$\boxed{\text{Exc. 23}}$ $\iint_S \vec{a}\circ\vec{n}\,dS = 0$ $\qquad \vec{a} = c_1\vec{i} + c_2\vec{j} + c_3\vec{k}$

$$\operatorname{div}\vec{a} = \frac{\partial c_1}{\partial x} + \frac{\partial c_2}{\partial y} + \frac{\partial c_3}{\partial z} = 0 + 0 + 0 \Rightarrow 0$$

$\boxed{\text{Exc. 24}}$ $V(E) = \frac{1}{3}\iint_S \vec{F}\circ d\vec{S}$ $\qquad F(x,y,z) = x\vec{i} + y\vec{j} + z\vec{k}$

$\operatorname{div}\vec{F} = 1 + 1 + 1 = 3$ $\qquad \iint_S \vec{F}\,d\vec{S} = \iiint_E 3\circ dV$

$$\iiint_E dV = V(E) = \frac{1}{3}\iint_S \vec{F}\,ds$$

$\boxed{\text{Exc. 25}}$ $\iint_S \operatorname{curl}\vec{F}\circ dS = 0$

$$\operatorname{curl}\vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial_x & \partial_y & \partial_z \\ P & Q & R \end{vmatrix} = \left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\right)\vec{i} + \left(\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}\right)\vec{j} + \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)\vec{k}$$

$$\operatorname{div}(\operatorname{curl}F) = \underline{\cancel{\frac{\partial^2 R}{\partial x\partial y}}} - \underline{\underline{\cancel{\frac{\partial^2 Q}{\partial x\partial z}}}} + \underline{\underline{\underline{\cancel{\frac{\partial^2 P}{\partial y\partial z}}}}} - \underline{\cancel{\frac{\partial^2 R}{\partial y\partial x}}} + \underline{\underline{\cancel{\frac{\partial^2 Q}{\partial z\partial x}}}} - \underline{\underline{\underline{\cancel{\frac{\partial^2 P}{\partial z\partial y}}}}}$$

$\boxed{\text{Exc. 26}}$ $\iint_S D_n f\,dS = \iiint \nabla^2 f\,dV$

$\nabla f = \frac{\partial f}{\partial x}\vec{i} + \frac{\partial f}{\partial y}\vec{j} + \frac{\partial f}{\partial z}\vec{k}$ $\qquad$ ~~$\nabla f = D_a \cdot \langle\vec{i},\vec{j},\vec{k}\rangle$~~ ~~$D_a \cdot \vec{a}$~~

$D_n(f) = \underline{\underline{\nabla f\cdot n}}$ $\qquad n = \langle\vec{i},\vec{j},\vec{k}\rangle$

$$\iint_S \nabla f\circ\underbrace{n\,ds}_{d\vec{S}} = \iiint_E \underbrace{\operatorname{div}(\nabla f)}\,dV = \iiint_E \nabla^2 f\,dV$$

$$\operatorname{div}(\nabla f) = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial y^2} = \underline{\nabla^2 f}$$

$\boxed{\text{Exc. 27}}$ $$\iint_S (f\nabla g)\cdot\vec{n}\,ds = \iiint_E (f\nabla^2 g + \nabla f\cdot\nabla g)\,dv$$

$$\operatorname{div}(f\cdot\vec{F}) = f\operatorname{div}\vec{F} + \vec{F}\cdot\nabla f$$
$$\operatorname{div}(f\nabla g) = f\operatorname{div}(\nabla g) + \nabla g\cdot\nabla f = f\cdot\nabla^2 g + \nabla f\cdot\nabla g$$

$\boxed{\text{Exc. 28}}$ $$\iint_S (f\nabla g - g\nabla f)\cdot\vec{n}\,ds = \iiint_E (f\nabla^2 g - g\nabla^2 f)\,dv$$

$$\operatorname{div}(f\nabla g - g\nabla f) = \operatorname{div} f\nabla g - \operatorname{div}(g\nabla f)$$
$$= f\operatorname{div}(\nabla g) + \cancel{\nabla g\cdot\nabla f} - (g\operatorname{div}\nabla f + \cancel{\nabla g\cdot\nabla g}) =$$

$\underline{\nabla g\cdot\nabla f = \nabla f\cdot\nabla g}$

$$= f\nabla^2 g - g\nabla^2 f \;//$$

$\boxed{\text{Exc. 29}}$ $$\iint_S f\,\vec{n}\,ds = \iiint_E \nabla f\,dV$$

$F = f\cdot\vec{c}$ ~~$F = \operatorname{div}(f\cdot\vec{c}) = \frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} + \frac{\partial f}{\partial z}$~~ [illegible]

~~$\vec{c} = \vec{i} + \vec{j} + \vec{k}$~~

$$\iint_S \vec{F}\cdot\vec{n}\,ds = \iint_S f\vec{c}\cdot\vec{n}\,ds$$

$$\operatorname{div}(f\vec{n}) = f\operatorname{div}\vec{n} + \vec{n}\cdot\nabla f$$
$$\vec{F} = f\cdot\vec{c} = f(c_1\vec{i} + c_2\vec{j} + c_3\vec{k})$$
$$\operatorname{div}\vec{F} = c_1\frac{\partial f}{\partial x} + c_2\frac{\partial f}{\partial y} + c_3\frac{\partial f}{\partial z} = \vec{c}\cdot\nabla f$$
$$\operatorname{div}(f\vec{c}) = f\underbrace{\operatorname{div}\vec{c}}_{0} + \boxed{\vec{c}\cdot\nabla f}$$

$$\iint_S f\vec{c}\cdot\vec{n}\,ds = \overset{\vec{c}}{\iiint_E} \nabla f\,dV$$

~~[illegible]~~

$$\iint_S (f\vec{c})\cdot\vec{n}\,ds = \iiint_E \nabla f\cdot\vec{c}\,dV$$

$$\vec{c} = c_1\vec{i} + c_2\vec{j} + c_3\vec{k}$$
$$\vec{n} = n_1\vec{i} + n_2\vec{j} + n_3\vec{k}$$

IF: $c = \vec{i}$

$$\iint_S (f\vec{i})\vec{n}\,ds = \iiint_E \nabla f\cdot\vec{i}\,dV \qquad \iint_S f\cdot n_1\,ds = \iiint_E \frac{\partial f}{\partial x}\,dV$$

$$\iint_S f\,n_2\,ds = \iiint_E \frac{\partial f}{\partial y}\,dV \qquad \iint f\,n_3\,ds = \iiint_E \frac{\partial f}{\partial z}\,dV$$

$$\underline{\underline{\iint_S f\vec{n}\,ds}} = \left(\iint_S f\,n_1\,ds\right)\vec{i} + \left(\iint_S f\,n_2\,ds\right)\vec{j} + \left(\iint_S f\,n_3\,ds\right)\vec{k}$$

$$= \left(\iiint_E \frac{\partial f}{\partial x}\,dV\right)\vec{i} + \left(\iiint_E \frac{\partial f}{\partial y}\,dV\right)\vec{j} + \left(\iiint_E \frac{\partial f}{\partial z}\,dV\right)\vec{k}$$

$$= \underline{\underline{\iiint_E \nabla f\,dV}}$$

$\boxed{\text{Exc. 30}}$ liquid with constant density $-\rho$
pressure at depth z is $\boxed{p = \rho g z}$
$g = 9.81\ m/s^2$

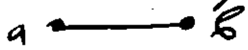
$$\vec{F} = -\iint_S p\,\vec{n}\,ds \Rightarrow \text{buoyant force}$$

$$\nabla p = \frac{\partial p}{\partial x}\vec{i} + \frac{\partial p}{\partial y}\vec{j} + \frac{\partial p}{\partial z}\vec{k} = \rho\cdot g\cdot\vec{k}$$

$$I = -\iint_S p\,\vec{n}\,ds = -\iiint_E \nabla p\,dV$$

$$I = -\vec{k}\cdot\rho\cdot g\iiint_E dV = -\rho\cdot g\cdot V\cdot\vec{k} = \left|\rho V = m\right| = \overset{\ominus}{-}\underbrace{m\cdot g}_{W\Rightarrow \text{WEIGHT OF LIQUID}}\cdot\vec{k} = \overset{\ominus}{-}W\vec{k}$$

- FUNDAMENTAL THEOREM OF CALCULUS: $\int_a^b F'(x)\,dx = F(b) - F(a)$

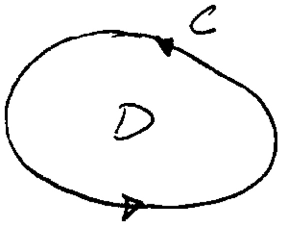


- FUNDAMENTAL THEOREM OF LINE INT.:

$$\int_c \nabla f\, d\vec{r} = \nabla f(r(b)) - \nabla f(r(a))$$

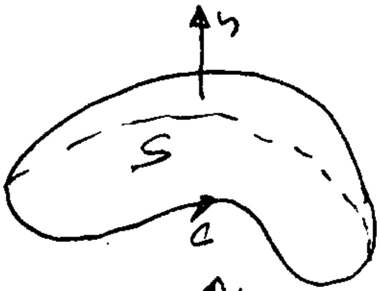
- GREEN'S THEOREM

$$\iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA = \int_C P\,dx + Q\,dy$$



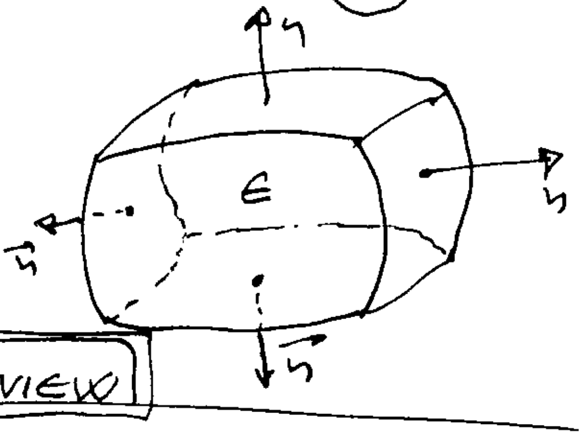
- STOKES' THEOREM

$$\iint_S \text{curl}\,\vec{F}\, d\vec{S} = \int_C \vec{F}\, d\vec{r}$$



- DIVERGENCE THEOREM

$$\iiint_E \text{div}\,\vec{F}\, dV = \iint_S \vec{F} \circ d\vec{S}$$



$\boxed{\text{CH. 16 REVIEW}}$

$\boxed{2}$ $\vec{F} = \nabla f$

f - POTENTIAL FUNCTION

$\vec{F}$ - VECTOR FIELD

$\boxed{3}$

2D: $\int_a^b f(x,\ ) \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx = \int_C f(x\ ) \sqrt{1 + y'^2}\,dx$

3D: $\int_a^b f(x,y) \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt = \int_a^b f(x(t),y(t)) \sqrt{x'^2 + y'^2}\,dt$

ⓒ $m = \int_a^b \rho(x,y) \sqrt{x'^2(t) + y'^2(t)}\,dt$

$\bar{m}_x = \frac{1}{m} \int_a^b \bar{x}(t)\,\rho(x,y) \sqrt{1 + x'^2(t) + y'^2(t)}\,dt$

$F(x,y,z) = 0$ $\qquad$ $F(x,y,z) + z = z$

③ $I = \int_C f(x,y,z)\,ds = \int_a^b \underset{\uparrow\, x(t),\, y(t),\, z(t)}{f(x,y,z)} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\,dt$

$$\boxed{I = \int_C f(r(t))\,|r'(t)|^2\,dt}$$

4 $\int_C \vec{F}\cdot d\vec{r}$ $\qquad$ $r(t) = x(t)\,\vec{i} + y(t)\,\vec{j} + z(t)\,\vec{k}$

$\vec{F} \Rightarrow$ FORCE $\qquad$ $W = \int_C \vec{F}\cdot d\vec{r} = \int_a^b \vec{F}\cdot |r'(t)|^2\,dt$

$$\int_C \vec{F}\cdot d\vec{r} = \int P\,dx + Q\,dy + R\,dx$$

5 $\int_C \vec{F}\,d\vec{r} = \int_C \nabla f\,d\vec{r} = \nabla f(r(b)) - \nabla f(r(a))$

$\boxed{\vec{F} = \nabla f}$ $\qquad$ ∇f CONTINUOUS ON C

6 INDEPENDENT OF PATH

$\oint_C \vec{F}\,d\vec{r} = 0$ $\qquad$ $\vec{F}$ CONSERVATIVE

7 $\int_C \vec{F}\,d\vec{r} = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA$

8 $Q = x$, $P = 0$ $\qquad$ $A = \iint_D dA = \int_C P\,dx + Q\,dy = \int_C Q\,dy = \int_C x\,dy$

$Q = 0$, $P = -y$ $\qquad$ $A = \int_C -y\,dx$

$P = -\frac{1}{2}y$, $Q = \frac{1}{2}x$ $\qquad$ $A = \int_C \frac{1}{2}x\,dy - \frac{1}{2}y\,dx = \frac{1}{2}\int x\,dy - y\,dx$

$\boxed{9}$ $\text{curl}\,\vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ P & Q & R \end{vmatrix}$ $\text{div}\,\vec{F} = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}$

$\boxed{10}$ $\int_C \vec{F}\, d\vec{r} = 0 \Rightarrow \vec{F}$ - conservative

$\text{curl}\,\vec{F} = 0 \Rightarrow \vec{F}$ conservative

$\boxed{\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = 0}$ $\boxed{\vec{F} = P\vec{i} + Q\vec{j}}$

- $\vec{F} = P\vec{i} + Q\vec{j} + R\vec{k}$

$\text{curl}\,\vec{F} = \underbrace{\left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\right)}_{0}\vec{i} + \underbrace{\left(\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}\right)}_{0}\vec{j} + \underbrace{\left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)}_{0}\vec{k}$

If $\vec{F}$ is conservative $\text{curl}\,\vec{F} = 0$

- PARAMETRIC surface

$\vec{r}(u,v) = x(u,v)\vec{i} + y(u,v)\vec{j} + z(u,v)\vec{k}$

GRID : fixed u or fixed v

$A(S) = \iint_S d\vec{S} = \iint_D |\vec{n}|\, dS = \iint_D |r_u \times r_v|\, dS$

$\vec{r}(u,v) = x\vec{i} + y\vec{j} + g(x,y)\vec{k}$

$z = g(x,y) \Rightarrow A(S) = \iint_D \sqrt{1 + g_x^2(x,y) + g_y^2(x,y)}\, dA$

$r_x = \vec{i} + g_x\vec{j}$ $r_y = \vec{j} + g_y\vec{k}$

$\boxed{12}$ $\iint_S f(x,y,z)\, dS = \iint_D f(x,y,g(x,y)) \sqrt{1 + \left(\frac{dz}{dx}\right)^2 + \left(\frac{dz}{dy}\right)^2}\, dA$

$= \iint_D f(r(u,v))\, \overset{|r_u \times r_v|^2}{\cancel{\sqrt{1 + \left(\frac{dx}{du}\right)^2 + \left(\frac{dz}{dv}\right)^2}}}\, dA$

$$I = \iint_{S} f(x,y,z)\,dS = \iint_{D} f(r(u,v))\,|r_u \times r_v|\,dA$$

$z = g(x,y)$

$$r = x\vec{i} + y\vec{j} + g(x,y)\vec{k}$$

$$I = \iint_{D} f(r(x,y))\,|r_x \times r_y|\,dA = \iint_{D} f(r(x,y))\sqrt{1+g_x^2+g_y^2}\,dA$$

$$|r_x \times r_y| = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & g_x \\ 0 & 1 & g_y \end{vmatrix} = \underline{-g_x\vec{i} - g_y\vec{j} + \vec{k}}$$

$$m = \iint_{D} \rho(x,y,\overset{g(x,y)}{z})\sqrt{1+g_x^2+g_y^2}\,dA$$

$$\boxed{\bar{m}_x = \frac{1}{m}\iint_{D} x\,\rho(x,y,g(x,y))\underbrace{\sqrt{1+g_x^2+g_y^2}\,dA}_{dS}}$$

- ORIENTED SURFACE

$$\iint_{S} \vec{F}\cdot d\vec{S} = \iint_{D} \vec{F}\cdot\vec{n}\,dA = \iint_{D} \vec{F}(r(u,v))\cdot(r_u \times r_v)\,dA$$

$z = g(x,y)$

$$r = x\vec{i} + y\vec{j} + g(x,y)\vec{k}$$

$$\iint_{S} \vec{F}\cdot d\vec{S} = \iint_{D}\left(-P\frac{\partial g}{\partial x} - Q\frac{\partial g}{\partial y} + R\right)dx\,dy$$

$$r_x = \vec{i} + g_x\vec{k} \qquad r_y = \vec{j} + g_y\vec{k}$$

$$r_x \times r_y = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & g_x \\ 0 & 1 & g_y \end{vmatrix} = -g_x\vec{i} - g_y\vec{j} + 1$$

- Stokes' $\displaystyle\iint_{S} \underset{\text{curl}}{\vec{F}}\,d\vec{S} = \int_{C} \vec{F}\,d\vec{r}$

- DIVERG. $\displaystyle\iiint_{E} \text{div}\,\vec{F}\,dV = \iint_{S} \vec{F}\cdot d\vec{S}$

DERIVATES
ON LEFT
SIDE OVER
THE REGION,
RIGHT SIDE
ONLY FUNCTIO
OVER THE
BOUNDARIES O
THE REGION

QUIZ 7:

$$I = \iint_S \vec{F}\, d\vec{S} \qquad S:\ x^2+y^2+z^2=1$$

$$\vec{F} = c_1\vec{i} + c_2\vec{j} + c_3\vec{k}$$

$$I = \int_0^{2\pi}\int_0^{\pi} c_1 \cdot \sin^2\theta \cos\varphi + c_2 \sin^2\theta \sin\varphi + c_3 \sin\theta\cos\theta \, d\theta\, d\varphi = 0$$

$D:\ x^2+y^2+z^2=1$

- $\operatorname{curl} \vec{F} = x\vec{i} + y\vec{j} + z \cdot k$

$$\vec{F} = P\vec{i} + Q\vec{j} + R\vec{k}$$

$$\begin{vmatrix} \vec{i} & \vec{j} & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ P & Q & R \end{vmatrix} = \underbrace{\left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\right)}_{x}\vec{i} + \underbrace{\left(\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}\right)}_{y}\vec{j} + \underbrace{\left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)}_{z}\vec{k}$$

$$\overset{(1)}{\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z} = x} \qquad \overset{(2)}{\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x} = y} \qquad \overset{(3)}{\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = z}$$

$R = 2x \cdot y \quad Q = xz \qquad \underline{\underline{2x - x = x}}$

$P = 3yz \qquad 3y - 2y = y$

$z - 3z = z \quad \#$

$R = a \cdot x \cdot y \qquad Q = b \cdot xz \qquad P = c \cdot y \cdot z$

$a \cdot x - b \cdot x = x \Rightarrow a - b = 1$

$c \cdot y - ay = y \Rightarrow c - a = 1 \qquad a = c - 1$

$bz - cz = z \Rightarrow b - c = 1 \qquad b = 1 + c$

$c - 1 - 1 - c = 1 \Rightarrow -2 = 1 \quad \#$

Ch.16 FINAL EXERCISES

$\boxed{\text{Exc.2}}$ $\quad J = \int_C x\, ds \qquad y = x^2 \quad (0,0) \ldots (1,1)$

$$I = \int_C x\,ds = \int_a^b x \sqrt{1+\left(\frac{dy}{dt}\right)^2}\,dx = \int_0^1 x \sqrt{1+(2x)^2}\,dt =$$

$$= \int_0^1 x \sqrt{1+4x^2}\,dx = \frac{5\sqrt{5}}{12} - \frac{1}{12}$$

$\boxed{Exc.5}$ $I = \int_C x^2 y\,dx - x\,dy$ $\quad x^2+y^2=1$ CONTERCLOCKWISE

$x = \cos\theta$

$y = \sin\theta$

$$I = \int_0^{2\pi} -\cos^2\theta \sin\theta \sin\theta\,d\theta - \cos\theta \cdot \cos\theta\,d\theta =$$

$$= -\int_0^{2\pi} \cos^2\theta \sin^2\theta + \cos^2\theta\,d\theta = -\pi$$

$\boxed{Exc.7}$ $I = \int_C x^2 z\,ds$ $\quad x = 2\sin t \quad y = t \quad z = 2\cos t \quad 0 \leq t \leq \frac{\pi}{2}$

$$I = \int_0^{\pi/2} 6\sin^2 t \cdot 2\cos t \sqrt{(2\cos t)^2 + 1 + (-2\sin t)^2}\,dt =$$

$$= \int_0^{\pi/2} 12\sin^2 t \cos t \sqrt{4+1}\,dt = 60\int_0^{\pi/2} \sin^2 t \cos t\,dt = \underline{\underline{15}}$$

$\boxed{Exc.9}$ $I = \int_C \vec{F} \circ d\vec{r} = ?$ $\quad \vec{F} = e^z \vec{i} + xz\vec{j} + (x+y)\vec{k}$

$r(t) = \underbrace{t^2}_{x}\vec{i} + \underbrace{t^3}_{y}\vec{j} - \underbrace{t}_{z}\vec{k} \quad 0 \leq t \leq 1$

$$I = \int_0^1 \left(e^z\vec{i} + xz\vec{j} + (x+y)\vec{k}\right)\left(t^2\vec{i} + t^3\vec{j} - t\vec{k}\right)'\,dt =$$

$$= \int_0^1 \left(e^z\vec{i} + xz\vec{j} + (x+y)\vec{k}\right)\left(2t\vec{i} + 3t^2\vec{j} - \vec{k}\right)\,dt =$$

$$= \int_0^1 e^z \cdot 2t + xz \cdot 3t^2 - (x+y)\,dt = \int_0^1 e^{-t} \cdot 2t - t^3 \cdot 3t^2 - (t^2+t^3)\,dt$$

$$= \int_0^1 2+e^{-t} - 3t^5 - t^2 - t^3\,dt = \frac{11}{12} - 4e^{-1}$$

$\boxed{\text{Exc. 10}}$ $\vec{F} = z\vec{i} + x\vec{j} + y k$ $W = ?$ MOVING PARTICLE

$(3,0,0) \to (0, \pi/2, 3)$

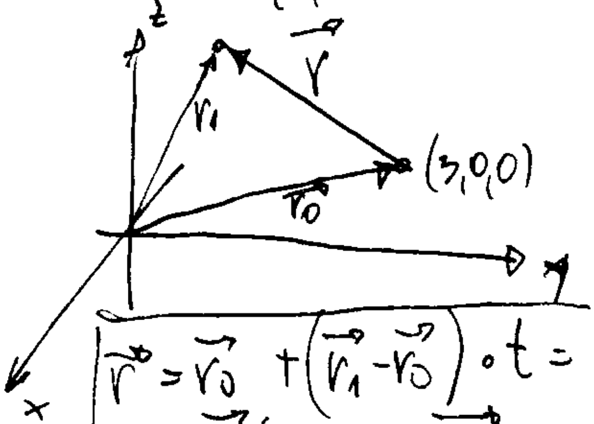
ⓐ straight line

$$W = \int_{C\ (0,\pi/2,3)} \vec{F}\, d\vec{r}$$

$$\frac{x-x_0}{a} = \frac{y-y_0}{b} = \frac{z-z_0}{c}$$

PLANE

$$(\vec{r}-\vec{r_0}) \cdot \vec{n} = 0$$



$$\vec{a} = \vec{r_1} - \vec{r_0} = (-3, \pi/2, 3)$$

$$\vec{r} = \vec{r_0} + \vec{a} \cdot t =$$

$$= \langle 3,0,0 \rangle + \langle -3, \pi/2, 3 \rangle t$$

$$\boxed{\begin{aligned}\vec{r} &= \vec{r_0} + (\vec{r_1} - \vec{r_0}) \cdot t = \\ &= \vec{r_0}(1-t) + \vec{r_1}\end{aligned}}$$

$\boxed{\text{LINE SEGMENT}}$

$$\vec{r} = \langle 3-3t, \tfrac{\pi}{2}t, 3t \rangle$$

$x = 3-3t$ $y = \frac{\pi}{2}t$ $z = 3t$

$t = 0, \dots 1$ $t = 0$ $\langle 3,0,0 \rangle$

$t = 1$ $\langle 0, \pi/2, 3 \rangle$

$$W = \int_0^1 \left(3t\vec{i} + (3-3t)\vec{j} + \frac{\pi}{2}t\vec{k}\right) \cdot \langle -3, \frac{\pi}{2}, 3 \rangle\, dt =$$

$$= \int_0^1 -9t + \frac{\pi}{2}(3-3t) + \frac{3\pi}{2}t\, dt = \int_0^1 -9t + \frac{3\pi}{2} - \cancel{\frac{3\pi}{2}t} + \cancel{\frac{3\pi}{2}t}\, dt$$

$$= -9\frac{t^2}{2}\Big|_0^1 + \frac{3\pi}{2}t\Big|_0^1 = -\frac{9}{2} + \frac{3\pi}{2} \doteq 0{,}2124 \,//$$

ⓑ helix: $x = 3\cos t$, $y = t$, $z = 3\sin t$

$$\vec{r}'(t) = -3\sin t\,\vec{i} + \vec{j} + 3\cos t\,\vec{k}$$

$$W = \int_0^{a=\pi/2} \left(-9t\sin t + (3-3t) + \frac{3\pi}{2}t\cos t\right) dt \doteq -5{,}3 \,//$$

$0 = 3\cos t \Rightarrow t = \pi/2$

x: $3 = 3\cos t$ $t = 0$

y: $0 \div \frac{\pi}{2}$ $t = 0 .. \pi/2$

$\boxed{\text{Exc. 11}}$ $f = ?$ T.S. $F = \nabla f$

$$\vec{F}(x,y) = (1+xy)e^{xy}\vec{\imath} + \left(e^{y} + x^2 e^{xy}\right)\vec{\jmath}$$

$$\left.\frac{\partial Q}{\partial x} = \frac{\partial P}{\partial y}\right\} \text{ conservative}$$

$$\cancel{\frac{\partial Q}{\partial x} = 2x e^{xy}\cdot y = 2xy e^{xy}}$$

$$\frac{\partial Q}{\partial x} = 2x e^{xy} + x^2 e^{xy}\cdot y \Longleftrightarrow$$

$$\frac{\partial P}{\partial y} = x e^{xy} + (1+xy) e^{xy}\cdot x$$
$$= x e^{xy} + x e^{xy} + x^2 y e^{xy}$$
$$= 2x e^{xy} + x^2 y e^{xy}$$

$$\Rightarrow \frac{\partial Q}{\partial x} = \frac{\partial P}{\partial y} \Rightarrow \vec{F} \text{ - CONSERVATIVE}$$

$$\vec{F} = \nabla f \qquad P = \frac{df}{dx} = f_x(x,y)$$

$$f(x,y) = \int P(x,y)\,dx = \int (1+xy)e^{xy}\,dx = xe^{xy} + g(y)$$

$$f_y(x,y) = x e^{xy}\cdot x + g_y(y) = x^2 e^{xy} + g_y(y)$$

$$\underline{g_y(y) = \cancel{x^2 e^{xy}}\, e^{y}} \qquad g(y) = \int e^{y}\,dy = e^{y}$$

$$\boxed{f(x,y) = x e^{xy} + e^{y}}$$ POTENTIAL FUNCTION!!

$$\nabla f(x,y) = \underbrace{\left(e^{xy} + xye^{xy}\right)}_{\circledast}\vec{\imath} + \left(x^2 e^{xy} + e^{y}\right)\vec{\jmath}$$

$$\circledast = e^{xy}(1+xy) \qquad \nabla f(x,y) = e^{xy}(1+xy)\vec{\imath} + \left(e^{y} + x^2 e^{xy}\right)\vec{\jmath}$$

$\boxed{\text{Exc. 12}}$

$$\vec{F}(x,y) = (4x^3y^2 - 2xy^3)\vec{\imath} + (2x^4y - 3x^2y^2 + 4y^3)\vec{\jmath}$$
$$\vec{r}(t) = (t + \sin\pi t)\vec{\imath} + (2t + \cos\pi t)\vec{\jmath} \quad 0 \le t \le 1$$

$$I = \int_C \vec{F}\,d\vec{r} = ? \qquad \boxed{\frac{\partial P}{\partial y} = 8x^3y - 6xy^2} = \boxed{\frac{\partial Q}{\partial x} = 8x^3y - 6xy^2.}$$

$$I = \int_0^1 (4x^3y^2 - 2xy^3)(1 + \pi\cdot\cos\pi t) + (2x^4y - 3x^2y^2 + 4y^3)(2 - \pi\sin\pi t)\,dt$$

$\boxed{I \overset{\circ}{=} 11.82 - 3.76\pi}$

$$\boxed{\int_C \nabla f \, dr = f(r(b)) - f(r(a))}$$

TREBA DA SE
REŠI SO POMOŠ
NA OVAA FORMULA

$$\vec{F}(x,y) = (4x^3y^2 - 2xy^3)\vec{i} + (2x^4y - 3x^2y^2 + 4y^3)\vec{j}$$

$$r(t) = (t + \sin \pi t)\vec{i} + (2t + \cos \pi t)\vec{j} \quad 0 \le t \le 1$$

$$f(x,y) = \int 4x^3y^2 - 2xy^3 dx = 4\frac{x^4}{4}y^2 - \frac{2x^2}{2}y^3 + g(y)$$
$$= x^4y^2 - x^2y^3 + g(y)$$

$$f_y(x,y) = 2x^4y - 3x^2y^2 + g_y(y) = 2x^4y - 3x^2y^2 + 4y^3$$

$g_y(y) = 4y^3 \qquad g(y) = \int 4y^3 dy = \frac{\cancel{4}y^4}{\cancel{4}} = y^4$

$\boxed{f(x,y) = x^4y^2 - x^2y^3 + y^4} \qquad \boxed{\vec{F} = \nabla f}$

$$I = \int_C \vec{F} d\vec{r} = \int_C \nabla f d\vec{r} = f(r(b)) - f(r(a)) =$$

$$= f(r(1)) - f(r(0))$$

$x(t) = t + \sin \pi t$

$t=0 \Rightarrow x(0) = 0$

$t=1 \Rightarrow x(1) = 1 + \cancel{\sin \pi}^{0} = 1$

$y(t) = 2t + \cos \pi t$

$t=0 \Rightarrow y(0) = 0 + 1 = 1$

$t=1 \Rightarrow y(1) = 2 + \cancel{\cos \pi}^{-1} = 1$

$f(r(1)) = \cancel{1}\cdot 1 - \cancel{1}\cdot 1 + 1 = 1 \qquad f(r(0)) = 1$

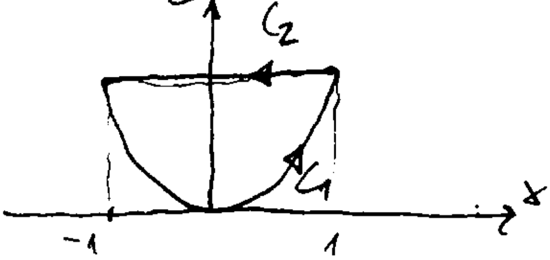
$\boxed{I = 1 - 1 = 0}$

Exc. 15

$$\int_C xy^2 dx - x^2y\,dy$$

$C_1: \; y = x^2 \quad (-1,1) \div (1,1)$

$C_2: \; L.S \quad (1,1) \div (-1,1)$



$$\int_C \vec{F} d\vec{r} = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA$$

$x = \pm\sqrt{y}$

$$I = \iint_{0\ -\sqrt{y}}^{1\ \sqrt{y}} \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dx\, dy = \int_0^1 \int_{-\sqrt{y}}^{\sqrt{y}} -2xy - 2xy\, dx\, dy = -\int_0^1 \int_{-\sqrt{y}}^{\sqrt{y}} 4xy\, dx\, dy$$

$$I = -\int_0^1 2\cancel{4}y \frac{x^2}{\cancel{2}} \Big|_{-\sqrt{y}}^{\sqrt{y}} dy = -\int_0^1 \cancel{2}y\,(y - y)\, dy = 0$$

ControlVault
Firmware

- PREKO LINISKI INTEGRAL

$$I = \underbrace{\int_{C_1} xy^2 dx - x^2 y\, dy}_{I_1} + \underbrace{\int_{C_2} xy^2 dx - x^2 y\, dy}_{I_2}$$

$$I_1 = \int_{-1}^{1} x \cdot x^4\, dx - x^2 \cdot x^2 \cdot 2x\, dx = -\int_{-1}^{1} x^5 dx = \frac{x^6}{6}\Big|_{-1}^{1} = \frac{1-1}{6} = 0$$

- $\vec{r} = \vec{r_0} + (\vec{r_1} - \vec{r_0})t = \vec{r_0}(1-t) + \vec{r_1}t = \langle 1,1 \rangle (1-t) + \langle -1,1 \rangle t$

$\vec{r} = \langle 1-1, 1+1 \rangle + \langle -1,-1 \rangle t = \langle 0, 2 \rangle + \langle -1,-1 \rangle t$

$\boxed{x = -t \qquad y = 2-t}$ $\qquad dx = -dt \qquad dy = -dt$

~~NETOČNA~~

$$I_2 = \int -t\,(2-t)^2 (-dt) - t^2 (2-t)(-dt) =$$

$$= \int t(2-2t+t^2) + 2t^2 - t^3\, dt = \int 2t - \cancel{2t^2} + \cancel{t^3} + \cancel{2t^2} - \cancel{t^3}\, dt$$

$$= \cancel{2}\frac{t^2}{\cancel{2}}\Big|$$

$\vec{r} = \langle 1,1 \rangle + \langle -1-1, -1+1 \rangle t = \langle 1,1 \rangle + \langle -2, 0 \rangle t$

$x = 1-2t \qquad y = 1 \qquad t = 0 \qquad x = 1 \quad y = 1$

$dx = -2dt \qquad dy = 0 \qquad t = 1 \qquad x = -1 \quad y = 1$

$$I_2 = \int_0^1 2(1-2t)\, dt - (1-2t)^2 \cdot 1 \cdot 0 = -2\int_0^1 (1-2t)\, dt = -2\left(1 - \frac{\cancel{2}t^2}{\cancel{2}}\Big|_0^1\right)$$

$$= -2(1-1) = -2 \cdot 0 = 0$$

$I_1 + I_2 = 0$ DOKAZANO!

$\boxed{\text{Exc.}21}$ C: simple closed plane curve

$$I = \oint_C f(x)\,dx + g(y)\,dy = 0$$

$$I = \iint_D \left(\overset{\to 0}{\frac{\partial g}{\partial x}} - \overset{\to 0}{\frac{\partial f}{\partial y}} \right) dA = 0$$

$\boxed{\text{Exc.}22}$ $\nabla^2(fg) = f\nabla^2 g + g\nabla^2 f + 2\nabla f \cdot \nabla g$

$$\nabla^2 = \nabla \circ \nabla(fg) = \nabla \circ \left(\frac{\partial(fg)}{\partial x}\vec{i} + \frac{\partial(fg)}{\partial y}\vec{j} + \frac{\partial(fg)}{\partial z}\vec{k} \right)$$

$$= \frac{\partial^2}{\partial x^2}(fg) + \frac{\partial^2(fg)}{\partial y^2} + \frac{\partial^2(fg)}{\partial z^2}$$

$$\nabla \circ \left(\left(g\frac{\partial f}{\partial x} + f\frac{\partial g}{\partial x} \right)\vec{i} + \left(g\frac{\partial f}{\partial y} + f\frac{\partial g}{\partial y} \right)\vec{j} + \left(g\frac{\partial f}{\partial z} + f\frac{\partial g}{\partial z} \right)\vec{k} \right)$$

$$= \underline{g_x \cdot f_x} + \underline{g f_{xx}} + \underline{f_x g_x} + \overset{\boxed{f g_{xx}}}{[illegible]} + \underline{g_y f_y} + \underline{g f_{yy}} + \underline{f_y g_y} + \boxed{f g_{yy}}$$

$$+ \underline{g_z f_z} + \underline{g f_{zz}} + \underline{f_z g_z} + \boxed{f g_{zz}} =$$

$$= f(g_{xx} + g_{yy} + g_{zz}) + g(f_{xx} + f_{yy} + f_{zz}) + 2g_x f_x + 2g_y f_y + 2g_z f_z$$

$$= f\nabla^2 g + g\nabla^2 f + 2\nabla f \cdot \nabla g$$

<u>DOKAZANO!!!</u>

$\boxed{\text{Exc.}23}$ $\nabla^2 f = 0 \Rightarrow I = \oint_C f_y\,dx - f_x\,dy$ — INDEPENDENT OF PATH $\quad 0 \quad \nabla^2 f = 0$

$$I = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA = \iint_D -f_{xx} - f_{yy}\,dA = \iint_D -\overbrace{(f_{xx} + f_{yy})}^{0}\,dA$$

$\boxed{\text{Exc. 24}}$ $C: x=\cos t \quad y=\sin t \quad z=\sin t \quad 0\le t\le 2\pi$

$$I=\int_C 2xe^{2y}dx + (2x^2e^{2y}+2y\cot z)dy - y^2\csc^2 z\, dz = ?$$

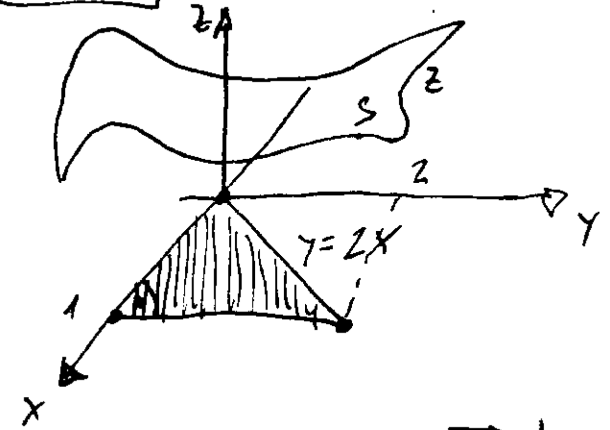
$$I=\iint_S \operatorname{curl}\vec{F}\,d\vec{S} = \int_C \vec{F}\,d\vec{r} \qquad \operatorname{curl}\vec{F}=\left(-2y\csc^2(z)+2y(1+\operatorname{ctg}^2 z)\right)\vec{i}\ \checkmark$$

$S:\ x=r\cos t \quad y=r\sin t \quad z=r\sin t \qquad r=0\cdots1,\ t=0\cdots2\pi$

$$\left.\begin{aligned}\vec{r}&=r\cos(t)\vec{i}+r\sin t\,\vec{j}+r\sin t\,\vec{k}\\ \vec{r}_r&=\cos(t)\vec{i}+\sin t\,\vec{j}+\sin t\,\vec{k}\\ \vec{r}_t&=-r\sin t\,\vec{i}+r\cos t\,\vec{j}+r\cos t\,\vec{k}\end{aligned}\right\}\ r_r\times r_t=-\rho\vec{j}+\rho\vec{k}$$

$$I=\iint_{D\ x^2+y^2\le1}\left(\underbrace{-2y\csc^2(z)+2y(1+\operatorname{ctg}^2 z)\vec{i}\circ(-\rho\vec{j}+\rho\vec{k}}_{0}\right)dA=\int_0^{2\pi}\int_0^1 0\,\rho\,d\rho\,d\theta=0$$

$\boxed{\text{Exc.25}}$ $z=x^2+2y$ TRIANGLE $(0,0);(1,0);(1,2);$



$$A(S)=\iint_D d\vec{S}=\iint_D |r_u\times r_v|\,dS$$

$$\vec{r}=x\vec{i}+y\vec{j}+(x^2+2y)\vec{k}$$
$$\vec{r}_x=\vec{i}+2x\vec{k}$$
$$\vec{r}_y=\vec{j}+2\vec{k}$$

$$|r_x\times r_y|=\begin{vmatrix}\vec{i}&\vec{j}&\vec{k}\\1&0&2x\\0&1&2\end{vmatrix}=|-2x\vec{i}\ \text{[illegible]}\ 2\vec{j}+\vec{k}|=\sqrt{4x^2+4+1}$$

$$A(S)=\int_0^1\int_0^{2x}\sqrt{4x^2+5}\,dy\,dx=\int_0^1 2x\sqrt{5+4x^2}\,dx=\frac{9}{2}-\frac{5\sqrt5}{6}$$

$\boxed{\text{Exc. 26}}$ TANGENT PLANE $(4,-2,1)$

ⓐ $r(u,v) = v^2\vec{\imath} - uv\vec{\jmath} + u^2\vec{k} \qquad 0 \leq u \leq 3 \quad -3 \leq v \leq 3$

$r_u = -v\vec{\jmath} + 2u\vec{k} \qquad \vec{r_v} = 2v\vec{\imath} - u\vec{\jmath}$

$$r_u \times r_v = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ 0 & -v & 2u \\ 2v & -u & 0 \end{vmatrix} = +2u^2\vec{\imath} + 4uv\vec{\jmath} + 2v^2\vec{k} = \vec{n}$$

PLANE: $(\vec{r} - \vec{r_0}) \circ \vec{n} = 0 \qquad \vec{r_0} = \langle 4,-2,1 \rangle$

$$\left.\begin{array}{l} v^2 = 4 \Rightarrow v = \pm 2 \\ -u \cdot v = -2 \quad u \cdot v = 2 \\ u^2 = 1 \Rightarrow u = \pm 1 \end{array}\right\} \quad \left.\begin{array}{l} v = 2 \Rightarrow u = 1 \\ u \cdot v = 2 \end{array}\right\} \text{OPT. 1} \quad \left.\begin{array}{l} v = -2 \quad u = -1 \\ u \cdot v = 2 \end{array}\right\} \text{OPT. 2}$$

$$\boxed{\vec{n}(1,2) = +2\vec{\imath} + 8\vec{\jmath} + 8\vec{k}}$$

PLANE: $\left(\langle x,y,z \rangle - \langle 4,-2,1 \rangle\right) \circ \langle +2, 8, 8 \rangle =$

$+2(x-4) + 8(y+2) + 8(z-1) = 0$

$+2x \cancel{-8} + 8y + 16 + 8z \cancel{-8} = 0$ ~~$+2x + 8y + 8z + 16 = 0$~~

~~T.P. $x - 4y - 4z - 16 = 0$~~ ~~$4z = x - 4y - 16$~~ ~~$z = \frac{1}{4}x - y - 4$~~

~~T.P. $x + 4y + 4z + 8 = 0$~~ ~~$z = -\frac{1}{4}(x + 4y + 8) = -\frac{1}{4}x - y - 2$~~

$2x + 8y + 8z + \cancel{16} - \cancel{16} = 0 \qquad \boxed{x + 4y + 4z = 0}$

$z = -\frac{1}{4}x - 7$ TANGENT PLANE

ⓒ $A(S) = \iint_D d\vec{S} = \iint_D |\vec{n}|\,ds = \iint_D |r_u \times r_v|\,ds$

$= \iint_D \sqrt{4 \cdot u^4 + 16u^2v^2 + 4v^4}\;du\,dv$ ~~$= \iint_D u\sqrt{4u^2 + 16v^2 +}$~~

$$A(S) = \int_{-3}^{3}\int_{0}^{3} \sqrt{4u^2(u^2+4v^2)+4v^4}\, du\, dv$$

ⓑ $$F(x,y,z) = \frac{z^2}{1+x^2}\vec{i} + \frac{x^2}{1+y^2}\vec{j} + \frac{y^2}{1+z^2}\vec{k}$$

$$\iint_S \vec{F}\, d\vec{S} = \iint \left(\frac{u^4}{1+v^4}\cdot 2u^2 + \frac{v^4}{1+u^2v^2}\cdot 4uv + \sqrt{\frac{u^2v^2}{1+u^4}} \cdot 2v^2 \right) du$$

$$I = \int_{-3}^{3}\int_{0}^{3} \left(\frac{2u^6}{1+v^4} + \frac{4uv^5}{1+u^2v^2} + \frac{2u^2v^4}{1+u^4} \right) du\, dv$$

$$= \underline{\underline{1524{,}01902}}$$

$$\left.\begin{array}{l} z = r\cos\theta \\ x = r\sin\theta \end{array}\right\} \begin{array}{lll} \theta = 0 & z = r & x = 0 \\ \theta = \frac{\pi}{2} & z = 0 & x = r \end{array}$$

Exe. 30 $$F(x,y,z) = x^2\vec{i} + xy\vec{j} + z\vec{k}$$

$z = x^2+y^2$ BELOW $z = 1$

$$\iint_{S_1} \vec{F}\, d\vec{S} = \iint_{D_1} \left(-P\frac{\partial z}{\partial x} - Q\frac{\partial z}{\partial y} + R \right) dA$$

$$\vec{r}_1(t) = x\vec{i} + y\vec{j} + (x^2+y^2)\vec{k}$$

$r_{1x} = \vec{i} + 2x\vec{k}$; $r_{1y} = \vec{j} + 2y\vec{k}$

$$r_{1x}\times r_{1y} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 2x \\ 0 & 1 & 2y \end{vmatrix} = -2x\vec{i} - 2y\vec{j} + 1\vec{k}$$

$$= -\frac{\partial z}{\partial x}\vec{i} - \frac{\partial z}{\partial y}\vec{j} + \vec{k}$$

$$I = \iint \left(-x^2(2x) - xy\cdot 2y + \overset{x^2+y^2}{\cancel{z}} \right) dx\, dy = \iint_D \left(-2x^3 - 2xy^2 + \overset{x^2+y^2}{\cancel{z}} \right) dx\, dy$$

MARE:

D: $x^2+y^2 \leq 1$

$$\left| \begin{array}{l} x = r\cos\theta \\ y = r\sin\theta \end{array} \right| = \int_0^{2\pi}\int_0^1 \left(-2r^3\cos^3\theta - 2r^3\cos\theta\sin^2\theta + r^2 \right) r\, dr\, d\theta$$

• DOKAZUV $\vec{S}$ e ORIENTIRANA NADOLU TOGAŠ

$$\vec{r}_{1y}\times\vec{r}_{1x} = \begin{vmatrix}\vec{i}&\vec{j}&\vec{k}\\0&1&2y\\1&0&2x\end{vmatrix} = 2x\vec{i}+2y\vec{j}\ominus\vec{k}$$

→ ORIENTACIJA NADOLU

$$I = \iint_D \left(2x^3+2xy^2 \; x^2 \; y^2\right)dx\,dy = -\frac{\pi}{2} \quad (\text{MAPLE})$$

