\frac{6M-5}{2}}} \sum_{k=0}^{M-1} \frac{G_1(k)}{k\cdot\Gamma(k)\cdot(M\delta+M+1)^{2k}}$$

$\Gamma(k+1) = k!$

$$Pe2 = \frac{1}{2} - \frac{\sqrt{M\delta}}{2\sqrt{\pi}\,(M\delta+M+1)^{\frac{6M-5}{2}}} \sum_{k=0}^{M-1} \frac{G_1(k)\,(M\delta)^{2-M}(M\delta+M+1)^{4M-6-2k}}{2\sqrt{\pi}\,\Gamma(k+1)}$$

$$= \frac{1}{2} - \frac{\sqrt{M\delta}}{(M\delta+M+1)^{\frac{6M-5}{2}}} \sum_{k=0}^{M-1} \frac{G_1(k)\,(M\delta)^k\cdot(M\delta)^{2-M-k}(M\delta+M+1)^{4M-6-2k}}{2\sqrt{\pi}\,\Gamma(k)}$$

$$= \frac{1}{2} - \frac{\sqrt{M\delta}}{(M\delta+M+1)^{\frac{6M-5}{2}}} \sum_{k=0}^{M-1} C(k)(M\delta)^k$$

$$C(k) = \frac{G_1(k)\cdot(M\delta+M+1)^{4M-2k-6}}{2\sqrt{\pi}\,\Gamma(k+1)\,(M\delta)^{M+k-2}}$$

$C_1(k,\bar{\gamma}) = (-1)^k M^k \Gamma(M-k)\Gamma\left(2k-M+\frac{1}{2}\right)\left(\frac{3}{2}+M^2\bar{\gamma}+\frac{3M}{2}-(2k+1)\bar{\gamma}M\right)\left(M-2k-\frac{3}{2}\right)\left(M-2k-\frac{1}{2}\right)$

$$Pe2 = \frac{1}{2} - \frac{\sqrt{M\bar{\gamma}}}{(M\bar{\gamma}+M+1)^{\frac{6M-5}{2}}} \sum_{k=0}^{M-1} \frac{C_1(\bar{\gamma},k)(M\bar{\gamma}+M+1)^{4M-2k-6}(M\bar{\gamma})^{k}}{2\sqrt{\pi}\,\Gamma(k+1)(M\bar{\gamma})^{M+k-2}}$$

$$Pe2 = \frac{1}{2} - \frac{\sqrt{M\bar{\gamma}}}{(M\bar{\gamma}+M+1)^{\frac{6M-5}{2}}} \sum_{k=0}^{M-1} \frac{C_2(k)\left[\frac{3}{2}+\frac{3M}{2}+M\bar{\gamma}(M-2k-1)\right](M\bar{\gamma}+M+1)^{4M-2k-6}}{2\sqrt{\pi}\,\Gamma(k+1)\,(M\bar{\gamma})^{M-2}}$$

48
18
66

$C_2(k) = (-1)^k M^k \Gamma(M-k)\Gamma\left(2k-M+\frac{1}{2}\right)\left(M-2k-\frac{3}{2}\right)\left(M-2k-\frac{1}{2}\right)$

$$C(M,k) = \frac{(-1)^k M^k \Gamma(M-k)\Gamma\left(2k-M+\frac{1}{2}\right)\left(M-2k-\frac{3}{2}\right)\left(M-2k-\frac{1}{2}\right)}{2\sqrt{\pi}\,\Gamma(k+1)}$$

$$Pe(2) = \frac{1}{2} - \frac{\sqrt{M\bar{\gamma}}}{(M\bar{\gamma}+M+1)^{\frac{6M-5}{2}}} \sum_{k=0}^{M-1} C(M,k)\, \underbrace{\frac{\left[M(M-2k-1)\bar{\gamma}+\frac{3(M+1)}{2}\right](M\bar{\gamma}+M+1)^{4M-2k-6}}{(M\bar{\gamma})^{M-2}}}_{x(\bar{\gamma},k,M)}$$

MMV

$M=2$

33·4 / 132    162 / 27 / 135    42·8 / 336

$$(*) = \sum_{k=0}^{1} C(2,k)\,x(\bar{\gamma},k,M) = C(2,0)\left[2(2-1)\bar{\gamma}+\frac{9}{2}\right](2\bar{\gamma}+3)^2 + C(2,1)\left[2(-1)\bar{\gamma}+\frac{9}{2}\right](2\bar{\gamma}+3)^0 = \frac{1}{2}\left(2\bar{\gamma}+\frac{9}{2}\right)\left(4\bar{\gamma}^2+12\bar{\gamma}+9\right) - \frac{3}{4}\left(-2\bar{\gamma}+\frac{9}{2}\right) =$$

$C(2,0) \to \frac{1}{2}$, $C(2,1) \to -\frac{3}{4}$

75·4 / 300

~~$(*) = \frac{1}{2}\left(16\bar{\gamma}^3+48\bar{\gamma}^2+32\bar{\gamma}+18\bar{\gamma}^2+54\bar{\gamma}+\frac{81}{2}\right)+\frac{3}{2}\bar{\gamma}-\frac{27}{8} =$~~

~~$= 8\bar{\gamma}^3+\frac{66}{2}\bar{\gamma}^2+43\bar{\gamma}+\frac{81}{4}+\frac{3}{2}\bar{\gamma}-\frac{27}{8} = 8\bar{\gamma}^3+\frac{66}{2}\bar{\gamma}^2+\frac{89}{2}\bar{\gamma}+\frac{135}{8}$~~

~~$= \frac{64\bar{\gamma}^3+264\bar{\gamma}^2+392\bar{\gamma}+135}{8}$~~

eUEl - MeUCl
CAM

$= \left(\bar{\gamma}+\frac{9}{4}\right)\left(4\bar{\gamma}^2+12\bar{\gamma}+9\right)+\frac{3}{2}\bar{\gamma}-\frac{27}{8} =$

$= 4\bar{\gamma}^3+12\bar{\gamma}^2+9\bar{\gamma}+9\bar{\gamma}^2+27\bar{\gamma}+\frac{81}{4}+\frac{3\bar{\gamma}}{2}-\frac{27}{8} =$

$= 4\bar{\gamma}^3+21\bar{\gamma}^2+\frac{75}{2}\bar{\gamma}+\frac{135}{8} = \frac{32\bar{\gamma}^3+168\bar{\gamma}^2+300\bar{\gamma}+135}{8}$ ✓

$3a+b=11$    $3a+7-2a=11$    $a=4$
$2a+b=7$    $b=7-2a$    $b=7-8=-1$

~~$3a+3-b=11$    $3a+3-2a+9=11$    $a=11-12=-1$~~
~~$2a+2-b=7$    $b=2a-9$    $b=-2-9=-11$~~
~~$-3M+M-11$~~

$3a+3-b=11$ $\quad 2a+2-7=6 \quad 2a - 5 = 6$ $\boxed{b=6-5=1}$

$2a+2-b=7$ $\quad 3a+3-2a+5=11$ $\boxed{a=11-8=3}$

$\boxed{3N+N-1}$ $\quad \boxed{4N-1}$

$$-M+\frac{4M-1}{2} = \frac{-2M+4M - 1}{2} = \frac{2M-1}{2} = M-\frac{1}{2}$$

STEPENOT NA $(M \cdot C + M + 1)^{B}$ $\quad \boxed{B = M - \frac{1}{2}}$

$3a+3-b=7$

$2a+2-b=5$ $\quad \boxed{b=2a-3}$ $\quad 3a+3-2a+3=7$ $\boxed{a=1}$

$\boxed{b=2-3=-1}$

$$Ma+M-b = M+M+1 = 2M+1$$

$$\boxed{M-\frac{2M+1}{2} = \frac{2M-2M-1}{2} = -\frac{1}{2}}$$

---

$$(z)_n = z(z+1)(z+2)\cdots(z+n-1)$$

$$\left(k-\frac{M}{2}+\frac{1}{4}\right)\left(k-\frac{M}{2}+\frac{3}{4}\right)\left(k-\frac{M}{2}+\frac{5}{4}\right) =$$

$$-\frac{1}{8}\left(M-2k-\frac{1}{2}\right)\left(M-2k-\frac{3}{2}\right)\left(M-2k-\frac{5}{2}\right) =$$

$$= \frac{1}{8}\left(\underbrace{2k-M+\frac{1}{2}}_{z}\right)\left(2k-M+\frac{3}{2}\right)\left(2k-M+\frac{5}{2}\right)$$

$z = \left(2k-M+\frac{1}{2}\right)$ $\quad (z)_{M \to 3} = \frac{1}{8}\left(2k-M+\frac{1}{2}\right)\left(2k-M+\frac{1}{2}+1\right)\cdot\left(2k-M+\frac{1}{2}+2\right)$

MMV

$$\to = \frac{1}{2^n} \;!!!$$

$$(z)_a = \frac{\Gamma(z+a)}{\Gamma(z)} \qquad \Gamma\left(\underbrace{2k}_{n}+\frac{1}{2}\right) = \frac{\sqrt{\pi}}{2^n}(2n-1)!!$$

$$= \frac{\sqrt{\pi}}{2^{2k}}(4k-1)!! = \boxed{(2n-1)!! = \frac{(2n)!}{2^n \cdot n!}} = \frac{\sqrt{\pi}}{2^{2k}}\frac{4k!}{2^{2k}\cdot 2k!}$$

$4M^4 \cdot c^2 r^2$ ZA $3 \times 1 \times 7$ $2^{M-1} \cdot M^M c^{M-1} r^{M-1}$

$2M^2 c \cdot r$ ZA $2 \times 1 \times 2$

---

$2 \times 1 \times 2$

$B_{M2}(M, K, r, c) = 2(M - 2k - 1) M \cdot r \cdot c + 3(M+1)$

$3 \times 1 \times 3$ $= 2(M^k - (2k+1)) M \cdot r \cdot c + 3(M+1)$ $\cdot Mcr$

$4(M^2 + 2(k+1)(2k+1) - (4k+3)M) M^2 c^2 r^2 + 20(M^2 - (2k+1)M - 2(k+1))$
$+ 15(M+1)^2$

$B_{M3}(r, c, M, k) =$ $15(M+1)^2$

$= 4[M^2 + 2(k+1)(2k+1) - (4k+3)M] M^2 c^2 r^2 + 20(M^2 - (2k+1)M - 2(k+1)) Mcr +$ $2^{M-2}$

GENERALIZACIJA:

$B_{2i2} = 2 \cdot A [M^{M-1} - (2k+1) M^{M-2}] M \cdot r \cdot c + 3 \cdot c (M+1)^{M-1}$

$2 \cdot a + 2 - b = 2$ $2a + 2 - 3a + 17 = 2$ $a = 17$ $51 - 17 = 34$

$3a + 3 - b = 20$ $b = 3a - 17$ $b = 51 - 17 = 34$

$17 \cdot M + M - 34 = 18M - 32$ $54 - 32 = 20$ FIRST ORDER COEFFICIENT

---

$2^a + 2 - b = 2$ $b = 2^a$

$3^a + 3 - b = 20$ $3^a + 3 - 2^a = 20$ $3^a - 2^a = 17$

$2^{2a} + 2 - b = 2$ $b = 2^{2a}$

$2^{3a} + 3 - b = 20$ $2^{3a} - 2^{2a} + 3 = 20$

$\Gamma(2a + 2 - b) = 2$ ?

$\Gamma(3a + 3 - b) = 20$ $39 - 23 = 16$ $36 - 21 = 15$

---

$2a + 2 - b = 3$ $b = 2a - 1$ $b = 22 - 1 = 21$

$3a + 3 - b = 15$ $3a + 3 - 2a + 1 = 15$ $a = 11$

$11M + M - 21$ $12M - 21$ ZERO ORDER

---

$2a + 2 - b = 0$ $b = 2(a+1)$ $a = 3$ $3M + M - 8$

$3a + 3 - b = 4$ $3a + 3 - 2a - 2 = 4$ $b = 8$ $4M - 8$ SECOND ORDER

$B_{MIM}(r,c,M,k)=$

$$= 4(M-2)\left[M^{M-1}+2(k+1)(2k+1)-(4k+3)M\right]M^2c^2r^2+2(9M-17)\left[M^{M-1}-(2k+1)\cdot M^{M-2}-2^{M-2}(k+1)\right]Mcr+\left((12M-21)\cdot(M+1)^{M-1}\right)$$

---

$$B_{MIM}(r,c,M,k)= \underbrace{4(M-2)\left[M^{M-1}+2(k+1)(2k+1)-(4k+3)M\right]}_{a_3}M^2c^2r^2+\underbrace{2(9M-17)\left[M^{M-1}-(2k+1)M^{M-2}-2^{M-2}(M-2)(k+1)\right]}_{a_2}Mcr+\underbrace{(12M-21)\cdot(M+1)^{M-1}}_{a_1}$$

MMV

---

OVA RABOTI NO SAMO ZA 313 I 212

TREBA DA SE GENERALIZIRA ZA 414

$$M^{M-1}-(4k+3)M^{M-2}= M^{M-2}(M-4k-3)$$

$$\left[M^{M-2}(M-4k-3)+2M^{M-3}(k+1)(2k+1)\right](Mcr)^{M-1}$$

$$M^{M-2}\left(M-(M+1)k-M\right)= M^{M-2}(M+1)k$$

$$\sum_{n=0}^{M-1} c_n\left[M^{M-2}(M+1)k+2(k+1)(2k+1)\right]M^n c^n r^n$$

MORA DA SE ODI SO „n"

$$a_3= 4(M-2)\left[M^{M-1}-(4k+3)M^{M-2}+2^{M-2}(k+1)(2k+1)\right]$$

$$a_2= 2(9M-17)\left[M^{M-1}-(2k+1)M^{M-2}-2^{M-2}(M-2)(k+1)\right]$$

$$a_1= (12M-21)(M+1)^{M-1}$$

---

- DA GO PRETSTAVAM 2x1x2 SO SUMA

$$B_{212}= 2(M-2k-1)M\cdot r\cdot c+3(M+1)=\sum_{n=0}^{M-1}c_n(n)\cdot\left(M-2(k+1)\cdot n+1\right)\cdot(Mrc)^n$$

$$= c_0\left(M-2(k+1)\cdot n+1\right)(M\cdot r\cdot c)^0+c_1\left(\underbrace{M-2k-2+1}_{M-2k-1}\right)=$$

$$= c_0(M+1)+c_1(M-2k-1)$$

• DA SE ODIDAM 3+1+3 DA GO PRETSTAVAM SO SUMA

$$B_{313} = 4\left[M^2 + 2(k+1)(2k+1) - (4k+3)M\right](Mcr)^2 + 20\left(M^2 - (2k+1)M - 2(k+1)\right)_{Mcr} + 15(M+1)^2$$

$$M^2 - 4kM - 3M = M(M-4k-3) = M(M - 2\cdot 2(k+1) + 1)$$

$$M^2 - 2kM - M = M(M - 2k - 1) = M(M - 2(k+1) + 1)$$

$$B_{313} = 4\left[M(M - 2\cdot 2(k+1)+1) + 2(k+1)(2k+1)\right](Mcr)^2 + 20\left[M(M - 2(k+1)+1) - 2(k+1)\right](Mcr) + 15(M+1)^2$$

$$\to 2\cdot(-1)^n (k+1)(2k+1)$$

$$M(M - 2\cdot n(k+1)+1) + 2\cdot c\cdot(k+1)$$

$$(M+1)^2 = M^2 + 2M + 1$$

$n=1$ $M(M - 2(k+1)+1)$

$n=2$ $M(M - 4(k+1)+1)$

$n=0$ $M(M+1) + (M+1)$ ?

$$2nk - (2n-1) = 2n(k-1)+1$$

1 $2k-1$

2 $4k-3$

$$\boxed{C = C(n)}$$

$$2\cdot C\cdot(k+1) = 2(k+1)(2k+1)$$

$$2\cdot C(k+1) = -2(k+1)$$

$$2\cdot C(k+1) = (M+1)$$

$$(k+1)^n \cdot (2k+1)^{n-1}$$

$$[(n-1)k+1][nk+1]$$

$$\boxed{M(M - 2n(k+1)+1) + (-1)^n 2[(n-1)k+1](nk+1)}$$ GENERAL

$\boxed{n=1}$

$$M(M - 2(k+1)+1) - 2(k+1)$$

$\boxed{n=2}$

$$M(M - 4(k+1)+1) + 2[(k+1)(2k+1)]$$

$\boxed{n=0}$

$$M(M+1) + 2(1-k)$$

$n=1 \quad 2k-1$

$n=2 \quad 4k-3$

$n=0$

$$m(m-2^n(k+1)+1)+2(-1)^n[(n-1)k+1][nk+1]$$

$$m(m-(k+1)+1)+2(1-k) = m(m-k)+2(1-k)$$

---

[illegible] $m(m-k)+2(1-k)(m-2) = m^2-km+(2-2k)(m-2) =$

$= m^2-km+2m-2mk-4+4k$

$(m-n+1)$ KOEFICIENTI

$m=2$ $\quad 2-0+1=3 \quad n=0.$

$\quad 2-1+1=2 \quad n=1$

$m=3$ $\quad (m-n+1)(m+2) = |n=0| = 4\cdot 5 = 20$

---

1. $a\cdot m+b\cdot n-c=2 \qquad m=2 \quad n=1$
2. $a\cdot m+b\cdot n-c=3 \qquad m=2 \quad n=0$

1. $2a+b-c=2 \qquad 2a+b-2a+3=2 \quad b=-1$
2. $2a-c=3 \rightarrow c=2a-3 \quad c=33$

[illegible]

3. $a\cdot m+b\cdot n-c=20 \qquad m=3 \quad n=1$

$3a+b-c=20 \qquad 3a-1-2a+3=20 \quad a=18$

$18m-n-33$

$m=2 \quad n=0 \rightarrow 36-0-33=3$

$m=2 \quad n=1 \quad 36-1-33=2$

$m=3 \quad n=1 \quad 54-1-33=54-34=20$

$m=3 \quad n=2 \quad 54-2-33=54-35=19$ ?

