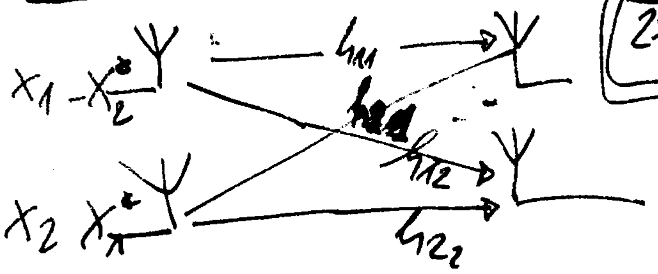
• OUTAGE PROBABILITY OF MULTI-ANTENNAS PLAZHOF SYSTEM

$$\boxed{\tilde{x} = X\cdot(2\Delta I) + \Xi} \qquad \boxed{\tilde{x}_1 = 2\Delta x_1 + \xi_1}$$

$$\hat{x}_1 = G\cdot\Lambda\cdot\tilde{x}_1 + \mu_1$$



$\boxed{2\times2}$

ANTENNA

$$y_1[1] = h_{11}x_1 + h_{21}x_2 + n_{①}[1]$$
$$y_1[2] = -h_{11}x_2^* + h_{21}x_1^* + n_1[2]$$
$$y_2[1] = h_{12}x_1 + h_{22}x_2 + n_2[1]$$
$$y_2[2] = -h_{12}x_2^* + h_{22}x_1^* + n_{②}[2]$$

ANTENNA

$$\tilde{x}_1 = h_{11}^* y_1[1] + h_{21}y_1^*[2] + h_{12}^* y_2[1] + h_{22}y_2^*[2]$$

$$\tilde{x}_1 = \sqrt{E}\,\Delta_2 x_1 + \eta_1 \qquad \Delta_2 = |h_{11}|^2 + |h_{12}|^2 + |h_{21}|^2 + |h_{22}|^2$$

$$\eta_1 = h_{11}^* n_1[1] + h_{21}n_1^*[2] + h_{12}^* n_2[1] + h_{22}n_2^*[2]$$
$$\eta_2 = h_{21}^* n_1[1] - h_{11}n_1^*[2] + h_{22}^* n_2[1] - h_{12}n_2^*[2]$$

$$\begin{bmatrix} n_1[1] & n_1^*[2] \\ n_2[1] & n_2^*[2] \end{bmatrix} \Big[$$

$$[y_1[1]\ y_{12}^*\ y_{21}\ y_{22}^*] = [x_1\ x_2]\left[\overbrace{\begin{matrix} h_{11} & +h_{21}^* \\ h_{21} & -h_{11}^* \end{matrix}}^{\Omega_1}\ \overbrace{\begin{matrix} h_{12} & h_{22}^* \\ h_{22} & -h_{12} \end{matrix}}^{\Omega_2}\right] + \begin{bmatrix} n_{11} \\ n_{12} \\ n_{21} \\ n_{22} \end{bmatrix}^T$$

$$[\tilde{x}_1,\ \tilde{x}_2] = \cancel{\begin{bmatrix} y_{11} & y_{12}^* \\ y_{21} & y_{22}^* \end{bmatrix}}\cdot\Omega^H = \begin{bmatrix} y_{11}\ y_{12}^* \dots \\ \dots y_{21}\ y_{22}^* \end{bmatrix}\cdot\begin{bmatrix} h_{11}^* & h_{21}^* \\ h_{21} & -h_{11} \\ h_{12}^* & h_{22}^* \\ h_{22} & -h_{12} \end{bmatrix} \begin{matrix} \Omega_1^H \\ \\ \Omega_2^H \end{matrix}$$

$$Y_a = [y_{11}\ y_{12}^*\ y_{21}\ y_{22}^*]$$

$$\tilde{x}_2 = h_{21}^* y_{11} - h_{11}y_{12}^* + h_{22}^* y_{21} - h_{12}y_{22}^*$$

$$[\tilde{x}_1, \tilde{x}_2] = \underbrace{[Y_{1a},\ Y_{2a}]}_{Y_a}\begin{bmatrix} \Omega_1^H \\ \Omega_2^H \end{bmatrix} \qquad \boxed{\tilde{x} = Y_a\cdot\underbrace{\begin{bmatrix} \Omega_1^H \\ \Omega_2^H \end{bmatrix}}_{\Omega^H}}$$

$$\tilde{x} = Y_a\cdot\Omega^H = X\cdot\Delta_2 I_{2\times2} + N_a\cdot\Omega^H$$

$$N_a = [n_{11}\ \bar{n}_{12}\ n_{21}\ n_{22}]$$

$$P_S = \Delta_2^2$$
$$P_N = \Delta_2\cdot N_0$$

VIDI
Molt.log MIMO
• MW
MMLE

$\boxed{4\times2 \text{ SYSTEM}}$ (VIDI MultiHop MIMO, MVO)

67

~~Reo~~ 0820039

$$N_{ai} = [n_{i1}, n_{i2}, n_{i3}, n_{i4}, \bar{n}_{i5}, \bar{n}_{i6}, \bar{n}_{i7}, \bar{n}_{i8}]$$

$$N_a = [N_{a1}, N_{a2}]$$

$$\boxed{Y_t = C\cdot H^T + N^T} \qquad Y = (C\cdot H^T + N^T)^T = X\Omega + N_a$$

R – CODE ~~RATE~~ OF FIRST HOP

$$H = \begin{bmatrix} h_{11} & h_{21} & h_{31} & h_{41} \\ h_{12} & h_{22} & h_{32} & h_{42} \end{bmatrix}$$

$$\boxed{\tilde{x} = Y_a\cdot\Omega^H = R_1^{-1} X\Delta\cdot I_{4\times4} + \overbrace{N_a\cdot\Omega^H}^{\Xi}} \; !!$$

$$\Delta = |h_{11}|^2 + |h_{21}|^2 + |h_{31}|^2 + |h_{41}|^2 + |h_{12}|^2 + |h_{22}|^2 + |h_{32}|^2 + |h_{42}|^2$$

$$\boxed{P_S = 4\cdot\Delta^2} \qquad \boxed{P_N = 4\cdot\Delta} \rightarrow \text{VIDI MANJE!!!}$$

$$\hat{X} = \frac{G_R}{R_2}\cdot\Lambda\cdot\tilde{X} + M = \frac{G_R}{R_2}\circ\Lambda\left[\frac{\Delta}{R_1}\overset{\sqrt{\epsilon}}{}X\cdot I_{4\times4} + N_a\Omega^H\right] + M$$

$$\hat{X} = \frac{\sqrt{\epsilon}G_R\Lambda\cdot\Delta}{R_1\cdot R_2}X\cdot I_{44} + \frac{G_R\Lambda}{R_2}\cdot N_a\underset{\text{FIRST HOP}}{\Omega_1^H} + \underset{\text{SECOND HOP}}{N_a\Omega_2^H}$$

$$P_S = \frac{\epsilon G_R^2\Lambda^2\cdot\Delta^2}{R_1^2R_2^2} \qquad P_N = \frac{G_R^2\Lambda^2}{R_2^2}\,\frac{\Delta N_0}{R_1^2} + \frac{\Lambda N_0}{R_2^2}$$

$$\gamma_G = \frac{\dfrac{\epsilon G_R^2\Lambda^2\Delta^2}{R_1^2\cancel{R_2^2}}}{\dfrac{G_R^2\Lambda^2\Delta N_0}{R_1^2\cancel{R_2^2}} + \dfrac{R_1^2\Lambda N_0}{R_1^2\cancel{R_2^2}}} = \frac{\epsilon G_R^2\Lambda^2\Delta^2}{G_R^2\Lambda^2\Delta N_0 + R_1^2\Lambda N_0}$$

$$\boxed{\begin{aligned} G_R &= R_1\sqrt{\frac{\epsilon}{\epsilon_S\cdot\Delta^2 + \Delta N_0}} \\ &= R_1\cdot G \end{aligned}} \Rightarrow \gamma_R = \frac{\epsilon\cancel{R_1^2}G^2\Lambda^{\cancel{2}}\Delta^2}{\cancel{R_1^2}G^2\Lambda^{\cancel{2}}\Delta N_0 + \cancel{R_1^2}\cancel{\Lambda}N_0}$$

$$\boxed{\gamma_R = \frac{\epsilon G^2\Lambda\Delta^2}{G^2\Lambda\Delta N_0 + N_0} = \frac{\epsilon}{N_0}\cdot\frac{G^2\Lambda\Delta^2}{G^2\Lambda\Delta\cancel{N_0} + 1}}$$

MMV

BEST

$4\times4\times4$

4×2

$4\times4\,2$

$\boxed{\begin{matrix}4\times4\times1\\4\times2\times2\end{matrix}}$

WORST $\downarrow$ $4\times2\times1$

$$\boxed{\begin{matrix}16 & 4\\ 8 & 4\end{matrix}}$$

- OSTBC SO 3 ANTENI

$$C_{348}=\begin{bmatrix} x_1 & x_2 & x_3\\ -x_2 & x_1 & -x_4\\ -x_3 & x_4 & x_1\\ -x_4 & -x_3 & x_2\\ x_1^* & x_2^* & x_3^*\\ -x_2^* & x_1^* & -x_4^*\\ -x_3^* & x_4^* & x_1^*\\ -x_4^* & -x_3^* & x_2^* \end{bmatrix}$$

$$\Omega=\left[\begin{array}{cccc|cccc} h_1 & h_2 & h_3 & h_4 & h_1^* & +h_2^* & h_3^* & h_4^*\\ h_2 & -h_1 & -h_4 & h_3 & h_2^* & -h_1^* & -h_4^* & h_3^*\\ h_3 & h_4 & -h_1 & -h_2 & h_3^* & h_4^* & -h_1^* & -h_2^* \end{array}\right]$$

$$\boxed{Y_a = X\circ\Omega + N_a}$$

$$\boxed{\tilde{X} = Y_a\cdot\Omega^H}$$

$$Y=\left(C\cdot H^T+N^T\right)^T$$

$$Y_a=[Y_1, Y_2, Y_3, Y_4, Y_5^*, Y_6^*, Y_7^*, Y_8^*]$$

$$X\cdot\Omega = Y_a - N_a \qquad \Omega=(Y_a-N_a)/X$$

$$\Omega = X^{-1}\cdot(Y_a-N_a)$$

$$\begin{bmatrix} h_1^* & h_2^* & h_3^* & 0\\ h_2^* & -h_1^* & 0 & -h_3^*\\ h_3^* & 0 & -h_1^* & h_2^*\\ 0 & h_3^* & -h_2^* & -h_1^*\\ h_1^* & h_2^* & h_3^* & [illegible]\\ h_2^* & -h_1^* & 0 & -h_3\\ h_3^* & -[illegible]^* & -h_1^* & h_2\\ 0 & h_3^* & -h_2^* & -h_1 \end{bmatrix}$$

$$=[Y_1, Y_2, Y_3, Y_4, Y_5^*, Y_6^*, Y_7^*, Y_8^*]\circ$$

$$\boxed{\tilde{X} = Y_a\cdot\Omega^H}$$

MMV OSTBC WITH 3 ANTENNAS

23 ÷ 27 // $\boxed{7.11.10 \div \;|\; 30.01.11}$

$$\begin{bmatrix} y_1 \\ y_2^* \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}^T \begin{bmatrix} h_{11} & h_{21}^* \\ h_{21} & -h_{11}^* \end{bmatrix} \cancel{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}} \Rightarrow$$

$$y_1 = x_1 h_{11} + x_2 h_{21}$$

$$y_2^* = x_1 h_{21}^* - x_2 h_{11}^*$$

$$y_2 = x_1^* h_{21} - x_2^* h_{11}$$

$$\gamma_\gamma = \frac{G_3^2 \Lambda_3 \Delta_\gamma^2}{G_\gamma^2 \Lambda_\gamma \Delta_3 + 1} = \frac{\frac{1}{\Delta_3^2} \cdot \Lambda_3 \cdot \Delta_\gamma^2}{\frac{\Lambda_3}{\Delta_\gamma} + 1} = \underline{\frac{\Lambda_3 \cdot \Delta_\gamma}{\Lambda_3 + \Delta_3}}$$

$$MGF(s) = \int_0^\infty p(x) \cdot e^{xs} ds = \underline{E[e^{+xs}]}$$

$$P_{out}(\gamma < \gamma_0) = \int_{-\infty}^{\gamma_0} p(\gamma) d\gamma \qquad \boxed{p(\gamma) = \frac{dP(\gamma)}{d\gamma}}$$

$$\mathcal{L}[p(x)] = \int_0^\infty p(x) \cdot e^{-xs} dx = \underline{M(-s)}$$

$$\boxed{\hat{p}(x) = M(-s)}$$

$$M(-s) = \int_0^\infty \frac{dP(\gamma)}{d\gamma} e^{-\gamma s} d\gamma = s \cdot P(s)$$

$$\mathcal{L}[f'(x)] = \mathcal{L}\left[\frac{df(x)}{dx}\right] = \underline{s \cdot F(s)}$$

$$\boxed{\frac{M(-s)}{s} = P(s)} \qquad \frac{M(-s)}{s} = \int_0^\infty P(\gamma) e^{-\gamma s} ds$$

$$\boxed{P(\gamma) = \mathcal{L}^{-1}\left[\frac{M(-s)}{s}\right]}$$

⊙ L. Yang PERFORMANCE ANALYSIS OF MIMO WITH OSTBC

C_1 – WITH DIMENSION N×T (3224100)

- Korston Hotel (Kosygina 15g) MyLoc2
- Sheremetjevo International - Terminal F MyLoc3

VOROBYOVY GORY = MyLOCATION 2

ULITSA VOZDVIZHENKA = MyLOCATION 3

PLOSTAT ARBATSKAYA = MyLOCATION 4 (ARBAT STREET)

SOBOREN HRAM HRISTA SPASITELA = MyLOCATION 5

TRETYAKOVSKAYA GALLERY = MyLOCATION 6

ROUTE WITH MyLOCATION 7 (TRETYAKOVSKAYA Ⓜ)

IZMAYLOVO MARKET

VICTORY PARK (MyLOCATION 1)

PUSHKIN FINE ARTS MUSEUM (MyLOCATION)

BOAT TOUR OF MOSCOW

BORODINO BATTLE KUTUZOVSKI PROSPECT 38

$$Y_{SR} = \sqrt{\frac{P}{N}} H_1 C_1 + W_1 \qquad Y_{SD} = \sqrt{\frac{P}{N}} H_0 C_1 + W_0$$

$$y_{R,l} = \sqrt{\frac{P}{N}} c\|H_1\|_F^2 s_l + \tilde{\omega}_{R,l} \qquad l = 1, 2, \ldots L$$

$$|E[y_{R,l}]|^2 = \frac{P}{N} c^2 \|H_1\|_F^4 + c\|H_1\|_F^2 N_0 = c\|H_1\|_F^2\left(\frac{P}{N} c\|H_1\|_F^2 + N_0\right)$$

$$r'_{R,l} = \frac{r_{R,l}}{\sqrt{E|r_{R,l}|^2}} = \frac{r_{R,l}}{\sqrt{c\|H_1\|_F^2\left(\frac{P}{N}c\|H_1\|_F^2+N_0\right)}} = \frac{r_{R,l}}{\sqrt{c}\,\|H_1\|_F\left(\frac{P}{N}c\|H_1\|_F^2+N_0\right)^{1/2}}$$

$$r'_{R,l} = \sqrt{\frac{P}{N}}\,c\|H_1\|_F^2\, s_l + \tilde{w}_{R,l}\,, \quad l=1,2,\dots L$$

$$r'_{R,l} = \frac{\sqrt{\frac{P}{N}}\sqrt{c}\,\|H_1\|_F}{\sqrt{\frac{P}{N}c\|H_1\|^2+N_0}}\,s_l + \frac{\tilde{w}_{R,l}}{\sqrt{c}\,\|H_1\|_F\sqrt{\frac{P}{N}c\|H_1\|_F^2+N_0}}$$

$$\boxed{Y_{RD} = \sqrt{\frac{P}{N}}\,H_2 C_2 + W_2}$$

$$r_{RD,l} = \sqrt{\frac{P}{N}}\,c\,\|H_2\|_F^2\, r'_{R,l} + \tilde{w}_{RD,l} =$$

$$= \frac{\frac{P}{N}\,c^{\frac{3}{2}}\|H_1\|_F\|H_2\|_F^2}{\sqrt{\frac{P}{N}c\|H_1\|^2+N_0}}\,s_l + \frac{\sqrt{\frac{P}{N}}\,c\,\|H_2\|_F^2\,\tilde{w}_{R,l}}{\sqrt{c}\,\|H_1\|_F\sqrt{\frac{P}{N}c\|H_1\|_F^2+N_0}} + \tilde{w}_{RD,l}$$

$$r_{SD,l} = \sqrt{\frac{P}{N}}\,c\,\|H_0\|_F^2\, s_l + \tilde{w}_{SD,l}\,, \quad l=1,2\dots L$$

- PDF OF THE OUTPUT SNR

$$\gamma_{OAF} = \bar{\gamma}X + \frac{\bar{\gamma}^2 YZ}{\bar{\gamma}Y+\bar{\gamma}Z+1} \doteq \bar{\gamma}X + \frac{\bar{\gamma}YZ}{Y+Z} = \gamma_0+\gamma_1$$

$$X = \|H_0\|_F^2 \qquad Y = \|H_1\|_F^2 \qquad Z = \|H_2\|_F^2 \qquad \bar{\gamma} = \frac{P}{NN_0}$$

10-2058/1 13.09.2010

BROJČE OD DAVANJETO ~~ZA~~ UVIDA OD GLADČIAR DOBIOCA.

$$\gamma_1 = \frac{\bar{\gamma}YZ}{Y+Z} \qquad \frac{\bar{\gamma}}{\gamma_1} = \frac{Y+Z}{YZ} = \frac{1}{Z}+\frac{1}{Y}$$

$$\boxed{\frac{1}{\gamma_1} = \frac{1}{\bar{\gamma}Z} + \frac{1}{\bar{\gamma}Y}} \Rightarrow$$

OVA E ISTO SO (42) OD WCNCM OCANAOT.

$$\boxed{f_{\gamma_1}(\gamma) = \frac{2\gamma^{2N^2-1} e^{-\frac{2\gamma}{\bar{\gamma}}}}{[\Gamma(N^2)]^2 \bar{\gamma}^{2N^2}} \sum_{i=0}^{2N^2} \binom{2N^2}{i} K_{N^2-i}\left(\frac{2\gamma}{\bar{\gamma}}\right)}$$

$\gamma_1^{MP} = \bar{\gamma} \min(Y, Z) \Rightarrow$ PDF OF γ_1^{MP}

$$\boxed{f_{\gamma_1}^{MP}(\gamma) = 2\frac{\Gamma(N^2, \frac{\gamma}{\bar{\gamma}})}{[\Gamma(N^2)]^2 \bar{\gamma}^{N^2}} \gamma^{N^2-1} e^{-\frac{\gamma}{\bar{\gamma}}}}$$

Es/No

SIMON & ALOUINI (8.23)

- PERFORMANCE AND DIVERSITY GAIN ANALYSIS

$$\boxed{P_s(\epsilon|\{\gamma_0,\gamma_1\}) = a\int_0^{b\pi} \exp\left(-\frac{g}{\sin^2\theta}(\gamma_0+\gamma_1)\right)d\theta}$$

CONDITIONAL SER FOR COHERENT MODULAT. SIGNALS

MNOGO VAZAN GENERALAN IZRAZ!!! ...

BPSK

$$P_e = \int_0^\infty p(e|\gamma)\, p(\gamma)\, d\gamma = \int_0^\infty Q(a\sqrt{\gamma})\, p(\gamma)\, d\gamma$$

BPSK $\boxed{p(e|\gamma) = Q(\sqrt{2\gamma})}$

$$\boxed{P_e = \int_0^\infty Q(\sqrt{2\gamma})\, p(\gamma)\, d\gamma = \frac{1}{\pi}\int_0^{\pi/2} M_\gamma\left(\frac{-1}{\sin^2\theta}\right)d\theta}$$

FOR MPSK $\boxed{a = \frac{1}{\pi} \quad b = \frac{M-1}{M} \quad g = \sin^2\left(\frac{\pi}{M}\right)}$ MMV

FOR BPSK $M = 2$ $\boxed{b = \frac{1}{2} \quad g = 1}$

$$P_s = \frac{1}{\pi}\int_0^{\pi/2} e^{-\frac{\gamma}{\sin^2\theta}}\, d\theta = \left| \begin{matrix} \gamma = \frac{x^2}{2} \\ x = \sqrt{2\gamma} \end{matrix} \right| = \frac{1}{\pi}\int_0^{\pi/2} e^{-\frac{x^2}{2\sin^2\theta}}\, d\theta$$

$$= Q(x) = \boxed{Q(\sqrt{2\gamma})}$$

UNIVERZALAN IZRAZ →

SIMON & ALOUINI (9.15)

$$\boxed{P_S = a\int_0^{b\pi} MGF\left(\frac{-g}{\sin^2\theta}\right)d\theta}$$

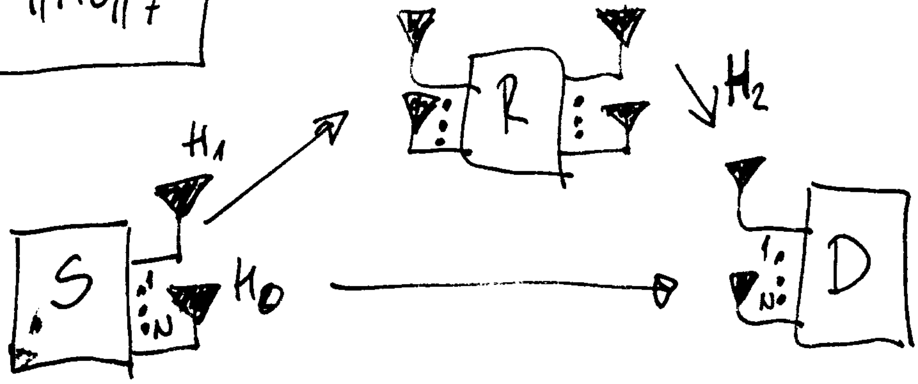
MMV

$$P_s^{CAF}(\epsilon) = E_{\gamma_0,\gamma_1}\left\{P_s(\epsilon|\gamma_0,\gamma_1)\right\} = a\int_0^{6\pi} I_0(\bar{\gamma},\theta)\, I_1(\gamma,\theta)\, d\theta$$

- X IS CHI-SQUARE RANDOM VARIABLE WITH 2N²

$$I_0(\gamma,\theta) = \int_0^{\infty} e^{-A\gamma} f_{\gamma_0}(\gamma)\, d\gamma$$

$$\boxed{X = \|H_0\|_F^2}$$



PO MOJATA FORMULA (35) (ICUMT_stosic_hadzivelkov V.4.pdf)

$$\boxed{f_{\gamma_0}(\gamma) = \frac{\gamma^{N^2-1}}{\bar{\gamma}^{N^2}\,\Gamma(N^2)}\, e^{-\frac{\gamma}{\bar{\gamma}}}}$$

$$\boxed{\text{GRADSHTEYN } 3{:}351.3}$$

$$M_{\gamma_0}(-s) = \int_0^{\infty} f_{\gamma_0}(\gamma)\, e^{-s\gamma}\, d\gamma = \underline{(1+\bar{\gamma}s)^{-N^2}} = \frac{1}{(1+\bar{\gamma}s)^{N^2}}$$

$$\boxed{M_{\gamma_0}(-s) = \frac{1}{\bar{\gamma}^{N^2}}\,\frac{1}{\left(\frac{1}{\bar{\gamma}} + s\right)^{N^2}} = \frac{1}{\bar{\gamma}^{N^2}}\left(s + \frac{1}{\bar{\gamma}}\right)^{-N^2}}$$

$$f_{\gamma_1}(\gamma) = \frac{2\gamma^{2N^2-1} e^{-2\frac{\gamma}{\bar{\gamma}}}}{[\Gamma(N^2)]^2\,\bar{\gamma}^{2N^2}} \sum_{i=0}^{2N^2} \binom{2N^2}{i} K_{N^2-i}\left(\frac{2\gamma}{\bar{\gamma}}\right)$$

$$\boxed{M_{\gamma_1}(-s) = \int_0^{\infty} e^{-s\gamma} f_{\gamma_1}(\gamma)\, d\gamma = \frac{2\sqrt{\pi}}{[\Gamma(N^2)]^2\,\bar{\gamma}^{2N^2}} \cdot \sum_{i=0}^{2N^2}\binom{2N^2}{i}\frac{\Gamma(3N^2-i)\,\Gamma(N^2+i)}{4^{i-N^2}\,\bar{\gamma}^{N^2-i}\,\Gamma(2N^2+0.5)}\left(s+\frac{4}{\bar{\gamma}}\right)^{i-3N^2} \cdot {}_2F_1\left(3N^2-i, N^2-i+0.5; 2N^2+0.5; \frac{s\bar{\gamma}}{s\bar{\gamma}+4}\right)}$$

MMV

- GRADSHTEYN <u>6.621.3</u> $\boxed{2N^2=\delta}$

$$\boxed{\int_0^\infty x^{\mu-1}e^{-\alpha x}K_\nu(\beta x)\,dx=\frac{\sqrt{\pi}\,(2\beta)^\nu}{(\alpha+\beta)^{\mu+\nu}}\frac{\Gamma(\mu+\nu)\Gamma(\mu-\nu)}{\Gamma(\mu+1/2)}F\left(\mu+\nu,\nu+\frac{1}{2};\mu+\frac{1}{2};\frac{\alpha-\beta}{\alpha+\beta}\right)\quad \mathrm{Re}\,\mu>\mathrm{Re}\,\nu\quad \mathrm{Re}(\alpha+\beta)>0}$$

$\boxed{\text{BPSK}}$

$$P_e=\frac{1}{\pi}\int_0^{\pi/2}M_\gamma\left(-\frac{1}{\sin^2\theta}\right)d\theta$$

$\boxed{i=0}$

$$f_{\gamma_1}(\gamma)\Big|_{i=0}=\frac{2\gamma^{2N^2-1}e^{-2\frac{\gamma}{\bar\gamma}}}{\Gamma^2(N^2)\,\bar\gamma^{2N^2}}K_{N^2}\left(\frac{2\gamma}{\bar\gamma}\right)$$

$$2\int_0^\infty e^{-\overbrace{\left(s+\frac{2}{\bar\gamma}\right)}^{a}\gamma}\cdot\frac{\cancel{\gamma}^{\overset{\mu}{(2N^2)}-1}K_{N^2}\left(\overset{\beta}{\frac{2\gamma}{\bar\gamma}}\right)}{\Gamma^2(N^2)\,\bar\gamma^{2N^2}}d\gamma=\frac{2}{\Gamma^2(N^2)\,\bar\gamma^{2N^2}}\int_0^\infty\gamma^{\overset{\mu}{(2N^2)}-1}e^{-\overbrace{\left(s+\frac{2}{\bar\gamma}\right)}^{\alpha}\gamma}\underset{\nu}{K_{(N^2)}}\left(\overset{\beta}{\frac{2}{\bar\gamma}}\gamma\right)d\gamma$$

$$=\frac{2\sqrt{\pi}\left(\frac{4}{\bar\gamma}\right)^{N^2}}{\left(s+\frac{2}{\bar\gamma}+\frac{2}{\bar\gamma}\right)^{3N^2}}\frac{\Gamma(3N^2)\Gamma(N^2)}{\Gamma(2N^2+0.5)}F\left(3N^2,N^2+0.5;2N^2+\frac{1}{2};\frac{s+\cancel{\frac{2}{\bar\gamma}}-\cancel{\frac{2}{\bar\gamma}}}{s+\frac{4}{\bar\gamma}}\right)$$

$$=\frac{2\sqrt{\pi}\,4^{N^2}}{\bar\gamma^{N^2}\left(s+\frac{4}{\bar\gamma}\right)^{3N^2}}\frac{\Gamma(3N^2)\Gamma(N^2)}{\Gamma(2N^2+0.5)}F\left(3N^2,N^2+0.5;2N^2+0.5;\frac{s}{s+\frac{4}{\bar\gamma}}\right)$$

IMAŠE GREŠKA VO FORMULATA VO ČLANAKOT PA JA PROVERAM

$$f^{m1}_{\gamma_1}(\gamma)=2\frac{\Gamma(N^2,\gamma/\bar\gamma)}{\left[\Gamma(N^2)\right]^2\bar\gamma^{N^2}}\gamma^{N^2-1}e^{-\gamma/\bar\gamma}$$

$$\Gamma(\alpha,x)=\int_x^\infty e^{-t}t^{\alpha-1}dt$$

$$\bar\gamma\gg1\Rightarrow\Gamma(N^2,\gamma/\bar\gamma)\to\Gamma(N^2,0)$$

$$f^{m1}_{\gamma_1}(\gamma)\doteq\frac{2}{\Gamma(N^2)\,\bar\gamma^{N^2}}\gamma^{N^2-1}e^{-\gamma/\bar\gamma}$$

$$M_{\gamma_1}(-s)=\int_0^\infty e^{-s\gamma}\frac{2}{\Gamma(N^2)\,\bar\gamma^{(N^2)}}\gamma^{(N^2)-1}e^{-\frac{\gamma}{\bar\gamma}}d\gamma=2\cdot\frac{1}{\left(1+s\bar\gamma\right)^{N^2}}$$

$$M_{\gamma_0} = \frac{1}{(1+\bar{\gamma}s)^{N^2}} \qquad M_{\gamma_1} = \frac{2}{(1+\bar{\gamma}s)^{N^2}} \qquad s = \frac{g}{\sin^2\theta}$$

$$\underline{g = \sin^2\left(\frac{\pi}{M}\right)}$$

$$P_s^{CAF}(\epsilon) = a\int_0^{b\pi} I_0 I_1 \, d\theta = 2a\int_0^{b\pi}(1+\bar{\gamma}s)^{-2N^2} d\theta$$

$$P_s^{CAF}(\epsilon) = \frac{2a}{\bar{\gamma}^{2N^2}}\int_0^{b\pi}\frac{1}{(\bar{\gamma}+1)^{2N^2}}d\theta = \frac{2a}{\bar{\gamma}^{2N^2}}\int_0^{b\pi}\left(\bar{\gamma}+\frac{g}{\sin^2\theta}\right)^{-2N^2}d\theta$$

- MAXIMUM AT $\theta = b\pi$

$$P_s^{CAF}(\epsilon) \doteq \frac{2a\,b\pi}{\bar{\gamma}^{2N^2}}\left(\bar{\gamma}+\frac{g}{\sin^2 b\pi}\right)^{-2N^2}$$

BPSK $a = \frac{1}{\pi}$; $b = \frac{1}{2}$ $g = 1$ (VIDI N2.PPT)

$$P_s^{CAF}(\epsilon) = \frac{1}{\bar{\gamma}^{2N^2}}\left(\bar{\gamma}+1\right)^{-2N^2} = \frac{1}{\bar{\gamma}^{2N^2}(\bar{\gamma}+1)^{2N^2}} = \frac{1}{(\bar{\gamma}^2+\bar{\gamma})^{2N^2}}$$

$\boxed{078466963}$

MGF OF ANY K×L MIMO S.

- BER 2×2

$$f_{\gamma_0}(\gamma) = \frac{\gamma^{N^2-1}}{\bar{\gamma}^{N^2}\Gamma(N^2)}e^{-\frac{\gamma}{\bar{\gamma}}}$$

$$\boxed{M_{\gamma_0}(-s) = \frac{1}{(1+\bar{\gamma}s)^{N^2}}}$$ MMV

$$P_e = \frac{1}{\pi}\int_0^{\pi/2} M_\gamma\left(-\frac{1}{\sin^2\theta}\right)d\theta = \frac{1}{\pi}\int_0^{\pi/2}\frac{d\theta}{\bar{\gamma}^{N^2}\left(\frac{1}{\bar{\gamma}}+\frac{1}{\sin^2\theta}\right)^{N^2}}$$

$$P_e = \frac{1}{\pi}\int_0^{\pi/2}\frac{\cancel{\bar{\gamma}^{N^2}}\cdot\sin^{2N^2}\theta\,d\theta}{\cancel{\bar{\gamma}^{N^2}}(\sin^2\theta+\bar{\gamma})^{N^2}} = \frac{1}{\pi}\int_0^{\pi/2}\frac{\sin^{2d}\theta\,d\theta}{(\sin^2\theta+\bar{\gamma})^d}$$

$$\boxed{P_e = \frac{1}{\pi}\int_0^{\pi/2}\frac{\sin^{2d}\theta}{(\sin^2\theta+\bar{\gamma})^d}d\theta} \Rightarrow$$

~~OUTAGE~~ PROBABILITY OF BIT
ERROR FOR K×L WITH
BPSK ~ RAYLEIGH
FADING

[redacted]

$$\boxed{P_e = \frac{\Gamma(d)\sqrt{\pi} - 2\sqrt{\gamma}\,\Gamma(d+0.5)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}+d;\frac{3}{2};-\bar{\gamma}\right)}{2\sqrt{\pi}\,\Gamma(d)}}$$

$$\boxed{P_e = \frac{1}{2} - \frac{\sqrt{\bar{\gamma}}\;\Gamma(d+0.5)}{\sqrt{\pi}\;\Gamma(d)}\;{}_2F_1\left(\frac{1}{2}, \frac{1}{2}+d; \frac{3}{2}, -\bar{\gamma}\right)}$$

BER PROBABILITY FOR POINT TO POINT SYSTEM WITH DIVERSITY "d"

$$\boxed{\begin{aligned} {}_2F_1(a,b;c;z) &= \sum_{n=0}^{\infty} \frac{(a)_n (b)_n}{(c)_n}\frac{z^n}{n!} = \frac{\Gamma(c)}{\Gamma(a)\Gamma(b)}\sum_{n=0}^{\infty}\frac{\Gamma(a+n)\Gamma(b+n)}{\Gamma(c+n)}\frac{z^n}{n!} \\ (a)_n &= a\cdot(a+1)(a+2)\cdots\cdots(a+n-1) \end{aligned}}$$

GAUSS HYPERGEOMETRIC FUNCTION

▣ REAL ORTHOGONAL DESIGN (4×1) (JAFARKHANI BOOK)

$$C = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ -x_2 & x_1 & -x_4 & x_3 \\ -x_3 & x_4 & x_1 & -x_2 \\ -x_4 & -x_3 & x_2 & x_1 \end{bmatrix} \qquad \Omega_t = \begin{bmatrix} h_1 & h_2 & h_3 & h_4 \\ h_2 & -h_1 & h_4 & -h_3 \\ h_3 & -h_4 & -h_1 & h_2 \\ h_4 & h_3 & -h_2 & -h_1 \end{bmatrix}$$

$$H = [h_1, h_2, h_3, h_4] \qquad Y = (C\cdot H^T + N^T)^*$$

$$\tilde{X} = Y^T\cdot\Omega_t$$

$$Y_1 = x_1h_1 + x_2h_2 + x_3h_3 + x_4h_4 + n_1$$

$$Y_2 = -x_2h_1 + x_1h_2 - x_4h_3 + x_3h_4 + n_2$$

$$\cdots$$

$$\underbrace{[Y_1, Y_2, Y_3, Y_4]}_{Y^+}\;\underbrace{\begin{bmatrix} h_1 & h_2 & h_3 & h_4 \\ h_2 & -h_1 & h_4 & -h_3 \\ h_3 & -h_4 & -h_1 & h_2 \\ h_4 & h_3 & -h_2 & -h_1 \end{bmatrix}}_{\Omega_t}$$

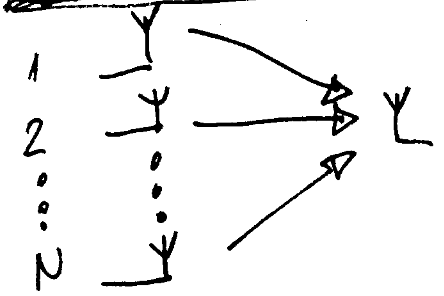
$$\tilde{x}_1 = Y_1h_1 + Y_2h_2 + Y_3h_3 + Y_4h_4 = (h_1^2+h_2^2+h_3^2+h_4^2)x_1 + \xi_1$$

$$\tilde{x}_2 = Y_1h_2 - Y_2h_1 \quad Y_3h_4 + Y_4h_3 = (h_1^2+h_2^2+h_3^2+h_4^2)x_2 + \xi_2$$

$$\cdots$$

- ZOSTO kaj Alamouti KxLxM_VarG_Lang.m DAVA PODOBRI SIMULACIONI REZULTATI OD TEORIJA VO SLUCAJ 2x2x2. Reseno!
- NE MI RABOTI SIMULACIJATA SO REALNI OSTBC !!? (Reseno! Vidi N9.28)

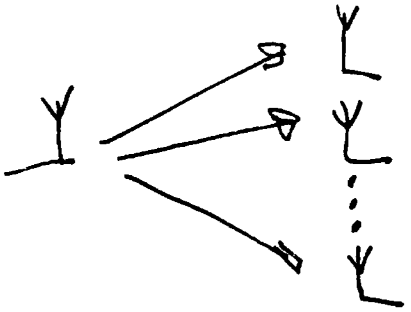
H. Jafarkhani : 4.9 PERFORMANCE ANALYSIS



$\boxed{\text{MISO}}$

$\alpha = [\alpha_1, \alpha_2, \ldots, \alpha_N]$

$\boxed{\text{CASE 1}}$



SIMO

$\boxed{\text{CASE 2}}$

- DECODER MINIMIZES $|\tilde{s}_k - s_k|^2$ for decoding "s_k"

$\boxed{\text{CASE 1}}$ $\boxed{\text{MISO}}$

$$\left.\tilde{s}_1 = r_1\alpha_1^* + r_2^*\alpha_2 = \sum_{n=1}^{2}|\alpha_n|^2 s_1 + N_1\right\} N=2$$

- N_1 – GAUSIAN RANDOM VARIABLE WITH ZERO MEAN AND $\frac{N_0}{2}\cdot\sum_{n=1}^{2}|\alpha_n|^2$ PER REAL DIMENSION
- POWER OF THE SIGNAL AT THE RECEIVER IS:

$$E_s\left[\sum_{n=1}^{2}|\alpha_n|^2\right]^2$$

- SNR

$$\left.\gamma_{\text{OSTBC}} = \frac{E_s\left[\sum_{n=1}^{2}|\alpha_n|^2\right]^2}{2\cdot\frac{N_0}{2}\sum_{n=1}^{2}|\alpha_n|^2} = \frac{E_s}{N_0}\sum_{n=1}^{2}|\alpha_n|^2\right\}$$ RECEIVE SNR OF THE FIRST SYMBOL

- SECOND CASE $\boxed{\text{MRC}}$

$\boxed{\text{POVTOPLVANJE ZA MRC}}$

$$(*)\ \boxed{\hat{x} = x + \overset{\boxed{\text{MMV}}}{\frac{h^H n}{h^H h}} = \frac{h^H\cdot y}{h^H\cdot h}}$$

$\boxed{y = hx + n}$

$$\hat{x}_e = \underset{\text{?– NE ŠE DOBRO!!}}{\textcircled{N}}\underbrace{x}_{\text{TRANSMITED SYMBOL}} + \sum_{i=1}^{N}\frac{h_i^H n_i}{|h_i|^2}$$

$$P_s = N^2 E_s$$ [illegible]

$$P_N = N_0\left[\sum_{i=1}^{N}\frac{h_i^H}{|h_i|^2}\right]^2$$

$$\gamma = \frac{E_s}{N_0}\frac{N^2}{\sum_{i=1}^{N}\frac{h_i^H}{|h_i|^2}}$$

$$\boxed{\gamma = \sum_{i=1}^{N}\frac{|h_i|^2 E_s}{N_0}}$$

VIDI N7.ppt
$y = [y_1, y_2, \cdots y_N]^T$
$h = [h_1, h_2, \cdots h_N]^T$
$n = [n_1, n_2, \cdots n_N]$

VIDI pp. 13 →

$$\boxed{h^H \cdot h = \sum_{i=1}^{N} |h_i|^2}$$

$$\hat{x} = x + \frac{h^H n}{\sum_{i=1}^{N} |h_i|^2}$$

TRANSMITED SYMBOL

RECEIVED SYMBOL

$$P_S = E_S \qquad P_N = \frac{(h^H)^2 N_0}{\left[\sum_{i=1}^{N} |h_i|^2\right]^2}$$

$$\gamma = \frac{E_S}{N_0} \frac{\left[\sum_{i=1}^{N} |h_i|^2\right]^2}{(h^H)^2}$$

$$n = \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

$$h = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} \qquad h^H \cdot n = h_1^* \cdot n_1 + h_2^* n_2$$

$$\hat{x} = x + \frac{h_1^* n_1 + h_2^* n_2}{\sum_{i=1}^{2} |h_i|^2}$$

MMV

$$\boxed{P_S = E_S \qquad P_N = \frac{|h_1|^2}{\left[|h_1|^2+|h_2|^2\right]^2} \cdot N_0 + \frac{|h_2|^2}{\left[|h_1|^2+|h_2|^2\right]^2} \cdot N_0}$$

$$P_N = \frac{\cancel{|h_1|^2}+|h_2|^2}{\left[|h_1|^2+|h_2|^2\right]^2} \cdot N_0 = \frac{N_0}{|h_1|^2+|h_2|^2}$$

$$\boxed{\gamma = \frac{E_S}{N_0} \cdot \left(|h_1|^2+|h_2|^2\right)} \rightarrow \text{FOR 2 ANTENNAS}$$

- FOR N ANTENNAS:

$$\boxed{\gamma = \frac{E_S}{N_0} \sum_{i=1}^{N} |h_i|^2}$$

MMV

- SECOND CASE CONTINUATION

$$\overline{J} = \sum_{m=1}^{2} |\alpha_m|^2 s + N'$$

ORA TI DICO ANCO IO ⊛ SE COI DEL NORMALIZZAZIONE

$$\boxed{\begin{aligned} &\hat{x} = h^H \cdot y_i \qquad y = hx + n\ i \\ &\hat{x} = \sum_{i=1}^{N} |h_i|^2 \cdot x + h^H \cdot n \end{aligned}}$$

N' - ZERO MEAN COMPLEX GAUSSIAN RANDOM VARIABLE WITH THIS VARIANCE PER REAL DIMENSION

$$\boxed{\frac{N_0}{2}\sum_{m=1}^{2}|\alpha_m|^2}$$

$$P_N = N_0 \sum_{m=1}^{2}|\alpha_m|^2 \qquad P_s = E_s\left[\sum_{m=1}^{2}|\alpha_m|^2\right]^2$$

$$\boxed{\gamma_{MRC} = \frac{P_s}{P_N} = \frac{E_s}{N_0}\cdot\sum_{m=1}^{2}|\alpha_m|^2}$$

$$\Rightarrow \boxed{\gamma_{OSTBC} \triangleq \gamma_{MRC}} \Rightarrow$$

OVA VAZI AKO [illegible]
SMAGA NA „S" VO
„CASE 1" e „N PATI POGOLE-
„MA OD VKUPNATA SMAGA
NA „S" VO VTORIOT SLUCAJ.

RATKA GONEVA

MARIJA PEOLOVA

$$\boxed{d^2 = (0-0)^2 + (0-1)^2 + (0-1)^2 \qquad d = \sqrt{2}}$$

EUCLIDEAN DISTANCE
x=[0,0,0;0,1,1]

HAMMING DISTANCE:
pdist(x, 'hamming') CALCULATES THE PERCENTAGE OF COORDINATES THAT DIFFER.

☐ RANK AND DETERMINANT CRITERIA (JAFARKHANI 3.2)

IN CASE OF OSTBC A CODEWORD IS TxN MATRIX

$$C^1 = \begin{bmatrix} C^1_{1,1} & C^1_{1,2} & \cdots & C^1_{1,N} \\ C^1_{2,1} & C^1_{2,2} & & C^1_{2,N} \\ \vdots & \vdots & & \vdots \\ C^1_{T,1} & C^1_{T,2} & & C^1_{T,N} \end{bmatrix}$$

TANAS OKRUA
070352444
Sef NA INSIGNIJA

- ERROR IF DECODER MISTAKENLY DECIDES THAT C^2 IS TRANSM.

$$C^2 = \begin{bmatrix} C^2_{1,1} & C^2_{1,2} & \cdots & C^2_{1,N} \\ C^2_{2,1} & C^2_{2,2} & \cdots & C^2_{2,N} \\ \vdots & & & \vdots \\ C^2_{T,1} & C^2_{T,2} & \cdots & C^2_{T,N} \end{bmatrix}$$

$P(C^1 \to C^2)$ – PAIRWISE PROBABILITY OF TRANSMITTING C^1 AND DETECTING C^2.

$$\boxed{P(\text{error} \mid C^1 \text{ is sent}) \le \sum_{i=2}^{I} P(C^1 \to C^i)}$$

CODEBOOK CONTAINS I CODEWORDS

11.11.2010 28-1-631 → PRIJAVATA DO INSPEKCIJA !!!

$r = C \cdot H + N$

$$\mathbf{r} = \begin{bmatrix} r_{1,1} & r_{12} & \cdots & r_{1M} \\ r_{21} & r_{22} & \cdots & r_{2M} \\ \cdots & & & \cdots \\ r_{T1} & r_{T2} & \cdots & r_{TM} \end{bmatrix}$$

PRO DESIGN

$\boxed{3050}$

2000

$$\mathbf{H} = \begin{bmatrix} \alpha_{1,1} & \alpha_{12} & \cdots & \alpha_{1M} \\ \alpha_{21} & \alpha_{22} & \cdots & \alpha_{2M} \\ \cdots & & & \\ \alpha_{N1} & \alpha_{N2} & & \alpha_{NM} \end{bmatrix}$$

$$\boxed{r = C \cdot H + N}$$

$$\mathbf{C} = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1N} \\ c_{21} & c_{22} & \cdots & c_{2N} \\ \cdots & & & \\ c_{T1} & c_{T2} & \cdots & c_{TN} \end{bmatrix}$$

- AVERAGE SYMBOL TRANSMISSION POWER FROM EACH ANTENNA IS:

$$\boxed{E_s = 1/N}$$

VARIANCE OF NOISE SAMPLE:

$$\boxed{E(|n_{t,m}|^2) = N_0 = \frac{1}{\gamma}}$$

MMV

DISTRIBUTION OF THE RECEIVED SIGNALS: $f(r \mid C, H)$

r – IS MULTIVARIATE MULTIDIMENSIONAL, GAUSSIAN RANDOM VARIABLE

$$f(r \mid c, H) = \frac{1}{(\pi N_0)^{\frac{M \times M}{2}}} \exp\left\{ - \frac{\operatorname{Tr}\left[(r - C \cdot H)^H (r - C \cdot H)\right]}{N_0} \right\} =$$

$$= \frac{\gamma^{\frac{M \times M}{2}}}{\pi} \exp\left\{ -\gamma \operatorname{Tr}\left[(r - CH)^H (r - CH)\right] \right\}$$

– FORBENEUS NORM OF A

$$\|A\|_F = \sqrt{\operatorname{Tr}(A^H \cdot A)} = \sqrt{\operatorname{Tr}(A \cdot A^H)}$$

$$\operatorname{tr}(A) = a_{11} + a_{22} + \cdots + a_{44} = \sum_{i=1}^{4} a_{ii}$$

TRACE OF MATRIX

$$f(r|C,H) = \left(\frac{\gamma}{\pi}\right)^{\frac{M\times M}{2}} \exp\left\{-\gamma\|r-CH\|_F^2\right\} =$$

$$= \left(\frac{\gamma}{\pi}\right)^{\frac{M\times M}{2}} \exp\left\{-\gamma\sum_{t=1}^{T}\sum_{m=1}^{M}\left|(r-CH)_{t,m}\right|^2\right\}$$

- MAXIMUM-LIKELIHOOD (ML) DECODING DECIDES IN FAVOUR OF CODEWORD THAT MAXIMIZES $f(r|C,H)$

- IF C^1 IS TRANSMITTED THEN

$$r^1 = C^1\cdot H + N^1$$

PAIRWISE PROBABILITY IS CALCULATED

$$P(C^1\to C^2|H) = P\left(\|r^1-C^1H\|_F^2 - \|r^1-C^2\cdot H\|_F^2 > 0 \,\middle|\, H\right)$$

[redacted]

$$= P\left\{Tr\left[(r^1-C^1\cdot H)^H(r^1-C^1\cdot H) - (r^1-C^2\cdot H)^H(r^1-C^2\cdot H)\right] > 0|H\right\}$$

$$= P\left(Tr\left\{\left[(C^1-C^2)\cdot H + N^1\right]^H\left[\underbrace{(C^1-C^2)}_{\to\textcircled{A}}H + N^1\right] - N^{1H}N^1\right\} < 0|H\right) = \textcircled{*}$$

$$\textcircled{II} = (r^{1H} - H^H C^{1H})(r^1 - C^1H) - (r^{1H} - H^H C^{2H})(r^1 - C^2H) =$$

$$= \cancel{r^{1H}}\cdot r^1 - H^H\cdot C^{1H}r^1 \overset{-r^{1H}C^1H}{+} H^H C^{1H}\cdot C^1H - \cancel{r^{1H}}\cdot r^1 + H^H C^{2H}r^1 + r^{1H}C^2H \overset{H^HC^{2H}C^2H}{-}$$

$$= -\underline{H^HC^{1H}r^1} - \underline{r^{1H}C^1H} + H^HC^{1H}C^1H + \underline{H^HC^{2H}r^1} + \underline{r^{1H}C^2H} - H^HC^{2H}C^2H$$

$$= (H^HC^{2H} - H^HC^{1H})r^1 + r^{1H}(C^2H - C^1H) + \overbrace{H^HC^{1H}C^1H - H^HC^{2H}C^2H}^{\textcircled{*}}$$

$$= |r^1 = C^1H + N^1| = (H^HC^{2H} - H^HC^{1H})(C^1H + N^1) +$$

$$+ (H^HC^{1H} + N^{1H})(C^2H - C^1H) + \textcircled{*} = \overset{\textcircled{1}}{\underline{H^HC^{2H}C^1H}} - \cancel{\underline{H^HC^{1H}C^1H}} + \overset{\textcircled{2}}{\underline{H^HC^{2H}N^1}} -$$

$$- \overset{\textcircled{3}}{\underline{H^HC^{1H}N^1}} + \overset{\textcircled{4}}{\underline{H^HC^{1H}C^2H}} + \overset{\textcircled{5}}{\underline{N^{1H}C^2H}} - \overset{\textcircled{6}}{\underline{H^HC^{1H}C^1H}} + [redacted] \; \overset{\textcircled{7}}{\underline{N^{1H}C^1H}} -$$

[redacted]

$$+ \underline{\cancel{H^HC^{1H}}C^1H} - \overset{\textcircled{8}}{\underline{H^HC^{2H}C^2H}}$$

$$\square = H^HC^{2H}(C^1-C^2)H + H^H(C^2-C^1)^H N_1 + H^HC^{1H}(C^2-C^1)H + N^{1H}(C^2-C^1)H$$

$$= H^HC^{2H}(C^1-C^2)H + H^H(C^2-C^1)^H N_1 + (H^HC^{1H} + N^{1H})(C^2-C^1)H \; ?$$

① → $(C^1H - C^2H + N^1)^H (C^1H - C^2H + N^1) - N^{1H}N^1 =$

$= (H^H C^{1H} - H^H C^{2H} + N^{1H})(C^1H - C^2H + N^1) - N^{1H}N^1 =$

$= \overset{-⑥}{H^H C^{1H} C^1 H} - \overset{-①}{H^H C^{2H} C^1 H} + \overset{-⑦}{N^{1H} C^1 H} - \overset{-④}{H^H C^{1H} C^2 H} + \overset{-⑧}{H^H C^{2H} C^2 H} -$

$- \overset{-⑤}{N^{1H} C^2 H} + \overset{-③}{H^H C^{1H} N^1} - \overset{-②}{H^H C^{2H} N^1} + \cancel{N^{1H} N^1} - \cancel{N^{1H} N^1} =$

IMAJĆI GI VO PREDVID PROJKITE SE POKAZUVA DEKA :

$(\square) = -(\Delta)$ → SE GUBI ZARADI PROMENA NA ZNAKOT ZA NESAKANOTO!!