$\boxed{\text{Exc. 72}}$ Stokes' TO CALCULATE $\iint_S \text{curl} F\, d\vec{S}$

$$F(x,y,z) = x^2yz\vec{i} + yz^2\vec{j} + z^3e^{xy}\vec{k}$$

$$S: \; x^2+y^2+z^2=5 \qquad z=1 \qquad \vec{r}(x,y) = x\vec{i}+y\vec{j}+\sqrt{5-x^2-y^2}\,\vec{k}$$

$$I=\int_C \vec{F}\,d\vec{r} = \iint_S \text{curl}\vec{F}\,d\vec{S}$$

C: $x^2+y^2+1=5 \qquad x^2+y^2=4 \qquad z=1$

C: $\boxed{\vec{r} = 2\cos\theta\,\vec{i}+2\sin\theta\,\vec{j}+\vec{k}}$ $\quad x=2\cos\theta$ $\quad y=2\sin\theta$

$$\boxed{\vec{r}\,' = -2\sin\theta\,\vec{i}+2\cos\theta\,\vec{j}}\,.$$

$$I=\int_0^{2\pi} -x^2y\cancel{z}^{1}\sin\theta + y\cancel{z}^{2\,1}\cos\theta\,d\theta = \int_0^{2\pi} 8\cos^2\theta\sin\theta+2\sin\theta\cos\theta\,d\theta$$

$$= +8\int_0^{2\pi}\cos^2\theta\,d\cos\theta \;\; 2\int_0^{2\pi}\cos\theta\,d\cos\theta = 8\frac{\cos^3\theta}{3}\Big|_0^{2\pi} - 2\frac{\cos^2\theta}{2}\Big|_0^{2\pi}$$

$$= \frac{1}{3}(1-1) - \frac{1}{2}(1-1) = 0 \qquad \iint_S \text{curl}\vec{F}\,d\vec{S}=0$$

CHECK! $\boxed{\text{curl}\vec{F} = \left(z^3xe^{xy}-2yz\right)\vec{i} + \left(x^2y-z^3ye^{xy}\right)\vec{j} - x^2z\vec{k}}$

$$I = \int_0^{2\pi}\int_0^{2} \text{curl}\vec{F}\,[\vec{r}_u\times\vec{r}_v]\,dA$$

$$\vec{r}(x,y) = x\,\vec{i} + y\,\vec{j} + \sqrt{5-x^2-y^2}\,\vec{k} \qquad x = \rho\cos\theta \quad y = \rho\sin\theta$$

$$\vec{r}(r,\theta) = \rho\cos\theta\,\vec{i} + \rho\sin\theta\,\vec{j} + \sqrt{5-\rho^2}\,\vec{k}$$

$$r_\rho \times r_\theta = \frac{\rho^2\cos\theta}{\sqrt{5-\rho^2}}\,\vec{i} + \frac{\rho^2\sin\theta}{\sqrt{5-\rho^2}}\,\vec{j} + \rho\,\vec{k}$$

~~$F(x,y,z) = F(\rho,\theta,\ldots)$~~ $$F(r(\rho,\theta)) = \rho^2\cos^2\theta\,\rho\sin\theta\,\vec{i} + \rho\sin\theta\,\vec{j} + e^{\rho^2\frac{\cos\theta}{\sin\theta}}\,\vec{k}$$

$$\underbrace{F(r(\rho,\theta))\circ(r_\rho \times r_\theta) = \frac{\rho^4\cos^3\theta\sin\theta}{\sqrt{5-\rho^2}} + \frac{\rho^3\sin^2\theta}{\sqrt{5-\rho^2}} + \rho\,e^{\rho^2\cos\theta\sin\theta}}_{\text{NE TREBA VAKA !!}}$$

TREBA: $\operatorname{curl}\left(F(r(\rho,\theta))\circ(r_\rho \times r_\theta)\right)$

OVA NE E DOBRO! ZEMAV 1 A NE TREBA TAKA TUKU TREBA SO SFERNI

$$\operatorname{curl}(F(r(\rho,\theta))) = \left(\rho\cos\theta\cdot e^{\rho^2\sin\theta\cos\theta} - 2\rho\sin\theta\right)\vec{i} + \left(\rho^3\cos^3\theta\sin\theta - \rho\sin\theta\, e^{\rho^2\frac{\sin\theta}{\cos\theta}}\right)\vec{j} - \rho^2\cos^2\theta\,\vec{k}$$

$$\int_0^{2\pi}\int_0^{2} \operatorname{curl}\vec{F}\,(r_\rho \times r_\theta)\,d\rho\,d\theta$$ } NE ZNAE DA GO REŠI MAPLE

SO SFERNI COORDINATI $\boxed{\rho=\sqrt{5}}$

$x = \cos\varphi\cdot\sin\theta \quad y = \sin\varphi\sin\theta$

$z = \sin\theta \quad z = 1 \Rightarrow \boxed{\theta = 0..\frac{\pi}{2}}$

$1 = \sin\theta$

$$|r_\varphi \times r_\theta| = \rho^2\sin\theta \qquad r_\varphi \times r_\theta = \rho^2\sin^2\theta\cos\varphi\,\vec{i} + \underbrace{\rho^2\sin^2\theta\sin\varphi\,\vec{j} + \rho^2\cos\theta\sin\theta\,\vec{k}}_{\text{NE E DOBRO}}$$

$$I = \int_0^{2\pi}\int_0^{\pi/2} \underbrace{\left(\rho\cos\theta\, e^{\rho^2\sin\theta\cos\theta} - 2\rho\sin\theta\right)}_{\text{NE E DOBRO}}\rho^2\sin^2\theta\cos\varphi + \left(\rho^3\cos^3\theta\sin\theta - \rho\sin\theta\, e^{\rho^2\frac{\sin\theta}{\cos\theta}}\right)\rho^2\sin^2\theta\sin\varphi\ \rho^4\cos^3\theta\sin\theta\ d\varphi\,d\theta$$

$$I = \int_0^{2\pi}\int_0^{\sqrt{5}} \left(\rho^2\cos^2\theta\cdot\rho\sin\theta\cos\varphi\, e^{\rho^2\sin^2\theta\sin\varphi\cos\varphi} - 2\rho^2\sin\theta\cos\theta\cdot\sin\varphi\right)\cdot\rho^2\sin^2\theta\cos\varphi +$$
$$+ \left(\rho^2\sin^4\theta\cdot\cos^2\varphi\cdot\rho\sin\theta\sin\varphi - \rho^2\cos^2\theta\cdot\rho\sin\theta\sin\varphi\, e^{\rho^2\sin^2\theta\sin\varphi\cos\varphi}\right)\cdot\rho^2\sin^2\theta\sin\varphi$$
$$- \rho^2\sin^2\theta\cos\varphi\cdot\rho\cos\theta\cdot\rho^2\cos\theta\sin\theta\ d\theta\,d\varphi$$

$\boxed{\text{Exc. 31}}$ $\vec{F} = x^2\vec{i} + y^2\vec{j} + z^2 k$ $z = 1 - x^2 - y^2$ ABOVE $z=0$

$$I = \int_C \vec{F}\, d\vec{r} = \iint_S \text{curl}\,\vec{F}\, d\vec{S} \qquad \text{curl}\,\vec{F} = 0$$

$$\iint_S \text{curl}\,\vec{F}\, dS = \int 0\, ds = 0$$

OVA TREBA DA E OD CURL, A NE OD $\vec{F}$!!

$$I = \iint_{D\ x^2+y^2\le 1} \left(-P\cdot\frac{\partial z}{\partial x} - Q\frac{\partial z}{\partial y} + R\right) dx dy = \iint_{x^2+y^2\le 1} -x^2(-2x) - y^2(-2y) + z^2\, dx dy$$

$$= \iint_D 2x^3 - 2y^3 + (1-x^2-y^2)^2 dx\, dy = \left| \begin{array}{l} \text{POLAR} \\ \text{COORDINATES} \\ x = r\cos\theta \\ y = r\sin\theta \end{array} \right| =$$

$$= \int_0^{2\pi}\int_0^1 \left(2\, r^3\cos^3\theta - 2r^3\sin^3\theta + (1-r^2)^2\right) r\, dr\, d\theta = \frac{\pi}{3}$$

OVA NEMA VEZE SO ZADAČATA

FORMULATA E OD POVRŠINSKI INTEGRALI

$$I = \int_C \vec{F}\, d\vec{r}$$

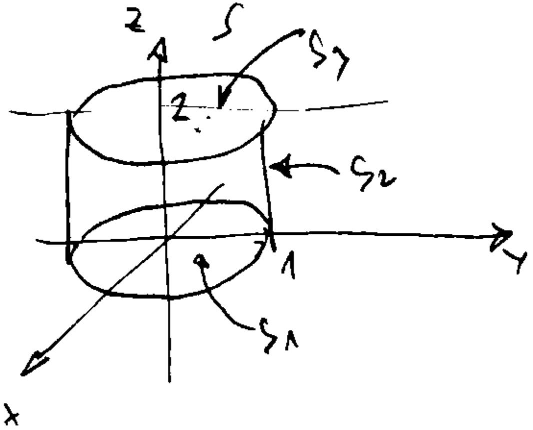
$$\boxed{\begin{array}{l} r = \cos\theta\,\vec{i} + \sin\theta\,\vec{j} + 0\,\vec{k} \\ r' = -\sin\theta\,\vec{i} + \cos\theta\,\vec{j} \end{array}}$$

$x = \cos\theta$
$y = \sin\theta$

$$I = \int_0^{2\pi} \left(\cos^2\theta\,\vec{i} + \sin^2\theta\,\vec{j} + 0\,\vec{k}\right) \circ \left(-\sin\theta\,\vec{i} + \cos\theta\,\vec{j}\right) d\theta$$

$$= \int_0^{2\pi} -\cos^2\theta\sin\theta + \sin^2\theta\cos\theta\, d\theta = 0$$

$\boxed{\text{Exc. 34}}$ $\iint_S \vec{F}\circ d\vec{S}$ $\vec{F} = x^3\vec{i} + y^3\vec{j} + z^3\vec{k}$ $E: \begin{cases} x^2+y^2=1 \text{ cilin.} \\ z=0 \\ z=2 \end{cases}$



$$I = \iint_S \vec{F}\, d\vec{S} = \iiint_E \text{div}\,\vec{F}\, dV$$

TYPE 1 SHAPE

$$\int_a^b \int_{g_1(x)}^{g_2(x)} \int_{u_1(x,y)}^{u_2(x,y)} f(x,y,z)\, dz\, dy\, dx$$

$$\operatorname{div}\vec{F} = 3x^2 + 3y^2 + 3z^2$$

$$I = \int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{0}^{2} 3x^2+3y^2+3z^2 \, dz \, dy \, dx$$

$x = r\cos\theta \qquad z = z$

$y = r\sin\theta$

$$J = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} & \frac{\partial x}{\partial z} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} & \frac{\partial y}{\partial z} \\ \frac{\partial z}{\partial r} & \frac{\partial z}{\partial \theta} & \frac{\partial z}{\partial z} \end{vmatrix} = \begin{vmatrix} \cos\theta & -r\cdot\sin\theta & 0 \\ \sin\theta & r\cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} = \underline{\underline{r}}$$

$$\left|\begin{array}{c|ccc|cc} \cos\theta & -r\sin\theta & 0 & & \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta & 0 & & \sin\theta & r\cos\theta \\ 0 & 0 & 1 & & 0 & 0 \end{array}\right| = r\cos^2\theta - 0 - 0 - r\cdot\sin^2\theta = r$$

$$I = \int_0^{2\pi}\int_0^{1}\int_0^{2} (3r^2\cos^2\theta + 3r^2\sin^2\theta + 3z^2)\, r\, dz\, dr\, d\theta = \overset{\text{MAPLE}}{\underline{\underline{11\pi}}}$$

⊛

---

so POVRŠINSKI INTEGRAL

$$I = \underbrace{\iint_{S_1} \vec{F}\, d\vec{S}}_{I_1} + \underbrace{\iint_{S_2} \vec{F}\, d\vec{S}}_{I_2} + \underbrace{\iint_{S_3} \vec{F}\, d\vec{S}}_{I_3}$$

$$I_1 = \iint \langle x^3, y^3, z^3 \rangle \cdot (-\vec{k})\, dS$$

$\vec{r} = \rho\cos\theta\,\vec{i} + \rho\sin\theta\,\vec{j} + 0\,\vec{k}$

$r_\rho \times r_\theta = \rho\,\vec{k} \qquad r_\theta \times r_\rho = -\rho\,\vec{k}$

$$I_1 = \underset{\substack{D:\\ x^2+y^2\le 1}}{\int_0^{2\pi}\int_0^{1}} \cancelto{0}{-z^3}\, dS = 0$$

$$I_3 = \iint \langle x^3, y^3, z^3 \rangle\, \overbrace{\vec{k}}^{r_\rho \times r_\theta}\, dS =$$

$$= \int_0^{2\pi}\int_0^{1} \cancelto{z^3=8}{z^3}\, \rho\, d\rho\, d\theta = 8\int_0^{2\pi}\int_0^{1} \rho\, d\rho\, d\theta = \underline{\underline{8\pi}}$$

$I_2 = \iint_D \langle x^3, y^3, z\rangle \circ (r_\theta \times r_z)\, dz\, d\theta \qquad S: \begin{cases} x^2+y^2=1 \\ z=0 \ldots 2 \end{cases}$

~~$\vec r = \cancel{g}^2\cos^2\theta\,\vec\imath + \cancel{g}^2\cdot\sin^2\theta\,\vec\jmath + z\vec k = \cos^2\theta\,\vec\imath + \sin^2\theta\,\vec\jmath + z\vec k$~~

~~$r_\theta \times r_z = 2\cos\theta\sin\theta\,\langle 1, 1, 0\rangle = 2\cos\theta\sin\theta\,\vec\imath + 2\cos\theta\sin\theta\,\vec\jmath$~~

~~$I_2 = \int_0^{2\pi}\int_0^2 \cos^6\theta\cdot 2\cos\theta\sin\theta + \sin^6\theta\cdot 2\cos\theta\sin\theta\; dz\, d\theta =$~~

~~$= 4\int_0^{2\pi} \sin\theta\cdot\cos\theta\left(\cos^6\theta + \sin^6\theta\right) d\theta =$~~

~~$= -4\int_0^{2\pi} \cos^7\theta\; d(\cos\theta) + 4\int_0^{2\pi} \sin^7\theta\; d(\sin\theta) =$~~

~~$= -4\,\frac{\cos^8\theta}{8}\Big|_0^{2\pi} \overset{1-1=0}{} + 4\cdot\frac{\sin^8\theta}{8}\Big|_0^{2\pi} \overset{0}{} = 0$~~

$\boxed{r = \cos\theta\,\vec\imath + \sin\theta\,\vec\jmath + z\vec k}$

$r_\theta = -\sin\theta\,\vec\imath + \cos\theta\,\vec\jmath \qquad r_z = \vec k$

$r_\theta \times r_z = \cos\theta\,\vec\imath + \sin\theta\,\vec\jmath$

$I_2 = \int_0^{2\pi}\int_0^2 \left(\cos^3\theta\,\vec\imath + \sin^3\theta\,\vec\jmath\right)\cdot \overbrace{\langle\cos\theta, \sin\theta\rangle}^{r_\theta\times r_z}\, dz\, d\theta$

$= \int_0^{2\pi}\int_0^2 \cos^4\theta + \sin^4\theta\; dz\, d\theta = 2\int_0^{2\pi}\left(\cos^4\theta + \sin^4\theta\right)d\theta \overset{\text{MAPLE}}{=} \underline{\underline{3\pi}}$

$I = I_1 + I_2 + I_3 = 0 + 8\pi + 3\pi = 11\pi \Rightarrow$ <u>ISTO SO</u> ⊗!!!

$\boxed{Exc. 35}$ $F(x,y,z) = x\vec{i} + y\vec{j} + z\vec{k}$ $\quad E: x^2+y^2+z^2 \le 1$

$2\pi\cdot 2r = 4\pi r$

$$I = \iint_S \vec{F}\, d\vec{S} = \iiint_E div\vec{F}\, dV \qquad divF = 1+1+1 = 3$$

$$I = 3\iiint_E dV = \cancel{3}\cdot\frac{4\overset{\nearrow 1}{r^3}\pi}{\cancel{3}} = 4\pi \quad ①$$

$x = r\sin\theta\cos\varphi$
$y = r\sin\theta\sin\varphi$
$z = r\cos\theta$
$|r_\theta \times r_\varphi| = r^2\sin\theta$

a — RADIUS NA SFERA

$$I = 3\int_0^{2\pi}\int_0^{\pi}\int_0^{a} r^2\sin\theta\, dr\, d\theta\, d\varphi = 3\cdot 2\pi \int_0^{\pi}\sin\theta\, d\theta \int_0^{1} r^2\, dr$$

$$= 3\cdot 2\pi(-\cos\theta)\Big|_0^{\pi}\ \frac{r^3}{3}\Big|_0^{a} = 3\cdot 2\pi\cdot 2\cdot\frac{a^3}{3} = 3\cdot\boxed{\frac{4\pi a^3}{3}}$$ — VOLUMEN NA SFERA

$$A(S) = \iint_S d\vec{S} = \int_0^{2\pi}\int_0^{\pi} |r_\theta\times r_\varphi|\, d\theta\, d\varphi = \int_0^{2\pi}\int_0^{\pi} a^2\sin\theta\, d\theta\, d\varphi$$

$$= 2\pi a^2\int_0^{\pi}\sin\theta\, d\theta = 2\pi a^2\cdot(-\cos\theta)\Big|_0^{\pi} = \boxed{4\pi a^2}$$ — POVRŠINA NA SFERA !!

$$\iint_S \vec{F}\, d\vec{S} = \int_0^{2\pi}\int_0^{\pi}(x\vec{i} + y\vec{j} + z\vec{k})\circ(\sin^2\theta\cos\varphi\,\vec{i} + \sin^2\theta\sin\varphi\,\vec{j} + \sin\theta\cos\theta\,\vec{k})\, d\theta\, d\varphi$$

$$= \int_0^{2\pi}\int_0^{\pi}\underbrace{\underbrace{\sin\theta\cos\varphi\cdot\sin^2\theta\cos\varphi}_{\sin^3\theta\cos^2\varphi} + \underbrace{\sin\theta\sin\varphi\sin^2\theta\sin\varphi}_{\sin^3\theta\sin^2\varphi}}_{\sin^3\theta} + \sin\theta\cos^2\theta\, d\theta\, d\varphi$$

$$= \int_0^{2\pi}\int_0^{\pi}\sin\theta\,(\underbrace{\sin^2\theta + \cos^2\theta}_{1})\, d\theta\, d\varphi = 2\pi\cdot(-\cos\theta)\Big|_0^{\pi} = 4\pi$$

ISTO SO ①

$\boxed{\text{Exc. 36}}$ $F(x,y,z) = \dfrac{x\vec{i}+y\vec{j}+z\vec{k}}{(x^2+y^2+z^2)^{3/2}}$ $4x^2+9y^2+6z^2=36$

ELIPSOID: $\left(\dfrac{x}{\frac{1}{4}}\right)^2 + \left(\dfrac{y}{\frac{1}{3}}\right)^2 + \left(\dfrac{z}{\frac{1}{\sqrt{6}}}\right)^2 = 36$

$$\frac{4x^2}{36}+\frac{9y^2}{36}+\frac{6z^2}{36}=1 \Rightarrow \frac{x^2}{9}+\frac{y^2}{4}+\frac{z^2}{6}=1$$

$$\boxed{\left(\frac{x}{3}\right)^2+\left(\frac{y}{2}\right)^2+\left(\frac{z}{\sqrt{6}}\right)^2=1}$$

$$\boxed{\begin{aligned} x &= 3\sin\theta\cdot\cos\varphi \\ y &= 2\cdot\sin\theta\sin\varphi \\ z &= \sqrt{6}\cdot\cos\theta \end{aligned}}$$

$\boxed{\operatorname{div}\vec{F}=0}$ ← MAKE

OUTWARD FLUX

$$I = \iint_S \vec{F}\,d\vec{S} = \iiint_E \underset{0}{\operatorname{div} F}\,dV \Rightarrow \underline{I=0}$$

~~$x^2+y^2+z^2 = 9\sin^2\theta\cdot\cos^2\varphi + 4\sin^2\theta\cdot\sin^2\varphi + 6\cos^2\theta$~~

~~$\sin^2\theta(9\cos^2\varphi + 4\sin^2\varphi) + 6\cos^2\theta$~~

POVRŠINA NA ELIPSOID

$$A(S) = \iint_S d\vec{S} = \iint_D |r_\theta \times r_\varphi|\,d\theta d\varphi$$

$$r_\theta \times r_\varphi = 2\sqrt{6}\sin^2\theta\cos\varphi\,\vec{i} + 3\sqrt{6}\sin^2\theta\sin\varphi\,\vec{j} + 6\cos\theta\sin\theta\,\vec{k}$$

$$|r_\theta\times r_\varphi| = \sqrt{6}\sqrt{\sin^2\theta(-5\cos^2\varphi + 5\cos^2\varphi\cdot\cos^2\theta + 9 - 3\cos^2\theta)} =$$

$$= \sqrt{6}\sqrt{\sin^2\theta\left[5\cos^2\varphi\underbrace{(\cos^2\theta-1)}_{-\sin^2\theta} + 9 - 3\cos^2\theta\right]}$$

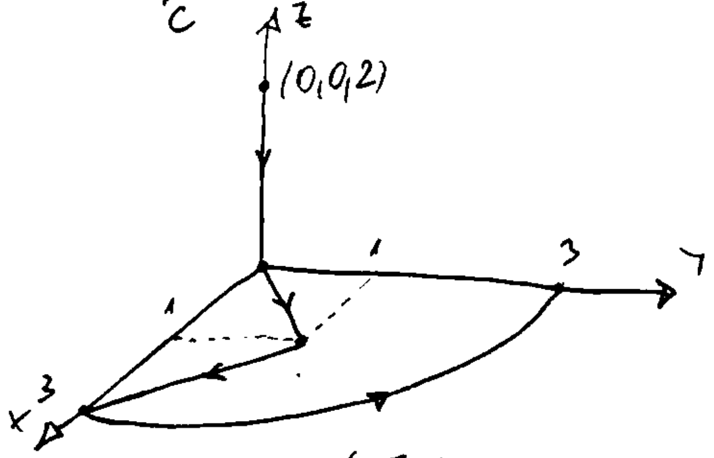
$$= \sqrt{6}\sqrt{\sin^2\theta\left[-5\sin^2\theta\cdot\cos^2\varphi + 9 - 3\cos^2\theta\right]}$$

$$\int_0^{2\pi}\int_0^{\pi}\sqrt{6}\sqrt{\sin^2\theta(9-3\cos^2\theta-5\sin^2\theta\cos^2\varphi}\,d\theta\,d\varphi =$$

MAKE NE ZNAE DA GO REŠI

$\boxed{\text{Exc. 37}}$ $F(x,y,z) = (3x^2yz - 3y)\vec{i} + (x^2z - 3x)\vec{j} + (x^2y + 2z)\vec{k}$

$$\int_C \vec{F}\,d\vec{r} = ?$$



$$\iint_S \text{curl}\,\vec{F}\,d\vec{S} = \int_C \vec{F}\,d\vec{r}$$

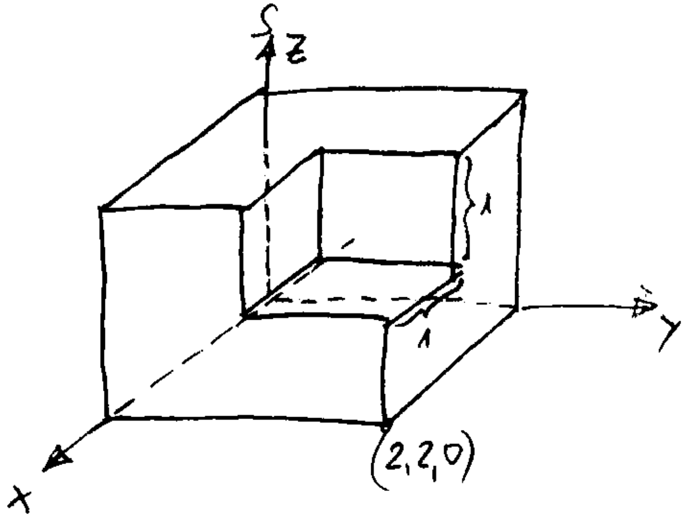
MAPLE

$$\boxed{\text{curl}\,\vec{F} = 0} \Rightarrow$$

$$\int_C \vec{F}\,dr = 0$$

$\boxed{\text{Exc. 38}}$ $\oint_C \vec{F}\,d\vec{r} = ?$ $\text{curl}\,\vec{F} = 0 \Rightarrow \oint_C \vec{F}\,d\vec{r} = \underset{=0}{\iint_S \text{curl}\,\vec{F}\,d\vec{S}}$

$\boxed{\text{Exc. 39}}$ $\iint_S \vec{F}\circ\vec{n}\,dS$ $\vec{F}(x,y,z) = x\vec{i} + y\vec{j} + z\vec{k}$



$$\text{div}\,\vec{F} = 1 + 1 + 1 = 3$$

$$I = \iint_S \vec{F}\,d\vec{S} = \iiint_E \text{div}\,\vec{F}\,dV$$

$$I = 3\iiint_E dV = 3\cdot V(E)$$

- VOLUMEN NA KOCKA SO STRANA a :

$V = a^3$ $V = V_2 - V_1 = a_2^3 - a_1^3 = 2^3 - 1 = 8 - 1 = 7$

$$\boxed{I = 3\cdot V(E) = 3\cdot 7 = 21}$$

$\boxed{\text{Exc 40}}$ $\iint_S \text{curl}\,\vec{F}\cdot d\vec{S} = 0$ I $\iint_S \text{curl}\,\vec{F}\,d\vec{S} = \iiint_E \underset{=0}{\text{div}(\text{curl}\,\vec{F})}\,dV$

$$\text{curl}\,\vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ P & Q & R \end{vmatrix} = \left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\right)\vec{i} + \left(\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}\right)\vec{j} + \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)\vec{k}$$

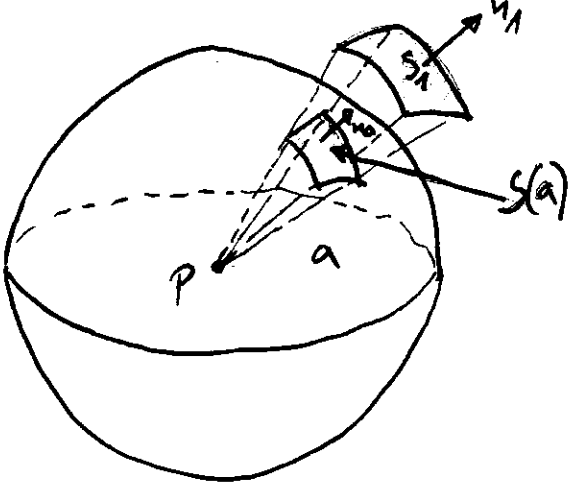
$$\text{div}(\text{curl}\,\vec{F}) = \cancel{\frac{\partial^2 R}{\partial x\partial y}} - \cancel{\frac{\partial^2 Q}{\partial x\partial z}} + \frac{\partial^2 P}{\partial y\partial z} - \cancel{\frac{\partial^2 R}{\partial y\partial x}} + \cancel{\frac{\partial^2 Q}{\partial z\partial x}} - \cancel{\frac{\partial^2 P}{\partial z\partial y}} = 0$$

PROBLEMS PLUS: $\Omega(S)$ SOLID ANGLE

- MEASURE OF SOLID ANGLE IN STERADIANS IS:

$$|\Omega(S)| = \frac{\text{area of } S(a)}{a^2}$$

SHOW THAT $|\Omega(S)| = \iint\limits_S \frac{\vec{r}\circ\vec{n}}{r^3}\,ds$



$$\iint\limits_S \vec{F}\,d\vec{s} = \iiint\limits_E \underline{\operatorname{div}\vec{F}}\,dV$$

$$\iint\limits_S \frac{\vec{r}}{r^3}\,d\vec{s} = \iiint\limits_E \operatorname{div}\left(\frac{\vec{r}}{r^3}\right)dV$$

$\boxed{r=|r|}$ $\boxed{S: \vec{r} = r_x\vec{i} + r_y\vec{j} + r_z\vec{k}}$

$$\vec{r} = \rho\sin\theta\cos\varphi\,\vec{i} + \rho\sin\theta\sin\varphi\,\vec{j} + \rho\cos\theta\,\vec{k}$$

$$|r| = \rho^2\cdot\sin\theta$$

AND: $\boxed{\frac{\vec{r}}{r^3} = \frac{x\vec{i} + y\vec{j} + z\vec{k}}{\sqrt{(x^2+y^2+z^2)^3}} \qquad \boxed{\operatorname{div}\vec{r}/r^3 = 0}}$

$$\iint\limits_{S_V} \vec{F}\,d\vec{s} = \iint\limits_{S_V} \frac{\vec{r}}{r^3}\,d\vec{s} = \iiint\limits_E \operatorname{div}\frac{r}{r^3} = 0$$

BOUNDARY SURFACE OF "E" $\boxed{S_V = S \cup S_a}$

$$\iint\limits_S \frac{\vec{r}\circ\vec{n}_1}{r^3}\,ds + \iint\limits_{S_a} \frac{r\circ(-\vec{n}_a)}{r^3}\,ds = 0$$

$$\underbrace{\vec{n}_a = \frac{\vec{r}}{|r|}}_{\text{SFERA}}$$

$$\iint\limits_S \frac{\vec{r}}{r^3}\,d\vec{s} = \iint\limits_{S_a} \frac{\vec{r}\circ\vec{n}_a}{r^3}\,ds$$

$$\iint_{S_a} \frac{\vec{r}\cdot}{r^3}\frac{\vec{r}}{r}\, ds = \iint_{S_a} \frac{r^2}{r^4}\, ds = \frac{1}{a^2}\iint_{S_a} ds = \underline{\underline{\frac{\text{area}(S_a)}{a^2}}}$$

$$\boxed{|\Omega(S)| = \frac{\text{area of } S(a)}{a^2} = \iint_S \frac{\vec{r}\cdot\vec{n}}{r^3}\, ds}$$

<u>DOKAZANO!!!</u>

PROBLEM 2 $C = ?$ FOR WHICH

$$\int_C (y^3 - y)\, dx - 2x^3\, dy = \text{MAX VALUE}$$

$$\int_C P\,dx + Q\,dy = \iint_D \underbrace{\left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)}_{f(x,y)} dA$$

$P = y^3 - y$ $Q = -2x^3$ $\frac{\partial P}{\partial y} = 3y^2 - 1$ $\frac{\partial Q}{\partial x} = -6x^2$

$$\boxed{f(x,y) = -6x^2 - 3y^2 + 1}$$

$x = x(t)$
$y = y(t)$

~~$\boxed{\frac{\partial f}{\partial t} = 12x(t)\cdot x'(t) - 2y(t) = 0}$~~

F: $\underline{x = r\cos\theta \quad y = r\sin\theta}$
$D \Rightarrow x^2 + y^2 = 1$

$$I > \int_0^{2\pi}\int_0^1 \left(-6r^2\cos^2\theta - 3r^2\sin^2\theta + 1\right) r\,dr\,d\theta$$

~~$-6r^2\cos^2\theta - 3r^2\sin^2\theta = 0$ $3r^2\sin^2\theta = -6r^2\cos^2\theta$~~

~~$\tan^2\theta = -2$ $\boxed{\tan\theta = j\sqrt{2}}$?~~

~~$\frac{d\, I(r,\theta)}{dr} = 0$ $\underbrace{\int_0^{2\pi}\int_0^1 \frac{d}{dr}\left[\left(1 - 3r^2\sin^2\theta - 6r^2\cos^2\theta\right)r\right] dr\,d\theta}$~~

~~$\frac{d}{dr}\left[r - 3r^3\sin^2\theta - 6r^3\cos^2\theta\right] = 0$ $1 - 9r^2\sin^2\theta - 24r^2\cos^2\theta = 0$~~

$3r^2(3\sin^2\theta + 6\cos^2\theta) = 1 \longleftarrow 9r^2\sin^2\theta + 24r^2\cos^2\theta = 1$

$r^2 = 1/(9\sin^2\theta + 24\cos^2\theta)$

~~$r^2 = \frac{1}{9\sin^2\theta + 24\cos^2\theta}$ $\theta = 0$ $r = \frac{1}{\sqrt{24}}$ $\theta = \pi/2$ $r = 1/\sqrt{9}$~~

~~$\frac{d}{d\theta}(9\sin^2\theta + 24\cos^2\theta) = 18\sin\theta \cdot \cos\theta - 48\cos\theta\sin\theta =$~~

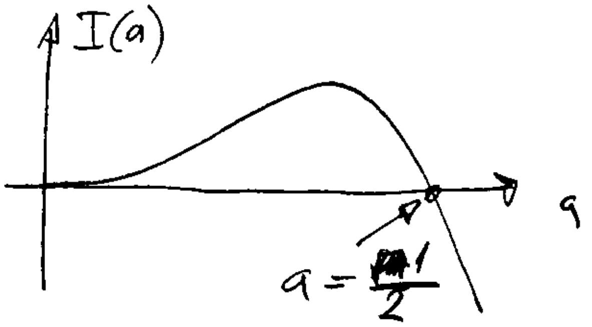
$$I(a) = \int_0^{2\pi}\int_0^a (1 - 6r^2\cos^2\theta - 3r^2\sin^2\theta)\, r\, dr\, d\theta =$$
$$= a^2\pi - 2a^4\pi$$

$$\frac{dI(a)}{da} = 2a\pi - 8a^3\pi = 0 \qquad 2a\pi(\cancel{\pi} - 4a^2) = 0$$

$a = 0$ #

$4a^2 = \cancel{\pi}$ $\quad a^2 = \frac{\cancel{\pi}}{4}$; $\boxed{a = \pm\frac{1}{2}}$ ~~$a = \pm\sqrt{\frac{\pi}{4}} = \pm\frac{\sqrt{\pi}}{2}$~~

$$\boxed{C: \quad x^2 + y^2 = a^2 \qquad a = \frac{1}{2}}$$ SOLUTION!!!



PROBLEM. 3 $\vec{n} = \langle a, b, c\rangle$ } VNIMAVAJ NE E UNIT NORMAL!!! (AKO TAKA PISUVA VO KNIGATA)

C lies IN PLANE WITH NORMAL $\vec{n}$

SHOW THAT PLANE AREA ENCLOSED BY C IS:

$$I = \frac{1}{2}\int_C (bz - cy)\,dx + (cx - az)\,dy + (ay - bx)\,dz \quad ⊛$$

$$I = \int_C P\,dx + Q\,dy = \iint_D \underbrace{\left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)}_{1} dA = \iint_D dA = A(D)$$

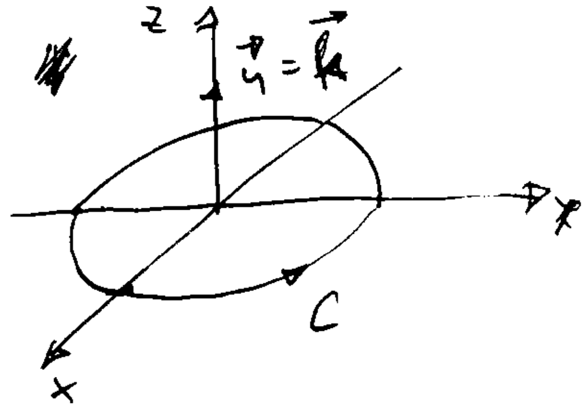
$Q = x \quad P = 0$ $\qquad I = \int_C x\,dy$

$Q = 0 \quad P = -y$ $\qquad I = -\int_C y\,dx$

$Q = \frac{x}{2} \quad P = -\frac{y}{2}$ $\qquad \boxed{I = \frac{1}{2}\int_C x\,dy - y\,dx}$

$$\iint_S \operatorname{curl}\vec F\, d\vec S = \int_C \vec F\, d\vec r \qquad A(S)=\iint_D |\vec n|\, dA = \iint_D |r_u \times r_v|\, dA$$

$$\iint_D f(x,y,g(x,y))\,\underline{\underline{\sqrt{1+g_x^2+g_y^2}}}\, dA = \iint_S f(x,y,z)\, dS$$



$z = 0$ PLANE

$\vec n = \langle \underset{a}{0}, \underset{b}{0}, \underset{c}{1} \rangle$

OD ⊛

$$I = \frac{1}{2}\int_C -y\,dx + x\,dy$$

AA

$$I = \int_C \overbrace{P\,dx + Q\,dy + R\,dz}^{\vec F\cdot d\vec r} = \iint_S \operatorname{curl}\vec F\, d\vec S = \iint_S \operatorname{curl}\vec F\cdot\vec n\, dS$$

$$\vec F = \overbrace{\left(\frac{bz-cy}{2}\right)}^{P}\vec i + \overbrace{\left(\frac{cx-az}{2}\right)}^{Q}\vec j + \overbrace{\left(\frac{ay-bx}{2}\right)}^{R}\vec k$$

$$\operatorname{curl}\vec F = \begin{vmatrix} \vec i & \vec j & \vec k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ P & Q & R \end{vmatrix} = \left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\right)\vec i + \left(\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}\right)\vec j + \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)\vec k$$

$$\operatorname{curl}\vec F = \frac{1}{2}(a+a)\vec i + \frac{1}{2}(b-(-b))\vec j + \frac{1}{2}(c+c)\vec k = \frac{2a}{2}\vec i + \frac{2b}{2}\vec j + \frac{2c}{2}\vec k$$
$$= a\vec i + b\vec j + c\vec k$$

$$I = \iint_S \operatorname{curl}\vec F\, d\vec S = \iint_S \langle a,b,c\rangle\, d\vec S = \iint_S \langle a,b,c\rangle \cdot d\vec S$$

~~$\neq \iint_S a^2+b^2+c^2\, dS \neq \iint_S |\vec n|^2\, dS$~~ ~~$\dfrac{\langle a,b,c\rangle}{\sqrt{a^2+b^2+c^2}}$~~

~~$a(x-x_0) + b(y-y_0) + c(z-z_0) = 0 \qquad cz - cz_0 = a(x-x_0) + b(y-y_0)$~~

~~$z = \frac{a}{c}(x-x_0) + \frac{b}{c}(y-y_0) + cz_0$~~

~~$\boxed{\frac{\partial z}{\partial x} = \frac{a}{c} \; ; \; \frac{\partial z}{\partial y} = \frac{b}{c}}$~~

$$I = \iint_S \text{curl}\vec{F}\, d\vec{S} = \iint_S \langle a, b, c\rangle \circ \vec{n}\, dS \qquad \vec{n} = \frac{\langle a, b, c\rangle}{\cancel{\sqrt{a^2+b^2+c^2}}}$$

$$I = \iint_S (a\vec{i} + b\vec{j} + c\vec{j}) \circ \frac{a\vec{i} + b\vec{j} + c\vec{j}}{\cancel{\sqrt{a^2+b^2+c^2}}}\, dS =$$

UNIT VECTOR $|\vec{n}|^2 = 1$

$$= \iint_S \frac{a^2+b^2+c^2}{\cancel{\sqrt{a^2+b^2+c^2}}}\, dS = \iint_S a^2+b^2+c^2\, dS = \cancel{\iint |\vec{n}|^2\, dS}$$

$$= \iint_S |\cancel{\ \ }|\, dS = A(S)$$

(**)

C: $x = \cos\theta \quad y = \sin\theta \quad z = \sin\theta$

S: $x = r\cos\theta \quad y = r\sin\theta \quad z = r\sin\theta$

$$\boxed{r_r \times r_\theta = -r\vec{j} + r\vec{k}}$$

$(\vec{r} - \vec{r_0}) \circ \vec{n} = 0 \Rightarrow$ PLANE

$\vec{r_0} = \langle 0, 0, 0\rangle \qquad \langle x, y, z\rangle \circ \langle 0, \overset{-1}{-r}, \overset{1}{+r}\rangle = 0$

$\boxed{-y + z = 0}$ $\boxed{z = y}$ PLANE!! $\boxed{a = 0 \quad b = -1 \quad c = 1} \Rightarrow$ NE VAKA TREBA $-\frac{1}{\sqrt{2}}$ i $\frac{1}{\sqrt{2}}$

$$A(S) = \iint_S |\cancel{\ \ }|\, d\vec{S} = \iint_D |r_r \times r_\theta|\, dA = \int_0^{2\pi}\int_0^1 |-r\vec{j} + r\vec{j}|\, dr\, d\theta$$

$$= \int_0^{2\pi}\int_0^1 \sqrt{r^2+r^2}\, dr\, d\theta = 2\pi \int_0^1 r\sqrt{2}\, dr = 2\pi \frac{r^2}{2}\Big|_0^1 \sqrt{2} = \frac{2\pi\sqrt{2}}{2}$$

$$= \pi\sqrt{2}$$

OD DRUGA STRANA: NE TREBA VAKA!! TREBA UNIT NORMAL T.E. $-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$ VIDI P.33

$$I = \frac{1}{2}\int_C (\underset{-1}{b}z - \underset{1}{c}y)dx + (\underset{1}{c}x - \underset{0}{a}z)dy + (\underset{0}{a}y - \underset{-1}{b}x)dz =$$

$$= \frac{1}{2}\int_C (z + y)dx + x\,dy + x\,dz = \frac{1}{2}\int_0^{2\pi} \cancel{(\sin\theta)\sin\theta\, d\theta} + \cos\theta\cos\theta\, d\theta + \cos\theta\cos\theta\, d\theta \quad ①$$

$$= \frac{1}{2}\int_0^{2\pi} \cos^2\theta + 1\, d\theta = \frac{3\pi}{2}$$

• SEPAK AKO „$\vec{n}$" E DEFINIRANO VAKA:

$$n = \frac{\langle a, b, c\rangle}{\sqrt{a^2+b^2+c^2}} \qquad I = \iint_S \operatorname{curl} \vec{F}\, d\vec{S} = \iint_S \langle a, b, c\rangle \frac{\langle a, b, c\rangle}{\sqrt{a^2+b^2+c^2}} dS$$

$$I = \iint_S \frac{(a^2+b^2+c^2)}{\sqrt{a^2+b^2+c^2}} dS = \iint_S \underbrace{\sqrt{a^2+b^2+c^2}}_{|n|} dS = A(S)$$

?

$$|n| = \sqrt{\frac{a^2}{a^2+b^2+c^2} + \frac{b^2}{a^2+b^2+c^2} + \frac{c^2}{a^2+b^2+c^2}} = 1$$

SEPAK POKAZOT E KAKO (4)

$$|n|^2 = 1$$

$$I = \iint_S \operatorname{curl} \vec{F}\, d\vec{S} = \iint_S \langle a, b, c\rangle \circ \langle a, b, c\rangle\, dS = \iint_S \cancel{a^2+b^2+c^2}^{\,1}\, dS$$

$$\boxed{I = \iint_S dS = A(S)}$$

$$\boxed{\iint_S 1\, dS = \iint_D |r_u + r_v|\, dA = A(S)}$$

---

① $$I = \frac{1}{2}\oint_C (-z - y)dx + x\,dy + x\,dz = \frac{1}{2}\int_0^{2\pi} (-\sin\theta - \sin\theta)(-\sin\theta)d\theta + \cos\theta\cos\theta\, d\theta + \cos\theta \cdot \cos\theta\, d\theta$$

$$I = \frac{1}{\cancel{2}}\int_0^{2\pi} \cancel{2}\sin^2\theta + \cancel{2}\cos^2\theta\, d\theta = \underline{\underline{2\pi}}$$

AREA OF ELIPSE
WITH a=2 b=1

VIDI (4*) NA N.7.11.22

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(11) $\boxed{S = r^2\pi = \pi}$

$y = \sin\theta$
$x = \cos\theta$

ZA SLUČAJ

$x = \cos\theta$
$y = \sin\theta$
$z = 0$

$$I = \frac{1}{2}\int_0^{2\pi} + \sin\theta\sin\theta\, d\theta + \cos\theta \cdot \cos\theta\, d\theta = \frac{2\pi}{2} = \pi$$

IF $y = a\sin\theta \quad x = a\cos\theta$

$$I = \frac{1}{2}\int_0^{2\pi} a^2\sin^2\theta\, d\theta + a^2\cos^2\theta\, d\theta = \pi a^2$$

S: $r(\rho,\theta) = \rho\cos\theta\,\vec{i} + \rho\sin\theta\,\vec{j} + 0\,\vec{k}$

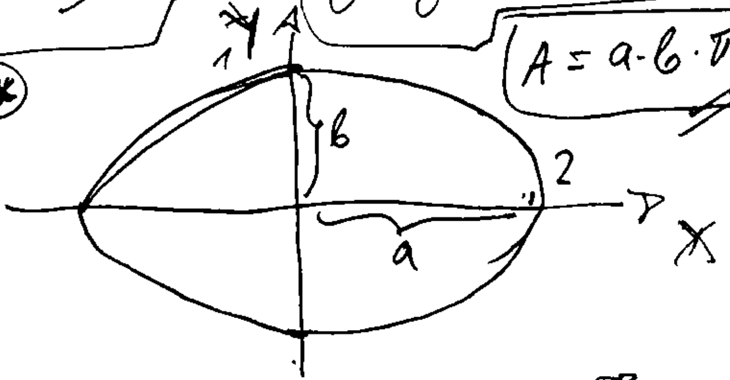
$r_\rho \times r_\theta = \rho\,\vec{k}$ $\quad |r_\rho \times r_\theta| = \rho$

ISTO SO (4A) SMARISKI DOKAZANO 6

$$\iint_S d\vec{s} = \int_0^{2\pi}\int_0^{a} \rho\,d\rho\,d\theta = 2\pi\cdot\frac{\rho^2}{2}\Big|_0^a = \pi\rho^2$$

---

④



$\boxed{A = a\cdot b\cdot\pi}$

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$$

$a = 2 \quad b = 1$

$x = 2\cos\theta$

$y = $ [illegible] $\sin\theta$

$r = 2\rho\cos\theta\,\vec{i} + \rho\sin\theta\,\vec{j}$

$r_\rho \times r_\theta = 2r\,e_z$ $\quad |r_\rho \times r_\theta| = 2r$

$$\iint_S ds = \iint_{D} 2r\,dr\,d\theta = \int_0^{2\pi}\int_0^{1} 2r\,dr\,d\theta = \cancel{2}\cdot 2\pi\cdot\frac{r^2}{\cancel{2}}\Big|_0^1 = 2\pi$$

$D: \left(\frac{x^2}{2}\right) + \left(\frac{y^2}{1}\right) \le 1$

$x = \rho\cos\theta \quad y = \rho\sin\theta \quad z = \rho\sin\theta$

$r(\rho,\theta) = \rho\cos\theta\,\vec{i} + \rho\sin\theta\,\vec{j} + \rho\sin\theta\,\vec{k}$

$$r_\rho \times r_\theta = \begin{vmatrix} \vec{i} & \vec{j} & k \\ \cos\theta & \sin\theta & \sin\theta \\ -\rho\sin\theta & \rho\cos\theta & \rho\cos\theta \end{vmatrix} = (\cancel{\rho\sin\theta\cos\theta} - \cancel{\rho\sin\theta\cos\theta})\vec{i} + (-\rho\sin^2\theta - \rho\cos^2\theta)\vec{j} + \rho\cos^2\theta + \rho\sin^2\theta\,\vec{k}$$

$$= -\rho\,\vec{j} + \rho\,\vec{k}$$

OK!!!

$$|r_\rho \times r_\theta| = \sqrt{\rho^2 + \rho^2} = \rho\sqrt{2}$$

$$\iint_S d\vec{s} = \int_0^{2\pi}\int_0^{1} |r_\rho \times r_\theta|\,d\rho\,d\theta = 2\pi\int_0^1 \rho\sqrt{2}\,d\rho = \cancel{2}\pi\,\frac{\rho^2}{\cancel{2}}\Big|_0^1\cdot\sqrt{2} = \pi\sqrt{2}$$

[redacted]

NE MI E ~~JASNO~~ ZOŠTO NE IZLEGUVA IDENTIČNO SO ④② T.E. ④ [redacted]

$\cancel{2}a = \sqrt{2^2 + 2^2} = \sqrt{8} = \cancel{2}\sqrt{2}$ $\quad \boxed{a = \sqrt{2}}$

VIDI MARKE!!