$m=3 \quad n=0 \quad 54-33=21$ ?

$$aM + bn + d\cdot M\cdot n + c = D$$

$M = 2$

$n=0$ $2a + c = 3$ (1)

$n=1$ $2a + b + 2d + c = 2$ (2)

$M = 3$

$n=0$ $3a + c = 15$ (3)

$n=1$ $3a + b + 3d + c = 20$ (4)

$n=2$ $3a + 2b + 6d + c = 4$ (5)

$$aM + bn + dM\cdot n + e\cdot M^n + c = D$$

$M = 2$

$n=0$ $2a + e + c = 3$ (1)

$n=1$ $2a + b + 2d + 2e + c = 2$ (2)

$M = 3$

$n=0$ $3a + e + c = 15$ (3)

$n=1$ $3a + b + 3d + e\cdot 3 + c = 20$ (4)

$n=2$ $3a + 2b + 6d + 9e + c = 4$ (5)

VO POMATAMOSTITE GENERALIZIRANI FORMI PO OVIOJ REDOSLED E DEFINIW „n"

$$C = \frac{1}{4}\left(48M - 73n + 45M\cdot n - 21M^n - 63\right)$$

$$B = M(M - 2n(k+1) + 1) + 2(-1)^n((n-1)k+1)(nk+1)$$

$n=0$ $B = (M+1)^2$

$$M(M - 2n(k+1) + 1) + 2(-1)^n((n-1)k+1)(nk+1))$$

$$M(M - 2^n(k+1) + 1) + 2\cdot(-1)^n\left[(2^n k+1)(2^{n+1}k+1)\right]$$

$$M(M - 2^n(k+1) + 1) + 2(-1)^n\left[((n-1)k+1)(nk+1)\right]$$

$n=0$ $M(M - 2(k+1) + 1) + 2\left[(-k+1)\cdot 1\right] =$

$M^2 - 2M(k+1) + M + 2 - 2k$

$$\text{pochhammer}\left(2k-m+\frac{1}{2}, m\right) = \frac{\Gamma\left(2k-m+\frac{1}{2}+m\right)}{\Gamma\left(2k-m+\frac{1}{2}\right)} = \frac{\Gamma\left(2k+\frac{1}{2}\right)}{\Gamma\left(2k-m+\frac{1}{2}\right)}$$

$$x^{m-\frac{1}{2}} = x^{\frac{15}{2}-m} \cdot Y \qquad Y = x^{m-\frac{1}{2}-\frac{15}{2}+m}$$

$$Y = x^{-\frac{16}{2}+2m} = x^{-8+2m}$$

$$\frac{x^{\frac{15}{2}-m}}{Y} = x^{m-\frac{1}{2}} \qquad Y = x^{\frac{15}{2}-m-m+\frac{1}{2}} = x^{8-2m}$$

$$\frac{15}{2}-m = \frac{15}{2}-4 = \frac{15-8}{2} = \frac{7}{2} \qquad m-\frac{1}{2} = \frac{8-1}{2} = \frac{7}{2}$$

$$\text{pochhammer}(z, a) = \frac{G(z+a)}{G(z)}$$

---

$q = \text{quo}(a, b, x)$ → $a = b \cdot q + r$ 070205477

$r = \text{rem}(a, b, x)$

$$\frac{\Gamma(x)}{\Gamma(x+a)} \qquad \frac{1}{\text{pochhammer}(x, a)}$$

---

• САКАМ ДА СЕ ОБИДАМ СО ГЕНЕТСКИ АЛГОРИТАМ ДА ГИ НАЈДАМ КОЕФИЦИЕНТИТЕ. НАЈДОБРО Е СО МУЛТИОБЈЕКТИВЕ ГА

$$f_1 = 2\Gamma(b) - \Gamma(b+2c+d)$$

$$f_2 = 3\Gamma(a+b) - \Gamma(a+b+2c+d)$$

$$f_3 = 4\Gamma(b) - \Gamma(b+3c+d)$$

$$f_4 = 20\,\Gamma(a+b) - \Gamma(a+b+3c+d)$$

test:=(n,m) → $2^{am-bn-c}$ pochhammer(dn+e, fm+g)

$$2-\frac{2^{2a+c}\Gamma(e+2f+g)}{\Gamma(e)}=0$$

[2, 3]

[4, 20, 15]

[8, 84, 210, 105]

**451**

$$3-\frac{2^{2a+b+c}\Gamma(d+e+2f+g)}{\Gamma(d+e)}=0$$

$$4-\frac{2^{3a+c}\Gamma(e+3f+g)}{\Gamma(e)}=0$$

$$20-\frac{2^{3a+b+c}\Gamma(d+e+3f+g)}{\Gamma(d+e)}=0$$

$$15-\frac{2^{3a+2b+c}\Gamma(2d+e+3f+g)}{\Gamma(2d+e)}=0$$

2
3
4
20
15
8
84
210
105
**451**

• a = 1, b = 1, c = 1, d = 2, e = 1, f = 1, g = −1

2+3+8+24+30+48+240+420+504 = 1279

initChrom = [1, −1, −1, 2, 1, 1, −1]

initFVAL = 451 − 1279 = −828

---

◘ GENERALNA UNIVERZALNA FORMULA BEZ CONST. COEFFICIENTI:

$$Pe_{N\times N}(r,c,m)=\frac{1}{2}-\frac{1}{2^m}\sum_{k=0}^{m-1}\sum_{n=0}^{n-1}\frac{(-m)^k\,\Gamma(n-k)\,\Gamma\left(2k+\frac{1}{2}\right)\cdot(-1)^{n+m+1}\cdot(m+1)^n\,(mrc)^{m+\frac{1}{2}}}{(2k)_{n-m+1}\sqrt{\pi}\,\Gamma^2(m)\,(mrc+m+1)^{m-\frac{1}{2}}\,\Gamma(k+1)\,(m\cdot rc+m+1)^{2k}}$$

$$Pe_{N\times N}(r,c,m)=\frac{1}{2}-\frac{(-1)^{m+1}(mrc)^{m+\frac{1}{2}}}{2^m\sqrt{\pi}\,\Gamma^2(m)\,(mrc+m+1)^{m-\frac{1}{2}}}\sum_{k=0}^{m-1}\sum_{n=0}^{m-1}\frac{c(n,k)(-1)^{k+n}\,\Gamma(m-k)\,\Gamma\left(2k+\frac{1}{2}\right)\cdot m^k\,(m+1)^{n-m}}{\Gamma(k+1)\,(mrc+m+1)^{2k}\,(2k)_{n-m+1}}$$

**c(n,k) = ?**

---

$$\Gamma\left(2k+\frac{1}{2}\right)=\frac{\sqrt{\pi}}{2^{2k}}(4k-1)!!=$$ | DOUBLE FACTORIAL WOLFRAM.COM $(2n-1)!!=\frac{(2n)!}{2^n n!}$ | $$=\frac{\sqrt{\pi}}{2^{2k}}\cdot\frac{(4k)!}{2^{2k}(2k)!}$$

GRADSHTEYN 8.339.2

$$=\frac{\sqrt{\pi}}{16^k}\cdot\frac{(4k)!}{(2k)!}=\frac{\sqrt{\pi}}{16^k}\cdot\frac{\Gamma(4k+1)}{\Gamma(2k+1)}$$

$$(*) = \frac{\Gamma(n-k)\,\Gamma(2k+\frac{1}{2})\cdot\Gamma(2k)}{\Gamma(k+1)} = \frac{\Gamma(n-k)\cdot\Gamma(2k)}{\Gamma(k+1)}\cdot\frac{\sqrt{\pi}}{16^k}\cdot\frac{\Gamma(4k+1)}{\Gamma(2k+1)}$$

$$\frac{\Gamma(2k)}{\Gamma(2k+1)} = \frac{2\cdot4\cdot6\cdot\ldots\cdot2k-1}{3\cdot5\cdot7\cdot\ldots\cdot2k+1} = \frac{\Gamma(2k)}{2k\cdot\Gamma(2k)}$$

$$\boxed{\Gamma(x+1) = x\cdot\Gamma(x)} \qquad \Gamma(\alpha) = \int_0^\infty t^{\alpha-1}e^{-t}\,dt$$

$$\Gamma(\alpha+1) = \int_0^\infty t^{\alpha+1-1}\cdot e^{-t}\,dt = \int_0^\infty t^{\alpha}\cdot e^{-t}\,dt$$

$$\Big/\, t = u-1 \quad dt = du \,\Big/ \quad t=0 \quad u=1 \qquad = \int_0^\infty (u-1)^{\alpha}\cdot e^{-u+1}\,du$$

---

INTEGRATION BY PARTS

$$\int_0^\infty t^{\alpha-1}e^{-t}\,dt = \left|\begin{array}{l} u = t^{\alpha-1} \\ du = (\alpha-1)t^{\alpha-2}\,dt \\ v = \int e^{-t}\,dt = -e^{-t}\end{array}\right| = u\cdot v - \int v\,du$$

$$= -t^{\alpha-1}\cdot e^{-t}\Big|_0^\infty + \int_0^\infty e^{-t}(\alpha-1)\,t^{\alpha-2}\cdot dt = \frac{0}{e^0} - \frac{(\infty)^\alpha}{e^{+\infty}} + (\alpha-1)\int_0^\infty t^{\alpha-2}e^{-t}\,dt$$

$$\int_0^\infty t^{\alpha-1}e^{-t}\,dt = (\alpha-1)\int_0^\infty t^{(\alpha-1)-1}e^{-t}\,dt = (\alpha-1)\,\Gamma(\alpha-1)$$

$$\Gamma(\alpha) = (\alpha-1)\,\Gamma(\alpha-1) \qquad \boxed{\Gamma(\alpha+1) = \alpha\cdot\Gamma(\alpha)}$$

---

$$(*) = \frac{\Gamma(n-k)\Gamma(2k)}{\Gamma(k+1)}\cdot\frac{\sqrt{\pi}}{2^{4k}}\cdot\frac{2\,4k\,\Gamma(4k)}{2k\cdot\Gamma(2k)}$$

$$(*) = \frac{\sqrt{\pi}}{2^{4k-1}}\,\frac{\Gamma(n-k)\,\Gamma(4k)}{\Gamma(k+1)}$$

$A = \text{pochhammer}(2k-n+1, m-1) = \dfrac{\Gamma(2k-n+1+n-1)}{\Gamma(2k-n+1)}$

$B = \text{pochhammer}(2k-n+1, n) = \dfrac{\Gamma(2k-n+1+n)}{\Gamma(2k-n+1)}$

$\dfrac{A}{B} = \dfrac{\Gamma(2k)}{\Gamma(2k-m+1+n)} = \dfrac{1}{\text{pochammer}(2k, m+1-n)}$

$24 = 6 \cdot 4 = \Gamma(3+1) \cdot 2^2$

| m\n | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 2 | 2 | 3 | | |
| 3 | 4 | 20 | 15 | |
| 4 | 8 | 84 | 210 | 105 |

$\dfrac{\Gamma(n+m+1)}{2^n}$

| m\n | 0 | 1 | 2 | 3 | ~~4~~ |
|---|---|---|---|---|---|
| 2 | 2 | 3 | | | |
| 3 | 6 | 12 | 30 | | |
| 4 | 24 | 60 | 180 | 630 | |

• $e(4,0)$

$\dfrac{\Gamma(0+5)}{x\,2^n} = 8$ , $n \to 1$

$\dfrac{24}{x \cdot 2^n} = 8$ $x = 3$ $\Gamma(5) = 24$

$e(m,n)$ $e(4,1) = \dfrac{\Gamma(1+5)}{x \cdot 2^1} \overset{?}{=} 84$ $\Gamma(6) = 120$ ?

~~$e(3,2) = 210 \quad \Gamma(7) = 720$~~

$\dfrac{\Gamma(6)}{x \cdot 2^1} = \dfrac{120}{x \cdot 2^1} = 84$ $x = \dfrac{120}{168} = \dfrac{5}{7} = \left(\dfrac{m+n}{m+n+2}\right)^n$

(?) $\dfrac{\Gamma(n+m+1)}{(m+2n-1)\,2^n} = \dfrac{(2n-1)}{(m+2n-1)} \cdot \dfrac{\Gamma(n+m+1)}{2^n}$

• $e(4,2) = 210$ $\Gamma(7) = 720$ $2^2 = 4$

$\dfrac{(2n-1)\,\Gamma(n+m+1)}{2 \cdot 2^n}$

$\dfrac{720}{4} = 60/2 = 30 \cdot 7$

mnozi so $7^n$, vo $e(4,1)$ i vo $e(4,2)$

$$\frac{(2m-1)\Gamma(n+m)}{(2m-2n-1)\,2^{2n-1}}$$

$[2,7]$, $[4,9,15]$ $[12, \frac{84}{5}, 35, \frac{315}{2}]$ ($\cdot 5 \cdot 6 \cdot \frac{2}{3}$)

40, 20, 15 $[8, 84, 210, 105]$

1 4 1

n : 0 1 2 3

$(2^{n+1}-1) =$ 1 3 5 7

$$\frac{(2m-1)\Gamma(n+m)}{(2m-2n-1)\,2^n\,\Gamma(n)}$$

$[1,3]$ $[1, \frac{5}{2}, 15]$ $[1, \frac{14}{5}, \frac{35}{3}, 105]$

2, 1   4, 8, 1   8, 30, 18, 1

- ~~$e(m,n)$ $e(4,3)$~~

$$\frac{(2m-1)\,\Gamma(8)}{(m+2n-1)\,2^3}$$

$\Gamma(8) = 5040$

$\frac{\Gamma(8)}{105} = 48$

$7 = 8(4+6-1) = 8\cdot 9 = 54$

$$\frac{\Gamma(8)}{x\cdot 2^3} = \frac{5040}{x\cdot 8} = 105$$

$x \cdot 8 = 48$

$x = 6$

$$e(4,3) = \frac{\Gamma(8)}{x\cdot 2^4} = \frac{\Gamma(8)}{6\cdot 2^4} = \frac{\Gamma(8)}{48} = 105$$

- $e(4,2)$ REVISITED

$$\frac{\Gamma(7)}{x\cdot 2^2} = 210$$

$$x = \frac{720}{4\cdot 210} = \frac{6}{7}$$

---

- GENERALIZED FORMULA

$$e(m,n) = \frac{\Gamma(n+m+1)}{x(n)\,2^n}$$

$\Gamma(1) = 1$
$\Gamma(2) = 1$
$\Gamma(3) = 2$
$\Gamma(4) = 6$

$x(n) = 3, \frac{5}{7}, \frac{6}{7}, 6$

1 · 1 · 2 · 6

$\Gamma(m-n)$, $m = 4$

$[6, 2, 1, 1]$ $\Gamma(m-n)$

$[1, 1, 2, 6]$ $\Gamma(n+1)$

| n\m | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 2 | 2 | 3 | | |
| 3 | 4 | 20 | 15 | |
| 4 | 8 | 84 | 210 | 105 |

e(3,0) $\frac{\Gamma(m+n+1)}{x\cdot 2^n} = 4$  $\frac{\Gamma(3+1)}{x\cdot 2^{n=0}} = 4$

$x = \frac{\Gamma(4)}{4} = \frac{6}{4} = \frac{3}{2}$

e(3,1) $\frac{\Gamma(5)}{x\cdot 2} = 20$  $x = \frac{24}{40} = \frac{12}{20} = \frac{6}{10} = \frac{3}{5}$

e(3,2) $\frac{\Gamma(6)}{x\cdot 2^2} = 15$  $x = \frac{120}{15\cdot 4} = \frac{120}{6} = 20$

$\frac{2m-1}{am+bn+c} = \frac{2}{3}$

$n=0 \Rightarrow$ $am+bn+c = \frac{3}{2}(2m-1)$

$n=1 \Rightarrow$ $am+bn+c = \frac{3}{5}(2m-1)$

$n=2 \Rightarrow$ $am+bn+c = 20(2m-1)$

$3a+c = \frac{15}{2}$  $3a+b+c = \frac{15}{5}$  $3a+2b+c = 100$

NEMA REŠENJE

$\frac{(2m-1)\Gamma(n+m)}{(2m-2n-1)2^{2n-1}} = [4, 20, 15]$

e(3,0) $\frac{5\cdot\Gamma(3)}{x\cdot 2^{2n-1}} = 4$  $x = \frac{5\cdot 2\cdot 2}{4} = 5$

e(3,1) $\frac{5\cdot\Gamma(4)}{x\cdot 2} = 20$  $x = \frac{5\cdot 3}{20} = \frac{15}{20} = \frac{3}{4}$

e(3,2) $\frac{5\,\Gamma(5)}{x\cdot 8} = 15$  $x = \frac{5\cdot 24}{15\cdot 8} = \frac{5\cdot 3}{15} = 1$

$am+bn+c = 5 / \frac{3}{4} / 1$

$3a+c = 5$  $3a+b+c = \frac{3}{4}$  $3a+2b+c = 1$

NEMA REŠENJE

e(2,0) : $\frac{(dm+ \quad +e)\Gamma(n+m)}{2^{2n-1}(am+bn+c)} = [2, 3]$

$e(2,0)$

$$\frac{(2d+e)\,\Gamma(2)}{2^{-1}\cdot(an+bn+c)} = 2$$

$a n^2 + b n + c = (2d+e)$ (with $n \to 0$)

$2a+c-2d-e=0$ (1)

$e(2,1)$

$$\frac{(2d+e)\,\Gamma(3)}{2^{1}\cdot(2a+b+c)} = 3$$

$3(2a+b+c) = 2(2d+e)$

(2) $6a+3b+3c-2d-e=0$

- od $e(3,n)$ se dobiva

$3a+c = \frac{3d+e}{5}$

$3a+c-3d-e=0$ (3)

$3a+b+c = \frac{3}{4}\cdot\frac{3d+e}{5}$

$20(3a+b+c) = 9d+3e$

$60a+20b+20c-9d-3e=0$ (4)

$3a+2b+c = \frac{1}{5}(3d+e)$

$15a+10b+5c-3d-e=0$ (5)

---

$a=-2,\ b=5\ \ c=11\ \ d=2\ \ e=-3\ \ f=3$

$$\frac{(-2m+5n+11)}{(2m-3n+3)}$$

$[43, -89, 7, 2, -17]\cdot\frac{1}{40}$

$54\cdot 3 = 162$

$197 - 162 = 25$

$197 - 108 = 89$

---

$16k^2 - 16mk + 108k + 4m^2 - 54m + 197$

$197 - 54 = 143$

$4(2k-m)^2 = 16k^2 - 16mk + 4m^2$

$108k - 54m + 197 = 54(2k-m+1) + 89$

$54(2k-m+2) + 89$

$54(2k-m+3) + 25$

$72 - 53 = 19$

$4(2k-m+6)^2 + 12k - 6m + 53 = 4(2k-m+6)^2 + 12(2k-m+6) - 19$

$= (2k-m+6)\left[4(2k-m+6)+12\right] - 19$

$= 4(2k-m+6)(2k-m+9) - 19$

$\cdot \quad (a,n) = a\cdot(a+1)\cdot(a+2)\cdots(a+n-1)$

$(7,-3) = 7\cdot$

$$\left.\begin{matrix} 1, & 7, & 15, & 105 \\ 1 & 7, & 7, & 1 \end{matrix}\right\} \times$$

---

$a(n) + b(n) + c = D$

$$\left.\begin{aligned} 3a + b + c &= \frac{7}{3} \\ 4a + b + c &= 3 \\ 4a + 2b + c &= 3 \end{aligned}\right\} \quad a = \frac{2}{3} \quad b = 0 \quad c = \frac{1}{3}$$

$\boxed{\frac{2}{3}m + \frac{1}{3} = D}$ $\qquad \boxed{\frac{1}{3}(2m+1)}$

$$(2n+1)!! = \frac{(2n+1)!}{2^n n!} = \frac{\Gamma(2n+2)}{2^n\cdot\Gamma(n+1)} \qquad n = 0,1,2,\dots$$

$\boxed{(am+b)\cdot n\cdot(m-n-1) = D}$

$m=3, n=1$: $(3a+b)\cdot 1\cdot(3-1-1) = \frac{10}{3}$ $\qquad 3a+b = \frac{10}{3}$

$m=3, n=1$: $(4a+b)\cdot 1\,(4-1-1) = 7$ $\qquad 4a+b = \frac{7}{2}$

$b = \frac{7}{2} - 4a$

$3a + \frac{7}{2} - 4a = \frac{10}{3}$ $\qquad \boxed{a = \frac{7}{2} - \frac{10}{3} = \frac{21-20}{6}}$ $\qquad$ ~~$b = \frac{19}{3}$~~

$\boxed{a = \frac{1}{6}}$

$\boxed{b = \frac{17}{6} = \frac{17}{6}}$ $\qquad b = \frac{7}{2} - \frac{4}{6} = \frac{21-4}{6}$

$\left(\frac{m}{6} + \frac{17}{6}\right)\cdot n\,(m-n-1)$

$\frac{1}{6}(m+17)\cdot n(m-n-1)\dots$

$3a+b = \frac{10}{3} - 1$ $\quad 3a+b = \frac{7}{3}$ $\quad b = \frac{7}{3} - 3a$

$2(4a+b) = 7 - 1 \Rightarrow$ $\quad 2(4a+b) = \frac{6}{2}$ $\quad 4a+b = 3$

$m=3$, $n=1$: $1\cdot(3a+b)(m-n-1) + c\cdot 1 = \frac{10}{3}$ $\quad 3a+b+c = \frac{10}{3}$