$✪ = P(\text{Tr}\{H^H (C^1-C^2)^H (C^1-C^2)\cdot H\} - X < 0 | H\}) =$

OČIGLEDNO SE DOBIVA SO [illegible] NA (Δ)

$= \Big| X = \text{Tr}\{N^{1H}\cdot(C^2-C^1)\cdot H + H^H (C^2-C^1)^H\cdot N\}$

↳ ZERO-MEAN GAUSSIAN VARIABLE WITH VARIANCE

$\sigma^2 = 2N_0 \|(C^2-C^1)\cdot H\|_F^2 = \frac{2}{\gamma}\cdot\|(C^2-C^1)\cdot H\|_F^2 \Big| =$

$= P(\|(C^1-C^2)\cdot H\|_F^2 < X | H) = P(X > \overbrace{\|(C^1-C^2)H\|_F^2}^{A} | H)$

- THEREFORE THE PAIRWISE ERROR PROBABILITY IS

$$P(C^1 \to C^2 | H) = Q\left(\frac{\|(C^2-C^1)\cdot H\|_F^2}{\sqrt{\frac{2}{\gamma}\|(C^2-C^1)\cdot H\|_F^2}}\right) = Q\left(\sqrt{\frac{\gamma}{2}\|(C^2-C^1)\cdot H\|_F^2}\right)$$

$Q(z) = \frac{1}{2}\text{erfc}\frac{z}{\sqrt{2}}$ $\quad P(C^1 \to C^2|H) = \frac{1}{2}\text{erfc}\left(\frac{\sqrt{\gamma}}{2}\|(C^2-C^1)\cdot H\|_F^2\right)$

BPSK (GENERALNO)

$P_e = Q(\sqrt{2\gamma})$ $\quad \gamma = \frac{E_b}{N_0} = \frac{A^2\cdot\cancel{T}}{2\sigma^2\cancel{T}}$

$P_b = Q\left(\sqrt{\frac{A^2}{\sigma^2}}\right) = Q\left(\frac{A}{\sqrt{\sigma^2}}\right)$ $\quad \boxed{Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-\frac{y^2}{2}}dy}$

$$P(X > A) = \int_A^\infty \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2}}dx = \left| \begin{array}{l} y = \frac{x}{\sigma} \\ dx = \sigma dy \\ x = A \\ y = \frac{A}{\sigma} \end{array} \right| = \frac{1}{\sigma\sqrt{2\pi}}\int_{A/\sigma} \sigma e^{-\frac{y^2}{2}}dy$$

$= Q\left(\frac{A}{\sigma}\right)$

$$\boxed{P(C^1 \to C^2 | H) = Q\left(\sqrt{\frac{\gamma}{2}} \left\| (C^2 - C^1) \cdot \mathbf{H} \right\|_F \right)}$$

[MMV]

$$P(C^1 \to C^2 | H) = Q\left(\sqrt{\frac{\gamma}{2} \mathrm{Tr}\left[H^H \cdot \underbrace{(C^2 - C^1)^H \cdot (C^2 - C^1)}_{\text{ORTHOGONAL DESIGN}} H\right]}\right)$$

BY USAGE OF DEFINITION OF ORTHOGONAL DESIGN (4.76)

$$P(C^1 \to C^2 | H) = Q\left(\sqrt{\frac{\gamma}{2} K \sum_{i=1}^{K} |s_k^2 - s_k^1|^2 \mathrm{Tr}\left[H^H \cdot H\right]}\right)$$

$$= Q\left(\sqrt{K \frac{\gamma}{2} \sum_{i=1}^{K} |s_k^2 - s_k^1|^2 \sum_{n=1}^{N} \sum_{m=1}^{M} |\alpha_{n,m}|^2}\right)$$

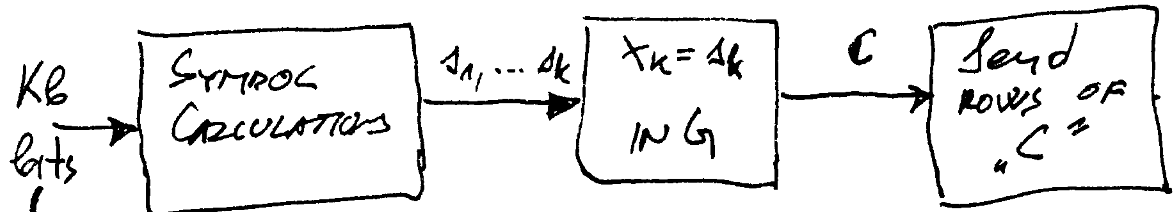
VIOL pp. 20 (KADE SE GLEDI 2-TO)

CONTINUE pp. 19

- GENERALIZED orthogonal DESIGN (SE NAVRAKAM NA OVAA GLAVA)

$$\boxed{G^H \cdot G = K\left(|x_1|^2 + |x_2|^2 + \dots + |x_K|^2\right) I_N = K \sum_{k=1}^{K} |x_k|^2 I_N}$$

$[G]_{T \times N}$ ⇒ GENERALIZED ORTHOGONAL DESIGN MATRIX WITH ENTRIES THAT ARE LINEAR COMBINATIONS OF INDETERMINATE VARIABLES $x_1, x_2, \dots, x_K$ AND THEIR CONJUGATES



U - UNITARY MATRIX i.e. $U^H \cdot U = I$

- $G' = U \cdot G$ IS ALSO GENERALIZED COMPLEX DESIGN (GCD)

$$G'^H \cdot G' = (U \cdot G)^H \cdot U \cdot G = G^H \cdot U^H \cdot U \cdot G = G^H \cdot G$$

- SIMILARLY $G' = G \cdot U$ IS GCD SINCE:

$$G'^H \cdot G' = U^H \cdot G^H \cdot G \cdot U = K \sum_{k=1}^{K} |x_k|^2 U^H \cdot I_N \cdot U = K \sum_{k=1}^{K} |x_k|^2 U^H U$$

$$\Rightarrow G'^H \cdot G' = K \sum_{k=1}^{K} |x_k|^2 I_N$$ ✓

$C_{t,n}$ $n = 1, 2, \dots, N$ ANTENNA
$t = 1, 2, \dots, T$ TIMESLOT

THEOREM 4.7.1 A COMPLEX STBC DESIGNED FROM A TxN GENERALIZED COMPLEX ORTHOGONAL DESIGN PROVIDES A DIVERSITY OF NM FOR M RECEIVE ANTENNAS AND SEPARATE ML DECODING OF ITS SYMBOLS

$$G(s_1, s_2, \dots, s_K) - G(s_1', s_2', \dots, s_K') \Rightarrow \text{full rank}$$

(NON-SINGULAR)

FOR ANY TWO DISTINCT SET OF INPUTS

$$(s_1, s_2, \dots, s_K) \neq (s_1', s_2', \dots, s_K')$$

$$G(s_1, s_2, \dots, s_K) - G(s_1', s_2', \dots, s_K') = G(s_1 - s_1', s_2 - s_2', \dots s_K - s_K')$$

$$⊛ = \det\left[G^H(s_1 - s_1', s_2 - s_2', \dots, s_K - s_K') \cdot G(s_1 - s_1', s_2 - s_2', \dots, s_K - s_K')\right] \neq$$

$$⊛ = K \sum_{k=1}^{K} |s_k - s_k'|^2$$

CONTINUE PP.18

$$d_E = \sqrt{\sum_{k=1}^{K} |s_k^2 - s_k'|^2}$$

EUCLIDIAN DISTANCE

- PEP IN TERMS OF EUCLIDIAN DISTANCE $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

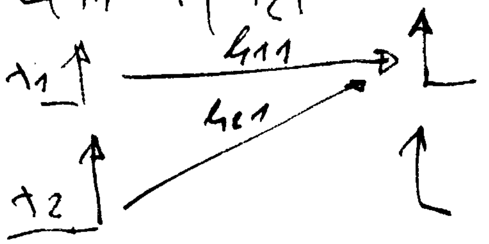
$$\boxed{P(C^1 \to C^2 | H) = Q\left(\sqrt{K \frac{\gamma}{2} d_E^2 \sum_{n=1}^{N} \sum_{m=1}^{M} |\alpha_{n,m}|^2} \right)}$$

- CRAIG FORMULATION OF Q FUNCTION

$$Q(x) = \frac{1}{\pi} \int_0^{\pi/2} \exp\left(\frac{-x^2}{2 \sin^2 \theta}\right) d\theta$$

$$\mathrm{Tr}\left\{ \begin{bmatrix} h_1^* & h_1^* \\ h_2^* & h_2^* \end{bmatrix} \begin{bmatrix} h_1 & h_2 \\ h_1 & h_2 \end{bmatrix} \right\} = \mathrm{Tr}\left\{ \begin{bmatrix} |h_1|^2 + |h_1|^2 & h_1^* h_2 + h_1^* h_2 \\ h_1 h_2^* + h_1 h_2^* & |h_2|^2 + |h_2|^2 \end{bmatrix} \right\} =$$

$$= 2|h_1|^2 + |h_2|^2$$



~~$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{11} \\ h_{21} \end{bmatrix} \begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix}$~~

$$y_1 = h_{11} x_1 +$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{21} \end{bmatrix} \begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix}$$

$$y_1 = h_{11} x_1 + h_{21} x_2$$

$$y_2 = -h_{11} x_2^* + h_{21} x_1^*$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{21} \end{bmatrix} \begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix} \qquad y_2 = -h_{11} x_2^* + h_{21} x_1^*$$

$$\Updownarrow$$

$$\begin{bmatrix} y_1 \\ y_2^* \end{bmatrix} = \begin{bmatrix} h_{11} & h_{21} \\ +h_{21}^* & -h_{11}^* \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad y_2^* = h_{21}^* x_1 - h_{11}^* x_2$$

$$H = \begin{bmatrix} h_{11} & h_{21} \\ h_{21}^* & -h_{11}^* \end{bmatrix} \qquad Tr\left[H^H \cdot H\right] = 2 \cdot \left(|h_{11}|^2 + \overset{\textcircled{\$}}{|h_{21}|^2}\right)$$

$$\begin{bmatrix} h_{11}^* & h_{21} \\ h_{21}^* & -h_{11} \end{bmatrix} \begin{bmatrix} h_{11} & h_{21} \\ h_{21}^* & -h_{11} \end{bmatrix} = \begin{bmatrix} |h_{11}|^2 + |h_{21}|^2 & a \\ b & |h_{21}|^2 + |h_{11}|^2 \end{bmatrix}$$

$$Q(x) = \frac{1}{\pi} \int_0^{\pi/2} \exp\left(\frac{-x^2}{2 \sin^2\theta}\right) d\theta$$

$$P(C^1 \to C^2/H) = Q\left(\sqrt{k \frac{\gamma}{2} d_e^2 \sum_{n=1}^{N} \sum_{m=1}^{M} |\alpha_{n,m}|^2}\right) \qquad \textcircled{\$A}$$

$$\underbrace{\begin{bmatrix} h_{11} & h_{21} \\ h_{12} & h_{22} \end{bmatrix}^H \cdot H}_{} = \begin{bmatrix} h_{11}^* & h_{12}^* \\ h_{21}^* & h_{22} \end{bmatrix} \circ \begin{bmatrix} h_{11} & h_{21} \\ h_{12} & h_{22} \end{bmatrix} = \begin{bmatrix} |h_{11}|^2 + |h_{12}|^2 & a \\ b & |h_{21}|^2 + |h_{22}|^2 \end{bmatrix}$$

$$Tr\left[H^H \cdot H\right] = |h_{11}|^2 + |h_{12}|^2 + |h_{21}|^2 + |h_{22}|^2$$

$$P(C^1 \to C^2/H) = \frac{1}{\pi} \int_0^{\pi/2} \exp\left(\frac{-k \cdot \gamma \cdot d_e^2 \sum_{n=1}^{N} \sum_{m=1}^{M} |\alpha_{n,m}|^2}{4 \sin^2\theta}\right) d\theta$$

$$= \frac{1}{\pi} \int_0^{\pi/2} \prod_{n=1}^{N} \prod_{m=1}^{M} \exp\left(\frac{-k\gamma d_e^2 |\alpha_{n,m}|^2}{4 \sin^2\theta}\right) d\theta \qquad ④$$

$$P(C^1 \to C^2) = \frac{1}{\pi} \int_0^{\pi/2} \left[\int_0^{\infty} \exp\left(\frac{-k\gamma d_E^2 x}{4 \sin^2\theta}\right) f_X(x)\, dx\right]^{MN} d\theta$$

$f_X(x) = e^{-x}, \; x > 0$ PDF OF $|\alpha_{n,m}|^2$

- MOMENT GENERATING FUNCTION OF RANDOM VARIAB.:

$$M(u) = E\left[e^{uX}\right]$$

- MGF OF EXPONENTIAL DISTRO $\boxed{u<1}$ IS:

$$M_x(u) = E[e^{ux}] = \int_0^\infty e^{ux} f_x(x)\,dx = \int_0^\infty e^{ux} e^{-x}\,dx$$

$$= \int_0^\infty e^{(u-1)x}\,dx = \left| \begin{matrix} y=(u-1)x \\ dy=(u-1)dx \\ x=0 \quad y=0 \end{matrix} \right| = \int_0^\infty e^{-y} \frac{dy}{-u+1}$$

$$= \frac{(-1)}{-u+1}\, e^{-y}\Big|_0^\infty = \frac{1}{u-1}\left(e^{-\infty} - e^{-0}\right) = \frac{-1}{u-1} = \frac{1}{1-u}$$

$$① \Rightarrow \int_0^\infty \exp\left(\frac{-k\gamma d_e^2 x}{4\sin^2\theta}\right) f_x(x)\,dx = \int_0^\infty \underline{\exp(+ux)\, e^{-x}}\,dx \quad \leftarrow u$$

$$= M(u) = \frac{1}{1-u} = \frac{1}{1+\frac{k\gamma d_e^2 x}{4\sin^2\theta}} = \frac{4\sin^2\theta}{4\sin^2\theta + k\gamma d_e^2}$$

$$= \frac{\sin^2\theta}{\sin^2\theta + k\frac{\gamma}{4}d_e^2}$$

$$\boxed{P(C^1 \to C^2) = \frac{1}{\pi}\int_0^{\pi/2} \left[\frac{\sin^2\theta}{\sin^2\theta + k\frac{\gamma}{4}d_e^2}\right]^{MN} d\theta}$$

$$\boxed{P(C^1 \to C^2) = \frac{1}{2}\left\{1 - \sqrt{\frac{a}{1+a}} \sum_{i=0}^{MN-1} \binom{2i}{i} \left[\frac{1}{4(1+a)}\right]^i \right\}}$$

CLOSED FORM

MMV

$\boxed{a = k\frac{\gamma}{4}d_e^2}$ K - DEFINED FROM THE STRUCTURE OF THE CODE.

EXAMPLE 4.9.1 ALAMOUTI CODE K=2 SYMBOLS IN T=2 TIMESLOTS FROM N=2 ANTENNAS

$E_s = \frac{1}{N} = \frac{1}{2}$ CONSTELLATION POINTS $\left\{\frac{-1}{\sqrt2}, \frac{1}{\sqrt2}\right\}$

$$C^1 = \left\{ \overset{x_1}{-\tfrac{1}{\sqrt2}},\ \overset{y_1}{\tfrac{1}{\sqrt2}} \right\}$$

$$C^2 = \left\{ \overset{x_2}{\tfrac{1}{\sqrt2}},\ \overset{y_2}{-\tfrac{1}{\sqrt2}} \right\}$$

TWO DIFFERENT SYMBOLS

$$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} = \sqrt{\left(\frac{1}{\sqrt2}+\frac{1}{\sqrt2}\right)^2 + \left(\frac{-2}{\sqrt2}\right)^2} = \sqrt{\frac{4}{2}+\frac{4}{2}} = \sqrt{4} = 2$$

$\boxed{d_e = 2}$ M=1 (one receive antenna): $a = 1\cdot\frac{\gamma}{4}\cdot 4 = \gamma$

$$P(C^1 \to C^2) = \frac{1}{2}\left\{1 - \sqrt{\frac{\gamma}{1+\gamma}} \sum_{i=0}^{2M-1} \binom{2i}{i} \left[\frac{1}{4(1+a)}\right]^i \right\}$$

$$\boxed{P(C^1\to C^2)=\frac{1}{2}\left\{1-\sqrt{\frac{\gamma}{1+\gamma}}\sum_{i=0}^{2M-1}\binom{2i}{i}\left[\frac{1}{4(1+\gamma)}\right]^i\right\}}$$

$$P(C^1\to C^2)=\frac{1}{2}\left\{1-\sqrt{\frac{\gamma}{1+\gamma}}\left[1+\frac{\cancel{2}}{2\cancel{4}(1+\gamma)}\right]\right\} \quad ①$$

$$\boxed{\underset{\textcircled{P_2}}{P(C^1\to C^2)}=\frac{1}{2}\left\{1-\sqrt{\frac{\gamma}{1+\gamma}}\left[1+\frac{1}{2(1+\gamma)}\right]\right\}} \quad (*)$$

- ONE OF CORRESPONDING SYMBOLS IS DIFFERENT

$$C^1=\{s_1^1,\ s_2^1\} \qquad C^2=\{s_1^2,\ s_2^2\}$$

$$s_1^1=s_1^2 \qquad s_2^1=-s_2^2=\frac{1}{\sqrt{2}}$$

$$\text{e.g.}\quad C^1=\left\{\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}}\right\} \qquad C^2=\left\{\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right\}$$

$$d_e=\sqrt{0^2+\left(\frac{2}{\sqrt{2}}\right)^2}=\sqrt{\frac{4}{2}}=\underline{\underline{\sqrt{2}}}$$

$$a=k\frac{\gamma}{4}d_e^2=\frac{\gamma}{4}\cdot 2=\textcircled{\frac{\gamma}{2}}$$

$$\boxed{\underset{\textcircled{P_1}}{P(C^1\to C^2)}=\frac{1}{2}\left\{1-\sqrt{\frac{\gamma}{2+\gamma}}\left[1+\frac{1}{2+\gamma}\right]\right\}} \quad \text{FOR } \boxed{M=1} \quad (**)$$

ABOVE ANALYSIS PROVIDES PEP FOR GIVEN PAIR OF CODEWORDS.

- ~~BA~~ SPORED MENE AKO PODEDNAKVO SE VKLUČUVAAT (*) I (**) UKUP~~NATA~~ VEROJATNOST NA GREŠKA BI BILA:

$$P_b=\frac{1}{4}\left\{1-\sqrt{\frac{\gamma}{1+\gamma}}\left[1+\frac{1}{1+\gamma}\right]\right\}+\frac{1}{4}\left\{1-\sqrt{\frac{\gamma}{2+\gamma}}\left[1+\frac{1}{2+\gamma}\right]\right\}$$

→ VO (*) FORMA:

$$\boxed{P(C^1\to C^2)=\frac{1}{2}\left\{1-\sqrt{\frac{\gamma}{2+\gamma}}\sum_{i=0}^{2M-1}\binom{2i}{i}\left[\frac{1}{2(2+\gamma)}\right]^i\right\}}$$

$P_1 \quad -1,1 \to \textcircled{+1,1 \quad -1,-1}$

$P_2 \quad -1,1 \to 1,-1$

$P_1 \quad 1,-1 \to \textcircled{-1,-1 \quad 1,1}$

$P_2 \quad 1,-1 \to -1,1$

$P_1 \quad -1,-1 \to \textcircled{1,-1 \quad -1,1}$

$P_2 \quad -1,-1 \to 1,1$

$P_1 \quad 1,1 \to \textcircled{-1,+1 \quad 1,-1}$

$P_2 \quad 1,1 \to -1,-1$

$$\boxed{P_b=\frac{8}{12}P_1+\frac{4}{12}P_2}$$

⇓ PAK E OVA MNOGU [illegible] OD SIMULACIJATA!
POLOŠO ISPADNALO ODI SO $\boxed{P_e=P_2}$ (2x1)

- AKO PRESMETUVAŠ SIMBOLNA GREŠKA KAKO DOSEGA E DA ODIŠ SO ⊕ KOJA VO SUŠTINA PRESMETUVA GREŠKA VO KODNIOT ZBOR (SAMO VO EDEN KODEN ZNAK), A TIE DVE NAČINI SE EKVIVALENTNI.

⊡ DA PROVERAM KAKO BI BILO ZA 4×1

$C^1 \; (1/2 \;\; 1/2 \;\; 1/2 \;\; 1/2)$

$C^2 \; (-\frac{1}{2} \; -\frac{1}{2} \; -\frac{1}{2} \; -\frac{1}{2})$

$$d_E = \sqrt{1^2+1^2+1^2+1^2} = \sqrt{4} = 2$$

$$a = K \frac{\gamma}{4} d_E^2 = \boxed{K \cdot \gamma}$$

$$P(C^1 \to C^2) = \frac{1}{2}\left\{1 - \sqrt{\frac{a}{1+a}} \sum_{i=0}^{MN-1} \binom{2i}{i} \left[\frac{1}{4(1+a)}\right]^i\right\}$$

IF $K = \frac{1}{2}$ (RATE OF THE CODE) $a = \gamma/2$

$$P(C^1 \to C^2) = \frac{1}{2}\left\{1 - \sqrt{\frac{\gamma}{2+\gamma}} \sum_{i=0}^{MN-1} \binom{2i}{i} \left[\frac{1}{2(2+\gamma)}\right]^i\right\}$$

IF $K=2$ $\quad a = 2\gamma$

$$\boxed{P(C^1 \to C^2) = \frac{1}{2}\left\{1 - \sqrt{\frac{2\gamma}{1+2\gamma}} \sum_{i=0}^{MN-1} \binom{2i}{i} \left[\frac{1}{4(1+2\gamma)}\right]^i\right\}}$$

OVA DAVA DOBRI REZULTATI (ZA 4×1)

$K = \frac{1}{\text{CodeRate}}$!!?

SEPAK OVA NE E TOČNO (VIDI ST 24)

KOEFICIENTOT „K" SE GLEDA OD MODOT (3.2)
VIDI IZVEDUVANJE VO MAPLE MultihopMIMO.mw

ISTATA VREDNOST SE DOBIVA ZA [illegible] MODELOT SO TRI ANTENI.

⊡ DIFFERENT METOD FOR ANALYSIS OF ERROR RATES

$G' = \sqrt{\xi}\, G$ } NORMALIZED VERSION GENERALIZED OF OSTBC

$G = G'(s_1, s_2, \dots, s_K)$ $\qquad \|C\|_F^2 = T$

r_m – T×1 VECTOR $m = 1, 2, \dots, M$ SIGNAL VECTOR RECEIVED AT THE m-th RECEIVE ANTENNA.

$$\boxed{r = C \cdot H + N}$$

$f_k(s_k)$ – $k = 1, 2, \dots K$ FUNCTION TO BE MINIMIZED FOR ML DECODING OF s_k

↳ CAN BE CALCULATED AS THE LINEAR COMBINATION OF RECEIVED SIGNALS

- OSTBC UTILIZING FOUR TRANSMIT ANTENNAS

$$G434 = \begin{bmatrix} x_1 & x_2 & x_3 & 0 \\ -x_2^* & x_1^* & 0 & x_3 \\ x_3^* & 0 & -x_1^* & x_2 \\ 0 & x_3^* & -x_2^* & -x_1 \end{bmatrix}$$

$R = 3/4$

$\boxed{K=1} \rightarrow$ SE GLEDA VO MALE OD FORMULITE

- OSTBC UTILIZING THREE TRANSMIT ANTENNAS

$$G334 = \begin{bmatrix} x_1 & x_2 & x_3 \\ -x_2^* & x_1^* & 0 \\ x_3^* & 0 & -x_1^* \\ 0 & x_3^* & -x_2^* \end{bmatrix}$$

[illegible]

$\boxed{R = \frac{3}{4}}$

$\boxed{K=1}$

$h_k(s_k)$ CAN BE WRITEN AS VECTOR PRODUCT OF

$$\left.\begin{matrix}\left[(r'_m)^k\right]^T & (\Omega_{mk})^H\end{matrix}\right\}$$ AT m-TH column OF $\mathbf{r}$ I.E. AT m-TH ANTENNA

$(r'_m)^k$. $T \times 1$ VECTOR derived from $\mathbf{r}_m$ BY REPLACING SOME ELEMENTS WITH ITS CONJUGATES

Ω_{mk} - $1 \times T$ VECTOR CONTAINING PATH GAINS FOR THE k-TH SYMBOL ($t = 1, 2 \ldots T$)

$$(r'_m)^k(t) = \begin{cases} r_m^*(t) & \text{IF } x_k^* \text{ OR } -x_k^* \text{ EXIST IN } t\text{-th ROW OF } G \\ r_m(t) & \text{OTHERWISE} \end{cases}$$

$$\Omega_{mk}(t) = \begin{cases} \alpha_{n,m} & \text{if } G_{t,n} = x_k \\ \alpha_{n,m}^* & \text{if } G_{t,n} = x_k^* \\ -\alpha_{n,m} & \text{if } G_{t,n} = -x_k \\ -\alpha_{n,m}^* & \text{if } G_{t,n} = -x_k^* \\ 0 & \text{OTHERWISE} \end{cases}$$

$\boxed{HMV}$ $\boxed{1*}$

EXAMPLE (ALAMOUTI CODE) $(r'_m)^k$ FOR ALL VALUE OF k K= IS SAME

$$r'_m = \begin{bmatrix} r_{1m} \\ r_{2m}^* \end{bmatrix} \qquad \Omega_m = \begin{bmatrix} \alpha_{1,m} & \alpha_{2,m}^* \\ \alpha_{2,m} & -\alpha_{1,m}^* \end{bmatrix}$$

POTVRDA NA MOJOT IZRAZ OD WCNC CLANAKOT (29)

$$\boxed{\sum_{m=1}^{M} \left[(r'_m)^k\right]^T \cdot (\Omega_{mk})^H = \sqrt{\xi}\, K \sum_{m=1}^{M} \sum_{n=1}^{N} |\alpha_{n,m}|^2 s_k + \sum_{m=1}^{M} N_{mk}}$$

N_{mk} - iid ZERO MEAN COMPLEX GAUSSIAN RANDOM VARIABLE WITH VARIANCE $\frac{K}{\delta} \sum_{n=1}^{N} |\alpha_{n,m}|^2$

NORMALIZACIJA ZA VKUPNATA SNAGA OD SITE ANTENI VO DADEN TAJM SLOT DA E 1

$$SNR = \frac{\xi K^2 \left[\sum_{m=1}^{M}\sum_{n=1}^{N}|\alpha_{n,m}|^2\right]^2}{\cancel{\frac{K}{\gamma}\sum_{m=1}^{M}\sum_{n=1}^{N}|\alpha_{n,m}|^2}} = \underline{\underline{\xi K\gamma \sum_{m=1}^{M}\sum_{n=1}^{N}|\alpha_{n,m}|^2}}$$

$$P(SE[illegible]|H) = P\left(\text{symbol error} \,\Big|\, SNR = \xi K \sum_{m=1}^{M}\sum_{n=1}^{N}|\alpha_{n,m}|^2\gamma\right)$$

$$SER \in \left[P\left(\text{symbol error} \,\Big|\, SNR = \xi K \sum_{m=1}^{M}\sum_{n=1}^{N}|\alpha_{n,m}|^2\gamma\right)\right].$$

- **L-PSK** constellation over Rayleigh channel

$$\boxed{\begin{aligned}SER = {} & \frac{L-1}{L} - \left(\frac{1}{\pi}\sqrt{\frac{\xi K\gamma \sin^2\frac{\pi}{L}}{1+\xi K\gamma\sin^2\frac{\pi}{L}}}\right)\left\{\left(\frac{\pi}{2}+\tan^{-1}\beta\right)\sum_{i=0}^{d-1}\frac{\binom{2i}{i}}{\left[4\left(1+\xi K\gamma\sin^2\frac{\pi}{L}\right)\right]^i}\right. \\ & \left. + \operatorname{sgn}(\tan^{-1}\beta)\sum_{i=1}^{d-1}\sum_{j=1}^{i}\frac{T_{ji}}{\left(1+\xi K\gamma\sin^2\frac{\pi}{L}\right)^i}\left[\cos(\tan^{-1}\beta)\right]^{2(i-j)+1}\right\} \\ & \beta = \sqrt{\frac{\xi K\gamma\sin^2\frac{\pi}{L}}{1+\xi K\gamma\sin^2\frac{\pi}{L}}}\cot\left(\frac{\pi}{L}\right), \quad T_{ji} = \frac{\binom{2i}{i}}{\binom{2(i-j)}{i-j}4^j\left[2(i-j)+1\right]} \\ & d = M\cdot N\end{aligned}}$$

Example 4.9.2 STBC using **BPSK** constellation

$$P(\text{symbol error}|H) = Q\left(\sqrt{2\xi K\sum_{m=1}^{M}\sum_{n=1}^{N}|\alpha_{n,m}|^2\gamma}\right)$$

$$SER = \in\left[Q\left(\sqrt{2\xi K\sum_{m=1}^{M}\sum_{n=1}^{N}|\alpha_{n,m}|^2\gamma}\right)\right]$$

(EXPECTATION OVER CHANNEL PATH GAINS)

$$\boxed{SER = \frac{1}{2}\left\{1-\sqrt{\frac{\xi K\gamma}{1+\xi K\gamma}}\sum_{i=0}^{MN-1}\frac{\binom{2i}{i}}{\left[4(1+\xi K\gamma)\right]^i}\right\}}$$

BPSK

MMV

- FOR ALAMOUTI CODE $N=2$, $M=1$, $K=1$, $\boxed{\xi = 1/2}$

$$SER = \frac{1}{2}\left\{1-\sqrt{\frac{\gamma}{2+\gamma}}\sum_{i=0}^{MN-1}\frac{\binom{2i}{i}}{\left[4(1+\xi K\gamma)\right]^i}\right\}$$

ξ NORMALIZIRA UKUPNA SNAGA DA BUDE ①

$\boxed{3X1}$

$$C_{334} = \begin{bmatrix} x_1 & x_2 & x_3 \\ -x_2^* & x_1^* & 0 \\ x_3^* & 0 & -x_1^* \\ 0 & x_3^* & -x_2^* \end{bmatrix} \qquad \Omega_t = \begin{bmatrix} h_1 & h_2 & h_3 \\ +h_2^* & -h_1^* & 0 \\ -h_3^* & 0 & +h_1^* \\ 0 & -h_3^* & +h_2^* \end{bmatrix}$$

$H = [h_1, h_2, h_3]$; $N = [n_1, n_2, n_3, n_4]$;

$\tilde{X} = Y_a \cdot \Omega^H$ $\qquad$ $Y = (C \cdot H^T + N^T)^T = [y_1, y_2, y_3, y_4]$

$Y_a = [y_1, y_2, y_3^*, y_4^*]$

$$\tilde{X} = [y_1\ y_2\ y_3^*\ y_4^*] \cdot \begin{bmatrix} h_1 & h_2 & h_3 \\ h_2^* & -h_1 & 0 \\ -h_3^* & 0 & h_1^* \\ 0 & -h_3 & h_2^* \end{bmatrix}$$

SPOREDBA NA (1*) OD #24 ZA [illegible]

$$G = \begin{array}{c} \xrightarrow{n} \\ t\downarrow \begin{bmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{bmatrix} \end{array} \qquad \Omega = \begin{array}{c} \xrightarrow{k} \\ t\downarrow \begin{bmatrix} \alpha_1 & \alpha_2^* \\ \alpha_2 & -\alpha_1^* \end{bmatrix} \end{array} \qquad \Omega_t = \begin{bmatrix} \alpha_1 & \alpha_2^* \\ \alpha_2^* & -\alpha_1^* \end{bmatrix}$$

$$\Omega_k(t) = \begin{cases} \boxed{\alpha_{n,1}} & \text{if } G_{t,n} = x_k \\ \alpha_{n,1}^* & \text{if } G_{t,n} = x_k^* \\ -\alpha_{n,1} & \text{if } G_{t,n} = -x_k \\ -\alpha_{n,1}^* & \text{if } G_{t,n} = -x_k^* \\ 0 & \text{otherwise} \end{cases} \qquad H = [\alpha_1, \alpha_2]$$

$\Omega_1(1) = \alpha_1$; $\Omega_2(1) = \alpha_2$ $\;|G_{1,2} = x_2|$

$\Omega_1(2) = \alpha_2^*$; $|G_{2,1} = x_1^*|$ $\qquad$ $\Omega_2(2) = -\alpha_1^*$ $\;|G_{2,1} = -x_2^*|$

$$\Omega = \begin{bmatrix} \alpha_1 & \alpha_2 \\ \alpha_2^* & -\alpha_1^* \end{bmatrix} \Bigg\}$$

OVA VSUŠTOST VO MOJATA REPREZENTACIJA ZA G:

$\boxed{\Omega_t}$

$$G = \begin{array}{c|ccc} t \downarrow & & n \rightarrow & \\ 1 & x_1 & x_2 & x_3 \\ 2 & -x_2^* & x_1^* & 0 \\ 3 & x_3^* & 0 & -x_1^* \\ 4 & 0 & x_3^* & -x_2^* \end{array}$$

$$\Omega_t = \begin{array}{c|ccc} t\downarrow & & \xrightarrow{k} & \\ & \Omega_1(1) & \Omega_2(1) & \Omega_3(1) \\ & \Omega_1(2) & \Omega_2(2) & \Omega_3(2) \\ & \Omega_1(3) & \Omega_2(3) & \Omega_3(3) \\ & \Omega_1(4) & \Omega_2(4) & \Omega_3(4) \end{array}$$

$H = [h_1, h_2, h_3]$

MMV

$k=1\ t=1$

$G_{1,\textcircled{1}} = G_{t,n} = x_1 \Rightarrow \Omega_1(1) = h_1$

$k=2\ t=1$

$G_{t,n} = G_{1,\textcircled{2}} = x_2 \quad \Omega_2(1) = h_2$

$k=3\ t=1$

$G_{t,n} = G_{1,\textcircled{3}} = x_3 \quad \Omega_3(1) = h_3$

$k=1\ t=2$

$G_{t,n} = G_{2,\textcircled{2}} = x_1^* \Rightarrow \Omega_1(2) = h_2^*$

$k=2\ t=2$

$G_{t,n} = G_{2,\textcircled{1}} = -x_2^* \quad \Omega_2(2) = -h_1^*$

$k=3\ t=2$

$G_{t,n} = G_{2,\textcircled{3}} = 0 \quad \Omega_3(2) = 0$

$k=1\ t=3$

$G_{t,n} = G_{3,\textcircled{3}} = -x_1^* \quad \Omega_1(3) = -h_3^*$

$k=2\ t=3$

$G_{t,n} = G_{3,\textcircled{2}} = 0 \quad \Omega_2(3) = 0$

$k=3\ t=3$

$G_{t,n} = G_{3,1} = x_3^* \quad \Omega_3(3) = h_1^*$

$k=1\ t=4$

$G_{t,n} = G_{4,1} = 0 \quad \Omega_1(4) = 0$

$k=2\ t=4$

$G_{t,n} = G_{4,3} = -x_2^* \quad \Omega_2(4) = -h_3^*$

$k=3\ t=4$

$G_{t,n} = G_{4,2} = x_3^* \quad \Omega_3(4) = h_2^*$

$$\Omega_t = \begin{bmatrix} h_1 & h_2 & h_3 \\ h_2^* & -h_1^* & 0 \\ -h_3^* & 0 & h_1^* \\ 0 & -h_3^* & +h_2^* \end{bmatrix}$$

MMV

$$Y_a^T = [y_1\ y_2^*\ y_3^*\ y_4]$$

$$(r_n')^k(t) = \begin{cases} r_n^*(t) & \text{if } x_k^* | {-x_k^*} \text{ exist in } t\text{=row OF } G \\ r_n(t) & \text{otherwise} \end{cases}$$

Ω^H

VIDI MARE !!

$$\tilde{X} = [y_1\ y_2^*\ y_3^*\ y_4^{\textcircled{*}}] \cdot \begin{bmatrix} h_1^* & h_2^* & h_3^* \\ h_2 & -h_1 & 0 \\ -h_3 & 0 & h_1 \\ 0 & -h_3 & h_2 \end{bmatrix} = \begin{bmatrix} y_1 h_1^* + y_2^* h_2 - y_3^* h_3, \\ y_1 h_2^* \ [illegible]\ y_2^* h_1 - y_4^* h_3, \\ y_1 h_3^* + y_3^* h_1 + y_4^* h_2 \end{bmatrix};$$

ZA DECODER VO MATLAB

334 ' '' ''' ' '' ''' ' '' ''' ' '' '''

~~000~~ 0 0 1 0 1 1 1 1 0

e1 = 1 0 0 1

e2 = 1 0 1 1

e3 = 1 1 1 0

$\Longrightarrow$ ERROR RATE

PER ~~[illegible]~~

ANTENA ARRET !!!

ZORICA MICEVA

070300993

070300119 MIRČE

⊡ REALNI OSTBC

- 4×1 (444 code)

$$C=\begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ -x_2 & x_1 & -x_4 & x_3 \\ -x_3 & x_4 & x_1 & -x_2 \\ -x_4 & -x_3 & x_2 & x_1 \end{bmatrix} \qquad \Omega_t=\begin{bmatrix} h_1 & h_2 & h_3 & h_4 \\ +h_2 & -h_1 & h_4 & -h_3 \\ +h_3 & -h_4 & -h_1 & h_2 \\ h_4 & h_3 & -h_2 & -h_1 \end{bmatrix}$$

- 3×1 (334 code)

$$C:=\begin{bmatrix} x_1 & x_2 & x_3 \\ -x_2 & x_1 & 0 \\ -x_3 & 0 & x_1 \\ 0 & -x_3 & x_2 \end{bmatrix} \qquad \Omega_t=\begin{bmatrix} h_1 & h_2 & h_3 \\ +h_2 & -h_1 & 0 \\ h_3 & 0 & -h_1 \\ 0 & h_3 & -h_2 \end{bmatrix}$$

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web2sms

sms.telekom.mk

- 3×1 (344 code)

$$C:=\begin{bmatrix} x_1 & x_2 & x_3 \\ -x_2 & x_1 & -x_4 \\ -x_3 & x_4 & x_1 \\ -x_4 & -x_3 & x_2 \end{bmatrix} \qquad \Omega_t=\begin{bmatrix} h_1 & h_2 & h_3 & 0 \\ h_2 & -h_1 & 0 & -h_3 \\ h_3 & 0 & -h_1 & h_2 \\ 0 & h_3 & -h_2 & h_1 \end{bmatrix}$$

$$M_\gamma(s)=\left(\frac{1}{2}\right)^2\cdot\left(\frac{1}{c8}\right)^3\psi^{-4}\left[9\cdot\left(\frac{1}{c8}\right)^3\frac{2^{10}}{7\psi^2}\,{}_2F_1\left(6,\frac{5}{2};\frac{9}{2};\Omega\right)+\right.$$

$$+3\left(\frac{1}{c8}\right)^2\frac{2^{12}}{35\psi}\,{}_2F_1\left(5,\frac{3}{2};\frac{9}{2};\Omega\right)-\frac{28}{3cg}\cdot{}_2F_1\left(4,\frac{3}{2};\frac{7}{2};\Omega\right)+$$

$$\left.+\frac{3\cdot 2^9}{35cg}\,{}_2F_1\left(4,\frac{1}{2};\frac{9}{2};\Omega\right)\right];$$

$$\boxed{c=\frac{L}{nS\cdot K\cdot\log_2 M}}$$

L → BLOCK LENGTH

M → CONSTELATION

nS → ANTENI

K → SIMBOLI

$$\Omega = \frac{\frac{3}{2c\cdot\bar{\gamma}} - \frac{1}{c\bar{\gamma}}\sqrt{2} + s}{\frac{3}{2c\bar{\gamma}} + \frac{1}{c\bar{\gamma}}\sqrt{2} + s} = \frac{3 - 2\sqrt{2} + 2\cdot s\, c\bar{\gamma}}{3 + 2\sqrt{2} + 2\cdot s\, c\bar{\gamma}}$$

$$\boxed{\psi = \frac{3}{2c\bar{\gamma}} + \frac{1}{c\bar{\gamma}}\sqrt{2} + s} \quad \boxed{\Omega = \frac{3 - 2\sqrt{2} + 2c\bar{\gamma}s}{3 + 2\sqrt{2} + 2c\bar{\gamma}s}} \quad \boxed{c = \frac{L}{nS\cdot K\cdot ldM}}$$

$$\boxed{M_{\gamma}(-s) = \frac{\psi^{-4}}{4c^3\bar{\gamma}^3}\left[\frac{9\cdot 2^{10}}{7c^3\bar{\gamma}^3\psi^2}\, {}_2F_1\left(6, \frac{5}{2}; \frac{9}{2}; \Omega\right) + \frac{3\cdot 2^{12}}{35c^2\bar{\gamma}^2\psi}\, {}_2F_1\left(5, \frac{1}{2}; \frac{9}{2}; \Omega\right) - \frac{2^8}{5c\bar{\gamma}}\cdot {}_2F_1\left(4, \frac{3}{2}; \frac{7}{2}; \Omega\right) + \frac{3\cdot 2^9}{35c\bar{\gamma}}\, {}_2F_1\left(4, \frac{1}{2}; \frac{9}{2}; \Omega\right)\right]}$$

matlab-symbolic

EsNo

$$\boxed{s = \frac{g}{\sin^2\theta}}$$

!!

JULIJANA ZAOVSKA

$$\boxed{\psi = \frac{3}{2c\bar{\gamma}} + \frac{\sqrt{2}}{c\bar{\gamma}} + \frac{g}{\sin^2\theta} \qquad \Omega = \frac{(3-2\sqrt{2})\sin^2\theta + 2c\bar{\gamma}\cdot g}{(3+2\sqrt{2})\sin^2\theta + 2c\bar{\gamma}\cdot g}}$$

$$f_{\gamma}^{NS}(\gamma) = \left(\frac{1}{c\bar{\gamma}}\right)^3 \frac{\gamma^2}{2\beta_1\beta_2} e^{-(\beta_1+2\beta_2)\gamma/(2c\bar{\gamma}\beta_1\beta_2)} \cdot$$

$$\left[\left(\frac{\beta_1+2\beta_2}{\beta_1\beta_2}\right)^2 \frac{\gamma}{2c\bar{\gamma}} K_2\left(\frac{\gamma}{c\bar{\gamma}}\sqrt{\frac{2}{\beta_1\beta_2}}\right) + \left\{\frac{2\gamma}{c\bar{\gamma}}\left(\frac{\beta_1+2\beta_2}{\beta_1\beta_2}\right)\sqrt{\frac{2}{\beta_1\beta_2}} - 2\sqrt{\frac{2}{\beta_1\beta_2}}\right\}\right.$$

$$\left. \times K_1\left(\frac{\gamma}{c\bar{\gamma}}\sqrt{\frac{2}{\beta_1\beta_2}}\right) + \frac{4\gamma}{c\bar{\gamma}\beta_1\beta_2} K_0\left(\frac{\gamma}{c\bar{\gamma}}\sqrt{\frac{2}{\beta_1\beta_2}}\right)\right].$$

$$\int_0^\infty x^{\mu-1} e^{-\alpha x} K_\nu(\beta x)\, dx = \frac{\sqrt{\pi}(2\beta)^\nu}{(\alpha+\beta)^{\mu+\nu}} \frac{\Gamma(\mu+\nu)\Gamma(\mu-\nu)}{\Gamma(\mu+\frac{1}{2})} F\left(\mu+\nu, \nu+\frac{1}{2}; \mu+\frac{1}{2}; \frac{\alpha-\beta}{\alpha+\beta}\right)$$

GRADSHTEYN 6.631.3

$$MGF(-s) = \int_{-\infty}^{\infty} f_\gamma^{NS}(\gamma) e^{-\gamma s} d\gamma$$

$$P_s = \frac{1}{\pi}\int_0^{(M-1)\pi/M} MGF\left(-\frac{g}{\sin^2\theta}\right) d\theta$$

$$g = \sin^2(\pi/M)$$

$\boxed{\beta_1=\beta_2=1}$

$$f_\gamma^{NB}(\gamma)=\left(\frac{1}{c\cdot\bar\gamma}\right)^3\frac{\gamma^2}{2}\cdot e^{-3\gamma/2c\bar\gamma}\left[\frac{9\gamma}{2c\bar\gamma}K_2\left(\frac{\gamma\sqrt2}{c\bar\gamma}\right)+\left(\frac{6\gamma\sqrt2}{c\bar\gamma}-2\sqrt2\right)K_1\left(\frac{\gamma\sqrt2}{c\bar\gamma}\right)+\frac{4\gamma}{c\bar\gamma}K_0\left(\frac{\gamma\sqrt2}{c\bar\gamma}\right)\right]$$

$768=2^9+2^8=(2+1)\cdot2^8=3\cdot2^8$

$\boxed{MGF2=MGF21+MGF22}$

$MGF(-1)=MGF1+MGF2+MGF3$

$$MGF1=\int_0^\infty\frac{\gamma^2}{2c^3\bar\gamma^3}e^{-\left(\frac{3}{2c\bar\gamma}+1\right)\gamma}\cdot\frac{9\gamma}{c\bar\gamma}K_2\left(\frac{\gamma\sqrt2}{c\bar\gamma}\right)d\gamma$$

$$=\frac{9}{2c^4\bar\gamma^4}\int_0^\infty\gamma^{\overset{\overset{④-1}{\mu-1}}{3}}\cdot e^{-\overbrace{\left(\frac{3}{2c\bar\gamma}+1\right)}^{\alpha}\gamma}K_{\underbrace{2}_{\nu}}\left(\gamma\underbrace{\frac{\sqrt2}{c\bar\gamma}}_{\beta}\right)d\gamma=\frac{9}{2c^4\bar\gamma^4}\int_0^\infty\gamma^{\mu-1}e^{-a\gamma}\cdot K_{\underset{\nu}{②}}(\beta\gamma)d\gamma$$

$\mu=4\quad\alpha=\left(\frac{3}{c\bar\gamma}+1\right)\quad\beta=\frac{\sqrt2}{c\bar\gamma}$

$$=\frac{9}{2c^4\bar\gamma^4}\cdot\frac{\sqrt\pi\left(2\cdot\frac{\sqrt2}{c\bar\gamma}\right)^2}{\left(\left(\frac{3}{2c\bar\gamma}+1\right)+\frac{2\sqrt2}{c\bar\gamma}\right)^{4+2}}\cdot\overset{=\frac{768}{7\sqrt\pi}}{\boxed{\frac{\Gamma(6)\cdot\Gamma(4)}{\Gamma\left(\frac{9}{2}\right)}}}F\left(6,\frac52;\frac92;\frac{\alpha-\beta}{\alpha+\beta}\right)$$

$$\boxed{\Psi=\frac{3}{2c\bar\gamma}+\frac{2\sqrt2}{c\bar\gamma}+1\qquad\Omega=\frac{\alpha-\beta}{\alpha+\beta}=\frac{\frac{3}{2c\bar\gamma}+1-\frac{\sqrt2}{c\bar\gamma}}{\frac{3}{2c\bar\gamma}+1+\frac{\sqrt2}{c\bar\gamma}}=\frac{3-2\sqrt2+2c\bar\gamma1}{3+2\sqrt2+2c\bar\gamma1}}$$

$$MGF1=\frac{9}{\cancel{2}c^4\bar\gamma^4}\cdot\frac{\cancel{\sqrt\pi}\,\frac{4\cdot2}{c^2\bar\gamma^2}}{\Psi^6}\,\frac{3\cdot2^{\cancel{8}7}}{7\cancel{\sqrt\pi}}\cdot F\left(6,\frac52;\frac92;\Omega\right)=$$

$$\boxed{=\frac{27\cdot2^{10}}{7\cdot c^6\bar\gamma^6\cdot\Psi^6}F\left(6,\frac52;\frac92;\Omega\right)}$$

$$MGF21=\int_0^\infty\frac{\gamma^2}{2c^3\bar\gamma^3}\frac{6\gamma\sqrt2}{c\bar\gamma}K_1\left(\frac{\gamma\sqrt2}{c\bar\gamma}\right)e^{-\overbrace{\left(\frac{3}{2c\bar\gamma}+1\right)}^{a}\gamma}d\gamma=\int_0^\infty\frac{6\gamma^3\sqrt2}{2c^4\bar\gamma^4}K_1\left(\frac{\gamma\sqrt2}{c\bar\gamma}\right)e^{-a\gamma}d\gamma$$

$$MGF21=\frac{3\sqrt2}{c^4\bar\gamma^4}\int_0^\infty\gamma^3K_1\left(\gamma\underbrace{\frac{\sqrt2}{c\bar\gamma}}_{\beta}\right)e^{-a\gamma}d\gamma=\left|\begin{matrix}\mu=4;\ \nu=1\\a=\frac{3}{2c\bar\gamma}+1\end{matrix}\right|=$$

$$=\frac{3\sqrt2}{c^4\bar\gamma^4}\cdot\frac{\sqrt\pi\,\frac{2\sqrt2}{c\bar\gamma}}{\Psi^5}\overset{\frac{768}{35\sqrt\pi}}{\frac{\Gamma(5)\cdot\Gamma(4)}{\Gamma\left(\frac92\right)}}\,{}_2F_1\left(5,\frac32;\frac92;\Omega\right)=\frac{12\sqrt\pi\cdot3\cdot2^8}{c^5\bar\gamma^5\cdot\Psi^5\cdot35\sqrt\pi}\,{}_2F_1(:)$$

$$=\underline{\frac{3^2\cdot2^{10}}{35c^5\bar\gamma^5\Psi^5}\,{}_2F_1\left(5,\frac32;\frac92;\Omega\right);}$$

$$MGF22 = -\int_0^\infty \frac{\cancel{2}\sqrt{2}\gamma^2}{\cancel{2}c^3\bar{\gamma}^3} e^{-\left(\frac{3\gamma}{2c\bar{\gamma}}+1\right)\gamma} K_1\left(\frac{\gamma\sqrt{2}}{c\bar{\gamma}}\right) d\gamma = -\frac{\sqrt{2}}{c^3\bar{\gamma}^3}\int_0^\infty \gamma^2 e^{-\left(\frac{3\gamma}{2c\bar{\gamma}}+1\right)\gamma} K(\beta\gamma)$$

$$\left| \mu = 3 \quad \nu = 1 \quad i \quad \alpha = \frac{3}{2c\bar{\gamma}} + 1 \right|$$

$$\frac{\sqrt{\pi}\,2\cdot\sqrt{2}}{c\bar{\gamma}\psi^4} \qquad \rightarrow \frac{16}{5\sqrt{\pi}}$$

$$MGF_{22} = -\frac{\sqrt{2}}{c^3\bar{\gamma}^3}\int_0^\infty \gamma^{3-1} e^{-\alpha\gamma} K_1(\beta\gamma)\, d\gamma = -\frac{\sqrt{2}}{c^3\bar{\gamma}^3}\frac{\Gamma(4)\cdot\Gamma(2)}{\Gamma\left(\frac{7}{2}\right)}\,{}_2F_1\left(4,\frac{3}{2};\frac{7}{2};\Omega\right)$$

$$= -\frac{4}{c^4\bar{\gamma}^4\psi^4}\frac{16}{5\cancel{\sqrt{\pi}}}\,{}_2F_1\left(4,\frac{3}{2};\frac{7}{2};\Omega\right) = -\frac{2^6}{5c^4\bar{\gamma}^4\psi^4}\,{}_2F_1\left(4,\frac{3}{2};\frac{7}{2};\Omega\right)$$

②

$$MGF_3 = \frac{1}{\cancel{2}c^3\bar{\gamma}^3}\frac{\cancel{4}}{c\bar{\gamma}}\int_0^\infty \gamma^3 e^{-\left(\frac{3\gamma}{2c\bar{\gamma}}+1\right)\gamma} K_0\left(\frac{\gamma\sqrt{2}}{c\bar{\gamma}}\right) d\gamma = \frac{2}{c^4\bar{\gamma}^4}\int_0^\infty \gamma^{4-1} e^{-\alpha\gamma} K_0(\beta\gamma)\, d\gamma$$

$$\frac{192}{35\sqrt{\pi}} \qquad 3\cdot 2^7$$

$$= \frac{2}{c^4\bar{\gamma}^4}\frac{\sqrt{\pi}}{\psi^4}\cdot\frac{\Gamma(4)\cdot\Gamma(4)}{\Gamma\left(\frac{9}{2}\right)}\,{}_2F_1\left(4,\frac{1}{2};\frac{9}{2};\Omega\right) = \frac{384}{35c^4\bar{\gamma}^4\psi^4}\,{}_2F_1\left(4,\frac{1}{2};\frac{9}{2};\Omega\right)$$

$$= \left| 2^8 + 2^7 = 384 = 3\cdot 2^7 \right| = \frac{3\cdot 2^7}{35c^4\bar{\gamma}^4\psi^4}\,{}_2F_1\left(4,\frac{1}{2};\frac{9}{2};\Omega\right)$$

$$\bar{\gamma} = E_sN_0$$

$$MGF = \frac{27\cdot 2^{10}}{7\cdot c^6\bar{\gamma}^6\psi^6}\,{}_2F_1\left(6,\frac{5}{2};\frac{9}{2};\Omega\right) + \frac{3^2\cdot 2^{10}}{35c^5\bar{\gamma}^5\psi^5}\,{}_2F_1\left(5,\frac{3}{2};\frac{9}{2};\Omega\right)$$

$$\frac{2^6}{5c^4\bar{\gamma}^4\psi^4}\,{}_2F_1\left(4,\frac{3}{2};\frac{7}{2};\Omega\right) + \frac{3\cdot 2^7}{35c^4\bar{\gamma}^4\psi^4}\,{}_2F_1\left(4,\frac{1}{2};\frac{7}{2};\Omega\right) =$$

$$= \frac{\psi^{-4}}{4c^3\bar{\gamma}^3}\Bigg[\frac{27\cdot 2^{12}}{7c^3\bar{\gamma}^3\psi^2}\,{}_2F_1\left(6,\frac{5}{2};\frac{9}{2};\Omega\right) + \frac{9\cdot 2^{12}}{35c^2\bar{\gamma}^2\psi}\,{}_2F_1\left(5,\frac{3}{2};\frac{7}{2};\Omega\right) +$$

$$\frac{2^8}{5c\bar{\gamma}}\,{}_2F_1\left(4,\frac{3}{2};\frac{7}{2};\Omega\right) + \frac{3\cdot 2^9}{35c\bar{\gamma}}\,{}_2F_1\left(4,\frac{1}{2};\frac{9}{2};\Omega\right)\Bigg]$$

ODLUKA
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I TOA NE
POREZENO
SO 4S!!!

- POVTORUVANJE: END-TO-END ANALYSIS... (I.H. LEE)

n_t^S – TRANSMIT ANTENNAS AT THE „S“

n_t^D – TRANSMIT -//- AT THE „D“

OSTBC WITH „K“ SYMBOLS $x_1, x_2, \dots, x_K$

$G_{n_t^S}(x)$ (NUMBER OF TRANSMIT ANTENNAS) X (CODE LENGTH)

$h^R = \{h_i^R\}_{1\times n_t^S}$ $\qquad h^D = \{h_i^D\}_{n_r^D\times 1}$

- SIGNAL AT THE RELAY

$$y^R = h^R \cdot G_{n_t^S} + e^R \qquad y^R = \{y_l^R\}_{1\times L} \qquad e^R = \{e_l^R\}_{1\times L}$$

L- BLOCK LENGTH OF THE OSTBC

- SIGNAL AT THE DESTINATION

$$Y^D = h^D x^R + \mathcal{E}^D \qquad Y^D = \{y_{il}^D\}_{n_r^D\times L}$$

$$\mathcal{E}^D = \{e_{il}^D\}_{n_r^D\times L}$$

$$x^R = \{\alpha\, y_l^R\}_{1\times L}$$

i – i-TH RECEIVE ANTENNA AT D

l – l-TH SYMBOL $1 \le l \le L$

$$\alpha = \sqrt{\frac{n_t^S}{\sum_{i=1}^{n_t^S} |h_i^R|^2}} \qquad r_K = \alpha^4 \cdot \Lambda^2 \cdot \Delta^2 \cdot E_s$$

$$\boxed{r_K \triangleq \alpha^4 \left(\sum_{i=1}^{n_r^D} |h_i^D|^2\right)^2 \left(\sum_{j=1}^{n_t^S} |h_j^R|\right)^2 \underbrace{E\left[|x_k|^2\right]}_{\triangleq E_s}}$$

AVERAGE SIGNAL POWER AT DESTINATION

$$\eta_K \triangleq \alpha^2 \underbrace{\left(\sum_{i=1}^{n_r^D} |h_i^D|^2\right)}_{\Lambda} \underbrace{\left(\sum_{i=1}^{n_t^S} |h_i^R|^2\right)}_{\Delta} \left\{\alpha^2 \Lambda \cdot \sigma^2 + \sigma^2\right\}$$

$$\eta_K \triangleq \alpha^2 \Lambda \cdot \Delta \left\{\alpha^2 \Lambda \cdot \sigma^2 + \sigma^2\right\} = \underline{\underline{\alpha^4 \Lambda^2 \Delta \cdot N_0 + N_0 \alpha^2 \Delta \Lambda}}$$

$$\gamma = SNR = \frac{\alpha^4 \cdot \Lambda^2 \cdot \Delta^2 \cdot E_s}{\cancel{\alpha^2}\cancel{\Lambda}\,\alpha^2 \Lambda^2 \cdot \Delta \cdot N_0 + N_0 \Delta} = \frac{E_s}{N_0} \frac{\alpha^2 \cdot \Delta^2 \cdot \Lambda}{\alpha^2 \cdot \Lambda \cdot \Delta + \Delta}$$

$$\gamma = \frac{E_s}{N_0} \frac{\alpha^2 \cdot \Delta \cdot \Lambda}{\alpha^2 \cdot \Delta \Lambda + 1}$$

$$P_x = E\left[|x_1|^2\right] = \dots = E\left[|x_K|^2\right] \qquad E_sN_0 = \frac{E_s}{N_0} = \frac{E_b \cdot LdM}{N_0}$$

NON REGENERATIVE SYSTEM

$$\gamma^{NS}(\rho) = \frac{\gamma_K}{\eta_K}\cdot\frac{1}{LdM} = c\cdot\rho\left[\frac{1}{\sum_{j=1}^{n_t^s}|h_j^R|^2} + \frac{1}{n_t^s\sum_{i=1}^{n_r^D}|h_i^D|^2}\right]^{-1}$$

SNR PER BIT

RECALL

$$\gamma = \frac{E}{N_0}\cdot\frac{G^2\cdot\Lambda\cdot\Delta^2}{G^2\cdot\Lambda\cdot\Delta+1} = \frac{E}{N_0}\,\frac{1}{\frac{1}{\Delta}+\frac{1}{G^2\cdot\Lambda\cdot\Delta^2}}$$

$$\boxed{G^2 = \frac{1}{\Delta}}$$

$$\gamma = \frac{E}{N_0}\,\frac{1}{\frac{1}{\Delta}+\frac{1}{\Lambda}} = \bar{\gamma}\,\frac{1}{\frac{1}{\Delta}+\frac{1}{\Lambda}}$$

$$\boxed{w = \frac{1}{\gamma} = \underbrace{\frac{1}{\bar{\gamma}\cdot\Delta}}_{\gamma_1} + \underbrace{\frac{1}{\bar{\gamma}\cdot\Lambda}}_{\gamma_2}} = \frac{1}{\gamma_1}+\frac{1}{\gamma_2}$$

ρ - TRANSMIT SNR $\quad \boxed{\rho = \frac{P}{\sigma^2}}$ $\quad \boxed{C = \frac{L}{n_t^s\,K\,LdM}}$

K - BROJ NA SIMBOLI VO KODEN ZBOR

L - BLOCK LENGTH OF OSTBC

$$\boxed{c\cdot\rho = E_b}$$

$$\gamma^{NS} = \bar{\gamma}\left[\frac{1}{\Delta}+\frac{1}{n_t^s\cdot\Lambda}\right]^{-1} \Rightarrow \frac{\bar{\gamma}}{\gamma^{NS}} = \frac{1}{\Delta}+\frac{1}{n_t^s\Lambda}$$

$$\boxed{\frac{1}{\gamma^{NS}} = \frac{1}{\bar{\gamma}\Delta}+\frac{1}{n_t^s\bar{\gamma}\cdot\Lambda}} \qquad \Delta = \sum_{i=1}^{n_t^s}|h_i^R|^2 \quad \Lambda = \sum_{i=1}^{n_r^D}|h_i^D|^2$$

- SNR OF REGENERATIVE OSTBC TRANSMISSION

$$Y^D = h^D\cdot\hat{\rho} + \epsilon^D$$

– RECEIVED SNRS FOR FIRST AND SECOND HOP ARE:

$$\gamma^{RS1}(\rho) = c\cdot\rho\,\|h^R\|^2 \qquad \underbrace{\gamma^{RS2}(\rho) = \rho\,\|h^D\|^2 / LdM}_{MRC}$$

$$f_\Delta(x) = \frac{x^{d_1-1}}{\Gamma(d_1)}e^{-x}, \quad f_\Lambda(x) = \frac{x^{d_2-1}}{\Gamma(d_2)}e^{-x}$$

$$z^R = \frac{1}{\Delta} \qquad z^D = \frac{1}{\Lambda\cdot n_t^s} \qquad M_{z^R}(s) = \frac{2}{\beta_1^{n_t^s}\Gamma(n_t^s)}\left(\frac{1}{\beta_1 s}\right)^{-n_t^s/2} K_{n_t^s}\left(2\sqrt{\frac{1}{\beta_1}}\right)$$

$$M_{Z^{\bullet}} = \mathcal{E}_{Z^P}\left(e^{-sz}\right) = \frac{2}{\Gamma(n_r^P)\left(n_t^S\right)^{n_r^P}\cdot \beta_2^{n_r^P}}\left(\frac{1}{n_t^S \beta_{2S}}\right)^{-\frac{n_r^P}{2}} K_{n_r^P}\left(2\sqrt{\frac{s}{n_t^S \beta_2}}\right)$$

$$M_W = M_{Z^R}\cdot M_{Z^D} = \frac{4}{\Gamma^2(N)\cdot N^N\cdot \beta_1^N \beta_2^N}\left(N\cdot\beta_2 s\right)^{\frac{N}{2}}\left(\beta_1 s\right)^{\frac{N}{2}} K_N\left(2\sqrt{\frac{s}{\beta_1}}\right)\cdot K_N\left(2\sqrt{\frac{s}{N\beta_2}}\right)$$

$$= \frac{4}{\Gamma^2(N)\cdot \underset{N/2}{N^N}\,\beta_1^{\frac{N}{2}}\beta_2^{\frac{N}{2}}}\cdot \cancel{N^{\frac{N}{2}}}\cdot s^N K_N\left(2\sqrt{\frac{s}{\beta_1}}\right)\cdot K_N\left(2\sqrt{\frac{s}{N\beta_2}}\right)$$

$$\boxed{M_W = \frac{4}{\Gamma^2(N)\cdot N^{N/2}\beta_1^{N/2}\beta_2^{N/2}}\cdot s^N K_N\left(2\sqrt{\frac{s}{\beta_1}}\right)K_N\left(2\sqrt{\frac{s}{N\beta_2}}\right)}$$

$\boxed{N = n_t^S = N_r^P}$ $\quad\gamma^{NS}(\rho) = c\cdot\rho/W$ CONTINUE ... P.53

FROM END-TO-END... PAPER FROM HASNA & ALOUINI

THEOREM 1: (HARMONIC MEAN OF TWO EXPONENTIAL RV)

LET X_1 AND X_2 ARE TWO INDEPENDENT EXPONENTIAL RV WITH PARAMETERS β_1, β_2 I.E $X_i \sim \mathcal{E}(\beta_i)$ $i = 1, 2$ THEN CDF OF $X = \mu_H(X_1, X_2)$ IS GIVEN BY:

$$\boxed{P_X(x) = 1 - x\sqrt{\beta_1\beta_2}\, e^{-\frac{x}{2(\beta_1+\beta_2)}} K_1\left(x\sqrt{\beta_1\beta_2}\right)}$$

$$\boxed{p_X(x) = \beta e^{-\beta x} U(x)}$$

VNIMAVAJ KAKO E DEFINIRANA EXPONENCIJALNATA PDF!

$$x = \frac{2\cdot x_1 x_2}{x_1 + x_2} \qquad \frac{1}{x} = \frac{2}{x_1} + \frac{2}{x_2}$$