SEPAK POLGOLEMA OSKA NA ELIPSATA E $a = \sqrt{2}$ TOGAŠ

④② $\Rightarrow$ $A = a\cdot b\cdot\pi = \sqrt{2}\,\pi$

ZNAČI OVA E TOČNO A NE ČINI ④

- IF $\boxed{a=0 \quad b=-\frac{1}{\sqrt{2}} \quad c=\frac{1}{\sqrt{2}}}$ VNIMAVA!!! VAKA TREBA UNIT NORMAL

$$|u| = \sqrt{a^2+b^2+c^2} = \sqrt{\frac{1}{2}+\frac{1}{2}} = \sqrt{1} = 1 \checkmark$$

$$I = \frac{1}{2}\int_C (\underset{-\frac{1}{\sqrt{2}}}{b}z-\underset{\frac{1}{\sqrt{2}}}{c}y)dx + (\underset{\frac{1}{\sqrt{2}}}{c}x-\underset{0}{a}z)dy + (\underset{0}{a}y-\underset{-\frac{1}{\sqrt{2}}}{b}x)dz$$

$$I = \frac{1}{2}\int_C \left(-\frac{z}{\sqrt{2}}-\frac{y}{\sqrt{2}}\right)dx + \frac{x}{\sqrt{2}}dy + \frac{x}{\sqrt{2}}dz$$

$x = \cos\theta \qquad y = \sin\theta \qquad z = \sin\theta$

$$I = \frac{1}{2}\int_0^{2\pi}\Big[_{2\pi} -\left(-\frac{2\sin\theta}{\sqrt{2}}\right)\sin\theta + \frac{\cos\theta\cos\theta}{\sqrt{2}} + \frac{\cos\theta\cos\theta}{\sqrt{2}}\Big]d\theta$$

OK!!!

$$= \frac{1}{\cancel{2}}\frac{1}{\sqrt{2}}\int_0^{2\pi} \cancel{2}\sin^2\theta + \cancel{2}\cos^2\theta \, d\theta = \frac{1}{\sqrt{2}}\int_0^{2\pi} d\theta = \frac{2\pi}{\sqrt{2}} = \underline{\underline{\sqrt{2}\pi}}$$

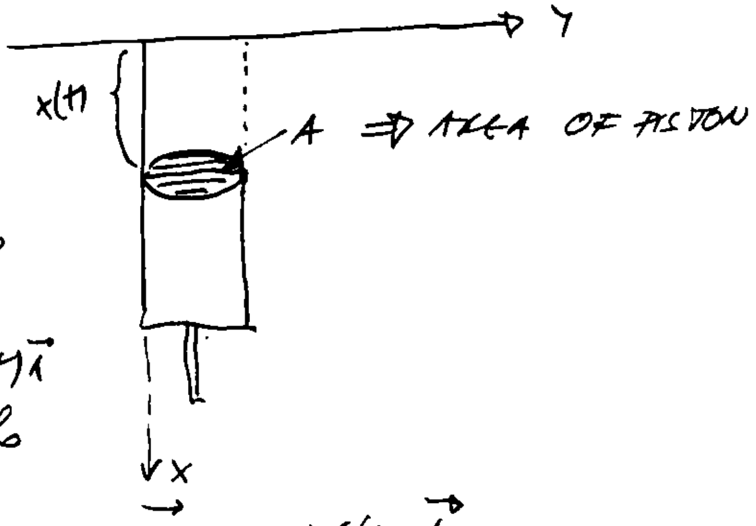
DOKAZAHMO EMPIRISKI !!!

PROBLEM 4 $P(t)$ & $V(t)$ PRESSURE AND VOLUME WITHIN A CYLINDER AT TIME t

$a \le t \le b$ } TIME FOR COMPLETE CYCLE

ⓐ SHOW THAT THE WORK DONE DURING ONE CYCLE IS

$$W = \int_C P\,dV$$



$\vec{F} = A \cdot P(t)\vec{i}$ } FORCE ON THE PISTON

$$W = \int_{C_1} \vec{F}\cdot d\vec{r} \qquad C_1: \vec{r}(t) = x(t)\vec{i}, \quad a \le t \le b$$

$$W = \int_{a_1}^{b} \left(A \cdot P(t)\vec{i}\right)\cdot \vec{r}'(t)\,dt \qquad \vec{r}'(t) = x'(t)\vec{i}$$

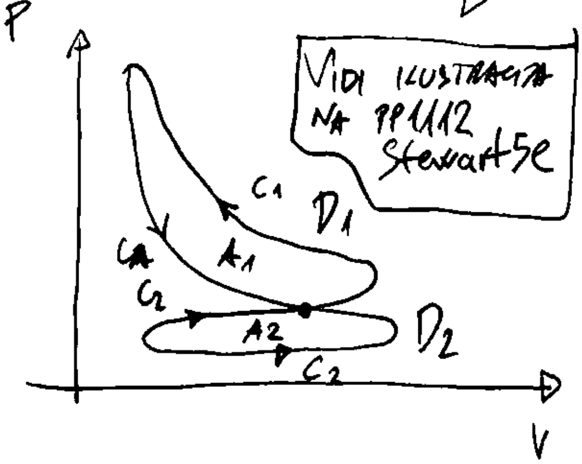
$$W = \int_a^b A\cdot P(t)\cdot x'(t)\,dt = \int_a^b P(t)\underbrace{A\cdot x'(t)\,dt}_{dV} = \int_a^b P(t)\,dV = \int_C P(t)\,dV$$

⑥ $A = \oint_C x\,dy = -\oint_C y\,dx = \frac{1}{2}\oint_C -y\,dx + x\,dy$

$$\iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA = \int_C P\,dx + Q\,dy$$

$$W = \int_C P\,dV = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA = \iint_{D_1}\left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA + \iint_{D_2}\left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA$$

HMV!!!



VIDI ILUSTRACIJA
NA PP1112
Stewart5e

$$W = \int_C P\,dV = \oint_{C_1} P\,dV + \oint_{-C_2} P\,dV$$

C_2 - COUNTERWISE ZATO A ZEMAM „$-C_2$"

$$= \oint_{C_1} P\,dV - \oint_{C_2} P\,dV = A_1 - A_2$$

OSNOVICA	TOTAL	KAMATA
28699	32760	4061

$$28699 \cdot \left(\frac{x}{100} + 1\right) = 32760$$

$$\frac{x}{100} + 1 = \frac{32760}{28699} \Rightarrow \frac{x}{100} = 0{,}1415 = 14{,}15\%$$

$$18 \cdot 1820 \left(\frac{x}{100} = 4061 \qquad \frac{406100}{1820 \cdot 18} = \underline{12.39}$$

1.699 RATA 18 RATI

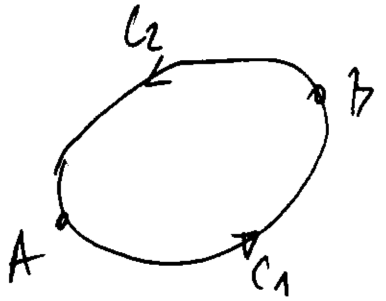
OSNOVICA	KAMATA
27999	2.583

$$\begin{array}{r} 27999 \\ 2583 \\ \hline 30582 \end{array}$$

20388

$$1000\left(1 + \frac{x}{100}\right) = 1032$$

$$\frac{30582}{27999} - 1 = \underline{0{,}092} \qquad \frac{x}{100} = 0{,}032 \Rightarrow x = 3\%$$



AND $\vec{F}$ e CONSERVATIVE

$$\oint_C \vec{F}\,d\vec{r} = \int_{C_1} \vec{F}\,dr + \int_{-C_2} \vec{F}\,dr$$

$$= \int_{C_1} \vec{F}\,dr - \int_{C_2} \vec{F}\,dr = 0$$

$$\int_{C_1} \vec{F}\,d\vec{r} = \int_{C_2} \vec{F}\,d\vec{r}$$

CONTINUE **DIGITAL COMMUNICATIONS OVER FADING CHANNELS**

SIMON & ALOUINI (02.03.2010

- GAUSSIAN Q FUNCTION

$$Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-\gamma^2/2}\,d\gamma$$

$$Q(z) = \operatorname{erfc}\frac{z}{\sqrt{2}}$$

$$\operatorname{erfc}(z) = \frac{2}{\sqrt{\pi}}\int_z^\infty e^{-x^2}\,dx$$

$$\operatorname{erfc}\left(\frac{z}{\sqrt{2}}\right) = \frac{2}{\sqrt{\pi}}\int_{z/\sqrt{2}}^\infty e^{-x^2}\,dx = \left|\begin{matrix} \mu = x\sqrt{2} \\ d\mu = \sqrt{2}\,dx \end{matrix} \xrightarrow{} x = \frac{\mu}{\sqrt{2}} \quad x = \frac{z}{\sqrt{2}} \;\; \mu = z\right|$$

$$= \frac{2}{\sqrt{\pi}}\int_z^\infty e^{-\frac{x^2}{2}}\frac{d\mu}{\sqrt{2}} = \frac{1}{\sqrt{2\pi}}\int_z^\infty e^{-x^2/2}\,d\mu$$

ALTERNATIVE REPRESENTATION

$$\boxed{Q(x) = \frac{1}{\pi}\int_0^{\pi/2} \exp\left(-\frac{x^2}{2\sin^2\theta}\right)d\theta}$$

- MARCUM'S Q FUNCTION (FIRST ORDER)

$$Q_1(s,\sqrt{\gamma}) = \int_{\sqrt{\gamma}}^\infty x \exp\left[-\left(\frac{x^2+s^2}{2}\right)\right] I_0(sx)\,dx$$

$$Q_1(\alpha,\beta) = \int_\beta^\infty x \exp\left[-\frac{x^2+\alpha}{2}\right] I_0(\alpha x)\,dx$$

• POTSETUVANJE

$$Q(\cancel{x}) = \frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-\frac{y^2}{2}}dy = \frac{1}{\pi}\int_0^{\pi/2} \exp\left(-\frac{\cancel{x}^2}{2\sin^2\theta}\right)d\theta$$

$$Q(z) = \overbrace{2\int_0^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}}dy}^{1} \cdot \int_z^\infty \frac{1}{\sqrt{2\pi}}\exp\left(-\frac{x^2}{2}\right)dx = \circledast$$

$$Q(z) = \operatorname{erfc}\left(\frac{z}{\sqrt{2}}\right) \qquad \int_{-\infty}^{\infty} e^{-x^2}dx = \sqrt{\pi}$$

$$\operatorname{erfc}(0) = \frac{2}{\sqrt{\pi}}\int_0^\infty e^{-x^2}dx \xrightarrow{} \frac{\sqrt{\pi}}{2} \quad = \frac{2}{\sqrt{\pi}}\cdot\frac{\sqrt{\pi}}{2} = 1$$

$$= \operatorname{erfc}(0) =$$

$$\frac{1}{\sqrt{2\pi}}\int_0^\infty e^{-y^2/2}dy = \left| \begin{matrix} y = x\cdot\sqrt{2} \\ dy = \sqrt{2}dx \end{matrix} \right| = \frac{1}{\sqrt{\cancel{2}\pi}}\int_0^\infty e^{-x^2}\cancel{\sqrt{2}}dx$$

$$= \frac{1}{\sqrt{\pi}}\int_0^\infty e^{-x^2}dx = \frac{1}{2}\cdot\operatorname{erfc}(0) = \frac{1}{2}$$

$$\circledast = \frac{1}{\pi}\int_0^\infty\int_z^\infty e^{-\frac{x^2+y^2}{2}}dx\,dy \qquad \left| \begin{matrix} x = r\cos\theta \\ y = r\sin\theta \\ dx\,dy = r\,dr\,d\theta \\ x = z \to r = \frac{z}{\cos\theta} \end{matrix} \right|$$

$$= \frac{1}{\pi}\int_0^{\pi/2} \underbrace{\int_{z/\cos\theta}^\infty e^{-\frac{r^2}{2}} r\,dr}_{I}\, d\theta$$

$\theta = 0 \ldots \frac{\pi}{2}$

ZATOA STO:

$$\left.\begin{matrix} x = 0 \ldots \infty \\ y = 0 \ldots \infty \end{matrix}\right\} \text{ I KVADRANT}$$

$$I = -\int_{z/\cos\theta}^\infty e^{-r^2/2}\,d\left(\frac{-r^2}{2}\right) = -e^{-\frac{r^2}{2}}\Big|_{\frac{z}{\cos\theta}}^{\infty} = e^{-\frac{z^2}{2\cos^2\theta}}$$

//

$$Q(z) = \frac{1}{\pi}\int_0^{\pi/2} e^{-\frac{z^2}{2\cos^2\theta}}\, d\theta = \left| \begin{array}{l} \varphi = \frac{\pi}{2} - \theta \quad d\varphi = -d\theta \\ \cos\left(\frac{\pi}{2} - \varphi\right) = \sin\varphi \\ \theta = 0 \;\; \varphi = \pi/2 \quad \theta = \frac{\pi}{2} \;\; \varphi = 0 \end{array} \right| =$$

$$= \frac{1}{\pi}\int_{\pi/2}^{0} e^{-\frac{z^2}{2\sin^2\varphi}} (-d\varphi) = \frac{1}{\pi}\int_0^{\pi/2} e^{-\frac{z^2}{2\sin^2\varphi}}\, d\varphi$$

$$\cdot \quad P_{out} = P(\gamma < \gamma_{th}) = \int_0^{\gamma_{th}} p_\gamma(\gamma)\, d\gamma \qquad p_\gamma(\gamma) = \frac{dP_\gamma(\gamma)}{d\gamma}$$

$$\hat{P}_\gamma(\gamma) = \frac{\hat{p}_\gamma(\gamma)}{s}$$ „^" LAPLACE TRANSFORM $$\hat{p}_\gamma(\gamma) = M_\gamma(-s)$$

$$P_{out} = \frac{1}{2\pi j}\int_{\sigma - j\infty}^{\sigma + j\infty} \frac{M_\gamma(-s)}{s} e^{s\gamma_{th}}\, ds$$

$$M_\gamma(s) = \int_{-\infty}^{\infty} p_\gamma(\gamma)\cdot e^{\gamma s}\, d\gamma \Longleftrightarrow \hat{p}_\gamma(\gamma) = \int_0^{\infty} p_\gamma(\gamma)\, e^{-\gamma s}\, d\gamma$$

USEFUL EXPRESSIONS FOR EVALUATING AVERAGE ERROR PROBABILITY PERFORMANCE

POTSETIUVANJE (ALTERNATIVE FORM OF MARCUM'S Q FUNCTION)

$$Q_1(\alpha, \beta) = \int_\beta^\infty x \exp\left(-\frac{x^2+\alpha^2}{2}\right) I_0(\alpha x)\, dx$$

$$\xi = \frac{\alpha}{\beta}$$

$\beta > \alpha$

$$Q_1(\alpha, \beta) = Q_1(\xi\beta, \beta) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{1 + \xi\sin\theta}{1 + 2\xi\sin\theta + \xi^2} \exp\left[-\frac{\beta^2}{2}(1 + 2\xi\sin\theta + \xi^2)\right] d\theta$$

$\alpha > \beta$

$$Q_1(\alpha, \beta) = Q_1(\alpha, \xi\alpha) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{\xi^2 + \xi\sin\theta}{1 + 2\xi\sin\theta\ \xi^2} \exp\left[-\frac{\alpha^2}{2}(1 + 2\xi\sin\theta + \xi^2)\right] d\theta$$

☐ INTEGRALS INVOLVING GAUSSIAN Q - FUNCTION

$$\boxed{I = \int_0^\infty Q(a\sqrt{\gamma})\, p_\gamma(\gamma)\, d\gamma} \quad (5.1)$$ ★

$$Q(x) = \frac{1}{\pi}\int_0^{\pi/2} \exp\left(\frac{-x^2}{2\sin^2\theta}\right) d\theta$$ $\boxed{\text{MMV}}$

$$Q(a\sqrt{\gamma}) = \frac{1}{\pi}\int_0^{\pi/2} \exp\left(-\frac{a^2\gamma}{2\sin^2\theta}\right) d\theta$$

$$I = \int_0^\infty \left[\frac{1}{\pi}\int_0^{\pi/2} \exp\left(-\frac{a^2\gamma}{2\sin^2\theta}\right) d\theta\right] p_\gamma(\gamma)\, d\gamma = \frac{1}{\pi}\int_0^{\pi/2} \underbrace{\int_0^\infty e^{-\frac{a^2\gamma}{2\sin^2\theta}} p_\gamma(\gamma)\, d\gamma}_{\text{FORM OF LAPLACE TRANSFORM}}\, d\theta$$

$$\boxed{I = \frac{1}{\pi}\int_0^{\pi/2} M_\gamma\left(-\frac{a^2}{2\sin^2\theta}\right) d\theta \quad (5.3) \quad \boxed{\text{MMV}}}$$

KLUČEN INTEGRAL ZA PRESMETKA NA BER

(*1)

☐ RAYLEIGH FADING (AVERAGE BER)

$$p_\gamma(\gamma) = \frac{1}{\gamma} e^{-\frac{\gamma}{\bar{\gamma}}} \qquad \boxed{\gamma \geq 0}$$

$$p_\alpha(\alpha) = \frac{2\alpha}{\Omega} e^{-\frac{\alpha^2}{\Omega}} \qquad p_\gamma(\gamma) = \frac{p_\alpha(\alpha)}{\frac{d\gamma}{d\alpha}}$$

$$\boxed{\bar{\gamma} = \frac{\Omega E_s}{N_0}} \qquad \frac{E_s}{N_0} = \frac{\bar{\gamma}}{\Omega} \qquad \boxed{\gamma = \frac{\alpha^2 E_s}{N_0}}$$

$$\gamma = \frac{\alpha^2 \cdot \bar{\gamma}}{\Omega} \qquad \boxed{\frac{d\gamma}{d\alpha} = \frac{2\alpha \cdot \bar{\gamma}}{\Omega}} \qquad \alpha^2 = \frac{\gamma\Omega}{\bar{\gamma}}$$

$$p_\gamma(\gamma) = \frac{\cancel{2\alpha}}{\cancel{\Omega}} \cdot e^{-\frac{\gamma\cancel{\Omega}}{\bar{\gamma}\cdot\cancel{\Omega}}} \cdot \frac{1}{\frac{\cancel{2\alpha}\cdot\bar{\gamma}}{\cancel{\Omega}}} = \frac{1}{\bar{\gamma}} \cdot e^{-\frac{\gamma}{\bar{\gamma}}}$$

$$M_\gamma(-s) = \int_0^\infty p_\gamma(\gamma)\cdot e^{-\gamma s}\,d\gamma = \hat{p}_\gamma(\gamma) = \mathcal{L}\{p_\gamma(\gamma)\}$$

$$p_\gamma(\gamma) = \frac{1}{\bar\gamma}\, e^{-\gamma/\bar\gamma} \Rightarrow \hat{p}_\gamma(\gamma) = \int_0^\infty \frac{1}{\bar\gamma}\, e^{-\gamma/\bar\gamma}\cdot e^{-\gamma s}\,d\gamma$$

$$= \frac{1}{\bar\gamma}\int_0^\infty e^{-\left(\frac{1}{\bar\gamma}+s\right)\gamma}\,d\gamma = \frac{1}{\bar\gamma}\,\frac{1}{-\left(\frac{1}{\bar\gamma}+s\right)}\cdot e^{-\left(\frac{1}{\bar\gamma}+s\right)\gamma}\Bigg|_0^\infty = \frac{1}{\cancel{\bar\gamma}}\cdot\frac{1}{\frac{1+\bar\gamma s}{\cancel{\bar\gamma}}}$$

$$\boxed{\hat{p}_\gamma(\gamma) = \frac{1}{1+\bar\gamma s}} \qquad \boxed{M_\gamma(-s) = \frac{1}{1+\bar\gamma s}} \quad s>0$$

FOR RAYLEIGH

$$I = \frac{1}{\pi}\int_0^{\pi/2} M_\gamma\left(-\frac{a^2}{2\sin^2\theta}\right)d\theta = \frac{1}{\pi}\int_0^{\pi/2}\frac{1}{1+\frac{a^2\bar\gamma}{2\sin^2\theta}}\,d\theta$$

$$= \frac{1}{\pi}\int_0^{\pi/2}\frac{2\sin^2\theta}{2\sin^2\theta + a^2\bar\gamma}\,d\theta = \frac{1}{2}\left(1-\frac{a\cdot\bar\gamma}{\sqrt{\bar\gamma(a^2\bar\gamma+2)}}\right)$$

$$= \frac{1}{2}\left(1-\sqrt{\frac{a^2\bar\gamma}{a^2\bar\gamma+2}}\right) = \boxed{\frac{1}{2}\left(1-\sqrt{\frac{a^2\bar\gamma/2}{1+\frac{a^2\bar\gamma}{2}}}\right)}$$

MMV

MAPLE

AVGN. BER IN RAYLEIGH CHAN.

$$I = \int\frac{\sin^2\theta}{2\sin^2\theta+1}\,d\theta = \left|\;\cancel{\begin{matrix} u = \sin\theta \\ du = \cos\theta\,d\theta \\ d\theta = \frac{du}{\cos\theta}\end{matrix}}\;\right| = \int\frac{u^2\,du}{(2u^2+1)\sqrt{1-u^2}}$$

$$\cancel{\sin^2\theta = u} \qquad \cancel{2\sin\theta\cdot\cos\theta\,d\theta = du} \qquad \cancel{\boxed{d\theta = \frac{du}{2\sin\theta\cos\theta}}}$$

$$u = \tan(\theta) \qquad du = \frac{\cos^2\theta+\sin^2\theta}{\cos^2\theta}\,d\theta = (1+tg^2\theta)\,d\theta$$

$$\cancel{d\theta = (1-u^2)\,du} \qquad \cancel{I = \int\frac{u^2}{2u^2+1}(1-u^2)\,du}$$

$$\cancel{I = \int\frac{u^2}{1+2u^2}\,du - \int\frac{u^4}{1+2u^2}\,du}$$

$du = (1+u^2)\,d\theta \qquad d\theta = \frac{du}{1+u^2}$

NE MOŽE VAKA!! $u = tg\theta$

$$I = \int \frac{\sin^2\theta}{2\sin^2\theta + 1}\,d\theta = \int \frac{u^2}{2u^2+1}\cdot\frac{du}{1+u^2} = \int \frac{u^2\,du}{4u^4+3u^2+1}$$

$$(2u^2+1)(1+u^2) = 2u^2+1+4u^4+u^2 = 4u^4+3u^2+1$$

$$\frac{u^2}{4u^4+3u^2+1} = \overbrace{\frac{1}{2u^2+2} - \frac{1}{6u^2+2}}^{\text{MALO. INTUITOV}} = \frac{\overset{4}{\cancel{6u^2+2-2u^2+2}}}{(2u^2+2)(6u^2+2)}$$

$$= \frac{4u^2}{12u^4+12u^2+4u^2+4} = \frac{4u^2}{12u^4+16u^2+4} = \frac{u^2}{3u^4+4u^2+1}$$

$$I = \int \frac{du}{2u^2+2} - \int \frac{du}{6u^2+2} = \frac{1}{2}\operatorname{arctg}(\overset{tg\theta}{\cancel{u}}) - \frac{1}{2}\int \frac{du}{3u^2+1}\ \otimes$$

$y = \operatorname{arctg} x \qquad x = tg\,y \qquad dx = (1+tg^2 y)\,dy = (1+x^2)\,dy$

$dy = \frac{dx}{1+x^2} \qquad \frac{dy}{dx} = \frac{1}{1+x^2} \qquad y = \operatorname{arctg} x \Rightarrow$

$$\int \frac{dx}{1+x^2} = \operatorname{arctg} x$$

$$\otimes = \int \frac{du}{3u^2+1} = \frac{1}{\sqrt{3}}\int \frac{d(\sqrt{3}u)}{(\sqrt{3}u)^2+1} = \frac{1}{\sqrt{3}}\operatorname{arctg}\sqrt{3}u$$

$$I = \frac{1}{2}\cancel{\operatorname{arctg}(tg\theta)}^{\theta} - \frac{1}{2}\,\frac{1}{\sqrt{3}}\operatorname{arctg}\sqrt{3}\,tg(\theta) = \frac{1}{2}\theta - \frac{1}{2\sqrt{3}}\operatorname{arctg}(\sqrt{3}tg\theta)$$

$$\boxed{I = \frac{\theta}{2} - \frac{\sqrt{3}}{6}\operatorname{arctg}(\sqrt{3}\,tg\theta)}$$

5.1.4 NAKAGAMI-m FADING CHANNEL

$$p_\gamma(\gamma) = \frac{m^m \gamma^{m-1}}{\bar{\gamma}^m \Gamma(m)}\,e^{-\frac{m\gamma}{\bar{\gamma}}} \qquad \hat{p}_\gamma(\gamma) = \int_0^\infty \frac{m^m \gamma^{m-1}}{\bar{\gamma}^m \Gamma(m)}\,e^{-\frac{m\gamma}{\bar{\gamma}}}\cdot e^{-s\gamma}\,d\gamma$$

$$\mathcal{L}\left\{p_\gamma(\gamma)\right\} = M_\gamma(-s) = \int_0^\infty \frac{m^m \gamma^{m-1}}{\bar\gamma^m \Gamma(m)} e^{-\left(\frac{m}{\bar\gamma}+s\right)\gamma} d\gamma = \frac{m^m}{(m+s\bar\gamma)^m}$$

$$\boxed{\hat{p}_\gamma(\gamma) = \frac{1}{\left(1+\frac{s\gamma}{m}\right)^m} = \left(1+\frac{s\gamma}{m}\right)^{-m}} \quad s>0$$

NAKAGAMI-m

PROBLEMOT SE SVEDUVA NA INTEGRALOT

$$\boxed{\int_0^\infty \gamma^n e^{-\gamma s} d\gamma = \frac{\Gamma(n+1)}{s^{n+1}} \quad s>0}$$

MMV

GRADSHTEIN TABLE OF LAPLACE TRANSFOR INTEGR. 3

$$\int_0^\infty \frac{m^m\gamma^{m-1}}{\bar\gamma^m\Gamma(m)} e^{-\left(\frac{m}{\bar\gamma}+s\right)\gamma} d\gamma = \overset{=C}{\frac{m^m}{\bar\gamma^m\Gamma(m)}} \int_0^\infty \gamma^{m-1} e^{-\left(s+\frac{m}{\bar\gamma}\right)\gamma} d\gamma =$$

$$= \left| s_1 = s+\frac{m}{\bar\gamma} \right| = C\int_0^\infty \gamma^{m-1} e^{-s_1\gamma} d\gamma = C\,\frac{\Gamma(m)}{s_1^m} = \frac{m^m}{\bar\gamma^m\cancel{\Gamma(m)}} \cdot \frac{\overset{1}{\cancel{\Gamma(m)}}}{s_1^m}$$

$$= \frac{1}{\left(s+\frac{m}{\bar\gamma}\right)^m}\cdot\frac{m^m}{\bar\gamma^m} = \frac{m^m}{(s\bar\gamma+m)^m} = \frac{1}{\left(\frac{s\bar\gamma}{m}+1\right)^m} = \left(1+\frac{s\bar\gamma}{m}\right)^{-m}$$

DOKAZANO DEKA: $M_\gamma(-s) = \left(1+\frac{s\bar\gamma}{m}\right)^{-m}$;

$$(*1) \Rightarrow I = \frac{1}{\pi}\int_0^{\pi/2}\left(1+\frac{\bar\gamma}{m}\frac{a^2}{2\sin^2\theta}\right)^{-m} d\theta = I_m(a,m,\bar\gamma)$$

$$I = \frac{1}{\pi}\int_0^{\pi/2}\left(\frac{2\sin^2\theta+\frac{\bar\gamma}{m}a^2}{2\sin^2\theta}\right)^{-m} d\theta = \frac{1}{\pi}\int_0^{\pi/2}\left(\frac{2\sin^2\theta}{2\sin^2\theta+\frac{\bar\gamma a^2}{m}}\right)^{m} d\theta$$

ZA $m=1$ SE SVEDUVA NA ISTIOT INTEGRAL KAKO ZA RAYLEIGH 11.39

$$\boxed{J_m(a,b) \triangleq \frac{a^m}{\Gamma(m)}\int_0^\infty e^{-at} t^{m-1} Q(\sqrt{bt})\, dt \quad \boxed{m\geq 0}}$$

(*1)

- CLOSED FORM OF ⊛1   3095284

$$J_n(a,b) = J_n(c) = \frac{\sqrt{c/\pi}}{2(1+c)^{n+1/2}} \frac{\Gamma(n+\frac{1}{2})}{\Gamma(n+1)} {}_2F_1\left(1, n+\frac{1}{2}; n+1; \frac{1}{1+c}\right)$$

- FROM T. ENG (COHERENT D-CDMA)

$$\boxed{P = \frac{a^b}{\Gamma(b)} \int_0^\infty e^{-at} t^{b-1} \phi(-\sqrt{ct})\, dt}$$

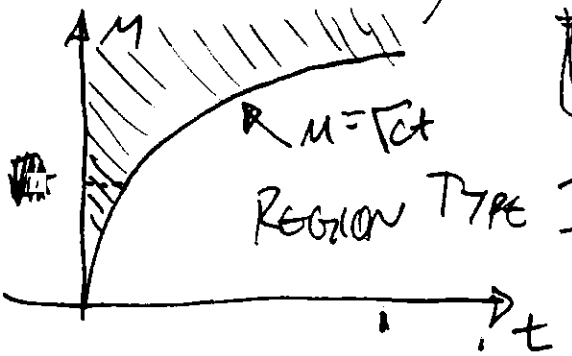
$$\phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-t^2/2} dt \qquad \phi(-x) = 1 - \phi(x) = \underbrace{\frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-t^2/2} dt}_{Q(x)}$$

GAUSS Q FUNCT. Q(x)

$$P = \frac{a^b}{\Gamma(b)} \frac{1}{\sqrt{2\pi}} \int_0^\infty e^{-at} t^{b-1} \int_{\sqrt{ct}}^\infty e^{-\mu^2/2} d\mu\, dt$$

REGION TYPE I

REGION OF INTEGRATION



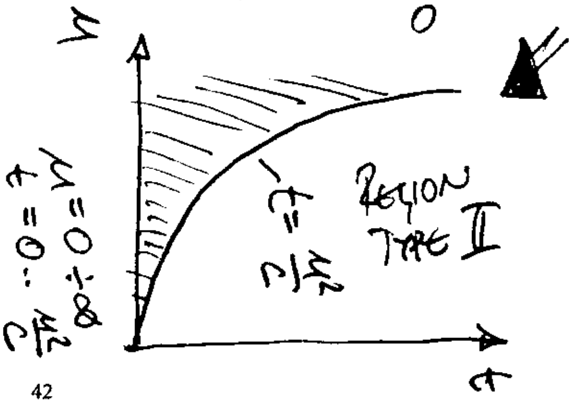
MMV

$\mu = \sqrt{ct}$ $\mu^2 = ct$
$t = \mu^2/c$

$\mu = \sqrt{ct} \ldots \infty$
$t = \frac{c t}{c} \ldots \infty$

- CHANGE ORDER OF INTEGRATION

$$P = \frac{a^b}{\Gamma(b)} \frac{1}{\sqrt{2\pi}} \int_0^\infty e^{-\mu^2/2} d\mu \int_0^{\mu^2/c} e^{-at} t^{b-1} dt\, d\mu$$



$x = at \quad dt = \frac{dx}{a}$ $t=0 \; x=0$ $t = \frac{\mu^2}{c} \; x = \frac{a\mu^2}{c}$

$$P = \frac{\cancel{a^b}}{\Gamma(b)} \frac{1}{\sqrt{2\pi}} \int_0^\infty e^{-\frac{\mu^2}{2}} d\mu \int_0^{\frac{a\mu^2}{c}} e^{-x} \frac{x^{b-1}}{\cancel{a^{b-1}}} \cdot \frac{dx}{\cancel{a}}$$

$$= \frac{1}{\Gamma(b)\sqrt{2\pi}} \int_0^\infty e^{-\frac{\mu^2}{2}} d\mu \int_0^{\frac{a\mu^2}{c}} \underbrace{e^{-x} x^{b-1} dx}_{\boxed{}}$$

$$\Gamma(\alpha,x) = \int_x^\infty e^{-t}t^{\alpha-1}dt \qquad \gamma(\alpha,x) = \int_0^x e^{-t}t^{\alpha-1}dt$$

$$\Gamma(x) = \gamma(\alpha,x) + \Gamma(\alpha,x)$$

$$④ = \gamma\left(\frac{a\mu^2}{c},x\right) = \Gamma(x) - \Gamma(\alpha,x)$$

$$\boxed{\gamma(b,z) = \int_0^z x^{b-1}e^{-x}dx = b^{-1}z^b e^{-z}\,\underbrace{{}_1F_1(1;b+1;z)}_{\text{CONFLUENT HYPERGEOM FUNCTION}}}$$

T.E Kummer_U

MAPLE & AMBROWITZ NOTAT.

HARMONIC MEAN OF x_1 & x_2

$$\mu_H = \frac{2}{\frac{1}{x_1}+\frac{1}{x_2}} = \frac{2x_1x_2}{x_1+x_2}$$

$$\Gamma(a)\cdot {}_1F_1(a,b,z) = \int_0^\infty e^{-zt}t^{a-1}(1+t)^{b-a-1}dt$$

$${}_1F_1(1,b+1,z) = \frac{1}{\Gamma(a)}\int_0^\infty e^{-zt}t^{1-1}\cdot(1+t)^{b-1}dt$$

$${}_1F_1(1,b+1,z) = \frac{1}{\Gamma(a)}\int_0^\infty e^{-zt}(1+t)^{b-1}dt = \text{Kummer\_U}(1,b+1,$$

FROM T. ENG & L.B. MILSTEIN: COHERENT DS-CDMA

$$P = \frac{b^{-1}}{\Gamma(b)\sqrt{2\pi}}\int_0^\infty e^{-\frac{a\mu^2}{c}}\cdot e^{-\frac{\mu^2}{2}}\,{}_1F_1\left(1,b+1,\frac{a\mu^2}{c}\right)\left(\frac{a\mu^2}{c}\right)^b d\mu$$

$$= \frac{(a/c)^b}{b\,\Gamma(b)\sqrt{2\pi}}\int_0^\infty e^{-\underbrace{\left(\frac{a}{c}+\frac{1}{2}\right)}_{\alpha}\mu^2}\cdot\mu^{2b}\cdot{}_1F_1\left(1,b+1,\frac{a\mu^2}{c}\right)d\mu$$

$$P = \frac{(a/c)^b}{b\,\Gamma(b)\sqrt{2\pi}}\int_0^\infty e^{-\alpha\mu^2}\cdot\mu^{2b}\cdot{}_1F_1\left(1,b+1,\frac{a\mu^2}{c}\right)d\mu$$

$y = \alpha\mu^2$

$dy = 2\alpha\mu\, d\mu$

$\mu^2 = \left(\frac{y}{\alpha}\right)$

$$P = \frac{(a/c)^b}{b\,\Gamma(b)\sqrt{2\pi}}\int_0^\infty e^{-y}\left(\frac{\cancel{y}}{\alpha}\right)^b {}_1F_1\left(1,b+1,\frac{ay}{\alpha c}\right)\frac{dy}{2\cancel{\alpha}\sqrt{\frac{y}{\cancel{\alpha}}}}$$

$$P = \frac{(a/c)^b}{b\Gamma(b)\sqrt{2\pi}} \frac{1}{2}\int_0^\infty e^{-y} \frac{y^{b-\frac{1}{2}}}{\alpha^{b+\frac{1}{2}}} \, {}_1F_1\left(1, b+1, \frac{a}{\alpha c} y\right) dy$$

$$\Gamma(b) = (b-1)! \qquad \underline{b\cdot(b-1)! = \Gamma(b+1)}$$

(OVA E ZA CELI BROEVI NO VAŽI I VO GENERALEN SLUČAJ)

$$P = \frac{(a/c)^b}{\Gamma(b+1)\sqrt{2\pi}} \left(\frac{1}{\alpha}\right)^{b+\frac{1}{2}} \frac{1}{2}\int_0^\infty e^{-y} y^{b-\frac{1}{2}} \, {}_1F_1\left(1, b+1, \frac{a}{\alpha c} y\right) dy \cdot$$

GAUSS HYPERGEOMETRIC FUNCTION

$\boxed{\text{LUKE book}}$

$${}_{p+1}F_q\left(\begin{matrix}\sigma, \alpha_p \\ \rho_q\end{matrix} \middle| w/z\right) = \frac{z^\sigma}{\Gamma(\sigma)} \int_0^\infty e^{-zt} t^{\sigma-1} \, {}_pF_q\left(\begin{matrix}\alpha_p \\ \rho_q\end{matrix} \middle| wt\right) dt$$

ZA $\underline{\underline{w<1}}$

$$\underbrace{\Gamma(\sigma)\, {}_2F_1(\sigma, p; q; w)}_{\text{GAUSS HYPERGEOMETRIC}} = \int_0^\infty e^{-y} y^{\sigma-1} \, {}_1F_1(p; q; wy) dy$$

$$P = \frac{(a/c)^b}{\Gamma(b+1)\sqrt{2\pi}} \left(\frac{1}{\alpha}\right)^{b+\frac{1}{2}} \frac{1}{2} \; \Gamma\left(b+\frac{1}{2}\right) \cdot {}_2F_1\left(b+\frac{1}{2}, 1; b+1, \frac{a}{\alpha c}\right)$$

$$\int_0^\infty e^{-y} y^{\overset{\sigma}{\widetilde{(b+\frac{1}{2})}}-1} \, {}_1F_1\left(1, b+1, \frac{a}{\alpha c} y\right) dy$$

$$\boxed{\begin{aligned} \gamma &= \frac{c}{2a} \\ \frac{c}{a} &= 2\gamma \end{aligned}}$$

$$\alpha = \frac{a}{c} + \frac{1}{2}$$

$$\alpha = \frac{1}{2\gamma} + \frac{1}{2} = \frac{1+\gamma}{2\gamma}$$

$$P = \frac{\left(\frac{1}{2\gamma}\right)^b}{2\Gamma(b+1)\sqrt{2\pi}} \left(\frac{2\gamma}{1+\gamma}\right)^{b+\frac{1}{2}} \Gamma\left(b+\frac{1}{2}\right) \cdot {}_2F_1\left(b+\frac{1}{2}, 1; b+1; \frac{1}{\cancel{2\gamma}\frac{(1+\gamma)}{\cancel{2\gamma}}}\right)$$

$$P = \frac{\Gamma\left(b+\frac{1}{2}\right)}{2\Gamma(b+1)\cancel{\sqrt{2\pi}}} \frac{1}{\cancel{(2\gamma)^b}} \cdot \left(\frac{\cancel{2\gamma}}{1+\gamma}\right)^b \cdot \sqrt{\frac{\cancel{2}\gamma}{1+\gamma}} \cdot {}_2F_1\left(b+\frac{1}{2}, 1; b+1; \frac{1}{1+\gamma}\right)$$

(*8) $$\boxed{P = \sqrt{\frac{\gamma}{1+\gamma}} \frac{(1+\gamma)^{-b}\,\Gamma\left(b+\frac{1}{2}\right)}{2\,\Gamma(b+1)\,\sqrt{\pi}} \cdot {}_2F_1\left(1, b+\frac{1}{2}; b+1; (1+\gamma)^{-1}\right)}$$

$b \equiv m \Rightarrow$ NONINTEGER $\qquad \gamma = c$

- Bx DEFF GAUSS HYPERGEOMETRIC FUNCTION

$$_2F_1(a,b;c;z)=\sum_{n=0}^{\infty}\frac{(a)_n(b)_n}{(c)_n}\frac{z^n}{n!}=\frac{\Gamma(a)}{\Gamma(a)\Gamma(b)}\sum_{n=0}^{\infty}\frac{\Gamma(a+n)\Gamma(b+n)}{\Gamma(c+n)}\frac{z^n}{n!}$$

$(z)_n = z(z+1)\cdots(z+n-1)=\frac{\Gamma(z+n)}{\Gamma(z)}$ Pochammer's symbol

$$_2F_1(1,n+\tfrac{1}{2};n+1;z)=\sum_{k=0}^{\infty}\frac{(1)_k(n+\frac{1}{2})_k}{(n+1)_k}\frac{z^k}{k!} \qquad ⊛$$

$$\frac{\Gamma(z+n)}{\Gamma(z)}=\frac{(z+n-1)!}{(z-1)!}=\frac{\cancel{(z-1)!}\,z\cdot(z+1)\cdots(z+n-1)!}{\cancel{(z-1)!}}$$

$$(1)_k = 1\cdot 2\cdots(1+k-1)=1\cdot 2\cdots k = k!$$

$$(n+1)_k=(n+1)(n+2)\cdots(n+\cancel{k-1+1})=\frac{n!(n+1)(n+2)\cdots(n+k)}{n!}$$

$$(n+1)_k=\frac{(n+k)!}{n!} \qquad (n+\tfrac{1}{2})_k=\frac{(\frac{1}{2})_{n+k}}{(\frac{1}{2})_n}$$

$$(n+\tfrac{1}{2})\cdot(n+\tfrac{1}{2}+1)\cdots(n+\tfrac{1}{2}+k-1)=(n+\tfrac{1}{2})\cdots(n+k-\tfrac{1}{2})=$$

$$\cancel{=\frac{(\frac{1}{2})_{n+k}}{(\frac{1}{2}+n)!}=\frac{(\frac{1}{2})(\frac{1}{2}+1)\cdots(\frac{1}{2}+n+k-1)}{(\frac{1}{2}+n)!}}$$

$$\left(\tfrac{1}{2}\right)_{n+k}=\underbrace{\tfrac{1}{2}\cdot(\tfrac{1}{2}+1)\cdots(\tfrac{1}{2}+n-1)}_{(\frac{1}{2})_n}\;\underline{\qquad\qquad}\;(\tfrac{1}{2}+n)(\tfrac{1}{2}+n+1)\cdots(\tfrac{1}{2}+n+k-1)$$

POKAZANO:

$$\left(n+\tfrac{1}{2}\right)_k=\frac{(\frac{1}{2})_{n+k}}{(\frac{1}{2})_n}=(n+\tfrac{1}{2})(n+\tfrac{1}{2}+1)\cdots(n+k-\tfrac{1}{2})$$

$$⊛\Rightarrow {}_2F_1(1,n+\tfrac{1}{2};n+1;z)=\sum_{k=0}^{\infty}\frac{\cancel{k!}\,(\frac{1}{2})_{n+k}}{\frac{(n+k)!}{n!}\,(\frac{1}{2})_n}\frac{z^k}{\cancel{k!}}=\sum_{k=0}^{\infty}\frac{(\frac{1}{2})_{n+k}\,n!}{(n+k)!\,(\frac{1}{2})_n}z$$

⊗

DCOFC APIENDIX 5A-1 (ŽE. VRAČAM VAMUŠ ~~[illegible]~~ POTSOSTUNGE)

$$\boxed{J_m(a,b) = \frac{a^m}{\Gamma(m)} \int_0^\infty e^{-at} t^{m-1} g(\sqrt{bt})\, dt \quad m>0}$$

$$J_m(a,b) = J_m(c) = \frac{\sqrt{c\pi}}{2(1+c)^{m+\frac{1}{2}}} \frac{\Gamma(m+\frac{1}{2})}{\Gamma(m+1)} \; {}_2F_1\left(1, m+\frac{1}{2}; m+1; \frac{1}{1+c}\right)$$

*8 $\begin{pmatrix} b \equiv m \\ c \equiv r \end{pmatrix}$

$$\boxed{c \triangleq \frac{b}{2a}}$$

m - NON INTEGER

$$⑧ \Rightarrow {}_2F_1\left(1, n+\frac{1}{2}; n+1; z\right) = \sum_{k=0}^{\infty} \frac{\left(\frac{1}{2}\right)_{n+k} n!}{(n+k)!\left(\frac{1}{2}\right)_n} z^k =$$

$$= \frac{n!}{\left(\frac{1}{2}\right)_n} z^{-n} \sum_{k=0}^{\infty} \frac{\left(\frac{1}{2}\right)_{n+k}}{(n+k)!} z^{k+n} = \frac{n!}{\left(\frac{1}{2}\right)_n} z^{-n} \left[\sum_{k=0}^{\infty} \left(\frac{1}{2}\right)_k \frac{z^k}{k!} - \sum_{k=0}^{n-1} \left(\frac{1}{2}\right)_k \frac{z^k}{k!}\right]$$

NE ZNAM KAKO GO DOBY !?

$$\left(\frac{1}{2}\right)_{n+k} = \underbrace{\frac{1}{2}\left(\frac{1}{2}+1\right)\cdot\left(\frac{1}{2}+2\right)\cdot \cdots \left(\frac{1}{2}+k-1\right)}_{\left(\frac{1}{2}\right)_k}\left(\frac{1}{2}+k\right)\left(\frac{1}{2}+k+1\right)\cdots\left(\frac{1}{2}+n+k-1\right)$$

$$(n+k)! = \underbrace{1\cdot 2\cdot 3 \cdots k}_{k!}\underbrace{(k+1)(k+2)\cdots(k+n)}$$

~~[illegible]~~ TAYLOR

$$f(x) = f(x_0) + \frac{f'(x_0)}{1!}(x-x_0) + \frac{f''(x_0)}{2!}(x-x_0)^2 + \cdots + \frac{f^{(n)}(x_0)}{n!}(x-x_0)^n + \cdots$$