$m=4$, $n=1$: $1\cdot(4a+b)(m-n-1) + c\cdot 1 = 7$ $\quad 2(4a+b)+c = 7$

$m=4$, $n=2$: $2(4a+b)(m-n-1) + c\cdot 2 = 7$ $\quad 2(4a+b)+2c = 7$

$a = \frac{1}{6}$ $\quad b = \frac{17}{6}$

$$\left(\frac{m}{6} + \frac{17}{6}\right)(m-n-1) = \frac{1}{6}(m+17)(m-n-1)$$

$m=4$, $n=3$: $3(4a+b)(4-3-1) + 3c = 1 \Rightarrow$ $\boxed{c = \frac{1}{3}}$

$3a+b+\frac{1}{3} = \frac{10}{3}$ $\quad 3a+b = 3$

$8a+2b+\frac{1}{3} = 7$ $\quad 8a+2b = \frac{21-1}{3} = \frac{20}{3}$

$\boxed{a = \frac{1}{3} \quad b = 2}$ $\quad \boxed{n\left(\frac{1}{3}m + 2\right)(m-n-1) + \frac{1n}{3}}$

$$\boxed{\frac{n}{3}\left[(m+6)(m-n-1)+1\right]}$$

NE DIVA GI ZAFRKNUVA KOEFICIENTITE ZA $\boxed{n=0}$

• NAJKOMPAKTEN IZLAZ KOJ IMA DO 4+1+4

$$P_e = \frac{1}{2} + \frac{(mrc)^{m-1/2}}{3\sqrt{\pi}\,\Gamma^2(m)\,(mrc+m+1)^{m-\frac{1}{2}}} \sum_{k=0}^{m-1}\sum_{n=0}^{m-1} \frac{(-1)^{n+k+m}\left((2m+1)n(m-n-1)+3\right)}{2^{4k-1}\,\Gamma(k+1)\,\Gamma(2k+n-m+1)} \cdot$$

$$\cdot \frac{\Gamma\left(n+\frac{3}{2}\right)\Gamma(m-k)\,\Gamma(4k)\,m^{k}\,(m+1)^{n}}{(mrc)^{n}\,(mrc+m+1)^{2k}}$$

$$\Gamma\left(n+\frac{1}{2}\right) = \frac{\sqrt{\pi}}{2^n}(2n-1)!! = \frac{\sqrt{\pi}}{2^n}\frac{(2n)!}{2^n n!} = \frac{\sqrt{\pi}}{2^n}\frac{\Gamma(2n+1)}{2^n\cdot\Gamma(n+1)}$$

$$\Gamma\left(n+2k+\frac{1}{2}\right) = \frac{\sqrt{\pi}}{2^{2n+4k}}\frac{\Gamma(2n+4k+1)}{\Gamma(n+2k+1)}$$

- POTSETUVANJE CDF ZA 2x2x2 SYSTEM

$$F = 1 - \frac{2}{\Gamma^2(t)\bar{\gamma}^t}\left\{\frac{d^{t-1}}{dw^{t-1}}\left[\frac{e^{-\frac{2}{\bar{\gamma}w}}}{w}K_t\left(\frac{2}{\bar{\gamma}w}\right)\right]\right\}$$

IEMT10-stosic-ladzivecov-final7.pdf ⇒ eq59

- $E\left[\Theta(\sqrt{k\gamma})\right] \to \frac{\prod_{i=1}^{t}(k_i-1)\, p_\gamma^{(t-1)}(0)}{2k^t\, t!}$

$$\Gamma\left(n+\frac{1}{2}\right) = \frac{\sqrt{\pi}}{2^n}(2n-1)!! = \frac{\sqrt{\pi}}{2^n}\frac{(2n)!}{2^n n!} = \frac{\sqrt{\pi}(2n)!}{2^{2n} n!}$$

26.090
- 9.000

17090
15000

32.000

- IZRAZITE NEKA ADAT SO GAMMA FUNKCII NAMESTO SO $\binom{n}{k}$

$$\binom{N-1}{n} = \frac{\Gamma(N)}{\Gamma(n+1)\Gamma(N-n)}$$

- POTVRDA NA ANALITICKITE REZULTATI SO NUMERICKA INTEGRACIJA
- PRIMER ZA DRUGO „$d$" NA PRIMER ZA QAM.

$$\mathrm{pochammer}(0,-3) = z\cdot(z+1)\cdot(z+2)\cdots(z+n-1)$$

$n-1=-3 \qquad \boxed{n=-4}$

$\boxed{n>0}$

$$\frac{\Gamma(m-k)}{\Gamma(m-n)} = \frac{1\cdot2\cdot3\cdots m-k-1}{1\cdot2\cdot3\cdots m-n-1} = (m-n)\cdots(m-k-1)$$

~~...~~ = pochammer

$$\frac{\text{pochhammer}(2k-m+1,\ m-1)}{\text{pochhammer}(2k-m+1,\ n)} = \frac{1}{\text{pochhammer}(2k-m+1,n)}$$

**N×1×N**

$$P_e^{N\times1\times N}(r,c,m,d) = \frac{1}{2} + \frac{\sqrt{dmcr}}{2\sqrt{\pi}\,\Gamma^3(m)} \sum_{k=0}^{m-1}\sum_{n=0}^{m-1} \frac{(-1)^{k+m+n}\cdot 2^{n+2k}\cdot\Gamma(m-k)\Gamma(2k)\Gamma(n+2k+\frac{1}{2})\Gamma(m)(m+1)\ldots}{\Gamma(2k+m-n+1)\cdot\Gamma(n+1)\Gamma(m-n)(dmcr+2m+2)^{n+2k+\frac{1}{2}}}$$

$$P_{eas}^{N\times1\times N}(r,c,m,d) = \frac{\Gamma(2m+1)\,(m^m+1)}{2^{m+1}\cdot\Gamma(m+1)^2\cdot N\cdot d^m\cdot c^m\cdot r^m}$$

MMV

**N×N×N**

$$P_e^{N\times N\times N}(r,c,N,d) = \frac{1}{2} + \frac{\sqrt{drc}}{2\sqrt{\pi}\,\Gamma(m^2)} \sum_{k=0}^{m^2-1}\sum_{n=0}^{m^2-1} \frac{(-1)^{m^2+n+k}\cdot 2^{2n+2k}\cdot\Gamma(m^2-k)\cdot\Gamma(n+2k+\frac{1}{2})\cdot\Gamma(2k)}{\Gamma(2k+n-m^2+1)\cdot\Gamma(k+1)\cdot\Gamma(n+1)\cdot\Gamma(m^2-n)(4+drc)^{n+2k+\frac{1}{2}}}$$

$$P_{ea}(r,c,m,d) = \frac{\Gamma(2m^2+1)}{2^{m^2}\Gamma(m^2+1)^2\cdot d^{m^2}\cdot r^{m^2}\cdot c^{m^2}}$$

MMV

$$c = \frac{T}{K\cdot m\cdot S\cdot \mathrm{ld}M}$$

(VO SIMULACIITE) OVA GO STAVAV DA MNOZI DA NE TREBA DA DELI?

LOGICNO E ŠTO MNOZI ZOŠTO IMPLICITNO SE ZGOLEMUVA SIGNALOT PO BIT KOGA SE PRODOLZUVA INTERVALOT ZA PRENESUVANJE.

- SIMULACIITE ZA 4×8 I 3×8 RABOTAT KOGA $E_s/N_0$ GO MNOZAM SO $1/\sqrt{mS}$ A NE SO „c"

$$c = \frac{T}{K\cdot mS\cdot \mathrm{ld}M} = \left|4\times4\right| = \frac{4}{3\cdot4\cdot\mathrm{ld}\,2} = \frac{1}{3}$$

$$M_w(-s) = \frac{4}{\Gamma^2(N)}\left(\frac{s}{\gamma}\right)^{N^2} K_{N^2}\left(2\sqrt{\frac{s}{\gamma}}\right)$$

$$COF = 1 - \mathcal{L}^{-1}\left\{\frac{M_w(-s)}{s}\right\}\Big|_{\omega=\frac{1}{\gamma}}$$

$$\mathcal{L}^{-1}\left\{K_d^2\left[2\sqrt{\frac{s}{\gamma}}\right]\right\} = \frac{1}{2\omega}\, e^{-\frac{2}{\omega\gamma}}\, K_d\left(\frac{2}{\gamma\omega}\right)$$

$$CDF = 1 - \mathcal{L}^{-1}\left[\frac{M_W(-s)}{s}\right]\Big|_{w=\frac{1}{\gamma}}$$

$$M_W(-s) = \frac{4}{\Gamma^2(d)} \frac{s^d}{\gamma^d} K_d^2\left(2\sqrt{\frac{s}{d}}\right)$$

$$\mathcal{L}^{-1}\left\{K_d^2\left[2\sqrt{\frac{s}{\gamma}}\right]\right\} = \frac{1}{2w} e^{-\frac{2}{w\gamma}} \cdot K_d\left(\frac{2}{\gamma w}\right)$$

$$\mathcal{L}^{-1}\left[\frac{M_W(-s)}{s^d}\right] = \frac{4}{\Gamma^2(d)} \cdot \frac{1}{\gamma^d} \mathcal{L}^{-1}\left[K_d^2\left(2\sqrt{\frac{s}{d}}\right)\right] =$$

$$= \frac{2}{\Gamma^2(d)} \cdot \frac{1}{\gamma^d} \cdot \frac{1}{w} e^{-\frac{2}{w\gamma}} K_d\left(\frac{2}{\gamma w}\right) = \frac{2 e^{-\frac{2}{w\gamma}}}{\Gamma^2(d)\,\gamma^d} \cdot K_d\left(\frac{2}{\gamma w}\right)$$

$$\boxed{CDF(\gamma) = 1 - \frac{2 e^{-\frac{2\gamma}{\bar\gamma}} \cdot \gamma}{\Gamma^2(d) \cdot \bar\gamma^d} K_d\left(\frac{2\gamma}{\bar\gamma}\right)}$$

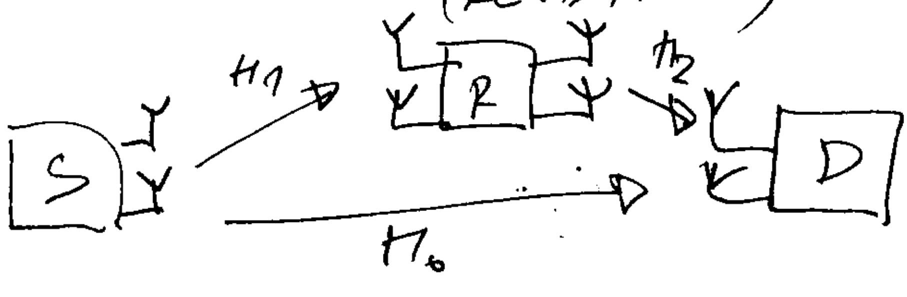
$$G_R = \frac{G}{k_1} \qquad P_S = k_1^2 k_2^2 \epsilon G_R^2 \Delta^2 \Lambda^2$$
$$P_N = k_2^2 G_R^2 \cdot \Lambda^2 k_1^2 \Delta N_0 + k_2^2 N_0$$

$$\gamma = \frac{P_S}{P_N} = \frac{k_1 k_2^2 \epsilon \cdot G_R \cdot \Delta^2 \Lambda^2}{k_2^2 G_R^2 \Lambda^2 k_1^2 \cdot \Delta N_0 + k_2^2 \Lambda N_0} =$$

$$= \frac{k_1^2 k_2^2 \cdot \epsilon \cdot \frac{G^2}{k_1^2} \Delta^2 \Lambda^2}{k_2^2 \frac{G^2}{k_1^2} \cdot \Lambda^2 k_1^2 \cdot \Delta N_0 + k_2^2 \Lambda N_0} = \frac{\epsilon \cdot G^2 \cdot \Delta^2 \Lambda^2}{G^2 \Lambda^2 \Delta N_0 + \Lambda N_0}$$

$$\boxed{\gamma = \frac{\epsilon}{N_0} \cdot \frac{G^2 \Delta^2}{G^2 \Lambda \Delta + 1}}$$

- L. YANG, et.al., PERFORMANCE ANALYSIS OF MIMO ... (REVISITED)



- S-R HAVE SAME TRANSMISSION RATE AS R-D I.E. ALL TERMINALS HAVE SAME NUMBER OF ANTENNAS.
- TRANSMITTED SYMBOLS: $s_1, s_2, \dots, s_L$

$C_1$ - OSTBC MATRIX $N \times T$

L SYMBOLS TRANSMITTED OVER T TIME INTERVALS

$$R = \frac{L}{T}$$

$$r_{SR} = \sqrt{\frac{P}{N}} H_1 \cdot C_1 + W_1$$

$$r_{SD} = \sqrt{\frac{P}{N}} H_0 C_1 + W_0$$

- THE EQUIVALENT SISO CHANNEL:

$$r_{R,l} = \sqrt{\frac{P}{N}} c \|H_1\|_F^2 \cdot s_l + \tilde{w}_{R,l} \qquad l = 1, 2, \dots L$$

c - CONSTANT DEPENDING ON STBC MATRIX NORMALIZATION SO: $\sqrt{E[r_{R,l}]}$

$$r'_{R,l} = \frac{\sqrt{\frac{P}{N}} \cdot c \|H_1\|_F^2 \cdot s_l}{\sqrt{\frac{P}{N} c^2 \|H_1\|_F^4 + \|H_1\|_F^2 \cdot N_0}} + \frac{\tilde{w}_{R,l}}{\sqrt{\frac{P}{N} c^2 \|H_1\|^4 + \|H_1\|_F^2 N_0}}$$

$$= \frac{\sqrt{\frac{P}{N}} \cdot \sqrt{c}\, \|H_1\|_F^2 \cdot s_l}{\sqrt{c} \|H_1\|_F \sqrt{\frac{P}{N} \cdot c \|H_1\|_F^2 + N_0}} + \frac{\tilde{w}_{R,l}}{\sqrt{c} \|H_1\|_F \sqrt{\frac{P}{N} c \|H_1\|^2 + N_0}}$$

$$= \frac{\sqrt{\frac{P}{N}} \cdot \sqrt{c} \|H_1\|_F \cdot s_l}{\sqrt{\frac{P}{N} \cdot c \|H_1\|_F^2 + N_0}} + \frac{\tilde{w}_{R,l}}{\sqrt{c} \|H_1\|_F \sqrt{\frac{P}{N} c \|H_1\|^2 + N_0}}$$

$l = 1, 2, \dots L$

- $r'_{R,l}$ ARE ENCODED BY STBC MATRIX $C_2$ WITH RATE $R = L/T$.

$$r_{RD,l} = \sqrt{\frac{P}{N}} \cdot c \cdot \|H_2\|_F^2 r'_{R,l} + \tilde{w}_{RD,l} = \frac{\frac{P}{N} c \sqrt{c} \|H_1\|_F \|H_2\|^2 \cdot s_l}{\sqrt{\frac{P}{N} \cdot c \|H_1\|_F^2 + N_0}}$$

$$+ \frac{\sqrt{\frac{P}{N}} \cdot c \|H_2\|_F^2 \cdot \tilde{w}_{R,l}}{\sqrt{c} \|H_1\|_F \sqrt{\frac{P}{N} \cdot c \|H_1\|_F^2 + N_0}} + \tilde{w}_{RD,l}$$

- VO GENERALEN SLUČAJ

$$G = \frac{\sqrt{E_s N_0 \cdot K \cdot C}}{\sqrt{E_s N_0 \cdot K \cdot C \cdot \Delta_{12}^2 + K \cdot \Delta_{12}}}$$

- SEGA VO RECEVO $P_{R,C}$ SE SUMIRA PO BROJOT NA ANTENI !!! TOA TREBA DA SE OGLEDA VO ČLANAKOT.

- RANDOM MATRIX THEORY AND WIRELESS COMM.

$$\boxed{y = H \cdot x \cdot n} \quad (1.1)$$

CAPACITY OF (1.1) DEPENDS ON THE DISTRIBUTION OF THE SINGULAR VALUES OF **H**.

5-6 CARINA 1-2. MVR

IVAN ILIEV

3100111

GORDANA NA GOLAN TRD

SNOUZE 1, SNOUZE 1, SUTI, TEOOVO

BITOLA → VLADIMI ZORI

$P_S = E G^2 \Delta^2 \lambda^2 \qquad P_N = G^2 \lambda^2 \Delta N_0 + \lambda N_0$

$$\gamma = \frac{P_S}{P_N} = \frac{E G^2 \Delta^2 \lambda^2}{G^2 \lambda^2 \Delta N_0 + \lambda N_0} = \frac{E}{N_0} \frac{G^2 \Delta^2 \lambda}{G^2 \lambda \Delta + 1}$$

---

POWER PER SYMBOL

$$C_{3\times4} = \overbrace{\begin{bmatrix} x_1 & x_2 & x_3 \\ -\bar{x}_2 & \bar{x}_1 & 0 \\ \bar{x}_3 & 0 & -\bar{x}_1 \\ 0 & \bar{x}_3 & -\bar{x}_2 \end{bmatrix}}^{P} \begin{matrix} \to \frac{P}{3} \\ \to \frac{P}{2} \\ \to \frac{P}{2} \\ \to \frac{P}{2} \end{matrix}$$

$$E = \frac{P \cdot L}{K \cdot N}$$

$L = 4 \quad K = 3$

$$E = \frac{P \cdot 4}{3 \cdot 3} = \frac{4P}{9}$$

SREDNA SNAGA PO ANTENA

$$\frac{\frac{P}{3} + 2\frac{P}{2}}{4} = \frac{4P}{4 \cdot 3} = \frac{P}{3}$$

Columns: $\frac{P}{3}, \frac{P}{2}, \frac{P}{2}, 0$ ; $\frac{P}{3}, \frac{P}{2}, 0, \frac{P}{2}$ ; $\frac{P}{3}, 0, \frac{P}{2}, \frac{P}{2}$

MMV

$$\frac{P}{3} + \frac{2 \cdot P}{2} = \frac{4P}{3}$$

PER ANTENNA

$$3 \cdot \frac{4P}{9} \cdot \frac{1}{4} = \frac{P}{3}$$

$$3 \cdot \frac{4P}{9} + (3)(2)\frac{4P}{9} = \frac{4P}{3} + \frac{8P}{3} = \frac{12P}{3} = 4P$$

(3) SLOTS, (2) SYMBOLS

VKUPNATA SNAGA VO BLOKOT TREBA DA BIDE $L \cdot P$

(*)
$$\frac{\frac{P}{3} + \frac{P}{2} + \frac{P}{2}}{(3)} = \frac{\frac{4P}{3}}{(3)} = \frac{4P}{9}$$

(3) SYMBOL, (3) SYMBOL — HMV

→ AVERAGE POWER PER SYMBOL!!!

- SEGA DA KAZE DEKA VO SEKOJ SLOT IMAŠ POWER LIMIT EDNAKOV NA „P" SO TOA SREDNATA SNAGA PO SIMBOL SE DOBIVA SOGLASNO (*)
- POENTATA E DEKA ZA 4 TIME-SLOTA SREDNATA EMITUVANA SNAGA OD PREDAVATELOT E P!!!

• (448)
$$E = P \cdot C = P \cdot \frac{L}{K \cdot N} = P \cdot \frac{8^2}{4 \cdot 4} = \frac{2P}{4}$$

• (348)
$$E = P \cdot \frac{L}{K \cdot N} = P \cdot \frac{8}{4 \cdot 3} = \frac{2P}{3}$$

JAFARKHANI 4.152

POWER PER SYMBOL

$$\frac{\frac{3 \cdot P}{3} + \frac{3 \cdot P}{3}}{3} = \frac{2P}{3}$$

→ TOTAL POWER PER SYMBOL

→ AVERAGED PER ANTENNA

STEMITR 3237801

• (434) JAFARKHANI 4.103

- POWER PER SYMBOL PER ANTENNA

$$\frac{\frac{4P}{3}}{4} = \frac{P}{3}.$$

$$E = P \cdot \frac{L}{K \cdot N} = \frac{4}{3 \cdot 4} = \frac{1}{3}$$

$$C_{434} = \begin{bmatrix} x_1 & x_2 & x_3 & 0 \\ -x_2^* & x_1^* & 0 & x_3 \\ x_3^* & 0 & -x_1^* & x_2 \\ 0 & x_3^* & -x_2^* & -x_1 \end{bmatrix}$$

GLAVNA POENTA: (VIDI $)

$$\frac{(3) \cdot \frac{4P}{9}}{(4)} = \frac{P}{3}$$

(3) SYMB, (4) SLOTS

→ SREDNATA SNAGA PO ANTENA OSTANUVA KAKO VO $.