PROOF. WE DEFINE AUXILIARY RANDOM VARIABLE. $Z =$

$$z = \frac{1}{2}\left(\frac{1}{x_1} + \frac{1}{x_2}\right) \qquad Z = \frac{1}{\mu_H(x_1, x_2)}$$

$$Y = \frac{1}{x} \Rightarrow p_Y(y) = \frac{\beta}{y^2} e^{-\beta/y} U(y)$$

$$M_Y(s) = \mathcal{E}_Y\left\{e^{-sy}\right\} = ?$$

- GRADSHTEIN (3.471.9)

$$\int_0^\infty x^{\nu-1} e^{-\frac{\beta}{x}-\gamma x}\,dx = 2\left(\frac{\beta}{\gamma}\right)^{\frac{\nu}{2}} K_\nu\left(2\sqrt{\beta\gamma}\right)$$

$$M_Y(s) = \int_0^\infty e^{-sy}\cdot\frac{\beta}{y^2} e^{-\frac{\beta}{y}}\,dy = \beta\int_0^\infty x^{-2} e^{-sx-\frac{\beta}{x}}\,dx =$$

$$= \beta\cdot 2\left(\frac{\beta}{s}\right)^{-\frac{1}{2}} \underbrace{K_{-1}}_{=K_1}\left(2\sqrt{\beta\cdot s}\right) = 2\beta\cdot\sqrt{\frac{s}{\beta}}\,K_1\left(2\sqrt{\beta s}\right)$$

$$\boxed{M_Y(s) = 2\sqrt{\beta s}\,K_1\left(2\sqrt{\beta s}\right)}$$

$$M_Z(s) = \frac{1}{2}\cdot M_{Y_1}(s)\cdot M_{Y_2}(s) = \sqrt{\beta_1 s}\,K_1\left(2\sqrt{\beta_1 s}\right)\cdot 2\sqrt{\beta_2 s}\,K_1\left(2\sqrt{\beta_2 s}\right)$$

$$\boxed{M_Z(s) = 2\sqrt{\beta_1\beta_2}\cdot s\,K_1\left(2\sqrt{\beta_1 s}\right)K_1\left(2\sqrt{\beta_2 s}\right)}$$

CDF OF $X = \mu_H(X_1, X_2)$ IS: MMV

$$P_X(x) = \Pr(X < x) = \Pr\left(\frac{1}{X} > \frac{1}{x}\right) = \Pr\left(Z > \frac{1}{x}\right) =$$

$$= 1 - \Pr\left(Z < \frac{1}{x}\right) = 1 - \underbrace{P_Z\left(\frac{1}{x}\right)}_{\text{CDF OF } Z = \alpha}$$

⊛ → PP. 36

$$M(s) = \mathcal{L}\{p(x)\} = \int_0^\infty p(x)\cdot e^{-sx}\,dx \qquad \cancel{\frac{dM(s)}{ds} = -s\int_0^\infty p(x)\,e^{-sx}\,dx}$$

$$P_X(x) = \int_0^x p(x)\,dx$$

$$\underline{M(-s)} = \int_0^\infty p(x)\,e^{sx}\,dx = \hat{p}(s)$$

$$p(x) = \frac{dP_X(x)}{dx} \qquad M(-s) = \int_0^\infty \frac{dP_X(x)}{dx}\,e^{-sx}\,dx$$

$$\mathcal{L}[f'(x)] = s\cdot F(s) \qquad s\cdot\hat{P}(s) = M(-s) \qquad \hat{P}(s) = \frac{\hat{p}(s)}{s}$$

$$\boxed{P_X(x) = \mathcal{L}^{-1}\left[\frac{M(-s)}{s}\right]}$$

$$F(s) = \int_{-\infty}^{\infty} f(x)\, e^{-sx}\, dx \qquad f(x) = \frac{1}{2\pi} \int_{\sigma - j\infty}^{\sigma + j\infty} F(s)\, e^{+sx}\, ds$$

$$\frac{d f(x)}{dx} = \frac{1}{2\pi} \int_{\sigma - j\infty}^{\sigma + j\infty} s \cdot F(s)\, e^{sx}\, ds \quad \Leftrightarrow$$

$$\mathcal{L}\left[\frac{d f(x)}{dt}\right] = \underline{s \cdot F(s)}$$

$$P_z(z) = \mathcal{L}^{-1}\left[\frac{M_z(-s)}{s}\right] \qquad P_x(x) = 1 - \mathcal{L}^{-1}\left[\frac{M_z(-s)}{s}\right]\Bigg|_{z = \frac{1}{x}}$$

$$P_x(x) = 1 - \mathcal{L}^{-1}\left[\frac{2\sqrt{\beta_1\beta_2}\, s \cdot K_1(2\sqrt{\beta_1 s})\, K_1(2\sqrt{\beta_2 s})}{s}\right]\Bigg|_{z = \frac{1}{x}}$$

$$\boxed{P_x(x) \doteq 1 - \mathcal{L}^{-1}\left[2\sqrt{\beta_1\beta_2}\; K_1(2\sqrt{\beta_1 s})\, K_1(2\sqrt{\beta_2 s})\right]\Bigg|_{z = \frac{1}{x}}} \quad ⑤$$

$$\boxed{P_x(x) = 1 - x\sqrt{\beta_1\beta_2}\; e^{-x/2(\beta_1+\beta_2)}\; K_1(x\sqrt{\beta_1\beta_2})}$$

- PRUDNIKOV VOL. 4 (3.16.6.6)

$$u_\pm = \sqrt{b}\,(\sqrt{p+a} \pm \sqrt{p-a}) \qquad \underline{v_\pm = \sqrt{p}\,(\sqrt{b+a} \pm \sqrt{b-a})} \qquad \boxed{p \triangleq 1}$$

$$\mathcal{L}\left[\frac{1}{x}\, e^{-b/x}\, K_\nu\left(\frac{a}{x}\right)\right] = \underline{2 K_\nu(v_-) \cdot K_\nu(v_+)}$$

$$\begin{cases} 2\sqrt{\beta_1} = \sqrt{b+a} + \sqrt{b-a} \\ 2\sqrt{\beta_2} = \sqrt{b+a} - \sqrt{b-a} \end{cases} \qquad \begin{aligned} v_+ &= 2\sqrt{s} \cdot \sqrt{\beta_1} \\ v_- &= 2\sqrt{s} \cdot \sqrt{\beta_2} \end{aligned}$$

$$2K_1(v_-) \cdot K_1(v_+) = 2K_1(2\sqrt{\beta_2 s}) \cdot K_1(2\sqrt{\beta_1 s})$$

$$= \mathcal{L}\left[\frac{1}{\cancel{z}}\, e^{-b/\cancel{z}}\, K_1\left(\frac{a}{\cancel{z}}\right)\right] = ①$$

$$2\,\cancel{4}\,\beta_1 = b + \cancel{a} + 2\sqrt{b^2 - a^2} + b - \cancel{a} = \cancel{2}b + \cancel{2}\sqrt{\cancel{\beta}^2 - a^2}$$

$$4\beta_2 = b + \cancel{a} - 2\sqrt{b^2 - a^2} + b - \cancel{a} = 2b - 2\sqrt{b^2 - a^2}$$

$$\cancel{2}\beta_1 = b + \sqrt{b^2 - a^2} \qquad \sqrt{b^2 - a^2} = 2\beta_1 - b \quad (*)$$

$$4\beta_2 = 2b - 2(2\beta_1 - b) = 2b - 4\beta_1 + 2b$$

$$\boxed{b = (\beta_1 + \beta_2)}$$

⊛ ⇒ $4(p_1+p_2)^2 - a^2 = 4p_1^2 - 2\overset{2(p_1+p_2)}{b}p_1 + b^2 = 4p_1^2 - 4(p_1+p_2)p_1 + 4(p_1+p$

$a^2 = \cancel{4p_1^2} + 4p_1p_2 - \cancel{4p_1^2}$ $\boxed{a = 2\sqrt{p_1p_2}}$

$$\boxed{\begin{aligned}\mathcal{L}^{-1}\left[2K_1(2\sqrt{p_1s})K_1(2\sqrt{p_1s})\right] &= \\ &= \frac{1}{z}e^{-b/z}K_1\left(\frac{a}{z}\right)\end{aligned}}\ \$$$

① $= \mathcal{L}\left[\frac{1}{z}e^{-\frac{2(p_1+p_2)}{z}}\cdot K_1\left(\frac{2\sqrt{p_1p_2}}{z}\right)\right]$

$$P_x(x) = 1 - \mathcal{L}^{-1}\left[\sqrt{p_1p_2}\ \underbrace{2K_1(2\sqrt{p_1s})K_1(2\sqrt{p_2s})}_{\#}\right]\Bigg|_{z=\frac{1}{x}}$$

$$P_x(x) = 1 - \sqrt{p_1p_2}\ \underbrace{\mathcal{L}^{-1}\left[2K_1(2\sqrt{p_1s})K_1(2\sqrt{p_2s})\right]}_{\#}\Bigg|_{z=\frac{1}{x}}$$

$$P_x(x) = 1 - \sqrt{p_1p_2}\cdot\frac{1}{z}e^{-\frac{2(p_1+p_2)}{z}}\cdot K_1\left(\frac{2\sqrt{p_1p_2}}{z}\right)\Bigg|_{z=\frac{1}{x}}$$

$$\boxed{P_x(x) = 1 - \sqrt{p_1p_2}\,x\,e^{-2x(p_1+p_2)}K_1(2x\sqrt{p_1p_2})}$$

⊛ ⇒ $\cancel{b^2} - a^2 = 4p_1^2 - 4p_1\cdot b + b^2 = 4p_1^2 - 8p_1(p_1+p_2) + \cancel{4(p_1+p_2)^2}$

$a^2 = 4p_1^2 - 8p_1^2 + 8p_2p_1 = 8p_2p_1 - 4p_1^2 = 4p_1(2p_2 - p_1)$

⊛ ⇒ $\cancel{b^2} - a^2 = 4p_1^2 - 4p_1b + \cancel{b^2}$ $a^2 = 4p_1(p_1+p_2) - 4p_1^2$

$a^2 = 4p_1p_2$ $\boxed{a = \pm 2\sqrt{p_1p_2}}$

MMV

\#

④ ⇒ $\boxed{\mathcal{L}^{-1}\left[2K_\nu(2\sqrt{p_2s})\cdot K_\nu(2\sqrt{p_1s})\right] = \frac{1}{z}e^{-\frac{p_1+p_2}{z}}\cdot K_\nu\left(\frac{2\sqrt{p_1p_2}}{z}\right)}$

\$ ⇒ $P_x(x) = 1 - \sqrt{p_1p_2}\,\mathcal{L}^{-1}\left[2K_1(2\sqrt{p_1s})K_1(2\sqrt{p_2s})\right]\Bigg|_{z=\frac{1}{x}}$

$$\boxed{P_x(x) = 1 - \sqrt{p_1p_2}\cdot x\,e^{-x(p_1+p_2)}\cdot K_1(2x\sqrt{p_1p_2})}$$

SE RAZLIKUVA
MALKU OD (12)
VO ~~END-TO-END~~
MASNA PAPER

- ALTERNATIVELY 3.16.6.3 (PRUDNIKOV)

$(2\sqrt{p_1s})^2 = (\sqrt{b}\sqrt{s+a} + \sqrt{b}\sqrt{s-a})^2 = b(s+a) + 2b\sqrt{s^2-a^2} + b(s-a)$

$(2\sqrt{p_2s})^2 = (\sqrt{b}\sqrt{s+a} - \sqrt{b}\sqrt{s-a})^2 = b(s+a) - 2b\sqrt{s^2-a^2} + b(s-a)$ | ±

$4(p_1+p_2)s = 2b(s+a) + 2b(s-a) = 2bs + \cancel{2ba} + 2bs - \cancel{2ba}$

$4(p_1+p_2) = 4b \Rightarrow b = (p_1+p_2)$

$\cancel{x}(p_1-p_2)s = \cancel{x}b\sqrt{s^2-a^2}$ $\quad (p_1-p_2)^2s^2 = b^2(s^2-a^2)$

$(p_1-p_2)^2s^2 = (p_1+p_2)^2s^2 - (p_1+p_2)^2a^2$

$(p_1+p_2)^2a^2 = \left[\underbrace{(p_1+p_2)^2-(p_1-p_2)^2}_{4p_1p_2}\right]s^2$

$$\boxed{a^2 = \frac{4p_1p_2}{(p_1+p_2)^2}\cdot s^2}$$

$4p_2\cdot s = b(s+a) - 2b\sqrt{s^2-a^2} + b(s-a)$

$4p_2s = 2bs - 2b\sqrt{s^2-a^2}$ $\quad \cancel{-4p_2s + 2(p_1+p_2)s = 2b\sqrt{s^2-a^2}}$

$2(p_1-p_2)s = 2b\sqrt{s^2-a^2}$

$\cancel{p_1^2s^2 = (p_1+p_2)^2(s^2-a^2)}$

PAK ISTOTO !!

$(p_1-p_2)s = b\sqrt{s^2-a^2}$

$(p_1^2-2p_1p_2+p_2^2)s^2 = (p_1^2+2p_1p_2+p_2^2)(s^2-a^2)$

$-2p_1p_2s^2 = 2p_1p_2s^2 - (p_1+p_2)^2a^2$

$$\boxed{a^2 = \frac{4p_1p_2}{(p_1+p_2)^2}s^2}$$

NE DAVA NIŠTO
SO 3.6.6.3 !!!

$$\boxed{P_x(x) = 1-\sqrt{p_1p_2}\,x\,e^{-x(p_1+p_2)}K_1(2x\sqrt{p_1p_2})}$$ (*1)

$p_x(x) = \frac{dP_x(x)}{dx}$ $\quad \mathcal{Z}'_\nu(z) = \mathcal{Z}_{\nu-1}(z) - \frac{\nu}{z}\mathcal{Z}_\nu(z)$

$$\boxed{\mathcal{Z}_\nu = I_\nu,\ e^{\nu\pi i}K_\nu}$$

$K_1'(x) = -K_0(x) - \frac{1}{x}\cdot K_1(x)$

$\cancel{e^{\pi i\nu}}^{-1}K_\nu'(z) = \cancel{e^{-\pi i\cdot 0}}^{1}K_0(z) - \frac{1}{z}\cancel{e^{-\pi i}}^{-1}K_1(z)$ $\quad -K_\nu'(z) = K_0(z) + \frac{K_1(z)}{z}$

$$\boxed{K_1'(z) = -K_0(z) - \frac{K_1(z)}{z}}$$

$P_x'(x) = -\sqrt{p_1p_2}\left(xe^{-x(p_1+p_2)}\right)'\cdot K_1(2x\sqrt{p_1p_2})$

$+\sqrt{p_1p_2}\,x\,e^{-x(p_1+p_2)}\cdot 2\sqrt{p_1p_2}\left(K_0(2x\sqrt{p_1p_2}) + \frac{K_1(2x\sqrt{p_1p_2})}{2x\sqrt{p_1p_2}}\right))$

$$p_x(x) = -\sqrt{\beta_1\beta_2}\left[e^{-x(\beta_1+\beta_2)}(\beta_1+\beta_2)x e^{-x(\beta_1+\beta_2)}\right]K_1(2x\sqrt{\beta_1\beta_2}) +$$

$$+ 2\beta_1\beta_2 x\, e^{-x(\beta_1+\beta_2)} K_0(2x\sqrt{\beta_1\beta_2}) + \cancel{\sqrt{\beta_1\beta_2}\, e^{-x(\beta_1+\beta_2)} K_1(2x\sqrt{\beta_1\beta_2})}$$

$$p_x(x) = \overset{\sqrt{\beta_1\beta_2}}{\vee}(\beta_1+\beta_2)x e^{-x(\beta_1+\beta_2)} K_1(2x\sqrt{\beta_1\beta_2}) + 2\beta_1\beta_2 x\, e^{-x(\beta_1+\beta_2)} K_0(2x\sqrt{\beta_1\beta_2})$$

$$\boxed{p_x(x) = \beta_1\beta_2 x\, e^{-x(\beta_1+\beta_2)}\left[\frac{\beta_1+\beta_2}{\sqrt{\beta_1\beta_2}} K_1(2x\sqrt{\beta_1\beta_2}) + 2K_0(2x\sqrt{\beta_1\beta_2})\right]}$$

→ PROVERENO OD: MALE OVA E DEFINITIVNO IZVODOT OD ⓶.

- AKO SE ODI SO IZRAZOT OD MASNA:

$$P_x(x) = 1 - x\sqrt{\beta_1\beta_2}\, e^{-\frac{x}{2(\beta_1+\beta_2)}} K_1(x\sqrt{\beta_1\beta_2})$$

MALE: $(\sqrt{\beta_1^3}\cdot\sqrt{\beta_2} + \sqrt{\beta_2^3}\cdot\sqrt{\beta_1})$

$$\frac{\partial P_x(x)}{\partial x} = \frac{1}{2}\,\frac{x\sqrt{\beta_1\beta_2}\, e^{-x/2(\beta_1+\beta_2)}}{\beta_1+\beta_2}\left[K_1(x\sqrt{\beta_1\beta_2}) + 2\,(\sqrt{\beta_1^3}\cdot\sqrt{\beta_2} + \sqrt{\beta_2^3}\cdot\sqrt{\beta_1})\, K_0(x\sqrt{\beta_1\beta_2})\right]$$

$$p_x(x) = \frac{x\sqrt{\beta_1\beta_2}\, e^{-x/2(\beta_1+\beta_2)}}{2}\left[\frac{K_1(x\sqrt{\beta_1\beta_2})}{\beta_1+\beta_2} + 2\,\frac{(\beta_1\sqrt{\beta_1}\sqrt{\beta_2} + \beta_2\sqrt{\beta_1\beta_2})}{(\beta_1+\beta_2)} K_0\right]$$

$$p_x(x) = \frac{1}{2} x\sqrt{\beta_1\beta_2}\, e^{-\frac{x}{2(\beta_1\beta_2)}}\left[\frac{K_1(x\sqrt{\beta_1\beta_2})}{\beta_1+\beta_2} + 2\sqrt{\beta_1\beta_2}\, K_0(x\sqrt{\beta_1\beta_2})\right]$$

$$p_x(x) = \frac{1}{2} x\,\beta_1\beta_2\, e^{-\frac{x}{2(\beta_1+\beta_2)}}\left[\frac{K_1(x\sqrt{\beta_1\beta_2})}{\sqrt{\beta_1\beta_2}\,(\beta_1+\beta_2)} + 2K_0(x\sqrt{\beta_1\beta_2})\right]$$

CONTINUES FROM T.34

- MGF-OT OD END-TO-END PATEKOT NA LEE

$$M_W(s) = \frac{4}{(N\beta_1\beta_2)^{N/2}\,\Gamma^2(N)}\, s^N K_N\!\left(2\sqrt{\frac{s}{\beta_1}}\right) K_N\!\left(2\sqrt{\frac{s}{N\beta_2}}\right)$$

$$\mathcal{L}^{-1}\left[\frac{M_W(s)}{s^N}\right] = \mathcal{L}^{-1}\left[\frac{4}{(N\beta_1\beta_2)^{N/2}\,\Gamma^2(N)} K_N\!\left(2\sqrt{\frac{s}{\beta_1}}\right) K_N\!\left(2\sqrt{\frac{s}{N\beta_2}}\right)\right]$$

$$= \frac{2}{(N\beta_1\beta_2)^{N/2}\,\Gamma^2(N)}\,\mathcal{L}^{-1}\left[2K_N\!\left(2\sqrt{\frac{s}{\beta_1}}\right) K_N\!\left(2\sqrt{\frac{s}{N\beta_2}}\right)\right]$$

⓷ pp.37

$$\boxed{\mathcal{L}^{-1}\left[2K_1(2\sqrt{\alpha_1 s})\cdot 2K_1(2\sqrt{\beta_1 s})\right] = \frac{1}{t}\, e^{-\frac{\alpha_1+\alpha_2}{t}} K_1\!\left(\frac{2\sqrt{\alpha_1\alpha_2}}{t}\right)}$$

$$\mathcal{L}^{-1}\left[\frac{M_W(s)}{s^N}\right] = \left|\begin{matrix}\alpha_1 = \frac{1}{\beta_1}\\ \alpha_2 = \frac{1}{N\beta_2}\end{matrix}\right| = \frac{2}{(N\beta_1\beta_2)^{N/2}\,\Gamma^2(N)}\cdot\frac{1}{t}\, e^{\frac{\frac{1}{\beta_1}+\frac{1}{N\beta_1}}{t}} K_1\!\left(\frac{2}{t\sqrt{N\beta_1\beta_2}}\right)$$

$$\mathcal{L}^{-1}\left[\frac{M_W(s)}{s^N}\right] = \frac{2\, e^{-\frac{\beta_1+N\beta_2}{\omega N \beta_1\beta_2}}}{(N\beta_1\beta_2)^{N/2}\,\Gamma^2(N)\,\omega}\cdot K_N\left(\frac{2}{\sqrt{N\beta_1\beta_2}\,\omega}\right)$$

$$F_{\gamma NS}(\gamma) = 1 - \frac{d^{(N-1)}}{d\omega^{(N-1)}}\mathcal{L}^{-1}\left[\frac{M_W(s)}{s^N}\right]\Bigg|_{\omega=\frac{c\rho}{\gamma}}$$

$\boxed{N=2}$

$$F_{\gamma NS}(\gamma) = 1 - \frac{d}{d\omega}\mathcal{L}^{-1}\left[\frac{M_W(s)}{s^N}\right]\Bigg|_{\omega=\frac{c\rho}{\gamma}}$$

$$F_{\gamma NS}(\gamma) = 1 - \frac{d}{d\omega}\left[\frac{\cancel{2}\, e^{-\frac{\beta_1+N\beta_2}{\omega\cancel{2}\beta_1\beta_2}}}{(\cancel{2}\beta_1\beta_2)^{\cancel{2}/2}\,\cancel{\Gamma^2(2)}\,\omega}\cdot K_2\left(\frac{\sqrt{2}}{\sqrt{\beta_1\beta_2}\,\omega}\right)\right]$$

① $-\frac{a}{\omega}$

$$= 1 - \frac{1}{\beta_1\beta_2}\left[\left(\frac{e^{-\frac{a}{\omega}}}{\omega}\right)'\cdot K_2\left(\frac{1}{\omega}\sqrt{\frac{2}{\beta_1\beta_2}}\right) + \sqrt{\frac{2}{\beta_1\beta_2}}\,\frac{e^{-\frac{a}{\omega}}}{\omega^3}\left(K_1\left(\frac{1}{\omega}\sqrt{\frac{2}{\beta_1\beta_2}}\right) + \frac{2}{\boxed{\omega}}K_2\left(\frac{1}{\omega}\sqrt{\frac{2}{\beta_1\beta_2}}\right)\right)\right]$$

TOCNA $\frac{1}{\omega}\sqrt{\frac{2}{\beta_1\beta_2}}$

$$\mathcal{Z}_\nu'(z) = \mathcal{Z}_{\nu-1}(z) - \frac{\nu}{z}\mathcal{Z}_\nu(z) \qquad \mathcal{Z}_\nu = e^{\nu\pi i}\cdot K_\nu$$

$$\cancel{e^{2\pi i}}\cdot K_2' = \cancel{e^{\pi i}}K_1(z) - \frac{2}{z}\cancel{e^{2\pi i}}K_2$$

$1 \qquad -1 \qquad 1$

$$\boxed{K_2' = -K_1(z) - \frac{2}{z}K_2(z)}$$

$$\boxed{a = \frac{\beta_1+\cancel{2}\beta_2}{2\beta_1\beta_2}} \qquad \boxed{K_1'(z) = -K_0(z) - \frac{K_1(z)}{z}}$$

$$\left(\frac{1}{\omega}\sqrt{\frac{2}{\beta_1\beta_2}}\right)' = -\sqrt{\frac{2}{\beta_1\beta_2}}\,\frac{1}{\omega^2}$$

$$\left(\frac{e^{-\frac{a}{\omega}}}{\omega}\right)' = +\frac{a}{\omega^2}\,\frac{e^{-\frac{a}{\omega}}}{\omega} - \frac{e^{-\frac{1}{\omega}}}{\omega^2} = \frac{a}{\omega^3}e^{-\frac{a}{\omega}}\left(1-\frac{\omega}{a}\right)$$

$$= \frac{a-\omega}{\omega^3}e^{-a/\omega}$$

$$F_{\gamma NS}(\gamma) = 1 - \frac{1}{\beta_1\beta_2}\left[\frac{a-\omega}{\omega^3}e^{-\frac{1}{\omega}}K_2\left(\frac{1}{\omega}\sqrt{\frac{2}{\beta_1\beta_2}}\right) + \sqrt{\frac{2}{\beta_1\beta_2}}\,\frac{e^{-\frac{a}{\omega}}}{\omega^3}\left[K_1\left(\frac{1}{\omega}\sqrt{\frac{2}{\beta_1\beta_2}}\right) + \frac{2}{\cancel{\omega}}K_2\left(\frac{1}{\omega}\sqrt{\frac{2}{\beta_1\beta_2}}\right)\right.\right.$$

PRODOLZUVA NA STR. 42

VAZA TOCNA $\frac{1}{\omega}\sqrt{\frac{2}{\beta_1\beta_2}}$

#8

$$\boxed{F_{\gamma NS}(\gamma) = 1 - \frac{1}{\beta_1\beta_2}\,\frac{e^{-\frac{a}{\omega}}}{\omega^3}\left[(a-\omega)K_2\left(\frac{1}{\omega}\sqrt{\frac{2}{\beta_1\beta_2}}\right) + \sqrt{\frac{2}{\beta_1\beta_2}}\,K_1\left(\frac{1}{\omega}\sqrt{\frac{2}{\beta_1\beta_2}}\right) + \frac{2}{\boxed{\omega}}\sqrt{\frac{2}{\beta_1\beta_2}}\,K_2\left(\frac{1}{\omega}\sqrt{\frac{2}{\beta_1\beta_2}}\right)\right.}$$

$$F_{\gamma NS}(\gamma) = 1 - \frac{1}{\beta_1\beta_2}\,\frac{e^{-\frac{a}{\omega}}}{\omega^3}\left[\underbrace{\left(a-\omega+\frac{2}{\omega}\sqrt{\frac{2}{\beta_1\beta_2}}\right)}_{①}K_2\left(\frac{1}{\omega}\sqrt{\frac{2}{\beta_1\beta_2}}\right) + \sqrt{\frac{2}{\beta_1\beta_2}}\,K_1\left(\frac{1}{\omega}\sqrt{\frac{2}{\beta_1\beta_2}}\right)\right]$$

$$① = \frac{\beta_1+\cancel{2}\beta_2}{2\beta_1\beta_2} - \omega + \frac{2}{\omega}\sqrt{\frac{2}{\beta_1\beta_2}} = \frac{\beta_1\omega+\cancel{2}\beta_2\omega - 2\beta_1\beta_2\omega^2 + 2\sqrt{\frac{2}{\beta_1\beta_2}}\cdot 2\beta_1\beta_2}{2\beta_1\beta_2\omega}$$

$$= \frac{\beta_1\omega+\cancel{2}\beta_2\omega - 2\beta_1\beta_2\omega^2 - 4\sqrt{2\beta_1\beta_2}}{2\beta_1\beta_2\omega}$$

$$\boxed{\omega = \frac{c\varrho}{\gamma}}$$

$$\boxed{F_\gamma^{NS}(\gamma) = 1 - \frac{\cdot\varrho^3}{\rho_1\rho_2}\,\frac{e^{-\frac{a\cdot\gamma}{c\varrho}}}{\gamma^3}\underbrace{\left[\left(a - \frac{c\varrho}{\gamma} + \frac{2\gamma}{c\varrho}\sqrt{\frac{2}{\rho_1\rho_2}}\right)K_2\left(\frac{\gamma}{c\varrho}\sqrt{\frac{2}{\rho_1\rho_2}}\right) + \sqrt{\frac{2}{\rho_1\rho_2}}\,K_1\left[\frac{\gamma}{c\varrho}\sqrt{\frac{2}{\rho_1\rho_2}}\right]\right]}_{f_1(\gamma)}}$$

$$f_\gamma^{NS}(\gamma) = \frac{d F_\gamma^{NS}(\gamma)}{d\gamma} = -\frac{c\cdot\varrho}{\rho_1\rho_2}\cdot\left[\left(\frac{e^{-\frac{a\gamma}{c\varrho}}}{\gamma^3}\right)'\cdot f_1 + \frac{e^{-\frac{a\gamma}{c\varrho}}}{\gamma^3} f_1'(\gamma)\right]$$

$$f_1'(\gamma) = \left(\frac{c\varrho}{\gamma^2} + \frac{2}{c\varrho}\sqrt{\frac{2}{\rho_1\rho_2}}\right)\cdot K_2\left(\frac{\gamma}{c\varrho}\sqrt{\frac{2}{\rho_1\rho_2}}\right) - \left(a - \frac{c\varrho}{\gamma} + \frac{2\gamma}{c\varrho}\sqrt{\frac{2}{\rho_1\rho_2}}\right)\cdot\frac{1}{c\varrho}\sqrt{\frac{2}{\rho_1\rho_2}}\cdot$$

$$\cdot\left[K_1\left(\frac{\gamma}{c\varrho}\sqrt{\frac{2}{\rho_1\rho_2}}\right) + \frac{2\cdot K_2\left(\frac{\gamma}{c\varrho}\sqrt{\frac{2}{\rho_1\rho_2}}\right)}{\frac{\gamma}{c\varrho}\sqrt{\frac{2}{\rho_1\rho_2}}}\right] - \sqrt{\frac{2}{\rho_1\rho_2}}\cdot\frac{1}{c\varrho}\sqrt{\frac{2}{\rho_1\rho_2}}\left[K_0\left(\frac{\gamma}{c\varrho}\sqrt{\frac{2}{\rho_1\rho_2}}\right) + \frac{K_1\left(\frac{\gamma}{c\varrho}\sqrt{\frac{2}{\rho_1\rho_2}}\right)}{\frac{\gamma}{c\varrho}\sqrt{\frac{2}{\rho_1\rho_2}}}\right]$$

$$f_1'(\gamma) = \underline{\left(\frac{c\varrho}{\gamma^2} + \frac{2}{c\varrho}\sqrt{\frac{2}{\rho_1\rho_2}}\right)\cdot K_2\left(\frac{\gamma}{c\varrho}\sqrt{\frac{2}{\rho_1\rho_2}}\right)} - \cancel{\left(a - \frac{c\varrho}{\gamma} + \frac{2\gamma}{c\varrho}\sqrt{\frac{2}{\rho_1\rho_2}}\right)} - \left(\frac{a}{c\varrho}\sqrt{\frac{2}{\rho_1\rho_2}} - \frac{1}{\gamma}\sqrt{\frac{2}{\rho_1\rho_2}} +\right.$$

$$\left. + \frac{2\gamma}{c^2\varrho^2}\,\frac{2}{\rho_1\rho_2}\right)\left[\underline{K_1\left(\frac{\gamma}{c\varrho}\sqrt{\frac{2}{\rho_1\rho_2}}\right)} + \underline{\frac{2c\varrho}{\gamma}\sqrt{\frac{\rho_1\rho_2}{2}}\,K_2\left(\frac{\gamma}{c\varrho}\sqrt{\frac{2}{\rho_1\rho_2}}\right)}\right] - \frac{1}{c\varrho}\,\frac{2}{\rho_1\rho_2}\left[K_0 + \underline{\frac{c\varrho}{\gamma}\sqrt{\frac{\rho_1\rho_2}{2}}\,K_1}\right]$$

$$= \left[\frac{c\varrho}{\gamma^2} + \frac{2}{c\varrho}\sqrt{\frac{2}{\rho_1\rho_2}} - \frac{2c\varrho}{\gamma}\sqrt{\frac{\rho_1\rho_2}{2}}\left(\frac{a}{c\varrho}\sqrt{\frac{2}{\rho_1\rho_2}} - \frac{1}{\gamma}\sqrt{\frac{2}{\rho_1\rho_2}} + \frac{2\gamma}{c^2\varrho^2}\,\frac{2}{\rho_1\rho_2}\right)\right]K_2\left(\frac{\gamma}{c\varrho}\sqrt{\frac{2}{\rho_1\rho_2}}\right)$$

$$- \left(\frac{a}{c\varrho}\sqrt{\frac{2}{\rho_1\rho_2}} - \frac{1}{\gamma}\sqrt{\frac{2}{\rho_1\rho_2}} + \frac{2\gamma}{c^2\varrho^2}\,\frac{2}{\rho_1\rho_2} + \frac{c\varrho}{\gamma}\sqrt{\frac{\rho_1\rho_2}{2}}\right)K_1\left(\frac{\gamma}{c\varrho}\sqrt{\frac{2}{\rho_1\rho_2}}\right) - \frac{2}{\rho_1\rho_2 c\varrho}K_0\left(\frac{\gamma}{c\varrho}\sqrt{\frac{2}{\rho_1\rho_2}}\right)$$

$$\left(\frac{e^{-\frac{a\gamma}{c\varrho}}}{\gamma^3}\right)' = -\frac{a}{c\varrho}\,\frac{e^{-\frac{a\gamma}{c\varrho}}}{\gamma^3} - 3\,\frac{e^{-\frac{a\gamma}{c\varrho}}}{\gamma^4} = -\frac{e^{-\frac{a\gamma}{c\varrho}}}{\gamma^3}\left(\frac{a}{c\varrho} + \frac{3}{\gamma}\right) = \frac{e^{-\frac{a\gamma}{c\varrho}}}{\gamma^3}\,\frac{a\gamma+3c\varrho}{c\varrho\gamma}$$

$$f_\gamma^{NS}(\gamma) = -\frac{c\varrho}{\rho_1\rho_2}\,\frac{e^{-\frac{a\gamma}{c\varrho}}}{\gamma^3}\left[\frac{a\gamma+3c\varrho}{c\varrho\gamma}\left(a - \frac{c\varrho}{\gamma} + \frac{2\gamma}{c\varrho}\sqrt{\frac{2}{\rho_1\rho_2}}\right)K_2 + \frac{a\gamma+3c\varrho}{c\varrho\gamma}\sqrt{\frac{2}{\rho_1\rho_2}}\,K_1 + f'(\gamma)\right]$$

$$= -\frac{c\varrho}{\rho_1\rho_2}\,\frac{e^{-\frac{a\gamma}{c\varrho}}}{\gamma^3}\left\{\left(\frac{c\varrho}{\gamma^2} + \frac{2}{c\varrho}\sqrt{\frac{2}{\rho_1\rho_2}} - \frac{2a}{\gamma} + \frac{2c\varrho}{\gamma^2} - \frac{a\sqrt{2}}{c\varrho\sqrt{\rho_1\rho_2}} + \frac{a^2\gamma+3ac\varrho}{c\varrho\gamma}\cdot\sqrt{\frac{2}{\rho_1\rho_2}} - \frac{a\gamma+3c\varrho}{\gamma^2}\right.\right.$$

$$\left.\frac{2(a\gamma+3c\varrho)}{c^2\varrho^2}\sqrt{\frac{2}{\rho_1\rho_2}}\right)K_2 - \underbrace{\left(\frac{a}{c\varrho}\sqrt{\frac{2}{\rho_1\rho_2}} - \frac{1}{\gamma}\sqrt{\frac{2}{\rho_1\rho_2}} + \frac{2\gamma}{c^2\varrho^2}\,\frac{2}{\rho_1\rho_2} + \frac{a\gamma+3c\varrho}{c\varrho\gamma} + \frac{c\varrho}{\gamma}\sqrt{\frac{\rho_1\rho_2}{2}}\right)}_{k_1}\cdot$$

$$\left.\cdot K_1\left(\frac{\gamma}{c\varrho}\sqrt{\frac{2}{\rho_1\rho_2}}\right) - \frac{2}{\rho_1\rho_2 c\varrho}K_0\left(\frac{\gamma}{c\varrho}\sqrt{\frac{3}{\rho_1\rho_2}}\right)\right\}$$

$$k_1 = \frac{a}{c\varrho}\sqrt{\frac{2}{\rho_1\rho_2}} + \frac{2\gamma}{c^2\varrho^2}\,\frac{2}{\rho_1\rho_2} - \frac{1}{\gamma}\sqrt{\frac{2}{\rho_1\rho_2}} + \frac{a\gamma+3c\varrho}{c\varrho\gamma}\sqrt{\frac{2}{\rho_1\rho_2}} + \frac{c\varrho}{\gamma}\sqrt{\frac{\rho_1\rho_2}{2}}\,)$$

$$k_1 = \sqrt{\frac{2}{\beta_1\beta_2}}\left(\frac{a}{c\varrho} + \frac{2\gamma}{c^2\varrho^2}\sqrt{\frac{2}{\beta_1\beta_2}} - \frac{1}{\gamma} + \frac{a\gamma+3c\varrho}{c\varrho\gamma} + \frac{c\varrho}{\gamma}\cdot\frac{\beta_1\beta_2}{2}\right)$$

$+ c^3\varrho^3\sqrt{\beta_1\beta_2\beta_1\beta_2}$

$$= \sqrt{\frac{2}{\beta_1\beta_2}}\ \frac{2a\cdot c\varrho\cdot\sqrt{\beta_1\beta_2}\,\gamma + 2\gamma\cdot 2\gamma - 2c^2\varrho^2\sqrt{\beta_1\beta_2} + 2c\varrho\sqrt{\beta_1\beta_2}\,(a\gamma+3c\varrho)}{2\cdot c^2\varrho^2\sqrt{\beta_1\beta_2}\ \gamma}$$

$$k_1 = \frac{2a\cdot c\varrho\sqrt{\beta_1\beta_2}\,\gamma + 4\gamma^2 - 2c^2\varrho^2\sqrt{\beta_1\beta_2} + 2ac\varrho\sqrt{\beta_1\beta_2}\cdot\gamma + 6c^2\varrho^2\sqrt{\beta_1\beta_2} + c^3\varrho^3\sqrt{\beta_1^3\beta_2^3}}{\sqrt{2}\,c^2\varrho^2\beta_1\beta_2\gamma}$$

$$k_1 = \frac{4ac\varrho\sqrt{\beta_1\beta_2}\,\gamma + 4c^2\varrho^2\sqrt{\beta_1\beta_2} + 4\gamma^2 + c^3\varrho^3\sqrt{\beta_1^3\beta_2^3}}{\sqrt{2}\,c^2\varrho^2\beta_1\beta_2\gamma} =$$

$$= \underline{\frac{2\sqrt{2}\,a}{c\varrho\sqrt{\beta_1\beta_2}}} + \underline{\underline{\frac{2\sqrt{2}}{\sqrt{\beta_1\beta_2}\,\gamma}}} + \underline{\frac{2\sqrt{2}\,\gamma}{c^2\varrho^2\beta_1\beta_2}} + \underline{\underline{\frac{c\varrho\sqrt{\beta_1\beta_2}}{\sqrt{2}\,\gamma}}} =$$

$$\cancel{= 4\sqrt{2}\,c\varrho\sqrt{\beta_1\beta_2}\,(a\gamma + 2c\varrho) \neq}$$

$$\cancel{\frac{2\sqrt{2}}{c\varrho\sqrt{\beta_1\beta_2}}\left(a + \frac{\gamma}{c\varrho\sqrt{\beta_1\beta_2}}\right) + \frac{2\sqrt{2}}{\gamma}\left(\frac{1}{\sqrt{\beta_1\beta_2}} + \frac{c\varrho\sqrt{\beta_1\beta_2}}{4}\right)}$$

$$= \frac{2\sqrt{2}}{c\varrho\sqrt{\beta_1\beta_2}}\cdot\frac{\beta_1+2\beta_2}{2\beta_1\beta_2} +$$

VO MAPLE: [MultiloadMIMO.mw (3.4)]

$$F(g) = \frac{-g^3e^{-\frac{ag}{c\varrho}}}{c^3\varrho^3b_1b_2}\sqrt{\frac{2}{b_1b_2}}\,K_1\left(\frac{g}{c\varrho}\sqrt{\frac{2}{b_1b_2}}\right) + 1 - \frac{e^{-\frac{ag}{c\varrho}}g^2}{b_1b_2c^2\varrho^2}\left(\frac{ag}{c\varrho}+1\right)K_2\left(\frac{g}{c\varrho}\sqrt{\frac{2}{b_1b_2}}\right)$$

$$F(g) = 1 - \frac{g^3e^{-\frac{ag}{c\varrho}}}{c^3\varrho^3b_1b_2}\sqrt{\frac{2}{b_1b_2}}\,K_1 - \frac{g^3e^{-\frac{ag}{c\varrho}}}{b_1b_2c^3\varrho^3}\left(a + \frac{c\varrho}{g}\right)K_2\left(\frac{g}{c\varrho}\sqrt{\frac{2}{\beta_1\beta_2}}\right)$$

$$F(g) = 1 - \frac{g^3e^{-\frac{ag}{c\varrho}}}{c^3\varrho^3b_1b_2}\left(\sqrt{\frac{2}{b_1b_2}}\,K_1\left(\frac{g}{c\varrho}\sqrt{\frac{2}{b_1b_2}}\right) + \left(a+\frac{c\varrho}{g}\right)K_2\left(\frac{g}{c\varrho}\sqrt{\frac{2}{\beta_1\beta_2}}\right)\right)$$

– KOREKCIJA NA IZVEDUVANJETO OD PP. 40

pp40 (**) ZA KRATI

$$F_{\gamma NS}(\gamma) = 1 - \frac{e^{-a/\omega}}{\beta_1\beta_2\omega^3}\left[(a-\omega)K_2\left(\frac{1}{\omega}\sqrt{\frac{2}{\beta_1\beta_2}}\right) + \sqrt{\frac{2}{\beta_1\beta_2}}\,K_1\left(\frac{1}{\omega}\sqrt{\frac{2}{\beta_1\beta_2}}\right) + 2\omega\sqrt{\frac{\beta_1\beta_2}{2}}\,K_2\left(\frac{\gamma}{c\varrho}\sqrt{\frac{2}{\beta_1\beta_2}}\right)\right]$$

$$\omega = \frac{c\varrho}{\gamma}\ \Big| = 1 - \frac{e^{ag/c*ro}\,\gamma^3}{\beta_1\beta_2c^3\varrho^3}\left[\left(a - \frac{c\varrho}{\gamma} + \frac{2c\varrho}{\gamma}\right)K_2\left(\frac{\gamma}{c\varrho}\sqrt{\frac{2}{\beta_1\beta_2}}\right) + \sqrt{\frac{2}{\beta_1\beta_2}}\,K_1\left(\frac{\gamma}{c\varrho}\sqrt{\frac{2}{\beta_1\beta_2}}\right)\right]$$

$$\boxed{F_\gamma^{NS}(\gamma) = 1 - \frac{\gamma^3e^{-ag/c*ro}}{\beta_1\beta_2c^3\varrho^3}\left[\left(a + \frac{c\varrho}{\gamma}\right)K_2\left(\frac{\gamma}{c\varrho}\sqrt{\frac{2}{\beta_1\beta_2}}\right) + \sqrt{\frac{2}{\beta_1\beta_2}}\,K_1\left(\frac{\gamma}{c\varrho}\sqrt{\frac{2}{\beta_1\beta_2}}\right)\right]}$$

#@

→ POTVRDENO VO MAPLE!!!

$$\cdot\ f_{\gamma NS}(\gamma) = \frac{dF_\gamma^{NS}(\gamma)}{d\gamma}$$

HMV MAPLE →

$$f_{\gamma^{NS}}(\gamma)=\frac{g^2 e^{-\frac{ag}{c\rho}}}{b_1b_2\sqrt{b_1b_2}\,c^4\rho^4}\left[\sqrt{2}(-c\rho+2ag)K_1\left(\frac{g}{c\rho}\sqrt{\frac{2}{b_1b_2}}\right)+\frac{g}{\sqrt{b_1b_2}}2K_0\left(\frac{g}{c\rho}\sqrt{\frac{2}{b_1b_2}}\right)+\right.$$

$$\left.b_1b_2a^2K_2\left(\frac{g}{c\rho}\sqrt{\frac{2}{b_1b_2}}\right)\right] \quad i$$

$$a=\frac{\mu_1+2\mu_2}{2\mu_1\mu_2}=\frac{b_1+2b_2}{2b_1b_2}$$

$$f_{\gamma^{NS}}(\gamma)=\frac{g^2 e^{-\frac{(b_1+2b_2)g}{2c\rho b_1b_2}}}{b_1b_2\sqrt{b_1b_2}c^4\rho^4}\left[\sqrt{2}(-c\rho+2ag)\cdot K_1+\frac{2g}{\sqrt{b_1b_2}}K_0+\sqrt{b_1b_2}\,g\cdot a^2K_2\right]=$$

$$\left\{\times\frac{2}{\sqrt{b_1b_2}\cdot c\rho}\right\}$$

$$=\frac{g^2\cdot e^{-\frac{(b_1+2b_2)g}{2c\rho b_1b_2}}}{2b_1b_2c^3\rho^3}\left[\frac{2\sqrt{2}}{\sqrt{b_1b_2}c\rho}\left(-c\rho+\frac{\cancel{2}(b_1+2b_2)g}{\cancel{2}b_1b_2}\right)K_1+\frac{4g}{b_1b_2c\rho}K_0+\frac{\cancel{4}g\,(b_1+2b_2)^2}{c\rho\,\cancel{4}\mu_1^2\mu_2^2}K_2\right]$$

$$f_{\gamma^{NS}}(\gamma)=\frac{g^2\cdot e^{-\frac{(b_1+2b_2)g}{2c\rho b_1b_2}}}{2b_1b_2c^3\rho^3}\left[\frac{g}{2c\rho}\left(\frac{b_1+2b_2}{b_1b_2}\right)^2\cdot K_2+\left(-\frac{2\sqrt{2}}{\sqrt{b_1b_2}}+\frac{2\sqrt{2}g}{c\rho\sqrt{b_1b_2}}\cdot\frac{b_1+2b_2}{b_1b_2}\right)K_1+\frac{4g}{b_1b_2c\rho}K_0\right]$$

$$\boxed{f_{\gamma^{NS}}(\gamma)=\frac{g^2\cdot e^{-\frac{(b_1+2b_2)g}{2c\rho b_1b_2}}}{2b_1b_2c^3\rho^3}\left[\frac{g}{2c\rho}\left(\frac{b_1+2b_2}{b_1b_2}\right)^2K_2+\left(\frac{2g}{c\rho}\cdot\frac{b_1+2b_2}{b_1b_2}\cdot\sqrt{\frac{2}{b_1b_2}}-2\sqrt{\frac{2}{b_1b_2}}\right)K_1+\frac{4g\,K_0}{b_1b_2c\rho}\right]}$$

POKAZAN IZRAZ (11) OD ČLANKOT END-TO-END... NA LEG

(k+)

- PRUDNIKOV 3.16.6.1

$$\mathcal{L}\left[x^{\mu}e^{\pm ax}K_\nu(ax)\right]=\Gamma\begin{bmatrix}1+\mu+\nu,\ 1+\mu-\nu\\ \mu+3/2\end{bmatrix}\frac{\sqrt{\pi}}{(2a)^{\mu+1}}{}_2F_1\left(1+\mu+\nu,1+\mu-\nu;\mu+\frac{3}{2};\delta-\frac{p}{2a}\right)$$

$$\delta=\begin{Bmatrix}0\\1\end{Bmatrix}\qquad \boxed{f_{\gamma^{NS}}(\gamma)=f_1+f_2+f_3}\qquad \boxed{a=\frac{b_1+2b_2}{2b_1b_2}}$$

$$f_3=\frac{g^2e^{-\frac{ag}{c\rho}}}{\cancel{2}b_1b_2}\cdot\frac{\overset{2}{\cancel{4}}g}{b_1b_2}K_0=\frac{2g^3\cdot e^{-\frac{ag}{c\rho}}}{b_1^2b_2^2}K_0\left(\frac{g}{c\rho}\sqrt{\frac{2}{b_1b_2}}\right)$$

$$MGF_3(s)=\frac{2}{b_1^2b_2^2}\int_0^\infty g^3e^{-\frac{ag}{c\rho}-sg}K_0\left(\frac{g}{c\rho}\sqrt{\frac{2}{b_1b_2}}\right)dg=\frac{2}{b_1^2b_2^2}\int_0^\infty g^3e^{-\left(\frac{a}{c\rho}+s\right)g}K_0\left(\frac{g}{c\rho}\sqrt{\frac{2}{b_1b_2}}\right)dg$$

$$a=\frac{b_1+2b_2}{2b_1b_2}=\left|\begin{matrix}b_1=1\\b_2=1\end{matrix}\right|=\frac{3}{2}\quad i\quad f_3=2g^3e^{-\frac{3g}{2c\rho}}K_0\left(\frac{g\sqrt{2}}{c\rho}\right)$$

$$\frac{b_1+2b_2}{2b_1b_2}=\sqrt{\frac{2}{b_1b_2}}\qquad\frac{(b_1+2b_2)^2}{4b_1^2b_2^2}=\frac{2}{\cancel{b_1b_2}}\qquad b_1^2+4b_2b_1+4b_2^2=$$

$$=\overset{}{\underset{4}{\cancel{8}}}b_1b_2$$

$$b_1^2-4b_1b_2+4b_2^2=0\qquad(b_1-2b_2)^2=0\qquad\boxed{b_1=2b_2}$$

$\underline{\text{If}}$: $\beta_1 = 2\beta_2 = 2\beta$ $\boxed{\beta_2 = \beta}$ $\boxed{\beta_1 = 2\beta}$ $a = \sqrt{\frac{2}{\beta_1\beta_2}} = \sqrt{\frac{2}{2\beta^2}} = \frac{1}{\beta}$

$$f_\gamma^{\beta_3}(\gamma) = \frac{\gamma^2}{4\beta^2c^3\rho^3}\, e^{-\frac{a\gamma}{c\rho}}\left[\frac{a^2\gamma}{c\rho}K_2\left(\frac{\gamma a}{c\rho}\right) + \left\{\frac{2\gamma a^2}{c\rho} - 2a\right\}K_1\left(\frac{a\gamma}{c\rho}\right) + \frac{a\gamma}{c\rho^2}K_0\left(\frac{a\gamma}{c\rho}\right)\right]$$

$$\boxed{\begin{aligned} f_1 &= \frac{\gamma^3a^2e^{-\frac{a\gamma}{c\rho}}}{4\beta^2c^4\rho^4}K_2\left(\frac{\gamma\cdot a}{c\rho}\right);\quad f_2 = \frac{\gamma^2\, e^{-\frac{a\gamma}{c\rho}}}{4\beta^2c^3\rho^3}\left\{\frac{2\gamma a^2}{c\rho} - 2a\right\}K_1\left(\frac{a\gamma}{c\rho}\right);\\ f_3 &= \frac{\gamma^3\, e^{-\frac{a\gamma}{c\rho}}}{2\beta^4c^4\rho^4}\,K_0\left(\frac{a\gamma}{c\rho}\right) \qquad f = f_1 + f_2 + f_3 \end{aligned}}$$

$\delta = \begin{Bmatrix}0\\1\end{Bmatrix}$

$$\mathcal{L}\left[x^\mu e^{\pm ax}K_\nu(ax)\right] = \Gamma\begin{bmatrix}1+\mu+\nu,\ 1+\mu-\nu\\ \mu+3/2\end{bmatrix}\frac{\sqrt{\pi}}{(2a)^{\mu+1}}\cdot {}_2F_1\left(1+\mu+\nu, 1+\mu-\nu; \mu+\tfrac{3}{2}; \delta - \frac{s}{2a}\right)$$

→ VIDI PRUDNIKOV NOTATIONS PR. 6.15

$$\cdot\ \mathcal{L}[f_3] = \frac{1}{2\beta^4c^4\rho^4}\mathcal{L}\left[\gamma^3e^{-\frac{a\gamma}{c\rho}}K_0\left(\frac{a\gamma}{c\rho}\right)\right] = \left[\Gamma(4)\Gamma(4)/\Gamma\left(3+\frac{3}{2}\right)\right]\frac{\sqrt{\pi}}{2\beta^4c^4\rho^4\left(\frac{2a}{c\rho}\right)^4}\ {}_2F_1\left(1+3, 4; \frac{9}{2}; 1 - \frac{s}{\frac{2a}{c\rho}}\right)$$

$$\mathcal{L}[f_3] = \frac{\sqrt{\pi}\,{}^{36}/\Gamma(4.5)}{2\beta^4 16\cdot a^4}\ {}_2F_1\left(4, 4; 4.5; 1 - \frac{c\rho}{2a}\cdot s\right) = \frac{\sqrt{\pi}\cdot 36}{32\ \Gamma(4.5)}\ {}_2F_1\left(4, 4; 4.5; 1 - \frac{c\rho\beta}{2}\cdot s\right)$$

$$\boxed{a = \frac{b_1 + 2b_2}{2b_1b_2} = \frac{2\beta + 2\beta}{4\beta^2} = \frac{1}{\beta}} \Leftarrow$$

$\boxed{3212672}$

$\boxed{026134688}$

$$\cdot\ \mathcal{L}[f_1] = \frac{a^2}{4\beta^2c^4\rho^4}\mathcal{L}\left[\gamma^3e^{-\frac{a\gamma}{c\rho}}K_2\left(\frac{\gamma a}{c\rho}\right)\right] = \frac{a^2 = \frac{1}{\beta^2}}{4\beta^2c^4\rho^4}\ \frac{\frac{\Gamma(6)\cdot\Gamma(2)}{\Gamma(4.5)}\sqrt{\pi}}{\frac{16}{c^4\rho^4\beta^4}}\ {}_2F_1\left(1+3+2, 1+3-2; 4.5; 1 - \frac{c\rho\beta}{2}\cdot s\right)$$

$$\boxed{\mathcal{L}[f_1] = 120\cdot\frac{\sqrt{\pi}}{64\ \Gamma(4.5)}\ {}_2F_1\left(6, 2; 4.5, 1 - \frac{c\rho\beta}{2}s\right)}$$

VH24
VGA DVI HDMI

$$\cdot\ \mathcal{L}[f_{21}] = \mathcal{L}\left[\frac{a^2\gamma^3e^{-\frac{a\gamma}{c\rho}}}{4\beta^2c^4\rho^4}K_1\left(\frac{a\gamma}{c\rho}\right)\right] = \frac{\frac{\Gamma(5)\cdot\Gamma(3)}{\Gamma(4.5)}}{4\beta^4c^4\rho^4}\ \frac{\sqrt{\pi}}{\frac{16}{c^4\rho^4\beta^4}}\cdot {}_2F_1\left(5, 3; 4.5; 1 - \frac{c\rho\beta}{2}s\right)$$

$$= \frac{48\sqrt{\pi}}{32\ \Gamma(4.5)}\ {}_2F_1\left(5, 3; 4.5; 1 - \frac{c\rho\beta}{2}s\right)$$

$$\cdot\ \mathcal{L}[f_{22}] = -\frac{1}{2\beta^3c^3\rho^3}\mathcal{L}\left[\gamma^2e^{-\frac{a\gamma}{c\rho}}K_1\left(\frac{a\gamma}{c\rho}\right)\right] = -\frac{\sqrt{\pi}\ \frac{\Gamma(4)\cdot\Gamma(2)}{\Gamma(3.5)} \xrightarrow{} ⑥}{2\beta^3c^3\rho^3\ \frac{8}{\beta^3c^3\rho^3}}\cdot {}_2F_1\left(4, 2; 3.5; 1 - \frac{c\rho\beta}{2}s\right)$$

$$\boxed{\begin{aligned}\mathcal{L}[f] = {} & 120\,\frac{\sqrt{\pi}\,\Gamma^{-1}(4.5)}{64}\ {}_2F_1\left(6, 2; 4.5; 1 - \frac{c\rho\beta}{2}s\right) + 48\cdot\frac{\sqrt{\pi}\,\Gamma^{-1}(4.5)}{32}\ {}_2F_1\left(5, 3; 4.5; 1 - \frac{c\rho\beta}{2}s\right) - \\ & - 6\cdot\frac{\sqrt{\pi}\,\Gamma^{-1}(3.5)}{16}\ {}_2F_1\left(4, 2; 3.5; 1 - \frac{c\rho\beta}{2}s\right) + 36\,\frac{\sqrt{\pi}\,\Gamma^{-1}(4.5)}{32}\ {}_2F_1\left(4, 4; 4.5; 1 - \frac{c\rho\beta}{2}s\right)\end{aligned}}$$

$$\boxed{\mathcal{L}[f(x)] = F(s) \qquad \mathcal{L}[e^{-ax} f(x)] = F(s+a)} \quad ⊛$$

- SO KORISTENJE NA ⊛ 3.16.6.1 OD PRUDNIKOV SE SVEDUVA NA 3.16.1.3

$$F_3(s) = \mathcal{L}[f_3] = \mathcal{L}\left[\frac{2\gamma^3}{\beta_1^2\beta_2^2 c^4\rho^4} e^{-\frac{a\gamma}{c\rho}} K_0\left(\frac{\gamma}{c\rho}\sqrt{\frac{2}{\beta_1\beta_2}}\right)\right] \qquad a = \frac{\beta_1+2\beta_2}{2\beta_1\beta_2}$$

(&⊛) p.43

$$F_3(s) = \frac{2}{\beta_1^2\beta_2^2 c^4\rho^4}\underbrace{\mathcal{L}\left[\gamma^3 K_0\left(\frac{\gamma}{c\rho}\sqrt{\frac{2}{\beta_1\beta_2}}\right)\right]}_{⊛} \qquad \#1$$

$$\boxed{s = s + \frac{a}{c\rho}}$$

PRUDNIKOV 3.16.1.3

$$\boxed{\mathcal{L}\left[x^{\mu} K_{\nu}(ax)\right] = \frac{(2a)^{\nu}\sqrt{\pi}}{(s+a)^{\mu+\nu+1}} \cdot \Gamma\left[\begin{matrix}\mu-\nu+1, \mu+\nu+1\\ \mu+\frac{1}{2}\end{matrix}\right] {}_2F_1\left(\mu+\nu+1, \nu+\frac{1}{2}; \mu+\frac{3}{2}; \frac{s-a}{s+a}\right)}$$

$$⊛ = \mathcal{L}\left[\gamma^3 K_0\left(\overset{a}{\frac{\gamma}{c\rho}\sqrt{\frac{2}{\beta_1\beta_2}}}\right)\right] = \left(\frac{2}{c\rho}\sqrt{\frac{2}{\beta_1\beta_2}}\right)^0 \frac{\sqrt{\pi}}{\underbrace{\left(s+\frac{1}{c\rho}\sqrt{\frac{2}{\beta_1\beta_2}}\right)}_{\psi}{}^4} \frac{\Gamma(4)\cdot\Gamma(4)}{\Gamma(3.5)} {}_2F_1\left(4, \frac{1}{2}; 4.5; \underbrace{\frac{s-\frac{1}{c\rho}\sqrt{\frac{2}{\beta_1\beta_2}}}{s+\frac{1}{c\rho}\sqrt{\frac{2}{\beta_1\beta_2}}}}_{\Omega}\right)$$

$$\Omega = \frac{s-\frac{1}{c\rho}\sqrt{\frac{2}{\beta_1\beta_2}}}{s+\frac{1}{c\rho}\sqrt{\frac{2}{\beta_1\beta_2}}} = \left| \underline{\underline{s = s+\frac{1}{c\rho}\frac{b_1+2b_2}{2b_1b_2}}} \right| = \frac{s+\frac{1}{c\rho}\frac{\beta_1+2\beta_2}{2\beta_1\beta_2}-\frac{1}{c\rho}\sqrt{\frac{2}{\beta_1\beta_2}}}{s+\frac{1}{c\rho}\frac{\beta_1+2\beta_2}{2\beta_1\beta_2}+\frac{1}{c\rho}\sqrt{\frac{2}{\beta_1\beta_2}}}$$