~~TAYLOR~~ MCLAURIN

$$f(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \cdots + \frac{f^{(n)}(0)}{n!}x^n + \cdots$$

$$\left.\sum_{k=0}^{\infty}\left(\frac{1}{2}\right)_k \frac{z^k}{k!}\right\} \quad \text{TAYLOR OF } \frac{1}{\sqrt{1-z}}$$

$$S = \sum_{k=0}^{3}\left(\frac{1}{2}\right)_k \frac{z^k}{k!} = \left(\frac{1}{2}\right)_0 \cdot 1 + \left(\frac{1}{2}\right)_1 \cdot \frac{z}{1} + \left(\frac{1}{2}\right)_2 \frac{z^2}{2} + \left(\frac{1}{2}\right)_3 \frac{z^3}{6}$$

$\left(\frac{1}{2}\right)_0 \to 1$, $\left(\frac{1}{2}\right)_1 \to \frac{1}{2}$, $\left(\frac{1}{2}\right)_2 \to 3/4$, $\left(\frac{1}{2}\right)_3 \to 15/8$

$$\left(\frac{1}{2}\right)_3 = \frac{1}{2}\cdot\frac{3}{2}\cdot\frac{5}{2} = \frac{15}{8} \qquad ~~(4)_2 = 4\cdot 5 \;\; 6-1 \;\; 20~~ \qquad \left(\frac{1}{2}\right)_2 = \frac{1}{2}\cdot\frac{3}{2} = \frac{3}{4}$$

$$(5)_3 = 5\cdot 6\cdot 7 = 30\cdot 7 = 210$$

$$(4)_3 = 4\cdot 5\cdot 6 = 20\cdot 6 = 120$$

$$S = 1 + \frac{z}{2} + \frac{3z^2}{8} + \frac{5\,\cancel{15}\,z^3}{\cancel{48}\,16} + \cdots$$

$$S = 1 + \frac{z}{2} + \frac{3z^2}{8} + \frac{5z^3}{16} + \cdots$$

$$\underbrace{\mathrm{taylor}\left(\frac{1}{\sqrt{1-z}},\ z=0,\ 4\right)}_{\text{MAPLE !!!}} = 1 + \frac{z}{2} + \frac{3z^2}{8} + \frac{5z^3}{16}$$

IDENTIČNO

$$⋈ \Rightarrow \left|\ \boxed{z = \frac{1}{1+\gamma}}\ \right| \quad ⋈ = \frac{n!}{\left(\frac{1}{2}\right)_n} z^{-n}\left[\frac{1}{\sqrt{1-z}} - \sum_{k=0}^{n-1}\left(\frac{1}{2}\right)_k \frac{z^k}{k!}\right] =$$

$$= \frac{n!}{\left(\frac{1}{2}\right)_n}\left(\frac{1}{(1+\gamma)^{n}}\right)^{-n}\left[\frac{1}{\sqrt{1-\frac{1}{1+\gamma}}} - \sum_{k=0}^{n-1}\left(\frac{1}{2}\right)_k \frac{1}{k!}\left(\frac{1}{1+\gamma}\right)^k\right]$$

$${}_2F_1\left(1, n+\tfrac{1}{2}; n+1; z\right) = \frac{n!\,(1+\gamma)^n}{\left(\frac{1}{2}\right)_n}\left[\sqrt{\frac{1+\gamma}{\gamma}} - \sum_{k=0}^{n-1}\left(\frac{1}{2}\right)_k \frac{(1+\gamma)^{-k}}{k!}\right]$$

$$\boxed{A8} \Rightarrow P = \sqrt{\frac{\gamma}{1+\gamma}}\ \frac{\cancel{(1+\gamma)^{-b}}\ \Gamma\left(b+\frac{1}{2}\right)}{2\sqrt{\pi}\ \Gamma(b+1)}\ \frac{b!\,\cancel{(1+\gamma)^{b}}}{\left(\frac{1}{2}\right)_b}\left[\sqrt{\frac{1+\gamma}{\gamma}} - \sum_{k=0}^{b-1}\left(\frac{1}{2}\right)_k \frac{(1+\gamma)^{-k}}{k!}\right]$$

$$\sqrt{\pi}\left(\frac{1}{2}\right)_n = \Gamma\left(n+\frac{1}{2}\right); \quad \Gamma(n+1) = n! \quad \boxed{b \equiv n} \qquad \frac{\left(\frac{1}{2}\right)_k}{k!} = \binom{2k}{k}\left(\frac{1}{4}\right)^k \Rightarrow$$

$$P = \sqrt{\frac{\gamma}{1+\gamma}}\ \frac{\cancel{\sqrt{\pi}\left(\frac{1}{2}\right)_n}}{2\cancel{\sqrt{\pi}}\cdot\cancel{n!}} \cdot \frac{\cancel{n!}}{\cancel{\left(\frac{1}{2}\right)_n}}\left[\sqrt{\frac{1+\gamma}{\gamma}} - \sum_{k=0}^{n-1}\binom{2k}{k}\left(\frac{1}{4}\right)^k \frac{1}{(1+\gamma)^k}\right]$$

$$P = \frac{1}{2}\left[1 - \sqrt{\frac{\gamma}{1+\gamma}}\sum_{k=0}^{n-1}\binom{2k}{k}\left(\frac{1}{4(1+\gamma)}\right)^k\right]$$

$$\underline{\underline{\mu}} = \sqrt{\frac{\gamma}{1+\gamma}} = \left|\gamma = \frac{c}{2a}\right| = \sqrt{\frac{\frac{c}{2a}}{1+\frac{c}{2a}}} = \sqrt{\frac{c}{2a+c}} \Rightarrow$$

$$\boxed{P = \frac{1}{2}\left[1 - \mu\sum_{k=0}^{n-1}\binom{2k}{k}\left[\frac{1}{4(1+\gamma)}\right]^k\right]} \quad \boxed{MMV}$$

GLAVNIOT IZRAZ OD T. Eng ŠTO SE KORISTI VO PCOFC!

$$\binom{2k}{k}$$ 2k NAD k!!

$$\boxed{▽▲} \quad \boxed{\frac{1}{4(1+\gamma)} = \frac{1-\mu^2}{4}}$$

NEXT PAGE

● ⇒ FOR m = INTEGER + $\boxed{\nabla\blacktriangle}$ $\boxed{8.8 \times 5.4}$

$\boxed{\text{DCOFU}}$ CONTINUE:

$$I(a,b) = J_m(c) = \frac{1}{2}\left[1 - \mu(c)\sum_{k=0}^{m-1}\binom{2k}{k}\left(\frac{1-\mu^2(c)}{4}\right)^k\right]$$

$$\mu = \sqrt{\frac{c}{1+c}} \qquad \frac{c}{1+c} = \mu^2 \qquad c(1-\mu^2) = \mu^2$$

$$c = \mu^2 + \mu^2 c \qquad c = \frac{\mu^2}{1-\mu^2}$$

$$P = \frac{1}{2}\left[1 - \mu\sum_{k=0}^{m-1}\binom{2k}{k}\left[\frac{1}{4\left(1+\frac{\mu^2}{1-\mu^2}\right)}\right]^k\right] = \frac{1}{2}\left[1-\mu\sum_{k=0}^{m-1}\binom{2k}{k}\left[\frac{1-\mu^2}{4}\right]^k\right]$$

$$\boxed{P = \frac{1}{2}\left[1-\mu\sum_{k=0}^{m-1}\binom{2k}{k}\left[\frac{1-\mu^2}{4}\right]^k\right]}$$

$\boxed{\text{MMV}}$ $\boxed{m \text{ integer}}$

$$\underbrace{J_m(c) = \left(\frac{1-\mu(c)}{2}\right)^m\sum_{k=0}^{m-1}\binom{m-1+k}{k}\left(\frac{1+\mu(c)}{2}\right)^k}_{\text{FORMULA OD Proakis Digital Comm.}}$$

m INTEGER

$Q(\sqrt{bt})$

$$J_m(a,b) = \frac{a^m}{\Gamma(m)}\int_0^\infty e^{-at}t^{m-1}\left(\frac{1}{\pi}\int_0^{\pi/2}e^{-\frac{bt}{2\sin^2\theta}}d\theta\right)dt =$$

$$= \frac{a^m}{\pi\cdot\Gamma(m)}\int_0^{\pi/2}\int_0^\infty e^{-\left(a+\frac{b}{2\sin^2\theta}\right)t}\,t^{m-1}\,dt\,d\theta$$

$$\boxed{\Gamma(m) = \alpha^m\int_0^\infty t^{m-1}e^{-\alpha t}dt = \int_0^\infty t^{m-1}e^{-t}dt}$$

ALTERNATIVNA FORMA NA GAMMA FUN.

$\boxed{\text{MMV}}$

Ambrowitz 6.1.1

$$\alpha = \left(a + \frac{b}{2\sin^2\theta}\right)$$

$$J_m(a,b) = \frac{a^m}{\pi\Gamma(m)}\int_0^{\pi/2}\frac{\Gamma(m)}{\left(a+\frac{b}{2\sin^2\theta}\right)^m}d\theta = \frac{1}{\pi}\int_0^{\pi}\frac{1}{\left(1+\frac{b}{2a\sin^2\theta}\right)^m}d\theta =$$

$$= \left|c = \frac{b}{2a}\right| = \frac{1}{\pi}\int_0^{\pi}\left(\frac{\sin^2\theta}{\sin^2\theta + c}\right)^m d\theta$$

$$\boxed{J_m(a,b) = \frac{1}{\pi}\int_0^{\pi/}\left(\frac{\sin^2\theta}{\sin^2\theta + c}\right)^m d\theta = \frac{1}{2}\left[1 - \mu\sum_{k=0}^{m-1}\binom{2k}{k}\left[\frac{1-\mu^2}{4}\right]^k\right] \quad \mu = \sqrt{\frac{c}{1+c}} \quad m - \text{integer}}$$

Blagojče Petrevš

$\boxed{071200349}$

$$\frac{1}{\pi}\int_0^{\pi}\left(\frac{\sin^2\theta}{\sin^2\theta + c}\right)^m d\theta = \sqrt{\frac{\gamma}{1+\gamma}}\ \frac{(1+\gamma)^{-m}\Gamma(m+\frac{1}{2})}{2\,\Gamma(m+1)\sqrt{\pi}}\ {}_2F_1\left(1, m+\tfrac{1}{2}; m+1; \tfrac{1}{1+\gamma}\right)$$

$\gamma \equiv c$

$$\boxed{\frac{1}{\pi}\int_0^{\pi}\left(\frac{\sin^2\theta}{\sin^2\theta + c}\right)^m d\theta = \mu\ \frac{(1+c)^{-m}\Gamma(m+\frac{1}{2})}{2\,\Gamma(m+1)\sqrt{\pi}}\ {}_2F_1\left(1, m+\tfrac{1}{2}; m+1; \tfrac{1}{1+c}\right)}$$

- FOR RAYLEIGH FADING: $\boxed{m=1}$

$c = \frac{b}{2a}$

$$\frac{1}{\pi}\int_0^{\pi}\frac{\sin^2\theta}{\sin^2\theta + c}\,d\theta = \frac{1}{2}\left[1 \blacksquare \mu\right] = \frac{1}{2}\left(1 - \sqrt{\frac{c}{1+c}}\right) \quad ⊛$$

$$\frac{1}{\pi}\int_0^{\pi/2}\frac{\sin^2\theta}{\sin^2\theta + c}\,d\theta = \frac{1}{\pi}\int_0^{\pi/4}\left(1 - \frac{c}{\sin^2\theta + c}\right)d\theta = \frac{1}{2} - \frac{1}{\pi}\int_0^{\pi/2}\frac{c}{\sin^2\theta + c}\,d\theta$$

$\boxed{\text{Gradshtern 2.562.1}}$

$$\begin{aligned}\int\frac{dx}{a + b\sin^2 x} &= \frac{\operatorname{sign}(a)}{\sqrt{a(a+b)}}\operatorname{arctg}\left(\sqrt{\frac{a+b}{a}}\,\operatorname{tg}x\right) \quad \left[\frac{b}{a} > -1\right]\\ &= \frac{\operatorname{sgn}(a)}{\sqrt{-a(a+b)}}\operatorname{arctgh}\left(\sqrt{-\frac{a+b}{a}}\,\operatorname{tg}x\right) \quad \left[\frac{b}{a} < -1 \quad \sin^2 x < -\frac{a}{b}\right]\\ &= \frac{\operatorname{sgn}(a)}{\sqrt{-a(a+b)}}\operatorname{arctgh}\left(\sqrt{-\frac{a+b}{a}}\,\operatorname{tg}x\right) \quad \left[\frac{b}{a} < -1 \quad \sin^2 x > -\frac{a}{b}\right]\end{aligned}$$

MMV

$$\frac{1}{\pi}\int_0^{\pi/2}\frac{\sin^2\theta}{\sin^2\theta + c}\,d\theta = \frac{1}{2} - \frac{c}{\pi}\,\frac{1}{\sqrt{c(1+c)}}\operatorname{arctg}\left(\sqrt{\frac{1+c}{c}}\,\operatorname{tg}\theta\right)\Bigg|_0^{\pi/2}$$

$$= \frac{1}{2} - \frac{1}{\pi}\sqrt{\frac{c}{1+c}}\operatorname{arctg}\left(\overset{\infty}{\cancel{\sqrt{\frac{1+c}{c}}}}\right) = \frac{1}{2} - \frac{1}{\pi}\sqrt{\frac{c}{1+c}}\,\frac{\pi}{2} = \frac{1}{2}\left(1 - \sqrt{\frac{c}{1+c}}\right)$$

EQUIVALENTNO NA ⊛

- QAM & MPSK OVER RAYLEIGH CHANNEL

$$p_\gamma(\gamma) = \frac{1}{\bar\gamma} e^{-\gamma/\bar\gamma} \qquad \bar P = \int_0^\infty P_b \, p_\gamma(\gamma)\, d\gamma$$

$$P_b = 2Q\left(\sqrt{\frac{2E_s}{N_0}} \sin\frac{\pi}{M}\right) = 2Q\left(\sqrt{2\gamma}\, \sin\frac{\pi}{M}\right) \Big\} \ \underline{\underline{MPSK}}$$

$$I = \int_0^\infty Q(a\sqrt{\gamma})\, p_\gamma(\gamma)\, d\gamma \qquad \boxed{y = \gamma} \quad \boxed{a = \sqrt{2}\sin\frac{\pi}{M}}$$

$$\bar P = \int_0^\infty 2\, Q\Big(\underbrace{\sin\frac{\pi}{M}\sqrt{2}}_{a}\sqrt{\gamma}\Big) \frac{1}{\bar\gamma} e^{-\gamma/\bar\gamma} d\gamma = 2\int_0^\infty Q(a\sqrt{\gamma}) \frac{1}{\bar\gamma} e^{-\gamma/\bar\gamma} d\gamma$$

$$= 2\,\frac{1}{\pi}\int_0^{\pi/2} M_\gamma\left(-\frac{a^2}{2\sin^2\theta}\right) d\theta = \frac{2}{\pi}\int_0^{\pi/2} \frac{2\sin^2\theta}{2\sin^2\theta + a^2\bar\gamma}\, d\theta$$

$$= \frac{2}{\pi}\int_0^{\pi/2} \frac{\sin^2\theta}{\sin^2\theta + \underbrace{\frac{a^2\bar\gamma}{2}}_{C}}\, d\theta = \frac{2}{\pi}\int_0^{\pi/2} \frac{\sin^2\theta}{\sin^2\theta + c}\, d\theta \quad *$$

- ZA QAM

$$P_{bc} = \frac{4}{k}\left(1 - \frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{3k}{M-1}}\sqrt{\overset{\gamma}{\cancel{\frac{E_b}{N_0}}}}\right)$$

$$P_{bc} = \frac{4}{k}\left(1 - \frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{3k}{M-1}\,\gamma}\right)$$

~~MX~~ ISTO!!

$$\frac{1}{\pi}\int_0^{\pi/4} \frac{\sin^2\theta}{\sin^2\theta + c}\, d\theta = \frac{1}{\pi}\int_0^{\pi/4}\left(1 - \frac{c}{\sin^2\theta + c}\right) d\theta = \frac{1}{\cancel{\pi}}\cdot\frac{\cancel{\pi}}{4} - \underbrace{\frac{1}{\pi}\int_0^{\pi/4} \frac{c}{\sin^2\theta + c}\, d\theta}_{I}$$

$$\int \frac{d\theta}{a + b\sin^2\theta} = \frac{\mathrm{sign}(a)}{\sqrt{a(a+b)}}\, \mathrm{arctg}\left(\sqrt{\frac{a+b}{a}}\, \mathrm{tg}(\theta)\right)$$

$$I = \frac{c}{\pi}\, \frac{1}{\sqrt{c(1+c)}}\, \mathrm{arctg}\left(\sqrt{\frac{c+1}{c}}\, \mathrm{tg}(\theta)\right)\Bigg|_0^{\pi/4} = \underline{\underline{\frac{1}{\pi}\sqrt{\frac{c}{1+c}}\, \mathrm{arctg}\left(\sqrt{\frac{1+c}{c}}\right)}}$$

$$\boxed{\frac{1}{\pi}\int_0^{\pi/4}\frac{\sin^2\theta}{\sin^2\theta+c}\,d\theta=\frac{1}{4}\left[1-\frac{4}{\pi}\sqrt{\frac{c}{1+c}}\,\operatorname{arctg}\left(\sqrt{\frac{1+c}{c}}\right)\right]}$$

QAM

ZNAČI 11 4 1 $c=\frac{\bar{\gamma}a^2}{2m}$

$$\frac{1}{\pi}\int_0^{\pi/2}\left(1+\frac{c}{\sin^2\theta}\right)^{-m}d\theta=\begin{cases}\frac{1}{2}\left[1-\mu(c)\sum_{k=0}^{m-1}\binom{2k}{k}\left(\frac{1-\mu^2(c)}{4}\right)^k\right], & m\text{-integer}\\[2mm] \frac{1}{2\sqrt{\pi}}\frac{\sqrt{c}}{(1+c)^{m+1/2}}\frac{\Gamma(m+\frac{1}{2})}{\Gamma(m+1)}\,{}_2F_1\left(1,m+\frac{1}{2};m+1;\frac{1}{1+c}\right) & m\text{-noninteger}\end{cases}$$

$$\boxed{\mu(c)=\sqrt{\frac{c}{1+c}}}$$

$c=\frac{\bar{\gamma}a^2}{2m}$

$$I_m(a,m,\bar{\gamma})=\begin{cases}\frac{1}{2\sqrt{\pi}}\frac{\sqrt{a^2\bar{\gamma}/2m}}{\left(1+\frac{a^2\bar{\gamma}}{2m}\right)^{m+1/2}}\frac{\Gamma(m+\frac{1}{2})}{\Gamma(m+1)}\,2F_1\left(1,m+\frac{1}{2};m+1;\frac{1}{1+\frac{a^2\bar{\gamma}}{2m}}\right) & \underline{m\text{-noninteger}}\\[2mm] \frac{1}{2}\left[1-\mu\left(\frac{a^2\bar{\gamma}}{2m}\right)\sum_{k=0}^{m-1}\binom{2k}{k}\left(\frac{1-\mu^2\left(\frac{a^2\bar{\gamma}}{2m}\right)}{4}\right)^k\right] & \underline{m\text{ - integer}}\end{cases}$$

MMV

NAKAGAMI AVG. BER IN NAKAGAMI CHANNEL ($P_e=Q(a\sqrt{\gamma})$)

$\boxed{m=1}$ (RAYLEIGH)

ISTO KO KK 91. >g

$$I_m(a,1,\bar{\gamma})=\frac{1}{2}\left[1-\mu\right]=\frac{1}{2}\left[1-\sqrt{\frac{\frac{a^2\bar{\gamma}}{2}}{1+\frac{a^2\bar{\gamma}}{2}}}\right]$$

BPSK VO RAYLEIGH

$$\boxed{Q(z)=\frac{1}{2}\operatorname{erfc}\frac{z}{\sqrt{2}}}$$

BPSK $P_e(\gamma)=\frac{1}{2}\operatorname{erfc}(\sqrt{\gamma})$

$$Q(z)=\frac{1}{\sqrt{2\pi}}\int_z^{\infty}e^{-\frac{x^2}{2}}dx \qquad P_e(\gamma)=\frac{1}{2}\operatorname{erfc}\left(\frac{\sqrt{2\gamma}}{\sqrt{2}}\right)=Q(\sqrt{2\gamma})$$

$$Q(\sqrt{2\gamma})=\frac{1}{\sqrt{2\pi}}\int_{\sqrt{2\gamma}}^{\infty}e^{-\frac{x^2}{2}}dx=$$

$y=\frac{x}{\sqrt{2}}$ $x=\sqrt{2\gamma}$ $y=\sqrt{\gamma}$

$dy=\frac{dx}{\sqrt{2}}$ $dx=\sqrt{2}\,dy$

$$Q(\sqrt{2\gamma})=\frac{1}{\sqrt{2\pi}}\sqrt{2}\int_{\sqrt{\gamma}}^{\infty}e^{-y^2}dy=2\frac{1}{\sqrt{\pi}}\frac{1}{2}\int_{\sqrt{\gamma}}^{\infty}e^{-y^2}dy=\frac{1}{2}\operatorname{erfc}(\sqrt{\gamma})$$

$$erfc(x) = 1 - \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt = \frac{2}{\sqrt{\pi}}\int_0^\infty e^{-t^2}dt - \frac{2}{\sqrt{\pi}}\int_0^x = \frac{2}{\sqrt{\pi}}\int_x^\infty e^{-t^2}dt$$

BPSK (DUOCAL)

$$\boxed{P_e = Q\left(\sqrt{2\gamma}\right)}$$

$$I = \int_0^\infty Q(a\sqrt{\gamma})\, p_\gamma(\gamma)\, d\gamma$$

$$I = \int_0^\infty M_\gamma\left(-\frac{a}{2\sin^2\theta}\right) d\theta = \frac{1}{2}\left[1 - \sqrt{\frac{\frac{a^2\bar{\gamma}}{2}}{1+\frac{a^2\bar{\gamma}}{2}}}\right]$$

$$\boxed{a = \sqrt{2}} \qquad \boxed{I = \frac{1}{2}\left[1 - \sqrt{\frac{\bar{\gamma}}{1+\bar{\gamma}}}\right]}$$

ODGOVARA NA IZRAZOT DOBIEN SO MAHLE (VIDI N4. pp100)

5.1.7 NAKAGAMI-N (RICE) FADING CHANNEL (AVERAGE BER)

$$p_\gamma(\gamma) = \frac{(1+K)}{\bar{\gamma}} e^{-K} \exp\left(-\frac{(1+K)\gamma}{\bar{\gamma}}\right) I_0\left(2\sqrt{\frac{K(1+K)\gamma}{\bar{\gamma}}}\right)$$

$$M_\gamma(-s) = \hat{p}_\gamma(\gamma) = \int_0^\infty \frac{1+K}{\bar{\gamma}} e^{-K} e^{-\frac{(1+K)\gamma}{\bar{\gamma}}} \cdot e^{-\gamma s}\, I_0\left(2\sqrt{\frac{K(1+K)\gamma}{\bar{\gamma}}}\right) d\gamma$$

$$\boxed{\text{GRADSHTEYN}} \quad 6.631.4$$

$$I = \int_0^\infty x^{\nu+1} e^{-\alpha x^2} J_\nu(\beta x)\, dx = \frac{\beta^\nu}{(2\alpha)^{\nu+1}} \exp\left(-\frac{\beta^2}{4\alpha}\right) \qquad (\Delta 8)$$

$$\hat{p}_\gamma(\gamma) = \frac{1+K}{\bar{\gamma}} e^{-K} \int_0^\infty e^{-\underbrace{\left[\frac{1+K}{\bar{\gamma}}+s\right]}_{a}\gamma}\, I_0\Big(\underbrace{2\sqrt{\frac{K(1+K)}{\bar{\gamma}}}}_{p_1}\sqrt{\gamma}\Big)\, d\gamma$$

$$\gamma = x^2 \qquad d\gamma = 2x\,dx \qquad a = \left[\frac{1+K}{\bar{\gamma}} + s\right] \qquad p_1 = 2\sqrt{\frac{K(1+K)}{\bar{\gamma}}}$$

$$\hat{p}_\gamma(\gamma) = \frac{1+K}{\bar{\gamma}} e^{-K} \int_0^\infty e^{-ax^2}\, I_0(p_1 x)\, 2x\,dx \qquad (\Delta *)$$

$$\hat{p}_\gamma(\bar\gamma) = \frac{2(1+K)}{\bar\gamma} e^{-K} \underbrace{\int_0^\infty x\, e^{-ax^2} I_0(\beta_1 x)\, dx}_{I} \qquad \boxed{\nu = 0} \qquad = J_0(\underbrace{j\beta_1}_{\beta} x)$$

$$I = \frac{1}{2a} \exp\left[+\cancel{4}\, \frac{K(1+K)}{\bar\gamma} \cdot \frac{1}{\cancel{4}\cdot a}\right] = \frac{1}{2\left(\frac{1+K}{\bar\gamma}+s\right)} \exp\left[+\frac{K(1+K)}{\cancel{\bar\gamma}} \frac{1}{\frac{1+K+s\bar\gamma}{\cancel{\bar\gamma}}}\right]$$

$$\boxed{I = \frac{\bar\gamma}{2(1+K+s\bar\gamma)} \exp\left[+\frac{K(1+K)}{1+K+s\bar\gamma}\right]} \Longrightarrow \text{~~Z~~A} \quad \nu = 0 \qquad a = \frac{1+K}{\bar\gamma} + s \qquad \beta_1 = 2\sqrt{\frac{K(1+K)}{\bar\gamma}}$$

~~ZATO~~ 6, +

$$\boxed{(j\beta_1)^2 = -\beta_1^2}$$

$$\hat{p}_s(\bar\gamma) = \frac{\cancel{2}(1+K)}{\cancel{\bar\gamma}} e^{-K} \frac{\cancel{\bar\gamma}}{\cancel{2}(1+K+s\bar\gamma)} \exp\left[+\frac{K(1+K)}{1+K+s\bar\gamma}\right]$$

$$M(-s) = \hat{p}_\gamma(\bar\gamma) = \frac{(1+K)e^{-K}}{1+K+s\bar\gamma} \exp\left[+\frac{K(1+K)}{1+K+s\bar\gamma}\right]$$

$$M(-s) = \frac{1+K}{1+K+s\bar\gamma} \exp\left[+\frac{\cancel{K(1+K)} \cancel{-K} - K^2 - K s\bar\gamma}{1+K+s\bar\gamma}\right] = \frac{1+K}{1+K+s\bar\gamma} e^{-\frac{K s\bar\gamma}{1+K+s\bar\gamma}}$$

~~[illegible]~~

~~$\exp\left[-K - j\frac{K(1+K)}{1+K+s\bar\gamma}\right] = \exp\left[\frac{-K^2 - K - s\bar\gamma K - (2K^2}{1+K+s\bar\gamma}\right]$~~

~~$= \exp\left[\frac{-2K^2 - 2K - s\bar\gamma K}{1+K+s\bar\gamma}\right]$~~

$$\boxed{\begin{aligned} I_\nu(z) &= e^{-\frac{1}{2}\nu\pi i} J_\nu\left(z e^{\frac{1}{2}\pi i}\right) && -\pi < \arg z < \frac{\pi}{2} \\ I_\nu(z) &= e^{\frac{3\nu\pi i}{2}} J_\nu\left(z e^{-3\pi i/2}\right) && \frac{\pi}{2} < \arg z < \pi \end{aligned}}$$

$$\nu = 0 \qquad \boxed{I_0(z) = J_0(jz)} \qquad \boxed{\text{AMBROWITZ 9.6.3}}$$

$$e^{j\frac{\pi}{2}} = \cos\frac{\pi}{2} + j\sin\frac{\pi}{2} = j$$

$$(1\Sigma) \quad \nu = 0 \qquad \int_0^\infty x^{\cancel{\mu}} \cdot e^{-ax^2} J_0(\beta x)\, dx$$

$$(1*) \quad \frac{2(1+K)e^{-K}}{\bar\gamma} \int_0^\infty x e^{-ax^2} I_0(\beta_1 x)\, dx = \frac{2(1+K)e^{-K}}{\bar\gamma} \int_0^\infty x e^{-ax^2} J_0(\underbrace{j\beta_1}_{\beta} x)\, dx$$

$$M_{\gamma}(-s) = \frac{1+K}{1+K+s\bar{\gamma}} \exp\left(-\frac{Ks\bar{\gamma}}{1+K+s\bar{\gamma}}\right)$$

$$I(a,\bar{\gamma}) = \frac{1}{\pi}\int_0^{\pi/2} M_{\gamma}\left(-\frac{a^2}{2\sin^2\theta}\right)d\theta \qquad \boxed{P_e = Q\left(a\sqrt{\gamma}\right)}$$

~~AWGN BER~~

$$I(a,K,\bar{\gamma}) = \frac{1}{\pi}\int_0^{\pi/2} \frac{1+K}{1+K+\frac{a^2}{2\sin^2\theta}\cdot\bar{\gamma}} \exp\left(-\frac{\frac{K\bar{\gamma}\cdot a^2}{2\sin^2\theta}}{1+K+\frac{a^2}{2\sin^2\theta}\cdot\bar{\gamma}}\right)d\theta$$

$$I(a,K,\bar{\gamma}) = \frac{1}{\pi}\int_0^{\pi/2} \frac{2(1+K)\sin^2\theta}{2\sin^2\theta+2K\sin^2\theta+a^2\bar{\gamma}} \exp\left(-\frac{K\bar{\gamma}\cdot a^2}{2\sin^2\theta+2K\sin^2\theta+a^2\bar{\gamma}}\right)d\theta$$

$$\boxed{I(a,K,\bar{\gamma}) = \frac{1}{\pi}\int_0^{\pi/2} \frac{(1+K)\sin^2\theta}{(1+K)\sin^2\theta+\frac{a^2\bar{\gamma}}{2}} \exp\left(-\frac{K\bar{\gamma}\cdot a^2/2}{(1+K)\sin^2\theta+\frac{a^2\bar{\gamma}}{2}}\right)d\theta}$$

AVG BER IN RICIAN CHANEL $\left[P_e = Q\left(a\sqrt{\gamma}\right)\right]$ $\boxed{MMV}$

IMPLEMENTIRANO VO MATLAB!! (quadl)

- SEGA SIMULIRAME SI VO MATLAB!!!

$$P_{out} = P\left(\gamma < \gamma_{th}\right) = \int_0^{\gamma_{th}} p(\gamma)\,d\gamma$$

$$\boxed{\frac{dP_{out}}{d\gamma} = p(\gamma)} \qquad \mathcal{L}\left(p(\gamma)\right) = \mathcal{L}\left(\frac{dP_{out}}{d\gamma}\right)$$

$$\mathcal{L}\left\{p_{\gamma}(\gamma)\right\} = \int_0^{\infty} p_{\gamma}(\gamma)\,e^{-\gamma s}\,d\gamma = \hat{p}_{\gamma}(s) = M(-s)$$

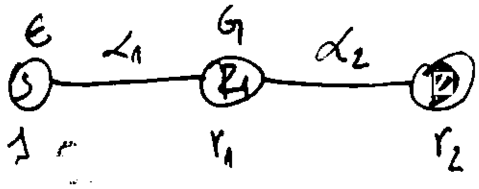
$$\mathcal{L}\left\{\frac{dP_{out}(\gamma)}{d\gamma}\right\} = \frac{s\,\hat{P}_{out}(s)}{\cancel{s}} \qquad \hat{p}_{\gamma}(s) = \frac{s\cdot\hat{P}_{out}(s)}{\cancel{s}}$$

$$\mathcal{L}\left\{p_{\gamma}(\gamma)\right\} = \int_0 \frac{dP_{\gamma}}{ds}\,e^{-\gamma s}\,d\gamma = s\cdot\hat{P}(s) \qquad \boxed{\hat{P}(s) = \frac{\hat{p}_{\gamma}(s)}{s}}$$

$P(s) = \frac{MGF(-s)}{s}$ $\boxed{P_{out} = \mathcal{L}^{-1}\left\{ \frac{M(-s)}{s} \right\}}$ GRAGZAMINOT $\bar{X}$

$$\boxed{\mathcal{L}\left[\int_0^{\gamma_{th}} p(\gamma)\, d\gamma\right] = \frac{\hat{p}(s)}{s}}$$

$\hat{P}_{out}(s) = \frac{\hat{f}(s)}{s} = \frac{M(s)}{s}$



$r_1 = \sqrt{E}\,\alpha_1 \cdot s + n_1(t) = \sqrt{E} \cdot \alpha_1 \cdot s + n_1(t)$

$r_2 = G\alpha_2 r_1 + n_2(t) = G\sqrt{E}\,\alpha_1 \cdot \alpha_2\, s + G\alpha_2 \cdot n_1 + n_2$

$$\boxed{r_2 = G\sqrt{E}\,\alpha_1 \cdot \alpha_2 \cdot s + G\alpha_2 \cdot n_1 + n_2}$$

- AVERAGE BER IN NAKAGAMI CHANNEL

$$P_e = \int_0^\infty P_b(\gamma)\, p_\gamma(\gamma)\, d\gamma$$

$$\boxed{p_\gamma = \frac{m^m \gamma^{m-1}}{\bar{\gamma}^m \Gamma(m)} e^{-m\gamma/\bar{\gamma}}}$$

DPSK: $\boxed{P_e = \frac{1}{2}\left(\frac{m}{m+\bar{\gamma}}\right)^m}$

$$\boxed{\mu = \sqrt{\frac{a^2\bar{\gamma}/2}{m + a^2\bar{\gamma}/2}}}$$

GENERAL : $P_b = Q(a\sqrt{\gamma})$

$$\boxed{P_e = \frac{1}{2}\left[1 - \mu\left(\frac{a^2\bar{\gamma}}{2m}\right) \sum_{k=0}^{m-1} \binom{2k}{k} \left(\frac{1 - \mu^2\left(\frac{a^2\bar{\gamma}}{2m}\right)}{4}\right)\right]}$$

$\binom{3}{1} = \frac{n!}{k!\,(n-k)!} = \frac{3!}{1\;2!} = \frac{6}{2} = 3$

$\boxed{\text{BPSK}} \Rightarrow P_b = Q\left(\sqrt{2\gamma}\right) \Rightarrow a = \sqrt{2}$

$$\boxed{\begin{array}{ll} \frac{1}{2}\,\text{erfc}(\sqrt{\gamma}) & Q(z) = \frac{1}{2}\,\text{erfc}\,\frac{z}{\sqrt{2}} \\ \hookrightarrow & \frac{1}{2}\,\text{erfc}\left(\frac{\sqrt{2\gamma}}{\sqrt{2}}\right) = Q\left(\sqrt{2\gamma}\right) \end{array}}$$

- COHERENT FSK

$$\frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{\gamma}{2}}\right) = \frac{1}{2}\,\mathrm{erfc}\,\frac{\sqrt{\gamma}}{\sqrt{2}} = Q(\sqrt{\gamma})$$

$$\boxed{P_{b\,CFSK} = Q(\sqrt{\gamma})} \qquad \boxed{a=1}$$

- MPSK

$$P_S(M) \doteq \underline{\underline{2Q}}\left(\sqrt{\frac{2E_s}{N_0}}\,\sin\frac{\pi}{M}\right) \Rightarrow \text{SYMBOL ERROR PROBABILITY (SKLAR)}$$

$$E_s = E_b\cdot\log_2 M \qquad P_e \doteq 2Q\left(\underbrace{\sqrt{2\cdot \mathrm{ld}M}\,\sin\frac{\pi}{M}}_{a}\cdot\sqrt{\gamma}\right)$$

$$\boxed{a = \sqrt{2\,\mathrm{ld}M}\,\sin\frac{\pi}{M}}$$

NO TREBA DA MNOŽIŠ SO 2

T.e. $P_E = 2I(a,m,\gamma)$

- PRODOLŽ

$$P_M = 2Q\left(\sqrt{2\gamma_s}\,\sin\frac{\pi}{M}\right) = 2Q\left(\sqrt{2k\gamma_b}\,\sin\frac{\pi}{M}\right)$$

$$\boxed{P_b = \frac{1}{\mathrm{ld}M}\cdot P_M = \frac{1}{\mathrm{ld}M}\cdot 2Q\left(\sqrt{2k\gamma_b}\,\sin\frac{\pi}{M}\right) \qquad \underline{k=\mathrm{ld}M}}$$

MMMV

Ova FORMULA ZA UPOTREBUV VO KOMBINACIJA ROČNE 5.18
VO BER Nakagami, MPSK.m ~~DAVA~~ MNOGU TOČNI REZULTATI!!!

ZA RAYLEIGH I RICIAN POTOČNO IZVRŠUVA. BEZ $P_M/\mathrm{ld}M$

- PAM

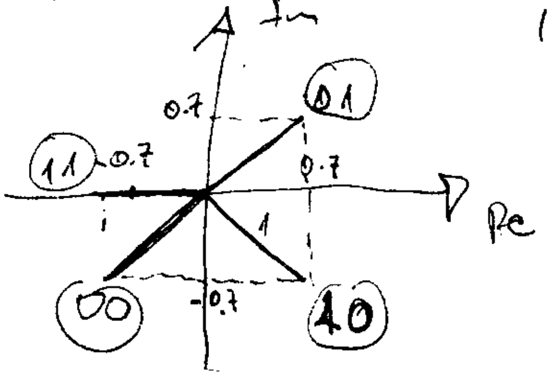
$$\boxed{P_M = \frac{2(M-1)}{M}\,Q\left(\sqrt{\frac{6\,\mathrm{ld}M\cdot\gamma}{(M^2-1)}}\right)} \qquad a = \sqrt{\frac{6\,\mathrm{ld}M}{(M^2-1)}}$$

$$\boxed{M=2} \quad P_b = Q\left(\sqrt{\frac{6^2\cdot\gamma}{3}}\right) = Q\left(\sqrt{2\gamma}\right) = \underbrace{Q\left(\sqrt{2}\sqrt{\gamma}\right)}$$

ISTO KAKO BPSK

OQPSK

3	-0.7	
0	-0.7	-0.7j
2	0.7	-0.7j
1	0.7	+0.7j



PROAKIS 14-4 DIVERSITY TECHNIQUES FOR FADING MULTIPATH CHANNELS

$$\lambda = \left(\frac{f\,[\mathrm{MHz}]}{300}\right)^{-1} = \frac{300}{f\,[\mathrm{MHz}]} = \frac{300}{900} = \frac{1}{3} = 0{,}33\,\mathrm{m}$$

$T_m W \Rightarrow$ NUMBER OF COMPONENTS

T_m → DELAY SPREAD $\quad T_m \approx \frac{1}{(\Delta f)_c}$ — coherence bandwidth

$$\boxed{\text{NUMBER OF RESOLVABLE COMPONENTS} = \frac{W}{(\Delta f)_c}}$$

ISM $\quad \lambda = \frac{300}{2500} = 0.12 = 12\,\mathrm{cm}$

14-4-1 BINARY SIGNALS (ERROR RATE PERFORMANCE FOR A BINARY DIGITAL COM. SYSTEM WITH DIVERSITY)

- L-DIVERSITY CHANNELS IN RAYLEIGH FADING AND AWGN CHANNEL

$$r_{lk} = a_k e^{-j\phi_k} s_{km}(t) + z_k(t) \qquad k = 1, 2, \dots L \qquad m = 1, 2$$

$\{a_k e^{-j\phi_k}\}$ - ATTENUATION FACTORS

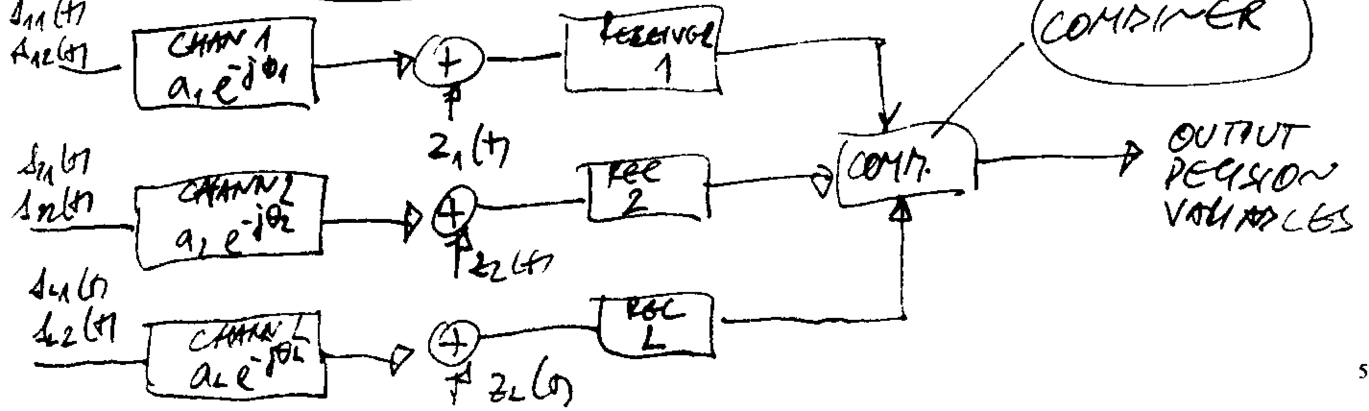
s_{km} - m-th signal on k-th channel

z_k - AWGN ON k-TH CHANNEL

OPTIMUM DEMODULATOR FOR SIGNAL RECEIVED IN k-TH CHANNEL (TWO MATCHED FILTERS)

$$\left.\begin{aligned} b_{k1}(t) &= s_{k1}^*(T-t) \\ b_{k2}(t) &= s_{k2}^*(T-t) \end{aligned}\right\}$$ IMPULSE RESPONSE OF THE MATCHED FILTERS.

FOR BPSK: $\boxed{s_{k1}(t) = -s_{k2}(t)}$



- OUTPUT OF MAXIMAL RATIO COMBINER

$$U = \mathrm{Re}\left(2\varepsilon \sum_{k=1}^{L} \alpha_k^2 + \sum_{k=1}^{L} \alpha_k N_k\right) = 2\varepsilon \sum_{k=1}^{L} \alpha_k^2 + \sum_{k=1}^{L} \alpha_k N_{kr}$$

N_{kr} - ~~COMPLEX~~ REAL PART OF COMPLEX GAUSSIAN NOISE VALUE.

$$N_k = e^{j\phi_k} \int_0^T z_k(t)\, s_k^*(t)\, dt$$

- BER IN RAYLEIGH FADING

$$\varepsilon(U) = 2\varepsilon \sum_{k=1}^{L} \alpha_k^2 \qquad \sigma_M^2 = 2\varepsilon N_0 \sum_{k=1}^{L} \alpha_k^2$$

- SNR PER BIT:

$$\gamma_b = \frac{\varepsilon}{N_0} \sum_{k=1}^{L} \alpha_k^2 = \sum_{k=1}^{L} \gamma_k \quad //$$

$\gamma_k = \dfrac{\varepsilon \alpha_k^2}{N_0}$ ⇒ INSTANTANEOUS SNR ON kth channel.