• 434 (CORTES CODE)

$$\frac{\frac{3}{8}+\frac{3}{8}+\frac{3}{8}+\frac{3}{8}}{4} = \frac{3}{8}$$

$$E = P \cdot \frac{L}{K \cdot N} = P \cdot \frac{4}{3 \cdot 4} = \frac{P}{3} \quad (4)$$

$$E[C_{4,4}^{\wedge 2}] = \begin{bmatrix} 1 & 1 & 1/2 & 1/2 \\ 1 & 1 & 1/2 & 1/2 \\ 1/2 & 1/2 & \frac{1}{2}+\frac{1}{2} & \frac{1}{2}+\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2}+\frac{1}{2} & \frac{1}{2}+\frac{1}{2} \end{bmatrix}$$

$$\frac{1+1+\frac{1}{2}\cdot 2+\frac{1}{2}\cdot 2}{4} = \frac{2+2}{4} = 1$$

– PRVA REDICA:

$$\frac{P}{3}+\frac{P}{3}+\frac{P}{6}+\frac{P}{6} = 3 \cdot \frac{P}{3} = P$$

• POWER PER ANTENA:

$$E = \frac{P}{3} \qquad \frac{\frac{P}{3}+\frac{P}{3}+2\cdot\frac{1}{2}\frac{P}{3}}{4} = \frac{\frac{3P}{3}}{4} = \frac{P}{4}$$

• POWER PER SYMBOL: $E = \frac{P}{3}$

ANTIHERMITIAN

$$A = \begin{bmatrix} x_1 & x_2 \\ -\bar{x}_2 & \bar{x}_1 \end{bmatrix} \qquad A^H = -A$$

$$A^H = \begin{bmatrix} x_1 & -\bar{x}_2 \\ x_2 & \bar{x}_1 \end{bmatrix}^* = \begin{bmatrix} \bar{x}_1 & -x_2 \\ \bar{x}_2 & x_1 \end{bmatrix} = -\begin{bmatrix} -\bar{x}_1 & x_2 \\ -\bar{x}_2 & -x_1 \end{bmatrix}$$

$$\gamma = \frac{P_s}{N} = \frac{\epsilon \eta^4 \cdot G_R^2 \Delta^2 \Lambda^2}{G_R^2 \eta^4 \Lambda^2 \Delta N_0 + \Lambda N_0 \eta^2} = \Big/ \; G_R^2 = \frac{1}{\eta^2 \cdot \Delta^2} \Big/$$

$$= \frac{\epsilon \cdot \eta^2 \frac{1}{\eta^2 \cdot \Delta^2} \cdot \Delta^2 \cdot \Lambda^2}{\frac{1}{\eta^2 \cdot \Delta^2} \cdot \eta^4 \cdot \Lambda^2 \cdot \Delta N_0 + \Lambda N_0 \cdot \eta^2} = \frac{\epsilon \cdot \eta^2 \cdot \Lambda^2 \cdot \Delta}{\eta^2 \Lambda^2 \cdot N_0 + \Delta \Lambda \cdot N_0 \eta^2}$$

$$= \frac{\epsilon}{N_0} \frac{\Lambda^2 \cdot \Delta}{\Lambda^2 + \Delta \Lambda}$$

$$\gamma = \frac{\epsilon \cdot \eta^2 \frac{G^2}{\eta^2} \cdot \Delta^2 \Lambda^2}{\frac{G^2}{\eta^2} \cdot \eta^2 \cdot \Lambda^2 \cdot \Delta \cdot N_0 + \Lambda N_0 \cdot \eta^2} = \frac{\epsilon}{N_0} \frac{G^2 \cdot \Delta^2 \cdot \Lambda^2}{G^2 \cdot \Lambda^2 \Delta + \Lambda \eta^2}$$

$$\gamma = \frac{\epsilon}{N_0} \frac{G^2 \cdot \Delta^2 \cdot \Lambda}{G^2 \cdot \Delta \cdot \Lambda + \Lambda} \qquad G = \frac{1}{\eta \Delta}$$

$$\gamma = \bar{\gamma}_e \frac{\frac{1}{\eta^2 \Delta^2} \Delta^2 \cdot \Lambda}{\frac{1}{(\eta \Delta)^2} \cdot \Delta \cdot \Lambda + 1} = \bar{\gamma}_e \frac{\frac{\Delta^2 \cdot \Lambda}{\eta^2 \Delta^2}}{\frac{\Delta \cdot \Lambda + \eta^2 \Delta^2}{\eta^2 \Delta^2}}$$

$$\gamma = \bar{\gamma}_e \frac{\Delta^2 \cdot \Lambda}{\Delta \cdot \Lambda + \eta^2 \Delta^2} \qquad \frac{\bar{\gamma}_e}{\gamma} = \frac{\Delta \Lambda + \eta^2 \Delta^2}{\Delta^2 \cdot \Lambda}$$

$$\frac{\bar{\gamma}_e}{\gamma} = \frac{1}{\Delta} + \frac{\eta^2}{\Lambda} \qquad \boxed{\frac{1}{\gamma} = \frac{1}{\gamma_e \Delta} + \frac{\eta^2}{\gamma_e \Lambda}}$$

$$\gamma = \gamma_e \cdot \left[ \frac{1}{\Delta} + \frac{\eta^2}{\Lambda} \right] \qquad \boxed{N \times N \times N}$$

$$\gamma = \gamma_e \cdot \left[ \frac{1}{\Delta} + \frac{\eta^2}{N_0 \Lambda} \right] \qquad \boxed{N \times 1 \times N}$$

- PETOK VO 18:00 [IVAN TENIS] BODI GI ORGANIZIRA GRUPITE.

N10.37

$$\mathcal{L}^{-1}\left[2K_\nu(2\sqrt{\alpha_1 s})\cdot K_\nu(2\sqrt{\alpha_2 s})\right] = \frac{1}{w} e^{-\frac{\alpha_1+\alpha_2}{w}} K_\nu\left(\frac{2\sqrt{\alpha_1\alpha_2}}{w}\right)$$

NUAN

$$\mathcal{L}^{-1}\left(\frac{M_W(-s)}{s^t}\right) = \frac{2\eta^t}{\Gamma^2(t)\bar{\gamma}^t}\mathcal{L}^{-1}\left[2K_t\left(2\sqrt{\frac{s}{\bar{\gamma}}}\right)K_t\left(2\sqrt{\frac{\eta^2 s}{\bar{\gamma}}}\right)\right] =$$

$\alpha_1 = \frac{1}{\bar{\gamma}}$ $\alpha_2 = \frac{\eta^2}{\bar{\gamma}}$

$$= \frac{2\eta^t}{\Gamma^2(t)\bar{\gamma}^t}\frac{1}{w} e^{-\frac{\frac{1}{\bar{\gamma}}+\frac{\eta^2}{\bar{\gamma}}}{w}} K_\nu\left(\frac{2}{w}\sqrt{\frac{\eta^2}{\bar{\gamma}^2}}\right)$$

$$\mathcal{L}^{-1}\left[\frac{M_W(-s)}{s^t}\right] = \frac{2\eta^t}{\Gamma^2(t)\bar{\gamma}^t}\frac{1}{w} e^{-\frac{\eta+1}{w\bar{\gamma}}} K_\nu\left(\frac{2\eta}{w\bar{\gamma}}\right)$$

$\eta = 1$

$$\mathcal{L}^{-1}\left[\frac{M_W(-s)}{s^t}\right] = \frac{2}{\Gamma^2(t)\bar{\gamma}^t}\frac{1}{w} e^{-\frac{2}{w\bar{\gamma}}} K_\nu\left(\frac{2}{w\bar{\gamma}}\right)$$

N×1×N

$$f_U(u) = \frac{u^{-t_1-1}}{\bar{\gamma}^{t_1}\Gamma(t_1)} e^{-\frac{1}{u\bar{\gamma}}} \qquad f_V(v) = \frac{\eta^{2t_2} v^{-t_2-1}}{\bar{\gamma}^{t_2}\cdot N^{t_2}\Gamma(t_2)} e^{-\frac{\eta^2}{v N\bar{\gamma}}}$$

$t_1 = t_2 = N$

$$f_U(u) = \frac{u^{-N-1}}{\bar{\gamma}^N\Gamma(N)} e^{-\frac{1}{u\bar{\gamma}}} \qquad f_V(v) = \frac{\eta^{2N} v^{-N-1}}{\bar{\gamma}^N N^N \Gamma(N)} e^{-\frac{\eta^2}{v N\bar{\gamma}}}$$

$$M_U(-s) = \frac{2}{\Gamma(N)}\left(\frac{s}{\bar{\gamma}}\right)^{\frac{N}{2}} K_N\left(2\sqrt{\frac{s}{\bar{\gamma}}}\right)$$

$$M_V(-s) = \frac{2}{\Gamma(N)}\left(\frac{s}{\bar{\gamma}}\right)^{\frac{N}{2}}\cdot\left(\frac{\eta^2}{N}\right)^{\frac{N}{2}}\cdot K_N\left(2\sqrt{\frac{\eta^2 s}{N\bar{\gamma}}}\right)$$

$$M_V(-s) = \frac{2\eta^N}{N^{N/2}\Gamma(N)}\cdot\left(\frac{s}{\bar{\gamma}}\right)^{\frac{N}{2}} K_N\left(2\sqrt{\frac{\eta^2 s}{N\bar{\gamma}}}\right)$$

$$M_W(-1) = \frac{4\eta^N}{N^{N/2}\Gamma^2(N)} \cdot \left(\frac{1}{\delta}\right)^N \cdot K_N\left(2\sqrt{\frac{1}{\delta}}\right) \cdot K_N\left(2\sqrt{\frac{\eta^2 1}{N\delta}}\right)$$

- SO OGLED NA TOA ŠTO SE MNOŽI SO sqrt(c) A NE SE DELI SO sqrt(N) TREBA DA SE VMETNE „c" NAMESTO „N" VO IZRAZITE T.E.:

- IZRAZOT PREKU „c" E:

$$f_V(v) = \frac{c^N \cdot \eta^{2N} v^{-N-1}}{\delta^N \cdot \Gamma(N)} e^{-\frac{c\eta^2}{v \cdot \delta}}$$

$$M_V(-1) = \frac{2}{\Gamma(N)} \left(c \cdot \eta^2\right)^{N/2} \left(\frac{1}{\delta}\right)^{N/2} \cdot K_N\left(2\sqrt{\frac{\eta^2 c 1}{\delta}}\right)$$

$$M_V(-1) = \frac{2c^{N/2} \cdot \eta^N}{\Gamma(N)} \cdot \left(\frac{1}{\delta}\right)^{N/2} \cdot K_N\left(2\sqrt{\frac{\eta^2 c 1}{\delta}}\right)$$

$$M_W(-1) = \frac{4c^{N/2} \cdot \eta^N}{\Gamma^2(N)} \left(\frac{1}{\delta}\right)^N K_N\left(2\sqrt{\frac{1}{\delta}}\right) K_N\left(2\sqrt{\frac{\eta^2 c 1}{\delta}}\right)$$

$$L(\omega) = \frac{2c^{t/2}\eta^t e^{-\frac{c\eta^2+1}{\delta\omega}}}{\omega\,\Gamma^2(t)\,\delta^t} K_t\left(\frac{2\eta\sqrt{c}}{\delta\omega}\right)$$

UNIVERZALEN IZRAZ

→ OVA E ES/No PER SYMBOL

- $\eta = 1$, $c = \frac{1}{N}$, $t = N$

$$L(\omega) = \frac{2e^{-\frac{N+1}{N\delta\omega}}}{N^{t/2}\omega\,\Gamma^2(t)\,\delta^t} K_t\left(\frac{2}{\delta\omega\sqrt{N}}\right)$$

$$L(\omega) = \frac{2e^{-\frac{N+1}{N\delta\omega}}}{N^{N/2}\omega\,\Gamma^2(N)\,\delta^N} K_N\left(\frac{2}{\delta\omega\sqrt{N}}\right) \Big\} N \times 1 \times N$$

- $\eta = 1$, $c = 1$, $t = N^2$

$$L(\omega) = \frac{2e^{-\frac{2}{\delta\omega}}}{\omega\,\Gamma^2(N^2)\,\delta^{N^2}} K_{N^2}\left(\frac{2}{\delta\omega}\right) \qquad N \times N \times N$$

· 348 coog

$$\frac{6 \cdot \frac{P}{3}}{8} = \frac{2P}{8} = \frac{P}{4}$$

$$\epsilon = P \cdot \frac{L}{K\,N} = \frac{8P}{12} = \frac{2P}{3}$$

(L = 8, K = 4, N = 3)

$$\epsilon = \frac{P}{3}$$

$$\begin{bmatrix} x_1 & x_2 & x_3 \\ -x_2 & x_1 & -x_4 \\ -x_3 & x_4 & x_1 \\ -x_4 & -x_3 & x_2 \\ x_1^* & x_2^* & x_3^* \\ -x_2^* & x_1^* & -x_4^* \\ -x_3^* & x_4^* & x_1^* \\ -x_4^* & -x_3^* & x_2^* \end{bmatrix}$$

348

$$8 \cdot \epsilon = 8 \cdot \frac{P}{3}$$

$$\frac{6 \cdot \frac{P}{3}}{6} = \frac{P}{3}$$

$$M(-1) = \int_0^\infty p(x)\, e^{-x \cdot 1}\, dx$$

434 $\quad c = \frac{L}{K \cdot N} = \frac{4}{3 \cdot 4} = \frac{1}{3}$

334 $\quad c = \frac{L}{K \cdot N} = \frac{4}{3 \cdot 3} = \frac{4}{9}$

$$8 \cdot \frac{P}{3} \Big/ 8 = \frac{P}{3}$$

$N \times 1 \times N$

$$\tilde{x}_k = \sqrt{E_s}\,\eta\,\Delta\, x_k + \xi_k \qquad \hat{x}_k = G_R \cdot \Lambda\, \tilde{x}_k + \mu_k$$

[illegible] $\quad \hat{x}_k = G_R \cdot \Lambda \left(\sqrt{E_s}\,\eta\,\Delta\, x_k + \xi_k\right) + \mu_k$

$$\hat{x}_k = \underbrace{G_R \Lambda \sqrt{E_s}\,\eta\,\Delta\, x_k}_{S} + \underbrace{G_R \cdot \Lambda \cdot \xi_k + \mu_k}_{N}$$

$$P_S = G_R^2 \cdot \Lambda^2 \cdot E_s \cdot \eta^2 \cdot \Delta^2 \qquad P_N = G_R^2 \cdot \Lambda^2 \cdot \eta^2\, \Delta\, N_0 + \Lambda^2 \cdot N_0$$

$$\frac{S}{N} = \frac{G_R^2 \cdot \Lambda^2 \eta^2 \cdot \Delta^2}{G_R^2 \cdot \Lambda^2 \cdot \eta^2 \cdot \Delta + \Lambda^2} \cdot \frac{E_s}{N_0} = \frac{E_s}{N_0}\, \frac{G_R^2 \cdot \eta^2 \Lambda \Delta^2}{G_R^2 \cdot \eta^2 \Lambda \Delta + 1}$$

$$G_R = \frac{1}{\sqrt{c}\cdot\eta\cdot\Delta} \qquad \frac{S}{N} = \bar\gamma \, \frac{\frac{1}{c\eta^2\Delta^2}\cdot\eta^2\lambda\Delta^2}{\frac{1}{c\eta^2\Delta^2}\cdot\eta^2\cdot\lambda\cdot\Delta+1} = \frac{\bar\gamma\cdot\frac{\lambda}{c}}{\frac{\lambda}{c\Delta}+1}$$

$$\gamma = \bar\gamma\,\frac{\frac{\lambda}{c}}{\frac{\lambda + c\Delta}{c\Delta}} = \frac{\lambda\Delta}{\lambda + c\Delta}$$

$$\boxed{\frac{\bar\gamma}{\gamma} = \frac{\lambda + c\Delta}{\lambda\Delta} = \frac{1}{\Delta} + \frac{c}{\lambda}} \qquad \boxed{\frac{1}{\gamma} = \frac{1}{\bar\gamma\Delta} + \frac{c}{\bar\gamma\lambda}}$$

N + 1 + N

• OPCIJA:

$$\boxed{G_R = \frac{1}{\sqrt{c}\cdot\Delta}}$$

$$\gamma = \bar\gamma\,\frac{\frac{1}{c\Delta^2}\cdot\eta^2\cdot\lambda\Delta^2}{\frac{1}{c\Delta}\cdot\eta^2\lambda\Delta+1} = \frac{\frac{\lambda\eta^2}{c}}{\frac{\lambda\eta^2}{c\Delta}+1}$$

$$\gamma = \bar\gamma\,\frac{\frac{\lambda\eta^2}{c}}{\frac{\lambda\eta^2 + c\Delta}{c\Delta}} = \frac{\Delta\lambda\eta^2}{\lambda\eta^2 + c\Delta}$$

$$\frac{\bar\gamma}{\gamma} = \frac{\lambda\eta^2 + c\Delta}{\Delta\lambda\eta^2} = \frac{1}{\Delta} + \frac{c}{\lambda\eta^2}$$

N × N × N

$$\tilde{x}_k = \sqrt{E_s}\,\eta\Delta x_k + \xi_k \qquad \hat{x}_k = G_R\,\eta\lambda\,\tilde{x}_k + \mu_k$$

$$\hat{x}_k = G_R\eta\lambda\left(\sqrt{E_s}\,\eta\Delta x_k + \xi_k\right) + \mu_k = \underbrace{\sqrt{E_s}\,\eta^2\lambda\Delta x_k}_{S} + \underbrace{G_R\eta\lambda\xi_k + \mu_k}_{N}$$

$N_0$

$$P_S = G_R^2 E_s\,\eta^4\lambda^2\Delta^2 \qquad P_N = G_R^2\,\eta^2\lambda^2\eta^2\cdot\Delta + \eta^2\cdot\lambda N_0$$

$$\gamma = \frac{G_R^2 E_s\,\eta^4\lambda^2\Delta^2}{G_R^2\cdot\eta^2\cdot\lambda^2\cdot\eta^2\cdot\Delta N_0 + \eta^2\cdot\lambda N_0} = \frac{E_s}{N_0}\,\frac{G_R^2\cdot\eta^2\cdot\lambda\cdot\Delta^2}{G_R^2\cdot\Delta\cdot\lambda\cdot\eta^2 + 1}$$

$$\gamma = \bar\gamma\cdot\frac{G_R^2\cdot\eta^2\cdot\lambda\cdot\Delta^2}{G_R^2\cdot\Delta\cdot\lambda\cdot\eta^2 + 1} \qquad G_R = \frac{1}{\sqrt{c}\cdot\eta\cdot\Delta}$$

$$\gamma = \bar\gamma\cdot\frac{\frac{1}{c\eta^2\Delta^2}\cdot\eta^2\cdot\lambda\cdot\Delta^2}{\frac{1}{c\eta^2\Delta^2}\cdot\Delta\cdot\lambda\cdot\eta^2 + 1} = \bar\gamma\,\frac{\frac{\lambda}{c}}{\frac{\lambda}{\Delta c}+1} = \bar\gamma\,\frac{\frac{\lambda}{c}}{\frac{\lambda + \Delta c}{\Delta c}}$$

$$\gamma = \bar{\gamma}\,\frac{\Delta\cdot 1}{1+\Delta c} \qquad \frac{\bar{\gamma}}{\gamma} = \frac{1+\Delta c}{\Delta\cdot 1} = \frac{1}{\Delta} + \frac{c}{1}$$

$$\boxed{\frac{1}{\gamma} = \frac{1}{\bar{\gamma}\cdot\Delta} + \frac{c}{\bar{\gamma}\,1}} \qquad \boxed{N+N+N}$$

$$\boxed{k=1}$$

$$\boxed{\frac{1}{\gamma} = \frac{1}{\bar{\gamma}\cdot\Delta} + \frac{1}{\bar{\gamma}\cdot 1}}$$

LENA ŽAČEVA / MIZKA / GORDANA / GYM ADAMTEL

070419100

PIVARA TIKVEŠ M6

TELEVIZOR NA RATE !!!