VIDI #1

ZNACI SE DOBIVA ISTOTO „Ω" KAKO VO IZRAZITE OD CLANAKOT NA I.H. LEE.

$$\boxed{\psi = s + \frac{1}{c\rho}\frac{b_1+2b_2}{2b_1b_2} + \frac{1}{c\rho}\sqrt{\frac{2}{\beta_1\beta_2}}} \qquad \boxed{⊛ = \frac{\sqrt{\pi}}{\psi^4}\overset{\Gamma\binom{4,4}{3.5}}{\frac{\Gamma(4)\Gamma(4)}{\Gamma(3.5)}} {}_2F_1\left(4, \frac{1}{2}; 4.5; \Omega\right)}$$

$$F_3(s) = M(-s) = \frac{2}{\beta_1^2\beta_2^2 c^4\rho^4}\frac{\sqrt{\pi}}{\psi^4}\underbrace{\Gamma\binom{4,4}{3.5}}_{=153.6=\frac{768}{5}=\frac{3\cdot 2^8}{5}} {}_2F_1\left(4, \frac{1}{2}; 4.5; \Omega\right) = \frac{3\cdot 2^8}{5}$$

$$= \frac{1}{4\beta_1^2\beta_2^2 c^3\rho^3}\psi^{-4}\left[\frac{8\sqrt{\pi}}{c\rho}\Gamma\binom{4,4}{3.5} {}_2F_1\left(4, \frac{1}{2}; 4.5; \Omega\right)\right] = \frac{\psi^{-4}}{4\beta_1^2\beta_2^2 c^3\rho^3}\left[\frac{153.6}{c\rho}{}_2F_1\left(4, \frac{1}{2}; 4.5; \Omega\right)\right.$$

$$\Gamma\left[\begin{matrix}(a_p)\\(b_q)\end{matrix}\right] = \Gamma\left[\begin{matrix}a_1, \ldots, a_n\\ b_1, \ldots, b_n\end{matrix}\right] = \frac{\prod_{k=1}^{n}\Gamma(a_k)}{\prod_{k=1}^{q}\Gamma(b_l)} \qquad \Gamma(a_1, \ldots a_n) = \prod_{k=1}^{n}\Gamma(a_k)$$

$$\Gamma\binom{4,4}{3.5} = \frac{\Gamma(4)\cdot\Gamma(4)}{\Gamma(3.5)}$$

PRUDNIKOV
LIST OF NOTATIONS p. 615

$$\bullet\; F_1(s) = \mathcal{L}\left[\frac{\gamma^3 e^{-\frac{\alpha\gamma}{c\rho}}}{b_1 b_2 c^4\rho^4}\,\alpha^2 K_2\left(\frac{\gamma}{c\rho}\sqrt{\frac{2}{b_1 b_2}}\right)\right] = \mathcal{L}\left[\frac{\gamma^3\alpha^2}{b_1 b_2 c^4\rho^4}K_2\left(\frac{\gamma}{c\rho}\sqrt{\frac{2}{b_1 b_2}}\right)\right]\Bigg|_{s=s+\frac{\alpha}{c\rho}}$$

$$\alpha = \frac{b_1+2b_2}{2b_1b_2}$$

$$F_1(s) = \frac{\alpha^2}{b_1b_2c^4\rho^4}\mathcal{L}\left[\gamma^3 K_2\left(\overset{a}{\frac{\gamma}{c\rho}\sqrt{\frac{2}{b_1b_2}}}\right)\right] = \frac{\alpha^2\sqrt{\pi}}{b_1b_2c^4\rho^4}\left(\frac{2}{c\rho}\sqrt{\frac{2}{p_1p_2}}\right)^2 \frac{1}{\underbrace{(s+a)}_{\psi}{}^6}\,\Gamma\begin{bmatrix}2,6\\4.5\end{bmatrix}\; {}_2F_1\left(6,\tfrac{5}{2};4.5;\Omega\right)$$

$$F_1(s) = \frac{(b_1+2b_2)^2\,\cancel{4}\cdot 2}{\cancel{4}b_1^2b_2^2c^6\rho^6\;p_1^2p_2^2}\,\frac{1}{\psi^6}\,\textcircled{64}\;{}_2F_1\left(6,\tfrac{5}{2};4.5;\Omega\right)$$

$$F_1(s) = \frac{(b_1+2b_2)^2\cdot 2^7}{c^6\rho^6 b_1^4b_2^4\;\psi^6}\;{}_2F_1\left(6,\tfrac{5}{2};4.5;\Omega\right) = \frac{\psi^{-4}}{4p_1^2p_2^2c^3\rho^3}\left[\left(\frac{b_1+2b_2}{b_1b_2}\right)^2\frac{2^9}{c^3\rho^3\psi^2}\;{}_2F_1\left(6,\tfrac{5}{2};\tfrac{9}{2};\Omega\right)\right]$$

✓

$$\bullet\; F_{21}(s) = \mathcal{L}\left[\frac{\gamma^2\cdot 2\gamma\,(b_1+2b_2)}{c^3\rho^3\,2p_1p_2\cdot c\rho\;b_1b_2}\sqrt{\frac{2}{b_1b_2}}\,K_1\left(\frac{\gamma}{c\rho}\sqrt{\frac{2}{b_1b_2}}\right)\right]\Bigg|_{s=s+\frac{\alpha}{c\rho}} =$$

$\Gamma(3,5) = 25.6$

$\Gamma(3.5) = \frac{128}{5}$

$$= \frac{(b_1+2b_2)}{c^4\rho^4 b_1^2b_2^2}\sqrt{\frac{2}{b_1b_2}}\;\mathcal{L}\left[\gamma^3 K_1\left(\frac{\gamma}{c\rho}\sqrt{\frac{2}{b_1b_2}}\right)\right] = \frac{b_1+2b_2}{c^4\rho^4b_1^2b_2^2}\sqrt{\frac{2}{b_1b_2}}\,\frac{2\sqrt{\pi}}{c\rho}\sqrt{\frac{2}{p_1p_2}}\,\frac{1}{\psi^5}\;{}_2F_1\left(5,\tfrac{3}{2};\tfrac{9}{2};\Omega\right)$$

$$F_{21}(s) = \frac{(b_1+2b_2)\cdot\overbrace{4\cdot 25.6}^{102.4}}{c^5\rho^5b_1^3b_2^3\;\psi^5}\;{}_2F_1\left(5,\tfrac{3}{2};\tfrac{9}{2};\Omega\right) = \frac{\psi^{-4}}{4p_1^2p_2^2c^3\rho^3}\left[\frac{b_1+2b_2}{b_1b_2}\,\frac{\overbrace{4\cdot 102.4}^{=409,6=\frac{2^{11}}{5}}}{c^2\rho^2\psi}\;{}_2F_1\left(5,\tfrac{3}{2};\tfrac{9}{2};\Omega\right)\right]$$

✓

$$\bullet\; F_{22} = \mathcal{L}\left[\frac{\gamma^2}{2c^3\rho^3\;p_1p_2}\,\gamma\cdot\sqrt{\frac{2}{p_1p_2}}\,K_1\left(\frac{\gamma}{c\rho}\sqrt{\frac{2}{b_1b_2}}\right)\right]\Bigg|_{s=s+\frac{\alpha}{c\rho}} = \frac{\sqrt{2}}{c^3\rho^3p_1p_2\sqrt{p_1p_2}}\,\mathcal{L}\left[\gamma^2K_1\left(\frac{\gamma}{c\rho}\sqrt{\frac{2}{b_1b_2}}\right)\right]$$

$$= \frac{\sqrt{2}}{c^3\rho^3p_1p_2\sqrt{b_1b_2}}\,\frac{2\sqrt{2}\sqrt{\pi}}{c\rho\sqrt{p_1p_2}}\,\frac{1}{\psi^4}\cdot\Gamma\begin{bmatrix}2,4\\2.5\end{bmatrix}\;{}_2F_1\left(4,\tfrac{3}{2};\tfrac{7}{2};\psi\right)$$

$\Gamma \to \frac{8}{\sqrt{\pi}}$

$32\cdot 4 = 2^5\cdot 2^2 = 2^7$

✓

$$F_{22}(s) = \frac{4\sqrt{\pi}}{c^4\rho^4p_1^2p_2^2}\cdot\frac{1}{\psi^4}\cdot\frac{8}{\sqrt{\pi}}\;{}_2F_1\left(4,\tfrac{3}{2};\tfrac{7}{2};\Omega\right) = \frac{\psi^{-4}}{4b_1^2b_2^2c^3\rho^3}\left[\frac{2^7}{c\rho}\;{}_2F_1\left(4,\tfrac{3}{2};\tfrac{7}{2};\Omega\right)\right]$$

$$\text{MGF} = M(-s) = \frac{\psi^{-4}}{4p_1^2p_2^2c^3\rho^3}\left[\left(\frac{b_1+2b_2}{b_1b_2}\right)^2\frac{2^9}{c^3\rho^3\psi^2}\;{}_2F_1\left(6,\tfrac{5}{2};\tfrac{9}{2};\Omega\right) + \frac{b_1+2b_2}{b_1b_2}\,\frac{2^{11}}{5c^2\rho^2\psi}\;{}_2F_1\left(5,\tfrac{3}{2};\tfrac{9}{2};\Omega\right) - \frac{2^7}{c\rho}\;{}_2F_1\left(4,\tfrac{3}{2};\tfrac{7}{2};\Omega\right) + \frac{3\cdot 2^8}{5c\rho}\;{}_2F_1\left(4,\tfrac{1}{2};\tfrac{9}{2};\Omega\right)\right]$$

?

$$b_1 = b_2 = 1 \Rightarrow \left(\frac{b_1+2b_2}{b_1b_2}\right)^2 = \frac{9}{1} \qquad \frac{b_1+2b_2}{b_1b_2} = 3$$

pp. 29:

$\psi = 3/(2c\rho) + \sqrt{2}/(c\rho) + g/\sin^2\theta$

$$\Omega = \frac{(3-2\sqrt{2})\sin^2\theta + 2c\rho g}{(3+2\sqrt{2})\sin^2\theta + 2c\rho g}$$

$$\boxed{\mathcal{L}\left[x^{\mu}K_{\nu}(ax)\right]=\frac{(2a)^{\nu}\sqrt{\pi}}{(p+a)^{\mu+\nu+1}}\,\Gamma\left[\begin{matrix}\mu-\nu+1,\ \mu+\nu+1\\ \mu+\frac{1}{2}\end{matrix}\right]{}_2F_1\left(\mu+\nu+1,\nu+\frac{1}{2};\mu+\frac{3}{2};\frac{p-a}{p+a}\right)}$$

PRUDNIKOV 3.16.1.3

IZGLEDA DEKA E GREŠKATA TREBA: $\mu+1+\frac{1}{2}=\mu+\frac{3}{2}$

GRADSHTEYN 6.621.3

@x

$$\int_0^{\infty}x^{\mu-1}e^{-\alpha x}K_{\nu}(\beta x)\,dx=\frac{\sqrt{\pi}(2\beta)^{\nu}}{(\alpha+\beta)^{\mu+\nu}}\frac{\Gamma(\mu+\nu)\Gamma(\mu-\nu)}{\Gamma(\mu+\frac{1}{2})}{}_2F_1\left(\mu+\nu,\nu+\frac{1}{2};\mu+\frac{1}{2};\frac{\alpha-\beta}{\alpha+\beta}\right)$$

• AKO SE ODI SO OVOJ IZRAZ TOGAŠ SE DOBIVA TOKMU IZRAZOT (12) OD Lee: (VIDI MAPLE MultipleMIMO.mw (3.19))

$$\underbrace{GF}_{N(-1)}=\frac{\psi^{-4}}{4b_1^2b_2^2c^3\rho^3}\left[\left(\frac{b_1+2b_2}{b_1b_2}\right)^2\frac{2^{10}}{7c^3\rho^3\psi^2}{}_2F_1\left(6,\frac{5}{2};\frac{9}{2};\Omega\right)+\frac{b_1+2b_2}{b_1b_2}\frac{2^{12}}{35c^2\rho^2\psi}F\left(5,\frac{3}{2};\frac{9}{2};\Omega\right)\right.$$

$$\left.-\frac{2^8}{5c\rho}{}_2F_1\left(4,\frac{3}{2};\frac{7}{2};\Omega\right)+\frac{3\cdot2^9}{35c\rho}{}_2F_1\left(4,\frac{1}{2};\frac{9}{2};\Omega\right)\right]$$

100% ISKLUČENO GRADSHTEYN str. 47-48

$\mu=4$

- AKO SE ODI SO @x SE DOBIVA

$$F_1(1)=\frac{1}{b_1\cdot b_2}\frac{(b_1+2b_2)^2}{4b_1^2b_2^2}\cdot\frac{1}{c^4\rho^4}\mathcal{L}\left[\gamma^3e^{-\frac{\alpha\gamma}{c\rho}}\cdot K_2(a\gamma)\right]=\frac{(b_1+2b_2)^2}{4b_1^3b_2^3c^4\rho^4}\cdot\frac{\sqrt{\pi}}{(\alpha'+a)^6}\left(\frac{2}{c\rho}\sqrt{\frac{2}{b_1b_2}}\right)^2$$

$$\cdot\frac{\Gamma(6)\cdot\Gamma(2)}{\Gamma\left(\frac{9}{2}\right)}{}_2F_1\left(6,\frac{5}{2};\frac{9}{2};\frac{\alpha'-a}{\alpha'+a}\right)$$

$$\boxed{\alpha'=\frac{\alpha}{c\rho}+1}\qquad\boxed{\alpha=\frac{b_1+2b_2}{2b_1b_2}}$$

$$\mathcal{L}\left[\gamma^3e^{-\frac{\alpha\gamma}{c\rho}}K_2(a\gamma)\right]=\int_0^{\infty}\gamma^3e^{-\left(\frac{\alpha}{c\rho}+1\right)\gamma}K_2(a\gamma)\,d\gamma$$

$$F_1(1)=\frac{(b_1+2b_2)^2}{4b_1^3b_2^3c^4\rho^4}\frac{\sqrt{\pi}}{\underbrace{\left(\frac{\alpha}{c\rho}+1+\frac{1}{c\rho}\sqrt{\frac{2}{b_1b_2}}\right)^6}_{\psi}}\frac{4}{c^2\rho^2}\frac{2}{b_1^2b_2^2}\frac{\Gamma(6)\Gamma(2)}{\Gamma(4.5)}{}_2F_1\left(6,\frac{5}{2};\frac{9}{2};\overbrace{\frac{\frac{\alpha}{c\rho}+1-\frac{1}{c\rho}\sqrt{\frac{2}{b_1b_2}}}{\frac{\alpha}{c\rho}+1+\frac{1}{c\rho}\sqrt{\frac{2}{b_1b_2}}}}^{\Omega}\right)$$

$$F_1(1)=\frac{(b_1+2b_2)^2}{4b_1^5b_2^5c^6\rho^6}\frac{1}{\psi^6}\boxed{\frac{8\sqrt{\pi}\Gamma(6)\Gamma(2)}{\Gamma(4.5)}}\xrightarrow{}\frac{2^{10}}{7}\;{}_2F_1\left(6,\frac{5}{2};\frac{9}{2};\Omega\right)$$

$$\boxed{F_1(1)=\frac{\psi^{-4}}{4b_1^2b_2^2c^3\rho^3}\left[\frac{2^{10}}{7c^3\rho^3\psi^2}{}_2F_1\left(6,\frac{5}{2};\frac{9}{2};\Omega\right)\right]}$$

U DOBIVAME!!!

$$\psi=\frac{b_1+2b_2}{2c\rho b_1b_2}+1+\frac{1}{c\rho}\frac{\sqrt{2}}{\sqrt{b_1b_2}}=\frac{(b_1+2b_2)+2c\rho b_1b_2+2\sqrt{2b_1b_2}}{2c\rho b_1b_2}$$

$$\psi=\frac{(b_1+2b_2)+2c\rho b_1b_2+2\sqrt{2}\sqrt{b_1b_2}}{2c\rho b_1b_2}=\left|\begin{matrix}b_1=b_2\\=1\end{matrix}\right|=\frac{3+2\sqrt{2}+2c\rho}{2c\rho}$$

$$\Omega=\frac{3-2\sqrt{2}+2c\rho}{3+2\sqrt{2}+2c\rho}\qquad\Omega=\frac{(b_1+2b_2)-2\sqrt{2}\sqrt{b_1b_2}+2c\rho b_1b_2}{(b_1+2b_2)+2\sqrt{2}\sqrt{b_1b_2}+2c\rho b_1b_2}$$

$$F_{21} = \frac{1}{c^4\varrho^4\cdot\rho_1\rho_2}\,\frac{\rho_1+2\rho_2}{\rho_1\rho_2}\sqrt{\frac{2}{\rho_1\rho_2}}\int_0^\infty \gamma^3 e^{-\left(\frac{\alpha}{c\varrho}+1\right)\gamma} K_1\left(\frac{\gamma}{c\varrho}\sqrt{\frac{2}{\rho_1\rho_2}}\right)d\gamma$$

$\mu=4 \quad \nu=1$

$$= \frac{(\rho_1+2\rho_2)\sqrt{2}}{\cancel{2}c^4\varrho^4\rho_1^2\rho_2^2\sqrt{\rho_1\rho_2}}\,\frac{\sqrt{\pi}}{\psi^5}\,\frac{\Gamma(5)\cdot\Gamma(3)}{\Gamma(4.5)}\,{}_2F_1\left(5,\frac{3}{2};4.5;\Omega\right)\cdot\frac{\cancel{2}}{c\varrho}\sqrt{\frac{2}{\rho_1\rho_2}} =$$

$$= \frac{(\rho_1+2\rho_2)}{4c^5\varrho^5\rho_1^3\rho_2^3\psi^5}\,\underbrace{\frac{16\sqrt{\pi}\,\Gamma(5)\cdot\Gamma(3)}{\Gamma(4.5)}}_{=\frac{2^{12}}{35}}\,{}_2F_1\left(5,\frac{3}{2};4.5;\Omega\right)$$

$$\boxed{F_{21} = \frac{\psi^{-4}}{4c^3\varrho^3\rho_1^2\rho_2^2}\left[\frac{(\rho_1+2\rho_2)\,2^{12}}{35\cdot\rho_1\rho_2\,c^2\varrho^2\psi}\cdot{}_2F_1\left(5,\frac{3}{2};4.5;\Omega\right)\right]}$$

✓ DOKAZANO!!

$$F_{22} = \frac{\cancel{2}}{\cancel{2}\rho_1\rho_2c^3\varrho^3}\sqrt{\frac{2}{\rho_1\rho_2}}\int_0^\infty \gamma^2 e^{-\left(\frac{\alpha}{c\varrho}+1\right)\gamma}K_1\left(\frac{\gamma}{c\varrho}\sqrt{\frac{2}{\rho_1\rho_2}}\right)d\gamma =$$

$\mu=3 \quad \nu=1$

$$= \frac{1}{\rho_1\rho_2c^3\varrho^3}\sqrt{\frac{2}{\rho_1\rho_2}}\,\frac{\sqrt{\pi}}{\psi^4}\,\frac{2}{c\varrho}\sqrt{\frac{2}{\rho_1\rho_2}}\cdot\frac{\Gamma(4)\cdot\Gamma(2)}{\Gamma(3.5)}\,{}_2F_1\left(4,\frac{3}{2};3.5;\Omega\right) =$$

$$= \frac{1}{4\cdot c^4\varrho^4\rho_1^2\rho_2^2\psi^4}\,\underbrace{\frac{16\sqrt{\pi}\,\Gamma(4)\Gamma(2)}{\Gamma(3.5)}}_{=\frac{2^8}{5}}\,{}_2F_1 = \frac{\psi^{-4}}{4c^3\varrho^3\rho_1^2\rho_2^2}\left[\frac{2^8}{5c\varrho}\cdot{}_2F_1\left(4,\frac{3}{2};3.5;\Omega\right)\right]$$

DOKAZANO!! ✓

$$F_3 = \frac{\overset{②}{\cancel{4}}}{\cancel{2}\rho_1^2\rho_2^2c^4\varrho^4}\int_0^\infty\gamma^3 e^{-\left(\frac{\alpha}{c\varrho}+1\right)\gamma}K_0\left(\frac{\gamma}{c\varrho}\sqrt{\frac{2}{\rho_1\rho_2}}\right)d\gamma =$$

$\mu=4 \quad \nu=0$

$$= \frac{2}{\rho_1^2\rho_2^2c^4\varrho^4}\,\frac{\sqrt{\pi}}{\psi^4}\left(\frac{2}{c\varrho}\sqrt{\frac{2}{\rho_1\rho_2}}\right)^0\frac{\Gamma(4)\cdot\Gamma(4)}{\Gamma(4.5)}\cdot{}_2F_1\left(4,\frac{1}{2};4.5;\Omega\right)$$

$$F_3 = \frac{1}{4c^4\varrho^4\rho_1^2\rho_2^2\psi^4}\,\underbrace{\frac{8\sqrt{\pi}\,\Gamma(4)\Gamma(4)}{\Gamma(4.5)}}_{\frac{3\cdot2^9}{35}}\,{}_2F_1\left(4,\frac{1}{2};4.5;-\Omega\right)$$

✓

$$\boxed{F_3 = \frac{\psi^{-4}}{4\cdot c^3\varrho^3\rho_1^2\rho_2^2}\left[\frac{3\cdot2^9}{35c\varrho}\,{}_2F_1\left(4,\frac{1}{2};4.5;-\Omega\right)\right]}$$

DOKAZANO!!

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434 OSTBC CODE

$\$*$

$$G_{t,n} = \begin{array}{c|c} & \xrightarrow{n} \\ \hline t\downarrow & \begin{bmatrix} x_1 & x_2 & x_3/\sqrt{2} & x_3/\sqrt{2} \\ -x_2^* & x_1^* & x_3/\sqrt{2} & -x_3/\sqrt{2} \\ x_3^*/\sqrt{2} & x_3^*/\sqrt{2} & \frac{-x_1-x_1^*+x_2-x_2^*}{2} & \frac{-x_2-x_2^*+x_1-x_1^*}{2} \\ x_3^*/\sqrt{2} & -x_3^*/\sqrt{2} & \frac{x_2+x_2^*+x_1-x_1^*}{2} & -\frac{x_1+x_1^*+x_2-x_2^*}{2} \end{bmatrix}\end{array}$$

$$\Omega_k(t) = \begin{cases} \alpha_{n,1} & G_{t,n}=x_k \\ \alpha_{n,1}^* & G_{t,n}=x_k^* \\ -\alpha_{n,1} & G_{t,n}=-x_k \\ -\alpha_{n,1}^* & G_{t,n}=-x_k^* \\ 0 & \text{otherwise} \end{cases}$$

$$\Omega_k^t = t\downarrow\overset{\xrightarrow{k}}{\left[\quad\right]}$$

$\Omega_1(1) = ?$ $\quad G_{1,1} = x_1 \Rightarrow \Omega_1(1) = h_1;$ $\quad$ $k=2\ t=1$ $\quad G_{1,2} = x_2 \Rightarrow \Omega_2(1) = h_2;$

Dragica Šajnovska

$\boxed{k=3\ t=1}$

$G_{t,4} = G_{t,3} = G_{1,3} = x_3/\sqrt{2} \quad \Omega_3(1) = h_3/\sqrt{2}$

$\boxed{k=4\ t=1}$

$G_{1,4} = x_3/\sqrt{2} \neq x_k \Rightarrow \Omega_4(1) = 0$

$k=1\ t=2 \quad G_{2,\textcircled{2}} = x_1^* \Rightarrow \Omega_1(2) = x_2^*$ | $k=2\ t=2 \quad G_{2,\textcircled{1}} = x_2^* \Rightarrow \Omega_2(2) = -h_1^*$

$k=3\ t=2 \quad G_{2,\textcircled{3}} = \frac{x_3}{\sqrt{2}} \Rightarrow \Omega_3(2) = \frac{h_1}{\sqrt{2}}$ | $k=4\ t=2 \quad G_{2,4} = -x_3 \neq x_4 \Rightarrow \Omega_4(2) = 0$

- OSTBC $\quad$ 434

$$t \downarrow \quad G = \begin{bmatrix} x_1 & x_2 & x_3 & 0 \\ -x_2^* & x_1^* & 0 & x_3 \\ x_3^* & 0 & -x_1^* & x_2 \\ 0 & x_3^* & -x_2^* & -x_1 \end{bmatrix} \qquad \Omega_t = \begin{bmatrix} h_1 & h_2 & h_3 & 0 \\ h_2^* & -h_1^* & \cancel{h_4} & 0 \\ -h_3^* & h_4 & h_1^* & 0 \\ -h_4 & -h_3^* & h_2^* & 0 \end{bmatrix}$$

$\boxed{k=1\ t=2} \quad G_{2,\textcircled{2}} = x_1^* \Rightarrow \Omega_1(2) = h_2^*$ | $\boxed{k=2\ t=2} \quad G_{2,\textcircled{1}} = -x_2^* \quad \Omega_2(2) = -h_1^*$

$k=3\ t=2 \quad G_{2,\textcircled{4}} = x_3 \Rightarrow \Omega_3(2) = h_4$ | $k=4\ t=2 \quad G_{2,?} = x_4 \Rightarrow \Omega_4(2) = 0$

$k=1\ t=3 \quad G_{3,\textcircled{3}} = -x_1^* \quad \Omega_1(3) = -h_3^*$ | $k=2\ t=3 \quad G_{3,4} = x_2 \rightarrow \Omega_2(3) = h_4$

$k=3\ t=3 \quad G_{3,\textcircled{1}} = x_3^* \quad \Omega_3(3) = h_1^*$ | $k=4\ t=3 \quad G_{3,?} = x_4 \rightarrow \Omega_4(3) = 0$

$k=1\ t=4 \quad G_{4,\textcircled{4}} = -x_1 \quad \Omega_1(4) = -h_4$ | $k=2\ t=4 \quad G_{4,\textcircled{3}} = -x_2^* \rightarrow \Omega_2(4) = -h_3^*$

$k=3\ t=4 \quad G_{4,2} = x_3^* \rightarrow \Omega_3(4) = h_2^*$ | $k=4\ t=4 \quad G_{4,?} = x_4 \rightarrow \Omega_4(4) = 0$

$$\boxed{\tilde{x}_1 = y_1 h_1^* + y_2^* h_2; \quad \tilde{x}_2 = y_1 h_2^* - y_2^* h_1 \,?}$$

$\Rightarrow$ ALAMOUTI CODE !!

- DECODING OF $\textcircled{\$*}$ (CORES_MIMO_STBC_MATLAB.pdf)

$$\tilde{x}_1 = y_1 \cdot h_1^* + y_2^* h_2 + \frac{(y_4 - y_3)(h_3^* - h_4^*)}{2} - \frac{(y_3^* + y_4^*)(h_3 + h_4)}{2}$$

$$\tilde{x}_2 = y_1 h_2^* - y_2^* h_1 + \frac{(y_4 + y_3)(h_3^* - h_4^*)}{2} + \frac{(y_4^* - y_3^*)(h_3 + h_4)}{2}$$

$$\tilde{x}_3 = (y_1 + y_2)\frac{h_3^*}{\sqrt{2}} + (y_1 - y_2)\frac{h_4^*}{\sqrt{2}} + (h_1 + h_2)\frac{y_3^*}{\sqrt{2}} + (h_1 - h_2)\frac{y_4^*}{\sqrt{2}}$$

MMU

$$\tilde{x}_1 = h_1^* y_1 + h_2 y_2^* + \underline{y_4 h_3^*} - \underline{y_4 h_4^*} - \underline{y_3 h_3^*} + \underline{y_3 h_4^*} - \underline{y_3^* h_3} - \underline{y_3^* h_4} - \underline{y_4^* h_3} - \underline{y_4^* h_4} = h_1^* y_1 + h_2 y_2^* + 2j[y_4^I h_3^R - y_4^R h_3^I] - 2[y_3^R h_3^R + y_3^I h_3^I] + 2j[y_3^I h_4^R - y_3^R h_4^I] - 2[y_4^R h_4^R + y_4^I h_4^I]$$

$a = a_1 + j a_2 \qquad b = b_1 + j b_2$

$(a \cdot \bar{b} + \bar{a} b) = 2 a_1 b_1 + 2 a_2 b_2 = 2\{R[a] \cdot R[b] + I[a] I[b]\}$

⊙ Simon & Alouni (8.1.1.3) M-ary PSK (AWGN)

$$P_k = \frac{1}{2\pi} \int_0^{\pi(1-(2k-1)/M)} \exp\left(-\frac{E_s}{N_0} \frac{\sin^2[(2k-1)\pi/M)}{\sin^2\theta}\right) d\theta -$$

$$- \frac{1}{2\pi} \int_0^{\pi(1-(2k+1)/M)} \exp\left(-\frac{E_s}{N_0} \frac{\sin^2[(2k+1)\pi/M]}{\sin^2\theta}\right) d\theta$$

ⓐ $\boxed{k=2}$

$1 - 3/4 = 1/4$

ⓑ $1 - 5/4 = -\frac{1}{4}$

$\sin(5\pi/4) = -\sqrt{2}$

$\boxed{k=3}$ ⓐ: $\sin(5\pi/4) = -\sqrt{2}$

ⓑ: $1 - 7/4 = -3/4$ $\quad \sin(7\pi/4) = -\sqrt{2}$

E_s/N_0

$P_b(e) = \frac{1}{2}(P_1 + 2P_2 + P_3) \qquad \boxed{M=4}$

$$P_{1a} = \frac{1}{2\pi} \int_0^{\pi(3/4)} \exp\left(-\frac{E_s}{N_0} \frac{\sin^2 \pi/4}{\sin^2\theta}\right) d\theta = \frac{1}{2\pi} \int_0^{3\pi/4} e^{-2\gamma/\sin^2\theta} d\theta = \frac{1}{2\pi} \int_0^{3\pi/4} e^{-2\gamma/\sin^2\theta} d\theta$$

$$P_{2a} = \frac{1}{2\pi} \int_0^{\pi/4} \exp\left(-\gamma \frac{\overset{2}{\sin^2(3\pi/4)}}{\sin^2\theta}\right) d\theta = \frac{1}{2\pi} \int_0^{\pi/4} e^{-\frac{2\gamma}{\sin^2\theta}} d\theta$$

$$P_{1b} = \frac{1}{2\pi} \int_0^{\pi/4} e^{-2\gamma/\sin^2\theta} d\theta \qquad P_{2b} = \frac{1}{2\pi} \int_0^{-\pi/4} e^{-2\gamma/\sin^2\theta}$$

$$P_{3a} = \frac{1}{2\pi} \int_0^{-\pi/4} e^{-2\gamma/\sin^2\theta} d\theta \qquad P_{3b} = \frac{1}{2\pi} \int_0^{-3\pi/4} e^{-2\gamma/\sin^2\theta} d\theta$$

$$P_b = \frac{1}{4\pi}\Bigg(\int_0^{3\pi/4} \overbrace{e^{-2\gamma/\sin^2\theta}}^{f(\theta)} d\theta - \int_0^{\pi/4} e^{-2\gamma/\sin^2\theta} d\theta + 2\int_0^{\pi/4} e^{-2\gamma/\sin^2\theta} d\theta - 2\int_0^{-\pi/4} e^{-2\gamma/\sin^2\theta} d\theta +$$

$$+ \int_0^{-\pi/4} e^{-2\gamma/\sin^2\theta} d\theta - \int_0^{-3\pi/4} e^{-2\gamma/\sin^2\theta} d\theta \Bigg) = \frac{1}{4\pi}\left(\int_{\pi/4}^{3\pi/4} f(\theta) d\theta + 2\int_{-\pi/4}^{\pi/4} f(\theta) d\theta + \int_{-3\pi/4}^{-\pi/4} f(\theta) d\theta \right)$$

$$\boxed{P_{b_1}(e/\gamma) = \frac{1}{4\pi} \int_{-3\pi/4}^{3\pi/4} e^{-2\gamma/\sin^2\theta} d\theta}$$

$$P_{b_1} = \int_{-\infty}^{\infty} P_{b_1}(e/\gamma)\, p_\gamma(\gamma)\, d\gamma = \frac{1}{4\pi} \int_{-\infty}^{\infty} \int_{-3\pi/4}^{3\pi/4} e^{-\frac{2\gamma}{\sin^2\theta}} p_\gamma(\gamma)\, d\theta\, d\gamma$$

$$= \frac{1}{4\pi} \int_{-3\pi/4}^{3\pi/4} \int_{-\infty}^{\infty} e^{-\frac{2\gamma}{\sin^2\theta}} p_\gamma(\gamma)\, d\gamma\, d\theta = \frac{1}{4\pi} \int_{-3\pi/4}^{3\pi/4} M\left(-\frac{2}{\sin^2\theta}\right) d\theta$$

$g = \sin^2\left(\frac{\pi}{M}\right) =$

$= \sin^2\frac{\pi}{4} = 2$ //

$P_{b_2}(E/\gamma) = \frac{1}{4\pi}\int_{-\pi/4}^{\pi/4} e^{-\frac{2\gamma}{\sin^2\theta}}\, d\theta \qquad P_{b_2} = \int_0^{\infty} P_{b_2}(E/\gamma)\, p_\gamma(\gamma)\, d\gamma = \frac{1}{4\pi}\int_{-\pi/4}^{\pi/4} M\left(-\frac{2}{\sin^2\theta}\right) d\theta$

$$\boxed{P_b = P_{b_1} + P_{b_2} = \frac{1}{4\pi}\int_{-3\pi/4}^{3\pi/4} M\left(-\frac{2}{\sin^2\theta}\right) d\theta + \frac{1}{4\pi}\int_{-\pi/4}^{\pi/4} M\left(-\frac{2}{\sin^2\theta}\right) d\theta} \quad \circledast$$

- DCoFC Eq. 8.120

$$P_b(E) \doteq \frac{2}{\max(\log_2 M, 2)} \sum_{i=1}^{\max(M/4,1)} \frac{1}{\pi}\int_0^{\pi/2} M_\gamma\left(-\frac{1}{\sin^2\theta}\frac{E_b \log_2 M}{N_0}\sin^2\frac{(2i-1)\pi}{M}\right) d\theta$$

$\boxed{M=4}$

$$\boxed{P_b(E) = \frac{1}{\pi}\int_0^{\pi/2} M_\gamma\left(-\frac{2E_bN_0\cdot 2}{\sin^2\theta}\right) d\theta = \frac{1}{\pi}\int_0^{\pi/2} M_\gamma\left(-\frac{4E_bN_0}{\sin^2\theta}\right) d\theta} \quad \circledast$$

☐ BER FOR REGENERATIVE $\boxed{2x1+2}$

- BER FOR REGENERATIVE 1+1+1 :

- USLOVNA VEROJATNOST DA SE SLUČI GREŠKA VO AWGN KANAL E!

$$\boxed{\underbrace{P(E/\gamma_1,\gamma_2) = P(E/\gamma_1) + P(E/\gamma_2) - 2P(E/\gamma_1)\cdot P(E/\gamma_2)}_{\text{SE PRAŠAM DALI OVA VAŽI ZA BPSK!?}}} \quad \text{VAŽI! ZA QPSK!!}$$

- SREDNATA BITSKA GREŠKA VO FEDING KANAL E!

$$\boxed{P_b = P_{b_1}(E_r) + P_{b_2}(E_r) - 2P_{b_1}(E_r)P_{b_2}(E_r)} \quad \text{T.E}$$

1x1x1 $P_{b_1} = P_{b_2} = P$

$$\boxed{P_b = P_b(E_1) + P_b(E_2) - 2P_b(E_1)P_b(E_2)} \qquad P_b = 2P - 2P^2 = 2(1-P)P$$

- ZA 2x1+2 $(\underbrace{2x1}_{P_{b_1}} + \underbrace{1x2}_{P_{b_2}})$

$P = \frac{1}{2}\left(1 - \sqrt{\frac{\bar\gamma}{\bar\gamma+1}}\right)$ ↳ ZA DPSK

$$\boxed{P_{b_1} = \left(\frac{1}{2}(1-\mu)\right)^L \sum_{k=0}^{L-1}\binom{L-1+k}{k}\left(\frac{1+\mu}{2}\right)^k} \qquad \boxed{\mu = \sqrt{\frac{0.5\bar\gamma}{1+0.5\bar\gamma}}} \quad \text{BPSK}$$

MNOŽAM SO (0.5) ZOŠTO OVOJ IZRAZ VSUŠNOST SE ODNESUVA NA MRC

$$\boxed{P_{b_2} = P_{b_1} \quad \text{NO} \quad \mu = \sqrt{\frac{\bar\gamma}{1+\bar\gamma}}} \quad \text{BPSK}$$

- POKAŽ SO SIMULACIJA : ber_Ray_OSTBC222_2x1x2_MPSK.m

- MRC MPSK

$$P_b = 0.5\left[1 - \frac{\mu}{\sqrt{2-\mu^2}} \sum_{k=0}^{L-1} \binom{2k}{k} \left(\frac{1-\mu^2}{4-2\mu^2}\right)^k\right]$$

- dsplog

$$P_e = p^N \sum_{k=0}^{N-1} \binom{N-1+k}{k} (1-p)^k \qquad p = \frac{1}{2} - \frac{1}{2}\left(1 + \frac{1}{E_b N_0}\right)^{-1/2}$$

~~TREBA~~ DA GI PROVERAM SO berfading() OD MATLAB.

$$P_{M_b} = \frac{(-1)^{L-1}(1-\mu^2)^L}{\pi (L-1)!} \left(\frac{\partial^{L-1}}{\partial b^{L-1}} \left\{ \frac{1}{b-\mu^2} \left[\frac{\pi}{M}(M-1) - \frac{\mu \sin(\pi/M)}{\sqrt{b-\mu^2\cos^2(\pi/M)}} \cot^{-1} \frac{-\mu \cos(\pi/M)}{\sqrt{b-\mu^2\cos^2(\pi/M)}} \right] \right\} \right)$$

$$\boxed{\mu = \sqrt{\frac{\bar{\gamma}_c}{1+\bar{\gamma}_c}}}$$

BER

1XL MRC M-PSK

MMV

070267310 VALENTINO

DTK

NS ! ISAV !! ~~XZKP~~ : ZA OSTBC BER VO PROAKIS (14-4-44)

$$\boxed{P_{M_b} = 1 - \int_0^\infty \frac{1}{(1+\bar{\gamma}_c)^L (L-1)!} x^{L-1} e^{-x/2\sigma_1^2} \left[1 - e^{-x/2\sigma_2^2} \sum_{k=0}^{L-1} \left(\frac{x}{2\sigma_2^2}\right)^k\right]^{M-1} dx}$$

- $$P_s(E/\gamma) = \frac{1}{\pi} \int_0^{(M-1)\pi/M} \exp\left\{\frac{\gamma\, g_{MPSK}}{\sin^2\theta}\right\} d\theta$$

$\Rightarrow$ OVAA FORMULA DA KORISTAT VO MATLAB.

$$g_{MPSK} = \sin^2\left(\frac{\pi}{M}\right)$$

$$P_s(E) = \frac{1}{\pi} \int_0^{(M-1)\pi/M} MGF\left(-\frac{g_{MPSK}}{\sin^2\theta}\right) d\theta$$

$$\frac{1}{2} - \frac{1}{M} = \frac{M-2}{2M}$$

SIMON & ALOUINI 8.22

$$P_s(E) = \frac{1}{\pi} \int_{-\pi/2}^{\pi\left(-\frac{1}{M}+\frac{1}{2}\right)} \exp\left(-\frac{E_s}{N_0} \frac{g_{PSK}}{\sin^2\theta}\right) d\theta$$

- DCoFC (8.23)

$$\boxed{P_s(E) = \frac{1}{\pi}\int_0^{(M-1)\pi/M} \exp\left(-\frac{E_s}{N_0}\frac{g_{PSK}}{\sin^2\theta}\right)d\theta}$$

$\rightarrow$ $\boxed{\text{MRC}}$

- AVERAGE SYMBOL ERROR RATE OF M-PSK SIGNALS

(DCoFC)(pp.322)

$$\underset{\nearrow}{P_s}\left(E \,\middle|\, \{\gamma_l\}_{l=1}^{L}\right) = \frac{1}{\pi}\int_0^{(M-1)/M} \exp\left(-\frac{g_{PSK}\gamma_t}{\sin^2\phi}\right)d\phi = \frac{1}{\pi}\int_0^{(M-1)/M} \prod_{l=1}^{L} \exp\left(-\frac{g_{PSK}\gamma_l}{\sin^2\phi}\right)d\phi$$

CONDITIONAL

$$\boxed{\gamma_t = \gamma_1 + \gamma_2 + \dots \gamma_L}$$

- DESIRED FORM OF Q FUNCTION

$$\boxed{Q(x) = \frac{1}{\pi}\int_0^{\pi/2} e^{-\frac{x^2}{2\sin^2\theta}}\,d\theta} \qquad Q\left(\sqrt{\frac{2E_s}{N_0}}\right) = \frac{1}{\pi}\int_0^{\pi/2} e^{-\frac{E_s}{N_0\sin^2\theta}}\,d\theta$$

AVERAGE BER:

$$P_s(E) = \frac{1}{\pi}\int_0^{(M-1)\pi/M} \prod_{l=1}^{L} M_{\gamma_l}\left(-\frac{g_{PSK}}{\sin^2\phi}\right)d\phi$$

UNIVERZALAN IZRAZ

$$\gamma_1 = \gamma_2 = \dots \gamma_L = \gamma \rightarrow \boxed{P_s(E) = \frac{1}{\pi}\int_0^{(M-1)\pi/M} M_\gamma^L\left(-\frac{g_{PSK}}{\sin^2\phi}\right)d\phi}$$

MMV

EsNo PER LINK

RAYLEIGH:

$$\boxed{M(s) = \frac{1}{1 - s\bar{\gamma}_l}}$$

$$M_\gamma\left(-\frac{g_{PSK}}{\sin^2\theta}\right) = \frac{\sin^2\phi}{\sin^2\phi + g_{PSK}\cdot\bar{\gamma}}$$

$$P_s(E) = \frac{1}{\pi}\int_0^{(M-1)\pi/M} \left(\frac{\sin^2\theta}{\sin^2\theta + g_{PSK}\bar{\gamma}}\right)^L d\theta \qquad g_{PSK} = \sin^2(\pi/M)$$

$\boxed{L=2}$

QPSK

$$g_{PSK} = \sin^2(\pi/4) = 2$$

$$P_s(E) = \frac{1}{\pi}\int_0^{3\pi/4} \left(\frac{\sin^2\theta}{\sin^2\theta + 2\bar{\gamma}}\right)^2 d\theta =$$

EsNo PER LINK (VIDI MATLAB ILI PRAKTIKUS)

(MAKE SO REŠAVA NO MOŽE I NUMERIČKI VO MATLAB)

$\boxed{L=1}$ POINT-TO-POINT

$$P_s(e) = \frac{1}{\pi}\int_0^{(M-1)\pi/M} \frac{\sin^2\theta}{\sin^2\theta + g_{PSK}\cdot\bar{\gamma}}\, d\theta \qquad g_{PSK} = \sin^2 \pi/M$$

$$\int_0^{b} \frac{\sin^2\theta}{\sin^2\theta+g_{PSK}\bar{\gamma}}\,d\theta = \int_0^{b} \frac{d\theta}{\sin^2\theta+g_{PSK}\bar{\gamma}} - \int_0^{b} \frac{\cos^2\theta\, d\theta}{\sin^2\theta+g_{PSK}\bar{\gamma}}$$

MAPLE: $a>0$ ⓧ$

$$\boxed{\int \frac{\sin^2\theta}{\sin^2\theta+a}\,d\theta = -\frac{a}{\sqrt{a(1+a)}}\arctan\frac{(1+a)\tan(\theta)}{\sqrt{a(1+a)}} + \theta}$$

$\theta = \operatorname{arctg}(x) \qquad x = \operatorname{tg}\theta$

$\theta' = ? \qquad dx = \left(\frac{\sin\theta}{\cos\theta}\right)' d\theta$

$$dx = \frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta}\,d\theta \qquad \left(\frac{\sin\theta}{\cos\theta}\right)' = \cancel{\cos\theta\cdot\frac{1}{\cos\theta}} + \frac{\sin^2\theta}{\cos^2\theta}$$

$$= 1 + \frac{\sin^2\theta}{\cos^2\theta} = 1 + \operatorname{tg}^2\theta = \underline{1+x^2}$$

$$\frac{dx}{d\theta} = \frac{1-x^2}{\cancel{\quad}} \qquad \boxed{\frac{d\theta}{dx} = \frac{1}{1+x^2}}$$

$$\int \frac{\sin^2\theta}{\sin^2\theta+a}\,d\theta = \int \frac{\cancel{\sin^2\theta+a}}{\cancel{\sin^2\theta+a}}\,d\theta - \int \frac{a}{\sin^2\theta+a}\,d\theta$$

$$= \theta - \int \frac{a}{\sin^2\theta+a}\,d\theta$$

$$\cancel{\theta} - \int \frac{a}{\sin^2+a}\,d\theta = -\frac{a}{\sqrt{a(1+a)}}\arctan\left(\frac{(1+a)\tan\theta}{\sqrt{a(1+a)}}\right) + \cancel{\theta}$$

GRADSHTEYN 2.562.1 ⇐ $$\boxed{\int \frac{d\theta}{\sin^2\theta+a} = \frac{1}{\sqrt{a(1+a)}}\arctan\left(\frac{(1+a)\tan\theta}{\sqrt{a(1+a)}}\right)}$$

• DO IZRAZOT ⊛$ MOZE DA SE DOJDE INDIREKTNO BEZ KORISTENJE NA MAPLE NO SO KORISTENJE NA IZRAZOT 2.562.1 OD GRADSHTEJN.

$$\frac{1}{\pi}\int_0^{(M-1)\pi/M}\frac{\sin^2\theta}{\sin^2\theta+g_{PSK}\bar\delta}\,d\theta=\int\frac{\cancel{\sin^2\theta}+g_{PSK}\bar\delta}{\sin^2\theta\cancel{+g_{PSK}\bar\delta}}\,d\theta-\int\frac{g_{PSK}\bar\delta}{\sin^2\theta+g_{PSK}\bar\delta}\,d\theta$$

$$=\frac{\theta}{\pi}-\frac{g_{PSK}\bar\delta}{\pi}\int_0^{(M-1)\pi/M}\frac{d\theta}{\sin^2\theta+g_{PSK}\bar\delta}=\left.\frac{\theta}{\pi}-\frac{g_{PSK}\bar\delta}{\pi\sqrt{g_{PSK}\bar\delta(1+g_{PSK}\bar\delta)}}\operatorname{arctg}\frac{(1+g_{PSK}\bar\delta)\operatorname{tg}\theta}{\sqrt{g_{PSK}\bar\delta(1+g_{PSK}\bar\delta)}}\right|_0^{(M-1)\pi/M}$$

$$=\frac{M-1}{M}-\frac{g_{PSK}\bar\delta}{\pi\sqrt{g_{PSK}\bar\delta(1+g_{PSK}\bar\delta)}}\cdot\operatorname{arctg}\left(\operatorname{tg}\theta\sqrt{\frac{1+g_{PSK}\bar\delta}{g_{PSK}}}\right)$$

$$\boxed{P_S=\frac{M-1}{M}-\frac{1}{\pi}\sqrt{\frac{g_{PSK}\bar\delta}{1+g_{PSK}\bar\delta}}\operatorname{arctg}\left(\operatorname{tg}\theta\sqrt{\frac{1+g_{PSK}\bar\delta}{g_{PSK}\bar\delta}}\right)}$$

SER FOR MPSK POINT-TO-POINT

$$\boxed{P_S=\frac{M-1}{M}\left[1-\frac{M}{(M-1)\pi}\sqrt{\frac{g_{PSK}\bar\delta}{1+g_{PSK}\bar\delta}}\operatorname{arctg}\left(\operatorname{tg}\theta\sqrt{\frac{1+g_{PSK}}{g_{PSK}}}\right)\right]}$$

SER FOR MPSK POINT-TO-POINT

$$\operatorname{arctg}\left(\frac{x}{y}\right)=z\qquad\frac{x}{y}=\tan(z)\qquad\boxed{\frac{y}{x}=\operatorname{ctg}(z)}$$

$$\operatorname{arctg}x+\operatorname{arcctg}(x)=\frac{\pi}{2}\rightarrow\operatorname{arctg}x=\frac{\pi}{2}-\operatorname{arccot}(x)$$

$$\operatorname{arccot}(x)=\operatorname{arctg}(1/x)\rightarrow\operatorname{arctg}x=\frac{\pi}{2}-\operatorname{arctg}\left(\frac{1}{x}\right)$$

$$\boxed{P_S=\frac{M-1}{M}\left[1-\frac{M}{(M-1)\pi}\sqrt{\frac{g_{PSK}\bar\delta}{1+g_{PSK}\bar\delta}}\left(\frac{\pi}{2}\ominus\operatorname{arctg}\left(\cot\theta\sqrt{\frac{g_{PSK}}{1+g_{PSK}}}\right)\right)\right]}$$

VO DCofC OVDE IMA ZNAK „+"?

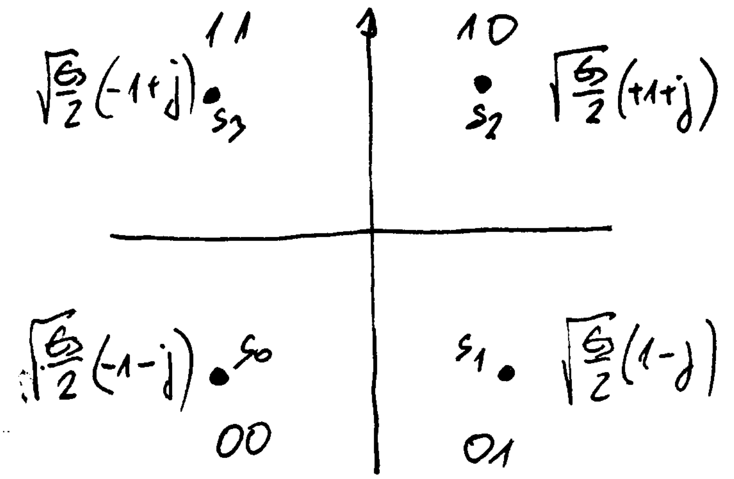
$$P_x=\mathcal{E}\left[|x_1|^2\right]=\cdots\mathcal{E}\left[|x_K|^2\right]$$

$$\boxed{P_x=\frac{\textcircled{P}L}{n_t^S\cdot K}}$$

VKUPNA SNAGA!!!

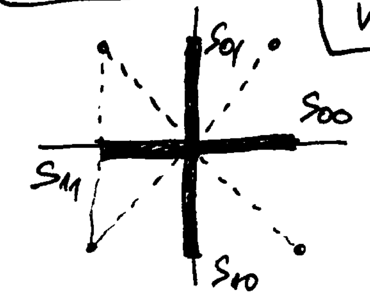
PO LINK

⊡ dsplog: SER for QPSK in AWGN



MOJA IMPLEMENTACIJA (N3-67)

ISTA TAKVA E MODULACIJATA VO MATLAB

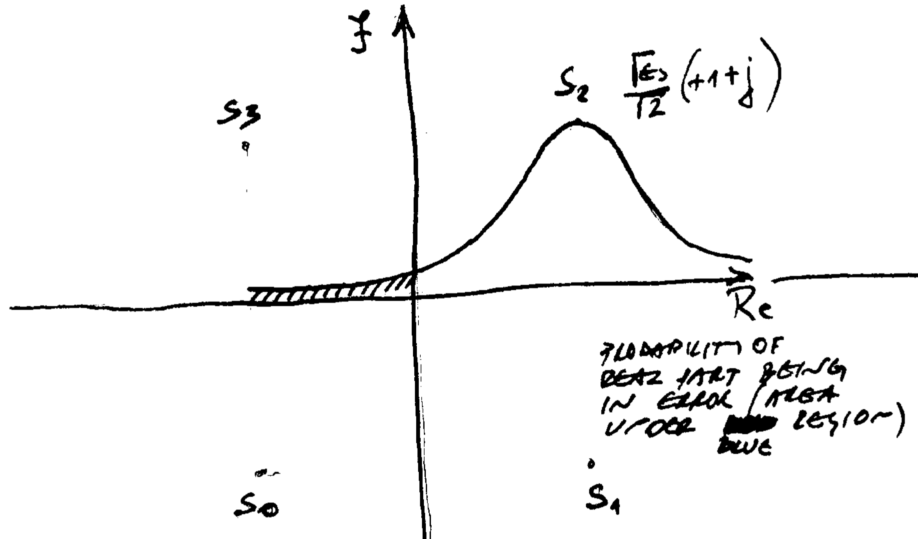


ELSAT

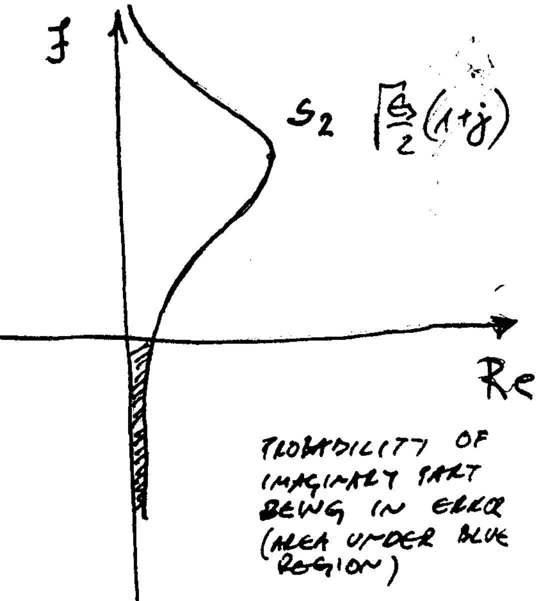
$$x=\left[\cos\left(\frac{2\pi m}{M}\right),\ \sin\left(\frac{2\pi m}{M}\right)\right]$$

$$x=\left[\underset{0}{(1,0)};\underset{1}{(0,1)};\underset{3}{(-1,0)};\underset{2}{(0,-1)}\right]$$

$\sqrt{\frac{E_s}{2}}$ ⇒ ZA NORMALIZIRANJE NA SREDNATA ENERGIJA NA ISPRATENITE SIMBOLI NA „1" POD PRETPOSTAVKA DEKA SITE TOČKI OD KONSTELACIJATA SE PODEDNAKVO VEROJATNI.



SL.1



SL.2

- THE CONDITIONAL PROBABILITY DISTRIBUTION FUNCTION GIVEN s_2 WAS TRANSMITTED

$$p(y/s_2)=\frac{1}{\sqrt{\pi N_0}}\,e^{-\frac{\left(y-\sqrt{\frac{E_s}{2}}\right)^2}{N_0}}$$

- S_2 IS DECODED CORRECTLY IF „Y" FALLS IN HASHED REGION

$$P(c/s_2)=P(\mathcal{R}[y]>0/s_2)\,P(\mathcal{I}[y]>0/s_2)$$

- PROBABILITY THAT REAL COMPONENT OF Y IS GREATER THAN 0 GIVEN s_2 IS TRANSMITTED IS (VIDI SL.1)

$$P(Re[y]>0/s_2)=1-\underbrace{\int_{-\infty}^{0}\frac{1}{\sqrt{\pi N_0}}\,e^{-\frac{\left(Re[y]-\sqrt{\frac{E_s}{2}}\right)^2}{N_0}}dy}_{I}\Rightarrow\quad \mathrm{erfc}(x)=\frac{2}{\sqrt{\pi}}\int_{x}^{\infty}e^{-x^2}dx$$

$$I = \frac{1}{\sqrt{\pi N_0}} \int_{-\infty}^{0} e^{-\frac{\left(Re[Y] - \sqrt{\frac{E_s}{2}}\right)^2}{N_0}} dy = \frac{1}{\sqrt{\pi N_0}} \int_{-\infty}^{0} e^{-x^2} dx$$

$$x = -\frac{Y - \sqrt{\frac{E_s}{2}}}{\sqrt{N_0}} \qquad dx = \frac{-1}{\sqrt{N_0}} dy \quad dy = -\sqrt{N_0} \cdot dx$$

$$Y = -\infty \quad x = +\infty \qquad Y = 0 \quad x = +\frac{1}{\sqrt{N_0}}\sqrt{\frac{E_s}{2}} = +\sqrt{\frac{E_s}{2N_0}}$$

$$I = \ominus \frac{\sqrt{N_0}}{\sqrt{\pi N_0}} \int_{+\infty}^{+\sqrt{E_s/2N_0}} e^{-x^2} dx = \frac{1}{\sqrt{\pi}} \int_{\sqrt{E_s/2N_0}}^{\infty} e^{-t^2} dt = \frac{1}{2} erfc\left(\sqrt{\frac{E_s}{2N_0}}\right)$$

$$\boxed{P\left(Re[Y] > 0 / s_2\right) = 1 - \frac{1}{2} erfc\left(\sqrt{\frac{E_s}{2N_0}}\right) = 1 - Q\left(\sqrt{\frac{E_s}{N_0}}\right)}$$

$$Q(z) = \frac{1}{2} erfc \frac{z}{\sqrt{2}} \qquad \frac{1}{2} erfc\left(\frac{\sqrt{\frac{E_s}{N_0}}}{\sqrt{2}}\right)$$

- SIMILARLY PROBABILITY OF IMAGINARY PART OF Y IS GREATER THAN 0, GIVEN S_2 WAS TRANSMITTED:

$$P\left(Im[Y] > 0 / s_2\right) = 1 - \frac{1}{\sqrt{\pi N_0}} \int_{-\infty}^{0} e^{-\frac{\left(Im[Y] - \sqrt{\frac{E_s}{2}}\right)^2}{N_0}} dy$$

$$= 1 - \frac{1}{2} erfc\left(\sqrt{\frac{E_s}{2N_0}}\right) = 1 - Q\left(\sqrt{\frac{E_s}{N_0}}\right)$$

- HENCE PROBABILITY OF s_2 BEEING DECODED CORECTLY IS:

$$\boxed{P(c/s_2) = \left(1 - \frac{1}{2} erfc\left(\sqrt{\frac{E_s}{2N_0}}\right)\right)^2 = 1 - erfc\left(\sqrt{\frac{E_s}{2N_0}}\right) + \frac{1}{4} erfc^2\left(\sqrt{\frac{E_s}{2N_0}}\right)}$$

- THE SYMBOL WILL BE IN ERROR IF ATLEAST ONE OF SYMBOLS IS DECODED INCORECTLY.

$$P_{QPSK}\Big|_{SYMBOL} = 1 - \left(1 - erfc\left(\sqrt{\frac{E_s}{2N_0}}\right) + \frac{1}{4} erfc^2\left(\sqrt{\frac{E_s}{2N_0}}\right)\right) = erfc\left(\sqrt{\frac{E_s}{2N_0}}\right) - \frac{1}{4} erfc^2\left(\sqrt{\frac{E_s}{2N_0}}\right)$$

$$\boxed{P_{QPSK} \doteq erfc\left(\sqrt{\frac{E_s}{2N_0}}\right) = 2Q\left(\sqrt{\frac{E_s}{N_0}}\right)}$$

SER FOR QPSK IN AWGN !!!

$$\boxed{E_s = Log2(M) \cdot E_b}$$

$$P_{QPSK}\Big|_{SYM} = 2Q\left(\sqrt{\frac{E_s}{N_0}}\right) - Q^2\left(\sqrt{\frac{E_s}{N_0}}\right) = 2Q\left(\sqrt{\frac{2E_b}{N_0}}\right) - Q^2\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

ISTO KAKO VO MATLAB !!!