$p(\gamma_b) = ?$

$\gamma_b \equiv \gamma_1$ FOR $L=1$ γ_b has chi-square PROBABILITY DENSITY FUNCTION

- CHARACTERISTIC FUNCTION OF γ_1 IS:

$$\boxed{\psi_{\gamma_1}(ju) = \varepsilon\left(e^{ju\gamma_1}\right) = \int_0^{\infty} p_{\gamma_1}(\gamma_1) \cdot e^{ju\gamma_1}\, d\gamma_1}$$

$$\boxed{p_\gamma(\gamma) = \frac{1}{\sqrt{2}\,\Gamma(1/2)}\, \gamma^{-\frac{1}{2}}\, e^{-\gamma/2}}$$

CHI-SQUARE FOR $k=1$

KARAKTERISTIČNA FUNKCIJA: $\psi_\gamma(\gamma, k) = (1-2j\gamma)^{-k/2}$ — STEPEN NA SLOBODA

$$\boxed{\psi_{\gamma_1}(\gamma_1) = \frac{1}{\sqrt{1-2j\gamma_1}}}$$ ⇒ GO DAVA MALE

~~OVA NE E DOBRO ZOŠTO CHI-SQUARE E DISTRIBUIRANA NA α A NE ZA γ~~

OVA NE E DOBRO ZOŠTO α^2 e CHI-SQUARE DISTRIBUIRAN SO $k=2$

- CHI SQUARE

$$\boxed{p_{\alpha^2}(\alpha^2, k) = \frac{1}{2^{k/2}\Gamma\left(\frac{k}{2}\right)} x^{\frac{k}{2}-1} e^{-\frac{\alpha^2}{2}} \qquad x > 0}$$

$k = 2$

$$p_{\alpha^2}(\alpha^2, 2) = \frac{1}{2\ \Gamma(1)} (\alpha^2)^{1-1} e^{-\frac{\alpha^2}{2}} = \frac{1}{2} e^{-\frac{\alpha^2}{2}}$$

POTSETUVANJE (RAYLEIGH)

$$p_\alpha(\alpha) = \frac{\alpha}{\sigma^2} e^{-\alpha^2/2\sigma^2}$$

$$\gamma = \frac{E_b}{N_0}\cdot\alpha^2 \qquad \Omega = 2\sigma^2$$

$$\bar{\gamma} = \frac{E_b}{N_0}\cdot E(\alpha^2) = \frac{E_b}{N_0}\cdot\Omega$$

$$\frac{E_b}{N_0} = \left(\frac{\Omega}{\bar{\gamma}}\right)^{-1} = \frac{\bar{\gamma}}{\Omega}$$

$$\gamma = \left(\frac{\Omega}{\bar{\gamma}}\right)^{-1}\cdot\alpha^2 \qquad \boxed{\alpha^2 = \frac{\Omega}{\bar{\gamma}}\cdot\gamma}$$

$$\gamma = \frac{\bar{\gamma}}{\Omega}\alpha^2$$

$$p_\gamma(\gamma) = \frac{p_\alpha(\alpha)}{\frac{d\gamma}{d\alpha}}\Bigg|_{\alpha = f(\gamma)} = \frac{\frac{\alpha}{\sigma^2} e^{-\frac{\alpha^2}{2\sigma^2}}}{\frac{\bar{\gamma}}{\Omega}\cdot 2\cancel{\alpha}} \qquad 2\sigma^2 = \Omega$$

$$\left|\frac{d\gamma}{d\alpha} = \frac{\bar{\gamma}}{\Omega} 2\cdot\alpha\right| \quad = \frac{\frac{2}{\cancel{\Omega}} e^{-\cancel{\frac{1}{\Omega}}\frac{\cancel{\Omega}\cdot\gamma}{\bar{\gamma}}}}{\frac{2\bar{\gamma}}{\cancel{\Omega}}} = \frac{1}{\bar{\gamma}}\cdot e^{-\frac{\gamma}{\bar{\gamma}}}$$

PROAKIS 2-1-4 CHI - SQUARE DISTRO

$Y = X^2$ $\qquad E(X) =$ [illegible] $\qquad Var(X) = \sigma^2$

$$\boxed{p_Y(y, 1) = \frac{1}{\sqrt{2}\ \underset{\sqrt{\pi}}{\Gamma\left(\frac{1}{2}\right)}\sqrt{y}} e^{-\frac{y}{2\sigma^2}} = \frac{1}{\sqrt{2\pi y}} e^{-\frac{y}{2\sigma^2}}}$$

$$\Psi_Y(j\mu) = E\left(e^{j\mu Y}\right) = \int_0^\infty \frac{1}{\sqrt{2\pi y}} e^{-\frac{y}{2\sigma^2}}\cdot e^{j\mu y}\,dy$$

$$\boxed{\Psi_Y(j\mu) = \frac{1}{(1 - j2\mu\sigma^2)^{1/2}}}$$

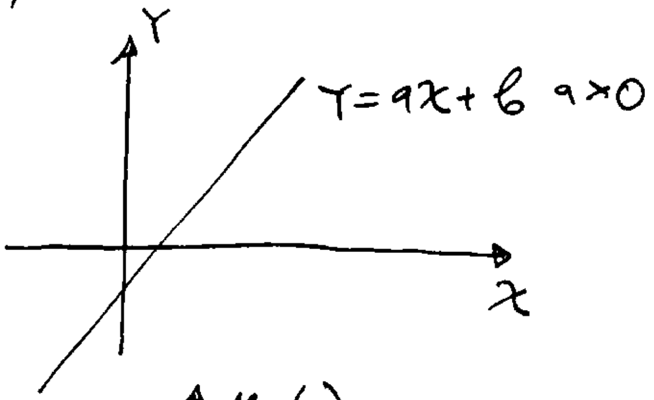
PROAKIS 2-1-2 FUNCTIONS OF RANDOM VARIABLES

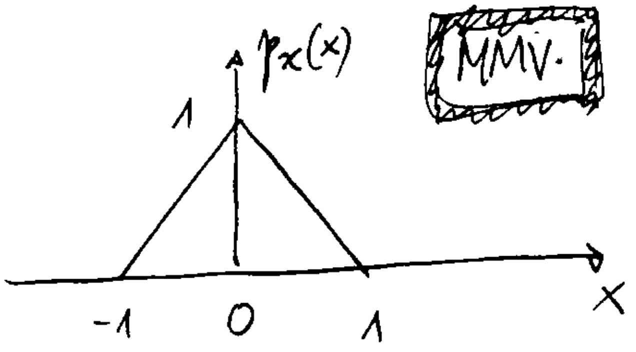
X CARACTERIZED WITH $p(x)$

DETERMINE PDF FOR: $Y = g(X)$

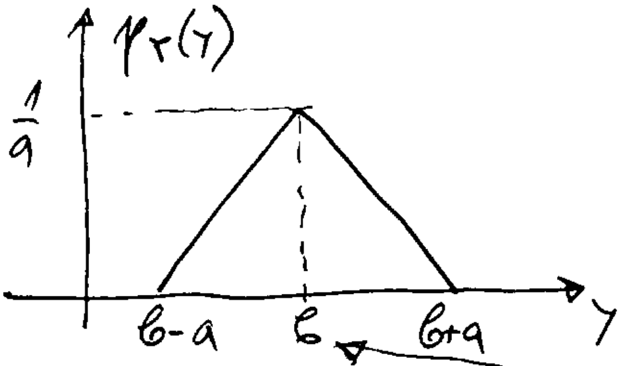
EXP. 2-1-1 $\quad Y = aX + b$

$a > 0$





MMV.



$$F_Y(y) = P(Y \le y) = P(aX + b \le y) = P\left(X \le \frac{y-b}{a}\right) =$$

$$= \int_{-\infty}^{(y-b)/a} p_x(x)\,dx = F_x\left(\frac{y-b}{a}\right)$$

$$F_Y(y) = F_x\left(\frac{y-b}{a}\right) \quad \Big/ \frac{d}{dy}$$

$$F_Y(y) = \int_{-\infty}^{y} p_Y(t)\,dt \qquad \frac{d}{dy}F_Y(y) = p_Y(y)$$

$$\boxed{\frac{d}{dy}\left(F_x\left(\frac{y-b}{a}\right)\right) = \frac{1}{a}\,p_x\left(\frac{y-b}{a}\right)}$$

$$p_Y(y) = \frac{1}{a}\,p_x\left(\frac{y-b}{a}\right)$$

$x = 0 = \frac{y-b}{a}$ $\quad \boxed{y = b}$

$x = -1 = \frac{y-b}{a}$ $\quad y = b - a$

$x = 1 = \frac{y-b}{a}$ $\quad y = a + b$

ZA $y = b$ $\quad p_Y(y) = \frac{1}{a}\overset{1}{\cancel{p_x(0)}} = \frac{1}{a}$

Exp. 2-1-2

$$Y = ax^3 + b \qquad a > 0$$

$$F_Y(y) = P(Y \leq y) = \int_{-\infty}^{y} p_Y(t)\,dt = P(ax^3+b \leq y) =$$

$$= P\left(x^3 \leq \frac{y-b}{a}\right) = P\left(x \leq \sqrt[3]{\frac{y-b}{a}}\right) = \int_{-\infty}^{\left(\frac{y-b}{a}\right)^{1/3}} p_x(t)\,dt$$

$$\frac{dF_Y(y)}{dy} = p_Y(y) \qquad \frac{dF_x\left(\left(\frac{y-b}{a}\right)^{1/3}\right)}{dy} = \underbrace{\left[\left(\frac{y-b}{a}\right)^{\frac{1}{3}}\right]'}_{\frac{dx}{dy}} \cdot p_x\left(\left(\frac{y-b}{a}\right)^{1/3}\right)$$

$$\frac{d}{dy}\left[\left(\frac{y-b}{a}\right)^{\frac{1}{3}}\right] = \frac{1}{3}\left(\frac{y-b}{a}\right)^{-\frac{2}{3}} \cdot \frac{1}{a} = \frac{1}{3a}\,\frac{1}{\left(\frac{y-b}{a}\right)^{\frac{2}{3}}} = \frac{1}{3a}\left(\frac{a}{y-b}\right)^{\frac{2}{3}}$$

$$p_Y(y) = \frac{1}{3a}\left(\frac{a}{y-b}\right)^{2/3} p_x\left(\left(\frac{y-b}{a}\right)^{1/3}\right)$$

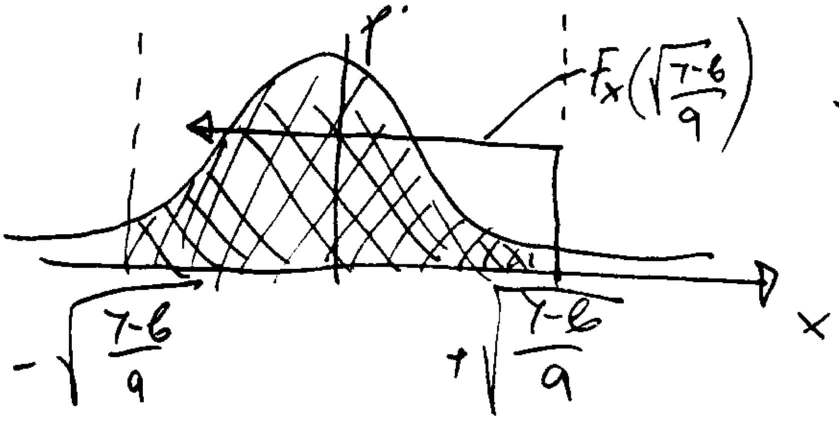
GENERALNA FORMULA: $\boxed{p_Y(y) = \left|\frac{dx}{dy}\right| \cdot p_x(x)\Big|_{x=f(y)}}$

Exp. 2-1-3

$$Y = ax^2 + b \qquad a > 0$$

$$F_Y(y) = P(Y \leq y) = P\left(x^2 \leq \frac{y-b}{a}\right) = P\left(|x| \leq \sqrt{\frac{y-b}{a}}\right)$$

$$= P\left(x \leq \sqrt{\frac{y-b}{a}}\right) - P\left(x \leq -\sqrt{\frac{y-b}{a}}\right) = F_x\left(\sqrt{\frac{y-b}{a}}\right) - F_x\left(-\sqrt{\frac{y-b}{a}}\right)$$



$$\frac{dF_Y(y)}{dy} = \frac{dF_x\left(\sqrt{\frac{y-b}{a}}\right)}{dy} - \frac{dF_x\left(-\sqrt{\frac{y-b}{a}}\right)}{dy} \ (*)$$

$$p_Y(y) = \frac{dx}{dy}\, p_x\left(\sqrt{\frac{y-b}{a}}\right) + \frac{dx}{dy} \cdot p_x\left(-\sqrt{\frac{y-b}{a}}\right)$$

$$= \frac{p_x(x)}{\frac{dy}{dx}}\Bigg|_{x=\sqrt{\frac{y-b}{a}}} + \frac{p_x(x)}{\frac{dy}{dx}}\Bigg|_{x=-\sqrt{\frac{y-b}{a}}}$$

$x = \pm\sqrt{\frac{y-b}{a}}$ $\quad y = ax^2 + b$

$$\circledast \Rightarrow F_Y(y) = F_X\left(\sqrt{\frac{y-b}{a}}\right) - F_Y\left(-\sqrt{\frac{y-b}{a}}\right) \Big/ \frac{d}{dy}$$

$$p_Y(y) = \frac{d}{dy}\left(\sqrt{\frac{y-b}{a}}\right)\underset{p_X\left(+\sqrt{\frac{y-b}{a}}\right)}{\cdot} + \frac{d}{dy}\left(\sqrt{\frac{y-b}{a}}\right)\cdot p_X\left(-\sqrt{\frac{y-b}{a}}\right)$$

$$\frac{d}{dy}\left(\sqrt{\frac{y-b}{a}}\right) = \frac{1}{2}\left(\frac{y-b}{a}\right)^{-\frac{1}{2}}\cdot\frac{1}{a} = \frac{1}{2a}\left(\frac{a}{y-b}\right)^{1/2} = \frac{1}{2\sqrt{a(y-b)}}$$

$$\boxed{p_Y(y) = \frac{p_X\left(\sqrt{(y-b)/a}\right)}{2\sqrt{a(y-b)}} + \frac{p_X\left(-\sqrt{(y-b)/a}\right)}{2\sqrt{a(y-b)}}} \ \boxed{\text{MMV}} \quad \textcircled{\Delta *}$$

$y = ax^2 + b$

$$g(x) = ax^2 + b = y \qquad \underbrace{x_1 = \sqrt{\frac{y-b}{a}} \qquad x_2 = -\sqrt{\frac{y-b}{a}}}_{\text{TWO REAL SOLUTIONS}}$$

$$p_Y(y) = \frac{p_X\left(x_1 = \sqrt{\frac{y-b}{a}}\right)}{\left|g'\left(x_1 = \sqrt{(y-b)/a}\right)\right|} + \frac{p_X\left(x_2 = -\sqrt{(y-b)/a}\right)}{\left|g'\left(x_2 = -\sqrt{(y-b)/a}\right)\right|}$$

$$g'(x) = \left|2ax\right|_{x = -\sqrt{\frac{y-b}{a}}} = \left|-2a\sqrt{\frac{y-b}{a}}\right| = \left|-2\sqrt{a(y-b)}\right| = 2\sqrt{a(y-b)}$$

- IN GENERAL

$$\boxed{p_Y(y) = \sum_{i=1}^{n} \frac{p_X(x_i)}{|g'(x_i)|}} \qquad \boxed{\begin{array}{l} x_i \ i = 1, 2, \dots n \\ \text{FUNCTIONS OF } y \end{array}}$$

- X_i $i = 1, 2, \dots n$ RANDOM VARIABLES WITH JOINT pdf $p_X(x_1, x_2, \dots x_n)$

$\left.Y_i \ i = 1, 2, \dots n\right\}$ ANOTHER SET OF RANDOM VARIABLES $i = 1, 2 \dots n$

$Y_i = g_i(x_1, x_2, \dots x_n)$

$x_i = g_i^{-1}(y_1, y_2, \ldots, y_n) \quad i = 1, 2, \ldots n \Big\}$ INVERTABLE FUNCTIONS

$p_Y(y_1, y_2, \ldots y_n) = ?$ given the $p_X(x_1, x_2, \ldots x_n)$

$$\iint\limits_{R_Y} \cdots \int p_Y(y_1, y_2, \ldots, y_n)\, dy_1\, dy_2 \cdots dy_n =$$

$$= \iint\limits_{R_X} \cdots \int p_X(x_1, x_2, \ldots, x_n)\, dx_1\, dx_2 \cdots dx_n$$

PROMENA NA PROMENLIVI

$x_i = g^{-1}(y_1, y_2, \ldots y_n) \qquad i = 1, 2, \ldots n$

$$\iint\limits_{R_Y} \cdots \int p_Y(y_1, y_2, \ldots y_n)\, dy_1\, dy_2 \cdots dy_n =$$

$$= \iint\limits_{R_Y} \cdots \int p_X(x_1 = g_1, x_2 = g_2, \ldots x_n = g_n)\, \underline{|J|\, dy_1\, dy_2 \ldots dy}$$

TEORIJA NA INFO VO [illegible] E OBRATNO DEFINIRAN (J) PA ZATOA TUKA DELI

$$J = \begin{vmatrix} \frac{\partial x_1}{\partial y_1} & \frac{\partial x_2}{\partial y_1} & \cdots & \frac{\partial x_n}{\partial y_1} \\ \frac{\partial x_1}{\partial y_2} & \frac{\partial x_2}{\partial y_2} & \cdots & \frac{\partial x_n}{\partial y_2} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{\partial x_1}{\partial y_n} & \frac{\partial x_2}{\partial y_n} & \cdots & \frac{\partial x_n}{\partial y_n} \end{vmatrix}$$

$$\boxed{p_Y(y_1, y_2, \ldots y_n) = p_X(x_1 = g_1, x_2 = g_2, \ldots x_n = g_n)\, |J|}$$

$$p_Y(y_1, y_2, \ldots y_n)\, dy_1\, dy_2 \ldots dy_n = p_X(x_1 = g_1, x_2 = g_2 \ldots)\, dx_1\, dx_2 \ldots dx$$

$$dx_1\, dx_2 \ldots dx_n = |J| \cdot dy_1\, dy_2 \ldots dy_n \Rightarrow$$

$$\boxed{p_Y(y_1, y_2 \ldots y_n) = |J| \cdot p_X(x_1 = g_1, x_2 = g_2, \ldots x_n = g_n)}$$

$$\boxed{\text{EXT. } 2\text{-}1\text{-}4}$$

$$Y_i = \sum_{j=1}^{n} a_{ij} X_j \qquad i = 1, 2, \dots, n$$

$$Y = A \cdot X \quad \text{(MATRIX)} \qquad X, Y \quad n\text{-DIMEN. VECTOR} \qquad A \quad n \times n \quad \text{MATRIX}$$

$$\boxed{X = A^{-1} Y} \qquad x_i = \sum_{j=1}^{n} b_{ij} Y_j \quad i = 1, 2, \dots n$$

$$\boxed{329\,1909}$$

$$\boxed{J = \frac{1}{\det A}}$$

$$p_Y(y_1, y_2, \dots, y_n) = p_x\left(x_1 = \sum b_{ij} y_j, \dots x_n = \sum_{j=1}^{n} b_{nj} y_j\right) \frac{1}{|\det A|}$$

$$\boxed{\text{PROAKIS: } 14\text{-}7}$$

α - RAYLEIGH DISTRO ⇒ α^2 ⇒ CHI-SQUARE DISTRO WITH TWO DEGREES OF FREDOM

$$y = \alpha^2 \qquad \boxed{p_Y(y) = ?} \qquad \boxed{\alpha = \pm\sqrt{y}}$$

$$p_\alpha(\alpha) = \frac{\alpha}{\sigma^2} \cdot e^{-\alpha^2/2\sigma^2} \qquad p_Y(y) = \frac{p_\alpha(\alpha)}{\left|\frac{dy}{d\alpha}\right|}\Bigg|_{\alpha=\sqrt{y}} + \frac{p_\alpha(\alpha)}{\left|\frac{dy}{d\alpha}\right|}\Bigg|_{\alpha=-\sqrt{y}}$$

$$p_Y(y) = \frac{\frac{\cancel{\alpha}^{\sqrt{y}}}{\sigma^2} \cdot e^{-\frac{\alpha^2}{2\sigma^2}}}{2\cancel{\alpha}^{\sqrt{y}}} + \frac{\frac{\alpha^{-\sqrt{y}}}{\sigma^2} e^{-\frac{\alpha^2}{2\sigma^2}}}{-2\sqrt{y}} = \frac{1}{2\sigma^2}\left(e^{-\frac{\alpha^2}{2\sigma^2}} + e^{-\frac{\alpha^2}{2\sigma^2}}\right)$$

$$\boxed{p_Y(y) = \frac{1}{2\sigma^2} \cdot e^{-\frac{y}{2\sigma^2}}} \quad ①$$

$$\boxed{\begin{array}{l}\text{CENTRAL}\\ \text{CHI - SQUARE}\\ \text{PROAKIS } 2\text{-}1\text{-}110\end{array}} \qquad \boxed{p_Y(y, K) = \frac{1}{\sigma^K 2^{K/2} \Gamma\left(\frac{K}{2}\right)} y^{\frac{K}{2}-1} e^{-\frac{y}{2\sigma^2}}} \quad \boxed{\text{MN}} \quad Ⓧ$$

$$\boxed{p_Y(y, 2) = \frac{1}{2\sigma^2} e^{-\frac{y}{2}}}$$

$$\boxed{\text{PROAKIS } 2\text{-}1\text{-}4} \qquad Y = X^2 \qquad \boxed{X - \text{GAUSSIAN}} \qquad \text{MNE}$$

$$x = \pm\sqrt{y}$$

$$p_X(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2}} \qquad p_Y(y) = \frac{p_X(x)}{\left|\frac{dy}{dx}\right|}\Bigg|_{x=\sqrt{y}} + \frac{p_X(x)}{\left|\frac{dy}{dx}\right|}\Bigg|_{x=-\sqrt{y}}$$

$$p_Y(y) = \left.\frac{\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2}}}{2x}\right|_{x=\sqrt{y}} + \left.\frac{\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2}}}{|2x|}\right|_{x=-\sqrt{y}} = \Leftarrow$$

MOŽE PIŠEMO OD (AX) PP.62

$$= \cancel{2} \frac{1}{\cancel{2}\sqrt{y}\sigma\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2}} = \frac{1}{\sigma\sqrt{2\pi y}} e^{-\frac{y}{2\sigma^2}}$$

$$\circledast \quad p_Y(y,1) = \frac{1}{\sigma\sqrt{2}\sqrt{\pi}} y^{-\frac{1}{2}} e^{-\frac{y}{2\sigma^2}} = \frac{1}{\sigma\sqrt{2\pi y}} e^{-\frac{y}{2\sigma^2}}$$

DOKAŽANO DEKA $Y = X^2$ X - GAUSS $\Rightarrow$ Y - CHI SQUARE

$$\boxed{p_Y(y) = \frac{1}{\sigma\sqrt{2\pi y}} e^{-\frac{y}{2\sigma^2}}}$$

CHI - SQUARE $\boxed{k=1}$

14-4-1 (PROAKIS) PRODOLŽENIE...

$$\gamma_b = \frac{\epsilon}{N_0} \sum_{k=1}^{L} a_k^2 = \sum_{k=1}^{L} \gamma_k \qquad \boxed{\gamma_k = \frac{\epsilon}{N_0} \alpha_k^2}$$

$L=1$ $\boxed{\gamma_b = \gamma_1}$ $\boxed{\Omega = 2\sigma^2}$

$$\bar{\gamma} = \epsilon(\gamma) = \epsilon\left(\frac{\epsilon_b}{N_0}\alpha^2\right) = \frac{\epsilon_b}{N_0}\epsilon(\alpha^2)$$

$$\frac{\bar{\gamma}}{\Omega} = \frac{\epsilon_b}{N_0}$$

$$\boxed{\gamma_1 = \frac{\epsilon_b}{N_0} \cdot \alpha_1^2}$$

$$(1^*) \quad \frac{\epsilon_b}{N_0} \cdot \alpha_1^2 = \frac{\bar{\gamma}}{\Omega} \cdot \alpha_1^2$$

$$a = \frac{\gamma}{\frac{\bar{\gamma}}{\Omega}} \Rightarrow p_\gamma = \frac{p_x\left(\sqrt{\frac{\gamma}{\bar{\gamma}} \cdot \Omega}\right)}{2\sqrt{\frac{\bar{\gamma}}{\Omega}(\gamma)}} + \frac{p_x\left(-\sqrt{\frac{\gamma}{\bar{\gamma}}\Omega}\right)}{2\sqrt{\frac{\bar{\gamma}}{\Omega} \cdot \gamma}}$$

$$\boxed{\alpha = \sqrt{\frac{\Omega}{\bar{\gamma}} \cdot \gamma}}$$

OVOJ ČLEN TREBA DA E 0 ($p(\alpha)=0$ ZA $\alpha<0$

$$p_\gamma = \frac{\frac{\sqrt{\frac{\gamma\Omega}{\bar{\gamma}}}}{\sigma^2} e^{-\frac{\cancel{\alpha^2}}{2\sigma^2}\frac{\gamma}{\bar{\gamma}}\cancel{\Omega}}}{2\sqrt{\frac{\bar{\gamma}}{\Omega}\cancel{\gamma}}} + \frac{\frac{\sqrt{\frac{\gamma\Omega}{\bar{\gamma}}}}{\sigma^2} e^{-\frac{1}{2\sigma^2}\frac{\gamma}{\bar{\gamma}}\cdot\cancel{\Omega}}}{2\sqrt{\frac{\bar{\gamma}}{\Omega}\cancel{\gamma}}} = \left|\sigma^2 = \frac{\Omega}{2}\right| =$$

$$= 2 \frac{\sqrt{\frac{\Omega}{\bar{\gamma}}}}{\frac{\Omega}{2} 2\sqrt{\frac{\bar{\gamma}}{\Omega}}} \cdot e^{-\frac{\gamma}{\bar{\gamma}}} = 2 \frac{\Omega}{\Omega\bar{\gamma}} \cdot e^{-\frac{\gamma}{\bar{\gamma}}} = \frac{2}{\bar{\gamma}} e^{-\frac{\gamma}{\bar{\gamma}}}$$

$$\boxed{p_\gamma = \frac{2}{\bar{\gamma}} e^{-\gamma/\bar{\gamma}}}$$

TREBA!

MMV !!! $\boxed{p_\gamma = \frac{1}{\bar{\gamma}} e^{-\gamma/\bar{\gamma}}}$ ZATOA ŠTO $p(\alpha) = 0$ ~~ZA~~ $\alpha < 0$

- ÚŠTE EDNAŠ:

$$\gamma = \frac{E_b}{N_0} \cdot \alpha^2$$

$$\alpha = \overset{+}{\ominus}\sqrt{\gamma \frac{N_0}{E_b}}$$

MINUSOT NE SE ZEMA VO PREDVID!!!

$p(\alpha)=0 \quad \alpha<0$

α - RAYLEIGH DISTRIBUTION

$$\bar{\gamma} = \frac{E_b}{N_0} \cdot \underbrace{E(\alpha^2)}_{\Omega = 2\sigma^2} = E(\gamma) = \frac{E_b}{N_0} \cdot \Omega$$

$$\frac{E_b}{N_0} = \frac{\bar{\gamma}}{\Omega} \qquad \sigma^2 = \frac{\Omega}{2}$$

$$\boxed{\alpha = \pm\sqrt{\frac{\Omega}{\bar{\gamma}}\gamma}}$$

$$\boxed{\gamma = \frac{\bar{\gamma}}{\Omega} \cdot \alpha^2}$$

$$p_\gamma(\gamma) = \frac{p_\alpha(\alpha_1)}{\left|\frac{d\gamma}{d\alpha}\right|}\Bigg|_{\alpha_1 = f_1(\gamma)} + \underbrace{\frac{p_\alpha(\alpha_2)}{\left|\frac{d\gamma}{d\alpha}\right|}\Bigg|_{\alpha_2 = f_2(\gamma)}}_{0}$$

$$p_\gamma(\gamma) = \frac{\frac{\alpha}{\sigma^2} \cdot e^{-\frac{\alpha^2}{\Omega}}}{\left|\frac{2\bar{\gamma}}{\Omega} \cdot \alpha\right|}\Bigg|_{\alpha = \sqrt{\frac{\Omega\gamma}{\bar{\gamma}}}} + \frac{\frac{\alpha}{\sigma^2} e^{-\frac{\alpha^2}{\Omega}}}{\left|\frac{2\bar{\gamma}}{\Omega}\alpha\right|}\Bigg|_{\alpha = -\sqrt{\frac{\Omega\gamma}{\bar{\gamma}}}} \rightarrow 0$$

$$p_\gamma(\gamma) = \frac{\cancel{\frac{2}{\Omega}} \cdot \cancel{\sqrt{\frac{\Omega\gamma}{\bar{\gamma}}}}\, e^{-\frac{1}{\Omega}\frac{\Omega \cdot \gamma}{\bar{\gamma}}}}{\cancel{\frac{2\bar{\gamma}}{\Omega}} \cdot \cancel{\sqrt{\frac{\Omega\gamma}{\bar{\gamma}}}}} + \frac{\frac{2}{\Omega}\sqrt{\frac{\Omega\gamma}{\bar{\gamma}}}\, e^{-\frac{\gamma}{\bar{\gamma}}}}{\frac{2\bar{\gamma}}{\Omega}\sqrt{\frac{\Omega\gamma}{\bar{\gamma}}}} \rightarrow 0$$

$p(\gamma)=0 \quad \gamma<0$

$$\boxed{p_\gamma(\gamma) = \frac{1}{\bar{\gamma}} e^{-\gamma/\bar{\gamma}} + \underbrace{\cancel{\frac{1}{\bar{\gamma}} e^{-\gamma/\bar{\gamma}}}}_{\text{OVA } = 0} = \frac{1}{\bar{\gamma}} e^{-\gamma/\bar{\gamma}}}$$

OVA =0 ZOŠTO $p(\alpha)=0 \quad \alpha<0$

MMV

$$p_\gamma(\gamma, \cancel{k}\,2) = \frac{1}{\sigma^k 2^{k/2} \Gamma\left(\cancel{\frac{k}{2}}\,1\right)} \gamma^{\frac{k}{2}-1} e^{-\gamma/2\sigma^2}$$

$$p_\gamma(\gamma, 2) = \frac{1}{\sigma^2 \cdot 2} e^{-\frac{\gamma}{2\sigma^2}} = \frac{1}{\Omega} e^{-\frac{\gamma}{\Omega}}$$

$$\boxed{p_\gamma(\gamma) = \frac{1}{\bar{\gamma}} e^{-\gamma/\bar{\gamma}}}$$

$\bar{\gamma}$ — AVERAGE SNR PER SYMBOL

⟹ VSUŠNOST OVA E KRAJČEN IZRAZ NA RELEJEVA RASPREDELBA PREKU SNR!!! T.E RASPREDELBA NA SNAGATA NA SIGNALOT ČIJA AMPLITUDI SE RASPREDELENI PO RELEJEVATA RASPREDELBA.

ZNAČI ~~ZA~~ RAYLEIGH:

$$\boxed{\bar{\gamma} = \Omega = 2\sigma^2} \quad \text{T.E}$$

$2\sigma^2 = P_r$ } VKUPNA SREDNA PRIEMNA SNAGA

$$\boxed{p_\gamma(\gamma) = \frac{1}{2\sigma^2} e^{-\gamma/2\sigma^2}}$$

- CHARACTERISTIC FUNCTION OF γ

$$p_\gamma(\gamma) = \frac{1}{\gamma} e^{-\gamma/\Gamma} \qquad \psi_\gamma(jM) = E\left(e^{jM\gamma}\right) = \int_0^\infty p_\gamma(\gamma) e^{jM\gamma} d\gamma$$

$$\boxed{\psi_\gamma(jM) = \frac{1}{1 - jM\cdot\bar{\gamma}}}$$

GO DOKAV VO MALE

$$\bar{\gamma} = \bar{\gamma}_C = \frac{E}{N_0} E(\alpha_k^2)$$

- KARAKTERISTIČNA FUNKCIJA

$$\gamma_b = \frac{E}{N_0}\sum_{k=1}^{L} \alpha^k = \sum_{k=1}^{L} \gamma_k$$

$$\boxed{\psi_{\gamma_b} = \frac{1}{(1 - jM\cdot\bar{\gamma}_c)^L}}$$

NE MOŽAM DA GO DOKAŽAM !!!

$$\gamma_b = \sum_{k=1}^{L} \overset{\nearrow 1}{a_k} \gamma_k$$

$$\Gamma_b = A \cdot \Gamma_k = \cancel{k} \cdot \Gamma_k$$

$$\gamma_b = [1, 1, 1 \ldots 1] \cdot \begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \vdots \\ \gamma_k \end{bmatrix} = \underline{\gamma_1 + \gamma_2 + \gamma_3 + \ldots \gamma_L}$$

$$p_{\gamma_b} = p(\gamma_1, \gamma_2, \ldots \gamma_L) \qquad p_{\gamma_k}(\gamma_k) = \frac{1}{\bar{\gamma}_k} \cdot e^{-\frac{\gamma_k}{\bar{\gamma}_k}}$$

$$\boxed{p_{\gamma_b} = \frac{1}{\bar{\gamma}^L} \cdot e^{-\frac{L\gamma}{\bar{\gamma}}}} \quad ?$$

$M = \gamma_1 + \gamma_2$

$v = \frac{\gamma_1}{\gamma_1 + \gamma_2}$

$$p(\gamma_1, \gamma_2) = \frac{1}{\bar{\gamma}_1} \cdot \frac{1}{\bar{\gamma}_2} \cdot e^{-\frac{\gamma_1}{\bar{\gamma}_1} \cdot \frac{\gamma_2}{\bar{\gamma}_2}}$$

N4. pp50

$$J = \begin{vmatrix} \frac{\partial x_1}{\partial M} & \frac{\partial x_1}{\partial v} \\ \frac{\partial x_2}{\partial M} & \frac{\partial x_1}{\partial v} \end{vmatrix} = |-M| \qquad g(M, v) = -M\, f(\gamma_1, \gamma_2) = -M\, p(\gamma_1, \gamma_2)$$

$$g(M, v) = |-\frac{M}{\bar{\gamma}_1 \bar{\gamma}_2} \cdot e^{-M} = \frac{\gamma_1 + \gamma_2}{\bar{\gamma}_1 \bar{\gamma}_2} e^{-(\gamma_1 + \gamma_2)}$$

$$\cancel{\frac{x+y}{(x+y)^2} - \frac{2x}{(x+y)^2} = \frac{-x+y}{(x+y)^2}}$$

$$\boxed{p(\gamma_1,\gamma_2) = \frac{1}{\bar{\gamma}_1\bar{\gamma}_2} e^{-\frac{\gamma_1}{\bar{\gamma}_1}-\frac{\gamma_2}{\bar{\gamma}_2}}}$$

$$\mu = \frac{\gamma_1}{\bar{\gamma}_1} + \frac{\gamma_2}{\bar{\gamma}_2} \qquad v = \frac{\gamma_1/\bar{\gamma}_1}{\frac{\gamma_1}{\bar{\gamma}_1} + \frac{\gamma_2}{\bar{\gamma}_2}} \qquad J = \begin{vmatrix} \frac{\partial \gamma_1/\bar{\gamma}_1}{\partial \mu} & \frac{\partial \gamma_1/\bar{\gamma}_1}{\partial v} \\ \frac{\partial \gamma_2/\bar{\gamma}_2}{\partial \mu} & \frac{\partial \gamma_2/\bar{\gamma}_2}{\partial v} \end{vmatrix} = -\frac{1}{\gamma_1\bar{\gamma}_2 + \gamma_2\bar{\gamma}_1}$$

$$p(\mu,v) = |J| \cdot p(\gamma_1,\gamma_2) = \frac{1}{\gamma_1\bar{\gamma}_2 + \gamma_2\bar{\gamma}_1} \cdot e^{-\mu} =$$

$$= \frac{1}{\bar{\gamma}_1\bar{\gamma}_2} \cdot \frac{1}{\frac{\gamma_1}{\bar{\gamma}_1} + \frac{\gamma_2}{\bar{\gamma}_2}} \cdot e^{-\mu} = \frac{1}{\bar{\gamma}_1\bar{\gamma}_2} \cdot \frac{1}{\mu} e^{-\mu}$$

$$\boxed{p(\mu,v) = \frac{1}{\bar{\gamma}_1\bar{\gamma}_2\mu} e^{-\mu}} \qquad p(\mu) = \int_0^1 p(\mu,v)\,dv$$

$$p(\mu) = \frac{1}{\bar{\gamma}_1\bar{\gamma}_2\mu} e^{-\mu} = \frac{\frac{1}{\bar{\gamma}_1\bar{\gamma}_1}}{\frac{\gamma_1}{\bar{\gamma}_1} + \frac{\gamma_1}{\bar{\gamma}_2}} e^{-\frac{\gamma_1}{\bar{\gamma}_1} - \frac{\gamma_2}{\bar{\gamma}_2}}$$

$$\Psi_{\gamma_b} = E\left(e^{+j\omega}\right) = \int_0^\infty \frac{1}{\bar{\gamma}_1\gamma_2\mu} e^{-\mu} \cdot e^{j\omega}\,d\mu$$

NAKAGAMI

$$\boxed{p_\alpha(\alpha) = \frac{2m^m \alpha^{2m-1}}{\Omega^m \Gamma(m)} e^{-\frac{\alpha^2}{\Omega}}} \qquad \boxed{p_\gamma(\gamma) = \frac{m^m \gamma^{m-1}}{\bar{\gamma}^m \Gamma(m)} e^{-\frac{\gamma m}{\bar{\gamma}}}}$$

$$\boxed{\Psi_{\gamma_b} = \frac{1}{(1-j\mu\bar{\gamma}_c)^L}}$$

CHARACTERISTIC FUNCTION FOR

$$\gamma_b = \sum_{k=1}^{L} \gamma_k$$

γ_k - CHI SQUARE WITH K=2

- OD DRUGA STRANA OVA E KARAKTERISTIČNA FUNKCIJA NA CHI-SQUARE DISTRIBUIRANA SLUČAJNA PROMENLIVA SO 2L DEGREES NA SLOBODA

$$p(\gamma_b) = \frac{1}{\sigma^{2L} 2^L \Gamma(L)} \gamma_b^{L-1} e^{-\frac{\gamma}{2\sigma^2}}$$

$$p(\gamma_b) = \frac{1}{\Gamma(L)\ \bar{\gamma}_c^L}\ \gamma_b^{L-1}\ e^{-\frac{\gamma}{\bar{\gamma}}}$$

$$\boxed{\bar{\gamma} = 2\sigma^2}$$

$$\Gamma(L) = (L-1)!$$

$$\boxed{p(\gamma_b) = \frac{1}{(L-1)!\ \bar{\gamma}_c^L}\ \gamma_b^{L-1}\ e^{-\frac{\gamma}{\bar{\gamma}_c}}}$$

$$\bar{\gamma}_b = L \cdot \bar{\gamma}_c$$

PER BIT

PER CHANNEL

$$\boxed{\bar{\gamma}_c = \frac{\mathcal{E}}{N_0} E(\alpha_k^2)}$$

FOR BINARY PSK

$$P_2 = \int_0^{\infty} P_2(\gamma_b)\, p(\gamma_b)\, d\gamma_b$$

$$\boxed{P_2(\gamma_b) = Q\left(\sqrt{2\gamma_b}\right)}$$

$$P_2 = \int_0^{\infty} Q\left(\sqrt{2\gamma_b}\right) \frac{1}{(L-1)!\ \bar{\gamma}^L}\ \gamma_b^{L-1}\ e^{-\frac{\gamma}{\bar{\gamma}}}\, d\gamma =$$

$$= \frac{1}{(L-1)!\ \bar{\gamma}^L} \int_0^{\infty} Q\left(\sqrt{2\gamma_b}\right)\ \gamma_b^{L-1}\ e^{-\frac{\gamma}{\bar{\gamma}}}\, d\gamma$$

BPSK

$$\boxed{P_2 = \left[\frac{1}{2}(1-\mu)\right]^L \sum_{k=0}^{L-1} \binom{L-1+k}{k} \left[\frac{1}{2}(1+\mu)\right]^k}$$

$$\mu = \sqrt{\frac{\bar{\gamma}}{1+\bar{\gamma}}}$$

FOR $\bar{\gamma} \gg 1$ $\quad \frac{1}{2}(1+\mu) \doteq 1 \quad \frac{1}{2}(1-\mu) \doteq \frac{1}{4\bar{\gamma}_c}$

$$\sum_{k=0}^{L-1} \binom{L-1+k}{k} = \binom{2L-1}{L}$$

FOR $\bar{\gamma} \geqslant 10$ dB $\rightarrow$

BPSK

$$\boxed{P_2 = \left(\frac{1}{4\bar{\gamma}_c}\right)^L \binom{2L-1}{L}}$$

KAKO RASTE „L" (BROJOT NA DIVERSITY GRANKI) TAKA SE NAMALUVA BER.

- **FSK** $\quad U_1 = \mathrm{Re}\left(2\mathcal{E} \sum_{k=1}^{L} \alpha_k^2 + \sum_{k=1}^{L} \alpha_k N_{k1}\right)$

$$U_2 = \mathrm{Re}\left\{ \sum_{k=1}^{L} \alpha_k N_{k2} \right\}$$

$$P_b = P(U_1 > U_2)$$

$$P_2(\gamma_b) = Q\left(\sqrt{\gamma_b}\right) \qquad \mu = \sqrt{\frac{\bar{\gamma}_c}{2+\bar{\gamma}_c}} \qquad P_2 = \left(\frac{1}{2\bar{\gamma}_c}\right)^L \binom{2L-1}{L}$$

⊙ **DPSK** OUTPUT OF MAXIMUM RATIO COMBINER (DECISION VARIAB)

$$U = Re\left[\sum_{k=1}^{L} \left(2E\alpha_k e^{-j\phi_k} + N_{k2}\right)\left(2E\alpha_k e^{j\phi_k} + N_{k1}\right)\right]$$

$$P_b = P(U < 0)$$

12-1-13
$$\left.\begin{aligned} P_b &= \frac{1}{2^{2L-1}} e^{-\gamma_b} \sum_{k=1}^{L-1} b_k \gamma_b^{k} \\ b_k &= \frac{1}{k!} \sum_{n=0}^{L-1-k} \binom{2L-1}{n} \end{aligned}\right\}$$ DPSK MULTICHANEL AWGN

AVERAGE OF CONDITIONAL PROBABILITY P_b OVER RAYLEIG FADING IS:

$$P_2 = \frac{1}{2^{2L-1}(L-1)!\,(1+\bar{\gamma}_c)^L} \sum_{k=0}^{L-1} b_k (L-1+k)! \left(\frac{\bar{\gamma}_c}{1+\bar{\gamma}_c}\right)^k$$

$$\boxed{\mu = \frac{\bar{\gamma}_c}{1+\gamma_c}}$$ for $\gamma_c \gg 1$ $$\boxed{P_2 = \left(\frac{1}{2\gamma_c}\right)^L \binom{2L-1}{L}}$$

AKO SAKAŠ DA ODIŠ SO 14-4-15

⊙ FSK (NONCOHERENT) SQUARE-LAW COMBINER

$$U_1 = \sum_{k=1}^{L} \left|2E\alpha_k e^{-j\phi_k} + N_{k1}\right|^2$$

$$U_2 = \sum_{k=1}^{L} |N_{k2}|^2 \qquad P_b = P(U_2 > U_1)$$

USE 14-4-15 WITH $\boxed{\mu = \frac{\bar{\gamma}_c}{2+\bar{\gamma}_c}}$

$$\boxed{p(U_1) = \frac{1}{(2\sigma_1^2)^L (L-1)!} U_1^{L-1} \exp\left(-\frac{U_1}{2\sigma_1^2}\right)}$$ CHI SQUARE

$$\sigma_1^2 = \frac{1}{2} E\left(\left|2E\alpha_k e^{-j\phi_k} + N_{k1}\right|^2\right) = 2E N_0 (1+\bar{\gamma}_c)$$

$$p(U_2) = \frac{2}{(2\sigma_2^2)^L (L-1)!} U_2^{L-1} \exp\left(-\frac{U_2}{2\sigma_2^2}\right) \qquad \sigma_2^2 = 2EN_0$$

$\gamma_c \gg 1$ $$\boxed{P_2 \doteq \left(\frac{1}{\bar{\gamma}_c}\right)^L \binom{2L-1}{L}}$$ FOR SQUARE-LAW DETECTED FSK

$$\boxed{\bar{\gamma}_b = L \cdot \bar{\gamma}_c}$$

$\bar{\gamma}_b$ - AVERAGE SNR PER BIT
$\bar{\gamma}_c$ - AVERAGE SNR PER CHANNEL

CH. 12 MULTICHANNEL AND MULTICARRIER SYSTEMS

12.1 MULTICHANNEL DIGITAL COMMUNICATION

$$s_m^{(n)}(t) = Re\left\{ s_{lm}^{n}(t) e^{j2\pi fd} \right\} \qquad 0 \leq t \leq T$$

$n = 1, 2, \ldots L \qquad m = 1, 2, \ldots M$

L - NUMBER OF CHANNELS
M - NUMBER OF WAVEFORMS

- EQUIVALENT LOWPASS SIGNALS RECEIVED FROM L CHANNELS

$$\boxed{r_l^{(n)}(t) = \alpha_n e^{j\phi_n} s_{lm}^{(n)}(t) + z_n(t)}$$

POTSEGUVANJE ZEPUCHIM

$$\boxed{\tilde{y}(t) = \sum_{k=1}^{K} a_k s(t - \tau_k)}$$

$$\boxed{\tilde{c}(\tau, t) = \sum_{k=1}^{K} \tilde{a}_k(t) \delta(\tau - \tau_k)}$$ IMPULSEN ODZIV

$$x_{RB} = \sum_{i=1}^{N} a_i e^{-j2\pi f_c \tau_i(t)} x(t - \tau_i(t)) = \underbrace{\sum_{i=1}^{N} a_i e^{-j\theta(t)}}_{h_b} x(t - \tau_i(t))$$

$h_b \Rightarrow$ LOWPASS EQUIVALENT IMPULSE RESPONSE

$g_n = a_n e^{-j\phi_n}$ $\quad \hat{g}_n$ - estimate of g_n

$$CM_m = \sum_{n=1}^{L} Re\left[\hat{g}_n^* \int_0^T r_l^{(n)}(t) \cdot s_{lm}^{(n)}(t) dt \right] \qquad m = 1, 2, \ldots M$$

CM_n - CORRELATION METRICS

- NONCOHERENT DETECTION

$$CM_m = \sum_{n=1}^{L} \left| \int_0^T r_l^{(n)}(t)\, s_m^{(n)*}(t) dt \right|^2 \qquad m = 1, 2, \ldots M$$

- BINARY SIGNALING $\quad s_{l1}^{(n)}, \; n = 1, 2, 3, \ldots L$

$CM_2 > CM_1 \Rightarrow$ error $\qquad D = CM_1 - CM_2 < 0$

$$D = \sum_{n=1}^{L} |X_n|^2 - |Y_n^2|$$

$$X_n = \int_0^T r_l^{(n)}(t) s_{l1}^{(n)*}(t) dt$$

$$Y_n = \int_0^T r_l^{(n)}(t) s_{l2}^{(n)*}(t) dt$$

$n = 1, 2, \ldots L$

. COHERENT DETECTION

$$D = \frac{1}{2}\sum_{n=1}^{L}\left(x_n y_n^* + x_n^* y_n\right)$$

$$y_n = \hat{g}_n \qquad n = 1, 2, \dots L$$

$$x_n = \int^{T} r_l^{(n)}(t)\left[s_{l1}^{(n)*}(t) - s_{l2}^{(n)*}(t)\right]dt$$

⊡ BINARY SIGNALS

$$D = \sum_{n=1}^{L}\left(A|x_n|^2 + B|y_n|^2 + C x_n y_n^* + C^* x_n^* y_n^*\right)$$

$$\boxed{P_b = P(D < 0)}$$

ISTO KAKO ZA JEDNOKANALNI PSK

- ANTIPODAL BINARY SIGNALS WITH PSK MODULATION

$$\boxed{P_b = Q\left(\sqrt{2\gamma_b}\right)} \qquad \gamma_b = \frac{E}{N_0}\sum_{n=1}^{L}|g_n|^2 = \frac{E}{N_0}\sum_{n=1}^{L}\alpha_n^2$$

- FOR IDENTICAL CHANNELS $\alpha_n = \alpha$

$$\boxed{\gamma_b = \frac{LE}{N_0}\alpha^2}$$

L·E – TOTAL TRANSMITTED SIGNAL ENERGY FOR L SIGNALS

∘ FOR DPSK B-21 IS SIMPLIFIED TO:

$$\boxed{P = \frac{1}{2^{2L-1}} e^{-\gamma_b}\sum_{n=1}^{L-1} c_n \gamma_b^{\,n} \qquad c_n = \frac{1}{n!}\sum_{k=0}^{L-1-n}\binom{2L-1}{k}}$$

12-1-2 M-ARY ORTHOGONAL SIGNALS

$s_{l1}^{(n)}(t)$ $\quad n = 1, 2, \dots L$

$$U_1 = \sum_{n=1}^{L}\left|2E\alpha_n + N_{n1}\right|^2 \qquad U_m = \sum_{n=1}^{L}\left|N_{nm}\right|^2 \quad m = 2, 3, \dots, M$$

- U_1 – NON CENTRAL CHI-SQUARE WITH 2L degrees of freedom

$$\boxed{s^2 = \sum_{n=1}^{L}\left(2E\alpha_n\right)^2 = 4E^2\sum_{n=1}^{L}\alpha_n^2} \Rightarrow$$

NONCENTRAL PARAMETER

- NON-CENTRAL CHI SQUARE PDF

$$\boxed{p_\gamma(y) = \frac{1}{\sqrt{2\pi}\,y} e^{-(y+s^2)/2\sigma^2}\cosh\left(\frac{\sqrt{y}\,s}{\sigma^2}\right) \qquad y \geq 0}$$

$$p(v_1) = \frac{1}{4\mathcal{E}N_0}\left(\frac{u_1}{s^2}\right)^{\frac{L-1}{2}} \exp\left(-\frac{s^2+u_1}{4\mathcal{E}N_0}\right) I_{L-1}\left(\frac{s\sqrt{u_1}}{2\mathcal{E}N_0}\right) \qquad u_1 > 0$$

$$p(u_m) = \frac{1}{(4\mathcal{E}N_0)^L (L-1)!} u_m^{L-1} e^{-\frac{u_m}{4\mathcal{E}N_0}} \qquad u_m \geq 0, \quad m = 2, 3, \dots, M$$

$$P_M = 1 - P_C = 1 - P(v_2 < v_1, v_3 < v_1, \dots, v_M < v_1) =$$

$$= 1 - \int_0^\infty \left[P(v_2 < u_1 \mid v_1 = u_1)\right]^{M-1} p(u_1)\, du$$

$$P(v_2 < u_1 \mid v_1 = u_1) = 1 - \exp\left(-\frac{u_1}{4\mathcal{E}N_0}\right) \sum_{k=0}^{L-1} \frac{1}{k!}\left(\frac{u_1}{4\mathcal{E}N_0}\right)^k$$

$$\boxed{P_M = 1 - \int_0^\infty \left(1 - e^{-v}\sum_{k=0}^{L-1}\frac{v^k}{k!}\right)^{M-1} \left(\frac{v}{\gamma}\right)^{\frac{L-1}{2}} e^{-(\gamma+v)} I_{L-1}(2\sqrt{\gamma v})\, dv}$$

$$\gamma = \mathcal{E}\sum_{n=1}^{L}\frac{\alpha_n^2}{N_0}$$

CAN BE EVALUATED NUMERICALLY

- $P_M < (M-1)P_2(L)$

$P_2(L)$ PROBABILITY OF ERROR IN CHOOSING BETWEEN v_1 AND ANY ONE OF THE DECISION VARIABLES $\{v_m\}_{m=1,2 \dots M}$

$$P_2(L) = \frac{1}{2^{2L-1}} e^{-k\gamma_b/2} \sum_{n=0}^{L-1} c_n \left(\frac{1}{2}k\gamma_b\right)^n$$

$$v = \frac{ds}{dt} \qquad s = \int_0^a v\, dt$$

12-2 MULTICARRIER COMMUNICATIONS

$$N = \frac{W}{\Delta f} \qquad \frac{|C(f)|^2}{\Phi_{nn}(f)} \approx \text{const} \quad \text{IN THE SUBBAND}$$

$$\int_W P(f)\, df \leq P_{av}$$

AVAILABLE AVERAGE POWER OF THE TRANSMITTER

12-2-1 CAPACITY OF NONIDEAL LINEAR FILTER CHANNEL

$$C = W \log_2\left(1+\frac{P_{av}}{WN_0}\right) \qquad C\,[\text{bit/sec}]$$

- IN MULTICARRIER SYSTEM WITH SUFFICIENTLY SMALL Δf SUBCHANNEL CAPACITY

$$C_i = \Delta f \log_2\left[1+\frac{\Delta f\, P(f_i)\,|C(f)|^2}{\Delta f\, \Phi_{nn}(f_i)}\right]$$

TOTAL CAPACITY OF CHANNEL

$$C=\sum_{i=1}^{N} C_i = \Delta f \sum_{i=1}^{N}\log_2\left[1+\frac{P(f_i)\,|C(f)|^2}{\Phi_{nn}(f_i)}\right]$$

$\Delta f \to 0$

$$C=\int_W \log_2\left[1+\frac{P(f)\,|C(f)|^2}{\Phi_{nn}(f)}\right]df$$

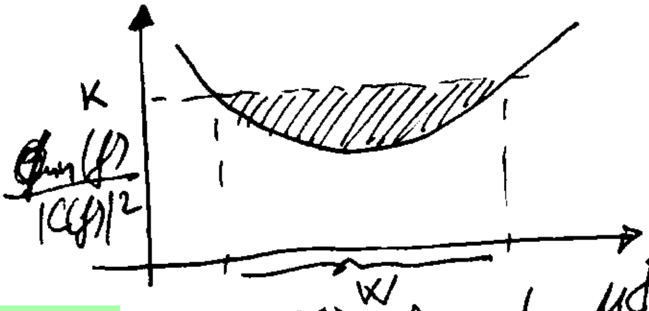
MAXIMIZING C =

$$\int_W \left\{\log_2\left[1+\frac{P(f)\,|C(f)|^2}{\Phi_{nn}(f)}\right]+\lambda P(f)\right\}df$$

λ - LAGRANGE MULTIPLIER

$$\frac{1}{|C(f)|^2 P(f)+\Phi_{nn}(f)}+\lambda=0$$

$$\boxed{P(f)=\begin{cases}K-\Phi_{nn}(f)/|C(f)|^2 & f\in W\\ 0 & f\notin W\end{cases}}$$



WATER-FILLING INTERPRETATION OF OPTIMUM POWER DISTRIBUTION.