---

PIRAZ:

$$\gamma = \bar{\gamma}\cdot\frac{G^2 1\Delta^2}{G^2\cdot 1\cdot\Delta + 1} = \left| G = \frac{1}{\sqrt{c\cdot\Delta}} \right| = \frac{\frac{1}{c\cdot\Delta^2}\cdot 1\cdot\Delta^2}{\frac{1}{c\Delta}\cdot 1\cdot\Delta + 1}$$

$$= \frac{\frac{1}{c}}{\frac{1}{c\Delta}+1} = \frac{\frac{1}{c}}{\frac{1+c\Delta}{c\Delta}} = \frac{\Delta\cdot 1}{1+c\Delta}$$

$$\boxed{\frac{\bar{\gamma}}{\gamma} = \frac{1+c\Delta}{\Delta\cdot 1} = \frac{1}{\Delta} + \frac{c}{1}} \qquad N+1+N$$

3109186

Za $N+N+N \qquad G = \frac{1}{\Delta}$

$$\gamma = \bar{\gamma}\,\frac{\frac{1}{\Delta^2}\cdot 1\cdot\Delta^2}{\frac{1}{\Delta^2}\cdot 1\cdot\Delta + 1} = \bar{\gamma}\,\frac{1}{\frac{1+\Delta}{\Delta}} = \bar{\gamma}\,\frac{1\Delta}{1+\Delta}$$

$$\boxed{\frac{\bar{\gamma}}{\gamma} = \frac{1+\Delta}{1\cdot\Delta} = \frac{1}{\Delta} + \frac{1}{1}} \qquad N+N+N$$

$$K_n(z) \doteq \frac{1}{2}\cdot\left(\frac{2}{z}\right)^n\cdot\sum_{k=0}^{n}(-1)^k\,\frac{(n-k-1)!}{k!}\left(\frac{2}{z}\right)^{2K}$$

IMA GREŠKA

$$K_n\left(\frac{2\sqrt{b}}{\bar{\gamma}w}\right) = \frac{1}{2}\left(\frac{2\bar{\gamma}w}{2\sqrt{b}}\right)^n\sum_{k=0}^{n}(-1)^k\,\frac{(n-k-1)!}{k!}\left(\frac{\bar{\gamma}w}{\sqrt{b}}\right)^{2K}$$

---

$$K_n(z) \doteq \frac{1}{2}\sum_{k=0}^{n-1}(-1)^k\,\frac{(n-k-1)!}{k!\left(\frac{z}{2}\right)^n}\left(\frac{z}{2}\right)^{2K}$$

OK!

$$K_n(z) = \frac{1}{2}\left(\frac{2}{z}\right)^n \sum_{k=0}^{n-1} (-1)^k \frac{(n-k-1)!}{k!} \cdot \left(\frac{z}{2}\right)^{2k}$$

$$K_n\left(\frac{2\sqrt{b}}{\gamma w}\right) = \frac{1}{2}\left(\frac{\gamma w}{\sqrt{b}}\right)^n \sum_{k=0}^{n-1} (-1)^k \frac{(n-k-1)!}{k!} \left(\frac{\sqrt{b}}{\gamma w}\right)^{2k}$$

$$L(w) = \frac{2 b^{n/2} e^{-\frac{b+1}{\gamma w}}}{w\,\Gamma^2(n)\,\gamma^n} K_n\left(\frac{2\sqrt{b}}{\gamma w}\right)$$

$$L(w) = \frac{b^{n/2} e^{-\frac{b+1}{\gamma w}}}{(w)\,\Gamma^2(n)\,\gamma^n} \left(\frac{\gamma w}{\sqrt{b}}\right)^n \sum_{k=0}^{n-1} (-1)^k \frac{(n-k-1)!}{k!} \left(\frac{\gamma w}{\sqrt{b}}\right)^{-2k}$$

$$F_\Gamma(\gamma) = 1 - \frac{d^{n-1}}{dw^{n-1}}\left[L(w)\right]\Bigg|_{w=1/\gamma} = \frac{b^{n/2}}{\Gamma^2(n)\gamma^n} \sum_{k=0}^{n-1} (-1)^k \frac{(n-k-1)!}{k!} \frac{d^{n-1}}{dw^{n-1}}\left[\frac{e^{-\frac{b+1}{\gamma w}}}{w}\left(\frac{\gamma w}{\sqrt{b}}\right)^{n-2k}\right]$$

$$F_\Gamma(\gamma) = 1 - \frac{b^{n/2}}{\Gamma^2(n)\gamma^n} \sum_{k=0}^{n-1} (-1)^k \frac{(n-k-1)!}{k!} \frac{d^{n-1}}{dw^{n-1}}\left[\frac{e^{-\frac{b+1}{\gamma w}}}{w}\left(\frac{\gamma w}{\sqrt{b}}\right)^{n-2k}\right]\Bigg|_{w=\frac{1}{\gamma}}$$

IZRAZ SO HYPERGEOM

$$P_{ap} = \frac{1}{2} + \frac{\sqrt{\bar{\gamma}\cdot d}}{2\sqrt{\pi}\,\Gamma^2(n)} \sum_{k=0}^{n-1} \frac{(-1)^{k+n} 2^{2k} \Gamma(n-k)\Gamma(2k)\Gamma\left(2k+\frac{1}{2}\right) c^k \cdot H}{\Gamma(k+1)\Gamma(2k+1-n)\left(2c+2+\bar{\gamma} d\right)^{2k+\frac{1}{2}}}$$

$$H = \mathrm{hypergeom}\left(\left[1-n, 2k+\frac{1}{2}\right], \left[2k+1-n\right], \frac{2c+2}{2c+2+\bar{\gamma} d}\right)$$

$$1-n = 2\alpha \qquad \alpha = \frac{1-n}{2}$$

$$2k+\frac{1}{2} = 2\beta \qquad \beta = \frac{4k+1}{4}$$

$$2k+1-n = \alpha+\beta+\frac{1}{2} \qquad \alpha+\beta+\frac{1}{2} = \frac{2-2n+4k+1}{4} + \frac{1}{2}$$

$$= \frac{3}{4} + k - \frac{n}{2} + \frac{1}{2} = \frac{5}{4} + k - \frac{n}{2}$$

$$F(b,a;c;z)=\left(\sum_{n=0}^{\infty}\frac{\Gamma(a+n)\Gamma(b+n)}{\Gamma(c+n)}\frac{z^n}{n!}\right)\cdot\frac{\Gamma(c)}{\Gamma(a)\Gamma(b)}$$

$$P=\frac{1}{2}+\frac{\sqrt{\delta\bar{\delta}}}{2\sqrt{\pi}\,\Gamma^2(N)}\sum_{k=0}^{m-1}\sum_{n=0}^{m-1}\frac{(-1)^{m+k+n}2^{n+2k}\Gamma(m-k)\Gamma(n+2k+\frac{1}{2})\Gamma(2k)\Gamma(n)(c+1)^{n}b^{k}}{\Gamma(2k+n-m+1)\Gamma(m-n)\Gamma(k+1)\Gamma(n+1)(c\bar{\delta}+2c+2)^{n+2k+\frac{1}{2}}}$$

$$H(X;Y)=\sum_{i=1}^{m}\sum_{j=1}^{r}P(x_i,y_i)\,ld\frac{1}{P(x_i,y_i)} \quad (*)$$

$$=\left|P(x_i/y_i)=\frac{P(x_i,y_i)}{P(y_i)}\right|=\sum\sum P(x_i,y_i)\,ld\frac{P(y_i)\,P(x_i)}{P(x_i,y_i)\,P(x_i)\,P(y_j)}$$

$$=\left|I(X;Y)=\sum\sum P(x_i,y_i)\,ld\frac{P(x_i,y_i)}{P(x_i)P(y_i)}\right|=$$

$$\overset{(*)}{=}\sum_{i=1}^{m}\sum_{j=1}^{r}P(x_i,y_i)\left[ld\left[\frac{1}{P(x_i)P(y_j)}\right]-ld\frac{P(x_i,y_i)}{P(y_i)P(x_i)}\right]$$

---

- Apriorna ентропија на изворната листа:

$$H(X)=\sum_{i=1}^{m}P(x_i)\cdot ld\frac{1}{P(x_i)}$$

- Апостериорна ентропија на „X“

$$H(X/y_j)=\sum_{i=1}^{m}P(x_i/y_j)\,ld\frac{1}{P(x_i/y_j)} \qquad j=1,2,\dots r$$

$$H(X/Y)=\overline{H(X/y_j)}=\sum_{j=1}^{r}H(X/y_i)P(y_i)=\sum_{i=1}^{m}\sum_{j=1}^{r}P(x_i/y_j)P(y_j)\,ld\frac{1}{P(x_i/y_j)}$$

$$H(X/Y)=\sum_{i=1}^{m}\sum_{j=1}^{r}P(x_i,y_j)\,ld\frac{1}{P(x_i/y_j)}$$

$$I(X;Y)=H(X)-H(X/Y)=\sum_{i=1}^{m}P(x_i)\,ld\frac{1}{P(x_i)}\cdot\sum_{j=1}^{r}P(y_i/x_i)-$$

$$-\sum_{i=1}^{m}\sum_{j=1}^{r}P(x_i,y_j)\,ld\frac{1}{P(x_i/y_j)}=\sum_{i=1}^{m}\sum_{j=1}^{r}P(x_i,y_i)\,ld\frac{P(x_i/y_j)}{P(x_i)}$$

$$=\sum_{i=1}^{m}\sum_{j=1}^{r}P(x_i,y_i)\,ld\frac{P(x_i,y_j)}{P(x_i)P(y_j)}$$

$$H(X;Y)=\sum_{i=1}^{m}\sum_{j=1}^{r}P(x_i)P(y_i/x_i)\,ld\frac{1}{P(x_i)}+\sum_{i=1}^{m}\sum_{j=1}^{r}P(y_j)P(x_i/y_j)\,ld\frac{1}{P(y_j)}-I(X;Y)$$

$$H(X;Y) = \sum_{i=1}^{n} P(x_i)\,\mathrm{ld}\,\frac{1}{P(x_i)} \underbrace{\sum_{j=1}^{r} P(y_j/x_i)}_{1} + \sum_{j=1}^{r} P(y_j)\,\mathrm{ld}\,\frac{1}{P(y_j)} \underbrace{\sum_{i=1}^{n} P(x_i/y_j)}_{1} - I(x;y)$$

$$\underbrace{H(X;Y)}_{\text{ZDRUŽENA ENTROPIJA}} = H(X) + H(Y) - \underbrace{I(X;Y)}_{\text{TRANSINFORMACIJA}}$$

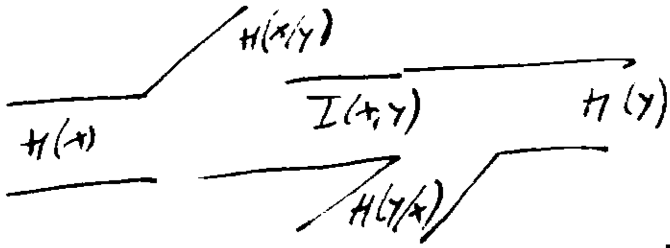
$$\mathrm{pochammer}(a,b) = \frac{\Gamma(a+b)}{\Gamma(a)}$$

H(Y) → PROSEČNO KOLIČESTVO PRIMENA INFOR

H(X) → PROSEČNO KOLIČESTVO PREDADENA INF.

$$I(x,y) = H(x) - H(x/y)$$

$$I(x,y) = H(y) - H(y/x)$$




$$H(x,y) = H(x) + H(y/x)$$
$$H(x,y) = H(y) + H(x/y)$$

- KAPACITET NA KANALOT:

$$C(x,y) = \upsilon(x,y)\cdot \max_{P(x_i)}\left[I(x,y)\right] \quad \left[\frac{Sh}{s}\right]$$

$$P = \frac{1}{2} + \frac{\sqrt{\bar{\gamma}d}}{2\sqrt{\pi}\,\Gamma^2(m)} \sum_{k=0}^{m-1}\sum_{n=0}^{m-1} \frac{(-1)^{k+n+1}\,2^{n+2k}\,\Gamma(m-k)\,\Gamma(2k)\,\Gamma(m)\,\Gamma(n+2k+\frac{1}{2})\,(c+1)^n c^k}{\Gamma(k+1)\,\Gamma(n+1)\,\Gamma(m-n)\,\Gamma(2k-m+n+1)\,(2c+2+\bar{\gamma}d)^{2k+n}}$$

$$\left(\frac{c+1}{2c+2+\bar{\gamma}d}\right)^n = \frac{1}{2} - \frac{1}{2}\,\frac{x}{2c+2+x}$$

$$\frac{\Gamma(n)\cdot\Gamma(2k+\frac{1}{2}+n)}{\Gamma(n+1)\,\Gamma(m-n)\,\Gamma(2k-m+n+1)}$$

$$\mathrm{pochamer}(a,b) = \frac{\Gamma(a+b)}{\Gamma(a)}$$

$$\Gamma(2k+\tfrac{1}{2}+n) = \Gamma(n)\cdot \mathrm{pochamer}(n, \ldots)$$

$$\Gamma(2k-m+1+n) = \Gamma(n)\cdot\mathrm{pochammer}(n,\ 2k-m+1)$$

$$\Gamma\left(n+\frac{1}{2}\right) = \frac{\sqrt{\pi}}{2^n}(2n-1)!! = \frac{\sqrt{\pi}}{2^n}\,\frac{(2n)!}{2^n\,n!}$$

$$X = \frac{\Gamma(n+a)}{\Gamma(n+b)} = ? \qquad X = \frac{\mathrm{pochamer}(n,a)}{\mathrm{pochamer}(n,b)}$$

$$F(-n,\beta;\beta;-z) = (1+z)^n \quad \text{GRADSHTEYN 9.121.1}$$

$$F(a,b;c;z) = {}_2F_1(a,b;c;z) = F(b,a;c;z) = \sum_{n=0}^{\infty} \frac{(a)_n (b)_n}{(c)_n}\,\frac{z^n}{n!}$$

$$= \left|\,(a)_n = \frac{\Gamma(a+n)}{\Gamma(a)}\right| = \frac{\Gamma(c)}{\Gamma(a)\Gamma(b)}\sum_{n=0}^{\infty} \frac{\Gamma(a+n)\,\Gamma(b+n)}{\Gamma(c+n)}\,\frac{z^n}{n!}$$

$$S_1 = \sum_{n=0}^{m-1} \frac{2^n (-1)^n \cdot \Gamma(m)\, \Gamma(n+2k+\frac{1}{2})}{\underbrace{\Gamma(n+1)}_{n!}\, \Gamma(m-n)\, \Gamma(2k-m+n+1)} \cdot \Bigg( \underbrace{\frac{c+1}{2c+2-8d}}_{z^n} \Bigg)^n$$

ONA ŠTO ZAVISI OD „n" N10.9z

$$S_1 = \sum_{n=0}^{m-1} \frac{2^n (-1)^n \cdot \Gamma(m)\, \Gamma(n+2k+\frac{1}{2})}{\Gamma(m-n)\, \Gamma(2k-m+n+1)} \quad \frac{z^n}{n!}$$

$$\Gamma(2k+\tfrac{1}{2}+n) = \Gamma(2k+\tfrac{1}{2})\,(2k+\tfrac{1}{2})_n \quad \Gamma(2k-m+1+n) = \Gamma(2k-m+1)\cdot(2k-m+1)_n$$

$$S_1 = \sum_{n=0}^{m-1} \frac{(-2)^n\, \Gamma(m) \cdot \Gamma(2k+\frac{1}{2})\, \mathrm{pochammer}(2k+\frac{1}{2}, n)}{\Gamma(m-n) \cdot \Gamma(2k-m+1)\, (2k-m+1)_n} \cdot \frac{z^n}{n!}$$

$$S_1 = \frac{\Gamma(2k+\frac{1}{2})}{\Gamma(2k-m+1)} \sum_{n=0}^{m} \frac{(-1)^n\, \Gamma(m) \cdot (2k+\frac{1}{2})_n}{\Gamma(m-n)\, (2k-m+1)_n} \cdot \frac{z^n}{n!}$$

$$\Gamma(m-n) = \Gamma(m) \cdot (m)_{-n}; \quad \Gamma(m) = \Gamma(m-n+n) = \Gamma(m-n)\cdot(m-n)_n$$

$$S_1 = \frac{\Gamma(2k+\frac{1}{2})}{\Gamma(2k-m+1)} \sum_{n=0}^{m-1} \frac{(-2)^n (m-n)_n \cdot (2k+\frac{1}{2})_n}{(2k-m+1)_n} \cdot \frac{z^n}{n!}$$

MOŽE UNUTRE DA UĐE!!!

$$\sum_{n=0}^{\infty} \frac{(m-n)_n \cdot (2k+\frac{1}{2})_n}{(2k-m+1)_n} \frac{z^n}{n!} = F\left(m-n,\, 2k+\tfrac{1}{2};\, 2k-m+1;\, z\right)$$

- $S(x) = \sum_{n=0} x^n = \frac{1}{1-x}$ → OSNOVEN IZRAZ

$(x^n)' = (n)x^{n-1}$

$\int x^2 dx = \frac{x^3}{3}$

$\int x\, dx = \frac{x^2}{2}$

$$S(x) = \sum_{n=1}^{\infty} n x^{n-1} \Big/ \int dx \qquad \int S(x)\,dx = \sum_{n=1}^{\infty} n \cdot \frac{x^n}{n}$$

$$\int S(x)\,dx = \sum_{n=1} x^n = \frac{1}{1-x} \qquad S(x) = \left(\frac{1}{1-x}\right)' = \left[(1-x)^{-1}\right]'$$

$$= (-1)\frac{1}{(1-x)^2}\cdot(-1)$$

$$S(x) = \frac{1}{(1-x)^2} \qquad (x^2)' = n \cdot x^{n-1}$$

- $S(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} x^{2n+1} \qquad S'(x) = \sum_{n=0}^{\infty} \frac{(-1)^n\, x^{2n}}{1}$

$S'(x) = \sum_{n=0}^{\infty} (-1)^n \cdot y^n$ ; $\boxed{y=x^2}$ ; $S'(x) = \sum_{n=0}^{\infty} (-y)^n = \frac{1}{1+y}$

$S'(x) = \frac{1}{1+x^2}$ $\qquad S(x) = \int \frac{1}{1+x^2}\,dx = \operatorname{arctg}(x) + C$

---

$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k \cdot b^{n-k}$ $\qquad \binom{2}{1} = \frac{2!}{(n-k)!\,k!} = \frac{2}{1\cdot 1} = 2$

$\boxed{n=2}$ $\quad (a+b)^2 = a^0 \cdot b^2 + 2\cdot a\cdot b + a^2 b^0$ ($a^0 \to 1$, $b^0 \to 1$)

$(a-b)^n = \sum_{n=0}^{n} \binom{n}{k} (-1)^k \cdot a^k \cdot b^{n-k} = b^2 - 2ab + a^2$

---

$\frac{1}{(1+x)^3} = \frac{1}{2} \sum_{n=2}^{\infty} (-1)^n \cdot n(n-1)\,x^{n-2}$ ??