- VO MATLAB VELAT DEKA (NAVODNO FROM PROAKIS)

$$\boxed{P_{bQPSK} = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)}$$

BER ZA QPSK VO AWGN KANAL.

- VO DCofC GO DAVAAT APROKSIMATIVNIOT IZRAZ ZA BER NA MPSK (8.32)

$$\boxed{P_b(\epsilon) \doteq \frac{2}{\max(\log_2(M),2)} \sum_{i=1}^{\max(M/4,1)} Q\left(\sqrt{\frac{2E_b\log_2 M}{N_0}}\sin\frac{(2i-1)\pi}{M}\right) = Q\left(\sqrt{2E_sN_0}\sin\frac{(2i-1)\pi}{M}\right)}$$

GENERALEN IZRAZ ŠTO VAŽI ZA SEKO KOJA VREDNOST NA „M".

ZA GOLEMI VREDNOSTI NA EbN0 I M>4

$$\boxed{P_b(\epsilon) \doteq \frac{2}{\log_2 M} Q\left(\sqrt{\frac{2E_s}{N_0}}\sin\frac{\pi}{M}\right)}$$

$$Q(z) = \frac{1}{2}\operatorname{erfc}\left(\frac{z}{\sqrt{2}}\right)$$

$$\boxed{P_b(\epsilon) = \frac{1}{\max(\log_2(M),2)} \sum_{i=1}^{\max(M/4,1)} \operatorname{erfc}\left(\sqrt{E_sN_0}\sin\frac{(2i-1)\pi}{M}\right)}$$

OVOJ E SUPER IZRAZ!!! FUNKCIONIRA ZA SEKOE M = 2, 4, 8, 16, 32. OSOBENO E TOČEN ZA EsN0 > 10 dB

- OD CLANAKOT NA Li YANG IZRAZOT ZA PDF NA NxNxN ZBIR E:

$$\boxed{f_{\gamma_1}(\gamma) = \frac{2\gamma^{2N^2-1}e^{-2\frac{\gamma}{\bar{\gamma}}}}{[\Gamma(N^2)]^2\,\bar{\gamma}^{2N^2}} \sum_{i=0}^{2N^2}\binom{2N^2}{i}K_{N^2-i}\left(\frac{2\gamma}{\bar{\gamma}}\right)}$$

$\boxed{N=2}$

$$f_{\gamma_1}(\gamma) = \frac{2\gamma^7 e^{-2\gamma/\bar{\gamma}}}{\Gamma^2(4)\,\bar{\gamma}^8}\sum_{i=0}^{8}\binom{8}{i}K_{N^2-i}\left(\frac{2\gamma}{\bar{\gamma}}\right)$$

$$f_{\gamma_1}(\gamma) = \frac{\gamma^7 e^{-2\gamma/\bar{\gamma}}}{18\,\bar{\gamma}^8}\left[K_4\left(\frac{2\gamma}{\bar{\gamma}}\right) + 8K_3\left(\frac{2\gamma}{\bar{\gamma}}\right) + 28K_2\left(\frac{2\gamma}{\bar{\gamma}}\right) + 56K_1\left(\frac{2\gamma}{\bar{\gamma}}\right) + \right.$$

$$+ 70 K_0\left(\frac{2\gamma}{\bar\gamma}\right) + 56 K_1\left(\frac{2\gamma}{\bar\gamma}\right) + 28 K_2\left(\frac{2\gamma}{\bar\gamma}\right) + 8 K_3\left(\frac{2\gamma}{\bar\gamma}\right) + K_4\left(\frac{2\gamma}{\bar\gamma}\right)\Bigg]$$

$$f_{\gamma_1}(\gamma) = \frac{\gamma^7 e^{-2\gamma/\bar\gamma}}{18\,\bar\gamma^8}\left[2K_4\left(\frac{2\gamma}{\bar\gamma}\right) + 16K_3\left(\frac{2\gamma}{\bar\gamma}\right) + 56K_2\left(\frac{2\gamma}{\bar\gamma}\right) + 112K_1\left(\frac{2\gamma}{\bar\gamma}\right) + 70K_0\right]$$

$$\boxed{f_{\gamma_1}(\gamma) = \frac{\gamma^2 e^{-2\gamma/\bar\gamma}}{9\,\bar\gamma^8}\left[K_4\left(\frac{2\gamma}{\bar\gamma}\right) + 8K_3\left(\frac{2\gamma}{\bar\gamma}\right) + 28K_2\left(\frac{2\gamma}{\bar\gamma}\right) + 56K_1\left(\frac{2\gamma}{\bar\gamma}\right) + 35K_0\left(\frac{2\gamma}{\bar\gamma}\right)\right]}$$

- GVE STO DAVA IZRAZOT (13) OD DCF CLANAKOT NA I.H. LEE

$\beta_1 = \beta_2 = 1$

$$f_{\gamma_A}^{DCF}(\gamma) = \left(\frac{1}{c\varrho}\right)^6 \boxed{\gamma^5} e^{-\frac{2\gamma}{c\varrho}}\Bigg[\overbrace{\frac{16\gamma^2}{18c^2\varrho^2}K_4\left(\frac{2\gamma}{c\varrho}\right)}^{\sim M_1} + \overbrace{\left[\frac{8\cdot4\cdot\gamma^2}{9c^2\varrho^2} - \frac{8\gamma}{3c\varrho}\right]K_3\left(\frac{2\gamma}{c\varrho}\right)}^{\sim M_2}$$

$$+ \underbrace{\left\{\frac{32\gamma^2}{c^2\varrho^2} - \frac{16\gamma}{3c\varrho} + \frac{2}{3}\right\}K_2\left(\frac{2\gamma}{c\varrho}\right)}_{\sim M_3} + \underbrace{\left\{\frac{32\gamma^2}{9c^2\varrho^2} - \frac{8\gamma}{3c\varrho}\right\}K_1\left(\frac{2\gamma}{c\varrho}\right)}_{M_4}$$

$$+ \underbrace{\frac{8\gamma^2}{9c^2\varrho^2}K_0\left(\frac{2\gamma}{c\varrho}\right)}_{\sim M_5}$$

$$f_{\gamma_A}^{DCF}(\gamma) = \frac{\gamma^7}{(c\varrho)^8}e^{-\frac{2\gamma}{c\varrho}}\Bigg[\frac{8}{9}K_4\left(\frac{2\gamma}{c\varrho}\right) + \frac{32}{9}K_3\left(\frac{2\gamma}{c\varrho}\right) + 32K_2\left(\frac{2\gamma}{c\varrho}\right) +$$

$$+ \frac{8}{9}K_0\left(\frac{2\gamma}{c\varrho}\right)\Bigg] + \frac{\gamma^5 e^{-\frac{2\gamma}{c\varrho}}}{(c\varrho)^6}\left[-\frac{8\gamma}{3c\varrho}K_3\left(\frac{2\gamma}{c\varrho}\right) + \left(-\frac{16\gamma}{3c\varrho} + \frac{2}{3}\right)K_2\left(\frac{2\gamma}{c\varrho}\right) - \frac{8\gamma}{3c\varrho}K_1\left(\frac{2\gamma}{c\varrho}\right)\right]$$

$$M(-s) = \int_{-\infty}^{\infty} f_{\gamma_A}^{DCF}(\gamma)\, e^{-\gamma s}\, d\gamma$$

$$\int_0^\infty x^{\mu-1}e^{-\alpha x}K_\nu(\beta x)\,dx = \frac{\sqrt{\pi}(2\beta)^\nu}{(\alpha+\beta)^{\mu+\nu}}\frac{\Gamma(\mu+\nu)\Gamma(\mu-\nu)}{\Gamma(\mu+\frac{1}{2})}F\left(\mu+\nu, \nu+\frac{1}{2}; \mu+\frac{1}{2}; \frac{\alpha-\beta}{\alpha+\beta}\right)$$

$$M_1 = \frac{1}{c^6\varrho^6}\int_0^\infty \frac{16\gamma^7}{18c^2\varrho^2}K_4\left(\frac{2\gamma}{c\varrho}\right)e^{-\frac{2\gamma}{c\varrho} - s\gamma}\,d\gamma = \frac{16}{18c^8\varrho^8}\int_0^\infty \gamma^7 e^{-\overbrace{\left(\frac{2}{c\varrho}+s\right)}^{\alpha}\gamma}K_4\Bigg(\underbrace{\frac{2}{c\varrho}}_{\beta}\gamma\Bigg)\,d\gamma$$

$\mu = 8 \quad \nu = 4$

$$M_1 = \frac{16}{18c^8\varrho^8}\frac{\sqrt{\pi}\left(\frac{4}{c\varrho}\right)^4}{\Bigg(\underbrace{\frac{2}{c\varrho}+s+\frac{2}{c\varrho}}_{\psi}\Bigg)^{12}}\frac{\Gamma(12)\Gamma(4)}{\Gamma\left(\frac{17}{2}\right)}\,{}_2F_1\left(12, \frac{9}{2}; \frac{17}{2}; \Omega\right)$$

$4^4 = 2^8$

$$\underline{\underline{M_1}} = \frac{2^{28}}{39\cdot c^8\varrho^8\psi^{12}\cdot c^4\varrho^4}\,{}_2F_1(12, 4.5; 8.5; \Omega) = \underline{\underline{\frac{2^{28}}{c^{12}\varrho^{12}\psi^{12}}\,{}_2F_1(12, 4.5; 8.5; \Omega)}}$$

$$\boxed{\psi = \frac{4}{c\varrho} + s \qquad \Omega = \frac{\cancel{\frac{2}{c\varrho}} + s - \cancel{\frac{2}{c\varrho}}}{\frac{2}{c\varrho}+s+\frac{2}{c\varrho}} = \frac{s}{\frac{4}{c\varrho}+s} = \frac{s\cdot c\varrho}{4+sc\varrho}}$$

- VO MAPLE IMA MOZNOST VO [inttrans] DA DODADES LINIJA VO TABELATA (OD GRADSHTEJN NA PRIMER) SO KORISTENJE NA „addtable"

- ISTO TAKA VO VRSKA SO DILEMATA ŠTO SE PUŠTA VNATRE VO MGF VO CLANCITE NA I. H. LEE OTKAKO SE NAVRATIV NA 2+2+2 CLANAKOT KONSTATIRAV DEKA I TAMU SE PUŠTA $E_sN_0 \cdot nS$

- JA IMPLEMENTIRAV FORMULATA 6.621.3 OD GRADSHTEJN VO MAPLE MultLoopMIMO.mw (3.10.5)

pp. 59 →

268435456

$$M1 = \frac{2^{28}}{39\cdot c^{12}\rho^{12}\psi^{12}}\;{}_2F_1(12, 4.5; 8.5; z) = \frac{2^{28}}{39\cdot c^{12}\rho^{12}\cdot\left(\frac{4}{c\rho}+s\right)^{12}}\cdot$$

$$\cdot\,{}_2F_1\left(12, 4.5; 8.5; \frac{s\cdot c\rho}{4+sc\rho}\right) = \underline{\frac{2^{28}\cdot {}_2F_1\left(12, 4.5; 8.5; \frac{sc\rho}{4+sc\rho}\right)}{39\cdot(4+sc\rho)^{12}}}$$

POVTORENA
~~RESENA~~ VO MAPLE (3.10.5)

$$M_2 = \mathcal{L}\left[\left(\frac{\gamma^2 32\cdot x}{9c^2\rho^2} - \frac{8\cancel{\gamma}\cdot x}{3c\rho}\right)K_3\left(\frac{2\gamma}{c\rho}\right)\right] = \underbrace{\mathcal{L}\left[\frac{\gamma^2 32\cdot x}{9c^2\rho^2}K_3\left(\frac{2\gamma}{c\rho}\right)\right]}_{M_{21}} +$$

$$+\underbrace{\mathcal{L}\left[\frac{8\gamma\cdot x}{3c\rho}K_3\left(\frac{2\gamma}{c\rho}\right)\right]}_{M_{22}} = M_{21} + M_{22}$$

$$\boxed{x = \frac{\gamma^5}{c^6\rho^6}e^{-\frac{2\gamma}{c\cdot\rho}}}$$

$$M_{21} = \frac{32}{9c^2\rho^2}\cdot\frac{1}{c^6\rho^6}\mathcal{L}\left[\gamma^7 e^{-\frac{2}{c\rho}\gamma}\cdot K_3\left(\frac{2\gamma}{c\rho}\right)\right]$$

(USE MAPLE 3.10.6)

~~536870912~~
1073741824

$$\boxed{M_{21} = \frac{2^{30}}{429\,[illegible]\,(sc\rho+4)^{[illegible]}}\;{}_2F_1\left([11, 3.5]; 8.5; \frac{sc\rho}{sc\rho+4}\right)}$$

$$M_{22} = -\frac{8}{3c^7\rho^7}\mathcal{L}\left[\gamma^6 e^{-\frac{2\gamma}{c\rho}}K_3\left(\frac{2\gamma}{c\rho}\right)\right]$$

50331648

$$\boxed{M_{22} = \ominus\frac{3\cdot 2^{24}}{143\,(sc\rho+4)^{10}}\;{}_2F_1\left([10, 3.5]; 7.5; \frac{sc\rho}{sc\rho+4}\right)}$$

$$M_3 = \mathcal{L}\left[\left(\frac{32\gamma^7 e^{-\frac{2\gamma}{c\rho}}}{c^8\rho^8} - \frac{16\gamma^6 e^{-\frac{2\gamma}{c\rho}}}{3c^7\rho^7} + \frac{2\gamma^5 e^{-\frac{2\gamma}{c\rho}}}{3c^6\rho^6}\right)K_2\left(\frac{2\gamma}{c\rho}\right)\right]$$

$$M_{31} = \frac{3\cdot 2^{27}}{143\,(sc\rho+4)^{10}}\ {}_2F_1\left([10, 2.5]; 8.5; \frac{sc\rho}{sc\rho+4}\right) \quad 3\cdot 2^{27} \rightarrow 402653184$$

$$M_{32} = \frac{-2^{25}}{429\,(sc\rho+4)^{9}}\ {}_2F_1\left([9, 2.5]; 7.5; \frac{sc\rho}{sc\rho+4}\right) \quad 2^{25} \rightarrow 33554432$$

$$M_{33} = \frac{2^{16}}{33\,(sc\rho+4)^{8}}\ {}_2F_1\left([8, 2.5]; 6.5; \frac{sc\rho}{sc\rho+4}\right) \quad 2^{16} - 65536$$

☐ $$M_4 = \mathcal{L}\left[\left(\frac{32\gamma^7 e^{-\frac{2\gamma}{c\rho}}}{9c^8\rho^8} - \frac{8\gamma^6 e^{-\frac{2\gamma}{c\rho}}}{3c^7\rho^7}\right)K_1\left(\frac{2\gamma}{c\rho}\right)\right]$$

$$M_{41} = \frac{2^{26}}{1287\,(sc\rho+4)^{9}}\ {}_2F_1\left([9, 1.5]; 8.5; \frac{sc\rho}{sc\rho+4}\right) \quad 2^{26} \rightarrow 67108864$$

$$M_{42} = \frac{-5\cdot 2^{19}}{427\,(sc\rho+4)^{8}}\ {}_2F_1\left([8, 1.5]; 7.5; \frac{sc\rho}{sc\rho+4}\right) \quad 5\cdot 2^{19} - 2621440$$

☐ $$\boxed{M_5 = \mathcal{L}\left[\frac{8\gamma^7 e^{-\frac{2\gamma}{c\rho}}}{9c^8\rho^8}K_0\left(\frac{2\gamma}{c\rho}\right)\right] = \frac{7\cdot 2^{19}\ {}_2F_1\left([8, 0.5]; 8.5; \frac{sc\rho}{sc\rho+4}\right)}{1287\,(sc\rho+4)^{8}}} \quad 7\cdot 2^{19} - 3670016$$

3220392

$$2^{19} + 2^{20} + 2^{21} = 2^{19}(1+2+4) = 7\cdot 2^{19}$$

$$\boxed{s = \frac{g}{\sin^2\theta}} \qquad sc\rho + 4 = \frac{gc\rho}{\sin^2\theta} + 4 = \frac{gc\rho + 4\sin^2\theta}{\sin^2\theta}$$

$$\frac{sc\rho}{sc\rho+4} = \frac{gc\rho}{gc\rho + 4\sin^2\theta} \qquad \underline{\underline{nS}}$$

$$\rho = \frac{P}{\sigma^2} \qquad c = \frac{1}{2\log_2 M} \qquad \text{T.e} \qquad c = \frac{L}{nS\cdot K\cdot \log_2 M}$$

BLOCK LENGHT (L)

SYMBOLS (K)

$L = 2 \quad K = 2$ } ~~ZA~~ ALAMOUTI CODE

$$\rho = E_sN_0 \qquad \rho\cdot c = \frac{E_sN_0}{nS\cdot\log_2 M} = \frac{E_bN_0}{nS}$$

E_bN_0 PER LINK

$$\mathcal{L}\left[\frac{d^{N-1}}{dx^{N-1}}(f(x))\right] = s^N\cdot F(s)$$

$$M(-s) = \int_0^\infty p(x)e^{-xs}\,dx = \hat{p}(s) = \mathcal{L}[p(x)]$$

$$P(x) = \int_0^x p(x)\,dx \qquad p(x) = \frac{dP(x)}{dx}$$

$p(x) = \frac{dP(x)}{dx}$ $\qquad M(-s) = \int_0^\infty \frac{dP(x)}{dx} e^{-xs} dx$

$M(-s) = \mathcal{L}\left[\frac{dP(x)}{dx}\right] = s \cdot \underline{\underline{\hat{P}(s)}}$ $\qquad \hat{P}(s) = \mathcal{L}[P(x)]$

$\mathcal{L}[P(s)] = \frac{M(-s)}{s}$ $\qquad \boxed{P(x) = \mathcal{L}^{-1}\left[\frac{M(-s)}{s}\right]}$

$\underline{\underline{M_W(s)}}$ $\qquad \boxed{W = \frac{1}{\gamma}}$

$P_W(W < x) = \mathcal{L}^{-1}\left[\frac{M_W(-s)}{s}\right]$

$P_\gamma = P_W\left(\frac{1}{\gamma} < x\right) = P_W\left(\frac{1}{x} < \gamma\right) = P_W\left(\gamma > \frac{1}{x}\right)$

$= 1 - \underline{P_W\left(\gamma < \frac{1}{x}\right)}$ $\qquad P_\gamma = 1 - P_W\left(\gamma < \frac{1}{x}\right)$

$\boxed{P_\gamma = 1 - \mathcal{L}^{-1}\left[\frac{M_W(-s)}{s}\right]\Big|_{W = \frac{1 \cdot s}{\gamma}}} = \frac{d^{N-1}}{dx^{N-1}} \mathcal{L}^{-1}\left[\frac{M_W(-s)}{s^N}\right]$

VIDI N9.63

- SO SIMBOLITE OD END-TO-END SLUČAJOT NA M.O. HASNA

$P_{\cancel{x}} = P_r(\chi < x) = P_r\left(\frac{1}{\chi} > \frac{1}{x}\right) = P_r\left(Z > \frac{1}{x}\right) = 1 - P_r\left(Z < \frac{1}{x}\right)$

$P_\chi(x) = 1 - P_r\left(Z < \frac{1}{x}\right) = 1 - P_Z\left(\frac{1}{x}\right)$

$P_Z = \mathcal{L}^{-1}\left[\frac{M_{\cancel{Z}}(-s)}{s}\right]$ $\qquad P_\chi(x) = 1 - \mathcal{L}^{-1}\left[\frac{M_{\cancel{Z}}(-s)}{s}\right]\Big|_{Z = \frac{1}{\cancel{x}}}$

$\mathcal{L}^{-1}\left[\frac{\Phi(s)}{s^{N-1}}\right] = ?$ $\qquad \boxed{\Phi(s) = \frac{M_Z(-s)}{s}}$

$$\mathcal{L}\left[\int \varphi(x)\,dx\right] = \frac{\Phi(s)}{s}$$

- FREQUENCY DIFERENCIATION OF LAPLACE TRANSFORM:

$$\mathcal{L}\left[x^n f(x)\right] = (-1)^n \frac{d^n F(s)}{ds^n} = (-1)^n F^{(n)}(s)$$

$$\underbrace{\frac{d^{n-1}}{dx^{n-1}} \mathcal{L}^{-1}\left[\frac{F(s)}{s^N}\right]}_{= \int_0^x f(t)\,dt} \qquad \mathcal{L}\Big[\underbrace{\int\int\cdots\int}_{n} f(x)\,dx\Big] = \frac{F(s)}{s^N}$$

$$\mathcal{L}\left[\int_0^x f(t)\,dt\right] = \frac{F(s)}{s}$$

$$\mathcal{L}\left[\int_0^t \boxed{\int_0^r f(\lambda)\,d\lambda}\,dr\right] \qquad \int_0^\infty \left[\int_0^t \int_0^r f(\lambda)\,d\lambda\,dr\right] e^{-st}\,dt$$

$$= \cancel{\int_0^t \int_0^\infty f(\lambda)e^{-st}d} \quad \int_0^t \left\{\int_0^\infty \left[\int_0^r f(\lambda)\,d\lambda\right] e^{-st}\,dt\right\} dr$$

$$\mathcal{L}\left[\frac{d^N}{dx^N} f(x)\right] = s^N \cdot F(s)$$

$$\mathcal{L}\left[\frac{d^{N-1}}{dx^{N-1}} f(x)\right] = s^{N-1} F(s)$$

$$\frac{d^{N-1}}{dx^{N-1}} \mathcal{L}^{-1}\left[\frac{F(s)}{s^N}\right] = \frac{d^{N-1}}{dx^{N-1}} \frac{1}{2\pi j}\int_{+\sigma-j\infty}^{\sigma+j\infty} \frac{F(s)}{s^N} e^{+xs}\,ds =$$

$$= \frac{1}{2\pi j}\int_{+\sigma-j\infty}^{\sigma+j\infty} \frac{F(s)}{s^N} \frac{d^{N-1}}{dx^{N-1}}\left(e^{xs}\right) ds = \frac{1}{2\pi}\int_{+\sigma-j\infty}^{\sigma+j\infty} \frac{F(s)}{s^{N}} s^{N-1} e^{xs}\,ds = \boxed{\frac{1}{2\pi}\int_{+\sigma-j\infty}^{\sigma+j\infty} \frac{F(s)}{s} e^{+s}\,ds}$$

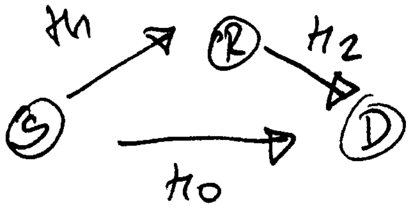
$$\to = \int_0^x f(t)\,dt = \text{CO}$$

DOKAŽIMO!

MMV

- L. Yang - Performance Analysis of MIMO ... (revisited)

$$Y_{SR} = \sqrt{\frac{P}{N}} H_1 C + W_1 \qquad Y_{SD} = \sqrt{\frac{P}{N}} H_0 C_1 + W_0$$



Y_{SR} Y_{SD} N×T MATRICES

$$r_{R,l} = \sqrt{\frac{P}{N}} c \|H_1\|_F^2 \Delta_l + \tilde{w}_{R,l} \qquad l=1,2,\dots L$$

"C" DEPENDS ON STBC MATRIX (C=1 FOR ALAMOUTI)

- RELAY NORMALIZES THE ~~RECEIVED SIGNAL~~

$$\boxed{r'_{R,l} = \frac{\sqrt{\frac{P}{N}}\sqrt{c}\|H_1\|_F \Delta_l}{\sqrt{c\|H_1\|_F^2 \frac{P}{N} + N_0}} + \frac{\tilde{w}_{R,l}}{\sqrt{c}\|H_1\|_F \sqrt{c\|H_1\|_F^2\frac{P}{N}+N_0}}} \quad (*)$$

$l=1,\dots,L$

SO OVA VSUŠNOST MNOŽI SO GAIN KAKO ŠTO ZAD IMAM MISLENO VO CLANCITE (e.g. WCNC11 Eq. 7)

$$\boxed{G_1 = \sqrt{\frac{G_R}{E_S\Delta_1^2+\Delta_1 N_0}}} \qquad \boxed{\Delta_1 \triangleq \|H_1\|_F^2}$$

GAINOT SOGLASNO L. YANG e: $G_1 = \dfrac{1}{\sqrt{c^2\|H_1\|_F^4\frac{P}{N} + cN_0\|H_1\|_F^2}}$

ZNACI MOJATA FORMULA PROŠIRENA SO "C" BI BILA

$$\boxed{G_1 = \sqrt{\frac{G_R}{E_S c^2\Delta_1^2 + c\Delta_1 N_0}}}$$

MMV

- SIGNAL AT THE RECEIVER (*) (YRD)

$$r_{RD,L} = \sqrt{\frac{P}{N}} c\|H_2\|_F^2 r'_{R,l} + \tilde{w}_{R,D,L}$$

$$r_{RD,L} = \frac{\sqrt{\frac{P}{N}}c\|H_2\|_F^2\sqrt{\frac{P}{N}}\sqrt{c}\|H_1\|_F\Delta_l}{\sqrt{c\|H_1\|_F^2\frac{P}{N}+N_0}} + \frac{\sqrt{\frac{P}{N}}\sqrt{c}\|H_1\|_F \tilde{w}_{R,l}}{\sqrt{c}\|H_1\|_F\sqrt{c\|H_1\|_F^2\frac{P}{N}+N_0}} + \tilde{w}_{RD,L}$$

$$\boxed{r_{SD} = \sqrt{\frac{P}{N}} c\|H_0\|_F^2\Delta_l + \tilde{w}_{SD,L}}$$

$$\gamma_{CAF} = \bar{\gamma}X + \frac{\bar{\gamma}^2 YZ}{\bar{\gamma}Y + \bar{\gamma}Z + 1} \approx \bar{\gamma}X + \frac{\bar{\gamma}YZ}{Y+Z} = \gamma_0 + \gamma_1$$

$$X = \|H_0\|_F^2 \qquad Y = \|H_1\|_F^2 \qquad Z = \|H_2\|_F^2 \qquad \boxed{\bar{\gamma} = \frac{P}{N N_0}}$$

- PDF OF APROXIMATE SNR γ_1 IS:

$$\boxed{f_{\gamma_1}(\gamma) = \frac{2\gamma^{2N^2-1} e^{-2\frac{\gamma}{\bar{\gamma}}}}{[\Gamma(N^2)]^2 \bar{\gamma}^{2N^2}} \sum_{i=0}^{2N^2} \binom{2N^2}{i} K_{N^2-i}\left(\frac{2\gamma}{\bar{\gamma}}\right)}$$

- UPER BOUND OF γ_1:

$$\boxed{\gamma_1^{UP} = \bar{\gamma} \min(Y, Z)}$$

$$f_{\gamma_1}^{UP}(\gamma) = 2 \frac{\Gamma(N^2, \gamma/\bar{\gamma})}{[\Gamma(N^2)]^2 \bar{\gamma}^{N^2}} \gamma^{N^2-1} e^{-\frac{\gamma}{\bar{\gamma}}}$$

- 6.621.3 → GRADSHTEYN

$$MGF(-1) = \int_0^\infty f_{\gamma_1}(\gamma) e^{-\gamma 1} d\gamma = \frac{2}{\Gamma^2(N^2) \bar{\gamma}^{2N^2}} \sum_{i=0}^{2N^2} \binom{2N^2}{i} \gamma^{2N^2-1} \cdot e^{-\frac{2\gamma}{\bar{\gamma}}} \cdot K_{N^2-i}\left(\frac{2\gamma}{\bar{\gamma}}\right)$$

$\mu = 2N^2 \quad \nu = N^2 - i \quad \beta = \frac{2}{\bar{\gamma}} \quad \alpha = \frac{2}{\bar{\gamma}} + 1$

$$= \frac{2}{\Gamma^2(N^2)\bar{\gamma}^{2N^2}} \sum_{i=0}^{2N^2} \frac{\cancel{\Gamma(2N^2)}\sqrt{\pi}\,(4/\bar{\gamma})^{N^2-i}}{\left(\frac{4}{\bar{\gamma}}+1\right)^{3N^2-i}} \frac{\Gamma(3N^2-i)\Gamma(N^2+i)}{\Gamma(2N^2+\frac{1}{2})} F\left(3N^2-i, N^2-i+\frac{1}{2}; 2N^2+\frac{1}{2}; \frac{1\bar{\gamma}}{4+1\bar{\gamma}}\right)$$

$$\left/ \frac{\alpha-\beta}{\alpha+\beta} = \frac{1}{\frac{4}{\bar{\gamma}}+1} = \frac{1\bar{\gamma}}{4+1\bar{\gamma}} \right/$$

$$= \frac{2\sqrt{\pi}}{\Gamma^2(N^2)\bar{\gamma}^{2N^2}} \sum_{i=0}^{2N^2} \binom{2N^2}{i} \left(\frac{4}{\bar{\gamma}}\right)^{N^2-i} \frac{\Gamma(3N^2-i)\Gamma(N^2+i)}{\Gamma(2N^2+\frac{1}{2})} \left(1+\frac{4}{\bar{\gamma}}\right)^{+i-3N^2} F\left(3N^2-i, N^2-i+\frac{1}{2}; 2N^2+\frac{1}{2}; \frac{1\bar{\gamma}}{4+1\bar{\gamma}}\right)$$

MMV

APPENDIX A

$$f_{\gamma_1}(\gamma) = \frac{d}{d\gamma} Pr\left(\frac{YZ}{Y+Z} < \frac{\gamma}{\bar{\gamma}}\right) = \frac{d}{d\gamma} Pr\left(\frac{\bar{\gamma}YZ}{Y+Z} \leq \gamma\right)$$

$$g(x) = \int_a^x f(t)\,dt \qquad f(x) = \frac{dg(x)}{dx}$$

FUNDAMENTAL THEOREM OF CALCULUS PART I

$$f_{\gamma_1}(\gamma) = \int_0^\infty P_r\left(\frac{Y z}{Y+z}\right) f_Z(z)\, dz \qquad f_Z(z) - \text{PDF of "}z\text{"}$$

- FOR RAYLEIGH FADING X, Y, Z ARE CHI-SQUARE VARIABLES WITH $2N^2$ degrees.

$$\boxed{f_Z(z) = \frac{z^{N^2-1}}{\Gamma(N^2)} e^{-z}} \Rightarrow$$

SOGLASNO MODATA NOMENKLATURA OVA E GAMMA DISTRO.

CHI-SQUARE E:

$$f(x;k) = \frac{1}{2^{k/2}\Gamma(k/2)} x^{\frac{k}{2}-1} e^{-\frac{x}{2}}$$

$$\boxed{z = \frac{x}{2}} \qquad \gamma(z;k) = \frac{f(x;k)}{\frac{\partial z}{\partial x}}\Bigg|_{x=2z} = \frac{\frac{(2z)^{\frac{k}{2}-1} e^{-z}}{2^{k/2}\Gamma(k/2)}}{\frac{1}{2}}$$

$$= \frac{\cancel{2}^{\frac{k}{2}}}{\cancel{2^{\frac{k}{2}}}} \frac{z^{\frac{k}{2}-1}}{\Gamma(k/2)} e^{-z} = \frac{z^{\frac{k}{2}-1}}{\Gamma\left(\frac{k}{2}\right)} e^{-z} = \left|k = 2N^2\right| = \underline{\underline{\frac{z^{N^2-1}}{\Gamma(N^2)} e^{-z}}}$$

SO OVAA ~~[illegible]~~ FUNKCIONALNA TRANSFORMACIJA PREOǴAŠ OD CHI-SQUARE VO GAMMA PDF.

CDF for $f_Z(z)$

$$\boxed{F_Z(z) = \frac{\gamma(N^2, z)}{\Gamma(N^2)}} \qquad F_Z(z) = P_r(Z \le z) = \int_{-\infty}^{z} f_Z(z)\, dz$$

$$= 1 - \sum_{i=0}^{N^2-1} \frac{z^i}{i!} e^{-z}$$

$$\boxed{F_Z(z) = 1 - \sum_{i=0}^{N^2-1} \frac{z^i}{i!} e^{-z}}$$

MMV

IZVEDUVANJE OD L.-L. YANG ERROR PROBABILITY...

$$f(\gamma) = \frac{d}{d\gamma} P\left(\frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2} \le \gamma\right) = \frac{d}{d\gamma} \underbrace{\int_0^\infty P\left(\frac{\gamma_1 x}{\gamma_1 + x} \le \gamma\right) f_{\gamma_2}(x)\, dx}_{I}$$

PDF OF: $\gamma = \frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2}$

$$f(\gamma) = \frac{d}{d\gamma}\left[\int_0^{\gamma} P[\gamma_1(x-\gamma) \leq x\gamma]\, f_{\gamma_2}(x)\,dx + \int_{\gamma}^{\infty} P[\gamma_1(x-\gamma) \leq x\gamma]\, f_{\gamma_2}(x)\,dx\right.$$

$$\frac{\gamma_1 x}{\gamma_1 + x} \leq \gamma \qquad \gamma_1 x \leq \gamma\gamma_1 + \gamma x$$

$$(\gamma_1 - \gamma)x \leq \gamma\gamma_1$$

$$\underline{\gamma_1(x-\gamma) \leq \gamma x}$$

$$I = \int_0^{\infty} P[\gamma_1(x-\gamma) \leq \gamma x]\, f_{\gamma_2}(x)\,dx$$

$0 < x < \gamma$ $\qquad P[\gamma_1(x-\gamma) \leq x\gamma] = 1$

$x = 0 \qquad P[-\gamma_1 \cdot \gamma \leq 0]$

$x = \gamma \qquad P[0 \leq \gamma^2]$

$$f(\gamma) = \frac{d}{d\gamma}\left[\int_0^{\gamma} f_{\gamma_2}(x)\,dx + \int_{\gamma}^{\infty} P[\gamma_1(x-\gamma) \leq x\gamma]\, f_{\gamma_2}(x)\,dx\right] =$$

$$= f_{\gamma_2}(\gamma) + \boxed{\frac{d}{d\gamma}\int_{\gamma}^{\infty} P\left[\gamma_1 \leq \frac{x\gamma}{x-\gamma}\right] f_{\gamma_2}(x)\,dx}$$

$\frac{x\gamma}{x-\gamma}$

$\$$ ●

OD OVDE
IDI DIREKTNO
NA PP. 73
NAZDOLU !!!

$$P\left[\gamma_1 \leq \frac{x\gamma}{x-\gamma}\right] = \int_0^{\frac{x\gamma}{x-\gamma}} f_{\gamma_1}(\gamma_1)\,d\gamma_1 = \frac{x}{x-\gamma}\int_0^{\gamma} f_{\gamma_1}\left(\frac{x}{x-\gamma}u\right)du$$

$$g(x) = \int_0^{x} f(t)\,dt \qquad \int_0^{a\cdot x} f(t)\,dt = ?$$

$$u = t/a \qquad \boxed{\begin{array}{ll} t = 0 & u = 0 \\ t = ax & u = x \end{array}}$$

$$du = dt/a$$

$$a\int_0^{x} f(au)\cdot du = \int_0^{x} f(au)\,d(au) = g(x)$$

$$\boxed{g'(x) = a \cdot f(ax)}$$

$$\int_0^{\frac{x\gamma}{x-\gamma}} f(\gamma_1)\,d\gamma_1 =$$

$$u = \frac{\gamma_1}{\frac{x}{x-\gamma}}$$

$\gamma_1 = 0 \qquad u = 0$

$\gamma_1 = \frac{x\gamma}{x-\gamma} \Rightarrow u = \gamma$

$$du = \frac{x-\gamma}{x}\,d\gamma_1 \qquad d\gamma_1 = \frac{x}{x-\gamma}\,du$$

$$P\left(\gamma_1 \le \frac{x\gamma}{x-\gamma}\right) = \int_0^{\frac{x\gamma}{x-\gamma}} f_{\gamma_1}(\gamma_1)\,d\gamma_1 = \frac{x}{x-\gamma}\int_0^{\gamma} f_{\gamma_1}\left(\frac{x}{x-\gamma}u\right)du$$

$\textcircled{*}$

$$\frac{d}{d\gamma}\left\{\int_\gamma^\infty \left(\frac{x}{x-\gamma}\int_0^\gamma f_{\gamma_1}\left(\frac{x}{x-\gamma}u\right)du\right)\cdot f_{\gamma_2}(x)\,dx\right\} =$$

$$= \int_\gamma^\infty f_{\gamma_2}(x)\,\underbrace{\frac{d}{d\gamma}\left[\frac{x}{x-\gamma}\int_0^\gamma f_{\gamma_1}\left(\frac{x}{x-\gamma}u\right)du\right]}_{\textcircled{\Delta}}dx = \textcircled{\$}$$

$$\textcircled{\Delta} = \frac{x\,\cancel{x}}{(x-\gamma)^2}\cdot\int_0^\gamma f_{\gamma_1}\left(\frac{x}{x-\gamma}u\right)du + \frac{x}{x-\gamma}f_{\gamma_1}\left(\frac{x}{x-\gamma}\gamma\right) =$$

$$\textcircled{\$} = +\int_\gamma^\infty f_{\gamma_2}(x)\cdot\frac{x}{(x-\gamma)^2}\int_0^\gamma f_{\gamma_1}\left(\frac{x}{x-\gamma}u\right)du + \int_\gamma^\infty \frac{x}{x-\gamma}f_{\gamma_1}\left(\frac{x\gamma}{x-\gamma}\right)f_{\gamma_2}(x)\,dx$$

$$\frac{d}{d\gamma}\left(\frac{a}{a-\gamma}\right) = a\frac{d}{d\gamma}(a-\gamma)^{-1} = -a\frac{(-1)}{(a-\gamma)^{+2}} = \frac{a}{(a-\gamma)^2}$$

$$\textcircled{*}\quad \frac{d}{d\gamma}\left[\int_\gamma^\infty P\left[\gamma_1 \le \frac{x\gamma}{x-\gamma}\right]f_{\gamma_2}(x)\,dx\right.$$

$$I = \int_0^\gamma f_{\gamma_1}\left(\frac{x}{x-\gamma}u\right)du = \left|\begin{array}{ll} \frac{x}{x-\gamma}u = v & u=0 \quad v=0 \\ du = \frac{x-\gamma}{x}\,dv & u=\gamma \quad v=\frac{x\gamma}{x-\gamma} \end{array}\right|$$

$$I = \frac{x-\gamma}{x}\int_0^{\frac{x\gamma}{x-\gamma}} f_{\gamma_1}(v)\,dv$$

$$\textcircled{\$} = \int_\gamma^\infty f_{\gamma_2}(x)\,\frac{\cancel{x}}{(x-\gamma)}\overbrace{\left(\int_0^{\frac{x\gamma}{x-\gamma}} f_{\gamma_1}(v)\,dv\right)}^{P\left(\gamma_1 \le \frac{x\gamma}{x-\gamma}\right)}dx + \int_\gamma^\infty \frac{x}{x-\gamma}f_{\gamma_1}\left(\frac{x\gamma}{x-\gamma}\right)f_{\gamma_2}(x)\,dx$$

$$\textcircled{\$} = \int_{\gamma}^{\infty} f_{\gamma_2}(x)\frac{1}{x-\gamma} P\left(\gamma_1 \le \frac{x\gamma}{x-\gamma}\right)dx + \int_{\gamma}^{\infty} \frac{x}{x-\gamma} f_{\gamma_1}\left(\frac{x\gamma}{x-\gamma}\right) f_{\gamma_2}(x)\,dx$$

$\gamma < x < \infty$

$x = \infty \quad P(\gamma_1 \le \gamma) = 0$

$x \to \gamma^+ \quad P\left(\gamma_1 \le \frac{\gamma^+\gamma}{\gamma^+ - \gamma}\right) = \underline{\underline{\lim_{x \to \gamma^+} P\left(\gamma_1 \le \frac{x\gamma}{x-\gamma}\right)}}$

$f(x) = \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{x^2}{2}} \qquad \mu = 0 \quad \sigma = 1 \Big\}$ NORMAL RASPRED

$P(x^2 < 1) = ? \qquad P(Y < 1) \qquad Y = x^2 \qquad \frac{dy}{dx} = \underline{\underline{2x}}$

$$f(y) = \frac{f(x)}{\left|\frac{dy}{dx}\right|}\Bigg|_{\boxed{x=+\sqrt{y}}} + \frac{f(x)}{\frac{dy}{dx}}\Bigg|_{x=-\sqrt{y}}$$

$$f(y) = \frac{\frac{1}{\sqrt{2\pi}} e^{-\frac{y}{2}}}{2\sqrt{y}} + \frac{\frac{1}{\sqrt{2\pi}} e^{-\frac{y}{2}}}{|-2\sqrt{y}|} = \frac{1}{\sqrt{2\pi y}} e^{-\frac{y}{2}}$$

$$P(x^2 < 1) = \underline{\underline{P(Y < 1)}} = \frac{1}{\sqrt{2\pi}} \int_0^1 \frac{e^{-\frac{y}{2}}}{\sqrt{y}} = \underline{\underline{~~\frac{\text{erf}(1)\sqrt{\pi}}{\sqrt{2\pi}} = \frac{1}{\sqrt{2}}\text{erf}(1)~~}}$$

$$= \cancel{\frac{1}{\sqrt{2\pi}}} \cdot \underbrace{\cancel{\sqrt{2\pi}}\,\text{erf}\left(\frac{\sqrt{2}}{2}\right)}_{\text{MAPLE}} = \underline{\underline{\text{erf}\left(\frac{\sqrt{2}}{2}\right)}}$$

- Ova vo MAPLE možeš da go rešiš so edna komanda:

```
>X:= RandomVariable(Normal(0,1)):
>Probability(x^2<1);= erf(1/2 √2)
```

- CHI-SQUARE PDF

$$f(x;1) = \frac{1}{\sqrt{2}\,\Gamma(1/2)} x^{-1/2} e^{-x/2} = \frac{1}{\sqrt{2x}\cdot\Gamma(1/2)} e^{-x/2} = \Big/ \Gamma(1/2) = \sqrt{\pi} \Big/ =$$

$$= \underline{\underline{\frac{1}{\sqrt{2\pi x}} e^{-x/2}}}$$

$$f(\gamma) = \frac{d}{d\gamma}\left[\int_0^{\gamma} f_{\gamma_2}(x)\,dx + \int_{\gamma}^{\infty} P\left(\gamma_1 \leq \frac{x\gamma}{x-\gamma}\right) f_{\gamma_2}(x)\,dx\right]$$

$$= f_{\gamma_2}(x) + \frac{d}{d\gamma}\left[\int_{\textcircled{\gamma}}^{\infty} P\left(\gamma_1 \leq \frac{x\gamma}{x-\gamma}\right) f_{\gamma_2}(x)\,dx\right] = \textcircled{1*}$$

$$P_r(\gamma \leq x) = \int_0^x p(t)\,dt = \left|\begin{matrix}\text{NORMAL}\\ \text{PDF}\end{matrix}\right| = \int_0^x \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{t^2}{2\sigma^2}}\,dt$$

$$= \frac{\sqrt{\sigma}}{2}\,\mathrm{erf}\left(\frac{x}{\sigma\sqrt{2}}\right)$$

$$P\left(\gamma_1 \leq \frac{x\gamma}{x-\gamma}\right) f_{\gamma_2}(x) = G(x)$$

$$\frac{d}{d\gamma}\int_{\gamma}^{\infty} G(x)\,dx = -\underline{G(\gamma)}$$

$$\frac{d}{d\gamma}\int_0^{\gamma} F(x)\cdot G(x)\,dx = F(g)\cdot G(g)$$

MAPLE: $f(x,\gamma) = \dfrac{x\gamma}{x-\gamma}$

$$\boxed{\frac{\partial}{\partial\gamma} f(x,\gamma) = \frac{x}{x-\gamma} + \frac{x\gamma}{(x-\gamma)^2}}$$

$$\boxed{\begin{aligned} P_{\mathcal{X}}(x) &= P_r(\mathcal{X} < x) = \int_0^x f_{\mathcal{X}}(t)\,dt \\ f_{\mathcal{X}}(x) &= \frac{dP_{\mathcal{X}}(x)}{dx}\end{aligned}}$$

$$\frac{d}{d\gamma}\int_0^{\gamma} f\left(\frac{x\gamma}{x-\gamma}\right)\cdot G(x)\,dx = \int_0^{\gamma} f'\left(\frac{xg}{x-g}\right)\left[\frac{x}{x-\gamma} + \frac{x\gamma}{(x-\gamma)^2}\right] G(x)\,dx$$

$$+ \lim_{x\to\gamma} f\left(\frac{x\gamma}{x-\gamma}\right) G(x)$$

$\boxed{\boxed{\text{MMV}}}$ $\boxed{\text{MAPLE}}$

$$\frac{d}{d\gamma}\left(\int_{\gamma}^{\infty} f\left(\frac{x\gamma}{x-\gamma}\right) G(x)\,dx\right) = \int_{\gamma}^{\infty} D f\left(\frac{x\gamma}{x-\gamma}\right)\left(\frac{x}{x-\gamma} + \frac{x\gamma}{(x-\gamma)^2}\right) G(x)\,dx$$

$$- \left(\lim_{x\to\gamma} f\left(\frac{\gamma\gamma}{x-\gamma}\right) G(x)\right)$$

$$\frac{d}{d\left(\frac{\gamma x}{x-\gamma}\right)} P\left(\gamma_1 \leq \frac{x\gamma}{x-\gamma}\right)$$

$$\textcircled{1*} = \int_{\gamma}^{\infty} f_{\gamma_1}\left(\frac{x\gamma}{x-\gamma}\right)\left(\frac{x}{x-\gamma} + \frac{x\gamma}{(x-\gamma)^2}\right) f_{\gamma_2}(x)\,dx - \lim_{x\to\gamma} P\left(\gamma_1 \leq \frac{x\gamma}{x-\gamma}\right) f_{\gamma_2}(x)$$

$$\frac{x}{x-\gamma} + \frac{x\gamma}{(x-\gamma)^2} = \frac{x^2 \cancel{-x\gamma} + \cancel{x\gamma}}{(x-\gamma)^2} = \frac{x^2}{(x-\gamma)^2}$$

$$(\ast\ast) = \int_{\gamma}^{\infty} f_{\gamma_1}\left(\frac{x\gamma}{x-\gamma}\right)\cdot\frac{x^2}{(x-\gamma)^2}\cdot f_{\gamma_2}(x)\,dx - \lim_{x\to\gamma} P\left(\gamma_1 \le \frac{x\gamma}{x-\gamma}\right) f_{\gamma_2}(x)$$

$$\frac{d}{d\gamma}\left(\int_{\gamma}^{\infty}\left(\frac{x\gamma}{x-\gamma}\right)^2 G(x)\,dx\right) \qquad f\left(\frac{x\gamma}{x-\gamma}\right)$$

$$D\left[\left(\frac{x\gamma}{x-\gamma}\right)^2\right]\left(\frac{x\gamma}{x-\gamma}\right) = \frac{dt^2}{dt}\Big|_{t=\frac{x\gamma}{x-\gamma}} = 2t = \underline{\underline{2\frac{x\gamma}{x-\gamma}}}$$

$$\ast \Bigg/ \frac{d}{dx}\left(\cos(x^2)\right) = \underline{\underline{-\sin(x^2)\cdot 2x}} \Bigg/ \ast$$

$$\frac{d}{d\gamma}\left[\left(\frac{x\gamma}{x-\gamma}\right)^2\right] = 2\,\frac{x\gamma}{x-\gamma}\left(\frac{x}{x-\gamma} + \frac{x\gamma}{(x-\gamma)^2}\right) =$$

$$= \frac{2x^2\gamma}{(x-\gamma)^2} + \frac{2x^2\gamma^2}{(x-\gamma)^3}.$$

$$h(g(x)) = \int_0^{g(x)} F(t)\,dt \qquad \underline{\underline{\frac{d}{dx}h(g(x)) = h'(g(x))\cdot g'(x)}}$$

$$\frac{d}{dx}\left[\int_0^{g(x)} F(t)\,dt\right] = \underline{\underline{F(g(x))\cdot g'(x)}}.$$

Differentiation under the integral sign

$$F(x) = \int_{a(x)}^{b(x)} f(x,t)\,dt \qquad F(x) = \int_a^b f(x)\,dx = \underline{\underline{F(b)}} - \underline{\underline{F(a)}}$$

АНАЛОГНО

$$\boxed{\frac{d}{dx}F(x) = \underbrace{\left(\frac{\partial F}{\partial b}\right)\frac{\partial b}{\partial x} - \frac{\partial F}{\partial a}\frac{\partial a}{\partial x}}_{\text{ДЕЛ ШТО СЕ ОДНЕСУВА НА ЛИМИТИТЕ}} + \underbrace{\int_{a(x)}^{b(x)}\frac{\partial}{\partial x}f(x,t)\,dt}_{\text{ОБИЧНОТО ЛАЈБНИЦ-ОВО ПРАВИЛО}}}$$

GENERALIZED LEIBNIZ RULE

[redacted]

DOKAZ NA OSNOVNATA
FORMA NA LEIBNIZ-VA FORMULA

$$h(x)=\int_a^b f(x,t)\,dt$$

$$h(x+\Delta x)-h(x)=\int_a^b f(x+\Delta x,t)\,dt-\int_a^b f(x,t)\,dt$$

$$\frac{h(x+\Delta x)-h(x)}{\Delta x}=\int_a^b\frac{f(x+\Delta x,t)-f(x,t)}{\Delta x}\,dt \Big/ \lim_{\Delta x\to 0}$$

$$\boxed{h'(x)=\int_a^b\frac{\partial f(x,t)}{\partial x}\,dt}$$

GENERALIZED
LEIBNIZ
RULE

$$f(\gamma)=\frac{d}{d\gamma}\left[\int_\gamma^\infty P\left(\gamma_1\le\frac{x\gamma}{x-\gamma}\right]f_{\gamma_2}(x)\,dx\right]+f_{\gamma_2}(\gamma)$$

$$\frac{d}{dx}\int_{a(x)}^{b(x)} f(x,t)\,dt=\underbrace{\frac{\partial F}{\partial b}}_{f(b,t)}\frac{\partial b}{\partial x}-\underbrace{\frac{\partial F}{\partial a}}_{f(a,t)}\frac{\partial a}{\partial x}+\int_{a(x)}^{b(x)}\frac{\partial}{\partial x}f(x,t)\,dt$$

$$f(\gamma)=f_{\gamma_2}(\gamma)+\underbrace{\frac{\partial}{\partial\gamma}\int^{\infty}\left[P\left(\gamma_1\le\frac{x\gamma}{x-\gamma}\right)\right]\cdot f_{\gamma_2}(x)}_{0\;-\;\lim_{x\to\gamma}\left\{P\left(\gamma_1\le\frac{x\gamma}{x-\gamma}\right)f_{\gamma_2}(x)\right\}}+\int_\gamma^\infty\frac{\partial}{\partial\gamma}\left[P\left(\gamma_1\le\frac{x\gamma}{x-\gamma}\right]f_{\gamma_2}(x)\,dx$$

$$f(\gamma)=f_{\gamma_2}(\gamma)-\lim_{x\to\gamma}P\left(\gamma_1\le\frac{x\gamma}{x-\gamma}\right)f_{\gamma_2}(\gamma)+\int_\gamma^\infty\frac{\partial P\left(\gamma_1\le\frac{x\gamma}{x-\gamma}\right)}{\partial\left(\frac{x\gamma}{x-\gamma}\right)}\cdot\frac{\partial\left(\frac{x\gamma}{x-\gamma}\right)}{\partial\gamma}\cdot f_{\gamma_2}(x)\,dx$$

$$f(\gamma)=f_{\gamma_2}(\gamma)-\lim_{x\to\gamma}P\left(\gamma_1\le\frac{x\gamma}{x-\gamma}\right)f_{\gamma_2}(\gamma)+\int_\gamma^\infty f_{\gamma_1}\left(\frac{x\gamma}{x-\gamma}\right)\left(\frac{x}{x-\gamma}+\frac{x\gamma}{(x-\gamma)^2}\right)f_{\gamma_2}\,dx$$

$$f(\gamma)=f_{\gamma_2}(\gamma)-\lim_{x\to\gamma}P\left(\gamma_1\le\frac{x\gamma}{x-\gamma}\right)f_{\gamma_2}(\gamma)+\int_\gamma^\infty f_{\gamma_1}\left(\frac{x\gamma}{x-\gamma}\right)\frac{x^2}{(x-\gamma)^2}f_{\gamma_2}(\gamma)\,dx$$

$$\lim_{x\to\gamma}P\left(\gamma_1\le\frac{x\gamma}{x-\gamma}\right)=P(\gamma_1\le\infty)=1\Rightarrow\boxed{f(\gamma)=\int_\gamma^\infty f_{\gamma_1}\left(\frac{x\gamma}{x-\gamma}\right)\frac{x^2}{(x-\gamma)^2}f_{\gamma_2}(\gamma)\,dx}$$

DOKAZANO !!!

- DOKAZ NA GENERALIZIRANATA FORMA NA LAJBNICOVA FORMULA

MMV

$$\int_a^b f(x,\alpha)\,dx = \phi(\alpha) \qquad a = a(\alpha) \quad b = b(\alpha) \qquad \Delta\alpha \to \begin{matrix}\Delta a\\ \Delta b\end{matrix}$$

$$\Delta\phi = \phi(\alpha+\Delta\alpha) - \phi(\alpha) = \int_{a+\Delta a}^{b+\Delta b} f(x,\alpha+\Delta\alpha)\,dx - \int_a^b f(x,\alpha)\,dx =$$

$$= \int_{a+\Delta a}^{a} f(x,\alpha+\Delta\alpha)\,dx + \int_a^b f(x,\alpha+\Delta\alpha)\,dx + \int_b^{b+\Delta b} f(x,\alpha+\Delta\alpha)\,dx - \int_a^b f(x,\alpha)\,dx =$$

$$= \int_{a+\Delta a}^{a} f(x,\alpha+\Delta\alpha)\,dx + \int_a^b \frac{f(x,\alpha+\Delta\alpha) - f(x,\alpha)}{1}\,dx + \int_b^{b+\Delta b} f(x,\alpha+\Delta\alpha)\,dx$$

MEAN VALUE THEOREM $\int_a^b f(x)\,dx = \underline{\underline{(b-a)\cdot f(\xi)}} \quad a<\xi<b$

$$\Delta\phi = -\Delta a\, f(\xi_1,\alpha+\Delta\alpha) + \int_a^b \left[f(x,\alpha+\Delta\alpha) - f(x,\alpha)\right]dx + \Delta b\, f(\xi_2,\alpha+\Delta\alpha)$$

$\Delta\alpha \to 0 \quad \xi_1 \to a \quad \xi_2 \to b \quad \Delta\alpha \to 0$

$$\lim_{\Delta\alpha\to 0}\frac{\Delta\phi}{\Delta\alpha} = -\frac{\partial a(\alpha)}{\partial\alpha} f(a,\alpha) + \int_a^b \frac{\partial f(x,\alpha)}{\partial\alpha}\,dx + \frac{\partial b(\alpha)}{\partial\alpha} f(b,\alpha)$$

$$\boxed{\frac{\partial\phi}{\partial\alpha} = \int_{a(\alpha)}^{b(\alpha)} \frac{\partial f(x,\alpha)}{\partial\alpha}\,dx + \frac{\partial b(\alpha)}{\partial\alpha} f(b,\alpha) - \frac{\partial a(\alpha)}{\partial\alpha} f(a,\alpha)}$$

GENERAL LIEBNIZ RULE **MMV**

11.17

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$$f(\gamma) = f_{\gamma_2}(\gamma) + \frac{\partial}{\partial\gamma}\left[\int_\gamma^\infty \underbrace{P\left(\gamma_1 \le \frac{x\gamma}{x-\gamma}\right) f_{\gamma_2}(x)}_{f(x,\gamma)}\,dx\right] = \quad \to 0 \quad \frac{d}{d\gamma}(\infty) = 0 \quad \cdot \lim_{x\to\gamma} P\left(\gamma_1 \le \frac{x\gamma}{x-\gamma}\right)$$

$$= f_{\gamma_2}(\gamma) + \int_\gamma^\infty \underbrace{\frac{\partial P\left(\gamma_1 \le \frac{x\gamma}{x-\gamma}\right)}{\partial\left(\frac{x\gamma}{x-\gamma}\right)} \cdot \frac{\partial\left(\frac{x\gamma}{x-\gamma}\right)}{\partial\gamma}}_{\text{CHAIN RULE}} \cdot f_{\gamma_2}(x) + \underbrace{\frac{\partial\infty}{\partial\gamma} P(\gamma_1 \le \gamma) f_{\gamma_2}(\infty)}_{0} - \frac{\partial\gamma}{\partial\gamma}\cdot f_{\gamma_2}(\gamma)$$

$$= f_{\gamma_2}(\gamma) + \int_\gamma^\infty f_{\gamma_1}\left(\frac{x\gamma}{x-\gamma}\right)\cdot\left(\frac{x}{x-\gamma} + \frac{x\gamma}{(x-\gamma)^2}\right)\cdot f_{\gamma_2}(x)\,dx + 0 - f_{\gamma_2}(\gamma)\cdot \overbrace{\lim_{x\to\gamma} P\left(\gamma_1 \le \frac{x\gamma}{x-\gamma}\right)}^{= P(\gamma_1 \le \infty) = 1}$$

$$= \cancel{f_{\gamma_2}(\gamma)} - \cancel{f_{\gamma_2}(\gamma)} + \int_\gamma^\infty f_{\gamma_1}\left(\frac{x\gamma}{x-\gamma}\right)\frac{x^2}{(x-\gamma)^2} f_{\gamma_2}(x)\,dx = \int_\gamma^\infty f_{\gamma_1}\left(\frac{x\gamma}{x-\gamma}\right)\frac{x^2}{(x-\gamma)^2} f_{\gamma_2}(x)\,dx$$

MMV DOKAZANO !!!