12-2-2 An FFT-Based Multicarrier System

FRAMES OF N_f - BITS (EACH FRAME N_f bits)

N_f bits PARSED IN $\bar N$ GROUPS $\qquad n_i$ - BITS PER GROUP

$$\sum_{i=1}^{\bar N} n_i = N_f$$

- EACH CHANNEL DINSTINCT QAM CONSTELATION

$M_i = 2^{q_i}$

$x_k, \ k = 0, 1, \dots, \bar{N}-1$ (information symbols)

$\bar{N}$ - subcarriers

$x_{N-k} = x_k^* \quad k = 1, 2, \dots, \tilde{N}-1$

$x_0 = Re(x_0) \quad X_n = Im(x_0)$

$$x_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} x_k e^{j \frac{2\pi n k}{N}} \qquad n = 0, 1, \dots, N-1$$

$1/\sqrt{N} \Rightarrow$ SCALE FACTOR

$$x(t) = \frac{1}{\sqrt{N}} \sum_{l=0}^{N-1} x_k e^{j \frac{2\pi k t}{T}} \qquad 0 \leq t \leq T$$

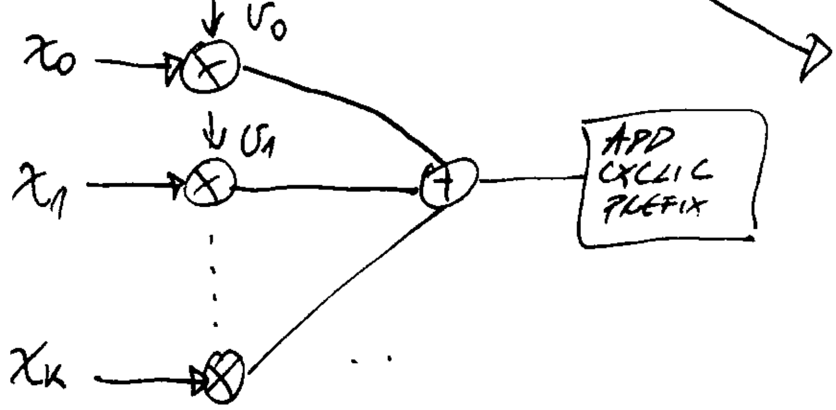
SUBCARRIER frequencies ARE : $f_k = \frac{k}{T} \quad k = 0, 1, \dots \tilde{N}$

$\{x_n\} \to$ SAMPLES OF $x(t)$ TAKEN AT $\boxed{t = n \frac{T}{N}}$

$n = 0, 1, \dots N-1$

$v_k = [v_{k0}, v_{k1}, \dots v_{k(N-1)}]$

$$v_{kn} = \frac{1}{\sqrt{N}} e^{j \frac{2\pi}{N} \cdot kn}$$



VSUŠNOS MISAM TREBA :

$\underbrace{v_n = [v_{n0}, v_{n1}, \dots v_{n(N-1)}]}$

n - fixno, a k se menuva

$$r(t) = x(t) \star h(t) + n(t) = x(t) \star h(t) + n(t)$$

$\star$ - CONVOLUTION

- CYCLIC PREFIX OF "V" SAMPLES $\{h_n, 0 < n \leq V\}$

$$H_k = H\left(\frac{2\pi k}{N}\right) = \sum_{n=0}^{V} h_n e^{-j \frac{2\pi n k}{N}}$$

$\boxed{\hat{X}_k \text{ - OUTPUT OF N-POINT DFT DEMODULATOR}}$

$$\hat{X}_k = H_k \cdot X_k + n_k \qquad k = 0, 1, \dots N-1$$

PROBLEM 12.1 $\quad v = \sum_{n=1}^{N} x_n$

$$E[v] = \sum_{n=1}^{N} E[x_n] = N\cdot m \quad ⊛$$

$$\sigma_v^2 = E[v^2] - E[v]^2 = \underbrace{E\left[\sum_n \sum_m x_n x_m\right]}_{N(\sigma^2+m^2)} - (N\cdot m)^2$$

$$⊛ = E\left(x_1 \sum_{m=1}^{N} x_m + x_2 \sum_{m=1}^{N} x_m + \;--\; x_N \sum_{m=1}^{N} x_m\right)$$

$\boxed{N=2}$

$$E[(x_1+x_2)^2] = E[x_1^2 + 2x_1x_2 + x_2^2] =$$

$$= E[x_1^2] + E[x_2^2] + 2E[x_1\cdot x_2] - \cancel{m}$$

$$\sigma_m^2 = E[v^2] - E[v]^2 = \overline{x_1^2} + \overline{x_2^2} + 2\,\overline{x_1x_2} - (2m)^2$$

$$= \overline{x_1^2} + \overline{x_2^2} + 2\,\overline{x_1x_2} - \underline{\underline{4m^2}} =$$

$$= \underbrace{\overline{x_1^2} - m^2}_{\sigma_1^2} + \underbrace{\overline{x_2^2} - m^2}_{\sigma_2^2} + 2\underbrace{\overline{x_1x_2}}_{\text{STATISTICALLY INDEPENDENT}} - 2m^2 =$$

$$= 2\,\overset{m}{\overline{x_1}}\cdot\overset{m}{\overline{x_2}}$$

$$= \sigma_1^2 + \sigma_2^2 + \cancel{2m^2} - \cancel{2m^2} = \sigma_1^2 + \sigma_2^2$$

$$⊛ = E(x_1^2) + (N-1)\cdot m^2 + E(x_2^2) + (N-1)m^2 + \dots + E(x_N^2) + (N-1)m^2 =$$

~~$= N\cdot\sigma^2 + N(N-1)m^2 = N\sigma^2 + N^2m^2 - Nm^2$~~

~~$\sigma_v^2 = N\sigma^2 + Nm^2 - Nm^2 - N^2m^2 = N(\sigma^2 - m^2)$~~ ↯

$$= N\left(E(x^2) - m^2\right) + N^2\cdot m^2 = N\cdot\sigma^2 + N^2\cdot m^2$$

$$\sigma_v^2 = N\cdot\sigma^2 + N^2\cdot m^2 - N^2\cdot m^2 = \underline{\underline{N\cdot\sigma^2}}$$

$$SNR_v = \frac{N^2\cdot m^2}{2\sigma_v^2} = \frac{N\cdot m^2}{N\cdot\sigma^2}$$

☐ DSPLog MAXIMAL RATIO COMBINING

$$h_i = \left(randn(N) + j\,randn(N)\right)\frac{1}{\sqrt{2}} \qquad \sigma_{h_i}^2 = \frac{1}{2}$$

$$p(n) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(n-\mu)^2}{2\sigma^2}} \qquad \mu = 0 \qquad \boxed{\sigma^2 = \frac{N_0}{2}}$$

$$\boxed{\gamma_i = \frac{|h_i|^2 E_b}{N_0}}$$

MATLAB SCRIPTATA SE NAOGJA VO WORK\dsplog
script_for_bpsk_rayleigh_channel_maximal_ratio_combining.m

i-th receiver antenna

$$\boxed{y_i = h_i x + n_i}$$

x – TRANSMITED SYMBOL

h_i – CHANNEL
i-TH

MATRIX FORM:

$$y = hx + n$$

$y = [y_1, y_2 \cdots y_N]^T$ – y – RECEIVED SYMBOL FROM ALL RECEIVE ANTENNA

$h = [h_1, h_2 \ldots h_N]^T$ – h – CHANNEL ON ALL THE RECEIVE ANTENNA

x – TRANSMITED SYMBOL

KAKO ZA BANDPASS FILTER ZEMA PA BIDE RAYLEIGH

$n = [n_1, n_2, \ldots n_N]^T$ – NOISE ON ALL RECEIVE ANTENNA

IZGLEDA ZNAČI „CONJ" VO MATLAB IMPLEMENT.

- EQUALIZED SYMBOL

$$\hat{x} = \frac{h^H y}{h^H h} = \frac{h^H h x}{h^H h} + \frac{h^H n}{h^H h} = x + \frac{h^H n}{h^H h}$$

$$h^H h = \sum_{i=1}^{N} |h_i|^2$$

- EFECTIVE BIT ENERGY TO NOISE IS

$$\boxed{\gamma = \sum_{i=1}^{N} \frac{|h_i|^2 E_b}{N_0} = \sum_{i=1}^{N} \gamma_i = N \cdot \gamma_i}$$

- MRC

$$p(\gamma_i) = \frac{1}{(E_b/N_0)} e^{-\gamma_i/(E_b/N_0)}$$

CHI SQUARE DISTRO WITH 2 DEGREES OF FREEDOM ZA $\boxed{\Omega = 1}$

CHI SQUARE (NZ. 1164 ⊛) WITH 2 DEGREES:

$$f_Y(y, 2) = \frac{1}{\sigma^2 \cdot 2} \cdot e^{-\frac{y}{2\sigma^2}} = \frac{1}{\Omega} \cdot e^{-\frac{y}{\Omega}}$$

$$\boxed{\gamma = \frac{E_b}{N_0}\cdot\alpha^2}$$

~~POTSTUVANZE~~

$$\alpha = \frac{\alpha}{\sigma^2}\, e^{-\frac{\alpha^2}{2\sigma^2}}$$

$$2\sigma^2 = N_0$$

$$p(\gamma) = \frac{p(\alpha)}{\frac{d\gamma}{d\alpha}} \;\Bigg|\; \alpha = (+)\sqrt{\frac{\gamma N_0}{E_b}} \; = \frac{\sqrt{\frac{\gamma N_0}{E_b}}\cdot\frac{2}{N_0}\, e^{-\frac{1}{N_0}\cdot\frac{\gamma N_0}{E_b}}}{\frac{E_b}{N_0}\cdot 2\cdot\sqrt{\frac{\gamma N_0}{E_b}}}$$

$$\boxed{p(\gamma) = \frac{1}{E_b}\cdot e^{-\frac{\gamma}{E_b}}}$$

$$E(\alpha^2) = 2\sigma^2 = \Omega$$

$$\boxed{\sigma^2 = \frac{\Omega}{2}}$$

$$\bar{\gamma} = \frac{E_b}{N_0}\cdot E(\alpha^2)$$

$$\bar{\gamma} = \frac{E_b}{N_0}\cdot\Omega \qquad \frac{E_b}{N_0} = \frac{\bar{\gamma}}{\Omega}$$

$$\gamma = \frac{\bar{\gamma}}{\Omega}\cdot\alpha^2 \qquad \frac{d\gamma}{d\alpha} = 2\frac{\bar{\gamma}}{\Omega}\cdot\alpha$$

$$p(\gamma) = \frac{\frac{\alpha}{\sigma^2}\cdot e^{-\frac{\alpha^2}{2\sigma^2}}}{2\frac{\bar{\gamma}}{\Omega}\cdot\alpha} \;\Bigg|\; \alpha = \sqrt{\frac{\gamma\Omega}{\bar{\gamma}}} = \frac{\frac{2}{\Omega}\sqrt{\frac{\gamma\Omega}{\bar{\gamma}}}\, e^{-\frac{1}{\Omega}\cdot\frac{\gamma\Omega}{\bar{\gamma}}}}{2\frac{\bar{\gamma}}{\Omega}\cdot\sqrt{\frac{\gamma\Omega}{\bar{\gamma}}}}$$

$$\boxed{p(\gamma) = \frac{1}{\bar{\gamma}}\cdot e^{-\gamma/\bar{\gamma}}}$$

$$\boxed{\begin{array}{l}\text{FOR } \Omega = 1\\ \bar{\gamma} = \frac{E_b}{N_0}\end{array}} \quad p(\gamma) = \frac{1}{E_b/N_0}\cdot e^{-\gamma/(E_b/N_0)}$$

$$\gamma = \sum_{i=1}^{N}\gamma_i \Rightarrow p(\gamma) - \text{chi SQUARE WITH } N \text{ DEGREES OF FREEDOM}$$

$$p(\gamma) = \frac{1}{(N-1)!\,(E_b/N_0)^N}\,\gamma^{N-1}\, e^{-\gamma/E_b/N_0} \qquad \gamma \geq 0$$

- BER IN BPSK AWGN

$$P_b = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right) = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{2E_b}{2N_0}}\right) = Q\left(\sqrt{2\frac{E_b}{N_0}}\right) = Q\left(\sqrt{2\gamma}\right)$$

SCALAR

$$Q(z) = \frac{1}{2}\,\mathrm{erfc}\,\frac{z}{\sqrt{2}}$$

$$P_e = \frac{1}{2}\int_0^\infty \frac{1}{2}\operatorname{erfc}(\sqrt{\gamma})\cdot p(\gamma)\,d\gamma = p^N \sum_{k=0}^{N-1}\binom{N-1+k}{k}(1-p)^k$$

$$\boxed{p = \frac{1}{2} - \frac{1}{2}\left(1+\frac{1}{E_b/N_0}\right)^{-1/2}}$$

$$\boxed{N_0 = 2\sigma^2 = 1}$$

$$\left.\frac{E_b}{N_0}\right|_{dB} = E_bN_{0dB} = 10\log\frac{E_b}{N_0}$$

$$n = \frac{\sqrt{N_0}}{\sqrt{2}}\cdot \operatorname{randn}(1,N)$$

ZA $\boxed{N_0 = 1}$ $\cancel{n = \frac{1}{\sqrt{2}}\operatorname{randn}(1,N)}$

VO GENERALEN SLUČAJ:

$$\boxed{n = \frac{\sqrt{N_0}}{\sqrt{2}}\operatorname{randn}(1,N)}$$

$$E_bN_{0}dB = 0:25$$

$$\Rightarrow \frac{E_b}{N_0} = 10^{0.1 E_bN_0dB}$$

$$N_0 = E_b\cdot 10^{-0.1E_bN_0dB}$$

AKO GO DRŽIŠ FIKSNO $E_b = 1 \Rightarrow N_0 = 10^{-0.1E_bN_0dB}$

$$\boxed{\sqrt{N_0} = 10^{-0.05E_bN_0dB}}$$

RECEIVE DIVERSITY

Ravi Raj Adve
Diversity Receive
Notes

MMV

2. THE MODEL

$x = h\cdot u(t) + n$

u - UNIT POWER SIGNAL ($\Omega = 1$)

h - CHANNEL (INCLUDING SIGNAL POWER)

$$\boxed{P = \frac{1}{T_s}\int_0^{T_s}|h_n(t)|^2|u(t)|^2dt = |h_n(t)|^2\frac{1}{T_s}\int_0^{T_s}|u(t)|^2dt = |h_n(t)|^2}$$

SLOW FADING!!

MMV

①⊛

$E\{|h_n(t)|^2\} = \sigma^2$

$$\boxed{\gamma_n = \frac{|h_n|^2}{\sigma^2}}$$

AT ELEMENT „n“

→ POWER OF THE SIGNAL OVER SINGLE SYMBOL PERIOD

$h_n = |h_n| e^{j\angle h_n}$ $\angle h_n$ UNIFORM $[0, 2\pi]$

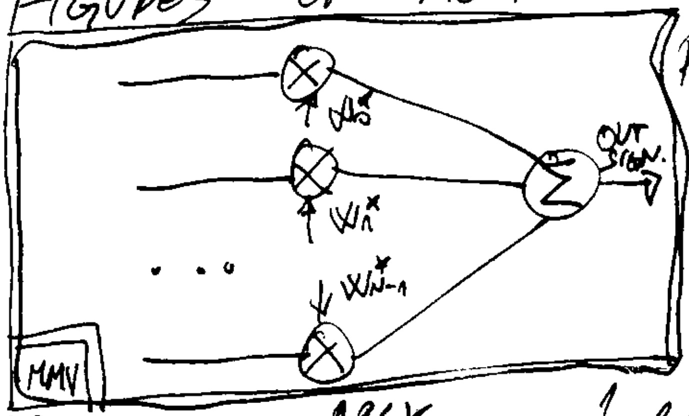
$h_n \to$ RAYLEIGH

$$|h_n| \sim \frac{2|h_n|}{P_0} e^{-|h_n|^2/P_0} \qquad \gamma_n \sim \frac{1}{\Gamma} e^{-\gamma_n/\Gamma}$$

$$\boxed{\Gamma = E[\gamma_n] = \frac{E[|h_n|^2]}{\sigma^2} = \frac{P_0}{\sigma^2}}$$

$\boxed{h_n \equiv \alpha_k \text{ OD PROAKIS}}$

- FIGURES OF MERIT



$$P_{out} = P(\gamma \le \gamma_s) = \int_0^{\gamma_s} \frac{1}{\Gamma} \cdot e^{-\gamma/\Gamma} d\gamma$$

$$= \left[1 - e^{-\gamma_s/\Gamma}\right]^{0}$$

$\Gamma \to \infty$ $P_{out} \sim \frac{1}{\Gamma}$

~~BER~~ FOR BPSK $\frac{1}{2} \operatorname{erfc}\sqrt{2\gamma} = Q\left(\frac{|h|}{\sigma}\right)$

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-\frac{t^2}{2}} dt$$

$$BER = \int_0^\infty \frac{2|h|}{P_0} e^{-\frac{|h|^2}{P_0}} Q\left(\frac{|h|}{\sigma}\right) d|h| = \frac{1}{2}\left(1 - \sqrt{\frac{\Gamma}{1+\Gamma}}\right)$$

⊙ SELECTION COMBINING

(SELECT ELEMENT WITH BEST γ)

- THE ELEMENT WITH GREATEST SNR IS CHOSEN FOR FURTHER PROCESSING

$$\boxed{w_k = \begin{cases} 1 & \gamma_k = \max_n\{\gamma_n\} \\ 0 & \text{otherwise} \end{cases}}$$

$\boxed{\gamma = \max_n \{\gamma_n\}}$ MMV

$$P_{out} = P[\gamma < \gamma_s] = P(\gamma_0 < \gamma_s, \gamma_1 < \gamma_s, \ldots \gamma_N < \gamma_s] = \prod_{i=0}^{N-1} P[\gamma_n < \gamma_s]$$

$$\boxed{P_{out}(\gamma_s) = \left[1 - e^{-\gamma_s/\Gamma}\right]^N}$$

$P_{out} \equiv CDF$

PDF:

$$\boxed{f_\Gamma(\gamma) = \frac{d P_{out}}{d\gamma} = N\left(1 - e^{-\gamma_s/\Gamma}\right)^{N-1} \cdot e^{-\frac{\gamma}{\Gamma}} \cdot \frac{1}{\Gamma} = \frac{N \cdot e^{-\frac{\gamma}{\Gamma}}}{\Gamma}\left(1 - e^{-\frac{\gamma}{\Gamma}}\right)^{N-1}}$$

MMV

- AVERAGE SNR

$$E[\gamma] = \int_0^\infty \gamma f_\Gamma(\gamma)\,d\gamma = \int_0^\infty \gamma \frac{N e^{-\frac{\gamma}{\Gamma}}}{\Gamma}\left(1-e^{-\frac{\gamma}{\Gamma}}\right)d\gamma = \Gamma\sum_{n=1}^{N}\frac{1}{n}$$

$$\Rightarrow E[\gamma] = \Psi(N+1) + \gamma \qquad \Psi = \frac{d}{dx}\ln(\Gamma(x)) = \frac{\frac{d\Gamma(x)}{dx}}{\Gamma(x)}$$

$$E[\gamma] \simeq \Gamma\left(C + \ln N + \frac{1}{2N}\right)$$

$\boxed{\text{IMPROVEMENT OF } \ln N}$

- AVERAGE BER

$$P_e = \int_0^\infty Q(\sqrt{2\gamma})\,\frac{N e^{-\gamma/\Gamma}}{\Gamma}\left(1-e^{-\gamma/\Gamma}\right)^{N-1} d\gamma$$

$\boxed{\text{NMV}}$

4. MAXIMAL RATIO COMBINING

$x(t) = h(t)\,m(t) + n(t)$

$h = [h_0, h_1, \ldots, h_{N-1}]^T$

$n = [n_0, n_1, \ldots, n_{N-1}]^T$

$y(t) = w^H \cdot x = w^H h\, m(t) + w^H n$

VIDI 80, (4*)

$H = \text{conj}(\ldots)$

$$\gamma = \frac{|w^H \cdot h|^2}{E\{|w^H n|^2\}}$$

NOISE POWER IN THE DENOMINATOR

$$P_n = E\{|w^H n|^2\} = E\{|w^H \cdot n \cdot n^H \cdot w|\} = w^H E\{|n \cdot n^H|\} \cdot w = \sigma^2 w^H \cdot w$$

$$= \sigma^2 \cdot w^H \cdot I_N \cdot w = \sigma^2 \cdot \|w\|^2$$

$$n \cdot n^H = \begin{bmatrix} n_{0r} + j n_{0I} \\ n_{1r} + j n_{1I} \\ \vdots \\ n_{Nr} + j n_{NI} \end{bmatrix} [n_{0r} - j n_{0I}, n_{1r} - j n_{1I}, \ldots n_{Nr} - j n_I]^T = \begin{bmatrix} |n_0|^2 \\ |n_1|^2 \\ \vdots \\ |n_N|^2 \end{bmatrix}$$

$$E[n \cdot n^H] = E[|n|^2] = \sigma^2 \qquad \boxed{E^2(n) = 0} \Rightarrow$$

AKO ~~SE~~ ZEMA DA SREDNATA VRED. NA SUMOT = 0

$$[1\ 2\ 3]\begin{bmatrix}1\\2\\3\end{bmatrix} = 1+4+9 = 14$$

$\|w\| = 1$ } SE PODEUVA ~~DA~~ BIDE 1 (Const)

$$\boxed{\gamma = \frac{|w^H \cdot h_1|^2}{\sigma^2}}$$

CAUCHY-SCHWARTZ THE MAXIMUM IS WHEN

$$\boxed{w = h}$$

ISTIOT GAIN OD KANALOT SO OBRATNA FAZE

AKO SE ZEME VO PREDVID ⊠

$$\gamma = \frac{|h^H \cdot h_1|^2}{\sigma^2\, h^H \cdot h}$$

NOISE POWER

$$\boxed{\gamma = \frac{h^H \cdot h}{\sigma^2} = \sum_{n=0}^{N-1} \frac{|h_n|^2}{\sigma^2} = \sum_{n=0}^{N-1} \gamma_n}$$

OUTPUT OF MAXIMUM RATIO COMBINER!

$$\boxed{E(\gamma) = N \cdot \Gamma}$$

MMV

EXPECTED VALUE OF SNR IS N TIMES THE AVERAGE PER CHANNEL (BETTER THEN IN SELECTION DIVERSITY WHERE WE HAD IMPROVEMENT OF $\ln N$)

- PDF OF THE SUM OF N INDEPENDENT ~~RVES~~ RANDOM VAR IS THE CONVOLUTION OF THE INDIVIDUAL PDFS

$$F_{\Gamma_n}(s) = E\{e^{-s\gamma_n}\} = \frac{1}{1+s\Gamma}$$

KARAKTERISTIČNA FUNKCIJA T.E MGF

$$F_\Gamma(s) = \left(\frac{1}{1+s\Gamma}\right)^N$$

$$\boxed{\mathcal{F}\left[\varphi_1(x) * \varphi_2(x) * \dots \varphi_N(x)\right] = \psi_1(j\mu)\cdot\psi_2(j\mu)\cdots\psi_N(j\mu)}$$

MMV

MMV

$$f_\Gamma(\gamma) = \mathcal{L}^{-1}\left[G_\Gamma(s)\right] = \frac{1}{2\pi}\int_{c-j\infty}^{c+j\infty} \frac{e^{s\gamma}}{(1+s\Gamma)^N}\,d\gamma = \frac{1}{(N-1)!}\,\frac{\gamma^{N-1}}{\Gamma^N}\,e^{-\frac{\gamma}{\Gamma}}$$

$$P_{out} = P(\gamma < \gamma_s) = \int_0^{\gamma_s} \frac{1}{(N-1)!}\,\frac{\gamma^{N-1}}{\Gamma^N}\,e^{-\gamma/\Gamma}\,d\gamma \Rightarrow$$

$$P_{out} = 1 - e^{-\gamma_s/\Gamma}\sum_{n=0}^{N-1}\left(\frac{\gamma_s}{\Gamma}\right)^n \frac{1}{n!}$$

FORMULATA VO PRAKSE POLESNO SE PRIMENI!!!

$$\boxed{P_e = \int_0^\infty \underbrace{\frac{1}{2}\mathrm{erfc}\left(\sqrt{2\gamma}\right)}_{Q(\sqrt{\gamma})} \frac{1}{(N-1)!}\,\frac{\gamma^{N-1}}{\Gamma^N}\,e^{-\frac{\gamma}{\Gamma}}\,d\gamma = \frac{1}{(N-1)!}\left(\frac{1-\mu}{2}\right)^N \cdot \left[\sum_{n=0}^{N-1}\frac{(N-1+n)!}{n!}\left(\frac{1+\mu}{2}\right)^n\right]}$$

- LARGE N

$$\boxed{P_e = \binom{2N-1}{N}\left(\frac{1}{4\Gamma}\right)^N}$$

$$\mu = \sqrt{\frac{\Gamma}{1+\Gamma}}$$

- DEFINITION FOR DIVERSITY ORDER

$$\boxed{D = \lim_{SNR\to\infty}\left(-\frac{\log P_e}{\log(SNR)}\right)}$$

$$\log(SNR^P) \sim \log\left(\frac{1}{P_e}\right)$$

$$\boxed{P_e \sim \frac{1}{SNR^P}}$$

$$\boxed{P_e \cong \frac{1}{\Gamma^N}}$$

5. EQUAL GAIN COMBINING

UNIT GAIN

$$w_n = ① e^{j\angle h_n} \Rightarrow w_n^* h_n = |h_n|$$

MRV

$$\boxed{w^H \cdot h = \sum_{n=0}^{N-1} |h_n|}$$

(♥O MRC IMAME $\sum_{n=0}^{N-1} |h_n|^2$)

- NOISE POWER : (VIDI N7.11.81 ◙)

$$P_n = w^H \cdot w \cdot \sigma^2 = N \cdot \sigma^2$$

$$[w_1^* \, w_2^* \dots w_n] \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix}$$

- INSTANTENIOUS SNR

$$\boxed{\gamma = \frac{\left[\sum_{n=0}^{N-1} |h_n|\right]^2}{N\sigma^2}}$$

VIDI Pl.76.

- FOR RAYLEIGH CHANEL

$$\boxed{E(|h_n|) = \sqrt{\pi P_0}} \rightarrow \sigma\sqrt{2\pi}$$

$$E(|h_n|^2) = P_0 = 2\sigma^2$$

NE ZNAM OD KADE 2O IZVATI

$$E(\gamma) = \frac{1}{②N\sigma^2} E\left\{\left[\sum_{n=0}^{N-1}|h_n|\right]^2\right\} = \frac{1}{②N\sigma^2} E\left\{\sum_{n=0}^{N-1}\sum_{m=0}^{N-1}|h_n||h_m|\right\} =$$

$$= \frac{1}{2N\sigma^2}\left[E\left[\sum_{n=0}^{N-1} h_n^2\right] + E\left[\sum_{n=0}^{N-1}\sum_{m=0}^{N-1}|h_n||h_m|\right]\right] =$$

$$\qquad\qquad\qquad\qquad\qquad\qquad n \neq m$$

$$= \frac{1}{2N\sigma^2}\left[\underbrace{\sum_{n=0}^{N-1} E[h_n^2]}_{②NP_0} + \underbrace{\sum_{n=0}^{N-1}\sum_{\substack{m=0\\ m\neq n}}^{N-1} E[|h_m|]\,E[|h_n|]}_{N(N-1)\frac{|\sqrt{\pi P_0}|^2}{2}}\right]$$

$$E[\gamma] = \frac{1}{2N\sigma^2}\left[2NP_0 + N(N-1)\frac{\pi P_0}{2}\right] = \left[\Gamma + (N-1)\frac{\pi\Gamma}{4}\right]$$

$$\boxed{E[\gamma] = \Gamma\left[1 + \frac{\pi(N-1)}{4}\right]}$$

IMPROVEMENT
$\sim N$ SIMILAR AS IN
MCR CASE

- $Pe = \frac{1}{2}\left[1 - \sqrt{\frac{\Gamma(\Gamma+2)}{\Gamma+1}}\right]$ $\boxed{N=2}$ · DBPSK

BPSK $\boxed{N=3}$

$$Pe = \frac{1}{2} - \frac{1}{2}\sqrt{\frac{\Gamma(2\Gamma+3)^2}{3(\Gamma+1)}} \times {}_2F_1\left(-\frac{1}{2}, \frac{1}{2}; \frac{1}{2}; \frac{\Gamma^2}{(2\Gamma+3)^2}\right) + \frac{\pi}{4}\sqrt{\frac{\Gamma^3}{27(\Gamma+1)^3}}$$

⊡ Dsplog's SELECTION DIVERSITY

⊙ POVTORUVANJE (PE ZA MRC)

- ADVE's NOTES

$$Pe = \frac{1}{(N-1)!}\left(\frac{1-\mu}{2}\right)^N \sum_{u=0}^{N-1} \frac{(N-1+u)!}{u!}\left(\frac{1+\mu}{2}\right)^u$$

$$\mu = \sqrt{\frac{\Gamma}{1+\Gamma}}$$

- PROAKIS DIGCOM

$$\boxed{Pe = \left[\frac{1}{2}(1-\mu)\right]^L \sum_{k=0}^{L-1} \binom{L-1+k}{k}\left[\frac{1}{2}(1+\mu)\right]^k}$$

OVAA IDE ZA KORISTAM VO MATLAB

$$\mu = \sqrt{\frac{\bar{\gamma}_c}{1+\bar{\gamma}_c}} \qquad \binom{L-1+k}{k} = \frac{(L-1+k)!}{(L-1)!\cdot k!} \Rightarrow$$

$$\boxed{Pe = \left(\frac{1-\mu}{2}\right)^L \cdot \frac{1}{(L-1)!} \sum_{k=0}^{L-1} \frac{(L-1+k)!}{k!} \cdot \left(\frac{1+\mu}{2}\right)^k}$$

IDENTIČNO!

- L = 1 $\sum_{u=0}^{0}(\dots)$ $u=0$

$$\frac{(1-1+0)!}{0!\cdot 1} \left(\frac{1+\mu}{2}\right)^0 = 1 \qquad \boxed{Pe = \frac{1-\mu}{2}}$$

$$\boxed{Pe = \frac{1}{2}\left[1 - \sqrt{\frac{\bar{\gamma}_c}{1+\bar{\gamma}_c}}\right]}$$

ODGOVARA NA SISO

- $L = 2$

$$P_e = \left(\frac{1-\mu}{2}\right)^2 \sum_{k=0}^{1} \frac{(1+k)!}{k!} \left(\frac{1+k}{2}\right)^k$$

~~$P_e|_L = P_e|_{L-1} + P_e|_1$~~

$$\boxed{g_m = \frac{E_b}{N_0} = \bar{\gamma}_c}$$

$$\boxed{\sigma^2 = \Omega = 1}$$

UNIT POWER

$$\gamma = \frac{E_b}{N_0} \alpha^2 \qquad \bar{\gamma} = \frac{E_b}{N_0} E(\alpha^2) \rightarrow$$

$$\boxed{\bar{\gamma} = \frac{E_b}{N_0}}$$

$$E_bN_0dB = 10\log(E_b/N_0) = 10\log(g_m)$$

$$\boxed{g_m = 10^{0.1\, E_bN_0\text{-}dB}}$$

- Vo dsplog PRIMEROT GO IMPLEMENTIRAAT NA SLEDNIOV NAČIN:

$$\boxed{P_{e1} = 0.5\left(1 - \left(1 + \frac{1}{\gamma}\right)^{-\frac{1}{2}}\right) = \frac{1}{2}\left(1 - \frac{1}{\sqrt{1 + 1/\gamma}}\right)}$$

NO DIVERSITY

$$p = \frac{1}{2} - \frac{1}{2}\left(1 + \frac{1}{\gamma}\right)^{-\frac{1}{2}}$$

$$\boxed{P_{e2} = p^2 \left(1 + 2(1-p)\right)}$$

dsplog's EQUAL GAIN COMBINING

$$\hat{y} = \sum_i \frac{y_i}{e^{j\theta_i}} = \sum_i \frac{|h_i| e^{j\theta_i} x + n_i}{e^{j\theta_i}} = \sum_i |h_i| \cdot x + \tilde{n}_i$$

$$\boxed{\tilde{n}_i = \frac{n_i}{e^{j\theta_i}}}$$

PROAKIS 14-4-2

MULTIPHASE SIGNALS

$$P_M = \frac{(-1)^{L-1}(1-\mu^2)^L}{\pi(L-1)!} \left(\frac{\partial^{L-1}}{\partial b^{L-1}} \left\{ \frac{1}{b-\mu^2} \left[\frac{\pi}{M}(M-1) - \frac{\mu \sin(\pi/M)}{\sqrt{b-\mu^2\cos^2(\frac{\pi}{M})}} \cot^{-1}\left(\frac{-\mu\cos(\pi/M)}{\sqrt{b-\mu^2\cos^2(\frac{\pi}{M})}}\right)\right]\right\}\right)\Bigg|_{b=1}$$

$$\boxed{\mu = \sqrt{\frac{\bar{\gamma}_c}{1+\bar{\gamma}_c}}}$$

COHERENT PSK

$$\boxed{\mu = \frac{\bar{\gamma}_c}{1+\bar{\gamma}_c}}$$

DPSK

$$\boxed{\overline{\gamma}_b = \frac{L\overline{\gamma}_c}{k} \qquad k = \log_2 M}$$

- PS~~K~~-4 & DPSK-4

$$\boxed{P_b = \frac{1}{2}\left[1 - \frac{\mu}{\sqrt{2-\mu^2}} \sum_{k=0}^{L-1} \binom{2k}{k} \left(\frac{1-\mu^2}{4-2\mu^2}\right)^k\right]}$$

⊡ M-ARY ORTHOGONAL SIGNALS (14-4-3)

- MULTIDIMENSIONAL SIGNALS (4-3-1)

$$\boxed{T_1 = N \cdot T}$$

NAVRAĆANJE NAZAD !!!

$$s_m(t) = \mathrm{Re}\left[s_{lm} e^{j2\pi f_c t}\right] \qquad m = 1, 2, \dots M \qquad 0 \le t \le T$$

$$s_m(t) = \sqrt{\frac{2E}{T}} \cos\left[2\pi f_c t + 2\pi m \Delta f t\right]$$

- LOWPASS EQUIVALENT WAVEFORMS

$$\boxed{s_{lm}(t) = \sqrt{\frac{2E}{T}} e^{j2\pi m \Delta f t} \qquad m = 1, 2, \dots, M \qquad 0 \le t \le T}$$

OVA E VSUŠNOST FSK.

- FOR EQUAL ENERGY WAVEFORMS THE CROSSCORREL.:

$$\rho_{km} = \frac{1}{\cancel{2E}} \cdot \frac{\cancel{2E}}{T} \int_0^T e^{j2\pi(m-k)\Delta f t} dt =$$

$$= \frac{1}{T} \frac{1}{j2\pi(m-k)\Delta f} \left(e^{j2\pi(m-k)\Delta f T} - 1\right) = \frac{\cancel{4} e^{j\pi(m-k)\Delta f T}}{j2\pi(m-k)\Delta f T} \left(e^{j\pi(m-k)\Delta f T} - e^{-j\pi(m-k)\Delta f T}\right)$$

$$= \frac{\sin(\pi(m-k)\Delta f T)}{\pi(m-k)\Delta f T} e^{j\pi(m-k)\Delta f T}$$

$$\rho_r = \frac{\sin(\pi(m-k)\Delta f T) \cdot \cos(\pi(m-k)\Delta f T)}{\pi(m-k)\Delta f T} = \frac{\sin(2\pi(m-k)\Delta f T)}{2\pi(m-k)\Delta f T}$$

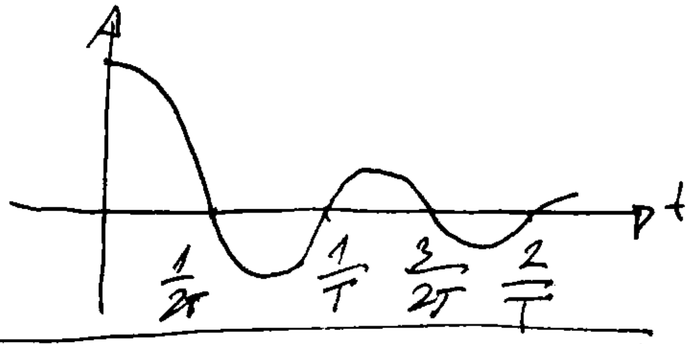
- SIMILARITY BETWEEN $s_m(t)$ AND $s_k(t)$ IS MEASURED BY NORMALIZED CROSS-CORRELATION

$$\frac{1}{\sqrt{E_m E_k}}\int_{-\infty}^{\infty} s_m(t)\, s_k(t)\, dt = Re\left\{\frac{1}{2\sqrt{E_m E_k}}\int_{-\infty}^{\infty} s_{lm}(t)\, s_{lk}^*(t)\, dt\right\}$$

~~FOR EQUAL ENERGY WAVEFORMS~~ | POTSETUVANJE

$Re(\rho_{km}) = 0$ $\boxed{\Delta f = \frac{1}{2T}}$ for $m \neq k$ $f_s = 2f_{max}$ $\frac{1}{T_s} = \frac{2}{T}$

$$\rho_r = \frac{\sin(2\pi(m-k)\Delta f T)}{2\pi(m-k)\Delta f T}$$



$\Delta f = \frac{1}{2T}$ ⇒ MINIMUM FREQUENCY SEPARATION BETWEEN ADJESENT SIGNALS FOR ORTHOGONALITY OF THE M SIGNALS

MMV

- EUCLIDIAN DISTANCE $\boxed{d_{km}^{(e)} = |s_m - s_k|}$

$$s_1 = [\sqrt{E}, 0, \dots 0, 0]$$
$$s_2 = [0, \sqrt{E}, \dots 0, 0]$$
$$\dots$$
$$s_N = [0, 0, \dots 0, \sqrt{E}]$$

$$v_1 = \sqrt{E}\,\vec{i} + 0\,\vec{j} + 0\,\vec{k} \qquad v_1 - v_2 = \sqrt{E}\,\vec{i} - \sqrt{E}\,\vec{j}$$
$$v_2 = 0\,\vec{i} + \sqrt{E}\,j + 0\,k \qquad |v_1 - v_2| = \sqrt{E + E} = \sqrt{2E}$$

- M-ARNOT ORTOGONALEN SISTEM MOŽE DA SE SMETA KAKO FSK SO MINIMALNA SEPARACIJA 1/T
- IZLEZOT OD MRC e:

$$\boxed{U_1 = \sum_{k=1}^{L} \left|2E a_k e^{-j\phi_k} + N_{k1}\right|^2}$$

→ FOR THE COMBINER CONTAININ TE SIGNAL

$$v_m = \sum_{k=1}^{L} |N_{km}|^2 \qquad \underline{\underline{m = 2,3,4,\dots M}}$$

- ZA FSK (14-4-31) pdf c:

$$p(v_1) = \frac{1}{(2\sigma^2)(L-1)!} v_1^{L-1} \exp\left(-\frac{v_1}{2\sigma_1^2}\right)$$

$$p(v_2) = \frac{2}{(2\sigma^2)(L-1)!} v_2^{L-1} \exp\left(-\frac{v_2}{2\sigma^2}\right)$$

$$P(v_2 < v_1) = \int_0^{v_1} p(v_2)\, dv_2 = 1 - \exp\left(-\frac{v_1}{2\sigma^2}\right) \sum_{k=0}^{L-1} \frac{1}{k!}\left(\frac{v_1}{2\sigma^2}\right)^k$$

$$P_M = 1 - \int_0^{\infty} \frac{1}{(2\sigma^2)(L-1)!} v_1^{L-1} \exp\left(-\frac{v_1}{2\sigma_1^2}\right) \underbrace{\left[1 - \exp\left(-\frac{v_1}{2\sigma^2}\right) \sum_{k=0}^{L-1} \frac{1}{k!}\left(\frac{v_1}{2\sigma^2}\right)^k\right]^{(M-1)}}_{\text{SE KREVA NA } (M-1) \text{ [illegible]}} dv_1$$

ŠTO: $P(v_2 < v_1, v_3 < v_1, \dots v_M < v_1)$

$$\boxed{P_M = 1 - \int_0^{\infty} \frac{1}{(1+\bar\gamma_c)^L (L-1)} v_1^{L-1} \exp\left(-\frac{v_1}{1+\bar\gamma_c}\right) \left(1 - e^{-v_1} \sum_{k=0}^{L-1} \frac{v_1^k}{k!}\right)^{M-1} dv_1}$$

↳ VEROJATNOST ZA GREŠKA ~~ZA~~ MULTIDIMENZIONALNI ORTOGONALNI SIGNALI !!!

$$\boxed{\gamma_b = \frac{L \cdot \gamma_c}{\mathrm{ld} M}}$$

$$\left(\sum_{k=0} \frac{v_1^k}{k!}\right)^m = \sum_{k=0}^{m(L-1)} \beta_{km} v_1^k$$

$$\boxed{P_M = \frac{1}{(L-1)!} \sum_{m=1}^{M-1} \frac{(-1)^{m+1}\binom{M-1}{m}}{(1+m+m\bar\gamma_c)^L} \cdot \sum_{k=0}^{m(L-1)} \beta_{km} (L-1+k)! \left(\frac{1+\bar\gamma_c}{1+m+m\bar\gamma_c}\right)^k}$$

AMV

- FOR NO DIVERSITY $L=1$

$$\boxed{P_M = \sum_{m=1}^{M-1} \frac{(-1)^{m+1}\binom{M-1}{m}}{1+m+m\bar\gamma_c}} \qquad \boxed{P_b = \frac{2^{k-1}}{2^k - 1}}$$