$\boxed{\begin{matrix}200\\376\end{matrix}}$

---

$S(x) = \sum_{n=1}^{\infty} n x^n \qquad \frac{S(x)}{x} = \sum_{n=1}^{\infty} n x^{n-1} \quad // \int dx$

$\int \frac{S(x)}{x}\,dx = \sum_{n=1}^{\infty} \frac{n x^n}{n} = \sum_{n=1}^{\infty} x^n = \underbrace{\sum_{n=0}^{\infty} x^n}_{1/1-x} - 1$

$= \frac{1}{1-x} - 1 = \frac{1-1+x}{1-x} = \frac{x}{1-x}$

$\frac{S(x)}{x} = \left(\frac{x}{1-x}\right)' = \frac{1-x+x}{(1-x)^2} = \frac{1}{(1-x)^2}$

$\boxed{S(x) = \frac{x}{(1-x)^2}}$

---

$S(x) = \sum_{n=1}^{\infty} n^2 x^{n-1} \Rightarrow S_1 = \int S(x)\,dx = \sum_{n=1}^{\infty} \frac{n^2 x^n}{n} = \sum_{n=1}^{\infty} n x^n$

$\int \frac{S_1(x)}{x}\,dx = \sum_{n=1}^{\infty} n \frac{x^{n}}{n}$ [redacted] $= \frac{1}{1-x} \qquad \frac{S_1(x)}{x} = \frac{1}{(1-x)^2} \qquad S_1(x) = \frac{x}{(1-x)^2}$

$S(x) = \left(\frac{x}{(1-x)^2}\right)' = \frac{(1-x)^2 + 2x(1-x)}{(1-x)^4} = \frac{1-2x+x^2+2x-2x^2}{(1-x)^4}$

$S(x) = \frac{1-x^2}{(1-x)^4} = \frac{(1+x)(1-x)}{(1-x)^4} = \frac{1+x}{(1-x)^3}$

---

$\operatorname{arctg}(1) = \frac{\pi}{4} = \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} \qquad \pi = 4 \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1}$

$$P_{ead} = \frac{1}{2} + \frac{\sqrt{\delta}\,\vartheta}{2\sqrt{\pi}\,\Gamma^2(n)} \sum_{k=0}^{n-1} \frac{(-1)^{k+n} 2^{2k} \Gamma(n-k)\Gamma(2k)\Gamma(2k+\frac{1}{2}) c^k F(-n+1, 2k+\frac{1}{2}; 2k+1-n, z)}{\Gamma(k+1)\Gamma(2k+1-n)(2c+2+\delta\vartheta)^{2k+\frac{1}{2}}}$$

$$X = \frac{\Gamma(n)}{\Gamma(n-q)} = \frac{1\cdot 2\cdot 3 \cdot \ldots \cdot n-1}{1\cdot 2\cdot 3 \cdot \ldots (n-q-1)} = (n-q)\cdot(n-q+1)\cdots(n-1)$$

$$X = (n-q)\cdot(n-q+1)\cdot(n-q+2)\cdot \ldots (n-q+q-1)$$
$$= (n-q)(n-q+1)\cdot(n-q+2)\cdots(n-1) = (n-q)_q$$

• MARKO TVRDI DRUGA

$$S_1 = \sum_{q=0}^{\infty} \frac{(-1)^q \Gamma(n) \cdot \Gamma(q+2k+\frac{1}{2})}{\Gamma(n-q)\cdot\Gamma(2k-n+q+1)} \frac{z^q}{q!} = \frac{\Gamma(2k+\frac{1}{2})}{\Gamma(2k-n+1)} F(-n+1, 2k+\frac{1}{2}; 2k-n+1, z)$$

$$(-1)\ (n-q+q-2)$$

$$(-1)^q \cdot X = (-1)(n-q)(-1)(n-q+1)(-1)(n-q+2)\cdots(-1)(n-1)$$

$$= (q-n)(q-n-1)(q-n-2)\cdots(1-n) \qquad n-q+q-1$$

$$(2-n)$$

$$= (1-n)(1-n+1)\cdot(1-n+2)\cdots(1-n+q-1) =$$

$$(q-n)$$

$$= (1-n)_q$$

MMV | DOKAZ KAKO SE DOLAZI DO PARAMETARA HYPERGEOM. FUN.

$$S_1 = \frac{\Gamma(2k+\frac{1}{2})}{\Gamma(2k-n+1)} \sum_{q=0}^{n-1} \frac{(-1)^q (n-q)_q (2k+\frac{1}{2})_q}{(2k-n+1)_q} \cdot \frac{z^q}{q!}$$

$$S_1 = \frac{\Gamma(2k+\frac{1}{2})}{\Gamma(2k-n+1)} \sum_{q=0}^{n-1} \frac{(1-n)_q (2k+\frac{1}{2})_q}{(2k-n+1)_q} \cdot \frac{z^q}{q!} \qquad \text{MMV} \quad (€)$$

$$S_1 = \frac{\Gamma(2k+\frac{1}{2})}{\Gamma(2k-n+1)} \sum_{q=0}^{n-1} \frac{\Gamma(q+1-n)\cdot\Gamma(q+2k+\frac{1}{2})}{\Gamma(1-n)\cdot\Gamma(2k+\frac{1}{2})} \cdot \frac{\Gamma(2k-n+1)}{\Gamma(q+2k-n+1)} \frac{z^q}{q!}$$

$$S_1 = \underbrace{\sum_{n=0}^{m-1} \frac{\Gamma(n+1-m)\,\Gamma(n+2k+\frac{1}{2})}{\Gamma(1-m)\cdot\Gamma(n+2k-n+1)} \frac{z^n}{n!}}$$

Во MultiLoopMIMO6.mw (24.5) се покажува дека оваа сума за ×m е еднаква на 0 па:

$$S_1 = \sum_{n=0}^{\infty} \frac{\Gamma(n+1-m)\,\Gamma(n+2k+\frac{1}{2})}{\Gamma(1-m)\cdot\Gamma(n+2k-m+1)} \frac{z^n}{n!}$$

- Ако се враќам назад на (Е) формата се добива дека:

$$S_1 = \frac{\Gamma(2k+\frac{1}{2})}{\Gamma(2k-m+1)} \sum_{n=0}^{\infty} \frac{(1-m)_n\,(2k+\frac{1}{2})_n}{(2k-m+1)_n} \cdot \frac{z^n}{n!} = $$

$$= \frac{\Gamma(2k+\frac{1}{2})}{\Gamma(2k-m+1)}\, {}_2F_1\left(1-m, 2k+\frac{1}{2}; 2k-m+1; z\right)$$

$$E_sN_0 = \frac{E_s}{N_0} \qquad 2\cdot\sigma^2 = \frac{E_s}{E_sN_0} \qquad \boxed{\sigma = \sqrt{\frac{E_s}{2 E_sN_0}}}$$

$$\sigma^2 = \frac{N_0}{2}$$

---

$$P_b(\gamma) = \frac{1}{\mathrm{ld} M} \sum_{j=1}^{\mathrm{ld}\sqrt{M}} \frac{2}{\sqrt{M}} \sum_{i=0}^{(1-2^{-j})\sqrt{M}-1} \left\{(-1)^{\lfloor 2^{j-1}\cdot i/\sqrt{M}\rfloor}\left(2^{j-1} - \left\lfloor \frac{2^{j-1} i}{\sqrt{M}} + \frac{1}{2}\right\rfloor\right) \cdot Q\left[(2i+1)\sqrt{\frac{3\gamma}{M-1}}\right]\right\}$$

$$\boxed{P_b(\gamma) = \frac{2}{\mathrm{ld}(M)\sqrt{M}} \sum_{j=1}^{\mathrm{ld}\sqrt{M}} \sum_{i=0}^{(1-2^{-j})\sqrt{M}-1} \left\{(-1)^{\lfloor 2^{j-1} i/\sqrt{M}\rfloor}\left(2^{j-1} - \left\lfloor \frac{2^{j-1} i}{\sqrt{M}} + \frac{1}{2}\right\rfloor\right) Q\left[(2i+1)\sqrt{\frac{3\gamma}{M-1}}\right]\right\}}$$

$$Q(\sqrt{d\gamma}) = Q\left[\sqrt{\frac{3(2i+1)^2}{M-1}\cdot\gamma}\right] \qquad \boxed{d = \frac{3(2i+1)^2}{M-1}}$$

$$Y = \sqrt{E}\begin{bmatrix} x_1 & x_2 & x_3 \\ -\bar{x}_2 & \bar{x}_1 & 0 \\ \bar{x}_3 & 0 & -\bar{x}_1 \\ 0 & \bar{x}_3 & -\bar{x}_2 \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix}$$

$$\boxed{Q(z) = \frac{1}{2}\,\mathrm{erfc}\,\frac{z}{\sqrt{2}}}$$

CEDOMIR MINDEROVIC

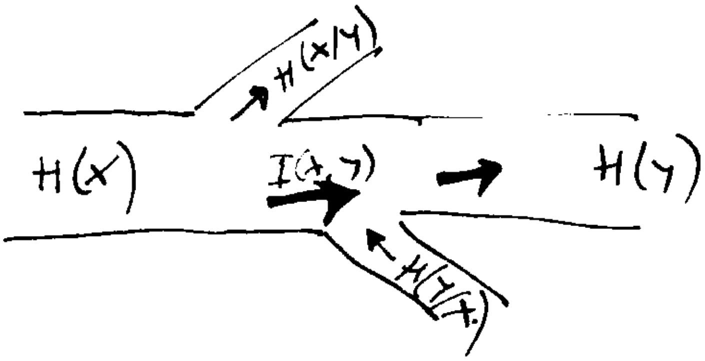
$$I(X;Y) = H(X) - H(X/Y)$$

$$I(X;Y) = \sum_{i=1}^{m} P(x_i)\, ld \frac{1}{P(x_i)} - \sum_{i=1}^{m}\sum_{j=1}^{r} P(x_i,y_j)\, ld \frac{1}{P(x_i/y_j)}$$

$$= \sum_{i=1}^{m} P(x_i)\, ld \frac{1}{P(x_i)} \sum_{j=1}^{r} P(y_j/x_i) - \;\text{〃}\; -$$

$$= \sum_{i=1}^{m}\sum_{j=1}^{r} P(x_i,y_j)\, ld \frac{P(x_i/y_j)}{P(x_i)} \qquad P(x_i,y_j) = P(y_j)\,P(x_i/y_j)$$

$$\boxed{I(X;Y) = \sum_{i=1}^{m}\sum_{j=1}^{r} P(x_i,y_j)\, ld \frac{P(x_i,y_j)}{P(x_i)\,P(y_j)}}$$ TRANSINFORM




$$I(X;Y) = H(X) - H(X/Y)$$

$$I(X;Y) = H(Y) - H(Y/X)$$

$$\boxed{C = v(x,y) \max_{P(x_i)} [I(X,Y)] \left[\frac{sh}{s}\right]}$$

KAKO SE PRESMETUVA MAX $[I(X,Y)]$

$$H(Y) = \sum_{j=1}^{r} P(y_j)\cdot ld \frac{1}{P(y_j)} = r\cdot\frac{1}{r}\cdot ld \frac{1}{\frac{1}{r}} = ld\, r$$

$\Downarrow$ (*)

$$H(X/y_j) = \sum_{i=1}^{m} P(x_i/y_j)\, ld \frac{1}{P(x_i/y_j)}$$

$$H(X/Y) = \overline{H(X/y_j)} = \sum_{j=1}^{r}\left(\sum_{i=1}^{m} P(x_i/y_j)\, ld \frac{1}{P(x_i/y_j)}\right) P(y_j)$$

$$H(X/Y) = \sum_{i=1}^{m}\sum_{j=1}^{r} P(x_i,y_j)\, ld \frac{1}{P(x_i/y_j)}$$

# ⊙ Капацитет на симетрични канали

$$C = v(x,y) \max_{P(x_i)} \left[ I(x,y) \right]$$

$$I(x,y) = H(y) - H(y/x)$$

- за симетрични канали $P(x_i) = \frac{1}{r} \Rightarrow P(y_i) = \frac{1}{r}$

$(*) \Rightarrow \quad H(y) = ld\, r \leftarrow H_{max}$

$$H(y/x) = \sum_{i=1}^{m} \sum_{j=1}^{r} P(x_i, y_j) \cdot ld \frac{1}{P(y_j/x_i)} =$$

$$= \sum_{i=1}^{m} P(x_i) \underbrace{\sum_{j=1}^{r} P(y_j/x_i)\, ld \frac{1}{P(y_j/x_i)}}_{const}$$

const (дефиниција за симетрични канали)

$$\Pi_c = \begin{matrix} & y_1 & y_2 & (j) & y_r \\ x_1 & P_{11} & P_{12} & \cdots & P_{1r} \\ x_2 & P_{21} & P_{22} & \cdots & P_{2r} \\ (i) & - & - & - & - \\ x_m & P_{m1} & P_{m2} & \cdots & P_{mr} \end{matrix}$$

$P_{ij} = P(y_j/x_i)$

$$P(y_j) = \sum_{i=1}^{m} P(x_i) \cdot P(y_j/x_i) \qquad j = 1, 2, \ldots r$$

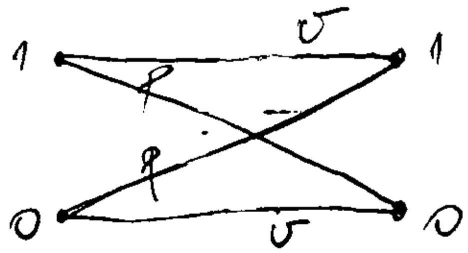
$$\Pi_y = \Pi_c^T \cdot \Pi_x$$

$$P_{ij} = P(y_j/x_i)$$

- капацитетот на симетричен канал е:

$$C = v(x,y) \cdot \left[ ld\, r - \sum_{j=1}^{r} P(y_j/x_i)\, ld \frac{1}{P(y_j/x_i)} \right]$$

• Бинарен симетричен канал (BSC)




$$C_{BSC} = v(x,y)\left[ ld\, 2 - \sum_{j=1}^{2} P(y_j/x_i)\, ld \frac{1}{P(y_j/x_i)} \right.$$

$$= v(x,y)\left[ 1 - v \cdot ld \frac{1}{v} - p\, ld \frac{1}{p} \right]$$

$$\Pi_{BSC} = \begin{bmatrix} v & p \\ p & v \end{bmatrix} \qquad \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} v & p \\ p & v \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$y_1 = v x_1 + p x_2$

$y_2 = p x_1 + v x_2$

$v = 1 - p$

$$C_{BSC} = \upsilon(x,y)\left[1 - \gamma\, ld\frac{1}{\gamma} - (1-\gamma)\, ld\frac{1}{1-\gamma}\right] = \upsilon(x,y)\cdot\left[1 - H(\gamma)\right]$$

ентрописка функција

• Мултипол BSC

$$\Pi^{(2)}_{BSC} = \begin{bmatrix} \upsilon & \gamma \\ \gamma & \upsilon \end{bmatrix}\begin{bmatrix} \upsilon & \gamma \\ \gamma & \upsilon \end{bmatrix} = \begin{bmatrix} \upsilon^2+\gamma^2 ; & 2\upsilon\gamma \\ 2\upsilon\gamma ; & \upsilon^2+\gamma^2 \end{bmatrix}$$

$$C^{(2)}_{BSC} = \upsilon(x,y)\cdot\left[1 - H(2\upsilon\gamma)\right]$$

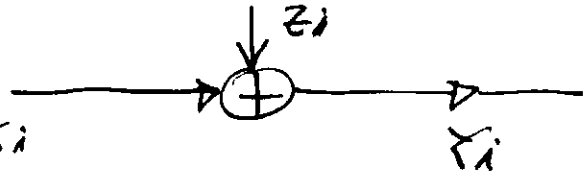
$$\Pi^{(3)}_{BSC} = \begin{bmatrix} \upsilon^2+\gamma^2 ; & 2\upsilon\gamma \\ 2\upsilon\gamma ; & \upsilon^2+\gamma^2 \end{bmatrix}\begin{bmatrix} \upsilon & \gamma \\ \gamma & \upsilon \end{bmatrix} = \begin{bmatrix} \upsilon^3+\upsilon\gamma^2+2\upsilon\gamma^2 ; & \upsilon^2\gamma+\gamma^3+2\upsilon^2\gamma \\ 2\upsilon^2\gamma+\upsilon^2\gamma+\gamma^3 ; & 2\upsilon\gamma^2+\upsilon^3+\upsilon\gamma^2 \end{bmatrix}$$

$$= \begin{bmatrix} \upsilon^3+3\upsilon\gamma^2 ; & \gamma^3+3\upsilon^2\gamma \\ \gamma^3+2\upsilon^2\gamma ; & 3\upsilon\gamma^2+\upsilon^3 \end{bmatrix}$$

$$C^{(3)}_{BSC} = \upsilon(x,y)\left[1 - H(\gamma^3+3\upsilon^2\gamma)\right]$$

• CAPACITY THEOREMS FOR RAYLEIGH CHANNELS (ова треба од членакот на T.M. Cover)

• CAPACITY OF GAUSSIAN CHANNEL (book T.M. Cover)




$Y_i = X_i + Z_i$

$Z_i \sim N(0,N)$

$f_s = 2\cdot f_g \Rightarrow T_s = ?$ $\quad \frac{1}{T_s} = \frac{2}{T_g} \quad T_g = 2\cdot T_s$

$f_s \geq 2 f_{max}$

• ПОТСЕТУВАЊЕ ОД ТЕОРИЈА НА ИНФОРМАЦИИ.

- КОНТИНУАЛЕН НЕПРЕКИНАТ КАНАЛ

$X \quad x \in (-\infty, \infty) \quad p_{\xi\eta}(x/y)$

$Y \quad y \in (-\infty, \infty) \quad p_{\xi}(x)$

$I(x,y) = H(Y) - H(Y/X)$

$I(x,y) = H(X) - H(X/Y)$

$$I(x;y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p_{\xi\eta}(x,y)\, ld\frac{p_{\xi\eta}(x,y)}{p_{\xi}(x)\cdot p_{\eta}(y)}\, dx\, dy$$

- СПЕЦИЈАЛЕН СЛУЧАЈ КОГА КАНАЛОТ Е СО АДИТИВЕН ШУМ:

$\eta = \xi + \nu \quad \nu = \{N\}$

$\boxed{n = y - x}$

$$H(Y/X) = \int_{-\infty}^{\infty} p(x)\,dx \int_{-\infty}^{\infty} p(y/x)\, ld \frac{1}{p(y/x)}\, dx$$

$$H(Y/X) = \int_{-\infty}^{\infty} p(y-x)\, ld \frac{1}{y-x}\, dy = \int_{-\infty}^{\infty} p(n) \cdot ld\left(\frac{1}{n}\right) dn$$

↳ IZRAZUVANJETO E ENTROPIJATA NA ŠUMOT ŠUM.

$$H(Y/X) = H(N) \qquad I(X;Y) = H(Y) - H(N)$$ } OVA OD SLIKATA SLEDI, NO NE ZNAM. MAT. DOKAZ

$$\max_{p(x)} [I(X;Y)] = \max_{p(x)} [H(Y)] - H(N) = ld\sqrt{2\pi e \overline{y^2}} - H(N)$$

VAŽI ZA $y \sim N(0, \sigma_y^2)$

---

$$H(Y) = \int_{-\infty}^{\infty} p(y)\, ld \frac{1}{p(y)}\, dy$$ ✓

$$p(y) = \frac{1}{\sigma_y\sqrt{2\pi}} e^{-\frac{y^2}{\sigma^2}}$$

$$H(Y) = \frac{\ln(2\pi) + 1 + 2\ln(\sigma_y)}{2\ln(2)} = \frac{\ln(2\pi \cdot \sigma_y^2) + 1}{2\ln(2)}$$ } VIDI MAPLE: MultiHarMMC Capacity.mw

$$\ln(e) = 1 \Rightarrow \quad H(Y) = \frac{\ln(2\pi \cdot e \cdot \sigma_y^2)}{2\ln(2)}$$

$$ld(x) = \frac{\ln x}{\ln 2} \qquad y = ld\, x \qquad x = 2^y$$

$$\boxed{x = e^{\ln x} \,/\, \ln} \qquad \boxed{\ln x = \ln x}$$

$$u = \ln v \qquad e^u = v \;/\, ld \qquad u \cdot ld\,e = ld\,v$$

$$u = \frac{ld\,v}{ld\,e}$$

$$\boxed{\ln v = \frac{ld\,v}{ld\,e}}$$

$$x = 2^y \;/\, \ln$$

$$\boxed{\ln x = y \cdot \ln 2}$$

$$\boxed{y = \frac{\ln x}{\ln 2}} \qquad \boxed{ld\,x = \frac{\ln x}{\ln 2}}$$

$$H(Y) = \frac{1}{2}\, ld(2\pi e \cdot \sigma_y^2) = ld\sqrt{2\pi e \sigma_y^2}$$

$$\boxed{\max_{p(x)} I(X;Y) = ld\sqrt{2\pi e \sigma^2} - H(N)} \leftarrow H(N) = ld\sqrt{2\pi e \sigma_x^2}$$

$$\max_{p(x)} \left[I(x;y)\right] = ld \sqrt{\frac{2\pi e \sigma_y^2}{2\pi e \sigma_n^2}} = \frac{1}{2} ld \frac{\sigma_y^2}{\sigma_n^2}$$

$$y = x + n \qquad E(y^2) = E\left[(x+n)^2\right] = E[x^2] + E[n^2]$$

STATISTIČKI NEZAVISNI SLUČAJNI PROCESI.

$$\boxed{E[y^2] = \sigma_x^2 + \sigma_n^2}$$

$$\max_{p(x)} \left[I(x;y)\right] = \frac{1}{2} ld \frac{\sigma_x^2 + \sigma_n^2}{\sigma_n^2} = \frac{1}{2} ld \left(1 + \frac{\sigma_x^2}{\sigma_n^2}\right)$$

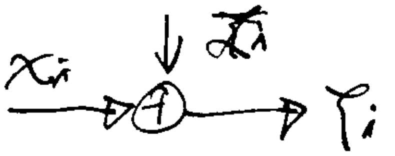
$$C = v(x,y) \cdot \max_{p(x)} \left[I(x;y)\right] (=) \left[\frac{sh}{s}\right]$$

$$v_0 = v(x;y) = 2 f_{max}$$

$$C = 2 f_{max} \cdot \frac{1}{2} ld \left(1 + \frac{\sigma_x^2}{\sigma_n^2}\right) \qquad \boxed{C = f_{max} \, ld \left(1 + \frac{\sigma_x^2}{\sigma_n^2}\right)}$$