- ZNAČI AKO $\boxed{\gamma = \frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2}}$ TOGAŠ PDF-OT NA OVAA SL. PROMENLIVA E:

$$\boxed{f_\gamma(\gamma) = \int_\gamma^\infty f_{\gamma_1}\left(\frac{\gamma x}{x-\gamma}\right)\frac{x^2}{(x-\gamma)^2} f_{\gamma_2}(x)\,dx}$$

MNOGU VAŽEN IZRAZ!!!

#@

- SEGA SE NAVRAĆAM NAZAD NA APPENDIX „A" OD L. Yang PERFORMANCE ANALYSIS ČLANAKOT.

$$\underbrace{f_{\gamma_1}(\gamma) = \frac{d}{d\gamma} P_Y\left(\frac{YZ}{Y+Z} \le \frac{\gamma}{\bar\gamma}\right) = \frac{d}{d\gamma}\int_0^\infty P_Y\left(\frac{Yz}{Y+z} \le \frac{\gamma}{\bar\gamma}\right) f_Z(z)\,dz}$$

OVOJ IZRAZ E IST SO IZRAZOT NA N7.66 SAMO NAMESTO „γ" OVDE IMAME „$\frac{\gamma}{\bar\gamma}$" → #@

OVA „γ_1" E VTORIOT ČLEN OD IZRAZOT (9) OD ČLANAKOT NA Yang

$\boxed{\xi = \frac{\gamma_1}{\bar\gamma}}$

$\boxed{\gamma_1 = \frac{\bar\gamma YZ}{Y+Z}}$

$$f_{\gamma_1}(\gamma) = \frac{d}{d\gamma}\int_0^\infty P_Y\left(\frac{Yz}{Yz} \le \xi\right) f_Z(z)\,dz = \int_\xi^\infty f_Y\left(\frac{\xi x}{x-\xi}\right)\frac{x^2}{(x-\xi)^2} f_Z(x)\,dx$$

- Y, Z ARE CHI-SQUARE PDFs WITH N^2 degrees OF FREED. (VIDI N9.66)
I.E THEY ARE GAMMA PDFs

$$\boxed{f_Z(z) = \frac{z^{N^2-1}}{\Gamma(N^2)} e^{-z}}$$

$$f_{\gamma_1}(\gamma) = \int_\xi^\infty f_Y\left(\frac{\xi x}{x-\xi}\right)\frac{x^2}{(x-\xi)^2}\,\frac{x^{N^2-1}}{\Gamma(N^2)} e^{-x}\,dx$$

$f_Y\left(\frac{\xi x}{x-\xi}\right) = ?$ $\quad w = \frac{\xi x}{x-\xi}$ $\quad f_W(w) \Rightarrow$ NE MI TREBA VO VAKVA FORMA!!

$$f_Y\left(\frac{\xi x}{x-\xi}\right) = \frac{\left(\frac{\xi x}{x-\xi}\right)^{N^2-1}}{\Gamma(N^2)} e^{-\frac{\xi x}{x-\xi}}$$

$$f_{\gamma_1}(\gamma) = \frac{1}{\Gamma^2(N^2)}\int_\xi^\infty \frac{\xi^{N^2-1}x^{N^2-1}}{(x-\xi)^{M-1}}\cdot\frac{x^2 e^{-\frac{\xi x}{x-\xi}}}{(x-\xi)^2}\,\frac{x^{N^2-1}}{1} e^{-x}\,dx$$

$$f_{\gamma_1}(\gamma) = \frac{\xi^{N^2-1}}{\Gamma^2(N^2)}\int_\xi^\infty \frac{x^{2N^2\cancel{-2}\cancel{+2}}}{(x-\xi)^{N^2+1}} e^{-\frac{\xi x}{x-\xi}-x}\,dx$$

$\frac{\xi x}{x-\xi}+x = \frac{\cancel{\xi x}+x^2-\cancel{\xi x}}{x-\xi} = \frac{x^2}{x-\xi}$

$$\boxed{f_{\gamma_1}(\gamma) = \underbrace{\frac{\gamma^{N^2-1}}{\bar{\gamma}^{N^2-1}\,\Gamma^2(N^2)}}_{K} \int_{\xi}^{\infty} \frac{x^{2N^2}}{(x-\xi)^{N^2+1}}\, e^{-\frac{x^2}{x-\xi}}\, dx}$$

ODI OD TUKA
DIREKTNO NA PP76 SLEDNA!

$$m = \frac{x^2}{x-\xi} \qquad \frac{dm}{dx} = \frac{2x}{x-\xi} - \frac{x^2}{(x-\xi)^2} = \frac{2x(x-\xi) - x^2}{(x-\xi)^2}$$

$$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - f(x)g'(x)}{g^2(x)} \qquad \frac{dm}{dx} = \frac{2x^2 - 2x\xi - x^2}{(x-\xi)^2} = \frac{x^2-2x\xi}{(x-\xi)^2}\ \boxed{\Delta D}$$

$$x^2 = mx - \xi m \qquad x^2 - mx + \xi m = 0$$

$$x_{1,2} = \frac{m \pm \sqrt{m^2 - 4\xi m}}{2} = \frac{m}{2} \pm \frac{1}{2}\sqrt{m^2 - 4\xi m}$$

$$\frac{dx}{dm} = \frac{(x-\xi)^2}{x(x-2\xi)} \qquad f_{\gamma_1} = \underbrace{\frac{\gamma^{N^2-1}}{\bar{\gamma}^{N^2-1}\,\Gamma^2(N^2)}}_{K} \int_{\xi}^{\infty} \cancel{x^2} \left(\frac{x^2}{x-\xi}\right)^{N^2} \cdot \frac{1}{\cancel{x-\xi}}\, e^{-m}\, \frac{(x-\xi)^{\cancel{2}}}{x(x-2\xi)}\, dm$$

$$f_{\gamma_1} = \frac{\gamma^{N^2-1}}{\bar{\gamma}^{N^2-1}\,\Gamma^2(N^2)} \int_{\xi}^{\infty} m^{N^2} \cdot \frac{e^{-m}}{\boxed{\frac{x^2}{x-\xi}} - \frac{2\xi x}{x-\xi}}\, dm = \frac{\gamma^{N^2-1}}{\bar{\gamma}^{N^2-1}\,\Gamma^2(N^2)} \cdot \int_{\xi}^{\infty} \frac{m^{N^2-1} e\, dm}{m - \frac{2\xi x}{x-\xi}}$$

?

$$\boxed{\Delta D} \Rightarrow \frac{dm}{dx} = \frac{x^2 - 2x\xi + \xi^2}{(x-\xi)^2} - \frac{\xi^2}{(x-\xi)^2} = 1 - \frac{\xi^2}{(x-\xi)^2}$$

$$f_{\gamma_1} = K \int_{\xi}^{\infty} \left(\frac{x^2}{\cancel{x}-\xi}\right)^{N^2} \frac{e^{-m}}{(x-\xi)}\, \frac{dm}{1 - \frac{\xi^2}{(x-\xi)^2}} = K \int_{\xi}^{\infty} m^{N^2} e^{-1}\, \frac{dm}{x - \xi - \frac{\xi^2}{x-\xi}}$$

$$x - \xi - \frac{\xi^2}{x-\xi} = \frac{x^2 - 2x\xi + \cancel{\xi^2} - \cancel{\xi^2}}{x-\xi} = \frac{x(x-2\xi)}{x-\xi}$$

$$v = \frac{x}{x-\xi} \qquad \frac{dv}{dx} = \frac{\cancel{x}-\xi-\cancel{x}}{(x-\xi)^2} = -\frac{\xi}{(x-\xi)^2}$$

$$v = \frac{x}{\sqrt{x-\xi}} \qquad \frac{dv}{dx} = \frac{\sqrt{x-\xi} - x\frac{1}{2}(x-\xi)^{\frac{1}{2}-1}}{(x-\xi)} = \frac{\sqrt{x-\xi} - \frac{x}{2\sqrt{x-\xi}}}{(x-\xi)}$$

$$\frac{dv}{dx} = \frac{2(x-\xi)-x}{2(x-\xi)\sqrt{x-\xi}} = \frac{2x - 2\xi - x}{2(x-\xi)\sqrt{x-\xi}} = \frac{x-2\xi}{2(x-\xi)\sqrt{x-\xi}};$$

$$\frac{dx}{dv} = \frac{2(x-\xi)\sqrt{x-\xi}}{x-2\xi} \qquad f_{\gamma_1}(\gamma) = K\int_{\xi}^{\infty} \frac{x^{2N^2}}{(x-\xi)^{N^2+1}} e^{-\frac{x^2}{x-\xi}} dx$$

$$v = \frac{x}{\sqrt{x-\xi}}$$

$$f_{\gamma_1}(\gamma) = K\int_{\xi}^{\infty} v^{2N^2} \cdot \frac{e^{-v^2}}{\cancel{(x-\xi)}} \cdot \frac{2\cancel{(x-\xi)}\sqrt{x-\xi}}{x-2\xi} dv$$

~~$\frac{2(x-\xi)\sqrt{x-\xi}}{x-2\xi} = \frac{2}{\frac{x-2\xi}{(x-\xi)\sqrt{x-\xi}}} = \frac{2}{\frac{1}{x-\xi} - \frac{2\xi}{(x-\xi)\sqrt{x-\xi}}}$~~

$$\frac{2\sqrt{x-\xi}}{x-2\xi} = \frac{2}{v - \frac{2\xi}{\sqrt{x-\xi}}} = \frac{2}{v - \frac{2\xi v}{x}} = \frac{2}{v\left(\frac{x-2\xi}{x}\right)} = \frac{2x}{v(x-2\xi)}$$

$$v^2 = \frac{x^2}{x-\xi} \qquad v^2x - v^2\xi = x^2 \qquad x^2 - v^2x + v^2\xi = 0$$

$$v^2 = \frac{x^2}{x-\xi}$$

~~SE VRAĆAM NA ČLANAKOT SO NAKAGAMI TRF 80 Lo Taug~~

VISTINSKA ZAMENA !!! HMV

$$\boxed{x-\xi = y} \qquad dx = dy \qquad \boxed{x = y+\xi} \qquad x^2 = y^2 + 2y\xi + \xi^2$$

$$e^{-\frac{x^2}{x-\xi}} = e^{-\frac{y^2+2y\xi+\xi^2}{y}} = e^{-\left(y + 2\xi + \frac{\xi^2}{y}\right)}$$

$$\boxed{x = \xi \quad y = 0 \qquad x = \infty \quad y = \infty}$$

$$f_{\gamma_1}(\xi) = K\int_0^{\infty} \frac{(y+\xi)^{2N^2}}{y^{N^2+1}} e^{-\left(y + \frac{\xi^2}{y}\right)} \cdot e^{-2\xi} dy$$

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k}$$

$$(a+b)^2 = \sum_{k=0}^{2} \binom{2}{k} a^k b^{2-k} = \binom{2}{0} b^2 + \binom{2}{1} a \cdot b + \binom{2}{2} a^2$$

$$= b^2 + 2ab + a^2$$

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$

$$(a+b)^3 = \binom{3}{0} a^3 + \binom{3}{1} a^2 b^1 + \binom{3}{2} a b^2 + \binom{3}{3} b^3 =$$

$$= a^3 + 3a^2b + 3ab^2 + b^3$$

$$f_{\gamma_1}(\gamma) = Ke^{-2\xi}\int_0^{\infty}\left(\sum_{k=0}^{2N^2} \binom{2N^2}{k} y^{2N^2-k} \cdot \xi^k\right) \frac{e^{-\left(y + \frac{\xi^2}{y}\right)}}{y^{N^2+1}} dy$$

$$f_{\gamma_1}(\xi) = K e^{-2\xi} \int_0^\infty \sum_{k=0}^{2N^2} \binom{2N^2}{k} y^{N^2-k-1} \xi^k \cdot e^{-\left(y + \frac{\xi^2}{y}\right)} dy$$

- GRADSHTEYN 3.471.9

$$\boxed{\int_0^\infty x^{\nu-1} e^{-\frac{\beta}{x} - \gamma x} dx = 2 \left(\frac{\beta}{\gamma}\right)^{\frac{\nu}{2}} K_\nu \left(2\sqrt{\beta\gamma}\right)}$$

$\beta > 0$
$\gamma > 0$

$$f_{\gamma_1}(\xi) = K \cdot e^{-2\xi} \sum_{k=0}^{2N^2} \binom{2N^2}{k} \xi^k \cdot \int_0^\infty y^{\overset{\nu}{N^2-k-1}} e^{-y - \frac{\overset{\beta}{\xi^2}}{y}} dy$$

$\xi^2 = \beta$

$$= K e^{-2\xi} \sum_{k=0}^{2N^2} \binom{2N^2}{k} \xi^k \cdot 2 \left(\frac{\xi^2}{1}\right)^{\frac{N^2-k}{2}} \cdot K_{N^2-k}\left(2\sqrt{\xi^2}\right)$$

$$f_{\gamma_1}(\xi) = \frac{2 \cdot \gamma^{N^2-1} e^{-\frac{2\gamma}{\bar\gamma}} \left(\frac{\gamma}{\bar\gamma}\right)^{N^2}}{\bar\gamma^{N^2-1} \Gamma^2(N^2)} \sum_{k=0}^{2N^2} \binom{2N^2}{k} K_{N^2-k}\left(\frac{2\gamma}{\bar\gamma}\right)$$

$$f_{\gamma_1}(\xi) = \frac{2 \gamma^{2N^2-1} e^{-\frac{2\gamma}{\bar\gamma}}}{\bar\gamma^{2N^2-1} \Gamma^2(N^2)} \sum_{k=0}^{2N^2} \binom{2N^2}{k} K_{N^2-k}\left(\frac{2\gamma}{\bar\gamma}\right)$$

NE MOŽE VAKA!! ZATOA ŠTO SO FUNKCIONALNA TRANSFORMACIJA NA PDF VIDI: @$

- VO ČLANAKOT E ISTO SAMO NAMESTO $\bar\gamma^{2N^2-1}$ IMA $\bar\gamma^{2N^2}$.
MOŽE ~~TAKA~~ TREBA ~~ZOŠTO~~ IZVEDUVANJATA ~~SE~~ ~~ZA~~ $\xi = \frac{\gamma}{\bar\gamma}$.

$$f_{\gamma_1}(\xi) = \frac{2 \xi^{2N^2-1} e^{-\frac{2\gamma}{\bar\gamma}}}{\Gamma^2(N^2)} \sum_{k=0}^{2N^2} \binom{2N^2}{k} K_{N^2-k}\left(\frac{2\gamma}{\bar\gamma}\right)$$

@$

$\xi = \frac{\gamma}{\bar\gamma}$ $\quad \gamma = \xi \cdot \bar\gamma$ $\quad \frac{\partial\gamma}{\partial\xi} = \bar\gamma$

$$f_{\gamma_1}(\gamma) \overset{!}{=} \left. \frac{f(\xi)}{\frac{\partial\gamma}{\partial\xi}} \right|_{\xi = \frac{\gamma}{\bar\gamma}} = \frac{1}{\bar\gamma} \cdot \frac{2\gamma^{2N^2-1} e^{\frac{-2\gamma}{\bar\gamma}}}{\bar\gamma^{2N^2-1} \cdot \Gamma^2(N^2)} \sum_{k=0}^{2N^2} \binom{2N^2}{k} K_{N^2-k}\left(\frac{2\gamma}{\bar\gamma}\right)$$

$$\boxed{f_{\gamma_1}(\gamma) = \frac{2 \gamma^{2N^2-1} e^{-\frac{2\gamma}{\bar\gamma}}}{\bar\gamma^{2N^2} \Gamma^2(N^2)} \sum_{k=0}^{2N^2} \binom{2N^2}{k} K_{N^2-k}\left(\frac{2\gamma}{\bar\gamma}\right)}$$

DOKAZAN IZRAZOT (10) PERFORMANCE ANALYSIS ČLANAKOT OD YANG

rate1 = K_1/T_1 $\quad N/\text{rate1}/T_1 = (N/\text{rate1})/T_1 = \frac{N}{\text{rate1} \cdot T_1} = \frac{N}{K_1}$

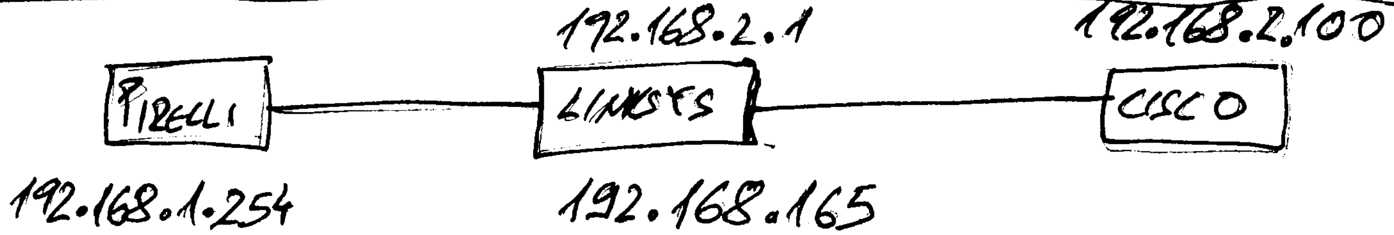
$8/4/2 = (8/4)/2 = \frac{8}{8} = 1$ $\quad c = \frac{L}{u_t^S \cdot K \cdot \log_2 M}$

TIMESLOTS

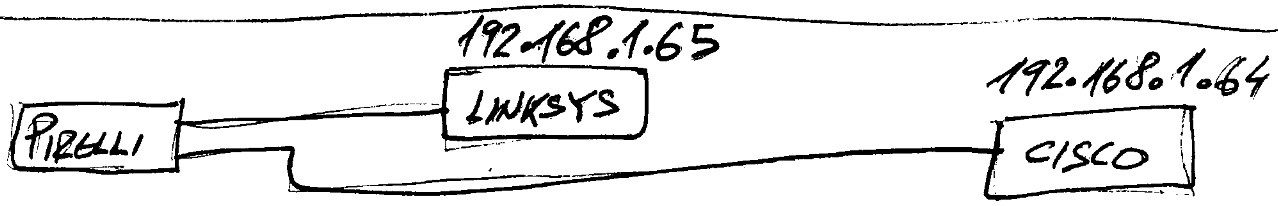
SYMBOLS

$\text{rate} = \frac{K}{T} = \frac{K}{L}$ $\quad \boxed{c = \frac{1}{u_t^S \cdot \text{rate} \cdot \log_2 M}}$

OLD   KOZLE NETWORK CONFIGURATION



NEW



- TANG APPROXIMATE FORMULA

$\boxed{\gamma_1^{up} = \bar{\gamma} \min(Y, Z)}$ $\gamma_1 = \frac{\bar{\gamma} Y Z}{Y + Z}$

$f_Z(z) = \frac{z^{N^2-1}}{\Gamma(N^2)} e^{-z}$ $f_Y(y) = \frac{y^{N^2-1}}{\Gamma(N^2)} e^{-z}$

→ UPPER BOUND OF γ_1 =

$\boxed{f_{\gamma_1}^{up}(\gamma) = 2 \frac{\Gamma(N^2, \gamma/\bar{\gamma})}{\left[\Gamma(N^2)\right]^2 \bar{\gamma}^{N^2}} \gamma^{N^2-1} e^{-\frac{\gamma}{\bar{\gamma}}}}$ (**)

$\Gamma(N^2, \gamma/\bar{\gamma}) \Rightarrow$ INCOMPLETE GAMMA FUN. $\gamma(\alpha, x) = \int_0^x e^{-t} t^{\alpha-1} dt$

$\boxed{\Gamma(\alpha, x) = \int_x^{\infty} e^{-t} t^{\alpha-1} dt}$ $\Gamma(\alpha) = \int_0^{\infty} e^{-t} t^{\alpha-1} dt$

- PROOF OF (**):

CDF OF $W = \min(Y, Z)$ IS GIVEN BY

$\boxed{F_W(w) = 1 - \left[e^{-w} \sum_{i=0}^{N^2-1} \frac{w^i}{i!}\right]^2 = 1 - \frac{\Gamma^2(N^2, w)}{\Gamma^2(N^2)}}$ (**)

$$f_W(w) = \frac{d F_W(w)}{dw} = 2 \frac{\Gamma(N^2, w)}{\Gamma^2(N^2)} w^{N^2-1} e^{-w}$$

$$\frac{dF_W(w)}{dw} = -\frac{d}{dw}\left[e^{-w} \sum_{i=0}^{N^2-1} \frac{w^i}{i!}\right] = e^{-w} \sum_{i=0}^{N^2-1} \frac{w^i}{i!} - e^{-w} \sum_{i=0}^{N^2-1} \frac{i w^{i-1}}{i!}$$

$$= e^{-w}\left[\sum_{i=0}^{N^2-1} \frac{w^i}{i!} - \sum_{i=0}^{N^2-1} \frac{w^{i-1}}{(i-1)!}\right]$$

$j = i - 1$ $\quad i = 0 \quad j = -1$

$\quad i = N^2-1 \quad j = N^2 - 2$

$$\frac{dF_W(\omega)}{d\omega} = e^{-\omega}\left[\sum_{i=0}^{N^2-1}\frac{\omega^i}{i!} - \sum_{j=-1}^{N^2-2}\frac{\omega^j}{j!}\right] = e^{-\omega}\left[\frac{\omega^{N^2-1}}{(N^2-1)!} - \frac{\omega^{-1}}{-1}\right]$$

$$\frac{dF_W(\omega)}{d\omega} = e^{-\omega}\left[\frac{\omega^{N^2-1}}{(N^2-1)!} + \frac{1}{\omega}\right]$$ → OVA NE E DOBRO ZOŠTO TREBA $[\cdots]^{②}$

- MATLE: $\frac{d}{d\omega}F(\omega) = \frac{\omega^{N^2-1}e^{-\omega}}{\Gamma(N^2)}$

- RELATION BETWEEN BINOMIAL COEFFICIENT AND GAMMA FCN:

$$\binom{z}{\omega} = \frac{z!}{\omega!\,(z-\omega)!} = \frac{\Gamma(z+1)}{\Gamma(\omega+1)\,\Gamma(z-\omega+1)}$$

$$\boxed{\Gamma(n+1,x) = \underset{\Gamma(n+1)}{n!}\,e^{-x}\sum_{m=0}^{n}\frac{x^m}{m!}}$$ GRADSHTEYN 8.352.1

(**) ⇒ $$F_W(\omega) = 1 - \left[\frac{\Gamma(n+1)}{\Gamma(n+1)}e^{-\omega}\sum_{i=0}^{\overbrace{N^2-1}^{=n}}\frac{\omega^i}{i!}\right]^2 =$$

$$= 1 - \left[\underbrace{\frac{\Gamma(N^2)}{\Gamma(N^2)}e^{-\omega}\sum_{i=0}^{N^2-1}\frac{\omega^i}{i!}}_{\Gamma(N^2,\omega)}\right]^2 = 1 - \frac{\Gamma^2(N^2,\omega)}{\Gamma^2(N^2)}$$

GRADSHTEYN 8.356.4

$$\boxed{\frac{d\Gamma(\alpha,x)}{dx} = -x^{\alpha-1}e^{-x}}$$

$$\frac{dF_W(\omega)}{d\omega} = +2\cdot\frac{\Gamma(N^2,\omega)}{\Gamma^2(N^2)}\,\omega^{N^2-1}\cdot e^{-\omega}$$

BY USING CHAIN RULE (ISTOTO SE DOBIVA I VO MATLE!!!)

DOKAZANO!!!

$\gamma_1 = \bar{\gamma}\cdot\omega$ $\quad f_W(\omega) = \frac{2\Gamma(N^2,\omega)}{\Gamma^2(N^2)}\omega^{N^2-1}e^{-\omega}$

$$f_{\gamma_1}(\gamma_1) = \frac{f_W(\omega)}{\underset{\bar{\gamma}}{\frac{\partial\gamma_1}{\partial\omega}}}\Bigg|_{\omega = \frac{\gamma_1}{\bar{\gamma}}} = \frac{2\Gamma(N^2,\gamma_1/\bar{\gamma})}{\bar{\gamma}\,\Gamma^2(N^2)}\left(\frac{\gamma_1}{\bar{\gamma}}\right)^{N^2-1}e^{-\gamma_1/\bar{\gamma}}$$

$$f_{\gamma_1}(\gamma_1) = \frac{2\Gamma(N^2,\gamma_1/\bar{\gamma})}{\Gamma^2(N^2)\,\bar{\gamma}^{N^2}}\gamma_1^{N^2-1}e^{-\gamma_1/\bar{\gamma}}$$ } DOKAZANO (**)

- FOR HIGH SNR THE APPROXIMATE PDF IS

$$f^{up}_{\gamma_1}(\gamma) = \frac{2\Gamma(N^2, \gamma/\bar{\gamma})}{\Gamma^2(N^2)\,\bar{\gamma}^{N^2}}\,\gamma^{N^2-1} e^{-\frac{\gamma}{\bar{\gamma}}} \underset{\bar{\gamma}\to\infty}{\doteq} \frac{2\ \gamma^{N^2-1}}{\Gamma(N^2)\,\bar{\gamma}^{N^2}} e^{-\frac{\gamma}{\bar{\gamma}}}$$

$$f^{up}_{\gamma_1}(\gamma) = 2\cdot \boxed{\frac{1}{\bar{\gamma}\,\Gamma(N^2)}\left(\frac{\gamma}{\bar{\gamma}}\right)^{N^2-1} e^{-\gamma/\bar{\gamma}}}$$

GAMMA DISTRIBUTION WITH N^2 SHAPE AND $\bar{\gamma}$ SCALE PARAMETER

- MGF FOR GAMMA DISTRO

$$M(s) = 2\cdot\left(\frac{1}{1-N^2\cdot s}\right)^{N^2} \qquad M(-s) = \frac{2}{(1+N^2 s)^{N^2}}$$

$$M_{\gamma_1}(s) = \int_0^\infty f_{\gamma_1}(\gamma)\cdot e^{-s\gamma}\,d\gamma = \int_0^\infty \frac{2\gamma^{N^2-1}}{\Gamma(N^2)\,\bar{\gamma}^{N^2}}\, e^{-\left(\frac{1}{\bar{\gamma}}+s\right)\gamma} d\gamma$$

$$M_{\gamma_1}(-s) = \frac{2}{\Gamma(N^2)\,\bar{\gamma}^{N^2}} \int_0^\infty \gamma^{\overset{u}{\overbrace{N^2-1}}} e^{-\left(\frac{1}{\bar{\gamma}}+s\right)\gamma} d\gamma$$

GRADSHTEYN 3.351.3

$$\boxed{\int_0^\infty x^u e^{-\mu x}\,dx = u!\,\mu^{-u-1}}$$

$$\boxed{\boxed{\text{MK SINESTAR}}\ \text{200BK}}\ \boxed{\text{AVR-141}}$$

$$M_{\gamma_1}(-s) = \frac{2}{\cancel{\Gamma(N^2)}\,\bar{\gamma}^{N^2}}\,\cancel{(N^2-1)!}\cdot\left(\frac{1}{\bar{\gamma}}+s\right)^{-N^2+\cancel{1}-\cancel{1}} = \frac{2}{\bar{\gamma}^{N^2}}\left(\frac{1+\bar{\gamma}s}{\bar{\gamma}}\right)^{-N^2}$$

$$\boxed{M_{\gamma_1}(-s) = \frac{2}{\cancel{\bar{\gamma}^{N^2}}}\cdot\frac{\cancel{\bar{\gamma}^{N^2}}}{(1+\bar{\gamma}s)^{N^2}} = \frac{2}{(1+\bar{\gamma}s)^{N^2}}}$$

$$s = \frac{g}{\sin^2\theta} \qquad g = \sin^2\pi/M$$

$$SER = \frac{1}{\pi}\int_0^{(M-1)\pi/M} M(-s)\,d\theta = \frac{1}{\pi}\int_0^{(M-1)\pi/M} \frac{2\cdot\sin^{2N^2}\theta}{(\sin^2(\theta)+\bar{\gamma}g)^{N^2}}\,d\theta$$

$$f_{\gamma_0}(\gamma) = \frac{\gamma^{N^2-1}}{\Gamma(N^2)\,\bar{\gamma}^{N^2}}\, e^{-\gamma/\bar{\gamma}} \qquad M_0(-s) = \int_0^\infty f_{\gamma_0}(\gamma)\, e^{-\gamma s}\, d\gamma =$$

$$= \int_0^\infty \frac{\gamma^{(N^2-1)}}{\Gamma(N^2)\,\bar{\gamma}^{N^2}}\, e^{-\left(1/\bar{\gamma}+s\right)\gamma} d\gamma = \frac{1}{\cancel{\Gamma(N^2)}\,\bar{\gamma}^{N^2}}\,\cancel{(N^2-1)!}\cdot\left(\frac{1+s\bar{\gamma}}{\bar{\gamma}}\right)^{-N^2} = \frac{1}{(1+s\bar{\gamma})^{N^2}}$$

$$\boxed{M_{\gamma_0}(s) = \left| \begin{array}{l} s = \frac{g}{\sin^2\theta} \\ g = \sin^2(\pi/M) \end{array} \right| = \frac{\sin^{2N^2}(\theta)}{\left(\sin^2(\theta) + \bar{\gamma}\cdot\sin^2(\pi/M)\right)^{N^2}}}$$

GOLIATH INTERNATIONAL - GOCE STEFANOSKI

ROSANA PERŠIĆ

- ASYMPTOTIC SER (ACORDING YANG'S PAPER)

$$M(-s) = M_{\gamma_0}\cdot M(\gamma_1) = \frac{1}{(1+s\bar{\gamma})^{N^2}}\cdot\frac{2}{(1+\bar{\gamma}s)^{N^2}} = \frac{2}{(1+s\bar{\gamma})^{2N^2}}$$

$$P_S^{CAF}(\epsilon) = \frac{1}{\pi}\int_0^{\pi(M-1)/M} M_{\gamma_1}\, d\theta = \frac{1}{\pi}\int_0^{\pi(M-1)/M} \frac{2\sin^{4N^2}\theta \; d\theta}{\left(\sin^2\theta + \sin^2\pi/M\;\bar{\gamma}\right)^{2N^2}} \qquad s = \frac{g}{\sin^2\theta} \qquad ①$$

OD CLANAKOT:

$$P_S^{CAF} = \frac{2a}{\bar{\gamma}^{2N^2}}\int_0^{b\pi}\left(\frac{g\cdot\bar{\gamma} + \sin^2\theta}{\bar{\gamma}\sin^2\theta}\right)^{-2N^2} d\theta = \cancel{\frac{2a}{\bar{\gamma}^{2N^2}}}\int_0^{b\pi}\frac{\cancel{\bar{\gamma}^{2N^2}}\sin^{4N^2}\theta}{(\sin^2\theta + g\bar{\gamma})^{2N^2}}$$

$$P_S^{CAF} = \frac{2}{\pi}\int_0^{\pi(M-1)/M}\frac{\sin^{4N^2}\theta \quad d\theta}{\left(\sin^2\theta + \sin^2\pi/M\;\bar{\gamma}\right)^{2N^2}} = ① \quad \text{POKAZANO!!!}$$

MMV

- DOPOLNITELNA APROXIMACIJA

ZOKI VIKA DEKA ZA PADTEČKA FUNKCIJA VAŽNA SE PRAVI. GORNATA GRANICA SE STAVA UMESTO PROMENLIVA.

$$P_S^{CAF} = \frac{2a}{\bar{\gamma}^{2N^2}}\int_0^{b\pi}\left(\underbrace{\frac{g}{\sin^2\theta} + \frac{1}{\bar{\gamma}}}_{\boxed{\leq 1}}\right)^{-2N^2} d\theta \leq \frac{2ab\pi}{\bar{\gamma}^{2N^2}}\left(\frac{g}{\sin^2 b\pi} + \frac{1}{\bar{\gamma}}\right)^{-2N^2}$$

$$\doteq \frac{2ab\pi}{\bar{\gamma}^{2N^2}}\left(\frac{g}{\sin^2 b\pi}\right)^{-2N^2} = \frac{2(M-1)/M}{\bar{\gamma}^{2N^2}}\left(\frac{\sin^2\pi/M}{\sin^2(M-1)\pi/M}\right)^{-2N^2}$$

$$\boxed{P_S^{CAF} \doteq \frac{2(M-1)}{M\bar{\gamma}^{2N^2}}\left(\frac{\sin^2(M-1)\pi/M}{\sin^2\pi/M}\right)^{2N^2}} = \frac{2(M-1)}{M}\left(\frac{\sin^2(M-1)\pi/M}{\bar{\gamma}\sin^2\pi/M}\right)^{2N^2}$$

$$f_{ave} = \frac{1}{b-a}\int_a^b f(x)dx \; ; \quad (b-a)\cdot f_{ave} = \int_a^b f(x)dx$$

TEOREMA

POVRSINATA POD OVO PRAVOAGOLNIK = POVRSINATA POD f(x) OD „a" DO „b"

- ТЕОРЕМА ЗА СРЕДНА ВРЕДНОСТ: АКО „f" е КОНТИНУ-АЛНА ВО ИНТЕРВАЛОТ [a, b] ТОГАШ ПОСТОИ „c" ВО [a, b] ТАКА ШТО

MMV

$$\boxed{\int_a^b f(x)dx = f(c)(b-a)}$$

→ ЗА МОНОТОНА РАСТЕЧКА ФУНКЦИЈА МАКСИМУМОТ ДАВА ВО ТОЧКА b е $\int_a^b f(x)dx \leq f(b)(b-a)$

$\boxed{\begin{array}{c}2.90\\2.79\end{array}}$

$$\boxed{P_s^{CAF}(\epsilon) = \frac{2(M-1)}{M\bar{\gamma}^{2N^2}} \frac{\sin^2\left(\frac{M-1}{M}\pi\right)}{\sin^2\frac{\pi}{M}}}$$

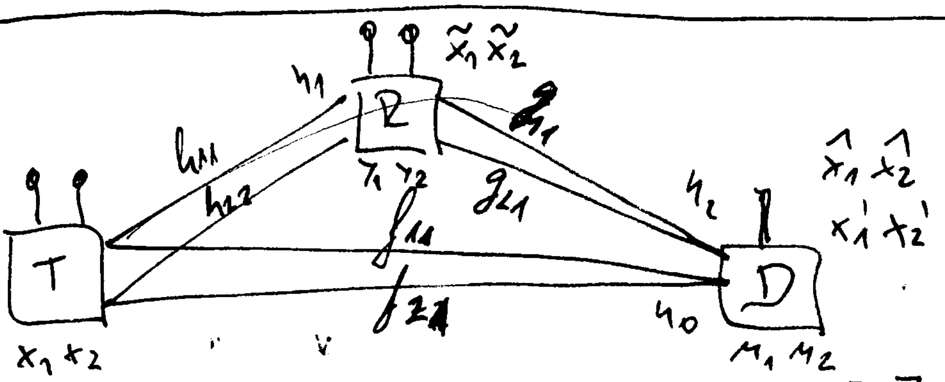
$$\boxed{\bar{\gamma} = \frac{E_s N_0}{nS}}$$

$$M=2: \quad P_s^{CAF}(\epsilon) = \frac{1}{\bar{\gamma}^{2N^2}} \cdot \frac{\cancel{\sin^2\left(\frac{\pi}{2}\right)}^{1}}{\cancel{\sin^2\frac{\pi}{2}}} = \frac{1}{\bar{\gamma}^{2N^2}}$$

$$M=4: \quad P_s^{CAF}(\epsilon) = \frac{6}{4\bar{\gamma}^{2N^2}} \cdot \cancel{\frac{\sin^2\frac{3\pi}{4}}{\sin^2\frac{\pi}{4}}}^{1} = \frac{3}{2\bar{\gamma}^{2N^2}}$$

- SYSTEM MODEL WITH „K" RELAYS
- TOTAL SNR AFTER MRC

$$\gamma_{ACAF} = \frac{1}{K+1}\left(\bar{\gamma}\chi + \sum_{k=1}^{K} \gamma_k\right) \approx \frac{\bar{\gamma}}{K+1}\left(\chi + \sum_{k=1}^{K} \frac{Y_k Z_k}{Y_k + Z_k}\right)$$



$$y_1[1] = \sqrt{E} h_{11} x_1 + \sqrt{E} h_{21} x_2 + n_1[1]$$
$$y_1[2] = -\sqrt{E} h_{11} x_2^* + \sqrt{E} h_{21} x_1^* + n_1[2]$$

$$y_2[1] = \sqrt{E} h_{12} x_1 + \sqrt{E} h_{22} x_2 + n_2[1]$$
$$y_2[2] = -\sqrt{E} h_{12} x_2^* + \sqrt{E} h_{22} x_1^* + n_2[2]$$

$$\tilde{x}_1 = h_{11}^* y_1[1] + h_{21} y_1^*[2] + h_{12}^* y_2[1] + h_{22} y_2^*[2];$$

$$\tilde{x}_2 = h_{21}^* y_1[1] - h_{11} y_1^*[2] + h_{22}^* y_2[1] - h_{12} y_2^*[2];$$

$$\boxed{\Delta_2 = \Delta_1 = |h_{11}|^2 + |h_{12}|^2 + |h_{21}|^2 + |h_{12}|^2}$$

$$\tilde{x}_{1R} = \sqrt{E}\Delta_2 x_1 + \eta_1 \qquad \tilde{x}_{2R} = \sqrt{E}\Delta_2 x_2 + \eta_2$$

$$m[1] = \sqrt{E} f_{11} x_1 + \sqrt{E} f_{21} x_2 + n_0[1]$$
$$m[2] = -\sqrt{E} f_{11} x_2^* + \sqrt{E} f_{21} x_2 + n_0[2]$$

$\hat{x}_1 = G_2 \Lambda_2 \tilde{x}_1 + \xi_1$ $\qquad$ $\xi_1 = g_{11}^* w_1[3] + g_{21} w_1^*[4]$

$\hat{x}_2 = G_2 \Lambda_2 \tilde{x}_2 + \xi_2$ $\qquad$ $\xi_2 = g_{21}^* w_1[3] - g_{11} w_1^*[4]$

$\Lambda_2 = |g_{11}|^2 + |g_{21}|^2$

$$\begin{cases} x_1' = \sqrt{E}\Delta_0 x_1 + v_1 \\ x_2' = \sqrt{E}\Delta_0 x_2 + v_2 \end{cases} \qquad \Delta_0 = |h_{11}|^2 + |h_{21}|^2$$

$\hat{x}_1 = G_2 \Lambda_2 \Delta_2 \sqrt{E} x_1 + G_2 \Lambda_2 n_1 + \xi_1$

$Rx = x_1' + \hat{x}_1 = \sqrt{E}(\Delta_0 + G_2\Lambda_2\Delta_2) x_1 + v_1 + G_2\Lambda_2 n_1 + \xi_1$

- ZANG'S PAPER – RELAY SELECTION IN MULTIPLE CHANNELS

2MAX – 2MAX Comfort + 2672883

BRANKO STANCEV 300006

U M

Во [illegible] е.

слот 1 : $u[1] = u'$

слот 2 : $u[2] = u''$

слот 3 : $v[3] = v'$

слот 4 : $v[4] = v''$

1	2	3	4

$\ell_0 = |h_{11}|^2$

$\overline{v_1^2} = \Delta_0 N_0$

$\sqrt{E}\Delta_0 X_1 + v_1$ $\qquad$ $\sqrt{E}\Delta_0' X_1 + v_1'$

$\sqrt{E}$

$\sqrt{E} X_1 (\Delta_0 + \Delta_0') + v_1 + n'$

$\gamma_1 = \dfrac{E(\Delta_0 + \Delta_0')^2}{N_0(\Delta_0^2 + (\Delta_0')^2)} \neq$ ~~MRC~~

$(\Delta_0 + \Delta_0')^2$

$\int_a^b f_1(x) f_2(x)\,dx$

$\int_a^b f_1(x)\,dx \le \int_a^b \underset{\uparrow \text{MAX}}{f_1}(x_0)\cdot dx$

$f_1(x) < f_1(x_0)$

$\int_a^b f_1(x)\,dx \le \int_a^b f_1(x_0)\,dx$

MEAN VALUE THEOREM → $= f_1(x_0)\cdot(b-a)$

$$\int_0^\infty f_1(x) \cdot e^{\sigma x \to 1} \cdot dx$$

ov. $\sigma \to 0 \Rightarrow \int_0^\infty$

• VIDI GO CLANAKOT NA I. BAMCECI. DOKAZOT NA LEMMA 1 KORISTI SLICNA VAKVA APROKSIMACIJA!

$$\frac{1}{(M-1)!}\int_0^v u^{M-1} e^{-u} du \le \frac{v^M}{M!}$$

VIDI N9.go !!!

OP

⊡ RELAY SELECTION IN MULTIPLE-RELAY CHANNELS

- K RELAYS ARE ADOPTED
- TOTAL POWER IS CONSTRAINED

$$\frac{P}{K+1}$$

$$\gamma_{ACAF} = \frac{1}{K+1}\left(\bar{\gamma}X + \sum_{k=1}^{K} \gamma_k\right) = \frac{\bar{\gamma}}{K+1}\left(X + \sum_{k=1}^{K} \frac{Y_k Z_k}{Y_k + Z_k}\right)$$

- DESTINATION SELECTS A TARGET RELAY ACCORDING TO THE FOLLOWING RULE:

$$\boxed{k^* = \arg \max_{1 \le k \le K} (\gamma_k)}$$

• ASYMPTOTIC ~~SER~~ EXPRESSION

- SNR OF THE COMBINER OUTPUT

$$\boxed{\gamma_{SCAF} = 0.5\left(\bar{\gamma}X + \bar{\gamma} \max_{1 \le k \le K} \frac{Y_k Z_k}{Y_k + Z_k}\right)}$$

• USTE EDNO POVTORUVANJE ZA MRC

x - TRANSMITED SYMBOL

$h = [h_1 \; h_2]^T$ - CHANNEL (RAYLEIGH)

$n = [n_1 \; n_2]^T$ — AWGN

$$Y = [h_1 \; h_2]^T \cdot x + [n_1, n_2]^T = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_1 x + n_1 \\ h_2 x + n_2 \end{bmatrix}$$

$$\hat{x} = \frac{h^H \cdot Y}{h^H \cdot h} = \frac{1}{|h_1|^2 + |h_2|^2} [h_1^* \; h_2^*] \cdot \begin{bmatrix} h_1 x + n_1 \\ h_2 x + n_2 \end{bmatrix} = (*)$$

$$h^H \cdot h = \begin{bmatrix} h_1^* \\ h_2^* \end{bmatrix}^T \cdot [h_1 \; h_2]^T = \underbrace{[h_1^* \; h_2^*]}_{h^H} \cdot \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = |h_1|^2 + |h_2|^2$$

$$\hat{x} = \frac{1}{|h_1|^2+|h_2|^2}\cdot\left(\underline{|h_1|^2\cdot x} + h_1^* n_1 + \underline{|h_2|^2 x} + h_2^*\cdot n_2\right)$$

$$\hat{x} = \frac{|h_1|^2+|h_2|^2}{|h_1|^2+|h_2|^2}x + \frac{h_1^* n_1 + h_2^*\cdot n_2}{|h_1|^2+|h_2|^2} = x + \frac{h^H\cdot n}{\sum_{i=1}^{2}|h_i|^2}$$

$$\hat{x} = x + \frac{1}{|h_1|^2+|h_2|^2}\cdot\left(h_1^* n_1 + h_2^*\cdot n_2\right)$$

$$P_S = E_S \qquad P_N = \frac{|h_1|^2}{\left(|h_1|^2+|h_2|^2\right)^2}\cdot N_0 + \frac{|h_2|^2\cdot N_0}{\left(|h_1|^2+|h_2|^2\right)^2}$$

$$P_N = \frac{N_0}{|h_1|^2+|h_2|^2} \quad : \quad SNR = \frac{P_S}{P_N} = \frac{E_S}{N_0}\left(|h_1|^2+|h_2|^2\right)$$

- VO GENERALEN SLUCAJ

$$\boxed{SNR = \frac{E_S}{N_0}\sum_{i=1}^{N}|h_i|^2}$$

- DA GO APLICIRAM NA TANG MODELOT:

$$\hat{x}_1 = \overbrace{G_2\Lambda_2\Delta_2\sqrt{E}}^{h_1}\,x_1 + \underbrace{G_2\Lambda_2 n_1 + \xi_1}_{n_1}$$

$$x_1' = \underbrace{\sqrt{E}\Delta_0}_{h_2}\,x_1 + \underbrace{\gamma_1}_{n_2}$$

$$n_1 = G_2\Lambda_2 n_1 + \xi_1$$

$$h_1 = G_2\Lambda_2\Delta_2\sqrt{E} \qquad h_2 = \sqrt{E}\Delta_0 \qquad n_2 = \gamma_1$$

$$\hat{x}_1 = h_1 x_1 + n_1 = y_1$$

$$x_1' = h_2 x_2 + n_2 = y_2$$

$$y = \begin{bmatrix}\hat{x}_1\\ x_1'\end{bmatrix} = \begin{bmatrix}y_1\\ y_2\end{bmatrix} = \begin{bmatrix}h_1 x_1 + n_1\\ h_2 x_2 + n_2\end{bmatrix}$$

$$\hat{x} = \begin{bmatrix}h_1^* & h_2^*\end{bmatrix}\cdot\begin{bmatrix}h_1 x_1 + n_1\\ h_2 x_1 + n_2\end{bmatrix}\cdot\frac{1}{|h_1|^2+|h_2|^2}$$

$$G_2\Lambda_2\Delta_2\sqrt{E} = \qquad \sqrt{E}\Delta_0$$

$$= /G_2 = 1/\Delta_2/ = \sqrt{E}\cdot\Lambda_2$$

$$\hat{x} = \frac{\sqrt{E}}{|h_1|^2+|h_2|^2}\begin{bmatrix}\Delta_0 & \overset{\text{LAMBDA!!!}}{\boxed{\Lambda_2}}\end{bmatrix}\cdot\begin{bmatrix}G_2\Lambda_2\Delta_2\sqrt{E}x_1 + G_2\Lambda_2 n_1 + \xi_1\\ \sqrt{E}\Delta_0 x_1 + \gamma_1\end{bmatrix}$$

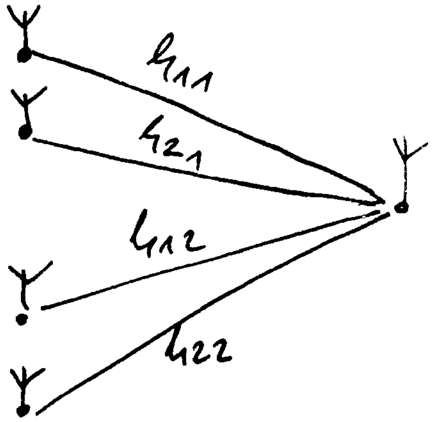
$$\hat{x} = \frac{\sqrt{E_s}}{\Delta_0^2 + \Lambda_2^2} [\Lambda_2 \ \Delta_0]^* \cdot \begin{bmatrix} G_2 \Lambda_2 \Delta_2 \sqrt{E} x_1 + G_2 \Lambda_2 n_1 + \xi_1 \\ \sqrt{E} \Delta_0 x_1 + V_1 \end{bmatrix}$$

$$\hat{x} = \frac{E_s}{\Delta_0^2 + \Lambda_2^2} \left(G_2 \Lambda_2^2 \Delta_2 x_1 + G_2 \Lambda_2^2 n_1 + \xi \Lambda_2 + \Delta_0^2 x_1 + \Delta_0 V_1 \right)$$

$$\hat{x} = \frac{E_s}{\Delta_0^2 + \Lambda_2^2} \left(G_2 \Lambda_2^2 \Delta_2 + \Delta_0^2 \right) x_1 + \frac{E_s \left(G_2 \Lambda_2^2 n_1 + \xi \Lambda_2 + \Delta_0 V_1 \right)}{\Delta_0^2 + \Lambda_2^2}$$

$$Y_1[1] = \sqrt{E_s} \left(h_{11} \tilde{x}_1 + h_{21} \tilde{x}_2 \right) + n_1[1]$$

$$Y_1[2] = \sqrt{E_s} \left(-h_{11} \tilde{x}_2^* + h_{21} \tilde{x}_1^* \right) + n_1[2]$$



?

$$Y_2[1] = \sqrt{E_s} \left(h_{12} x_1 + h_{21} x_2 \right) + n_2$$

$$Y_2[2] = \sqrt{E_s} \left(-h_{12} x_2^* + h_{22} x_1^* \right) + n_2$$

$$+ n_1[1] + n_2[1]$$

$$Y_1[1] = Y_1[1] + Y_2[1] = \sqrt{E_s} \left(h_{11} \tilde{x}_1 + h_{21} \tilde{x}_2 + h_{12} x_1 + h_{21} x_2 \right)$$

$$\gamma_{SCAF} = \boxed{0.5} \left(\bar{\gamma} X + \bar{\gamma} \max_{1 \le k \le K} \frac{Y_k Z_k}{Y_k + Z_k} \right)$$

→ SOURCE AND THE SELECTED RELAY USE 1/2 TRANSMIT POWER

- APROXIMATE PDF FOR MULTIPLE RELAY CHANNELS WITH RELAY SELECTION IS:

$$f_{SCAF}(\gamma) = \frac{2^{(K+1)N^2 + K} \Gamma(KN^2 + 1)}{\left[(N^2)! \right]^K \bar{\gamma}^{(K+1)N^2} \Gamma(KN^2 + N^2)} \cdot \gamma^{(K+1)N^2 - 1}$$

- APPENDIX C (PROOF)

$$F_{SCAF}(\gamma) = P_r \left(\frac{\bar{\gamma}}{2} \underset{\downarrow \ \sum_{i=1}^{N^2} |h_i|^2}{X} + \frac{\bar{\gamma}}{2} \max_k \frac{Y_k Z_k}{Y_k + Z_k} \le \gamma \right) = P_r \left(X + \max_k \frac{Y_k Z_k}{Y_k + Z_k} \le \frac{2\gamma}{\bar{\gamma}} \right)$$

$$F_{SCAF}(\gamma) = P_r\left(\max_k \frac{\gamma_k Z_k}{\gamma_k + Z_k} \leq \frac{2\gamma}{\bar{\gamma}} - x \right) =$$

$$= \int_0^{\frac{2\gamma}{\bar{\gamma}}} P_r\left(\max_k \frac{\gamma_k Z_k}{\gamma_k + Z_k} \leq \frac{2\gamma}{\bar{\gamma}} - x \right) f_x(x)\, dx =$$

ICCMTv4 Eq 34

$$\boxed{f_x(x) = \frac{x^{N^2-1}}{\Gamma(N^2)} e^{-x}} \qquad \boxed{f_x(x) = \frac{x^{N^2-1}}{\Gamma(N^2)} e^{-x}}$$

$$= \frac{1}{\bar{\gamma}^{N^2}\Gamma(N^2)} \int_0^{\frac{2\gamma}{\bar{\gamma}}} P_r\left(\max_k \frac{\gamma_k Z_k}{\gamma_k + Z_k} \leq \frac{2\gamma}{\bar{\gamma}} - x \right) x^{N^2-1} e^{-\frac{x}{\bar{\gamma}}}\, dx$$

$$= \Big/ \lambda = \frac{\bar{\gamma}}{2\gamma} x \qquad dx = \frac{2\gamma}{\bar{\gamma}} d\lambda \qquad x = \frac{2\gamma}{\bar{\gamma}} \quad \lambda = 1 \Big/ =$$

$$x = \frac{2\gamma}{\bar{\gamma}} \cdot \lambda$$

$$= \frac{1}{\Gamma(N^2)} \left(\frac{2\gamma}{\bar{\gamma}}\right)^{N^2} \int_0^1 P_r\left(\max_k \frac{\gamma_k Z_k}{\gamma_k + Z_k} \leq \frac{2\gamma}{\bar{\gamma}} (1-\lambda) \right) \cdot \lambda^{N^2-1} e^{-\frac{2\gamma}{\bar{\gamma}}\lambda} d\lambda$$

CONTINUE PP.38

$$F_{SCAF}(\gamma) = \frac{1}{\Gamma(N^2)} \left(\frac{2\gamma}{\bar{\gamma}}\right)^{N^2} \int_0^1 P_r\left(\max_k \frac{\gamma_k Z_k}{\gamma_k + Z_k} \leq \frac{2\gamma}{\bar{\gamma}} (1-\lambda) \right) \lambda^{N^2-1} e^{-\frac{2\gamma}{\bar{\gamma}}\lambda} d\lambda$$

- $\boxed{U = \max X_i}$ $\quad \boxed{F_U(u) = \prod_{i=1}^{N} F_{X_i}(u)}$ MMV

$\boxed{X_{max} = W_N}$

X_i OBSERVATIONS

$$\boxed{W_1 < W_2 < \dots < W_N} \Big\} \text{ ORDER OF THE OBSERVATIONS !!}$$

$$X_{min} = W_1 = \min(\underbrace{X_1, X_2, \dots X_N}_{\text{n - INDEPENDENT CONTINUOUS OBSERVATIONS}})$$

$$X_{max} = W_N = \max(X_1, X_2, \dots, X_N)$$

$F_X(x)$ – CDF OF X_i — i.i.d

VEROJATNOSTA
$X_{max} < x$ PODRA-
ZBIRA DEKA :
$W_1 < x$, $W_2 < x$ . . .
. . $W_N < x$. SITE DA SE
POMALI OD X

$$F_{max}(x) = P(X_{max} < x) = P(W_1 < x) \cdot P(W_2 < x) \cdots$$

$$\cdot P(W_N < x) = \prod_{i=1}^{N} P(X_i < x)$$

- $V = \min(X_i) \quad i = 1 \dots M$

$W_1, W_2, \dots, W_M$ } ORDERED OBSERVATIONS OF X_i RANDOM VARIABLES IN A GIVEN MOMENT

$W_1 < W_2 < W_3 \dots < W_N$

$$F_V^{(x)} = P_r(V < x) = P_r(\min(X_i) < x) = P_r(X_{\min} < x)$$

$$= P_r(W_1 < x) = 1 - P_r(W_1 > x) = 1 - P_r(W_1 > x) \cdot P_r(W_2 > x) \dots P_r(W_N > x)$$

$$\boxed{F_V^{(x)} = 1 - \prod_{i=1}^{M} P_r(X_i > x)}$$

$$f_{X_i}(\gamma) = \frac{\gamma^{N^2-1}}{\bar{\gamma}^{N^2}\Gamma(N^2)} e^{-\frac{\gamma}{\bar{\gamma}}}$$

[illegible]

$\mu = \frac{1}{\bar{\gamma}}$

- $$P_r(X > x) = \int_x^\infty \frac{\gamma^{N^2-1}}{\bar{\gamma}^{N^2}\Gamma(N^2)} e^{-\frac{\gamma}{\bar{\gamma}}} d\gamma = \frac{1}{\bar{\gamma}^{N^2}\Gamma(N^2)} \int_x^\infty \overset{n}{\gamma^{N^2-1}} e^{-\frac{\gamma}{\bar{\gamma}}} d\gamma$$

- GRADSHTEYN 3.351.2

$$\boxed{\int_u^\infty x^n e^{-\mu x} dx = e^{-u\mu} \sum_{k=0}^{n} \frac{n!}{k!} \frac{u^k}{\mu^{n-k+1}} = \mu^{-n-1}\Gamma(n+1, \mu u)}$$

$$P_r(X > x) = \frac{1}{\bar{\gamma}^{N^2}\cancel{\Gamma(N^2)}} e^{-\frac{x}{\bar{\gamma}}} \sum_{k=0}^{N^2-1} \frac{\cancel{(N^2-1)!}}{k!} \frac{x^k}{\left(\frac{1}{\bar{\gamma}}\right)^{N^2-k}} = \left(\frac{1}{\bar{\gamma}}\right)^{-N^2} \frac{\Gamma(N^2, \frac{x}{\bar{\gamma}})}{\bar{\gamma}^{N^2}\Gamma(N^2)}$$

$$P_r(X > x) = \frac{e^{-\frac{x}{\bar{\gamma}}}}{\bar{\gamma}^{N^2}} \sum_{k=0}^{N^2-1} \frac{\bar{\gamma}^{N^2-k} x^k}{k!} = \cancel{\bar{\gamma}^{N^2}} \frac{\Gamma(N^2, \frac{x}{\bar{\gamma}})}{\cancel{\bar{\gamma}^{N^2}}\Gamma(N^2)}$$

$$F_V(x) = 1 - \left[\frac{e^{-\frac{x}{\bar{\gamma}}}}{\bar{\gamma}^{N^2}} \sum_{k=0}^{N^2-1} \frac{\bar{\gamma}^{N^2-k} x^k}{k!}\right]^M = 1 - \left[\frac{\Gamma(N^2, \frac{x}{\bar{\gamma}})}{\Gamma(N^2)}\right]^M$$

$\boxed{M = 2}$ Eq.29 OD ČLANKOT NA ZANG

$$F_V(x) = 1 - \left[\frac{e^{-\frac{x}{\bar{\gamma}}}}{\bar{\gamma}^{N^2}} \sum_{k=0}^{N^2-1} \frac{\bar{\gamma}^{N^2-k} x^k}{k!}\right]^2 = 1 - \left[\frac{\Gamma(N^2, \frac{x}{\bar{\gamma}})}{\Gamma(N^2)}\right]^2$$

- NOMENKLATURA SOGLASNO ČLANAKOT NA ŽANG

$$W = \min(Y, Z) \qquad f_Z(z) = \frac{1}{\Gamma(N^2)} z^{N^2-1} e^{-z} \sim f_Y(y)$$

$$F_W(\omega) = 1 - P_r(Z > w) \cdot P_r(Y > w)$$

$$P_r(Z > w) = \int_w^{\infty} \frac{z^{N^2-1}}{\Gamma(N^2)} e^{-z} dz = \frac{e^{-w}}{\cancel{\Gamma(N^2)}} \sum_{k=0}^{N^2-1} \frac{\cancel{(N^2-1)!} \cdot w^k}{k!} = \frac{\Gamma(N^2, w)}{\Gamma(N^2)}$$

$$F_W(\omega) = 1 - \left[e^{-w} \sum_{k=0}^{N^2-1} \frac{w^k}{k!}\right]^2 = 1 - \left[\frac{\Gamma(N^2, \omega)}{\Gamma(N^2)}\right]^2$$

POKAZANO!! ...

$$1 - \sum_{k=0}^{a-1} \frac{x^k}{k!} e^{-x} \leq \frac{x^a}{a!}$$

$$e^{-\mu m} \sum_{k=0}^{n} \frac{n!}{k!} \frac{m^k}{\mu^{n-k+1}} = \frac{\Gamma(n+1, \mu m)}{\mu^{n+1}}$$

$$\Gamma(n+1, \mu m) = \cancel{\mu^{n+1}} e^{-m\mu} \sum_{k=0}^{n} \frac{n!}{k!} \frac{m^k}{\mu^{n+1-k}}$$

$$\Gamma(n+1, \mu m) = e^{-m\mu} \sum_{k=0}^{n} \frac{n!}{k!} (\mu m)^k$$

	GRADSHTEYN
$\Gamma(n+1, v) = e^{-v} \sum_{k=0}^{n} \frac{n!}{k!} v^k$	8.352.2
$\Gamma(n, x) = (n-1)!\, e^{-x} \sum_{n=0}^{n-1} \frac{x^n}{n!}$	8.352.4
$\gamma(n+1, x) = n! \left[1 - e^{-x} \sum_{m=0}^{n} \frac{x^m}{m!}\right]$	8.352.1

- I. BAHCECI PAPER: „ANTENNA SELECTION ..."