- BANDWIDTH

$$\boxed{B_C = \frac{L \cdot M}{\mathrm{ld}\, M}}$$

- CHERNOFF BOUND FOR BINARY ORTHOGONAL SIGNALING

$$P_2(L) = P(U_2 - U_1 > 0) = P(X > 0) = \int_0^\infty p(x)\, dx$$

$$x = U_2 - U_1 = \sum_{k=1}^{L} \left(|N_{k2}|^2 - |2\mathcal{E}a_k + N_{k1}|^2 \right)$$

$$\boxed{S(x) = \begin{cases} 1 & x \geq 0 \\ 0 & x < 0 \end{cases}}$$

$S(x)$ - UNIT STEP FUNCTION

$$P_2(L) = E[S(x)]$$

- <u>CHERNOFF BOUND IS OBTAINED BY OVERBOUNDING THE UNIT STEP FUNCTION BY EXPONENTIAL FUNCTION</u>

$$S(x) \leq e^{\xi x} \quad \xi > 0$$

$$P_2(L) = E[S(x)] \leq E[e^{\xi x}]$$

$$P_2(L) \leq \prod_{k=1}^{L} E\left(e^{\xi |N_{k2}|^2}\right) E\left(e^{-\xi |2\mathcal{E}a_k + N_{k1}|^2}\right)$$

$$E\left(e^{\xi |N_{k2}|^2}\right) = \frac{1}{1 \,\text{[illegible]}\, 2\xi\sigma_2^2} \qquad \xi < \frac{1}{2\sigma_2^2}$$

$$E\left(e^{-\xi |2\mathcal{E}a_k + N_{k1}|^2}\right) = \frac{1}{1 + 2\xi\sigma_1^2} \qquad \xi > -\frac{1}{2\sigma_1^2}$$

$$\sigma_2^2 = 2\mathcal{E}N_0 \qquad \sigma_1^2 = 2\mathcal{E}N_0(1 + \gamma_c)$$

$$P_2(L) \leq \left[\frac{1}{(1 - 2\xi\sigma_2^2)(1 + 2\xi\sigma_1^2)} \right]^L \qquad 0 \leq \xi \leq \frac{1}{2\sigma_2^2}$$

$$\left. \frac{d P_2(L)}{d\xi} = 0 \right| \xi_{min} = \ldots$$

$$P_2(L) \le \left[\frac{1}{(1-2\xi\sigma_2^2)(1+2\xi\sigma_1^2)}\right]^L \qquad 0 \le \xi \le \frac{1}{2\sigma_2^2}$$

$$\frac{dP_2(L)}{d\xi} = L\left[\frac{1}{(1-2\xi\sigma_2^2)(1+2\xi\sigma_1^2)}\right]^{L-1}\left(+\frac{2\cdot\sigma_2^2}{(1-2\xi\sigma_2^2)^2}\frac{1}{(1+2\xi\sigma_1^2)} \quad \frac{2\cancel{\xi}\sigma_1^2}{(1+2\xi\sigma_1^2)^2}\cdot\frac{1}{(1-2\xi\sigma_2^2)}\right) = 0$$

$$\frac{\cancel{2}\sigma_2^2(1+2\xi\sigma_1^2) - \cancel{2}\cancel{\xi}\sigma_1^2\cdot(1-2\xi\sigma_2^2)}{\cancel{(1-2\xi\sigma_2^2)^2}\ \cancel{(1+2\xi\sigma_1^2)^2}} = 0$$

$$\cancel{2}\sigma_2^2 + 2\xi\sigma_1^2\sigma_2^2 - \cancel{2}\sigma_1^2 + 2\xi\sigma_1^2\sigma_2^2 = 0$$

$$\cancel{4}\xi\sigma_1^2\sigma_2^2 = \cancel{2}\sigma_1^2 - \sigma_2^2 \qquad \boxed{\xi = \frac{\sigma_1^2-\sigma_2^2}{4\sigma_1^2\sigma_2^2}}$$

$$P_2^{1/L}(L) \le \frac{1}{\left(1-\cancel{2}\frac{\sigma_1^2-\sigma_2^2}{2\sigma_1^2}\right)\left(1+\frac{\sigma_1^2-\sigma_2^2}{2\sigma_2^2}\right)} =$$

$$= \frac{\cancel{4}\sigma_1^2\sigma_2^2}{(\cancel{2\sigma_1^2} - \sigma_1^2+\sigma_2^2)(2\sigma_2^2+\sigma_1^2-\sigma_2^2)} = \frac{4\sigma_1^2\sigma_2^2}{(\sigma_1^2+\sigma_2^2)(\sigma_1^2+\sigma_2^2)}$$

$$\sigma_1^2 = 2\epsilon N_0 \qquad \sigma_2^2 = 2\epsilon N_0(1+\bar{\gamma}_c)$$

$$P_2^{1/L}(L) = \frac{4\cdot 4\epsilon^2N_0^2(1+\bar{\gamma}_c)}{(4\epsilon N_0+\gamma_c 2\epsilon N_0)^2} = \frac{4\cdot\cancel{4\epsilon^2N_0^2}(1+\bar{\gamma}_c)}{\cancel{4\epsilon^2N_0^2}(2+\bar{\gamma}_c)^2}$$

$$\boxed{P_2(L) \le \left[\frac{4(1+\bar{\gamma}_c)}{(2+\bar{\gamma}_c)^2}\right]^L}$$

BER FOR BINARY ORTHOGONAL SIGNALING WITH DIVERSITY ($\le L$)

$$\boxed{P_2(L) \le \left[4p(1-p)\right]^L}$$

$$\boxed{p = \frac{1}{2+\bar{\gamma}_c}} \rightarrow$$

BER FOR BINARY ORTHOGONAL SIGNALING WITHOUT DIVERSITY

MMV

- EXACT BER FOR BINARY ORTHOGONAL SIGNALING

$$\boxed{P_2(L) = \left(\frac{1}{1+\bar{\gamma}_c}\right)^L \sum_{k=0}^{L-1} \binom{L-1+k}{k} \left(\frac{1+\bar{\gamma}_c}{2+\bar{\gamma}_c}\right)^k = p^L \sum_{k=0}^{L-1} \binom{L-1+k}{k} (1-p)^k}$$

- BOUND FOR M-ARY ORTHOGONAL SIGNALING

$$\boxed{P_M \le (M-1) P_2(L)}$$

14-5 DIGITAL SIGNALING OVER FREQUENCY-SELECTIVE, SLOWLY FADING CHANNEL

$\boxed{T_m \cdot B_d \ll 1}$

$W \ll (\Delta f)_c \Rightarrow$ FREQ NON SELECTIVE

$T \ll (\Delta t)_c \Rightarrow$ SLOW FADING

$\boxed{\frac{1}{(\Delta f)_c} \cdot (\Delta t)_c \ll 1}$

$\boxed{\text{LOWPASS}}$

14-5-1 A Tapped-Delay Channel Model

s_l – EQUIVALENT LOWPASS SIGNAL WITH BANDWIDTH $|f| \le \frac{W}{2}$

$$\underbrace{s_l(t) = \sum_{n=-\infty}^{\infty} s_l\left(\frac{n}{W}\right) \frac{\sin[\pi W(t-n/W)]}{\pi W(t-n/W)}}$$

$\boxed{f_s = W}$

ANALOGNO NA FORMULATA ZA RECONSTRUKCIJA OD DIGITAL SIGNAL PROCESSING USING MATLAB.

$$\boxed{S_l(f) = \begin{cases} \frac{1}{W} \sum_{n=-\infty}^{\infty} s_l(n/W) e^{-j2\pi f n/W} & |f| \le \frac{1}{2}W \\ 0 & |f| > \frac{1}{2}W \end{cases}}$$

$$r_l(t) = \int_{-\infty}^{\infty} C(f;t) S_l(f) e^{j2\pi f t} df$$

$C(f;t)$ – TIME VARIANT TRANSFER FUNCTION

$$r_l(t) = \frac{1}{W} \sum_{n=-\infty}^{\infty} s_l\left(\frac{n}{W}\right) \underbrace{\int_{-\infty}^{\infty} C(f;t) e^{-j2\pi f(t-\frac{n}{W})} df}_{c(t-\frac{n}{W};t)}$$

$$r_l(t) = \frac{1}{W}\sum_{n=-\infty}^{\infty} s_l(n/W)\,\underbrace{c\left(t-\frac{n}{W};t\right)}_{c(\tau;t)}$$

$c(\tau;t)$ – TIME VARIANT IMPULSE RESPONSE

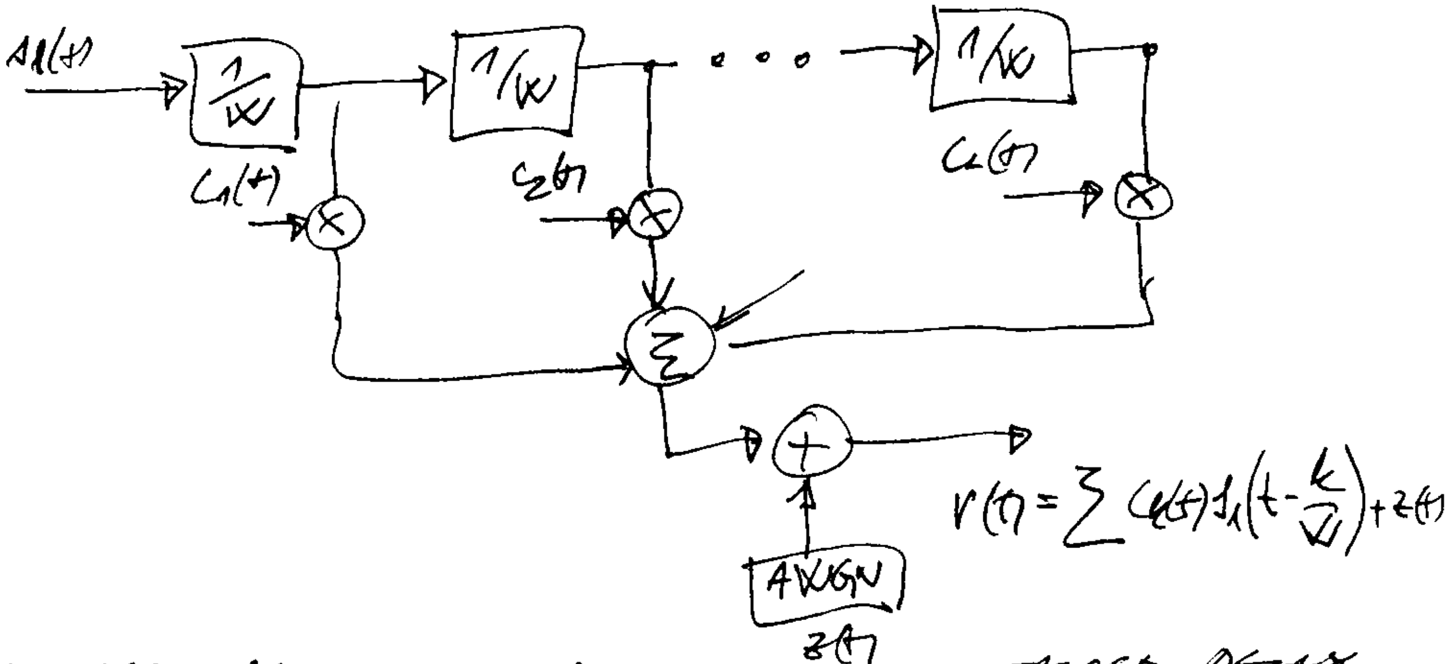
ALTERNATIVELY:

$$r_l(t) = \frac{1}{W}\sum_{n=-\infty}^{\infty} s_l\left(t-\frac{n}{W}\right) c\left(\frac{n}{W};t\right)$$

$$c_n(t) = \frac{1}{W}\, c\left(\frac{n}{W};t\right) \qquad r_l(t) = \sum_{n=-\infty}^{\infty} c_n(t)\cdot s_l\left(t-\frac{n}{W}\right)$$

$$\boxed{c(\tau;t) = \sum_{n=-\infty}^{\infty} c_n(t)\,\delta\left(\tau-\frac{n}{W}\right) \qquad C(f;t) = \sum_{n=-\infty}^{\infty} c_n(t)\, e^{-j2\pi f n/W}}$$

↳ LOWPASS IMPULSE RESPONSE & CORRESPONDING TIME TRANSFER FUNCTION



FOR ALL PRACTICAL PURPOSES THE TAPPED DELAY LINE MODEL FOR THE CHANNEL CAN BE TRUNCATED AT $\boxed{L = \lceil T_m W \rceil + 1}$ TAPS

$$r_l(t) = \sum_{n=1}^{L} c_n(t)\, s_l\left(t-\frac{n}{W}\right) \qquad \tau = \frac{n}{W} \quad n = 1, 2, \ldots L$$

$\boxed{14-5-2}$ THE RAKE DEMODULATOR

BINARY SIGNALING: $s_{l1}(t)$ & $s_{l2}(t)$

$$r_l(t) = \sum_{k=1}^{L} c_k(t)\, s_{li}(t - k/W) + z(t) = v_i(t) + z(t) \qquad 0 < t \le T \quad i = 1,2$$

$$\boxed{U_m = Re\left[\int_0^T r_l(t) * v_m^*(t)\,dt\right] = Re\left[\sum_{k=1}^{L} \int_0^T r_l(t)\, c_k^*(t)\, s_{lm}^*\left(t - \frac{k}{W}\right) dt\right]_{m=1,2}}$$

RAKE RECEIVER FORMULA

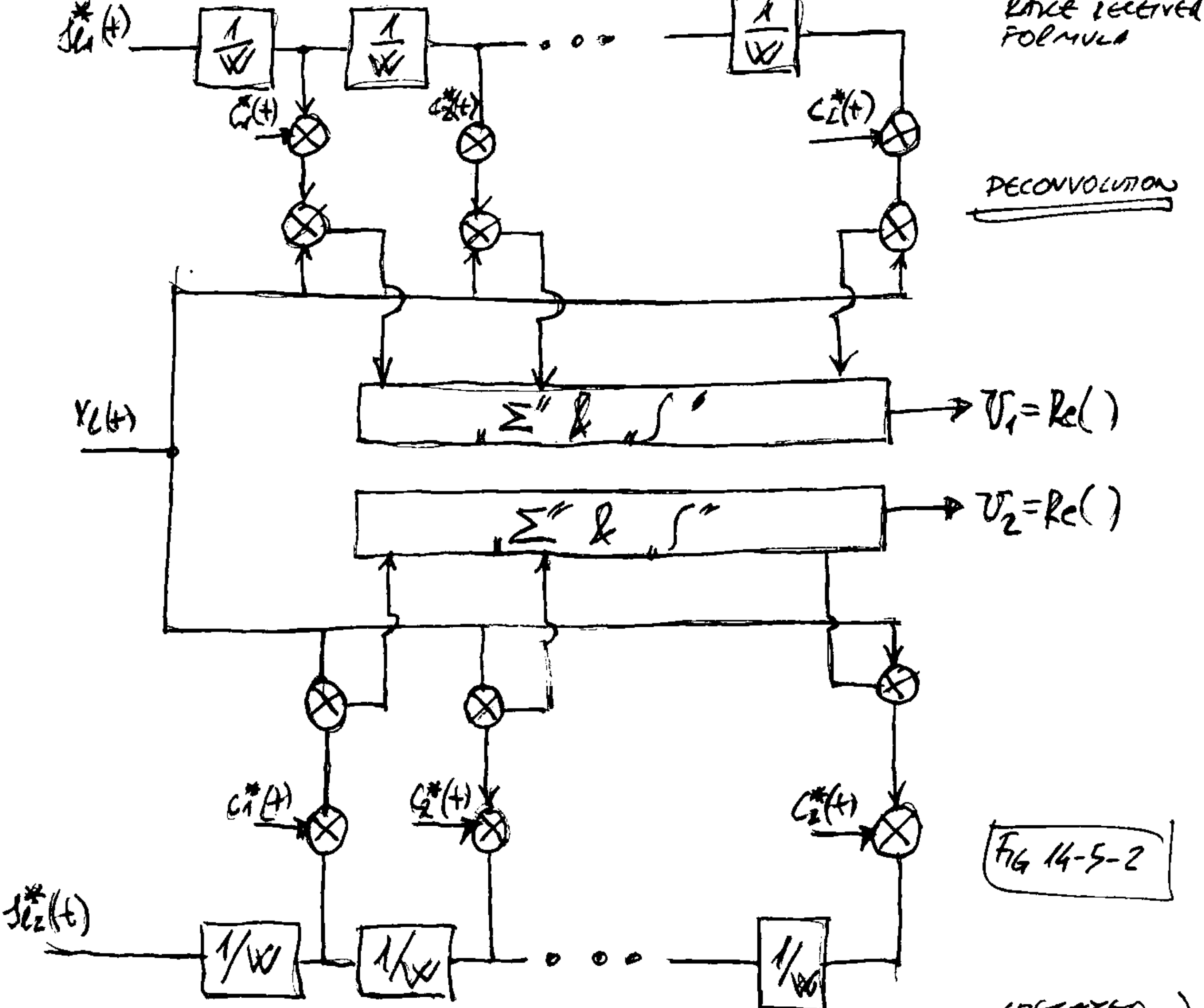


FIG 14-5-2

OPTIMUM DEMODULATOR FOR WIDEBAND BINARY SIGNALS (DELAYED ~~REFERENCE~~ CONFIGUR.)

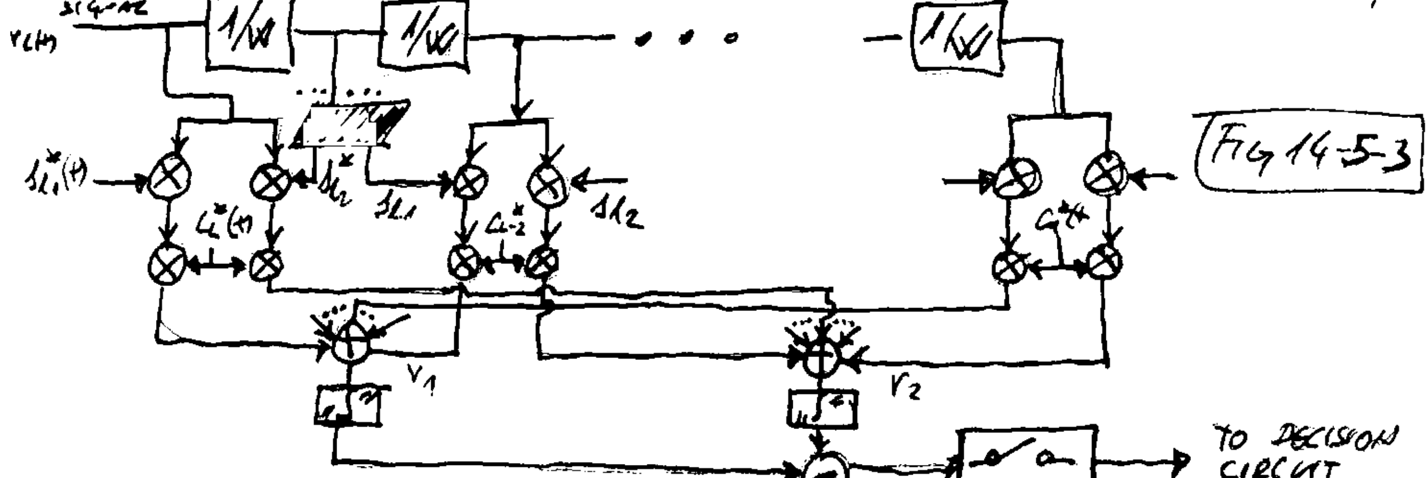


FIG 14-5-3

OPTIMUM DEMODULATOR FOR WIDEBAND SIGNALS (DELAYED RECEIVED SIGNAL CONFIG)

▣ PERFORMANCE OF RAKE RECEIVER

$$U_m = Re\left[\sum_{k=1}^{L} c_k^* \int_0^T r_l(t)\, s_{lm}^*(t-k/W)\,dt\right] \qquad \underline{\underline{m=1,2}}$$

$$r_l(t) = \sum_{n=1}^{L} c_n s_{l1}\left(t-n/W\right) + z(t) \qquad 0 \le t \le T$$

$$U_m = Re\left[\sum_{k=1}^{L} c_k^* \sum_{n=1}^{L} c_n \int_0^T s_{l1}(t-n/W)\, s_{lm}^*(t-k/W)\,dt\right] +$$

$$+ Re\left[\sum_{k=1}^{L} c_k^* \int_0^T z(t)\cdot s_{lm}^*(t-k/W)\,dt\right] \qquad m=1,2$$

$$\underbrace{\int_0^T s_{li}(t-n/W)\, s_{li}^*(t-k/W)\,dt \doteq 0}_{\text{NE SE KORELIRANI T.E. DE SE ORTOGONALNI}} \qquad \begin{matrix} k\neq n \\ i=1,2 \end{matrix}$$

- UNDER THIS CONDITION

$$U_m = Re \sum_{k=1}^{L} |c_k|^2 \int_0^T s_{l1}(t-n/W)\, s_{lm}^*(t-k/W)\,dt +$$

$$+ Re\left[\sum_{k=1}^{L} c_k^* \int_0^T z(t)\cdot s_{lm}(t-k/W)\,dt\right. \qquad m=1,2$$

- WHEN BINARY SIGNALS ARE ANTIPODAL THEN:

$$U_1 = Re\left[2E\sum_{k=1}^{L} a_k^2 + \sum_{k=1}^{L} a_k N_k\right] \qquad a_k = |c_k|$$

$$N_k = e^{j\phi_k}\int^T z(t)\cdot s_l(t-k/W)\,dt$$

- WHEN $E(a_k^2)$ ARE SAME FOR ALL k THAN 14-4-15 APPLIES

$$\boxed{P_2 = \left[\frac{1}{2}(1-\mu)\right]^L \sum_{k=0}^{L-1} \binom{L-1+k}{k} \left[\frac{1}{2}(1+\mu)\right]^k \qquad \mu = \sqrt{\frac{\bar{\gamma}_c}{1+\bar{\gamma}_c}}}$$

MMV SAME AS MRC

- P_2 FOR BINARY ANTIPODAL SIGNALS AND ORTHOGONAL SIGNALS UNDER THE CONDITION $E(a_k^2)$ IS DIFERENT FOR DIFER. k =

$$\boxed{P_2(\gamma_b) = Q\left(\sqrt{\gamma_b(1-\rho_r)}\right)}$$

SOGLASNO SLIKA 3.68

$\rho=-1$ ZA ANTIPODAL

$\rho=0$ ZA ORTHOGONAL

$$\gamma_b = \frac{E}{N_0}\sum_{k=1}^{L} a_k^2 = \sum_{k=1}^{L}\gamma_k \qquad p(\gamma_k) = \frac{1}{\bar{\gamma}_k} e^{-\gamma_k/\bar{\gamma}_k}$$

$$\bar{\gamma}_k = \frac{E}{N_0} E(a_k^2)$$

$$\boxed{\psi_{\gamma_k}(jv) = \frac{1}{1-jv\bar{\gamma}_k}}$$

$$\boxed{\psi_{\gamma_b} = \prod_{k=1}^{L}\frac{1}{1-jv\bar{\gamma}_k}}$$

DA SE IDENTIČNO DISTRIBUIRANI

$\underline{\underline{IID}}$ $\quad \gamma_{\gamma_b} = \dfrac{1}{(1-jv\gamma_c)^L}$ KAKO IOLANO!!

$$\psi_{\gamma_k}(jv) = \int_0^{\infty}\frac{1}{\gamma_k}e^{-\frac{\gamma_k}{\bar{\gamma}_k}}e^{j\gamma_k v}\,d\gamma_k$$

$$\boxed{\pi_k = \prod_{\substack{i=1\\ i\neq k}}^{L}\frac{\bar{\gamma}_k}{\bar{\gamma}_k-\bar{\gamma}_i}}$$

①

$$\boxed{p_{\gamma_b} = \frac{1}{2\pi}\int_0^{\infty}\prod_{k=1}^{L}\frac{1}{1-jv\bar{\gamma}_k}e^{-j\gamma_k v}\,dv = \sum_{k=1}^{L}\frac{\pi_k}{\bar{\gamma}_k}e^{-\gamma_b/\bar{\gamma}_k} \quad \gamma_b \geq 0}$$

$\boxed{\text{MMV}}$

~~ZA~~ $L=2$ MALE DONA

$$(*) = \frac{\left(-e^{g/g_1}+e^{g/g_2}\right)e^{\frac{-g(g_2+g_1)}{g_1g_2}}}{g_1-g_2} = \frac{1}{g_1-g_2}\cdot\left(-e^{\frac{g}{g_1}-\frac{gg_2}{g_1g_2}-\frac{gg_1}{g_1g_2}} + e^{\frac{g}{g_2}-\frac{g}{g_1}-\frac{g}{g_2}}\right)$$

$$(*) = \frac{1}{g_1-g_2}\left(-e^{-\frac{g}{g_2}}+e^{-\frac{g}{g_1}}\right) = \frac{1}{g_1-g_2}e^{-g/g_1}+\frac{1}{g_2-g_1}e^{-g/g_2}$$

$\Updownarrow$

$L=2$ ① ⇒ [illegible]

$$\pi_1 = \frac{\bar{\gamma}_1}{\bar{\gamma}_1-\bar{\gamma}_2} \qquad \pi_2 = \frac{\bar{\gamma}_2}{\bar{\gamma}_2-\gamma_1}$$

$$p_{\gamma_b} = \frac{\pi_1}{\bar{\gamma}_1}e^{-\gamma_b/\bar{\gamma}_1}+\frac{\pi_2}{\bar{\gamma}_2}e^{-\gamma_b/\bar{\gamma}_2} = \frac{1}{\bar{\gamma}_1-\bar{\gamma}_2}e^{-\gamma_b/\bar{\gamma}_1}+\frac{1}{\bar{\gamma}_2-\bar{\gamma}_1}e^{-\gamma_b/\bar{\gamma}_2}$$

$$\boxed{P_2 = \int_0^{\infty}P_2(\gamma_b)\,p_{\gamma_b}(\gamma_b)\,d\gamma_b = \frac{1}{2}\sum_{k=1}^{L}\pi_k\left[1-\sqrt{\frac{\bar{\gamma}_k(1-\rho_r)}{2+\bar{\gamma}_k(1-\rho_r)}}\right]}$$

MMV

- IF $\bar{\gamma}_k \gg 1$

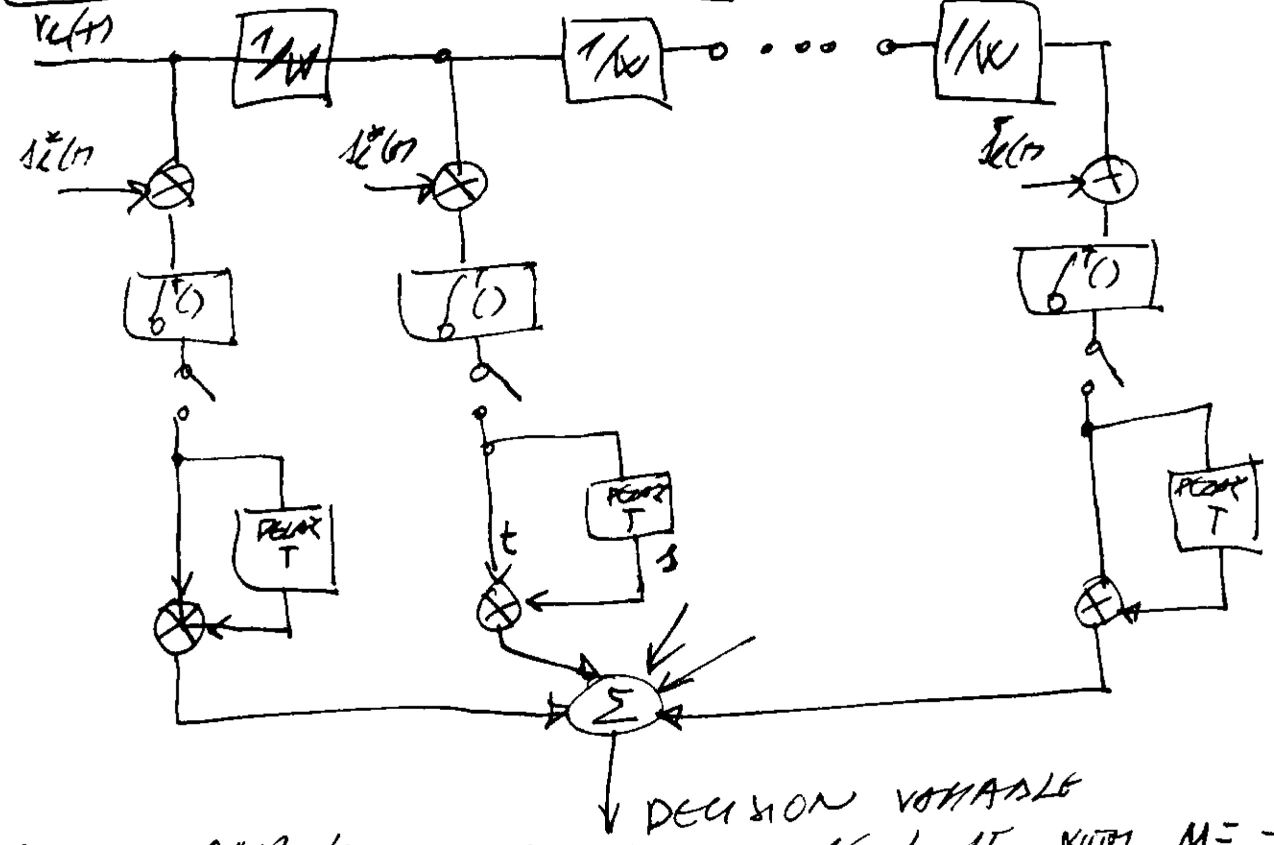
$$\boxed{P_2 \doteq \binom{2L-1}{L} \prod_{k=1}^{L} \frac{1}{2\bar{\gamma}_k(1-\rho_r)}}$$

$\rho_r = -1$ (ANTIPODAL)

$$\boxed{P_2 = \binom{2L-1}{L} \left(\frac{1}{4\bar{\gamma}}\right)^L}$$

SAME RESULT WAS OBTAINED FOR EQUAL SNRs/PATH (PROAKIS 14-4-18)

⓪ RAKE RECEIVER FOR DPSK



- EQUAL SNR/PATH SAME AS 14-4-15 WITH $\mu = \frac{\bar{\gamma}_c}{1+\bar{\gamma}_c}$

$$P_2 = \left[\frac{1}{2}(1-\mu)\right]^L \sum_{k=0}^{L-1} \binom{L-1+k}{k} \left[\frac{1}{2}(1+\mu)\right]^k$$

- NON-EQUAL SNR/PATH

$$P_2(\gamma_b) = \left(\frac{1}{2}\right)^{2L-1} e^{-\gamma_b} \sum_{k=0}^{L-1} b_k \gamma_b^2$$

$$P_2 = \int_0^\infty P_2(\gamma_b)\, p_{\gamma_b}(\gamma_b)\, d\gamma_b = \left(\frac{1}{2}\right)^{2L-1} \sum_{m=0}^{L-1} m!\, b_m \sum_{k=1}^{L-1} \frac{\pi_k}{\bar{\gamma}_k} \left(\frac{\bar{\gamma}_k}{1+\bar{\gamma}_k}\right)^{m+1}$$

$$\boxed{P_2 = \left(\frac{1}{2}\right)^{2L-1} \sum_{m=0}^{L-1} m!\, b_m \sum_{k=1}^{L-1} \frac{\pi_k}{\bar{\gamma}_k} \left(\frac{\bar{\gamma}_k}{1+\bar{\gamma}_k}\right)^{m+1}}$$

- BINARY ORTHOGONAL SIGNALING OVER FREQ-SELECTIVE CHANNEL WITH SQUARE-LAW DETECTION AT RECEIVER.

BER IS GIVEN IN 14-4-15 WITH $\boxed{\mu = \frac{\bar{\gamma}_c}{2+\bar{\gamma}_c}}$

$$P_2 = \left(\frac{1}{2}\right)^{2L-1} \sum_{m=0}^{L-1} m!\, b_m \sum_{k=1}^{L-1} \frac{\pi_k}{\bar{\gamma}_{k/2}} \left(\frac{\bar{\gamma}_{k/2}}{1+\bar{\gamma}_{k/2}}\right)^{m+1}$$

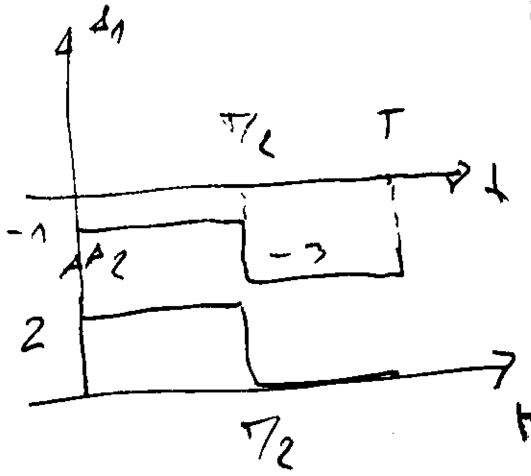
SKLAR EX. 3.1

$$s_1 = a_{11}\psi_1 + a_{12}\psi_2$$
$$s_2 = a_{21}\psi_1 + a_{22}\psi_2$$

$$a_{11} = \frac{1}{T}\int_0^T s_1(t)\psi_1(t)dt = 1$$
$$a_{12} = \frac{1}{T}\int_0^T s_1(t)\psi_2(t)dt = -2$$

$$\boxed{\begin{aligned} s_1 &= \psi_1 + 2\psi_2 \\ s_2 &= 2\psi_1 - \psi_2 \end{aligned}}$$

$$\int_0^T s_1(t)\cdot s_2(t) = ?$$



$$\int_0^{T/2} -1\cdot 2\,dt + \cancel{\int_{T/2}^{T} 0\cdot(-3)\,dt} \xrightarrow{} 0 = \underline{\underline{-T}}$$

$s_1(t)$ and $s_2(t)$ are not orthogonal

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$$h(t) = \sum_{i=1}^{M} a_i \delta(t-\tau_i) \qquad \tilde{s}(m;t) = s(m;t) * h(t) = \sum_{i=1}^{M} a_i s(m; t-\tau_i)$$

- RECEIVED SIGNAL

$$y(t) = (u*h)(t) + n(t) = \sum_m b[m]\,\tilde{s}(m; t-mT) + n(t)$$

$$Z[k] = \int y(t)\,\tilde{s}^*(m; t-mT)\,dt = \sum_{i=1}^{L} a_i^* \int y(t)\, s^*(m; t-mT-\tau_i)\,dt$$

- DESPREADING

$$Z(\tau) = \int y(t)\, s^*(t-\tau)\,dt$$

$$\boxed{y = \text{filter}(b, a, x)} \qquad \sum_{k=0}^{N} a_k\, y(n-k) = \sum_{m=0}^{M} b_m\, x(n-m) \quad \forall n$$

DIFERENCE EQUATION OF ORDER $\underline{\underline{N}}$

$$y(n) = \sum_{m=0}^{M} b_m\, x(n-m) - \sum_{k=1}^{N} a_k\, y(n-k)$$

$N = 0 \qquad a_0 = 1$

$$\boxed{y(n) = \sum_{m=0}^{M} b_m\, x(n-m)}$$

become $\qquad a = [1\ 2\ 3]$
$\qquad\qquad b = [2, 8, 17, 27, 19, 15]$

$$\begin{vmatrix} i & j & k \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{vmatrix} = \underbrace{(12-15)}_{-3}\, i + \underbrace{(12-6)}_{6}\, \vec{j} + \underbrace{(5-8)}_{-3}\, \vec{k}$$

$\boxed{\text{CROSS PROD}}$

$$\begin{array}{ccccc} 1 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 \\ 3 & 2 & 1 & 0 & 0 \\ 4 & 3 & 2 & 1 & 0 \\ \hline 5 & 4 & 3 & 2 & 1 \\ 6 & 5 & 4 & 3 & 2 \\ 7 & 6 & 5 & 4 & 3 \\ \ldots & & & & \end{array}$$

$c = a \cdot b$

$a = a_1 \vec{i} + b_2 \vec{j} + a_3 \vec{k}$

$b = b_1 \vec{i} + b_2 \vec{j} + b_3 \vec{k}$

$a \cdot b = a_1 b_1 + a_2 b_2 + a_3 b_3 = c_1$

$$\underbrace{\begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_N \end{bmatrix}}_{C} = \underbrace{\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ - & - & - \\ a_{N1} & a_{N2} & a_{N3} \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} b_{11} & \cancel{b_{21} \ \cdots \ b_{N1}} \\ b_{12} & \cancel{b_{22} \ \cdots \ b_{N2}} \\ b_{13} & \cancel{b_{23} \ \cdots \ b_{N3}} \end{bmatrix}}_{B}$$

$$\boxed{A^{-1} \cdot C = B}$$

$$\underbrace{\begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_N \end{bmatrix}}_{C} = \underbrace{\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ - & - & - \\ a_{N1} & a_{N2} & a_{N3} \end{bmatrix}}_{A.} \underbrace{\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}}_{D} \qquad \boxed{C = A * B}$$

$$\boxed{B = A^{-1} * C}$$

· **QPSK**

$$s_i(t) = \sqrt{\frac{2E}{T}} \cos\left(2\pi f_c t + (2i-1)\frac{\pi}{4}\right) \qquad i = 1, 2, 3, 4$$

$$s_i(t) = \sqrt{\frac{2E}{T}} \cos\left[(2i-1)\frac{\pi}{4}\right] \cdot \cos(2\pi f_c t) - \sqrt{\frac{2E}{T}} \sin\left[(2i-1)\frac{\pi}{4}\right] \sin(2\pi f_c t)$$

$$\left\{ \cos\left[(2i-1)\frac{\pi}{4}\right], \sin\left[(2i-1)\frac{\pi}{4}\right] \right\} \qquad i = 1, 2, 3, 4$$

$$i = 1 \quad \left[\cos(\pi/4) \quad \sin\left(\frac{\pi}{4}\right)\right] = \left[\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right] = \underline{\underline{[1, 1]}}$$

$$i = 2 \quad \left[\cos(3\pi/4) \quad \sin\left(\frac{3\pi}{4}\right)\right] = \left[-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right] = [-1, 1]$$

$$i = 3 \quad \left[\cos(5\pi/4) \quad \sin\left(\frac{5\pi}{4}\right)\right] = \left[-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right] = [-1, -1]$$

$$i = 4 \quad \left[\cos(7\pi/4) \quad \sin(7\pi/4)\right] = \left[\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right] = [1, -1]$$

BINGHAMTONS' RAKE RECEIVER FOR CDMA

$P = 64$ – SPREADING GAIN $\quad K = 5$ – # OF USERS

$Nd = 500$ – SYMBOL AMOUNT $\quad Lh = 6$ – CHANNEL LEGTH

$L_1 = 3Lh$; $\quad fd = \text{round}(L1/2)$ – EQULIZER LENGTH & DELAY

$tao = [0 \ \text{floor}(\text{rand}(1, K-1) * P)]$ – USER DELAY $\left(\text{length}(tao) = 5\right)$

$\text{size}(Mh) = 18 \times 23$ USER 1's CHANNEL MATRIX $H1$

$sf = (0:1:K-1) * 100$ USERS USE SAME LONG CODE WITH VARIOUS OFFSETS

$h = \text{randn}(K, Lh) + j\,\text{randn}(K, Lh)$

K = 5 USERS
Lh = 6 CHANNELS

$h(1,:) = [1, 2, 3, 4, 5, 6]$

$$Mh = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 & 4 & 5 & \cdots & & & & & & \\ 0 & 0 & 1 & 2 & 3 & 4 & \cdots & & & & & & \\ \vdots & & & & & & & & & & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 2 & 3 & 4 & 5 & 6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 1 & 2 & 3 & 4 & 5 & 6 \end{bmatrix}$$

18×23 $\quad \boxed{i = 1 : Lh}$ $\quad Mh(i, i : i + Lh - 1) = h(1,:)$

SHIFTANJE VO LEVO

- SPREDIRANJE NA SIGNALOT

$x = lcode(1 : \underset{500}{\overset{\shortparallel}{Nd}} * \underset{64}{\overset{\shortparallel}{P}}) \Rightarrow size(x) = 1 \times 32000$

MMV

$i = 1..Nd$ $\quad \overset{\circledast}{x}((i-1)*P+1 : 1 : i*P) = b(1,i) * x((i-1)P+1 : i*P)$

$size(b) = 5 \times 50$

ZA PRVIOT USER BIT OD SIGNALOT GO SPREDIRA SO 64 CHIPS OD lcode

OVA E REALIZACIJA NA OVAA SPREADING VO MATLAB. VIDI blind4rake.m

filter(b, a, x); filter(b, 1, x)

$$y(n) = b_0 x + b_1 \cdot x(n-1) + b_2 x(n-2) + \ldots = \sum_{m=0}^{M} b_m x(n-m)$$

$a = 1$

$$y(n) = \sum_{m=0}^{M} b_m x(n-m) \Rightarrow filter(b, 1, x) = conv(b, x)$$

- SIGNALOT OD DRUGITE KORISNICI (ISTIOT SPREADING CODE NO RAZLIČNI ŠIFT.)