- CAPACITY OF GAUSSIAN CHANNEL T.M. COVER BOOK

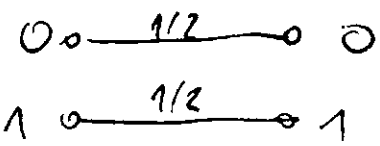
$$Y_i = X_i + Z_i \qquad Z_i \sim N(0, N)$$




---

$$C = \max_{p(x)} I(x;y)$$

- NOISELESS BINARY CHANNEL

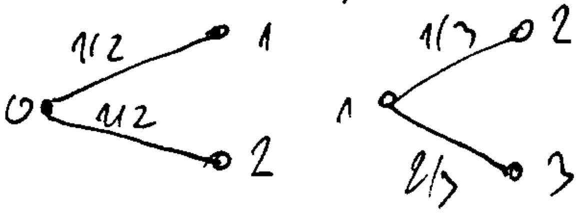



$$C = \max_{p(x)} I(x;y) \quad \nabla\theta$$

$$I(x;y) = H(y) - H(y/x)$$

$$H(y) = \sum_{j=1}^{r} P(y_j) \cdot ld \frac{1}{P(y_j)} = \frac{1}{2} \cdot ld 2 + \frac{1}{2} ld 2 = 1$$

- NOISY CHANNEL WITH NONOVERLAPPING OUTPUTS




$$H(y) = \frac{1}{3} ld 3 + \frac{2}{3} ld \frac{3}{2} = 1$$

$$= \frac{1}{3} ld 3 + \frac{2}{3} ld 3 - \frac{2}{3} ld 2 =$$

$$= \frac{3}{3} ld 3 - \frac{2}{3}$$

• LIMITATION ON THE INPUT – POWER CONSTRAINT

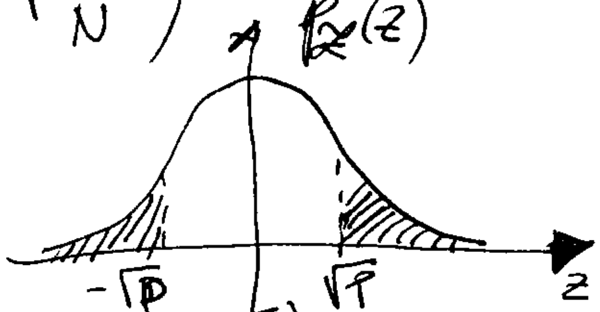
$$\frac{1}{n}\sum_{i=1}^{n} x_i^2 \leq P \qquad (x_1, x_2, \dots, x_n) \qquad \boxed{Y_i = X_i + Z_i} \quad Z \sim N(0,N)$$

$$P_e = \frac{1}{2}\Pr(Y<0 \mid X = +\sqrt{P}) + \frac{1}{2}\Pr(Y>0 \mid X = -\sqrt{P}) =$$

$$= \frac{1}{2}\Pr(Z < -\sqrt{P} \mid X = +\sqrt{P}) + \frac{1}{2}\Pr(Z > \sqrt{P} \mid X = -\sqrt{P}) =$$

$$= \Pr(Z > \sqrt{P}) = 1 - \phi\left(\sqrt{\frac{P}{N}}\right)$$

$$P_Z(z) = \frac{1}{\sqrt{2\pi N}} \cdot e^{-\frac{z^2}{2N}}$$




$$\Pr(Z > \sqrt{P}) = 1 - \Pr(Z < \sqrt{P}) = \underline{1 - \phi\left(\sqrt{\frac{P}{N}}\right)}$$

$$\phi = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt$$

$$\Pr(Z < \sqrt{P}) = \int_{-\infty}^{\sqrt{P}} \frac{1}{\sqrt{2\pi N}} e^{-\frac{t^2}{2N}} dt = \Bigg/ \quad v = \frac{t}{\sqrt{N}}$$

$$\Bigg/ \; dv = \frac{dt}{\sqrt{N}} \qquad t = -\infty \quad v = -\infty$$

$$t = \sqrt{P} \quad v = \sqrt{\frac{P}{N}} \Bigg/$$

$$dt = \sqrt{N} \cdot dv$$

$$\Pr(Z < \sqrt{P}) = \int_{-\infty}^{\sqrt{P/N}} \frac{1}{\sqrt{2\pi N}} e^{-\frac{v^2}{2}} \sqrt{N} \cdot dv = \int_{-\infty}^{\sqrt{P/N}} \frac{1}{\sqrt{2\pi}} e^{-\frac{v^2}{2}} dv = \phi\left(\sqrt{\frac{P}{N}}\right)$$

$$\boxed{P_e = 1 - \phi\left(\sqrt{\frac{P}{N}}\right)}$$

# 9.1 GAUSSIAN CHANNEL : DEFFINITIONS

$$C = \max_{f(x):\, E[X^2] \le P} I(X;Y)$$

$$I(X;Y) = h(Y) - h(Y/X) = h(Y) - h(X+Z/X) \Rightarrow (*)$$

$$H(\underbrace{X+Z}_{Y}/X) = \sum_{i=1}^{m} \sum_{j=1}^{r} P(x_i, \underbrace{y_j}_{z_j}) \, ld \frac{1}{P(y_j/x_i)}$$

---

$$H(Y/x_i) = \sum_{j=1}^{r} P(y_j/x_i) \, ld \frac{1}{P(y_j/x_i)} \quad , i = 1, 2, \ldots m$$

$$H(Y/X) = \overline{H(Y/x_i)} = \sum_{i=1}^{m} P(x_i) \sum_{j=1}^{r} P(y_j/x_i) \, ld \frac{1}{P(y_j/x_i)}$$

$$= \sum_{i=1}^{m} \sum_{j=1}^{r} P(x_i, y_j) \, ld \frac{1}{P(y_j/x_i)}$$

---

$$(*) \Rightarrow I(X;Y) = h(Y) - h(Z/X) = h(Y) - h(Z)$$

$$H(Z/X) = \overline{H(Z/x_i)} = \sum_{i=1}^{m} P(x_i) \sum_{j=1}^{r} P(z_j/x_i) \, ld \frac{1}{\underline{P(z_j/x_i)}}$$

$z_i$ NE ZAVISI OD $x_i$ $\Rightarrow$ $P(z_i/x_i) = P(z_i)$

$$H(Z/X) = \sum_{j=1}^{r} P(z_j) \, ld \frac{1}{P(z_j)} = H(Z)$$

OSNOVI NA TEORIJA NA VEROJATNOST
VIDI TEOR. INF. SKRIPTA

---

$$H(X+Z|X) = \overline{H(Y/x_i)} \qquad H(Y/x_i) = \sum_{j=1}^{r} P(y_j/x_i) \cdot ld \frac{1}{P(y_j/x_i)}$$

$$P(y_j/x_i) = P(x_j + z_j | x_i) = \overbrace{P(x_j|x_i)}^{1} + \underbrace{P(z_j|x_i)}_{P(z_j)} - P(x_j z_j)$$

~~$\frac{P(x_i, x_j)}{P(x_i)}$~~

$$P(Y_j/X_i) = 1 + P(z_j) - P(x_i)P(z_j)$$

$$H(Y/X_i) = -\sum_{j=1}^{m} \left(1 + P(z_j) - P(x_i)P(z_j)\right)\left[ld\, 1 + ld\, P(z_j) - ld\, P(x_i)P(z_j)\right] \to 0$$

$$= -\sum_{j=1}^{m} \left[1 + P(z_j) - P(x_i)P(z_j)\right] ld \frac{P(z_j)}{P(x_i)P(z_j)}$$

---

$$H(X+Y) = \sum_{i=1}^{m} P(x_i + y_i)\, ld \frac{1}{P(x_i + y_i)} =$$

$$= \sum_{i=1}^{m} \left[P(x_i) + P(y_i)\right] ld \left[\frac{1}{P(x_i) + P(y_i)}\right]$$

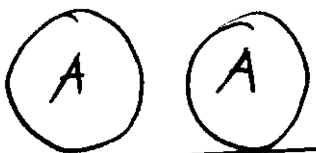
---

$$P(x_i + z_j | x_i) = \underbrace{P(x_i|x_i)}_{1} + P(z_j|x_i) - P(x_i \cap z_j) \to 0$$

$$H(Y|X_i) = -\sum_{j=1}^{r} \left[1 + \underbrace{P(z_j|x_i)}_{P(z_j)}\right] ld\left(1 + P(z_j)\right)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \qquad P(A|A) = \frac{P(A \cap A)}{P(A)} = \frac{P(A)}{P(A)} = 1$$

(with $P(A \cap A) \to P(A)$)




$$P(A \cap A) = P(A)$$

PRESEK OD DADENO MNOŽESTVO SO SAMOTO SEBE E SAMOTO MNOŽESTVO

CONTINUE ... pp. 112

ENTROPY (T.M. COVER BOOK)

---

- JOINT ENTROPY & CONDITIONAL

- DEFINITION: $(X, Y) \sim p(x,y)$ CONDITIONAL ENTROPY:

$$H(Y|X) = \sum_{x \in X} p(x) H(Y|X=x) = -\sum_{x \in X} p(x) \sum_{y \in Y} p(y|x)\, ld\, p(y|x) =$$

$$= -\sum_{x \in X} \sum_{y \in Y} p(x,y) \log_2 p(y|x) = -E\left[ld\, p(Y|X)\right]$$

- CHAIN RULE: $\boxed{H(X,Y) = H(X) + H(Y|X)}$

$$H(X+Z|X) = H(X+Z, X) - H(X)$$

Proof: $H(X,Y) = -\sum_{x \in X} \sum_{y \in Y} p(x,y)\, ld\, p(x,y)$

$$H(X,Y) = -\sum_{x\in X}\sum_{y\in Y} p(x,y)\, ld\big(p(x)\, p(y|x)\big) =$$

$$= -\sum_{x\in X}\sum_{y\in Y} p(x)\cdot p(y)\, ld\,(p(x)) - \underbrace{\sum_{x\in X}\sum_{y\in Y} p(x,y)\, ld\big(p(y|x)\big)}_{H(Y|X)}$$

$$= -\underbrace{\sum_{y\in Y} p(y)}_{1}\ \underbrace{\sum_{x\in X} p(x)\, ld[p(x)]}_{-H(X)} + H(Y|X)$$

$$\Rightarrow \boxed{H(X,Y) = H(X) + H(Y|X)}$$

- **Corollary**

$$H(X,Y|Z) = H(X|Z) + H(Y|X,Z)$$

$$H(X,Y) = -\sum_{x\in X}\sum_{y\in Y} p(x)\cdot p(y)\cdot ld[p(y)\cdot p(x/y)] =$$

$$= -\sum_{x\in X}\sum_{y\in Y} p(x)\cdot p(y)\cdot ld[p(y)] - \underbrace{\sum_{x\in X}\sum_{y\in Y} p(x,y)\, ld[p(x/y)]}_{-H(X/Y)}$$

S7: BARRE CHORDS ON THE 6TH STRING
S8: BARRE " " 5TH "

$$H(X,Y) = H(Y) + H(X|Y)$$

$$H(X,Y|Z) = H(X,W) = H(W) + H(X|W)$$

$$H(X,Y|Z) = H(Y|Z) + H(X|Y|Z)$$

---

$$H(X,Y|Z) = -\sum_{x\in X}\sum_{w\in W} p(x,w)\, ld[p(x,w)]$$

$$H(Y|Z) = -\sum_{y\in Y}\sum_{z\in Z} p(y,z)\cdot ld\,[p(y|z)]$$

**EXAMPLE 2.2.1** — JOINT DISTRIBUTION OF $(X,Y)$

| X \ Y | 1 | 2 | 3 | 4 | |
|---|---|---|---|---|---|
| 1 | 1/8 | 1/16 | 1/32 | 1/32 | |
| 2 | 1/16 | 1/8 | 1/32 | 1/32 | |
| 3 | 1/16 | 1/16 | 1/16 | 1/16 | |
| 4 | 1/4 | 0 | 0 | 0 | |
| | | | | | |

$$H(X) = -\sum_{x\in X} p(x)\, ld\, p(x)$$

$$H(X|Y) = -\sum_{x\in X}\sum_{y\in Y} p(x,y)\, ld[p(x|y)]$$

$P(B|A) = ?$ A – odd number
B – DA ... (3)

$$P(B|A) = \frac{P(A,B)}{P(A)} = \frac{\frac{1}{6}}{\frac{1}{2}} = \frac{1}{3}$$ $P(A,B) = P(A \cap B)$

---

- MARGINAL DISTRIBUTION OF $X$

$$p(X) = \sum_{y \in Y} p(X, Y=y) = \left[\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{8}\right]$$

$$p(Y) = \sum_{x \in X} p(X=x, Y) = \left[\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right]$$

$$H(X) = -\sum_{x \in X} p(x)\, ld\, p(x) = +\left[\frac{1}{2}\cdot 1 + \frac{1}{4}\cdot 2 + \frac{1}{8}\cdot 6\right] = \frac{2}{2} + \frac{6}{8} = 1 + \frac{3}{4} = \frac{7}{4}$$

$H(X) = \frac{7}{4}$ bits

$H(Y) = 2$ bits

$$H(Y) = \sum_{y \in Y} p(y)\, ld \frac{1}{p(y)} = 4 \cdot \frac{1}{4} \cdot 2$$

$$H(X|Y) = \sum_{y \in Y} P(Y=y) \cdot H(X|y) = \sum_{i=1}^{4} P(Y=i) H(X|Y=i)$$

$$H(X|Y=1) = \sum_{j=1}^{4} P(x_j = i|1) \cdot ld \frac{1}{P(X=i|1)} = \frac{1}{8}\cdot 3 + \frac{1}{16}\cdot 4 + \frac{2}{32}\cdot 5 = \frac{12+8+10}{32} = \frac{30}{32} = \frac{15}{16}$$

$$H(X|Y=2) = \frac{1}{16}\cdot 4 + \frac{1}{8}\cdot 3 + \frac{2}{32}\cdot 5 = \frac{8+12+10}{32} = \frac{30}{32} = \frac{15}{16}$$

$$H(X|Y=3) = \frac{1}{16}\cdot 4 \cdot 4 = 1 \qquad H(X|Y=4) = \frac{1}{4}\cdot 2 = \frac{1}{2}$$

$$H(X|Y) = \frac{1}{4}\cdot\frac{15}{16} + \frac{1}{4}\cdot\frac{15}{16} + \frac{1}{4}\cdot 1 + \frac{1}{4}\cdot\frac{1}{2} = \frac{15}{32} + \frac{1}{4} + \frac{1}{8}$$

$$H(X|Y) = \frac{15+8+4}{32} = \frac{27}{32} = \ldots$$

• CONDITIONAL DISTRIBUTION $P(X|Y=y) = ?$

$$P(X|Y) = \frac{P(X,Y)}{P(Y)} =$$

$P(Y)$: $\left[\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right]$

| Y\X | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | 1/2 | 1/4 | 1/8 | 1/8 |
| 2 | 1/4 | 1/2 | 1/8 | 1/8 |
| 3 | 1/4 | 1/4 | 1/4 | 1/4 |
| 4 | 1 | 0 | 0 | 0 |

$$H(X|Y=1) = \sum_{j=1}^{4} P(X=j|1)\, ld \frac{1}{P(X=j|1)} = \frac{1}{2} + \frac{1}{4}2 + \frac{2}{8}\cdot 3 = \frac{4+4+6}{8} = \frac{14}{8} = \frac{7}{4}$$

$$H(X|Y=2) = \frac{1}{4}\cdot 2 + \frac{1}{2} + \frac{2}{8}\cdot 3 = \frac{4+4+6}{8} = \frac{7}{4}$$

$$H(X|Y=3) = \sum_{j=1}^{4} P(X=j|3)\, ld \frac{1}{P(X=j|3)} = \frac{1}{4}\cdot 2\cdot 4 = 2$$

$$H(X|Y=4) = 1\cdot ld\, \emptyset = \emptyset$$

$$H(X|Y) = \sum_{i=1}^{4} P(Y)\cdot H(X|Y=i) = \frac{7}{4}\cdot\frac{1}{4} + \frac{7}{4}\cdot\frac{1}{4} + \frac{1}{4}\cdot 2 = \frac{7+7+8}{16} = \frac{22}{16} = \frac{11}{8}$$

---

CONTINUE FROM 109

$$H(X+Z/X) = \sum_{x\in X} P(x) \sum_{y\in Y} P(X+Z|X=x)\, ld \frac{1}{P(X+Z|X=x)}$$

$$= \sum_{x\in X} P(x) \sum_{y\in Y} \left[P(X|X=x_i) + P(Z|X=x_i) + \underbrace{P(X,Z|X=x)}_{\text{STAT. NEZ}}\right] ld \frac{1}{P}$$

(the last term marked $\to \emptyset$)

$$= \sum_{x\in X} P(x) \left[\underbrace{\sum_{x\in X} P(X|X=x)\cdot ld\frac{1}{P}}_{1} + \sum_{z\in Z} \underbrace{P(Z|X=x)}_{P(z)} ld\frac{1}{P}\right]$$

$$I(X;Y) = H(Y) - H(Y/X) = H(Y) - H(X+Z|X) =$$
$$= H(Y) - H(Z|X) = H(Y) - H(Z)$$

$$Z \sim \mathcal{N}(0,N) \qquad p_Z(z) = \frac{1}{\sqrt{2\pi N}} e^{-\frac{z^2}{2N}} \qquad H(Z) = \int_{-\infty}^{\infty} p_Z(z)\, ld\frac{1}{p_Z(z)}\, dz$$

$$H(z) = \frac{1}{\sqrt{2\pi N}} \int_{-\infty}^{\infty} e^{-\frac{z^2}{2N}} \frac{\ln\left(\sqrt{2\pi N}\, e^{+\frac{z^2}{2N}}\right)}{\ln 2}\, dz = \frac{1}{\ln 2 \sqrt{2\pi N}} \int_{-\infty}^{\infty} \frac{z^2}{2N} e^{-\frac{z^2}{2N}} dz$$

$$I_1 = \frac{1}{\ln 2 (2N) \cdot \sqrt{2\pi N}} \int_{-\infty}^{\infty} z^2 e^{-\frac{z^2}{2N}} dz \quad + \underbrace{\frac{1}{\ln 2 \sqrt{2\pi N}} \int_{-\infty}^{\infty} \ln \sqrt{2\pi N}\, e^{-\frac{z^2}{2N}} dz}_{I_2}$$

$$\int_{-\infty}^{\infty} x^2 e^{-\frac{x^2}{2N}} dx = \left| \begin{array}{l} u = x \\ v = \int x e^{-\frac{x^2}{2N}} dx = N \int e^{-\frac{x^2}{2N}} d\left(\frac{x^2}{2N}\right) = -N \cdot e^{-\frac{x^2}{2N}} \end{array} \right|$$

$$= x \cdot N e^{-\frac{x^2}{2N}} \Big|_{-\infty}^{\infty} + N \int_{-\infty}^{\infty} e^{-\frac{x^2}{2N}} dx = x N e^{-\frac{x^2}{2N}} + N\sqrt{2N} \int_{-\infty}^{\infty} e^{-\left(\frac{x}{\sqrt{2N}}\right)^2} d\left(\frac{x}{\sqrt{2N}}\right)$$

$$= \left| \int_{-\infty}^{\infty} e^{-u^2} du = \sqrt{\pi} \right| = \underbrace{x \cdot N \cdot e^{-\frac{x^2}{2N}} \Big|_{-\infty}^{\infty}}_{0} + N\sqrt{2N \cdot \pi}$$