- LEMMA 1

$$g(v) = 1 - e^{-v} \sum_{m=0}^{M-1} \frac{x^m}{m!}$$

$$\gamma(n, x) = (n-1)! \left(1 - e^{-x} \sum_{m=0}^{n-1} \frac{x^m}{m!}\right)$$

$$1 - e^{-x} \sum_{m=0}^{n-1} \frac{x^m}{m!} = \frac{1}{(n-1)!} \gamma(n, x)$$

$$g(v) = \frac{1}{(M-1)!}\cdot \gamma(M,v) = \frac{1}{(M-1)!}\int_0^v u^{M-1} e^{-u}\,du \le \frac{1}{(M-1)!}\int_0^v u^{M-1}\,du$$

$\boxed{\text{CONTINUE N7.98}}$

$$\int u\,dv = u\cdot v - \int v\,du$$

$$\int x e^{x}dx = \left| \begin{matrix} u = x \\ v = \int e^{x}dx = e^{x} \end{matrix}\right| = xe^{x} - \int e^{x}dx = (x-1)e^{x}$$

$$\int x\cdot e^{ax}dx = \left|\begin{matrix} u = x \\ v = \int e^{ax}dx = \frac{1}{a}e^{ax}\end{matrix}\right| = x\cdot\frac{e^{ax}}{a} - \frac{1}{a}\int e^{ax}dx$$

$$= xe^{ax} - \frac{1}{a^2}\int e^{ax}d(ax) = \frac{xe^{ax}}{a} - \frac{1}{a^2}\cdot e^{ax} = \left(\frac{x}{a} - \frac{1}{a^2}\right)e^{ax}$$

$$\int u(t)\underbrace{f(t)\,dt}_{dv(t)} \qquad \underline{\underline{\frac{dv(t)}{dt} = f(t)}} \qquad v(t) = \int f(t)\,dt$$

$$\int x^{M-1}e^{-x}dx = \left|\begin{matrix} x^{M-1} = u \\ du = (M-1)\cdot x^{M-2}dx \\ v = \int e^{-x}dx = -e^{-x}\end{matrix}\right| = -x^{M-1}\cdot e^{-x} + \int e^{-x}(M-1)x^{M-2}dx$$

$$\frac{1}{(M-1)!}\int_0^v u^{M-1}du = \frac{1}{(M-1)!}\cdot\frac{u^{M-1+1}}{M-1+1}\Big|_0^v = \frac{u^M}{M!}\Big|_0^v = \frac{v^M}{M!}$$

$$\int_0^\infty t^{\alpha}\cdot e^{-\lambda t}dt \qquad \boxed{\int_0^\infty t^{\alpha-1}e^{-t}dt = \Gamma(\alpha)}$$

$$\Gamma(\alpha+1) = \int_0^\infty t^{\alpha+1-1}e^{-t}dt = \int_0^\infty t^{\alpha}e^{-t}dt$$

$$\frac{1}{\lambda}\int_0^\infty t^{\alpha}\cdot e^{-\lambda t}d(\lambda t) = \frac{1}{\lambda}\int_0^\infty \frac{(\lambda t)^{\alpha}}{\lambda^{\alpha}}\cdot e^{-\lambda t}d(\lambda t) = \frac{1}{\lambda^{\alpha+1}}\cdot\Gamma(\alpha+1)$$

- WIKIPEDIA (BIG-O i.e. BIG-OH NOTATION)

$f(x) = O(g(x))$ AS $x \to \infty$

IF AND ONLY if for sufficiently large values of „x", $f(x)$ is at most constant multiplied by abs$(g(x))$:

$$|f(x)| \le M\,|g(x)| \quad \text{for al } x > x_0$$

$$e^x = 1 + x + \frac{x^2}{2} + O(x^3) \quad x \to 0$$

$$e^x - \left(1 + x + \frac{x^2}{2}\right) \le K \cdot |x^3|$$

BLITZ POVTORUVANJE ZA NIZI I REDOVI OD MOJATA SKRIPTA. KOGA ĆE IMAM VREME TREBA DA ZA POMINAM GLAVATA OD SEMESTAR.

- KOŠIEV KRITERIUM ZA KONVERGENCIJA NA NIZA:

$$|a_{n+p} - a_n| < \varepsilon \quad \text{ZA} \quad n > N(\varepsilon) \quad \forall p \in N$$

$$\forall \varepsilon > 0,\ \exists N(\varepsilon) \ \text{T.S.} \ |S_{n+p} - S_n| < \varepsilon \ \text{ZA} \ \forall n > N(\varepsilon),\ p \in N$$

$$\lim_{n \to \infty} S_n = S$$

- KRITERIUM ZA SPOREDUVANJE

$$\{S_n^*\} = a_1 + a_2 + \cdots a_n$$

$$\{S_n^{**}\} = b_1 + b_2 + \cdots b_n$$

$$S_n^{**} \le f(S_n^*) < f(S^*) \qquad S_{n+1}^{**} - S_n^{**} > 0 \qquad \text{M.T}$$

- KOŠIEV RADIKALEN KRITERIUM

$$\lim_{n \to \infty} |a_n|^{1/n} = \lim_{n \to \infty} \sqrt[n]{a_n} = r < 1 \qquad \sum_{n=1}^{\infty} a_n$$

REDOT E KONVERGENTEN

- RED SO POZITIVNI ČLENOVI

$$\sum_{n=1}^{\infty} a_n \qquad a_n > 0$$

$$\lim \frac{|a_{n+1}|}{|a_n|} = d$$

$d < 1$ REDOT E KONVERGENTEN

KRITERIUM NA DALAMBER

- RABOV KRITERIUM

$$\lim_{n\to\infty} n\left(\frac{a_n}{a_{n+1}}-1\right)=l \qquad \boxed{l>1} \rightarrow \sum_{n=1}^{\infty} a_n \text{ e konvergenten}$$

- KRITERIUM ZA SPOREDUVANJE 1

$$\sum_{n=1}^{\infty} a_n \qquad \sum_{n=1} b_n \qquad a_n \leq b_n$$

AKO $\sum b_n$ e konvergenten → $\sum a_n$ e convergent.

- KRITERIUM ZA SPOREDUVANJE 2

$$\lim \frac{a_n}{b_n}=l \quad (b_n \neq 0) \quad \text{togaš}$$

$\sum a_n$ i $\sum b_n$ se istovremeno ili konvergentni ili divergentni

- INTEGRALEN KOŠIEV KRITERIUM

AKO $f(x)$ e nenegativna monotono opagjačka funkcija vo intervalot $[1,\infty]$ togaš redot $\sum_{n=1}^{\infty} f(n)$ konvergira, ako postoi $\int_1^{\infty} f(x)\,dx$

- ZNAČI:

1° $f(x)>0$, 2° $f'(x)<0$, 3° $\int_1^{\infty} f(x)\,dx$ postoi

- ALTERNATIVNI REDOVI:

$$a_1-a_2+a_3-\dots(-1)^{n-1}a_n+\cdots \;;\; a_1,a_2,\dots a_n \geq 0$$

- KRITERIUM NA LIEBNIZ ZA ALTERNATIVNI REDOVI

$$a_1,a_2,\dots,a_n>0$$

$$\lim_{n\to\infty} a_n=0$$

$$a_1 \geq a_2 \geq a_3 \geq \dots \geq a_n \geq a_{n+1}$$

- KOŠIEV PROIZVOD NA DVA REDA

$$a_0b_0+a_1b_1+a_2b_2+\dots+a_nb_n$$

$\sum a_n$ i $\sum b_n$ se apsolutno konvergentni togaš gorniot red e apsolutno konvergenten so suma: $\underline{a \cdot b}$

□ KRITERIUMI ZA USLOVNO KONVERGENTNI REDOVI

- KRITERIUM NA ABEL

$$\sum_{j=n}^{m} a_j b_j = a_{m+1} S_m - a_n S_{n+1} + \sum_{j=n}^{m} (a_j \, a_{j+1}) s_j.$$

- REDOT $\sum_{n=1} a_n b_n$ E KONVERGENTEN AKO $\sum_{n=1}^{\infty} b_n = S$ E KONVERGENTEN I AKO ČLENOVITE a_n FORMIRAAT MONOTONA (M↑ ILI M↓) KOJA E OGRANIČENA T.E. KONVERGENTNA ($\lim a_n = 0$)

- KRITERIUM NA ~~[illegible]~~ DIRIHLE;

$\sum_{n=1} a_j$ $\sum_{1}^{\infty} b_j$ $|s_j| = \left| \sum_{n=1}^{j} b_n \right| \le \beta$

PRI OVAA PRETPOSTAVKA $\sum a_n b_n$ ĆE BIDE KONVERGENTEN AKO:

ⓐ AKO $\lim_{n\to\infty} a_n = 0$ I $\sum_{n=1}^{\infty} |a_n \cdot a_{n+1}|$ [illegible]

ILI:

ⓑ NIZATA $\{a_n\}$ MONOTONO STREMI KON 0 OD NEKOE $\underline{\underline{n = n_0}}$

◉ FUNKCIONALNI REDOVI

• $\overset{=a_1/x_0}{f_1(x)}, \overset{=a_2/x_0}{f_2(x)}, \ldots, \overset{=a_n/x_0}{f_n(x)}$ SE OPREDELENI NA

AKO SITE NIZI NA FUNKCII „x_0" VO TOJ INTERVAL INT $[a,b]$ TOČKI POTPOI FIKNO TAKA ŠTO:

$$\underset{a_1}{\underset{\parallel}{f_1(x_0)}}, \underset{a_2}{\underset{\parallel}{f_2(x_0)}}, \ldots, \underset{a_n}{\underset{\parallel}{f_n(x_0)}} \quad ⊛$$

$\lim_{n\to\infty} f_n(x_0) = f(x_0)$ AKO OVAA BROJNA NIZA E KONVERGENTNA TOGAŠ $f(x)$ KONVERGIRA VO FUNKCIONALNATA NIZA TOČKATA „x_0": T.E:

$\forall \varepsilon > 0 \quad \exists \, n_0(\varepsilon, x_0)$ T.S.

$|f_n(x_0) - f(x_0)| < \varepsilon$ ZA $n \ge n_0(\varepsilon, x_0)$

- AKO NIZATA ⊛ KONVERGIRA ZA SEKOE „x" VO $[a,b]$ SO LIMESOT SE DOBIVA GRANIČNA FUNKCIJA NA NIZATA

$\lim_{n\to\infty} f_n(x) = f(x)$

– ЗА НЕПРЕКИНАТИ ФУНКЦИИ

$$\lim f_n'(x) = f'(x)$$

ПРИМЕР: $x, \frac{x}{2}, \frac{x}{3}, \dots \frac{x}{n}$ ДАЛИ Е РАМНОМЕРНО КОНВЕРГЕНТНА

$$f_n(x) = \frac{x}{n} \qquad f(x) = \lim_{n\to\infty}\frac{x}{n} = 0 \qquad \forall x$$

$$|f_n(x) - f(x)| = \left|\frac{x}{n}\right| = \frac{|x|}{n} < \varepsilon \qquad n > \frac{|x|}{\varepsilon}$$

- $\forall \varepsilon > 0 \quad \exists n_0(x, \varepsilon)$ Т.Ѕ.

  $|f_n(x) - f(x)| < \varepsilon$ КОГА $n_0(x, \varepsilon) = \left[\frac{|x|}{\varepsilon}\right] + 1$

- ЗА ИНТЕРВАЛ $[0, 10]$

  $n_0 = \left[\frac{10}{\varepsilon}\right] + 1$ НАЈГОЛЕМО n_0 ВО ИНТЕРВАЛОТ

⊙ КОШИЕВ КРИТЕРИУМ ЗА РАМНОМЕРНА КОНВЕРГЕНЦИЈА НА ФУНКЦИОНАЛНИ НИЗИ

$$f_1(x), f_2(x), \dots f_n(x)$$

$$|f_n(x) - f_m(x)| < \varepsilon \quad \text{ЗА} \quad m, n \geq n_0(\varepsilon)$$

⊙ ФУНКЦИОНАЛНИ РЕДОВИ

$$f_1(x) + f_2(x) + f_3(x) + \dots + f_n(x) + \dots = \sum_{k=1}^{\infty} f_k(x)$$

$f(x) = \sum_{k=1}^{\infty} f_k(x) \Rightarrow$ СУМА ЗА ДАДЕНИОТ РЕД

$\forall \varepsilon > 0$, ЗА ДАДЕНО $x \in D$ $\exists n_0(\varepsilon, x)$ Т.Ѕ.

$$|S_n(x) - f(x)| < \varepsilon \qquad n \geq n_0(\varepsilon, 1)$$

$$S_n(x) = f_1(x) + f_2(x) + \dots + f_n(x)$$

⊙ КОШИЕВ КРИТЕРИУМ ЗА РАМНОМЕРНА КОНВЕРГЕНЦИЈА НА Ф-НИ РЕДОВИ

$$|S_m(x) - S_n(x)| < \varepsilon \qquad m, n \geq n_0(\varepsilon)$$

ПРИМЕР: $m > n$

$$S_m(x) = S_n(x) + f_{n+1}(x) + f_{n+2}(x) + \dots + f_m(x)$$

$$|S_m(x) - S_n(x)| = |f_{n+1}(x) + f_{n+2}(x) + \dots + f_m(x)| < \varepsilon$$

⊡ KRITERIUM NA VAERŠTRAS

⊛ $\sum_{n=1}^{\infty} f_n(x), \; x \in D$

AKO POSTOI BROEN RED: ① $\sum_{k=1}^{\infty} M_k$ TAKA ŠTO VAŽI:

$|f_k(x)| \leq M_k$ ZA $\forall k$ $\left(|f_1(x)| \leq M_1; \; f_2(x) \leq M_2, \dots \right)$

TOGAŠ AKO E KONVERGENTEN ① KONVERGENTEN KJE BIDE I ⊛.

- KOŠIEV PROIZVOD NA DVA REDA

$\sum_0^{\infty} a_n \quad \sum_0^{\infty} b_n \qquad \sum_0^{\infty} c_n = \sum_0^{\infty} a_n \cdot \sum_0^{\infty} b_n$

$c_n = \sum_{k=0}^{n} a_k \cdot b_{n-k}$ } KOŠIEV PROIZVOD NA 2 REDA

- FUNKCIONALNATA NIZA KONVERGIRA RAMNOMERNO

AKO: $|f_n(x) - f(x)| < \varepsilon$ ZA $n > N(\varepsilon)$ ZA $\forall \varepsilon$

$f_n(x) \rightrightarrows f(x) \iff \lim_{n \to \infty} \sup_{x \in X} (r_n(x)) = 0$

$r_n(x) = |f_n(x) - f(x)|$ $\quad \boxed{X - \text{OBLAST}}$

- ODREDUVAME NA sup$(r_n(x)$

$r_n'(x) = 0 \Rightarrow x_{1,2} = \dots$

$r_n''(x) = 0 \qquad r_n''(x_{1,2}) < 0 \Rightarrow$ SE RABOTI ZA MAXIMUM PA GO TOA BARAŠ LIMES.

$S_n(x) \rightrightarrows S(x) \iff \lim_{n \to \infty} \sup_{x \in X} r_n(x) = 0$ $\qquad S_n(x) = \sum_1^{n} \dots$

$r_n(x) = |S_n(x) - S(x)|$ $\qquad S(x) = \sum_1^{\infty} \dots$

⊙ STEPENASTI REDOVI

$a_0 + a_1(x-c) + a_2(x-c)^2 + \dots + a_n(x-c)^n + \dots$

T.E. $a_0 + a_1 t + a_2 t^2 + \dots + a_n t^n + \dots$

$t = x - c$

- РАДИУС И ОБЛАСТ НА КОНВЕРГЕНЦИЈА НА СТЕПЕНАСТ РЕД

$$\sum_{1}^{\infty} a_n x^n = \sum_{1}^{\infty} u_n(x)$$

$$\boxed{\frac{1}{R} = \underbrace{\lim_{n\to\infty} |a_n|^{\frac{1}{n}}}_{\text{ТЕОРЕМА НА КОШИ}} = \underbrace{\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|}_{\text{ДАЛАМ КРИТЕРИУМ}}}$$

$$\lim_{n\to\infty} |u_n|^{1/n} < 1$$

- **ТЕЈЛОРОВА ФОРМУЛА**

$$f(x) = f(x_0) + \frac{f'(x_0)}{1!}(x-x_0) + \frac{f''(x_0)}{2!}(x-x_0)^2 + \dots + \frac{f^{(n-1)}(x_0)}{(n-1)!}(x-x_0)^{n-1} + \frac{f^{(n)}(\xi)}{n!}(x-x_0)^n$$

$\xi = x_0 + \theta \Delta x$ $\Delta x = x - x_0$ $0 < \theta < 1$

- **МАКЛОРЕНОВА ФОРМУЛА**

$$f(x) = f(\underset{\xi}{0}) + \frac{f'(0)}{1!} \cdot x + \frac{f''(0)}{2!} x^2 + \dots + \frac{f^{(n)}(0)}{n!}$$

$\xi = \theta \cdot x$, $0 < \theta < 1$

⊙ ОСОБИНИ НА СТЕПЕНАСТИТЕ РЕДОВИ

⊛ $f(x) = a_0 + a_1 x + \dots + a_n x^n + \dots$ $x \in (-R, R)$

$$\int_a^b f(x)\,dx = \sum_{n=0}^{\infty} \int_a^b a_n x^n\,dx = \int_a^b \left(\sum_{n=0}^{\infty} a_n x^n\right) dx$$

● $f'(x) = a_1 + a_2 x + \dots + n a_n x^{n-1} + (n+1) a_{n+1} x^n + \dots$

$$f^{(n)}(x) = n \cdot (n-1) \cdots 1\, a_n + (n+1) n \cdots 2 \cdot a_{n+1} \cdot x + \dots$$

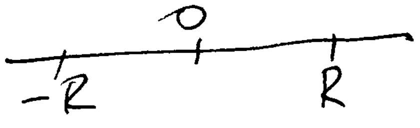
$f(0) = a_0$; $f'(0) = a_1$; $\dots$ $f^{(n)}(0) = n!\, a_n$

$\boxed{a_n = \frac{f^{(n)}(0)}{n!}}$ — ОПШТ ЧЛЕН НА МАКЛОРЕНОВИОТ РЕД.

- СЕКОЈ СТЕПЕНЕСТ РЕД ОД ОБЛИКОТ ⊛ Е ВСУШНОСТ МАКЛОРЕНОВ РЕД.

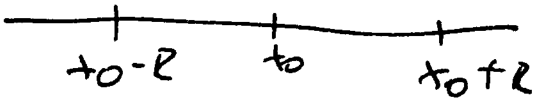
$f(x) = a_0 + a_1 x + \dots + a_n x^n + \dots$ $x \in (-R, R)$

РАДИУС НА КОНВЕРГЕНЦИЈА



- $\varphi(x) \doteq a_0 + a_1(x-x_0) + a_2(x-x_0)^2 + \cdots + a_n(x-x_0)^n + \cdots$

KONVERGIRA VO REGIONOT:



$$\varphi(x_0) = a_0 \quad \varphi'(x_0) = a_1 \quad \cdots \quad \varphi^{(n)}(x_0) = a_n \cdot n!$$

$$a_n = \frac{\varphi^{(n)}(x_0)}{n!}$$

- VRSKA MEGJU TAYLOROVA FORMULA I TAYLOROV RED:

$$f(x) = f(x_0) + \frac{f'(x_0)}{1!}(x-x_0) + \cdots + \frac{f^{(n-1)}(x_0)(x-x_0)^{n-1}}{(n-1)!} + R_n$$

$$R_n = \frac{f^{(n)}(\xi)}{n!}(x-x_0)^n \qquad \xi = x_0 + \theta\Delta x \qquad \theta \in (0,1)$$

$f(x)$ - GRANICA NA TAYLOR-OT ~~RED~~ T.E. SAMATA FUNKCIJA

$$R_n = f(x) - \sum_{i=0}^{n-1} \frac{f^{(i)}(x_0)}{i!}(x-x_0)^i$$

- AKO FUNKCIJATA IMA TAKVA OSOBINA DA: $n \to \infty$ $R_n \to 0$ ~~SE~~ DOBIVA:

$$\boxed{\text{BOJAN } 071369781}$$

$$f(x) = \lim_{n\to\infty} \sum_{i=0}^{n-1} \frac{f'(x_0)}{i!}(x-x_0)^i$$

PRIMER: $e^x = 1 + x + \frac{x^2}{2!} + \cdots + \frac{x^{n-1}}{(n-1)!} + R_n$

$R_n = \frac{e^{\xi} x^n}{n!}$ $\quad n \to \infty \quad R_n \to 0$ ~~PA ZATOA~~:

$$e^x = 1 + x + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!} + \cdots \quad \Bigg\} \text{ TAYLOROV T.E. MAKLORENOV RED.}$$

☐ POZNATI SUMI

$$s(x) = \sum_{n=1}^{\infty} n x^{n-1} \quad // \int dx$$

$$I = \int s(x)\,dx = \sum_{n=1}^{\infty} n \int x^{n-1}dx = \sum_{n=1}^{\infty} \frac{\cancel{n} \cdot x^n}{\cancel{n}} = \sum_{n=1}^{\infty} x^n$$

$$I = \frac{1}{1-x} \quad \boxed{s(x) = \left(\frac{1}{1-x}\right)' = \frac{1}{(1-x)^2}}$$

$$\boxed{\sum_{n=1}^{\infty} x^n = \frac{1}{1-x}} \longrightarrow \text{OSNOVA!!}$$

$$A=\int_0^{v} x^{\alpha-1}e^{-x}\,dx \;\boxed{\le}\; \int_0^{v} x^{\alpha-1}\,dx = \left.\frac{x^{\alpha}}{\alpha}\right|_0^{v} = \frac{v^{\alpha}}{\alpha}$$

ZOŠTO OVA VAŽI VIDI SCIENTIFIC NOTES 2.94.

$$B=\int_0^{v} x^{\alpha-1}e^{-x}\,dx = \int_0^{\infty} x^{\alpha-1}e^{-x}\,dx - \int_v^{\infty} x^{\alpha-1}e^{-x}\,dx = \underline{\Gamma(\alpha)-\Gamma(x,\alpha)}$$

– GI SPOREDUVAM OVIE DVA REZULTATI VO NAREDATA. NAVISTINA $\boxed{B \le A}$. MMV

$$\gamma(n,x)=(n-1)!\left[1-e^{-x}\sum_{n=0}^{n-1}\frac{x^n}{n!}\right]=(n-1)!-(n-1)!\,e^{-x}\sum_{0}^{n-1}\frac{x^n}{n!}$$

- ALTERNATIVE DEFINITIONS OF GAMMA FUNCTION

$$\Gamma(z)=\lim_{n\to\infty}\frac{n!\,n^{z}}{z(z+1)\cdots(z+n)}=\frac{1}{z}\prod_{n=1}^{\infty}\frac{\left(1+\frac{1}{n}\right)^{z}}{1+\frac{z}{n}}$$

$$\Gamma(z)=\frac{e^{-\gamma z}}{z}\prod_{n=1}^{\infty}\left(1+\frac{z}{n}\right)^{-1}e^{\frac{z}{n}}$$

$\gamma \doteq 0.577216$ – EULER-MASCHERONI CONSTANT

- $$g(v)=1-e^{-v}\sum_{n=0}^{M-1}\frac{x^n}{n!}=\frac{1}{(M-1)!}\int_0^{v}u^{M-1}e^{-u}\,du \le \frac{1}{(M-1)!}\int_0^{v}u^{M-1}\,du$$

$$\boxed{g(v)\le\frac{M^{v}}{M!}}$$

– SEGA SE VRAĆAM NAZAD NA IZRAZOT OD Yang.

$$\boxed{1-\sum_{k=0}^{a-1}\frac{x^k}{k!}e^{-x}\le\frac{x^a}{a!}} \quad (\ast)$$

$$F_{SCAF}(\delta)=\frac{1}{\Gamma(N^2)}\left(\frac{2\gamma}{\delta}\right)^{N^2}\int_0^{1}\prod_{k=1}^{K}\Pr\left[\frac{Y_k Z_k}{Y_k+Z_k}\le\frac{2\gamma}{\delta}(1-\lambda)\right]\lambda^{N^2-1}e^{-\frac{2\gamma}{\delta}\lambda}\,d\lambda$$

$$\Pr\left[\frac{Y_k Z_k}{Y_k+Z_k}\le\frac{2\gamma}{\delta}(1-\lambda)\right]\doteq 1-\left[e^{-\frac{2\gamma}{\delta}(1-\lambda)}\sum_{i=0}^{N^2-1}\frac{\left[\frac{2\gamma}{\delta}(1-\lambda)\right]^{i}}{i!}\right]^{2}$$

$$\Pr(\gamma) \doteq 1 - \left[e^{-\frac{2\gamma}{\bar{\gamma}}(1-\lambda)} \sum_{i=0}^{N^2-1} \left(\frac{2\gamma}{\bar{\gamma}}\right)^i \frac{(1-\lambda)^i}{i!} \right]^2$$

[illegible]

$$\circledast \Rightarrow \boxed{\sum_{k=0}^{a-1} \frac{x^k}{k!} e^{-x} \geq 1 - \frac{x^a}{a!}}$$

$$\left(1 - \frac{x^a}{a!}\right)^2 = 1 - \frac{2x^a}{a!} + \cancel{\frac{x^{2a}}{(a!)^2}} \doteq 1 - \frac{2x^a}{\boxed{a!}}$$

$$\Pr(\gamma) = \cancel{1} - \cancel{1} + \frac{2}{N^2!} \cdot \left(\frac{2\gamma}{\bar{\gamma}}\right)^{N^2} (1-\lambda)^{N^2}$$

$$\boxed{\Pr(\gamma) \doteq \frac{2}{N^2!} \left(\frac{2\gamma}{\bar{\gamma}}\right)^{N^2} (1-\lambda)^{N^2}}$$

$$\Pr(\gamma) = \frac{2}{N^2!} \left(\frac{2\gamma}{\bar{\gamma}}\right)^{N^2} (1-\lambda)^{N^2}$$

$$\boxed{\lambda = \frac{\bar{\gamma}}{2\gamma} \cdot x}$$

$$f_{SCAF}(\gamma) = \frac{d}{d\gamma} F_{SCAF}(\gamma) \doteq ?$$

$$F_{SCAF}(\gamma) = \frac{1}{\Gamma(N^2)} \left(\frac{2\gamma}{\bar{\gamma}}\right)^{N^2} \int_0^1 \prod_{l=1}^{K} \frac{2}{N^2!} \left(\frac{2\gamma}{\bar{\gamma}}\right)^{N^2} (1-\lambda)^{N^2} \lambda^{N^2-1} e^{-\frac{2\gamma}{\bar{\gamma}}\lambda} d\lambda$$

$$(\;)^K$$

! OVA NE E DOBRO !!

$$= \underbrace{\frac{2^K (2\gamma)^{KN^2}}{\Gamma(N^2)\, \Gamma^K(N^2+1)\, \bar{\gamma}^{KN^2}}}_{C} \int_0^1 \left[(1-\lambda)^{N^2} \lambda^{N^2-1} e^{-\frac{2\gamma}{\bar{\gamma}}\lambda} \right]^K d\lambda =$$

NAJDOBRO ODI NA PP. 10 >

$$= C \cdot \int_0^1 (1-\lambda)^{KN^2} \lambda^{KN^2-K} e^{-\frac{2K\gamma}{\bar{\gamma}}\lambda} d\lambda =$$

② OVA NE E DOBRO!!!...

[illegible]

$$= C \underbrace{\int_0^1 \lambda^{KN^2-K} e^{-\frac{2K\gamma}{\bar{\gamma}}\lambda} \cdot d\lambda}_{I1} - C \underbrace{\int_0^1 \lambda^{2KN^2-K} e^{-\frac{2K\gamma}{\bar{\gamma}}\lambda} d\lambda}_{I2}$$

$$I1 = \left| \begin{array}{lll} \frac{2K\gamma}{\bar{\gamma}}\lambda = t & d\lambda = \frac{\bar{\gamma}}{2K\gamma} dt & \lambda = 0 \quad t = 0 \\ \lambda = \frac{\bar{\gamma}}{2K\gamma} t & & \lambda = 1 \quad t = \frac{2K\gamma}{\bar{\gamma}} \end{array} \right|$$

$$I1 = \int_0^{\frac{2K\gamma}{\bar\gamma}} \left(\frac{\bar\gamma}{2K\gamma} t\right)^{KN^2-K} e^{-t} \frac{\bar\gamma}{2K\gamma}\, dt = \left(\frac{\bar\gamma}{2K\gamma}\right)^{KN^2-K+1} \int_0^{\frac{2K\gamma}{\bar\gamma}} t^{KN^2-K} e^{-t}\, dt$$

$\alpha - 1 = KN^2 - K$ $\alpha = KN^2 - K + 1$

$$I1 = \left(\frac{\bar\gamma}{2K\gamma}\right)^{KN^2-K+1} \cdot \underbrace{\gamma\left(KN^2-K+1, \frac{2K\gamma}{\bar\gamma}\right)}$$ INCOMPLETE GAMMA

$$I2 = \int_0^1 \lambda^{2KN^2-K} e^{-\frac{2K\gamma}{\bar\gamma}\lambda}\, d\lambda$$

$t = \frac{2K\gamma}{\bar\gamma}\lambda$ $d\lambda = \frac{\bar\gamma}{2K\gamma} dt$

$\lambda = \frac{\bar\gamma}{2K\gamma} \cdot t$ $t=0$ $\lambda=0$

$\lambda=1$ $t = \frac{2K\gamma}{\bar\gamma}$

$$I2 = \int_0^{\frac{2K\gamma}{\bar\gamma}} \left(\frac{\bar\gamma}{2K\gamma}\right)^{2KN^2-K} \cdot t^{2KN^2-K} \cdot e^{-t} \cdot \left(\frac{\bar\gamma}{2K\gamma}\right) \cdot dt$$

$$I2 = \left(\frac{\bar\gamma}{2K\gamma}\right)^{2KN^2-K+1} \int_0^{\frac{2K\gamma}{\bar\gamma}} t^{2KN^2-K} \cdot e^{-t}\, dt = \left(\frac{\bar\gamma}{2K\gamma}\right)^{2KN^2-K+1} \cdot \gamma\left(2KN^2-K+1, \frac{2K\gamma}{\bar\gamma}\right)$$

$$F_{CAF}(\gamma) = c \cdot \left[\left(\frac{\bar\gamma}{2K\gamma}\right)^{KN^2-K+1} \cdot \gamma\left(KN^2-K+1, \frac{2K\gamma}{\bar\gamma}\right) - \left(\frac{\bar\gamma}{2K\gamma}\right)^{2KN^2-K+1} \gamma\left(2KN^2-K+1, \frac{2K\gamma}{\bar\gamma}\right)\right]$$

$$F_{CAF}(\gamma) = c \cdot \left(\frac{\bar\gamma}{2K\gamma}\right)^{KN^2-K+1} \left[\gamma\left(KN^2-K+1, \frac{2K\gamma}{\bar\gamma}\right) - \left(\frac{\bar\gamma}{2K\gamma}\right)^{KN^2} \gamma\left(2KN^2-K+1, \frac{2K\gamma}{\bar\gamma}\right)\right]$$

$$\gamma(\alpha, x) = \int_0^x t^{\alpha-1} e^{-t}\, dt \qquad \frac{d\gamma(\alpha, x)}{dx} = x^{\alpha-1} e^{-x}$$

$$\gamma(\alpha, bx) = \int_0^{bx} t^{\alpha-1} e^{-t}\, dt \qquad \frac{d\gamma(\alpha, bx)}{dx} = \underbrace{b\,(bx)^{\alpha-1} e^{-bx}}$$

CHAIN RULE

DOKAZ: $bx = y$

$$\boxed{\frac{d}{dx}\gamma(\alpha, bx) = \frac{d}{dx}\int_0^y t^{\alpha-1} e^{-t}\, dt = (\underbrace{bx}_{y})^{\alpha-1} e^{-\overbrace{bx}^{y}} \cdot \frac{dy}{dx} = b\,(bx)^{\alpha-1} e^{-bx}}$$

$$f_{CAF}(\gamma) = \frac{dF_{CAF}(\gamma)}{dx} = c\left(\frac{\bar\gamma}{2K\gamma}\right)^{KN^2-K+1}\left[\frac{2K}{\bar\gamma} \cdot \left(\frac{2K\gamma}{\bar\gamma}\right)^{KN^2-K} e^{-\frac{2K}{\bar\gamma}\cdot\gamma} - \right.$$

$$\boxed{- \left(\frac{\bar{\gamma}}{2K\gamma}\right)^{KN^2} \cdot \frac{2K}{\bar{\gamma}} \cdot \left(\frac{2K\gamma}{\bar{\gamma}}\right)^{2KN^2-K} \cdot e^{-\frac{2K}{\bar{\gamma}}\gamma} \Bigg]} \qquad \boxed{\lambda = \frac{\bar{\gamma}}{2\gamma} \cdot x}$$

$$F_{CAF}(\gamma) = C \int_0^1 (1-\lambda)^{KN^2} \lambda^{KN^2-K} e^{-\frac{2K\gamma}{\bar{\gamma}}\lambda} d\lambda \qquad C = \frac{2^K (2\gamma)^{KN^2}}{\Gamma(N^2)\Gamma^K(N^2+1)\bar{\gamma}^{KN^2}}$$

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^n \cdot b^{n-k}$$

$$(a-b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} \cdot b^k \cdot (-1)^k \qquad \boxed{(1-\lambda)^n = \sum_{k=0}^{n} \binom{n}{k} (-1)^k \lambda^k}$$

$$(1-\lambda)^2 = 1 - 2\lambda + \lambda^2 \qquad \binom{2}{0} b^0 - \binom{2}{1} b + \binom{2}{2} b^2 = 1 - 2b + b^2$$

$$F_{CAF}(\gamma) = C \int_0^1 \left(\sum_{i=0}^{KN^2} \binom{KN^2}{i} (-1)^i \lambda^i \right) \lambda^{KN^2-K} e^{-\frac{2K\gamma}{\bar{\gamma}}\lambda} d\lambda =$$

$$= C \sum_{i=0}^{KN^2} \binom{KN^2}{i} (-1)^i \underbrace{\int_0^1 \lambda^{KN^2-K+i} e^{-\frac{2K\gamma}{\bar{\gamma}}\lambda} d\lambda}_{I}$$

$$I = \left| \begin{array}{ll} \frac{2K\gamma}{\bar{\gamma}}\lambda = t & \lambda = 0 \quad t = 0 \\ \lambda = \frac{\bar{\gamma}}{2K\gamma} \cdot t & \lambda = 1 \quad t = \frac{2K\gamma}{\bar{\gamma}} \\ d\lambda = \frac{\bar{\gamma}}{2K\gamma} dt & \end{array} \right| = \int_0^{\frac{2K\gamma}{\bar{\gamma}}} \left(\frac{\bar{\gamma}}{2K\gamma}\right)^{KN^2-K+i} t^{KN^2-K+i} e^{-t} \frac{\bar{\gamma}}{2K\gamma} dt$$

$$= \left(\frac{\bar{\gamma}}{2K\gamma}\right)^{KN^2-K+i+1} \int_0^{\frac{2K\gamma}{\bar{\gamma}}} t^{KN^2-K+i} e^{-t} dt = \left(\frac{\bar{\gamma}}{2K\gamma}\right)^{KN^2-K+i+1} \underline{\gamma\left(KN^2-K+i+1, \frac{2K\gamma}{\bar{\gamma}}\right)}$$

$$F_{CAF}(\gamma) = C \cdot \sum_{i=0}^{KN^2} \binom{KN^2}{i} (-1)^i \left(\frac{\bar{\gamma}}{2K\gamma}\right)^{KN^2-K+i+1} \gamma\left(KN^2-K+i+1, \frac{2K\gamma}{\bar{\gamma}}\right)$$

$$f_{CAF}(\gamma) = \frac{dF_{CAF}(\gamma)}{d\gamma} = \frac{d}{d\gamma} \left[\frac{2^K (2\gamma)^{KN^2}}{\Gamma(N^2)\Gamma^K(N^2+1)\bar{\gamma}^{KN^2}} \sum_{i=0}^{KN^2} \binom{KN^2}{i} (-1)^i \left(\frac{\bar{\gamma}}{2K\gamma}\right)^{KN^2-K+i+1} \gamma\left(\ldots, \frac{2K\gamma}{\bar{\gamma}}\right) \right.$$

$$= \frac{d}{d\gamma} \left[\frac{2^K (2\gamma)^{KN^2}}{\Gamma(N^2)\Gamma^K(N^2+1)\bar{\gamma}^{KN^2}} \left(\frac{\bar{\gamma}}{2\gamma K}\right)^{KN^2-K+1} \sum_{i=0}^{KN^2} (-1)^i \binom{KN^2}{i} \left(\frac{\bar{\gamma}}{2K\gamma}\right)^i \gamma\left(KN^2-K+i+1, \frac{2K\gamma}{\bar{\gamma}}\right) \right]$$

$$= \frac{d}{d\gamma} \left[\frac{2^K}{\Gamma(N^2)\Gamma^K(N^2+1)} \left(\frac{1}{K}\right)^{KN^2-K+1} \cdot \left(\frac{\bar{\gamma}}{2\gamma}\right)^{-K+1} \sum_{i=0}^{KN^2} (-1)^i \binom{KN^2}{i} \left(\frac{\bar{\gamma}}{2K\gamma}\right)^i \gamma\left(\ldots, \frac{2K\gamma}{\bar{\gamma}}\right) \right.$$

$$f_{CAF}=\frac{d}{d\gamma}\left[\frac{2^K}{\Gamma(N^2)\Gamma^K(N^2+1)}\left(\frac{1}{K}\right)^{KN^2-K+1}\left(\frac{\bar{\gamma}}{2\gamma}\right)^{-K+1}\sum_{i=0}^{KN^2}(-1)^i\binom{KN^2}{i}\left(\frac{\bar{\gamma}}{\cancel{2K\gamma}}\right)^i\gamma\left(\dots,\frac{2K\gamma}{\bar{\gamma}}\right)\right)$$

$$=\underbrace{\frac{2^K}{\Gamma(N^2)\Gamma^K(N^2+1)\,K^{KN^2-K+1}}}_{C1}\frac{d}{d\gamma}\sum_{i=0}^{KN^2}\left(\frac{-1}{K}\right)^i\left(\frac{\bar{\gamma}}{2\cancel{K}}\right)^{i-K+1}\boxed{\frac{\gamma\left(\dots,\frac{2K\gamma}{\bar{\gamma}}\right)}{\gamma^{i-K+1}}}$$

$$f_{CAF}=C1\cdot\frac{d}{d\gamma}\sum_{i=0}^{KN^2}\left(-\frac{1}{K}\right)^i\left(\frac{\bar{\gamma}}{2}\right)^{i-K+1}\cdot\frac{\gamma\left(KN^2-K+i+1,\frac{2K\gamma}{\bar{\gamma}}\right)}{\gamma^{i-K+1}}$$

$$f_{CAF}=C_1\frac{d}{d\gamma}\sum_{i=0}^{KN^2}(-1)^i\frac{\gamma\left(KN^2-K+i+1,\frac{2K\gamma}{\bar{\gamma}}\right)}{\left(\frac{2K\gamma}{\bar{\gamma}}\right)^{i-K+1}}=\left|\frac{d}{dx}\gamma(\alpha,bx)=b(bx)^{\alpha-1}e^{-bx}\right|$$

$$=C_1\sum_{i=0}^{KN^2}(-1)^i\frac{\frac{2K}{\gamma}\cdot\left(\frac{2K\gamma}{\bar{\gamma}}\right)^{KN^2-K+i}e^{-\frac{2K\gamma}{\bar{\gamma}}}\cdot\left(\frac{2K\gamma}{\bar{\gamma}}\right)^{i-K+1}-\gamma\left(KN^2-K+i+1,\frac{2K\gamma}{\bar{\gamma}}\right)\cdot(i-K+1)\left(\frac{2K\gamma}{\bar{\gamma}}\right)^{i-K}}{\left(\frac{2K\gamma}{\bar{\gamma}}\right)^{2i-2K+2}}$$

$$F_{CAF}=c\int_0^1(1-\lambda)^{KN^2}\lambda^{KN^2-K}e^{-\frac{2K\gamma}{\bar{\gamma}}\lambda}d\lambda\qquad\lambda=\frac{\bar{\gamma}}{2\gamma}\cdot x$$

$$\boxed{c=\frac{2^K(2\gamma)^{KN^2}}{\Gamma(N^2)\Gamma^K(N^2+1)\bar{\gamma}^{KN^2}}}$$

$$f(x)=(1-x)^K\qquad f(x)=f(0)+\frac{f'(0)}{1!}\cdot x+\frac{f''(0)}{2!}x^2+O(x^3)$$

$$f(x)=1-K\cdot(1-0)^{K-1}\cdot x+K(K-1)(1-0)^{K-2}\frac{x^2}{2}+O(x^3)$$

$$\left((1-x)^K\right)'=-K(1-x)^{K-1};\quad\left((1-x)^K\right)''=+K(K-1)(1-x)^{K-2}$$

$$\boxed{f(x)=1-Kx+K(K-1)\frac{x^2}{2}+O(\lambda^3)}\quad ⊡$$

- СО ОГЛЕД НА ТОА ШТО $0<\lambda<1$ ВАЖ СЛЕДНАВА АПРОКСИМАЦИЈА: $(1-\lambda)^{KN^2}\le 1$

$$F_{CAF}\doteq c\int_0^1\lambda^{KN^2-K}e^{-\frac{2K\gamma}{\bar{\gamma}}\lambda}d\lambda\qquad t=\frac{2K\gamma}{\bar{\gamma}}\lambda\quad\begin{matrix}\lambda=0\\t=0\end{matrix}\quad\lambda=1\;\; t=\frac{2K\gamma}{\bar{\gamma}}$$

$$\lambda=\frac{\bar{\gamma}}{2K\gamma}t\qquad d\lambda=\frac{\bar{\gamma}}{2K\gamma}dt$$

$$F_{CAF} \doteq c\int_0^{\frac{2K\gamma}{\bar\gamma}} \left(\frac{\bar\gamma}{2K\gamma}\right)^{KN^2-K} \cdot t^{KN^2-K} e^{-t} \frac{\bar\gamma}{2K\gamma}\, dt = c\left(\frac{\bar\gamma}{2K\gamma}\right)^{KN^2-K+1} \int_0^{\frac{2K\gamma}{\bar\gamma}} t^{KN^2-K} e^{-t}\, dt$$

$$F_{CAF} \doteq \frac{2^K (2\gamma)^{KN^2}}{\Gamma(N^2)\Gamma^K(N^2+1)\,\bar\gamma^{KN^2}} \cdot \left(\frac{\bar\gamma}{\cancel{2K}\gamma}\right)^{KN^2-K} \gamma\left(KN^2-K+1, \frac{2K\gamma}{\bar\gamma}\right)$$

$$F_{CAF} \doteq \frac{2^K}{\Gamma(N^2)\Gamma^K(N^2+1)} \left(\frac{\bar\gamma}{2\gamma}\right)^{\cancel{-KN^2}} \cdot \left(\frac{\bar\gamma}{2\gamma}\right)^{\cancel{KN^2}-K} \cdot K^K \gamma\left(KN^2-K+1, \frac{2K\gamma}{\bar\gamma}\right)$$

$$F_{CAF} \doteq \frac{2^K \cdot 2^K}{\Gamma(N^2)\Gamma^K(N^2+1)\,\bar\gamma^K} \cdot \gamma^K \cdot \gamma\left(KN^2-K+1, \frac{2K\gamma}{\bar\gamma}\right)$$

$$Pr\left[\frac{Y_K Z_K}{Y_K+Z_K} \le \frac{2\gamma}{\bar\gamma}(1-\lambda)\right] \doteq \frac{2}{(N^2)!}\left(\frac{2\gamma}{\bar\gamma}\right)^{N^2}(1-\lambda)^{N^2}$$

OVA G DOBRO !!! IZVEDUVANE

$$F_{SCAF}(\gamma) = \frac{1}{\Gamma(N^2)}\left(\frac{2\gamma}{\bar\gamma}\right)^{N^2}\int_0^1 \left[\frac{2}{(N^2)!}\left(\frac{2\gamma}{\bar\gamma}\right)^{N^2}(1-\lambda)^{N^2}\right]^K \cdot \lambda^{N^2-1} e^{-\frac{2\gamma}{\bar\gamma}\lambda}\, d\lambda$$

$$= \frac{1}{\Gamma(N^2)}\left(\frac{2\gamma}{\bar\gamma}\right)^{N^2} \cdot \frac{2^K}{\Gamma^K(N^2+1)}\left(\frac{2\gamma}{\bar\gamma}\right)^{KN^2}\int_0^1 (1-\lambda)^{KN^2}\lambda^{N^2-1}e^{-\frac{2\gamma}{\bar\gamma}\lambda}\, d\lambda$$

$$= \left(\frac{2}{(N^2)!}\right)^K \cdot \left(\frac{2}{\bar\gamma}\right)^{(K+1)N^2} \frac{\gamma^{(K+1)N^2}}{\Gamma(N^2)} \cdot \underbrace{\int_0^1 (1-\lambda)^{KN^2}\lambda^{N^2-1}e^{-\frac{2\gamma}{\bar\gamma}\lambda}\, d\lambda}_{I}$$

($*)

OK

$$I = \int_0^1 \underbrace{(1-\lambda)}_{\le 1}^{KN^2}\lambda^{N^2-1}e^{-\frac{2\gamma}{\bar\gamma}\lambda}\, d\lambda \doteq \int_0^1 \lambda^{N^2-1}e^{-\frac{2\gamma}{\bar\gamma}\lambda}\, d\lambda =$$

$t = \frac{2\gamma}{\bar\gamma}\lambda$

$\lambda = \frac{\bar\gamma}{2\gamma}t$

$d\lambda = \frac{\bar\gamma}{2\gamma}dt$

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$$= \int_0^{\frac{2\gamma}{\bar\gamma}}\left(\frac{\bar\gamma}{2\gamma}\right)^{N^2-1}\cdot t^{N^2-1}\cdot e^{-t}\frac{\bar\gamma}{2\gamma}\, dt = \left(\frac{\bar\gamma}{2\gamma}\right)^{N^2}\underbrace{\int_0^{\frac{2\gamma}{\bar\gamma}} t^{N^2-1}e^{-t}\, dt}_{\gamma\left(N^2, \frac{2\gamma}{\bar\gamma}\right)}$$

(□Δ)

$$\frac{\gamma\left(N^2, \frac{2\gamma}{\bar\gamma}\right)}{\left(\frac{\bar\gamma}{2\gamma}\right)^{N^2}}$$

$$F_{SCAF}(\gamma) \doteq \left(\frac{2}{(N^2)!}\right)^K\cdot\left(\frac{2}{\bar\gamma}\right)^{(K+1)N^2}\gamma^{(K+1)N^2}\, \frac{\gamma\left(N^2, \frac{2\gamma}{\bar\gamma}\right)}{\Gamma(N^2)}\left(\frac{\bar\gamma}{2\gamma}\right)^{N^2}.$$

Ⓘ ⇒

$$\int_0^1 KN^2\cdot\lambda^{N^2}e^{-\frac{2\gamma}{\bar\gamma}\cdot\lambda}\, d\lambda = KN^2\cdot\left(\frac{\bar\gamma}{2\gamma}\right)^{N^2+1}\gamma\left(N^2+1, \frac{2\gamma}{\bar\gamma}\right)$$

VTOROT CLEN OD I

$$F_{SCAF}(\gamma) \doteq \left(\frac{2}{(N^2)!}\right)^K\left(\frac{2}{\cancel{\bar\gamma}}\right)^{(K+1)N^2}\frac{\cancel{\gamma^{(K+1)N^2}}}{\Gamma(N^2)}\left(\frac{\bar\gamma}{2\gamma}\right)^{N^2}\left[\gamma\left(N^2, \frac{2\gamma}{\bar\gamma}\right) - KN^2\frac{\bar\gamma}{2\gamma}\gamma\left(N^2+1, \frac{2\gamma}{\bar\gamma}\right)\right]$$

- MOŽEBI TREBA DA PROBAM USTE EDNAŠ SO SUMA

$$F_{SCAF}(\gamma) = \left(\frac{2}{N^2!}\right)^K \left(\frac{2}{\bar\gamma}\right)^{(K+1)N^2} \frac{\gamma^{(K+1)N^2}}{\Gamma(N^2)} \left(\frac{\bar\gamma}{2\gamma}\right)^{N^2} \left[\gamma\left(N^2, \frac{2\gamma}{\bar\gamma}\right) - KN^2 \frac{\bar\gamma}{2\gamma}\gamma\left(N^2+1, \frac{2\gamma}{\bar\gamma}\right)\right]$$

$$= \left(\frac{2}{(N^2)!}\right)^K \left(\frac{2}{\bar\gamma}\right)^{(K+1)N^2} \frac{\gamma^{K\cdot N^2}}{\Gamma(N^2)} \left[\gamma\left(N^2, \frac{2\gamma}{\bar\gamma}\right) - KN^2 \frac{\bar\gamma}{2\gamma}\gamma\left(N^2+1, \frac{2\gamma}{\bar\gamma}\right)\right]$$

$$\varphi = \gamma^{KN^2}\gamma\left(N^2, \frac{2\gamma}{\bar\gamma}\right) - KN^2\frac{\bar\gamma}{2\cancel{\gamma}} \cdot \gamma^{\overset{KN^2-1}{KN^2}}\gamma\left(N^2+1, \frac{2\gamma}{\bar\gamma}\right)$$

$$\varphi = \gamma^{KN^2}\gamma\left(N^2, \frac{2\gamma}{\bar\gamma}\right) - \frac{\bar\gamma KN^2}{2}\cdot \gamma^{KN^2-1}\gamma\left(N^2+1, \frac{2\gamma}{\bar\gamma}\right)$$

$$\frac{d\varphi}{d\gamma} = KN^2\gamma^{KN^2-1}\gamma\left(N^2, \frac{2\gamma}{\bar\gamma}\right) + \gamma^{KN^2}\left(\frac{2\gamma}{\bar\gamma}\right)^{N^2-1} \cdot e^{-\frac{2\gamma}{\bar\gamma}} \cdot \frac{2}{\bar\gamma} - \frac{\bar\gamma KN^2}{2}\cdot(KN^2-1)\gamma^{KN^2-2} \cdot$$

$$\cdot\gamma\left(N^2+1, \frac{2\gamma}{\bar\gamma}\right) - \frac{\bar\gamma KN^2}{2}\gamma^{KN^2-1}\left(\frac{2\gamma}{\bar\gamma}\right)^{N^2}\cdot e^{-\frac{2\gamma}{\bar\gamma}}\cdot\frac{2}{\bar\gamma} =$$

$$= KN^2\gamma^{KN^2-1}\left[\gamma\left(N^2, \frac{2\gamma}{\bar\gamma}\right) - \frac{\bar\gamma}{2}(KN^2-1)\gamma^{-1}\cdot\gamma\left(N^2+1, \frac{2\gamma}{\bar\gamma}\right)\right] +$$

$$+ e^{-\frac{2\gamma}{\bar\gamma}}\cdot\frac{2}{\bar\gamma}\underbrace{\left[\gamma^{KN^2}\left(\frac{2\gamma}{\bar\gamma}\right)^{N^2-1} - \frac{\bar\gamma KN^2}{2}\gamma^{KN^2-1}\left(\frac{2\gamma}{\bar\gamma}\right)^{N^2}\right]}_{*}$$

$$* = \gamma^{KN^2}\left(\frac{2\gamma}{\bar\gamma}\right)^{N^2-1} - \frac{KN^2}{2}\,\frac{2^{N^2}\cdot\gamma^{KN^2+N^2-1}}{\bar\gamma^{N^2-1}} = \frac{2^{N^2-1}\cdot\gamma^{KN^2+N^2-1}}{\bar\gamma^{N^2-1}} - \frac{KN^2 2^{N^2-1}\gamma^{KN^2+N^2-1}}{\bar\gamma^{N^2-1}}$$

$$= \frac{2^{N^2-1}\gamma^{KN^2+N^2-1}}{\bar\gamma^{N^2-1}}(1-KN^2)$$

$$f_{CAF}(\gamma) = \left(\frac{2}{(N^2)!}\right)^K\left(\frac{2}{\bar\gamma}\right)^{(K+1)N^2}\left\{\frac{\gamma^{K\cdot N^2-1}\cdot KN^2}{\Gamma(N^2)}\left[\gamma\left(N^2, \frac{2\gamma}{\bar\gamma}\right) - \frac{\bar\gamma}{2}\frac{(KN^2-1)}{\gamma}\gamma\left(N^2+1, \frac{2\gamma}{\bar\gamma}\right)\right]\right.$$

$$\left. + \frac{\cancel{2}\cdot e^{-\frac{2\gamma}{\bar\gamma}}\cdot 2^{N^2-\cancel{1}}\gamma^{KN^2+N^2-1}}{\cancel{\bar\gamma}\,\Gamma(N^2)\quad\bar\gamma^{N^2-\cancel{1}}}(1-KN^2)\right\}$$

$$\boxed{f_{CAF}(\gamma) = \left(\frac{2}{(N^2)!}\right)^K\left(\frac{2}{\bar\gamma}\right)^{(K+1)N^2}\frac{\gamma^{KN^2-1}}{\Gamma(N^2)}\left\{KN^2\left[\gamma\left(N^2, \frac{2\gamma}{\bar\gamma}\right) + \frac{\bar\gamma(1-KN^2)}{2\gamma}\gamma\left(N^2+1, \frac{2\gamma}{\bar\gamma}\right)\right] + \overset{\gamma^{N^2}\cdot(1-KN^2)}{\frac{2^{N^2}e^{-\frac{2\gamma}{\bar\gamma}}}{\bar\gamma^{N^2}}}\right\}}$$

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$$F_{SCAF}(\gamma) = \left(\frac{2}{(N^2)!}\right)^K\left(\frac{2}{\bar\gamma}\right)^{(K+1)N^2}\frac{\gamma^{(K+1)N^2}}{\Gamma(N^2)}\int_0^1(1-\lambda)^{KN^2}\lambda^{N^2-1}e^{-\frac{2\gamma}{\bar\gamma}\lambda}\,d\lambda$$

$$(1-\lambda)^{KN^2} = \sum_{i=0}^{KN^2}\binom{KN^2}{i}(-1)^i\cdot\lambda^i$$

$$\boxed{C = \left(\frac{2}{(N^2)!}\right)^K\cdot\left(\frac{2}{\bar\gamma}\right)^{(K+1)N^2}}$$

$$F_{CAF}(\gamma)=\underbrace{\left(\frac{2}{(N^2)!}\right)^K\left(\frac{2}{\bar\gamma}\right)^{(K+1)N^2}}_{C}\frac{\gamma^{(K+1)N^2}}{\Gamma(N^2)}\int_0^1\sum_{i=0}^{KN^2}\binom{KN^2}{i}(-1)^i\lambda^i\lambda^{N^2-1}e^{-\frac{2\gamma}{\bar\gamma}\lambda}d\lambda$$

$$=C\frac{\gamma^{(K+1)N^2}}{\Gamma(N^2)}\sum_{i=0}^{KN^2}\binom{KN^2}{i}(-1)^i\int_0^1\lambda^{N^2+i-1}e^{-\frac{2\gamma}{\bar\gamma}\lambda}d\lambda$$

$t=\frac{2\gamma}{\bar\gamma}\lambda \quad d\lambda=\frac{\bar\gamma}{2\gamma}dt$

$\lambda=0 \quad t=0$

$\lambda=1 \quad t=\frac{2\gamma}{\bar\gamma}$

$$=C\frac{\gamma^{(K+1)N^2}}{\Gamma(N^2)}\sum_{i=0}^{KN^2}\binom{KN^2}{i}(-1)^i\int_0^{\frac{2\gamma}{\bar\gamma}}\left(\frac{\bar\gamma}{2\gamma}\right)^{N^2+i-1}t^{N^2+i-1}e^{-t}\left(\frac{\bar\gamma}{2\gamma}\right)dt$$

$$=C\frac{\gamma^{(K+1)N^2}}{\Gamma(N^2)}\sum_{i=0}^{KN^2}\binom{KN^2}{i}(-1)^i\left(\frac{\bar\gamma}{2\gamma}\right)^{N^2+i}\cdot\gamma\left(N^2+i,\frac{2\gamma}{\bar\gamma}\right)=$$

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$$=\frac{C}{\Gamma(N^2)}\sum_{i=0}^{KN^2}\binom{KN^2}{i}(-1)^i\left(\frac{\bar\gamma}{2}\right)^{N^2+i}\gamma^{(K+1)N^2-N^2-i}\gamma\left(N^2+i,\frac{2\gamma}{\bar\gamma}\right)$$

$$F_{CAF}(\gamma)=\frac{C}{\Gamma(N^2)}\sum_{i=0}^{KN^2}\binom{KN^2}{i}(-1)^i\left(\frac{\bar\gamma}{2}\right)^{N^2+i}\gamma^{KN^2-i}\gamma\left(N^2+i,\frac{2\gamma}{\bar\gamma}\right)$$

$$f_{CAF}(\gamma)=\frac{dF_{CAF}}{d\gamma}=\frac{C}{\Gamma(N^2)}\sum_{i=0}^{KN^2}\binom{KN^2}{i}(-1)^i\left(\frac{\bar\gamma}{2}\right)^{N^2+i}\Big[(KN^2-i)\gamma^{KN^2-i-1}\gamma\left(N^2+i,\frac{2\gamma}{\bar\gamma}\right)$$
$$+\gamma^{KN^2-i}\left(\frac{2\gamma}{\bar\gamma}\right)^{N^2+i-1}\cdot e^{-\frac{2\gamma}{\bar\gamma}}\cdot\frac{2}{\bar\gamma}\Big]$$

$$f_{CAF}(\gamma)=\frac{C}{\Gamma(N^2)}\sum_{i=0}^{KN^2}\binom{KN^2}{i}(-1)^i\frac{\bar\gamma^{N^2+i}(KN^2-i)\gamma^{KN^2-i-1}}{2^{N^2+i}}\gamma\left(N^2+i,\frac{2\gamma}{\bar\gamma}\right)+$$
$$+\underbrace{\frac{C}{\Gamma(N^2)}\sum_{i=0}^{KN^2}\binom{KN^2}{i}(-1)^i\frac{\cancel{\bar\gamma^{N^2+i-1}}\gamma^{KN^2-i}\cancel{2^{N^2+i}}\cdot\gamma^{N^2+i-1}}{\cancel{2^{N^2+i}}\cancel{\bar\gamma^{N^2+i-1}}}}_{S2}=S_1+S_2$$

MAPLE

$$S2=\frac{C}{\Gamma(N^2)}\sum_{i=0}^{KN^2}\binom{KN^2}{i}(-1)^i\cdot\gamma^{KN^2+N^2-1}=\frac{C}{\Gamma(N^2)}\gamma^{(K+1)N^2-1}\sum_{i=0}^{KN^2}(-1)^i\binom{KN^2}{i}\rightarrow 0$$

$K\cdot N^2$ - even

$$\sum_{i=0}^{4}(-1)^i=1-1+1-1+1=1$$

$$\cancel{S2=\left(\frac{2}{(N^2)!}\right)^K\left(\frac{2}{\bar\gamma}\right)^{(K+1)N^2}\frac{\gamma^{(K+1)N^2-1}}{\Gamma(N^2)}}$$

$$f_{CAF}(\gamma)=\frac{C}{\Gamma(N^2)}\sum_{i=0}^{KN^2}\binom{KN^2}{i}(-1)^i\left(\frac{\bar\gamma}{2}\right)^{N^2+i}(KN^2-i)\gamma^{KN^2-i-1}\gamma\left(N^2+i,\frac{2\gamma}{\bar\gamma}\right)$$

DOBRO PROVJERAVANE!

$\gamma\gg 1 \quad \gamma\left(N^2+i,\frac{2\gamma}{\bar\gamma}\right)=\Gamma(N^2+i)$

$$f_{CAF}(\gamma)\doteq\frac{C}{\Gamma(N^2)}\sum_{i=0}^{KN^2}\binom{KN^2}{i}(-1)^i\left(\frac{\bar\gamma}{2}\right)^{N^2+i}(KN^2-i)\gamma^{KN^2-i-1}\cdot\Gamma(N^2+i)$$

$$=\frac{C}{\Gamma(N^2)}\left(\frac{\bar\gamma}{2}\right)^{N^2}\gamma^{KN^2-1}\sum_{i=0}^{KN^2}\binom{KN^2}{i}\left(\frac{-\bar\gamma}{2\gamma}\right)^i(KN^2-i)\gamma\left(N^2+i,\frac{2\gamma}{\bar\gamma}\right)$$

$$f_{CAF} = \frac{c}{\Gamma(N^2)} \left(\frac{\bar\gamma}{2}\right)^{N^2} \gamma^{KN^2-1} \sum_{i=0}^{KN^2} \underbrace{\binom{KN^2}{i} \left(-\frac{\bar\gamma}{2\gamma}\right)^{i} (KN^2-i)\, \gamma\left(N^2+i, \frac{2\gamma}{\bar\gamma}\right)}_{(*)}$$

DOBAR IZRAZ!

$$\gamma\left(N^2+i, \frac{2\gamma}{\bar\gamma}\right) \doteq \Gamma(N^2+i)$$

$$\binom{KN^2}{i} = \frac{(KN^2)!}{i!\,(KN^2-i)!}$$

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$$f_{CAF} = \frac{c}{\Gamma(N^2)} \left(\frac{\bar\gamma}{2}\right)^{N^2} \gamma^{KN^2-1} (KN^2)! \sum_{i=0}^{KN^2} \frac{(-1)^i}{i!\cdot(KN^2-i-1)!} \left(\frac{\bar\gamma}{2\gamma}\right)^{i} \cdot \gamma\left(N^2+i, \frac{2\gamma}{\bar\gamma}\right)$$

$$\boxed{\gamma(n+1,x) = n!\left[1 - e^{-x}\sum_{m=0}^{n} \frac{x^m}{m!}\right]} \qquad \boxed{\Gamma(n+1,x) = n!\, e^{-x} \sum_{m=0}^{n} \frac{x^m}{m!}}$$

$$\boxed{\int_0^1 \lambda^{\alpha-1} e^{-b\lambda}\, d = b^{-\alpha}\cdot\gamma(\alpha, b)}$$

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GRADSHTEIN 8.352.1-2

POVTORI OD POSLEDNIOT IZRAZ NA STR. 105 PA DO OVDE!!! OK!!!

$$\frac{(-1)^i}{i!\,(KN^2-i-1)!}\cdot\Gamma(N^2+i) = \frac{(-1)^i}{i!\,(KN^2-i-1)!}\cdot(N^2+i-1)! =$$

$$(z)_n = z(z+1)(z+2)\cdots(z+n-1) = \frac{\Gamma(z+n)}{\Gamma(z)}$$

$$KN^2-i-1-(N^2+i-1) = KN^2-i\cancel{-1}-N^2-i\cancel{+1} \qquad \underline{\underline{(K-1)N^2-2i}}$$

$$= \frac{(-1)^i}{i!\,\Gamma(KN^2-i)}\,\Gamma(N^2+i) = \frac{(-1)^i}{i!\,\frac{\Gamma(\overbrace{KN^2-i}^{z+n})}{\Gamma(\underbrace{N^2+i}_{z})}}$$

$$z+n-z = n \qquad \underbrace{KN^2-i}_{z+n} - N^2 - i = \cancel{KN^2}\,\underline{\underline{(K-1)N^2-2i}}$$

$$= \frac{(-i)^i}{i!\,\cancel{(K-1)N^2-2i}_{\,N^2+i}\,(N^2+i)_{(K-1)N^2-2i}}$$

- ABRAMOWITZ : ASYMPTOTIC EXPANSION OF INCOMPLETE GAM+

$$\boxed{\gamma(a,z) \sim \sum_{n=0}^{\infty} \frac{(-1)^n z^{a+n}}{(a+n)\cdot n!}}$$

MMV

$$\gamma\left(N^2+i, \frac{2\gamma}{\bar\gamma}\right) \doteq \sum_{n=0}^{\infty} \frac{(-1)^n \left(\frac{2\gamma}{\bar\gamma}\right)^{N^2+i+n}}{(N^2+i+n)\,n!}$$

$$\boxed{\gamma(a,x) = \Gamma(a) - \Gamma(a,x)}$$ ABRAMOWITZ 6.5.2

$$\Gamma(n+1,x) = \Gamma(n+1)\, e^{-x} \sum_{m=0}^{n} \frac{x^m}{m!}$$ $N = n+1$

$$\Gamma(N,x) = \Gamma(N) e^{-x} \sum_{m=0}^{N-1} \frac{x^m}{m!} \qquad \gamma(N,x) = \Gamma(N)\left[1 - e^{-x} \sum_{m=0}^{N-1} \frac{x^m}{m!}\right]$$

$$\boxed{\gamma(N,x) = \Gamma(N) - \Gamma(N,x)}$$ (#)

VRZ OSNOV NA IZRAZITE OD GRADSHTEYN (1.9106) GO DOBIVAM GORNIOT IZRAZ OD ABRAMOWITZ

$$f_{CAF} = \underbrace{\frac{c}{\Gamma(N^2)}\left(\frac{\bar{\gamma}}{2}\right)^{N^2} \gamma^{KN^2-1} (KN^2)!}_{C1} \sum_{i=0}^{KN^2} \frac{(-1)^i}{i!\cdot(KN^2-i-1)!}\left(\frac{\gamma}{2\bar{\gamma}}\right)^i \sum_{j=0}^{\infty} \frac{(-1)^j}{(N^2+i+j)\, j!}\left(\frac{2\gamma}{\bar{\gamma}}\right)^{N^2+i}$$

$$j=0 \rightarrow \frac{1}{(N^2+i)}\left(\frac{2\gamma}{\bar{\gamma}}\right)^{N^2+i}$$

$$= C1 \cdot \sum_{i=0}^{KN^2} \frac{(-1)^i}{i!\,(KN^2-i-1)!} \cancel{\left(\frac{\gamma}{2\bar{\gamma}}\right)^i} \cdot \frac{1}{(N^2+i)} \left(\frac{2\gamma}{\bar{\gamma}}\right)^{N^2} \cdot \cancel{\left(\frac{2\gamma}{\bar{\gamma}}\right)^i} \quad \boxed{\text{ZA } j=0}$$

[illegible]

$$f_{CAF} = \frac{c}{\Gamma(N^2)}\left(\frac{\bar{\gamma}}{2}\right)^{N^2} \gamma^{KN^2-1} (KN^2)! \sum_{i=0}^{KN^2} \frac{(-1)^i}{i!\,(KN^2-i-1)!} \cancel{\left(\frac{\gamma}{2\bar{\gamma}}\right)^i} \left(\frac{2\gamma}{\bar{\gamma}}\right)^{N^2+i} \sum_{j=0}^{\infty} \frac{(-1)^j}{(N^2+i+j)}\left(\frac{2\gamma}{\bar{\gamma}}\right)$$

$$f_{CAF} = \frac{c}{\Gamma(N^2)} \gamma^{(K+1)N^2-1} (KN^2)! \sum_{i=0}^{KN^2} \frac{(-1)^i}{i!(KN^2-i-1)!} \sum_{j=0}^{\infty} \frac{(-1)^i}{(N^2+i+j)}\left(\frac{2\gamma}{\bar{\gamma}}\right)^j$$

$$f_{CAF} = \frac{c\,\gamma^{(K+1)N^2-1}}{\Gamma(N^2)} (KN^2)! \sum_{i=0}^{KN^2}\sum_{j=0}^{\infty} \frac{(-1)^{i+j}}{i!\,(KN^2-i-1)!\,(N^2+i+j)}\left(\frac{2\gamma}{\bar{\gamma}}\right)^j$$

$$\rightarrow \frac{\Gamma(KN^2+1)}{\Gamma(KN^2+N^2)}$$ (??)

- MAPLE DEFINICIJA NA [illegible] INCOMPLETE GAMMA

$$\Gamma(a,z) = \Gamma(a) - \frac{z^a\, {}_1F_1(a, 1+a, -z)}{a}$$ ← (#)

IZLEGUVA DEKA:

$$\boxed{\gamma(a,z) = \frac{z^a}{a}\, {}_1F_1(a, 1+a, -z)}$$

$$f_{CAF} = \frac{C}{\Gamma(N^2)}\left(\frac{\gamma}{2}\right)^{N^2}\gamma^{KN^2-1}\sum_{i=0}^{KN^2}\binom{KN^2}{i}(-1)^i\left(\frac{\gamma}{2\gamma}\right)^i\left(KN^2-i\right)\,\gamma\left(N^2+i,\frac{2\gamma}{\gamma}\right)$$

MAPLE & Abramowitz 6.5.12

$$\boxed{\gamma(a,z) = \frac{z^a}{a}\,\overset{\boxed{\text{KUMMER}}}{1F1}(a,1+a,-z)} \longrightarrow$$

KUMMER, HYPERGEOM

$$\gamma\left(N^2+i,\frac{2\gamma}{\gamma}\right) = \left(\frac{2\gamma}{\gamma}\right)^{N^2+i}\cdot\frac{1}{N^2+i}\,1F1\left(N^2+i,N^2+i+1,-\frac{2\gamma}{\gamma}\right)$$

$$f_{CAF} = \frac{C}{\Gamma(N^2)}\left(\frac{\gamma}{2}\right)^{N^2}\gamma^{KN^2-1}\sum_{i=0}^{KN^2}\binom{KN^2}{i}(-1)^i\cancel{\left(\frac{\gamma}{2\gamma}\right)^i}\left(KN^2-i\right)\cdot\left(\frac{2\gamma}{\gamma}\right)^{N^2+\cancel{i}}\frac{1}{N^2+i}\,1F1(\cdots)$$

$$f_{CAF} = \frac{C}{\Gamma(N^2)}\,\gamma^{(K+1)N^2-1}\sum_{i=0}^{KN^2}\frac{(KN^2)!}{i!\cdot(KN^2-i-1)!\cdot(N^2+i)}\,1F1\left(N^2+i,N^2+i+1,-\frac{2\gamma}{\gamma}\right)$$

PP106 ⑧ $\Rightarrow$

$$f_{CAF}(\gamma) = \frac{C}{\Gamma(N^2)}\left(\frac{\gamma}{2}\right)^{N^2}\gamma^{KN^2-1}(KN^2)!\sum_{i=0}^{KN^2}\frac{(-1)^i}{i!\,(KN^2-i-1)!}\left(\frac{\gamma}{2\gamma}\right)^i\left(N^2+i-1\right)!$$

MAPLE

$$= \frac{\overset{\text{pp.104}}{\textcircled{C}}}{\cancel{\Gamma(N^2)}}\left(\frac{\gamma}{2}\right)^{N^2}\gamma^{KN^2-1}\cancel{(KN^2)!}^{\,KN^2}\cdot\frac{\cancel{\Gamma(N^2)}}{\cancel{\Gamma(KN^2)}}\cdot\text{hypergeom}\left([N^2,-KN^2+1],[\,],\frac{\gamma}{2\gamma}\right)$$

GAUSS HYPERGEOM

$$f_{CAF}(\gamma) \doteq \left(\frac{2}{(N^2)!}\right)^K\left(\frac{2}{\gamma}\right)^{(K+1)N^2}\gamma^{KN^2-1}\cdot KN^2\cdot 2F1\left([N^2,-KN^2+1],[\,],\frac{\gamma}{\cancel{2\gamma}}\right)$$

$$\boxed{\Gamma(a,z) = e^{-x}\cdot U(1-a,1-a,x)}$$

CONFLUENT
T.E KUMMER U

$$M(a,c,z) = \lim_{b\to\infty}{}_2F_1(a,b;c;b^{-1}z)$$

!@

$${}_2F_1(a,b;c;z) = \sum_{n=0}^{\infty}\frac{(a)_n(b)_n}{(c)_n}\frac{z^n}{n!} = \frac{\Gamma(c)}{\Gamma(a)\Gamma(b)}\sum_{n=0}^{\infty}\frac{\Gamma(a+n)\Gamma(b+n)}{\Gamma(c+n)}\frac{z^n}{n!}$$

$$(a)_n = a\cdot(a+1)(a+2)\cdots(a+n-1) = \frac{(a+n-1)!}{(a-1)!} = \frac{\Gamma(a+n)}{\Gamma(a)}$$

$$(b)_n = b\cdot(b+1)(b+2)\dots(b+n-1) = \frac{(b+n-1)!}{(b-1)!} = \frac{\Gamma(b+n)}{\Gamma(b)}$$

$$\sum_{n=0}^{KN^2}\frac{(-1)^n(N^2+n-1)!\left(\frac{\gamma}{2\gamma}\right)^n}{n!\,(KN^2-n-1)!} = \sum_{n=0}^{KN^2}\frac{(-1)^n\,\Gamma(N^2+n)}{\Gamma(KN^2-n)}\cdot\frac{1}{n!}\left(\frac{\gamma}{2\gamma}\right)^n =$$

$$= \sum_{n=0}^{KN^2}\frac{(-1)^n\,\Gamma(-KN^2+1+n)\cdot\Gamma(N^2+n)}{\Gamma(n+1-KN^2)\cdot\Gamma(-(n-KN^2))}\cdot\frac{1}{n!}\left(\frac{\gamma}{2\gamma}\right)^n$$

$\boxed{\text{MALE}}$

$$\circledast = \sum_{n=0}^{\infty} \frac{\Gamma(M+n)\cdot\Gamma(N+n)}{\Gamma(-1+n)}\cdot\frac{x^n}{n!} = \frac{\Gamma(M+2)\Gamma(N+2)}{2}\, x^2\, {}_2F_1\left([N+2, M+2],[3];\right.$$

$$\Gamma(-1+n) = \Gamma(n-1)$$

- $\boxed{m = n-1}$ $\quad n=0 \quad m = 1$ $\quad n=\infty \quad m=\infty$

$n = m+1$

$$\sum_{m=1}^{\infty} \frac{\Gamma(M+1+m)\Gamma(N+1+m)}{\Gamma(m)} \frac{x^{m+1}}{(m+1)!}$$

$$= x \sum_{m=1}^{\infty} \frac{\Gamma(M+1+m)\Gamma(N+1+m)}{\underbrace{\Gamma(m)}_{(m-1)!}} \frac{x^{m}}{m!\,(m+1)} = x \sum_{m=1}^{\infty} \frac{\Gamma(M+1+m)\Gamma(N+1+m)}{\underbrace{\Gamma(1+m)}_{m!}} \frac{x^m}{m!}$$

[redacted]

NE MOŽE DA SE SPOJAT 1A ZATOA ZEMI $m=n-2$

$\emptyset$

$$\lim_{m\to -1^+} \frac{\Gamma(M+1+m)\Gamma(N+1+m)}{\Gamma(1+m)\cdot\Gamma(m-1)}\cdot\frac{1}{x} = \frac{\Gamma(M)\cdot\Gamma(N)}{\Gamma(0)}\cdot\frac{1}{x}\cdot\lim_{m\to -1^+}\frac{1}{\Gamma(m-1)}$$

$$\circledast = x \sum_{m=0}^{\infty} \frac{\Gamma(\overset{a}{M+1+m})\Gamma(\overset{e}{N+1+m})}{\Gamma(\underset{c}{1+m})} \frac{x^m}{m!} = x\cdot\frac{\Gamma(M+1)\Gamma(N+1)}{\Gamma(1)}\cdot {}_2F_1(M+1; N+1; 1; x$$

$$(-1)^n = \left(e^{-j\pi}\right)^n = \underline{\underline{e^{-jn\pi}}} = \cos(n\pi) - j\cancel{\sin(n\pi)}^{\to 0}$$

- $\boxed{m = n-2}$ $\quad \underline{n = m+2} \quad n=0 \quad \boxed{m=-2}$

$$\circledast = x^2 \sum_{m=-2}^{\infty} \frac{\Gamma(M+2+m)\Gamma(N+2+m)}{\Gamma(-1+2+m)} \frac{x^m}{(m+2)!} = x^2 \sum_{m=-2}^{\infty} \frac{\Gamma(M+2+m)\Gamma(N+2+m)}{\underbrace{\Gamma(1+m)\cdot(m+2)(m+1)}_{(m+2)!}} \frac{x^m}{m!}$$

$$= x^2 \sum_{m=-2}^{\infty} \frac{\Gamma(M+2+m)\Gamma(N+2+m)}{\Gamma(3+m)} \frac{x^m}{m!}$$

} ČLENOVITE ZA $m=-2$; $m=-1$; se 0 ZATOA ŠTO $m! = 0$ т.е. $\lim_{m\to -2} (\Gamma(m+1))$

⇓ KORISTI $\circled{!e}$

$$\circledast = x^2 \frac{\Gamma(M+2)\cdot\Gamma(N+2)}{\Gamma(3)} \cdot {}_2F_1(M+2, N+2; 3; x)$$

DOKAŽANO!!!

— ZNAČI DEFINITIVNO

$$\boxed{\sum_{n=0}^{\infty} \frac{\Gamma(M+n)\Gamma(N+n)}{\Gamma(-1+n)} \frac{x^n}{n!} = \frac{\Gamma(M+2)\Gamma(N+2)}{2 \leftarrow \Gamma(3)} x^2\, {}_2F_1(M+2, N+2; 3; x)}$$

$$f_{CAF}=\left(\frac{c}{\Gamma(N^2)}\right)\left(\frac{\bar\gamma}{2}\right)^{N^2}\bar\gamma^{-KN^2-1}\underbrace{\sum_{i=0}^{KN^2}\binom{KN^2}{i}(-1)^i\left(\frac{\bar\gamma}{2\bar\gamma}\right)^i(KN^2-i)\,\gamma\left(N^2+i,\frac{2r}{\bar\gamma}\right)}_{①}$$

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5 m² 20cm

$$① = \left|\begin{matrix}K=1\\N=2\end{matrix}\right| = \sum_{i=0}^{4}\binom{4}{i}(-1)^i\left(\frac{\bar\gamma}{2\bar\gamma}\right)^i(4-i)\,\gamma\left(4+i,\frac{2r}{\bar\gamma}\right)=$$

$$=\sum_{i=0}^{4}\frac{4!\,(-1)^i}{i!\,(4-i-1)!}\left(\frac{\bar\gamma}{2\bar\gamma}\right)^i\gamma\left(4+i,\frac{2r}{\bar\gamma}\right)=24\sum_{i=0}^{4}\frac{(-1)^i\gamma\left(4+i,\frac{2r}{\bar\gamma}\right)}{i!\,(3-i)!}\underbrace{\left(\frac{\bar\gamma}{2\bar\gamma}\right)^i}_{x}=$$

$$=24\left(\frac{\gamma(4,x)}{6}-\frac{\gamma(5,x)}{2}x+\frac{\gamma(6,x)x^2}{2}\ [illegible]\ \frac{\gamma(7,x)x^3}{6}+\frac{\gamma(8,x)x^4}{24}\right)$$

- GAMMA INCOMPLETE RECURSION FORMULA:

$$\boxed{\gamma(a+1,x)=a\gamma(a,x)-x e^{-x}}$$

≣

$$\gamma(5,x)=\cancel{5}\,\gamma(4,x)-x^4\cdot e^{-x}$$
$$\gamma(6,x)=5\gamma(5,x)-x^5\cdot e^{-x}$$
$$\gamma(7,x)=6\gamma(6,x)-x^6\cdot e^{-x}$$

Ana Sofronieva 070227111

$$+\frac{\gamma(6,x)}{2}x^2-\frac{x^3}{6}\left(6\gamma(6,x)-x^6e^{-x}\right)$$

$$①=24\left[\frac{\gamma(4,x)}{6}-\frac{x}{2}\left(4\gamma(4,x)-x^4e^{-x}\right)+[illegible]+\frac{\gamma(8,x)x^4}{24}\right]$$

$$=24\left[\gamma(4,x)\left(\frac{1}{6}-2x\right)+\frac{x^5}{2}e^{-x}+\gamma(6,x)\left(\frac{x^2}{2}-x^3\right)+\frac{\gamma(8,x)}{24}x^4+\frac{x^5}{2}e^{-x}+\frac{x^9}{6}e^{-x}\right]$$

$$\gamma(a,z)\sim\sum_{n=0}^{\infty}\frac{(-1)^n z^{a+n}}{(a+n)\,n!}=\frac{1}{a}\cdot z^a-\frac{z^{a+1}}{(a+1)}+\frac{z^{a+2}}{(a+2)2}-\dots$$

ASYMPTOTIC APROXIMATION

$$\gamma(n+1,x)=n!\left[1-e^{-x}\left(\sum_{m=0}^{n}\frac{x^n}{n!}\right)\right]$$

- DOSTA DOBRA APROXIMACIJA JE DOBIVA AKO SE
- ZEME SAMO PRVIOT CLEN T.E:

$$\boxed{\gamma(a,z)\sim\frac{z^a}{a}}$$

MMV

ASYMPTOTICALLY EQUAL

VIDI MAKE MultiHopMIMO.mw (3.11.6)

$$f_{CAF}=\frac{c}{\Gamma(N^2)}\left(\frac{\bar\gamma}{2}\right)^{N^2}\bar\gamma^{-KN^2-1}\sum_{i=0}^{KN^2}\binom{KN^2}{i}(-1)^i\left(\frac{\bar\gamma}{2\bar\gamma}\right)^i(KN^2-i)\cdot\left(\frac{2r}{\bar\gamma}\right)^{N^2+i}\cdot\frac{1}{N^2+i}$$

$$f_{CAF}(\gamma) = \frac{c}{\Gamma(N^2)} \left(\cancel{\frac{\bar{\gamma}}{2}}\right)^{N^2} \gamma^{(\cancel{K}+1)N^2-1} \left(\cancel{\frac{2}{\bar{\gamma}}}\right)^{N^2} \sum_{i=0}^{KN^2} \binom{KN^2}{i} (-1)^i \frac{(KN^2-i)}{N^2+i}$$

$$f_{CAF}(\gamma) = \frac{c}{\Gamma(N^2)} \cdot \gamma^{(K+1)N^2-1} \sum_{i=0}^{KN^2} \frac{(KN^2)! \; (-1)^i}{(KN^2-i)! \cdot i!} \; \frac{KN^2-i}{N^2+i}$$

$$\boxed{f_{CAF}(\gamma) = \frac{c \, (KN^2)!}{\Gamma(N^2)} \cdot \gamma^{(K+1)N^2-1} \sum_{i=0}^{KN^2} \frac{(-1)^i}{(KN^2-i-1)! \; i! \; (N^2+i)}} \quad ①$$

→ NA PP. 110 DOBIV SLIČEN IZRAZ SO KummerM(...)

- SO KONSTENZE NA MAPLE SE DOBIVA (ZA PRESMETKA NA SUMATA VO ①)

$$f_{CAF}(\gamma) = \frac{C \cdot KN^2}{\Gamma(N^2)} \gamma^{(K+1)N^2-1} \frac{\Gamma(N^2+1)}{N^2 \, \Gamma((K+1)N^2)}$$

$$f_{CAF} = \left(\frac{2}{(N^2)!}\right)^K \left(\frac{2}{\bar{\gamma}}\right)^{(K+1)N^2} \cdot \frac{\gamma^{(K+1)N^2-1}}{\Gamma(N^2)} (KN^2)! \; \frac{\Gamma(N^2+1)}{\cancel{N^2} \, \Gamma((K+1)N^2)}$$

DOBRO IZVEDUVANJE!!

$$f_{CAF} = \frac{2^{(K+1)N^2+K} \, (KN^2)! \, \Gamma(N^2+1)}{N^2 \left[(N^2)!\right]^K \bar{\gamma}^{(K+1)N^2} \Gamma(KN^2+N^2) \, \Gamma(N^2)} \gamma^{(K+1)N^2-1}$$

$$(KN^2)! = \Gamma(KN^2+1) \qquad \frac{\Gamma(N^2+1)}{\Gamma(N^2)} = \frac{(N^2)!}{(N^2-1)!} = N^2$$

$$f_{CAF}(\gamma) = \frac{2^{(K+1)N^2+K} \, \Gamma(KN^2+1) \, \overset{\cancel{N^2}}{\cancel{\Gamma(N^2+1)}}}{\cancel{N^2} \left[(N^2)!\right]^K \bar{\gamma}^{(K+1)N^2} \Gamma(KN^2+N^2) \, \cancel{\Gamma(N^2)}} \cdot \gamma^{(K+1)N^2-1} =$$

$$\boxed{f_{CAF}(\gamma) = \frac{2^{(K+1)N^2+K} \; \Gamma(KN^2+1)}{\left[(N^2)!\right]^K \bar{\gamma}^{(K+1)N^2} \; \Gamma(KN^2+N^2)} \cdot \gamma^{(K+1)N^2-1}}$$

DOKAZANO!!!

- SAMO NE MI E JASNO KAKO MAPLE OD SUMATA ① DOAĐA DO IZRAZOT SO $\Gamma(..)$ FUNKCII !? VEROJATNO PRAVI NEKOI KOMBINACII KAKO ŠTO SUM PRAVEL JAS NA PP. 109.

- MOŽEBI TREBA DA SE IMA VO PREDVID OVAA:

$$\sum_{i=0}^{KN^2} (-1)^i \binom{KN^2}{i} = 0$$

$$\sum_{i=0}^{KN^2} \binom{KN^2}{i} (-1)^i \frac{KN^2-i}{N^2+i} = \frac{\Gamma(N^2+1)}{N^2\,\Gamma(N^2(K+1))}\;{}^{*} \quad \Big\}$$ DA SE DOKAŽE!!!

$$\sum_{i=0}^{KN^2} \frac{(KN^2)!\,(-1)^i}{i!\,(KN^2-i)!} \frac{KN^2-i}{N^2+i} = \text{①*}$$

$i = n - KN^2$

$n = KN^2 - i$ $i = 0$ $n = KN^2 \;\|\; i = KN^2$ $n = 0$

$$\sum_{n=0}^{KN^2} \frac{(KN^2)!\,(-1)^{n-KN^2}}{(n-KN^2)!\,n!} \frac{n}{N^2+n-KN^2}$$

$K=1$ $N=2$ $K\cdot N^2 = 1\cdot 4 = 4$

$$ⓔ = \sum_{i=0}^{4} \frac{4!\,(-1)^i}{i!\,(4-i)!} \frac{4-i}{4+i} = \frac{4!\cdot 1}{0!\,4!}\cdot\frac{4}{4} + \frac{4!\,(-1)^1}{1!\cdot 3!}\frac{3}{5}$$

$$+ \frac{4!\,(-1)^2}{2!\,(4-2)!}\frac{2}{6} + \frac{4!\,(-1)^3}{3!\,1!}\cdot\frac{1}{7} + \frac{4!\,(-1)^4}{4!\,0!}\frac{0}{8}$$

$$= 1 - \frac{4!}{2!}\cdot\frac{1}{5} + \frac{4!}{2!\,2!}\frac{1}{3} - \frac{4!}{3!}\frac{1}{7} + 0$$

$$(a)_n = a\cdot(a+1)\cdots(a+n-1) = \frac{(a+n-1)!}{(a-1)!} = \frac{\Gamma(a+n)}{\Gamma(a)}$$

$$\boxed{\Gamma(z) = \int_1^{\infty} e^{-t} t^{z-1}\,dt + \sum_{k=0}^{\infty} \frac{(-1)^k}{k!\,(z+k)} \Rightarrow = \int_0^1 e^{-t} t^{z-1}\,dt}$$

GRADSHTEYN 8.314

$$(KN^2-i)_i = (KN^2-i)\cdot(KN^2-i+1)\cdots KN^2-\cancel{i}+\cancel{i}-1$$
$$= (KN^2-i)\,(KN^2-i+1)\cdots KN^2-1$$

$$(KN^2-i+1)_i = (KN^2-i+1)\cdot(KN^2-i+2)\cdots KN^2\cancel{-i}\cancel{+1}\cancel{+i}-1$$
$$= (KN^2-i+1)\,(KN^2-i+2)\cdots(KN^2) = \frac{(KN^2)!}{(KN^2-i)!}$$

$$\text{①*} = \sum_{i=0}^{KN^2} \frac{(KN^2-i+1)_i\,(-1)^i}{i!} \frac{(KN^2-i)}{N^2+i}$$

$$ⓐ = 1 - \frac{4!}{2!}\frac{1}{5} + \frac{4!}{2!\,2!}\frac{1}{3} - \frac{4!}{3!}\frac{1}{7} + 0 =$$

$$= 1 - \frac{\cancel{1\cdot 2}\cdot 3\cdot 4}{\cancel{1\cdot 2}\cdot 5} + \frac{\cancel{1\cdot 2}\cdot\cancel{3}\cdot 4}{\cancel{1\cdot 2\cdot 1\cdot 2\cdot 3}} - \frac{\cancel{1\cdot 2\cdot 3}\cdot 4}{\cancel{1\cdot 2\cdot 3}\cdot 7} + 0 =$$

$$= 1 - \frac{12}{5} + \frac{4}{2} - \frac{4}{7} = 1 + 2 - \frac{12\cdot 7 + 5\cdot 4}{35} = 1 + 2 - \frac{84+20}{35} = 3\ \frac{104}{35}$$

$$= \underline{\underline{1/35}}$$

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1310 FILES
86 FOLDERS

$$\frac{K\,\Gamma(N^2+1)\,\Gamma(KN^2)}{\Gamma((K+1)N^2)} =$$

$$\Gamma(KN^2) = (KN^2-1)!$$

$$\overbrace{}^{(KN^2)!}$$

$$= \frac{KN^2\,\Gamma(N^2+1)\,\Gamma(KN^2)}{N^2\,\Gamma((K+1)N^2)} = \frac{\Gamma(N^2+1)\,\Gamma(KN^2+1)}{N^2\,\Gamma((K+1)N^2)}$$

$$\sum_{i=0}^{KN^2} \frac{(KN^2)!\,(-1)^i}{i!\,(KN^2-i)}\,\frac{KN^2-i}{N^2+i}$$

$$\sum_{i=0}^{\infty} x^i = \frac{1}{1-x}$$

$$S(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1}\cdot x^{2n+1} \qquad (\ )'$$

$$S'(x) = \sum_{n=0}^{\infty} (-1)^n \cdot x^{2n}$$

[redacted]

MMV

$$\boxed{x^2 = y} \qquad S'(x) = \sum_{n=0}^{\infty} (-1)^n \cdot y^n = \sum_{n=0}^{\infty} (-y)^n = \frac{1}{1-(-y)}$$

[redacted]

$$= \text{[redacted]} \ \frac{1}{1+y} = \frac{1}{1+x^2}$$

$$S'(x) = \frac{1}{1+x^2}$$

$$y = \operatorname{arctg} x \qquad y' = ? \qquad x = \operatorname{tg} y \quad (\ )' \qquad dx = \frac{\cos^2 y + \sin^2 y}{\cos^2 y}\,dy$$

$$dx = (1 + \operatorname{tg}^2 y)\,dy \qquad \frac{dy}{dx} = \frac{1}{1+\operatorname{tg}^2 y} = \frac{1}{1+x^2}$$

$$\Rightarrow \boxed{S(x) = \operatorname{arctg} x + C}$$

$$S = \sum_{n=0}^{\infty} x^n = 1 + \cancel{x} + \cancel{x^2} + \dots - x^n + \dots$$

$$xS = \cancel{x} + \cancel{x^2} + x^3 + \dots x^{n+1} - -$$

$$S(1-x) = 1 \qquad \boxed{S = \frac{1}{1-x}}$$

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e-nabavki

$$S = \sum_{i=0}^{KN^2} \frac{(KN^2)!\,(-1)^i}{i!\,(KN^2-i)!} \frac{KN^2-i}{N^2+i} x^i = \sum_{i=0}^{KN^2} \frac{(KN^2)!}{i!\,(KN^2-i)!} \frac{KN^2-i}{N^2+i} (-x)^i$$

$$S'(x) = -\sum_{i=0}^{KN^2} \frac{(KN^2)!}{i!\,(KN^2-i)!} \frac{KN^2-i}{N^2+i}\, i \cdot (-x)^{i-1} \qquad ?$$

$$\begin{array}{r} 125 \\ 92 \\ \hline 277 \\ \hline 365 \\ \boxed{642} \end{array} \qquad \begin{array}{r} 160 \\ 120 \\ 85 \\ \hline 365 \end{array}$$

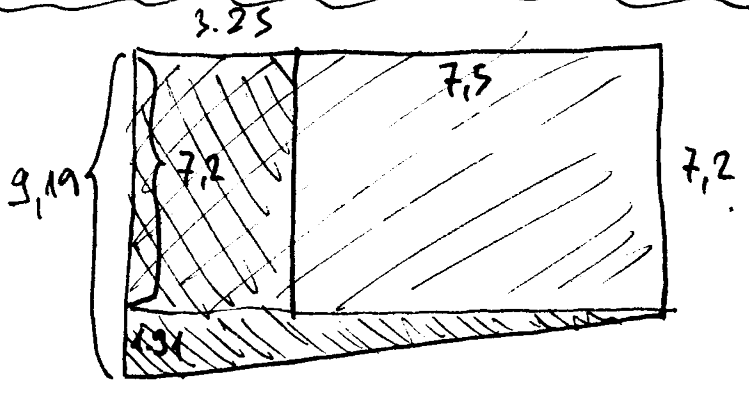
$$\left.\lim_{i \to \infty} \frac{(KN^2)!\,(-1)^i}{i!\,(KN^2-i)!} \frac{KN^2-i}{N^2+i} = 0 \right\}$$ REDOT E KONVERGENTEN.

$$S = (KN^2)! \left[\sum_{i=0}^{KN^2} \frac{(-1)^i\, KN^2}{i!\,(KN^2-i)!\,(N^2+i)} - \sum_{i=0}^{KN^2} \frac{(-1)^i\, i}{i!\,(KN^2-i)!\,(N^2+i)} \right] =$$

$$(KN^2)! \left[KN^2 \sum_{i=0}^{KN^2} \frac{(-1)^i}{i!\,(KN^2-i)!\,(N^2+i)} + \sum_{i=0}^{KN^2} \frac{(-1)^{i+1}}{(i-1)!\,(KN^2-i)!\,(N^2+i)} \right]$$

$j = i-1 \quad i = j+1 \qquad i = 0 \;\; i = KN^2 \qquad j = -1 \;\; j\; KN^2 - 1 \qquad \Gamma(i) = (i-1)! \qquad \Gamma(0) = \text{infinity} = \infty$

$$\sum_{j=-1}^{KN^2-1} \frac{(-1)^j}{j!\,(KN^2-j-1)!\,(N^2+j+1)} = \sum_{j=0}^{KN^2-1} \frac{(-1)^j}{j!\,(KN^2-j-1)!\,(N^2+j+1)}$$



$$\begin{array}{r} 7,5 \\ 3,25 \\ \hline 10,75 \end{array}$$

$10,75 \times 7,2 = 77,4$

$$-\begin{array}{r} 9,19 \\ 7,20 \\ \hline 1,91 \end{array} \qquad \frac{1.91 \cdot 10,75}{2} = 10,26$$

$$\boxed{\begin{array}{r} 77,4 \\ 10,26 \\ \hline 87,66 \end{array}}$$

$$S = \sum_{i=0}^{KN^2} \frac{(KN^2)!\,(-1)^i}{i!\,(KN^2-i)!}\,\frac{KN^2-i}{N^2+i} = \sum_{i=0}^{KN^2} \frac{(KN^2)!\,(-1)^i}{i!\,(KN^2-i-1)!}\,\frac{1}{N^2+i}$$

$$\boxed{\Gamma(z-n) = \frac{(-1)^n\,\Gamma(z)}{(1-z)_n}} \qquad (KN^2-i-1)! = \Gamma(KN^2-i)$$

$$S = \sum_{i=0}^{KN^2} \frac{(KN^2)!\,(-1)^i}{i!\,\Gamma(KN^2-i)\,(N^2+i)} = \sum_{i=0}^{KN^2} \frac{(KN^2)!\,\cancel{(-1)^i}}{i!\,\frac{\cancel{(-1)^i}\,\Gamma(KN^2)}{(1-KN^2)_i}\,(N^2+i)}$$

$$= \sum_{i=0}^{KN^2} \frac{\overset{KN^2}{\cancel{(KN^2)!}}\,(1-KN^2)_i}{i!\,\Gamma(\cancel{KN^2})\,(N^2+i)} = \sum_{i=0}^{KN^2} \frac{KN^2\,(1-KN^2)_i}{i!\,(N^2+i)}$$

$$(1-\overset{-3}{4})_3 = -3(\overset{-3}{1-4}+1)\cdot(-3+2) = \overset{-3}{(-2)}(-1) = 3\cdot 2 = -6$$ OK!!!

$$(a)_n = a(a+1)\cdot(a+2)(a+3)\cdots(a+n-1)$$

$${}_2F_1(a,b;c;z) = \sum_{n=0}^{\infty} \frac{(a)_n(b)_n}{(c)_n}\,\frac{z^n}{n!} = \frac{\Gamma(c)}{\Gamma(a)\Gamma(b)} \sum_{n=0}^{\infty} \frac{\Gamma(a+n)\,\Gamma(b+n)}{\Gamma(c+n)}\,\frac{z^n}{n!}$$

$$(1-KN^2)_i = (1-KN^2)(1-KN^2+1)+\ldots(\cancel{1}-KN^2+i-\cancel{1})$$

$$= \frac{(-KN^2)!\cdot(1-KN^2)_i}{\Gamma(-KN^2+1)} = \frac{(-KN^2+i)!}{\Gamma(1-KN^2)} = \frac{\Gamma(-KN^2+i+1)}{\Gamma(1-KN^2)}$$

$$= \sum_{i=0}^{KN^2} \frac{KN^2\,\Gamma(-KN^2+i+1)}{i!\,(N^2+i)\,\Gamma(1-KN^2)} = \sum_{i=0}^{KN^2} \frac{KN^2\,\Gamma(i-KN^2+1)}{\Gamma(i+1)\,\Gamma(1-KN^2)}$$

$$= \sum_{i=0}^{KN^2} \frac{KN^2\,\Gamma(i+1-KN^2)}{\Gamma(i+1)\,\Gamma(1-KN^2)} = \qquad \left|\; \frac{\Gamma(z-n)}{\Gamma(z)} = \frac{(-1)^n}{(1-z)_n} = \prod_{k=1}^{n} \frac{1}{z-k} \right|$$

$$= \sum_{i=0}^{KN^2} \frac{KN^2\,(-1)^n}{(1-i-1)_{KN^2}\cdot\Gamma(1-KN^2)} = KN^2 \sum_{i=0}^{KN^2} \frac{(-1)^n}{(-i)_{KN^2}\,\Gamma(1-KN^2)}$$

$$\frac{\Gamma(z-n)}{\Gamma(z)} = \frac{(-1)^n}{(1-z)_n} = \prod_{k=1}^{n} \frac{1}{z-k}$$

[redacted]

$$\Gamma(z-n) = (z-n-1)! = 1\cdot 2\cdot 3 \cdots (z-n-1)!$$

$$\Gamma(z) = (z-1)! = 1\cdot 2\cdot 3 \cdots (z-n-1)\cdot \underbrace{(z-n)\cdot(z-n+1)\cdot(z-1)}_{n}$$

$$\frac{\Gamma(z-n)}{\Gamma(z)} = \frac{1}{(z-n)(z-n+1)(z-n+2)\cdots(z-1)} = \prod_{k=1}^{n}\frac{1}{z-k} =$$

$$= \frac{1}{[-(n-z)][-(n-z-1)][-(n-z-2)]\cdots[-(1-z)]} = \frac{(-1)^n}{(1-z)\cdots(1-z+n-1)}$$

$$\Rightarrow \boxed{\frac{\Gamma(z-n)}{\Gamma(z)} = \frac{(-1)^n}{(1-z)_n}}$$

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$$S = \sum_{i=0}^{KN^2} \frac{(-1)^n}{(-i)_{KN^2}\,\Gamma(1-N^2)} = \sum_{i=0}^{KN^2} \frac{(-1)^{n+KN^2}}{(i)_{KN^2}\,\Gamma(1-N^2)}$$

$$\Gamma(1-N^2) = (1-N^2-1)! = (-N^2)! = (-1)^{N^2}\cdot N^2!$$

$$S = \sum_{i=0}^{KN^2} (-1)^{n+KN^2}$$

$$\boxed{\Gamma(-z) = -\frac{\pi \csc(\pi z)}{z\,\Gamma(z)}}$$

$$\Gamma(1-N^2) = -\frac{\pi}{\underbrace{\sin[(N^2-1)\pi]}_{\to 0}(N^2-1)\cdot\Gamma(N^2-1)} = \infty$$

$$\Gamma(-(N^2-1))$$

- DVATA CLANKA OD ZHAO POMINI GI
- TRUDOVITE OD ICCMT DA SE KESAT APROKSIMATIVNO
- VIDI GO CLANAKOT ZA TWO-WAY RELAYING. Y. Han, et-al, Performance Bounds...
- ANALIZATA ZA GOLEM ODNOS SIGNAL ŠUM
- NE E IDENTITATA DA IMA SUMA
  - SER-OT ZA CLANAKOT OD YANG

PRAVI GO SOGLASNO IZRAZ (4) OD TRUDOT ZA ZHAO. ZAVISI I MODULACIJATA. ZA DPSK IZRAZOT OD ZHAO E MNOGU PODOBAR (VSUŠNOST NE OVOJ TUKU TOJ ŠTO IM IMA IZLEZENO VO SLIKITE)

- PDF-OT OD YANG STAVI GO VO ZHAO I KJE GO DOBIEŠ ISTIOT IZRAZ ZA SER.

⊡ Y. ZHAO, R. ADVE **SYMBOL ERROR RATE OF SELECTION AMPLIFY AND FORWARD RELAY SYSTEMS**

- ZA DIVERZITET ZOKI REČE DEKA TOA E STEPENOT ŠTO SE ZAVRŠUVA VO IZRAZOT ZA SER NA MOMENTALNIOT SNR T.E. SREDNIOT SNR.
- SOURCE NODE: TRANSMITS TO DESTINATION OVER „m" RELAYS
- IN AP-AF AFTER MRC THE RECEIVED SNR IS:

$$\gamma_r^{AP}=\frac{|h_{s,d}|^2E_s}{N_{s,d}}+\sum_{i=1}^{m}\frac{\frac{|h_{s,i}|^2E_s}{N_{s,i}}\,\frac{|h_{i,d}|^2E_i}{N_{i,d}}}{\frac{|h_{s,i}|^2E_s}{N_{s,i}}+\frac{|h_{i,d}|^2E_i}{N_{i,d}}+1}$$

E_s, E_i - AVERAGE ENERGY TRANSMITTED AT THE SOURCE AND i-TH RELAY. IF EACH TRANSMITION HAS UNIT DURATION IT CAN BE CONSIDERED AS TRANSMISSION POWER.

- IN LINEAR MODULATION CASE SER IN AWGN IS:

$$SER=Q\left(\sqrt{c\gamma_r}\right)\qquad c=2 \text{ FOR PSK}$$

$$Q(z)=\frac{1}{2}\,\mathrm{erfc}\,\frac{z}{\sqrt{2}}\qquad Q(z)=\frac{1}{\sqrt{2\pi}}\int_z^{\infty}e^{-\frac{x^2}{2}}\,dx$$

$$\mathrm{erfc}(t)=\frac{2}{\sqrt{\pi}}\int_t^{\infty}e^{-x^2}\,dx=\left|\begin{matrix}x=\frac{z}{\sqrt{2}}\\ dx=\frac{dz}{\sqrt{2}}\\ x=t\quad z=t\sqrt{2}\end{matrix}\right|=\frac{1}{\sqrt{2}}\,\frac{2}{\sqrt{\pi}}\int_{t\sqrt{2}}^{\infty}e^{-\frac{z^2}{2}}\,dz$$

$$\mathrm{erfc}(t)=\frac{2}{\sqrt{2\pi}}\int_{t\sqrt{2}}^{\infty}e^{-z^2/2}\,dz;\quad Q(t)=\frac{1}{2}\,\mathrm{erfc}\left(\frac{t}{\sqrt{2}}\right)=\frac{1}{\sqrt{2\pi}}\int_t^{\infty}e^{-z^2/2}\,dz$$

- AVERAGE SER OVER DISTRIBUTION OF RECEIVED SNR IS:

$$P_e=E_{\gamma_r}\left\{Q\left(\sqrt{c\gamma_r}\right)\right\}$$ γ - RECEIVED SNR

- PDF & CDF OF RECEIVED SNR - γ_r ARE:

$$f_{\Gamma_R}()\qquad F_{\Gamma_R}()\qquad X\sim N(0,1)\qquad P_e=P\left(X>\sqrt{c\gamma_r}\right)=P\left(\gamma_r<\frac{X^2}{c}\right)$$

$$=E_X\left\{F_{\Gamma_R}\left(\frac{X^2}{c}\right)\right\}$$

$$Q(\sqrt{c\gamma_r}) = \frac{1}{\sqrt{2\pi}} \int_{\sqrt{c\gamma_r}}^{\infty} e^{-\frac{x^2}{2}} dx$$

$$\boxed{P_e = \int_0^{\infty} Q(\sqrt{c\gamma_r})\, p(\gamma_r)\, d\gamma_r = \mathcal{E}_{\gamma_r}\{Q(\sqrt{c\gamma_r})\}}$$

$$\underline{\chi \sim \mathcal{N}(0,1)} \Rightarrow \quad p_\chi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}}$$

$$P(\chi < x) = \frac{1}{\sqrt{2\pi}} \int^{x} e^{-\frac{t^2}{2}} dt \qquad P(\chi > x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-t^2/2} dt$$

MMV

$$P(\chi > \sqrt{c\gamma_r}) = \frac{1}{\sqrt{2\pi}} \int_{\sqrt{c\gamma_r}}^{-\infty\ \infty} e^{-t^2/2} dt \qquad P_e = \mathcal{E}_{\gamma_r}\{P(\chi > \sqrt{c\gamma_r})\}$$

$$= \mathcal{E}_{\gamma_r}\{P(\chi^2 > c\gamma_r)\} = \mathcal{E}_{\chi}\left\{P\left(\gamma_r < \frac{\chi^2}{c}\right)\right\} = \mathcal{E}_{\chi}\left\{F_{\Gamma_r}\left(\frac{\chi^2}{c}\right)\right\}$$

- ASUMING THAT ALL NOISE VARIANCES ARE EQUAL i.e

$$\boxed{N_{s,d} = N_{s,i} = N_{i,d} = N_0 = 1/\gamma}$$

VEROJATNO ŠO OVA GO DELEZI AVERAGE SNR

$$\alpha_0 = |h_{s,d}|^2 E_s \qquad \alpha_i = |h_{s,i}|^2 E_s \qquad \beta_i = |h_{i,d}|^2 E_i$$

$$\gamma_r^s = \alpha_0 \gamma + \max_i \frac{\alpha_i \beta_i \gamma^2}{\alpha_i \gamma + \beta_i \gamma + 1}$$

- APROXIMATION OF HIGH SNR CDF FOR S-AF (IN [4])

$$\boxed{F_{\Gamma_R}(\gamma_r) \approx \frac{\lambda_0 \prod_{i=1}^{m} (\lambda_i + \xi_i)}{m+1} \left(\frac{\gamma_r}{\gamma}\right)^{m+1}}$$

TREBA DA NAUČAM KAKO ŠO DOBIVA OVA!!!

$$\boxed{f_{\alpha_0}(x) = \lambda_0 e^{-\lambda_0 x}}$$

$$\boxed{\int_0^{\infty} x^{2n} e^{-px^2} dx = \frac{(2n-1)!!}{2(2p)^n} \sqrt{\frac{\pi}{p}}}$$

GRADSHTEYN 3.461.2

$$P_e = \mathcal{E}_{\chi}\left\{F_{\Gamma_r}\left(\frac{\chi^2}{c}\right)\right\} \doteq \frac{1}{\sqrt{2\pi}} \int_0^{\infty} \frac{\lambda_0 \prod_{i=1}^{m} (\lambda_i + \xi_i)}{m+1} \left(\frac{x^2}{c\cdot\gamma}\right)^{m+1} e^{-x^2/2} dx$$

$$= \frac{1}{\sqrt{2\pi}} \frac{\lambda_0 \prod_{i=1}^{m} (\lambda_i + \xi_i)}{m+1} \cdot \frac{1}{(c\gamma)^{m+1}} \int_0^{\infty} x^{2(m+1)} e^{-x^2/2} dt =$$

$$= \cancel{\frac{1}{\sqrt{2\pi}}} \frac{\lambda_0 \prod_{i=1}^{m} (\lambda_i + \xi_i)}{m+1} \frac{1}{(c\gamma)^{m+1}} \cdot \frac{(2\cdot m+2-1)!!}{2\cdot 1^{m+1}} \cdot \cancel{\sqrt{\frac{\pi}{1/2}}}$$

$$P_e = \lambda_0 \prod_{i=1}^{n} (\lambda_i + \xi_i) \frac{1}{(c\gamma)^{n+1}} \frac{(2n+1)!!}{2(n+1)}$$

$$n!! = n\cdot(n-2)\cdot(n-4)\cdots 4\cdot 2 \qquad n\text{-even}$$

- RELATION BETWEEN DOUBLE FACTORIAL AND FACTORIAL

$$(2n+1)!! \cdot 2^n \cdot n! = [(2n+1)(2n-1)\cdots 1]\cdot[2n]\cdot[2(n-1)][2(n-2)]\cdots 2$$

$$= [(2n+1)(2n-1)\cdots 1][(2n)(2n-2)(2n-4)\cdots 2\cdot 1] =$$

$$= (2n+1)(2n)(2n-1)(2n-2)(2n-3)(2n-4)\cdots 2\cdot 1 = \underline{\underline{(2n+1)!}}$$

$$(2n+1)!!\cdot 2^n\cdot n! = (2n+1)!$$

$$\boxed{(2n+1)!! = \frac{(2n+1)!}{2^n\cdot n!}}$$

MMV

$$P_e = \lambda_0 \prod_{i=1}^{n} (\lambda_i + \xi_i) \frac{1}{(c\gamma)^{n+1}} \frac{(2n+1)!}{2\cdot 2^n\, n!\,(n+1)}$$

$$\boxed{P_e = \lambda_0 \prod_{i=1}^{n} (\lambda_i + \xi_i) \frac{(2n+1)!}{(2c\gamma)^{n+1}(n+1)!}}$$

DIVERSITY ORDER OF n+1 → FULL DIVERSITY ORDER IS ACHIEVED.

- TREBA DA GO IMPLEMENTIRAM CLANAKOT OD ZHAO VO MATLAB.

☐ SER IMPROVEMENT : S-AF VS AP-AF

$$\boxed{G_s = G_i = \frac{1}{n+1}} \qquad \underline{\text{AP-AF}}$$

$$\boxed{G_s = G_{i\_chosen} = \frac{1}{2}} \qquad \text{S-AF}$$

- IF INSTANTENOUS SNR CAN BE WRITEN AS:

$$P_e|\gamma_r = Q\left(\sqrt{c\,\gamma_r}\right)$$

AND THE PDF OF γ_r

$$f_{\gamma_r} = a\gamma_r^{t} + O(\gamma_r^{t+\varepsilon})$$

CAN BE APROXIMATED AS: ⇗ OD CLANAKOT NA GIANNAKIS.

$$P_e \approx \frac{\prod_{i=1}^{t+1}(2i-1)}{2(t+1)\,c^{t+1}\,t!} \frac{\partial^t f_{\gamma_r}}{\partial \gamma_r^t}(0) \qquad \frac{\partial^t f_{\gamma_r}(0)}{\partial \gamma_r^t} = \lim_{t\to 0^+} \frac{\partial^t f_{\gamma_r}}{\partial \gamma_r^t}$$

- AP-AF SATISFIES

$$f_{\gamma_r} = a\,\gamma_r^{t} + o\left(\gamma_r^{t+\varepsilon}\right) \qquad t=m$$

- IN ORDER TO OBTAIN m-th order derivate of PDF OF γ_r CONSIDER TWO RVs:

$$Y_1 = \frac{x_0 + \sum_{i=1}^{m} x_i}{m+1} \qquad Y_2 = \frac{x_0 + \max_i x_i}{2} = \frac{x_0+V}{2}$$

$\{x_i : i = 0 \dots m\}$ INDEPENDENT RVs WITH:

$F_{x_i}(0) = 0 \qquad f_{x_i}(0) \neq 0$

$Z = X + Y \qquad X \sim N(0,1) \qquad Y \sim N(0,1)$

$$M_X(s) = \int_0^\infty p_X(x)\cdot e^{xs}\,dx \qquad M_Y(s) = \int_0^\infty p_Y(y)\,e^{ys}\,dy$$

$$\underbrace{M_Z(s) = \int_0^\infty p_Z(z)\cdot e^{zs}\,dz = \int_0^\infty\int_0^\infty p_X(x)\cdot p_Y(y)\cdot e^{(x+y)s}\,dx\,dy = M_X(s)\cdot M_Y(s)}_{\text{NE TREBA VAKA TUKU SO KONVOLUCIJA!}}$$

- TEORIJA NA INFORM. MOJA SKRIPTA

$\zeta = \xi + \eta \qquad z = x + y \qquad p_\zeta(z) = ?$

$\zeta' = \xi \qquad z = x + y \xrightarrow{\text{INV.F}} y = z - x'$

$\qquad\qquad x' = x \xrightarrow{\text{INV.F}} x = x'$

$$J = \begin{vmatrix} \frac{\partial z}{\partial x} & \frac{\partial z}{\partial y} \\ \frac{\partial x'}{\partial x} & \frac{\partial x'}{\partial y} \end{vmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} = -1 \qquad |J| = 1$$

$$p_{\zeta\zeta'}(z,x') = \frac{p_{\xi\eta}(x,y)}{|J|}\Bigg|_{\substack{x=x' \\ y=z-x'}} = p_{\xi\eta}(x', z-x')$$

$$p_\zeta(z) = \int_0^\infty p_{\zeta\zeta'}(z,x')\,dx' = \int_0^\infty p_{\xi\eta}(x', z-x')\,dx' = \int_0^\infty p_{\xi\eta}(x, z-x)\,dx$$

$$= \int_0^\infty p_\xi(x)\cdot p_\eta(z-x)\,dx = p_\xi \cancel{(z)} * p_\eta(z) \qquad M_Z(s) = \int_0^\infty p_\xi(z) * p_\eta(z)\,e^{sz}\,dz = M_{\cancel{X}(\xi)}(s)\cdot M_{(\eta)}(s$$