$\boxed{uv - \int v\, du = I}$

$$\lim_{x \to \infty} \frac{x \cdot N}{e^{x^2/2N}} = 0 \qquad \lim_{x \to \infty} \frac{x \cdot N}{e^{+x^2/2N}} = \lim_{x \to \infty} \frac{N}{\frac{2x}{2N} \cdot e^{\frac{x^2}{2N}}}$$

$$= \frac{-\infty}{\infty} = 0$$

$$I_1 = \frac{1}{\ln 2 (2N) \sqrt{2\pi N}} \cdot N \sqrt{2N \cdot \pi} = \frac{1}{2 \ln 2} = \frac{\frac{1}{2} \ln e}{\ln 2} = \frac{\ln \sqrt{e}}{\ln 2}$$

$$I_2 = \frac{\sqrt{2N}}{(\ln 2) \sqrt{2\pi N}} \cdot \ln \sqrt{2\pi N} \underbrace{\int_{-\infty}^{\infty} e^{-\left(\frac{z}{\sqrt{2N}}\right)^2} d\left(\frac{z}{\sqrt{2N}}\right)}_{\sqrt{\pi}} = \frac{\ln \sqrt{2\pi N} \cdot \sqrt{\pi}}{\ln 2 \sqrt{\pi}}$$

$$H(z) = I_1 + I_2 = \frac{\ln \sqrt{e} + \ln \sqrt{2\pi N}}{\ln 2} = \frac{\ln \sqrt{2e\pi N}}{\ln 2}$$

$\boxed{H(z) = \mathrm{ld} \sqrt{2e\pi N}}$ DOKAZANO!!!

$$E[\zeta^2] = E[(X+Z)^2] = E[X^2] + \underbrace{E[2X \cdot Z]}_{0} + E[Z^2]$$

$\boxed{E[\zeta^2] = P + N}$

$I(X;Y) = H(Y) - H(Z)$

MAXIMUM OF ENTROPY $H(Y)$ IS OBTAINED FOR $X \sim N(0, P+N)$

$H(Y) = \frac{1}{2} ld(2\pi e(P+N)) = \frac{1}{2} ld[2\pi e \overline{Y^2}]$ $\quad \overline{Y^2} = P+N$

$\max[I(X;Y)] = \frac{1}{2} ld(2\pi e(P+N)) - \frac{1}{2} ld(2\pi e N)$

$$\max[I(X;Y)] = \frac{1}{2} ld\left(1 + \frac{P}{N}\right)$$

$$C = \max_{E[X^2] \le P} I(X;Y) = \frac{1}{2} ld\left(1 + \frac{P}{N}\right)$$

MAXIMUM IS ATTAINED WHEN: $X \sim N(0, P)$

---

$H(X) = -\sum_{x \in \mathcal{X}} p(x)\, ld\, p(x)$ — $p(x)$: PROBABILITY MASS FUNCTION

$ld\, x = \frac{\ln(x)}{\ln 2}$ $\quad \ln(x) = \ln 2 \cdot ld(x)$

$\log_e(x) = \log_e(2) \log_2(x)$

Lemma 2.1.2 $\quad H_b(X) = \log_b(a) \log_a(X)$

$$X = \begin{cases} 1 & \text{WITH PROBABILITY } p \\ 0 & \text{WITH PROBABILITY } 1-p \end{cases}$$

$H(X) = -p\, ld\, p - (1-p)\, ld(1-p)$ — ENTROPIJSKA FUNKCIJA

---

- POVTORUVANJE OD SKRIPTA (STATISTIČKO KODIRANJE)

$s_1, s_2, \dots s_q$ → S → STATIST KODER → X $\{x_1, x_2, \dots x_m\}$

$s_i$ - IZVORNI SYMBOLI

$X_i$ - KODNI ZNACI (SYMBOLI)

$s_i \to X_i \; \{x_{j1}, x_{j2}, \dots x_{jk}, \dots x_{jn}\}$ $\quad i = 1, 2, \dots q$

BROJ NA KODNI ZBOROVI

$x_{jk} \in \{x_1, x_2, \dots, x_m\} \Rightarrow$ KODNA LISTA

$m^n = q$ $\quad n = \log_m q$

- PROSEČNA DOLŽINA NA KODNI ZBOROVI: $\overline{L}_m(X) = \sum_{i=1}^{q} l_i P(s_i) = \sum_{i=1}^{q} l_i P(s_i)$

($P(s_i)$ - VEROJATNOST NA POJAVUVANJE NA KODEN ZBOR)

⊙ NEKOI OSOBINI NA KODOVITE

- EDNOZNAČNO DEKODIRACEN

| $s_i$ | $x_i$ |
|---|---|
| $s_1$ | 0 |
| $s_2$ | 010 |
| $s_3$ | 01 |
| $s_4$ | 10 |

01 0010 10

OPT1: $s_2\ s_3\ s_3\ s_1$

OPT2: $s_3\ s_1 s_1\ s_4\ s_4$

ZNAČI KODOT NE E EDNOZNAČNO DEKODIRACEN

- METOD NA SARDINAS-PETERSON ZA OPREDELUVANJE NA EDNOZNAČNOST NA KODOT

$S = \{s_1, s_2, \dots, s_7\}$ IZVORNA LISTA SO 7 SIMBOLI

$X = \{a, b, c, d, e\}$

- DOLŽINA NA ZADOCNUVANJE PRI DEKODIRANJE:

$l_{min} \le l_d \le l_{max}$ $l_d$ – ZA MOMENTALEN KOD

$$\left\lceil \frac{k}{2} \right\rceil l_{min} \le l_d \le l_{max} \left\lceil \frac{k+1}{2} \right\rceil$$

(k) – BROJ NA KLASI NA KODOT.

PRIMER:

| $s_i$ | $x_i = S_0$ | $S_1$ | ... |
|---|---|---|---|
| $s_1$ | 00 | / | / |
| $s_2$ | 11 | / | |
| $s_3$ | 100 | / | |
| $s_4$ | 101 | / | |

$1 < l_d < 3$

- POKOLKU KODOT E DEZUMNO MOMENTALEN TOGAŠ NA PR $S_0, S_1, S_2$ ($k=3$ SEGMENTNI KLASI)

$$\left\lceil \frac{3}{2} \right\rceil 2 \le l_d \le \left\lceil \frac{4}{2} \right\rceil l_{max}$$

$$4 \le l_d \le 6$$

⊙ KRAFTOVO NERAVENSTVO

(A) $$\sum_{i=1}^{q} m^{-l_i} \le 1$$

$l_i = \{l_1, l_2, \dots, l_q\}$ – DOLŽINA NA KODNITE ZBOROVI

(B) $$\sum_{l_i=1}^{l_{max}} N(l_i) \le q$$

$N(l_i) \Rightarrow$ BROJ NA KODNI ZBOROVI SO DOLŽINA $l_i$

⊙ OSNOVNA TEOREMA NA STATISTIČKO KODIRANJE

- MINIMALNA MOŽNA GRANICA NA PROSEČNATA DOLŽINA NA KODNITE ZBOROVI

$$L_n(X) \ge \frac{H(S)}{ld\, m}$$

$$L_n(X) = \sum_{i=1}^{q} P(x_i) \cdot l_i$$

$$L_m(x)_{min} = \frac{H(S)}{ld\,m} = H_m(S)$$ } M-ARNA ENTROPIJA !!! NA IZVOROT PO ... LISTA NA SIMBOLI, S =

(*)

$$H_m(S) = \sum_{i=1}^{q} P(S_i) \log_m \frac{1}{P(S_i)}$$

$y = ld\,x$
$x = 2^y$

$\log_m x = ld\,m \cdot ld\,x$

$$ld\,x = \frac{\ln x}{\ln 2}$$

$\cdot y$

$\ln x = 2^y / \ln \;=\; y \cdot \ln 2 = ld\,x \cdot \ln 2$

$m^y = x \;/ld$ $\quad y\,ld\,m = ld\,x$

$\log_m x \quad \log_m x \cdot ld\,m = ld\,x$

$$\boxed{\log_m x = \frac{ld\,x}{ld\,m}}$$

$$H_m(S) = -\sum_{i=1}^{q} P(S_i) \frac{ld\,P(S_i)}{ld\,m} = \frac{H(S)}{ld\,m}$$

- MINIMALNATA DOLZINA (*) NA KODNITE ZBOROVI VAZI AKO I SAMO AKO:

**I** $\boxed{\sum_{i=1}^{q} m^{-l_i} = 1}$ **II** $\boxed{P(S_i) = m^{-l_i}}$

MAKSIMALNO ZASIEN

- KODOT E IDEALNO KOMPAKTEN DOKOLKU $P(S_i)$ E VO OBLIK II

$$L_m(x) \geq \frac{H(S)}{ld\,m}$$

BINAREN KOD:

$$L_2(x) \geq \frac{H(S)}{ld\,2} = H(S)$$

II $\Rightarrow$ $ld\,P(S_i) = -l_i\,ld\,m$ $\quad \boxed{l_i = \frac{1}{ld\,m} \cdot ld\,\frac{1}{P(S_i)}}$

- DOKOLKU $P(S_i)$ NE E VO OBLIK II:

$$\boxed{\frac{-ld\,P(S_i)}{ld\,m} \leq l_i \leq -\frac{ld\,P(S_i)}{ld\,m} + 1} \quad (*)$$

• Доколку $P(s_i)$ не е во облик II т.е. $l_i$ не е цел број тогаш:

$$l_i = \left\lceil \log_m \frac{1}{P(s_i)} \right\rceil = \left\lceil -\frac{ld\, P(s_i)}{ld(m)} \right\rceil$$

• (*) $/ \cdot P(s_i) \sum \Rightarrow$

$$-\sum_{i=1}^{q} \frac{P(s_i)\, ld\, P(s_i)}{ld\, m} \le \sum_{i=1}^{q} l_i P(s_i) \le -\frac{\sum_{i=1}^{q} P(s_i)\, ld\, P(s_i)}{ld\, m} + 1$$

$$\boxed{\frac{H(S)}{ld\, m} \le L_m(l_i) \le \frac{H(S)}{ld\, m} + 1}$$

• ПРВА ШЕНОНОВА ТЕОРЕМА: Може произволно да се приближиме до $L_m(x)_{min}$ доколку изворот го прошириме:

$$\frac{H(S^n)}{ld\, m} \le L_m(X^n) \le \frac{H(S^n)}{ld\, m} + 1$$

– Ентропијата на проширениот извор е $n$-пати поголема од ентропијата на оригиналот

$$n\frac{H(S)}{ld\, m} \le L_m(X^n) \le n\frac{H(S)}{ld\, m} + 1 \quad / n$$

$$\frac{H(S)}{n\, ld\, m} \le \frac{L_m(X^n)}{n} \le \frac{H(S)}{ld\, m} + \frac{1}{n}$$

просечен број на кодни знаци по симбол.

$$\boxed{\lim_{n \to \infty} \frac{L_m(X^n)}{n} = \frac{H(S)}{ld\, m}}$$

→ ПРВА SHANNON-ова ТЕОРЕМА

Апсолутна долна граница за должината на кодот на изворот S: $\frac{H(S)}{ld\, m}$

EXAMPLE 2.1.2

$$X = \begin{cases} a & \text{WITH PROB. } 1/2 \\ b & \text{WITH } -\text{"}- \ 1/4 \\ c & \text{WITH } -\text{"}- \ 1/8 \\ d & \text{WITH } -\text{"}- \ 1/8 \end{cases}$$

$$H(x) = \frac{1}{2}\cdot 1 + \frac{1}{4}\cdot 2 + \frac{1}{8}\cdot 3 + \frac{1}{8}\cdot 3 = \frac{4+4+6}{8} = \frac{14}{8} = \frac{7}{4} = 1.75$$

$H(X) = 1.75$ bits

3201471

- DETERMINE $X$ WITH MINIMUM NUMBER OF QUESTIONS.

1.) EFFICIENT FIRST QUESTION: Is $X=a$? (SPLITS PROBABILITY IN HALF)

2.) IF ANSWER OF 1.) IS „NO" ⇒ Is $X=b$?

3.) IF ANSWER OF 2.) IS „NO" ⇒ Is $X=c$!

MINIMUM EXPECTED NUMBER OF BINARY QUESTIONS REQUIRED TO DETERMINE THE VALUE OF X!!!

$H(X) \leq$ MINIMUM EXPECTED NUMBER OF BINARY QUESTIONS $\leq H(X)+1$

- DIFERENCIAL ENTROPY

$$h(X) = -\int_S f(x)\, ld\, f(x)\, dx$$

VESNA POPACOVA

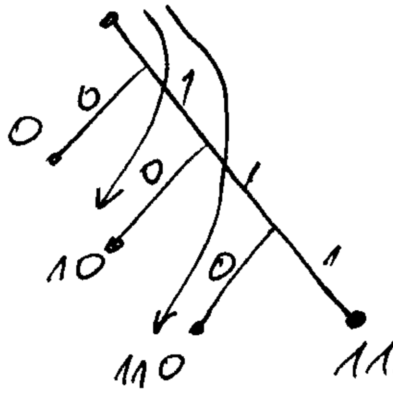
DW THMK

▶ OVA ODGOVARA NA FANOVA-TA POSTAPKA

ZOKI SIIRKOVSKI

0702007-13

| $s_i$ | $P(s_i)$ | | | | $x_i$ |
|---|---|---|---|---|---|
| $s_1$ | 1/2 | I | | | 0 |
| $s_2$ | 1/4 | II | I | | 10 |
| $s_3$ | 1/8 | | II | I | 110 |
| $s_4$ | 1/8 | | | II | 111 |




(I) & (II)

„m" GRUPI SO ISTA „SUMA NA VEROJATNOST"

$P(s_i) = 2^{-l_i}$ $P(s_1) = 2^{-1}$; $P(s_2) = 2^{-2}$; $P(s_3) = 2^{-3}$; $P(s_4) = 2^{-3}$

$$L(x) = \sum_{}^{2} l_i P(s_i) = 1\cdot\frac{1}{2} + 2\cdot\frac{1}{4} + 3\cdot\frac{1}{8} + 3\frac{1}{8} = \frac{4+4+6}{8} = \frac{7}{4}$$

$$H(S) = \sum_{x=1}^{x_q=n} P(s_i)\cdot ld\frac{1}{P(s_i)} = \frac{1}{2}\cdot 1 + \frac{1}{4}\cdot 2 + \frac{1}{8}\cdot 3\cdot 2 = \frac{4+4+6}{8} = \frac{7}{4}$$

$L(x) \stackrel{?}{=} H(S)$ } SAMO NUMERIČKI EDNAKVI DIMENZIONO

$L(x)\,[=]\,\frac{\text{BITS}}{\text{SYMBOL}}$ $H(S)\,[=]\,\frac{\text{SH}}{\text{SYMBOL}}$

ZATOA VO T.M.COVER ZA ENTROPIJATA ZEMAAT BITS/SIM.

• MUTUAL INFORMATION TURNS OUT TO BE SPECIAL CASE OF MORE GENERAL QUANTITY CALLED RELATIVE ENTROPY $D(p\|q)$ WHICH IS THE MEASURE OF THE DISTANCE BETWEEN TWO PROBABILITY MASS FUNCTIONS $p$ & $q$. IT IS DEFINED AS:

$$D(p\|q) = \sum_{x\in X} p(x) \log \frac{p(x)}{q(x)} = E_p\left[\log \frac{p(x)}{q(x)}\right].$$

$D(p\|q) = 0$ IF AND ONLY IF $p = q$

• MUTUAL INFORMATION (TRANSINFORMATION) IS MEASURE OF AMOUNT OF INFORMATION THAT ONE RANDOM VARIABLE CONTAINS ABOUT ANOTHER VARIABLE

$$I(X;Y) = \sum_{x\in X}\sum_{y\in Y} p(x,y)\, \mathrm{ld} \frac{p(x,y)}{p(x)\cdot p(y)} = D(p(x,y)\|p(x)p(y))$$
$$= E_{p(x,y)}\left[\mathrm{ld} \frac{p(x,y)}{p(x)p(y)}\right]$$

EXAMPLE: 2.3.1 $\mathcal{X} = \{0,1\}$ CONSIDER TWO DISTRIBUTIONS ON $\mathcal{X}$:

$p(0) = 1-r \quad p(1) = r$

$q(0) = 1-s \quad q(1) = s$

$$D(p\|q) = \sum_{x\in X} p(x)\, \mathrm{ld} \frac{p(x)}{q(x)} = (1-r)\, \mathrm{ld} \frac{(1-r)}{(1-s)} +$$
$$+ r\, \mathrm{ld} \frac{r}{s}$$

$$D(q\|p) = \sum_{x\in X} q(x)\, \mathrm{ld} \frac{q(x)}{p(x)} = (1-s)\, \mathrm{ld} \frac{(1-s)}{(1-r)} + s\, \mathrm{ld} \frac{s}{r}$$

$\boxed{r=s}$ $D(p\|q) = 0$

$\left.\begin{matrix} r = \frac{1}{2} & s = \frac{1}{4}\end{matrix}\right\}$ $D(p\|q) = \frac{1}{2}\, \mathrm{ld} \frac{\frac{1}{2}}{\frac{3}{4}} + \frac{1}{2}\, \mathrm{ld} \frac{\frac{1}{2}}{\frac{1}{4}}$

$$D(p\|q) = \frac{1}{2}\, \mathrm{ld} \frac{2}{3} + \frac{1}{2}\, \mathrm{ld}\, 2 = 0.5 + \frac{1}{2}(\mathrm{ld}\, 2 - \mathrm{ld}\, 3) = 0.5 - 0.29$$
$$2.079 \qquad = 0.20752$$

• RELATIONSHIP BETWEEN MUTUAL INFORMATION AND ENTROPY

$$I(X;Y) = \sum_{x\in X}\sum_{y\in Y} p(x,y)\, \mathrm{ld} \frac{p(x,y)}{p(x)\cdot p(y)} = \sum_{x,y} p(x,y)\, \mathrm{ld} \frac{p(x|y)}{p(x)}$$

$$I(X;Y) = \underbrace{-\sum_{x,y} p(x,y)\, ld\, p(x)}_{(*)} + \underbrace{\sum_{x,y} p(x,y)\, ld\, p(x|y)}_{-H(X|Y)}$$

$$(*) = \sum_{x\in X} ld\, p(x) \underbrace{\sum_{y \in Y} p(x,y)}_{p(x)} = \sum_{x \in X} p(x)\, ld\, p(x)$$

$I(X;Y) = -\sum_{x\in X} p(x)\, ld\, p(x)$ [redacted] $\sum_{x,y} p(x,y)\, ld \frac{1}{p(x|y)} = H(X) - H(X|Y)$

- АНАЛОГНО НА ТОА: $I(X;Y) = H(Y) - H(Y|X)$
- $I(X;Y) = H(X) - H(X|Y)$ → THE MUTUAL INFORMATION IS THE REDUCTION OF UNCERTAINTY OF $X$ DUE TO THE KNOWLEDGE OF $Y$.

$I(X;Y) = H(Y) - H(Y|X)$ → $X$ SAYS AS MUTCH ABOUT $Y$ AS $Y$ SAYS ABOUT $X$.

$$I(X;X) = H(X) - \underline{H(X|X)} = H(X) \qquad (H(X|X) \to 0)$$

$$H(X|X) = \sum_{x\in X} p(x,x)\, ld \frac{1}{p(x|x)} = \sum_{x \in X} p(x,x)\, ld\, 1 = 0$$

$$p(x|x) = \frac{p(x,x)}{p(x)} = \frac{p(x)}{p(x)} = 1$$

$p(x,x) = p(x \cap x) = p(x)$, $x \cap x = x$

$P(X+Y) = P(X) + P(Y) - \underbrace{P(X\cap Y)}_{P(X,Y)}$

~~$H(X|X) = \sum_{x\in X} ld\, p(x) = \sum_{i=1}^{4} ld\, p(x_i) = ld \prod_{i=1}^{4} p(x_i)$~~

~~$P(X) = \{\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\}$ $\prod_{i=1}^{4} p(x_i) = ld \frac{1}{4^4} = ld \frac{1}{2^8} = -8\, ld\, 2 = -8$~~

$X \cap X = X$ ⇒ ПРЕСЕК НА ЕДНО МНОЖЕСТВО СО САМОТО СЕБЕ ∈ САМОТО МНОЖЕСТВО!!!

- $I(X;X) = H(X)$ MUTUAL INFORMATION OF RANDOM VARIABLE WITH ITSELF IS THE ENTROPY OF THE RANDOM VARIABLE!!!

THIS IS THE REASON THAT ENTROPY IS SOMETIMES REFERRED AS SELF-INFORMATION.

**Theorem 2.4.1** (MUTUAL INFORMATION & ENTROPY)

$$I(X;Y) = H(X) - H(X|Y);\quad I(X;Y) = H(Y) - H(Y|X)$$
$$I(X;Y) = H(X) + H(Y) - H(X,Y);\quad I(X;Y) = I(Y;X);$$
$$I(X;X) = H(X)$$