$i = 1..Nd$, $j = 2..K$ $\quad x1((i-1)*P+1 : i*P) = Pow(j) * b(j,i) * lcode(sf(j) + (i-1)*P+1 : 1/(j) + i*P$

SLIČNO NA $\circledast$, SAMO SPREADING KODOT GO ZEMA SO OFFSET!

$$\underbrace{x(tao(j)+1:Nd*P)}_{\text{ASYNCHRONOUS DELAY}} = x(tao(j)+1:Nd*P) + x1(1:Nd*P-tao(j))$$

73
56
17

- AWGN $vn = randn(size(x)) + j\,randn(size(x))$ → SNR

$vn = (vn/norm(vn)) \cdot norm(x) \cdot 10^{-dB/20}$ $(\Rightarrow dB = 20)$

$x = x + vn$

ⓧ ✓ Ⓨ ✓

⊙ SAMPLE MATRICES ✓ AND DESPREADING MATRICES ✓

$x(:,P-j+1) = x(i*P+j-1+\underbrace{fd}_{9}:-1:i*P+j+fd-\underbrace{L1}_{18})$ $\boxed{17+1=18}$

$i=1\ j=1$ $x(:,64) = x(64+9:-1:64-8) = \underline{x(73:-1:56)}$

despreading:

$n = x*Ccode((i+1)*P:-1:i*P+1).'/P$

- RAKE RECEIVER : CHANNEL & SYMBOL ESTIMATION

$R = Y*Y'/(Nd-3)$ $size(R) = 18\times18$
$size(Y) = 18\times497$

$[Ur, Sr, Vr] = svd(R)$ } CHANNEL ESTIMATION
$sr, svd =$

$hb1 = Ur(:,1);$

$hb1 = \dfrac{hb1}{hb1'*Mh(:,fd)/abs(hb1'*Mh(:,fd))}$

- SYMBOL ESTIMATION

$\boxed{sy = hb1'*Y}$

$fh = hb1'*Mh$ (VIDI ZA Mh NA STR.100 OČIGLEDNO E DELA INAME fliplr(Cx))

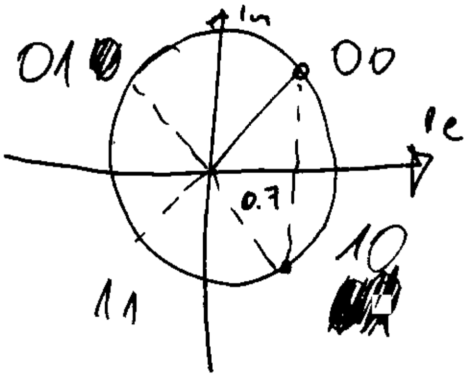
$i = find(abs(fh) = max(abs f(h)));$

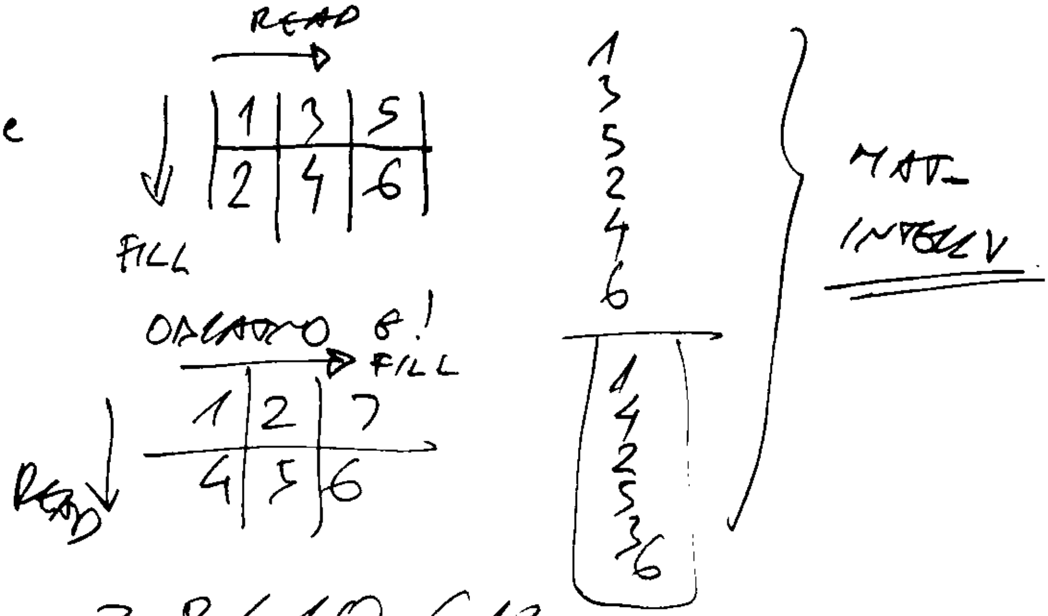
$\boxed{sy1 = sy/fh(i)}$ $sy2 = sign(real(sy1)) + j\,sign(\frac{[illegible]}{imag(sy1)})$

$(\Delta t)_c > T_m \Rightarrow$ SLOW FADING

$(\Delta f)_c > 1/T_m \Rightarrow$ NON-FRQ SELEC

$(\Delta f)_c < 1/T_m \Rightarrow$ FRQ-SELECTIVE





2	4	6
8	10	12

2 8 4 10 6 12

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \\ 3 & 3 \end{bmatrix} \xrightarrow{?} \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 2 & 2 \\ 2 & 2 \\ 3 & 3 \\ 3 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 2 & 2 \\ 3 & 3 \end{bmatrix}$$

PROAKIS DSP PR. 4.8

$$b \quad \frac{z^2+z+1+z^{-1}+z^{-2}+z^{-3}}{z+2+z^{-1}} = z-1+2z^{-1}-2z^{-2}+\frac{3z^{-2}+3z^{-3}}{z+2+z^{-1}}$$

$[q, nq, r, nr] \overset{a}{=} \text{deconv\_m}(b, nb, a, na)$

$nq = nb(1) - na(1)$; $nb(\text{end}) - na(\text{end}))$

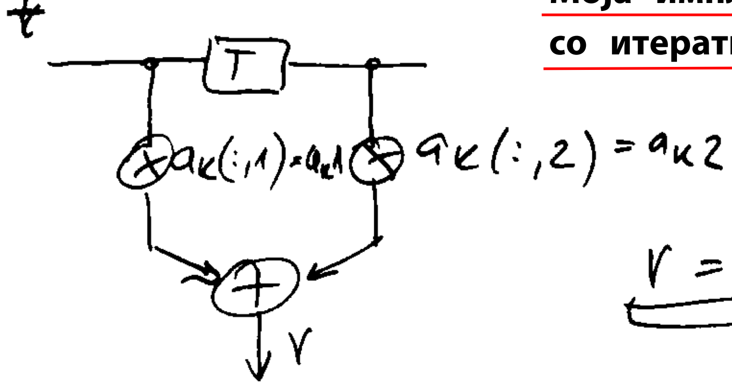
$-2 \; -1 \; 0 \; 1 \; 2 \; \boxed{3}$: nb

$-1 \; 0 \; \boxed{1}$: na

$-1 \; : \; \boxed{2}$

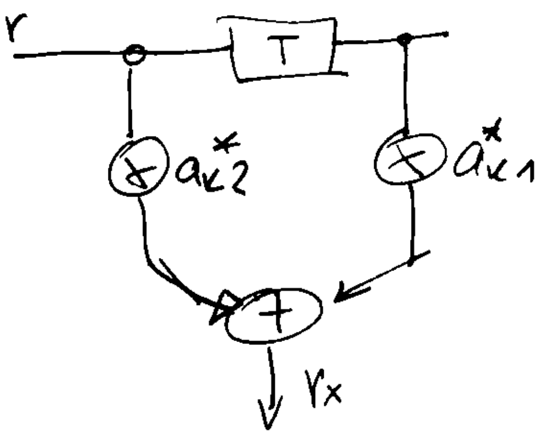
Моја имплементација на Rake Receiver со итеративна постапка

TRANSMITER



$$\boxed{r = a_{K1}\cdot t(n) + a_{K2}\, t(n-1)}$$

- RECEIVER



$$\boxed{t = a_{K2}^*\cdot r(n) + a_{K1}^*\, r(n-1)}$$

$$t = \begin{array}{ccc} 1 & 2 & 3 \\ [-1 & 0 & -1] \\ t(1) & t(2) & t(3) \end{array}$$

$$a_k = \begin{array}{ccc} & 1 & 2 \\ & \left[\begin{matrix} -0,31+0,84i & 0,2+0,12i \\ -1,18-0,03i & -0,81+0,13i \\ 0,08+0,23i & 0,84+0,51i \end{matrix}\right] & \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} \end{array}$$

$r(1) = a_k(1,1)\cdot t(1) + a_k(1,2)\cdot \cancel{t(-1)}^{0}$ = 0,3[illegible] - 0,84i

$r(2) = a_k(2,1)\cdot t(2) + a_k(2,2)\cdot t(1)$ = -0,3[illegible] + 0,1[illegible]i

$r(3) = a_k(3,1)\cdot t(3) + a_k(3,2)\cdot t(2) = 0,75 + 0,28i$

$r(2) = (-1,18-0,03i)\cdot \cancel{0}^{-1} + (-0,81+0,13i)\, t(1)^{=1} = \underline{\underline{-0,3671 + 0,1054i}}$

$t(1) = rx(1)$

$rx(1) = r(1) / a_k(1,1)$

$$rx(2) = \left(r(2) - a_K(2,2)\frac{r(1)}{a_K(1,1)}\right)\frac{1}{a_K(2,1)} = \frac{r(2)}{a_K(2,1)} - \frac{a_K(2,2)\, r(1)}{a_K(1,1)\, a_K(2,1)}$$

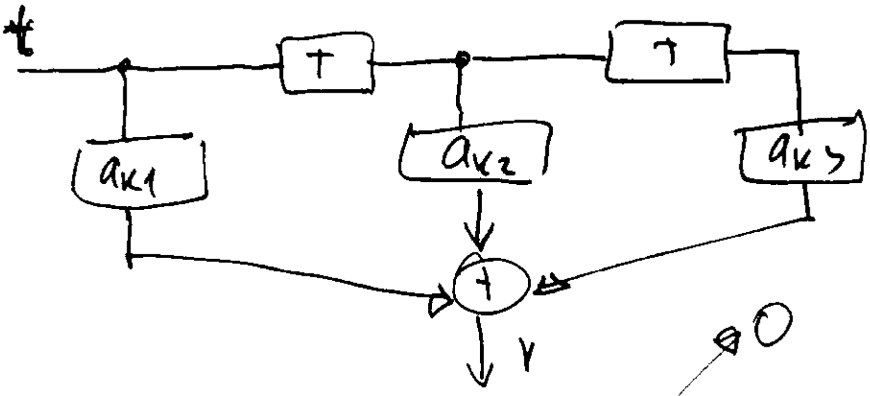
$$rx(3) = \frac{r(3)}{a_K(3,1)} - \frac{a_K(3,2)}{a_K(3,1)}\cdot t(2) = \text{[illegible]}$$

$t(2) \to rx(2)$

$$rx(2) = \frac{r(2)}{a_K(2,1)} - \frac{a_K(2,2)}{a_K(2,1)}\cdot rx(1)$$

- $\boxed{N_t = 3}$

$\boxed{95, 99, 100}$



$t = [-1\ 1\ -1]$

0 ×0

$$r(1) = a_k(1,1)\cdot t(1) + a_k(1,2)\, t(\cancel{4}) + a_k(1,3)\, t(-2)$$

$$r(2) = a_k(2,1)\, t(2) + a_k(2,2)\, t(1) + a_k(2,3)\, t(-1)$$

$$r(3) = a_k(3,1)\, t(3) + a_k(3,2)\, t(2) + a_k(3,3)\, t(1)$$

$$\underset{\equiv t(1)}{r_{\cancel{x}}(1)} = \frac{r(1)}{a_k(1,1)}$$

$$\underset{\equiv t(2)}{r_x(2)} = \frac{r(2)}{a_k(2,1)} - \frac{a_k(2,2)}{a_k(2,1)}\, r_x(1)$$

$$r_x(3) = \frac{r(3)}{a_k(3,1)} - \frac{a_k(3,2)}{a_k(3,1)}\, r_x(2) - \frac{a_k(3,3)}{a_k(3,1)}\, r_x(1)$$

$\boxed{\text{MMV}}$

VO PROCESS SOURCE CODING PREDIKTIRANJE t^* GI DEZEZI $\hat{t}$ NAMESTO SO r_x?

$$r(4) = a_k(4,1)\, t(4) + a_k(4,2)\, t(3) + a_k(4,3)\, t(2)$$

$$r(i) = a_k(i,1)\, t(i) + a_k(i,2)\, \underline{t(i-1)} + a_k(i,3)\, t(i-2)$$

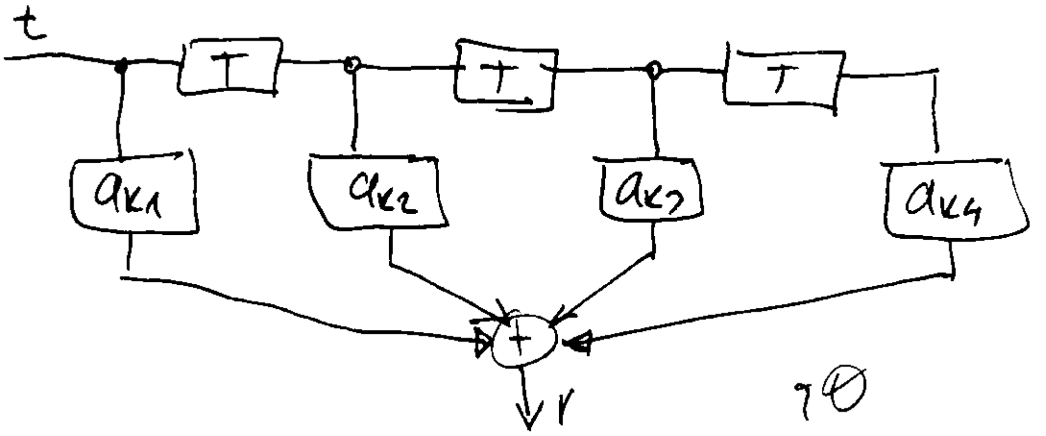
$$r_x(i) = \frac{r(i)}{a_k(i,1)} - \frac{a_k(i,2)}{a_k(i,1)}\cdot t(i-1) - \frac{a_k(i,3)}{a_k(i,1)}\, t(i-2)$$

$x =$ 1 0 $\qquad$ $-2x$ $\qquad$ -2 0

$\Updownarrow \quad \Updownarrow$ $\qquad$ $+1$ $\qquad$ -1 1

$y =$ -1 1

$$\boxed{s_m(t) = \sum_{n=-\infty}^{\infty} a_n\, g(t - nT_d)}$$



[-1 1 -1]

$$r(1) = a_k(1,1)\,t(1) + a_k(1,2)\cancelto{0}{t(-1)} + a_k(1,3)\cancelto{0}{t(-2)} + a_k(1,4)\cancelto{0}{t(-3)}$$

$$r(2) = a_k(2,1)\,t(2) + a_k(2,2)\,t(1) + a_k(2,3)\,t(-1) + a_k(2,4)\,t(-2)$$

$$r(3) = a_k(3,1)\,t(3) + a_k(3,2)\,t(2) + a_k(3,3)\,t(1) + a_k(3,4)\,t(-1)$$

$$r(4) = a_k(4,1)\,t(4) + a_k(4,2)\,t(3) + a_k(4,3)\,t(2) + a_k(4,4)\,t(1)$$

$\boxed{MMV}$

$$r_x(1) = r(1)/a_k(1,1)$$

$$r_x(2) = r(2)/a_k(2,1) - \frac{a_k(2,2)}{a_k(2,1)}\,r_x(1)$$

$$r_x(3) = r(3)/a_k(3,1) - \frac{a_k(3,2)}{a_k(3,1)}\,r_x(2) - \frac{a_k(3,3)}{a_k(3,1)}\,r_x(1)$$

$$r_x(4) = \frac{r(4)}{a_k(4,1)} - \frac{a_k(4,2)}{a_k(4,1)}\,r_x(3) - \frac{a_k(4,3)}{a_k(4,1)}\,r_x(2) - \frac{a_k(4,4)}{a_k(4,1)}\,r_x(1)$$

- FLAT FADING

MMV: Овие матрици најдобро се имплементираат со toeplitz()

$$\begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \end{bmatrix} = [a_{k1}, a_{k2}, a_{k3}, a_{k4}] \circledast \begin{bmatrix} t_1 & 0 & 0 & 0 \\ t_2 & t_1 & 0 & 0 \\ t_3 & t_2 & t_1 & 0 \\ t_4 & t_3 & t_2 & t_1 \end{bmatrix} \quad (*)$$

$$= \underbrace{[a_{k1}, a_{k2}, a_{k3}, a_{k4}]}_{A_k} \circledast \begin{bmatrix} t_1 & t_2 & t_3 & t_4 \\ 0 & t_1 & t_2 & t_3 \\ 0 & 0 & t_1 & t_2 \\ 0 & 0 & 0 & t_1 \end{bmatrix}$$

$$= \begin{bmatrix} a_{k1} & 0 & 0 & 0 \\ a_{k2} & a_{k1} & 0 & 0 \\ a_{k3} & a_{k2} & a_{k1} & 0 \\ a_{k4} & a_{k3} & a_{k2} & a_{k1} \end{bmatrix} \begin{bmatrix} t_1 \\ t_2 \\ t_3 \\ t_4 \end{bmatrix} \quad (**) \quad \Rightarrow \quad \boxed{t = A_k^{-1} \circ r}$$

$\boxed{MMV}$

OVA E MOŽ
SAMO AKO
ak IMA IST
BROJ ELEMENTI
KOLKO t.

NAMEN
SO OV
ODI SE
DECON
r.e
filter

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \qquad B = \begin{bmatrix} 4 & 4 & 4 & 4 & 4 \\ 3 & 3 & 3 & 3 & 3 \\ 2 & 2 & 2 & 2 & 2 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$X = \begin{bmatrix} 5 & 4 & 3 & 2 & 1 \\ 5 & 4 & 3 & 2 & 1 \\ - & - & - & - & - \\ 5 & 4 & 3 & 2 & 1 \end{bmatrix}$$

- IMPLEMENTACIJA SO INV. MATRICA (3 TAPS 5 SIMBOLS)

$$\begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \end{bmatrix} = \begin{bmatrix} a_{k1} & 0 & 0 & 0 & 0 \\ a_{k2} & a_{k1} & 0 & 0 & 0 \\ a_{k3} & a_{k2} & a_{k1} & 0 & 0 \\ 0 & a_{k3} & a_{k2} & a_{k1} & 0 \\ 0 & 0 & a_{k3} & a_{k2} & a_{k1} \end{bmatrix} \begin{bmatrix} t_1 \\ t_2 \\ t_3 \\ t_4 \\ t_5 \end{bmatrix}$$

$$\boxed{T = A^{-1} \cdot R} \qquad R = A * T \qquad \boxed{T = \cancel{R}\, A \backslash R}$$

- NEKA $a_{k1} = 1$ $a_{k2} = 2$ $a_{k3} = 3$

$$A_k = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 \\ 3 & 2 & 1 & 0 & 0 \\ 0 & 3 & 2 & 1 & 0 \\ 0 & 0 & 3 & 2 & 1 \end{bmatrix}$$

$$\boxed{Ax = b} \qquad A \cdot t = r \qquad \boxed{t = A \backslash b}$$

◎ BINGHAMPTON RAKE RECEIVER (THEORY)

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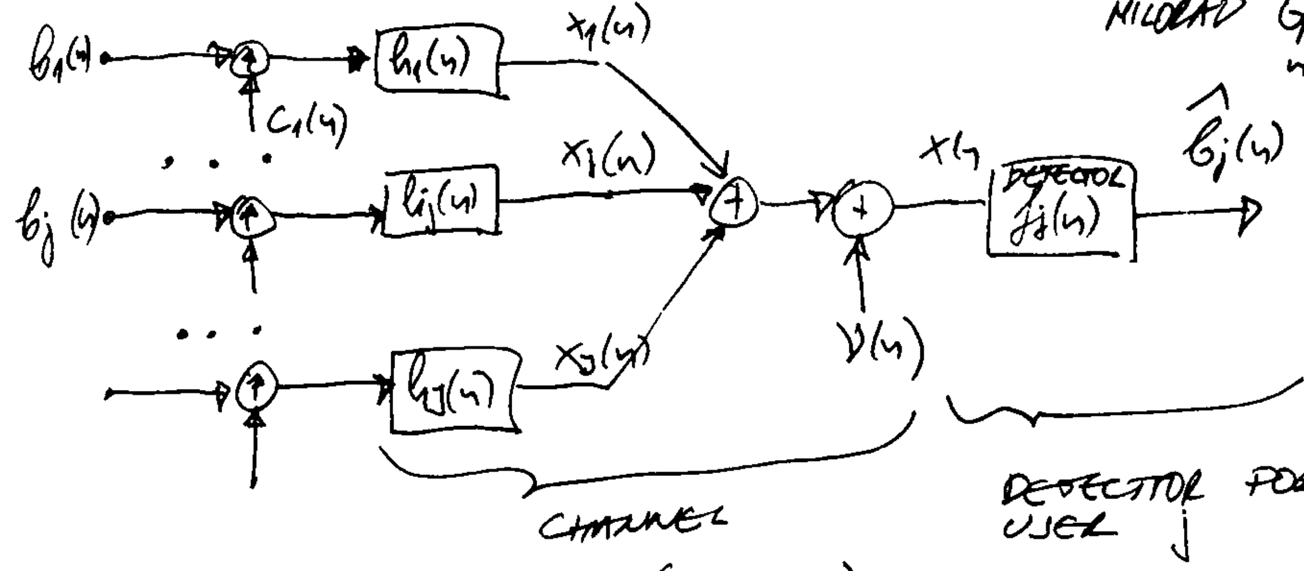
trajkovski
@uk.gle



- SIGNAL FROM USER j $(1 \leq j \leq J)$

$$s_j(n) = c_j(n) \cdot b_j\left(\left\lfloor \frac{n}{P} \right\rfloor\right)$$

$\left\lfloor \frac{n}{P} \right\rfloor$ – MAXIMUM INTEGER LESS THEN n/P

$$x_j(n) = [h_j(0) \dots h_j(L)] \begin{bmatrix} s_j(n) \\ \vdots \\ s_j(n-L) \end{bmatrix}$$

$\boxed{L \Rightarrow \text{DOLZINA NA KANALOT}}$

T.E. BROJ NA TAPOVI

- RECEIVED SIGNAL

$$x(n) = \sum_{j=1}^{J} x_j(n) + v(n)$$

◎ RAKE RECEIVER:

- FROM TIMING OF USER j CONSTRUCT ~~SPACE~~ MATRIX

for $n = k \cdot P + L$ $L = 0, 1, 2, \dots$

$$\underset{(L+1)\times 1}{x(n)} = \begin{bmatrix} x(n) \\ \vdots \\ x(n-L) \end{bmatrix} = \sum_{j=1}^{J} \begin{bmatrix} h_j(0) & \dots & h_j(L) & & \\ & \ddots & & \ddots & \\ & & h_j(0) & \dots & h_j(L) \end{bmatrix} \begin{bmatrix} s_j(n) \\ \vdots \\ s_j(n-L) \end{bmatrix} + \underline{V}(n)$$

$$\underset{(L+1)\times 1}{\underline{X}(n)} = j \sum_{j=1}^{J} \underset{(L+1)\times(2L+1)}{H_j} \underline{S}_j(n) + \underline{V}(n) \quad 2.L_q \times 1$$

$$\underset{((L+1)\times p)}{\underline{\bar{X}}(n)} = [\underline{X}(n) \dots \underline{X}(n+p)] = \sum_{j=1}^{J} \underset{L+1\times(2L+1)}{H_j} \underset{(2L+1)\times p}{\underline{S}_j(n)} + \underline{V}(n)$$

$\boxed{L+1}$

$$\underline{\bar{X}}(n) = H_j \begin{bmatrix} c_j(n)\, b_j(k+1) \dots c_j(n+\gamma)\, b_j(k+1) \\ c_j(n-L)\, b_j(k) \dots c_j(n-L+\gamma)\, b_j(k) \\ c_j(n-2L)\, b_j(k-1) \dots c_j(n-2L+\gamma)\, b_j(k) \end{bmatrix} + \sum_{\substack{i=1 \\ i \neq j}}^{J} H_i \underline{S}_i(n) + \underline{V}(n)$$

- Detector

$$\underline{f_i}(n) = \begin{bmatrix} c_j(n-L) \\ \vdots \\ c_j(n-L+\gamma) \end{bmatrix} \frac{1}{\gamma}$$

i) $\underset{(L+1)\times 1}{y(n)} = \underset{(L+1)\times\gamma}{\bar{X}(n)}\, \underset{\gamma\times 1}{\underline{f_i}(n)} = \begin{bmatrix} h_j(L) \\ \vdots \\ h_j(0) \end{bmatrix} \cdot b(k) +$ residual ISI MAI

ii) Channel estimation from

MMV

$$\boxed{R_y = E\left[\underline{y}(n)\, \underline{y}^H(n)\right] \to \begin{bmatrix} h_j(L) \\ \vdots \\ h_j(0) \end{bmatrix} \left[h_j^*(L) \dots h_j^*(0)\right]}$$

AUTOVETTORE

estimated channel $\underline{\hat{h}} \to \begin{bmatrix} h_j(L) \\ \vdots \\ h_j(0) \end{bmatrix}$

iii) SYMBOL ESTIMATION

$$\boxed{\hat{b}_j(k) = \underline{\hat{h}}^H \underline{y}(n) = \underline{\hat{h}}^H \underline{\bar{X}}(n)\, \underline{f_i}(n)}$$

$[x(n) \dots x(n-L)]$

$\underbrace{[x(10) \dots x(5)]}_{\text{LENGHT } \boxed{L+1}}$

$n = 10$

$L = 5$

$\boxed{i = 1 : N-3}$ $\boxed{j = 1 : P}$

$\boxed{\begin{matrix} i=1 \\ j=1 \end{matrix}}$ $iP + j + fd - L1 = 8 + 1 + 9 - 18 = 0$

$X(1 : iP + j - 1 + fd,\ P - j + 1) =$

$X(1 : 8 + 1 - 1 + 9,\ 8) = X(1:17, 8) =$

$= r(17:1)$

$\boxed{\begin{matrix} i=1 \\ j=2 \end{matrix}}$ $X(:, 7) = r(18:-1:1)$

1 2 3 4 5 6

1 2 3 4 5 6

B 1 2 3 4 5

$i=1$, $j=3$: $x(:,6) = r(19:-1:1)$

$i=1$, $j=8$: $x(:,1) = r(24:-1:7)$

$\boxed{\text{size}(x) = 18 \times P}$

$i = 2$

~~$\lambda = 1$~~

~~$$x = \begin{bmatrix} r_1 & r_1 & r_2 & \cdots & r_7 \\ r_2 & r_2 & r_3 & & r_8 \\ \vdots & \vdots & \vdots & & \\ & r_{18} & r_{17} & & r_{24} \end{bmatrix}$$~~

$i = 1$

$$x = \begin{bmatrix} r_{24} & r_{23} & r_{22} & r_{21} & r_{20} & r_{19} & r_{18} & r_{17} \\ r_{23} & r_{22} & & & & & r_{17} & r_{16} \\ & & & & & & & \vdots \\ r_8 & r_7 & r_6 & r_5 & r_4 & r_3 & r_2 & r_1 \\ r_7 & r_6 & r_5 & r_4 & r_3 & r_2 & r_1 & 0 \end{bmatrix}$$

$\boxed{i=2}$

$$x = \begin{array}{c|cccccc} & 1 & 2 & \cdots & & 7 & 8 \\ \hline 1 & r_{32} & r_{31} & - & - & r_{26} & r_{25} \\ 2 & r_{31} & r_{32} & - & - & r_{25} & r_{24} \\ & \vdots & \vdots & & & \vdots & \vdots \\ 17 & r_{14} & r_{15} & - & - & r_{10} & r_9 \\ 18 & r_{15} & r_{14} & - & - & r_9 & r_8 \end{array}$$

$\mathrm{size}(r) = 1 \times 160$

$24 - 17 = 7 + 1 = 8$

$\boxed{P=8}$

DESPREADING:

$\boxed{M = x * sp(24:17).'}$

$$M = \underset{18 \downarrow}{} \overset{\xrightarrow{8}}{\begin{bmatrix} r_{32} & \cdots & r_{26} & r_{25} \\ r_{31} & & r_{25} & r_{24} \\ \vdots & & \vdots & \vdots \\ r_{14} & - & r_{10} & r_9 \\ r_{15} & - & r_9 & r_8 \end{bmatrix}} \begin{bmatrix} -1 \\ -1 \\ 1 \\ -1 \\ 1 \\ -1 \\ -1 \\ -1 \end{bmatrix}$$

$$\begin{array}{r} 500 \\ - \quad 9 \\ \hline 492 \end{array}$$

$$R = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix} * \begin{bmatrix} 1^* & 1^* & 1^* \\ 2^* & 2^* & 2^* \\ 3^* & 3^* & 3^* \end{bmatrix} = S * S'$$

$[Ur, Sr, Vr] = svd(R);$ $\quad ak1 = Ur(:,1) \}$ ESTIMIRANI COEFICIENTI!

$$iP + j + fd - L_1 - (iP + j - 1 + fd) = \cancel{iP} + \cancel{j} + \cancel{fd} - L_1 - \cancel{iP} - \cancel{j} + 1 - \cancel{fd} =$$

$$= -L_1 + 1$$

$$\cancel{iP} + \cancel{j} - 1 + \cancel{fd} - \cancel{iP} - \cancel{j} - \cancel{fd} + L_1 = \boxed{L_1 - 1}$$

LENGHT
$L1 - 1 + 1 = L_1$

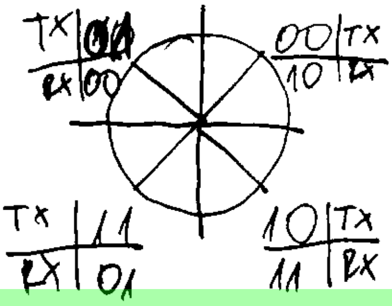
$rd = ak1' * r$;
$fh = ak1' * Mh$; $i = find(abs(fh) == max(abs(fh)))$
$rd = rd / fh(i)$ ⇒ SCALING AND AMBIGUITY PHASE REMOVING

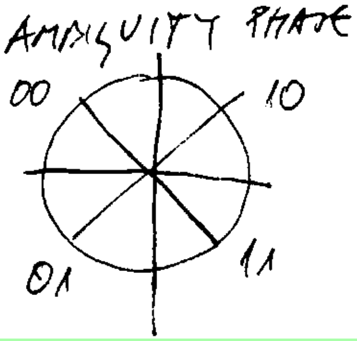
RX TX
2 ⟺ 0
3 ⟺ 2
0 ⟺ 1
1 ⟺ 3

→ OVA SE JAVUVA ZARADI AMBIGUITY PHASE

076689170
DUDE

Berm. ILE OD GODINE





ZNAČI POMESTUVANETO VO FAZATA E: $\frac{\pi}{2}$

POVTORUVANJE NA EQUILIZACIJA OD SKLOP

$$H_c(f) = |H_c(f)|\, e^{j\theta_c(f)}$$

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$$H_{RC}(f) = H_t(f)\, H_c(f)\, H_r(f)\, H_e(f)$$

- TRANSMIT & RECEIVED FILTERS ARE CHOSEN TO BE MATCHED

$$H_{RC}(f) = H_t(f)\cdot H_r(f)$$

SQUARE ROOT OF RCOSINE SQUARE ROOT OF RCOSINE

$$\boxed{H_e = \frac{1}{H_c(f)} = \frac{1}{|H_c(f)|}\cdot e^{-j\theta_c(f)}}$$

(2N+1) TAPS $c_{-N}, c_{-N+1}, \dots, c_{N-1}, c_N$

$$z(k) = \sum_{n=-N}^{N} x(k-n)\, c_n$$

$k = -2N \dots, 2N$ $n = -N \dots N$

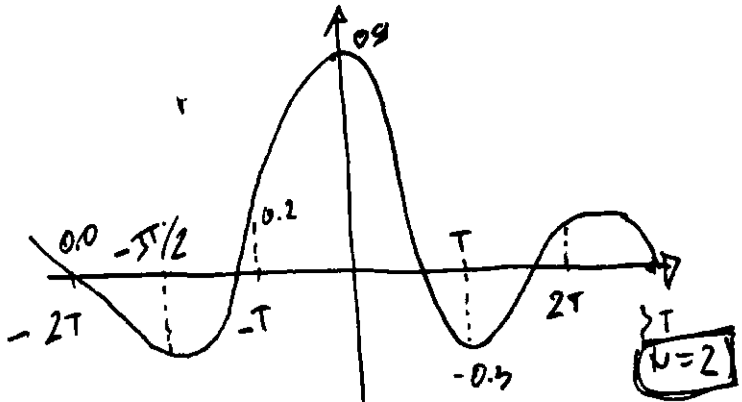
$k = 0, \pm 1, \pm 2 \dots$ (TIME INDEX)

$\underline{z} = \underline{X}\cdot\underline{c}$ $\boxed{\underline{c} = \underline{X}^{-1}\cdot\underline{z}}$

- ZERO FORCING SOLUTION

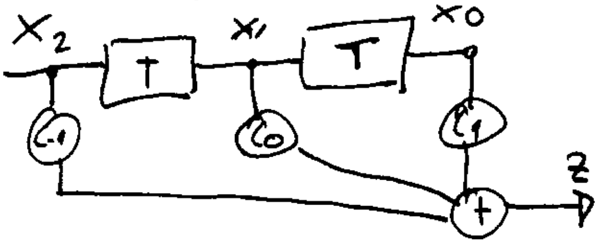
$$z(k) = \begin{cases} 1 & k=0 \\ 0 & k=\pm 1, \pm 2; \pm 7, \dots \end{cases}$$

EXAMPLE 3.5 ZERO-FORCING Three-Tap Equilizer



$$\{C_{-1}, C_0, C_1\} = ?$$

$$z(k) = \{z(-1)=0,\ z(0)=1,\ z(1)=0\}$$



$$x(k) = \{\overset{-2}{0.0}, \overset{-1}{0.2}, \overset{0}{0.9}, \overset{1}{-0.3}, \overset{2}{0.1}\}$$

$$\begin{bmatrix} z(-3) \\ z(-2) \\ z(-1) \end{bmatrix} = \begin{bmatrix} x(-2) & 0 & 0 \\ x(-1) & x(-2) & 0 \\ x(0) & x(-1) & x(-2) \end{bmatrix} \begin{bmatrix} C(-1) \\ C(0) \\ C(1) \end{bmatrix}$$

$$\begin{bmatrix} z(2) \\ z(3) \end{bmatrix} = \begin{bmatrix} 0 & x(2) & x(1) \\ 0 & 0 & x(2) \end{bmatrix} \begin{bmatrix} C(- \\ C(0 \\ C(1 \end{bmatrix}$$

$$\begin{bmatrix} z(-1) \\ z(0) \\ z(1) \end{bmatrix} = \begin{bmatrix} x(0) & x(-1) & x(-2) \\ x(1) & x(0) & x(-1) \\ x(2) & x(1) & x(0) \end{bmatrix} \begin{bmatrix} C(-1) \\ C(0) \\ C(1) \end{bmatrix}$$

$$z(-1) = C(-1)\,x(0) + C(0)\cdot x(-1) + C(1)\cdot x(-2)$$

$$z(0) = C(-1)\,x(1) + C(0)\cdot x(0) + C(1)\cdot x(-1)$$

$$z(1) = C(-1)\,x(2) \quad C(0)\,x(1) + C(1)\cdot x(0)$$

$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{array}{c} \begin{bmatrix} 0.9 & 0.2 & 0.0 \\ -0.3 & 0.9 & 0.2 \\ 0.1 & -0.3 & 0.9 \end{bmatrix} \\ \boxed{-1} \quad \boxed{0} \quad \boxed{1} \end{array} \begin{bmatrix} C(-1) \\ C(0) \\ C(1) \end{bmatrix}$$

$$\boxed{C = X^{-1} \cdot Z}$$

MATRAD ILI
TRI RAVENKI SO
TRI NEPOZNATI

$$\Rightarrow C = [-0.214;\ 0.9631;\ 0.3448]$$

$$z(-3) = x(-2)\cdot C(-1) = 0$$

$$z(-2) = x(-1)\cdot C(-1) + x(-2)\cdot C(0) = 0.2\cdot(-0.214) + 0 = -0.428$$

$$z(2) = \cancel{x(3)}^{0}\cdot C(-1) + \underset{0.1\cdot 0.9631}{x(2)\cdot C(0)} + \underset{-0.3\cdot 0.3448}{x(1)\cdot C(1)} = 0.09631 - 0.1034 = -0.0071$$

$$z(3) = C(1)\cdot X(2) = 0.3448 \cdot 0.1 = 0.03448$$

$$\{z(-3)\,z(-2)\,z(-1)\,z(0)\,z(1)\,z(2)\,z(3)\} = \{0, \underbrace{-0.428}_{\text{GREATEST}}, 0, 1, 0, -0.0071, 0.035\}$$

☐ MINIMUM MSE SOLUTION

(OVA ŠO KORISTI VO BINGHAMTON ~~BAZE~~ IMPLEMENTACIJA)

$$\boxed{\sigma^2 = \overline{(\xi - \bar{\xi})^2} = \overline{\xi^2} - \bar{\xi}^2}$$

$$\boxed{\text{MMV}} \rightarrow \boxed{R_{xz} = R_{xx} \cdot C}$$

$$z = X \cdot C \quad / \cdot X^T \qquad X^T \cdot z = X^T \cdot X C$$

$R_{xz} = X^T z$ – CROSS CORRELATION VECTOR

$R_{xx} = X^T \cdot X$ – AUTOCORRELATION MATRIX

$$\boxed{C = R_{xx}^{-1} R_{xz}}$$

- PRETPOSTAVKA E DENA ZA COMPLEKS, X^T TREBA X DA BIDE KONJUGIRANO I TRANSPONIRANO !!!

☐ PROAKIS PROBLEM 3.38

$\{x(n)\}$ STATIONARY STOHASTIC SEQUENCE WITH ZERO MEAN AND AUTOCORRELATION

$$\phi(n) = \begin{cases} 1 & n = 0 \\ 1/2 & n = \pm 1 \\ 0 & \text{OTHERWISE} \end{cases}$$

ⓐ PREDICTION COEFFICIENT OF THE FIRST-ORDER MSE PREDICTOR = ?

$$\boxed{\hat{x}(n) = a_1 x(n-1)}$$

ⓑ REPEAT (a) FOR SECOND ORDER PREDICTOR

$$\boxed{\hat{x}(n) = a_1 x(n-1) + a_2 x(n-2)}$$

CONTINUE PP. 114

- SOLUTION

AUTOCORRELATION/CROSSCORRELATION

$$\boxed{r_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n) \cdot x(n-l)}$$

DEFINITION FROM DSP USING MATLAB

$$x = [7, 11, 2, \underset{\uparrow}{0}, -1, 4, 2] \qquad y(n) = x(n-2) + w(n)$$

$$r_{xx}(l) = \sum_{n=1}^{7} x(n) \cdot x(n-l)$$

$r_{xx}(1) = x(1) \cdot x(0) + x(2) \cdot x(1) + x(3) \cdot x(2) + x(4) \cdot x(3) \ldots x(7) \cdot x(6)$

$r_{xx}(2) = x(1) \cdot x(-2) + x(2) \cdot x(0) + x(3) \cdot x(1) + x(4) \cdot x(2) \ldots x(7) \cdot x(5)$

$x = [\overset{1}{3}, \overset{2}{11}, \overset{3}{7}, \overset{4}{0}, \overset{5}{-1}, \overset{6}{4}, \overset{7}{2}]$ $y = x(n-2) = [\overset{1}{0}, \overset{2}{0}, \overset{3}{3}, \overset{4}{11}, \overset{5}{7}, \overset{6}{0}, \overset{7}{-1}] = y(n)$

$$r_{xy}(l) = \sum_{n=1}^{7} x(n) \cdot y(n-l)$$

$$r_{xy}(1) = \sum_{n=1}^{7} x(n) \cdot y(n-1) = \sum_{n=1}^{7} x(n)\, x(n-3) = \overset{y(3)}{\overset{\parallel}{}} \qquad \overset{y(6)}{\overset{\parallel}{}}$$

$$= x(1) \cdot x(-2) + x(2) \cdot x(-1) + x(3) \cdot x(0) + x(4) \cdot x(1) + \dots + x(7) \cdot x(4)$$

$$r_{xy}(2) = \sum_{n=1}^{7} x(n) \cdot x(n-4) = x(1) \cdot x(-3) + x(2) \cdot x(-2) + x(3)\, x(-1) + x(4) \cdot x(0) + x(5) \cdot x(1) + x(6) \cdot x(2) + x(7)\, x(3)$$

$$r_{xy}(7) = \sum_{n=1}^{7} x(n) \cdot x(n-7) = x(1) \cdot x(-6) + x(2) \cdot x(-5) + \dots + x(6) \cdot x(-1) + x(7) \cdot x(7-7)$$

ⓧ

[3, 11, 7, 0, -1, 4, 2]

7 0 -1
7 0 -1
11 7 0 -1
3 11 7 0 -1
0 3 11 7 0 -1
0 3 11 7 0 -1
0 0 3 11 7 0 -1
0 0 0 3 11 7 0 -1

MMV

VIDI:
WORK\DIVERSITY\
RAKE RECEIVER\SUPORTING
FILES\CROSSCORRELATIONFOR
COMPARINGSEQUENCES.m

$r_{xy}(1) = x_1 \cdot y_7 = -3$
$r_{xy}(2) = x_1 \cdot y_6 + x_2 \cdot y_7 = 3 \cdot 0 + 11 \cdot (-1) = -11$
$r_x(3) = x_1 \cdot y_3 + x_2 \cdot y_6 + x_3 \cdot y_7 = 3 \cdot 7 + 11 \cdot 0 + 7 \cdot (-1) = 21 - 7 = 14$

AKO IZBERES
Y = X(n-0)
OVA E MAKSIMUMOT

$r_x(13) = x_7 \cdot y_1 = 2 \cdot (0) = 0$
$r_x(12) = x_6 \cdot y_1 + x_7 \cdot y_2$
$r_x(11) = x_5 \cdot y_1 + x_6 \cdot y_2 + x_7 \cdot y_3$
$r_x(10) = x_4 \cdot y_1 + x_5\, y_2 + x_6 \cdot y_3 + x_7 \cdot y_4$
$r_x(9) = x_3 \cdot y_1 + x_4 \cdot y_2 + x_5 \cdot y_3 + x_6 \cdot y_4 + x_7 \cdot y_5$
$r_x(8) = x_2 \cdot y_1 + x_3 \cdot y_2 + x_4 \cdot y_3 + x_5 \cdot y_4 + x_6 \cdot y_5 + x_7 \cdot y_6$
$\boxed{r_x(7) = x_1 \cdot y_1 + x_2 \cdot y_2 + x_3 \cdot y_3 + x_4 \cdot y_4 + x_5 \cdot y_5 + x_6 \cdot y_6 + x_7 \cdot y_7}$
$r_x(4) = x_1 \cdot y_4 + x_2 \cdot y_5 + x_3 \cdot y_6 + x_4 \cdot y_7$
$\boxed{r_x(5) = x_1 \cdot y_3 + x_2 \cdot y_4 + x_3 \cdot y_3 + x_4 \cdot y_6 + x_5 \cdot y_7}$
$r_x(6) = x_1 \cdot y_2 + x_2 \cdot y_3 + x_3 \cdot y_4 + x_4 \cdot y_5 + x_5 \cdot y_6 + x_6 \cdot y_7$

→ ZA $x = y(n-2)$ OVA E MAKSIMUMOT ZOŠTO SEKOJ SO SEKOGO ~~SE~~ MNOZI ⇒ MAKSIMALNA KORELIRANOST

· PROBLEM (PROAKIS) 3.38 CONTINUE... (COEFFICIENTS OF MINIM. MSE PREDICTOR)

$$\phi(n) = \begin{cases} 1 & n=0 \\ 1/2 & n=\pm 1 \\ 0 & \text{OTHERWISE} \end{cases}$$

$\boxed{MMV}$

$\boxed{MSE = E[(\hat{X}-X)^2]}$

ⓐ $\hat{x}(n) = a_{11}x(n-1)$

a_{11} THAT MINIMIZES THE MSE IS FOUND FROM ORTHOGONALITY OF PREDICTION ERROR TO THE PREDICTION DATA:

$$E[e(n)x(n-1)] = 0 \qquad E\left[\left(x(n) - a_{11}x(n-1)\right)\cdot x(n-1)\right] = 0 \Rightarrow$$

$$E\left[x(n)\cdot x(n-1) - a_{11}x(n-1)\cdot x(n-1)\right] = \phi(1) - a_{11}\phi(0) = 0$$

MMV

$$\boxed{a_{11} = \frac{\phi(1)}{\phi(0)} = 1/2}$$

MINIMUM MSE:

$$\mathcal{E}_1 = \phi(0)\left(1 - a_{11}^2\right) = 3/4$$

ZERO MEAN

$$\mathcal{E}_1 = E\left\{\left[\hat{X} - E(\hat{X})\right]^2\right\} = E\left\{\left[\hat{X}^2 - 2\hat{X}\cdot \overset{0}{\cancel{E(\hat{X})}} + \overset{0}{\cancel{E^2(\hat{X})}}\right]\right\}$$

$$= E[\hat{X}^2] = E\left[a_{11}^2\,x(n-1)\cdot x(n-1)\right] = a_{11}^2\cdot\phi(0)$$

$$\mathcal{E}_1 = \phi(0)\left(1 - a_{11}^2\right) \leftarrow$$

DOKAZOT E NA DNOTO OD OVAA STRANICA !!!

ⓑ SECOND ORDER PREDICTOR

$$\hat{x}(n) = a_{21}x(n-1) + a_{22}x(n-2)$$

$\boxed{MMV}$

LEVINSON - DURBIN ALGORITHM (EQ 3-5-25)

$$a_{22} = \frac{\phi(2) - \sum_{k=1}^{1} a_{1k}\,\phi(2-k)}{\mathcal{E}_1} = \frac{0 - a_{11}\cdot\phi(1)}{3/4} = \frac{0 - \frac{1}{2}\cdot\frac{1}{2}}{3/4}$$

$$a_{22} = -\frac{1}{3}; \qquad a_{21} = \overset{1/2}{a_{11}} - \overset{\left(-\frac{1}{3}\right)}{a_{22}}\,\overset{\frac{1}{2}}{a_{11}} = 2/3 = \frac{1}{2} + \frac{1}{6} = \frac{3+1}{6} = \frac{4}{6}$$

$$\boxed{a_{21} = 4/6 = 2/3}$$

$$\mathcal{E}_2 = \mathcal{E}_1\left(1 - a_{22}^2\right) = \frac{3}{4}\left(1 - \frac{1}{9}\right) = :$$

2/3

$$\mathcal{E}_2 = \frac{\cancel{3}}{\cancel{4}}\cdot\frac{\overset{4}{\cancel{16}}}{\cancel{9}_3} = \frac{4}{3}$$

ESTIMATOR

ESTIMATED VALUE OF QUANTITY

TRUE VALUE $\boxed{MMV}$

$$\mathcal{E}_1 = E\left[\left(\underbrace{a_{11}x(n-1)}_{\hat{X}} - x(n-1)\right)^2\right] = E\left[a_{11}^2\cdot x^2(n-1) - \underbrace{2x^2(n-1) + x^2(n-1)}_{-x^2(n-1)}\right]$$

$$= E\left[a_{11}^2 x^2(n-1) - x^2(n-1)\right] = a_{11}^2\phi(0) - \phi(0) = (a_{11}^2 - 1)\phi(0)$$

$\boxed{MMV}$

$$x = \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \left[\begin{array}{cccc} 1 & 2 & 3 & 4 \end{array}\right] \end{array} \qquad r_{xy}(l) = \sum_{i=1}^{N=4} x(i)\cdot y(\underline{i}-\underline{l}) \Big\}$$

$$y = \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \left[\begin{array}{cccc} 0 & 1 & 2 & 3 \end{array}\right] \end{array}$$

[KOOKES] DEFINICIJA DSP WITH MAT.

$$r_{xy}(1) = x_1\cdot y_0 + x_2\cdot y_1 + x_3\cdot y_2 + x_4\cdot y_3 = 0 + 2\cdot 1 + 3\cdot 2 + 4\cdot 3 = 20$$

$$r_{xy}(2) = x_1\cdot y_{-1} + x_2\cdot y_0 + x_3\cdot y_1 + x_4\cdot y_2 = 3\cdot 0 + 4 = \textcircled{4}$$

$$r_{xy}(3) = x_1\, y_{-2} + x_2\cdot y_{-1} + x_3\cdot y_0 + x_4\cdot y_1 = 0$$

- MATLAB DEFINITION

$$r_{xy}(m) = E\{x_{n+m}\, y_n^*] = E[x_n\, y_{n-m}^*]$$

$$\hat{R}_{xy}(m) = \begin{cases} \sum\limits_{n=0}^{N-m-1} x_{n+m}\, y_n^* & m \geq 0 \\ \hat{R}_{yx}^*(-m) & m < 0 \end{cases}$$

$$r_{xy}(1) = \sum_{n=0}^{4-1-1=2} x_{n+1}\, y_n^* = x_1\cdot y_0 + x_2\cdot y_1 + x_3\cdot y_2 = 3\cdot 1 = 3$$

$$r_{xy}(2) = \sum_{n=0}^{1} x_{n+2}\, y_n^* = x_2\cdot y_0 + \underline{x_3\cdot y_1} + \cancel{x_4 y_2} = \underline{\underline{3\cdot 0\ \cancel{} = 0}}$$

$$r_{xy}(3) = \sum_{n=0}^{0} x_{n+1}\, y_n^* = x_1\cdot y_0 = 0$$

MMV

OBIČNA KONVOLUCIJA

$$C_{xy}(k) = \sum_{n=1}^{N} x(n)\, y(k-n) = \boxed{\sum_{n=1}^{4} x(n)\cdot y(k-n)}$$

GREŠKA MORA [illegible]
$k = 0, \dots N-1$
$n = 0, \dots N-1$

$$C_{xy}(1) = x_1\cdot y(\overset{\to 0}{0}) + x_2\cdot y(-1) + x_3\cdot y(-2) + x_4\cdot y(-3) = 0$$

$$= x_1\, \underset{n=1}{y(-(n-1))} + x_2\cdot \underset{n=2}{y(-(n-1))} + x_3\, \underset{n=3}{y(-(n-1))} + x_4\cdot \underset{n=4}{y(-(n-1))}$$

~~[illegible] OVO [illegible] MATEMATIČKO~~
~~PREDSTAVLJANJE (NE OBJESK) [illegible]~~

$$y(k-n) = y(-(n-k))$$

x(n)				1	2	3	4	
y([illegible])	3	2	1	0				
y(-(n-2))		3	2	1	0			
y(-(n-3))			3	2	1	0		
y(-(n-4))				3	2	1	0	
y(-(n-5))					3	2	1	0
y(-(n-6))						3	2	1
y(-(n-7))							3	2

$\boxed{OK}$

EMPIRIJSKA PREYMETKA NA CONV

$$C_{xy}(1) = 0$$

$$C_{xy}(2) = 1$$

$$C_{xy}(3) = 4$$

$$C_{xy}(4) = 3 + 4 + 3 = 10$$

$$C_{xy}(5) = 3\cdot 2 + 2\cdot 3 + 4\cdot 1 = 16$$

$$C_{xy}(6) = 9 + 8 = 17$$

$$C_{xy}(7) = 12$$

$$C_{xy} = [0, 1, 4, 10, 16, 17, 12]$$

$x = [\overset{1}{1} \ \overset{2}{2} \ \overset{3}{3} \ \overset{4}{4}]$

$y = [0 \ 1 \ 2 \ 3]$

$$C_{xy}(k) = \sum_{n=1}^{4} x(n)\, y(k-n)$$

GREŠKA!!!

TREBA
k=0...
n=0...

$C_{xy}(2) = \underbrace{x_1 \cdot \underbrace{y(2-1)}_{y_1}}_{0} + x_2 \underbrace{\underbrace{y(2-2)}_{y_0}}_{0} + x_3 \underbrace{y(2-3)}_{y(-1)} + x_4 \underbrace{y(2-4)}_{y(-2)} = 0$

$C_{xy}(3) = \underbrace{x_1 \cdot y_2}_{0} + \underbrace{x_2 \cdot y_1}_{2} + x_3 \cdot y_0 + x_4 \cdot y_{-1} = 2$

$C_{xy}(4) = \underset{2}{\overset{0}{x_1 \cdot y_3}} + \underset{2 \cdot 1}{x_2 \cdot y_2} + \underset{0}{x_3 \cdot y_1} + \underbrace{x_4 \cdot y_0}_{0} = 4$

$C_{xy}(5) = x_1 y_4 + x_2 \cdot y_3 + x_3 \cdot y_2 + x_4 \cdot y_1 = 3+4+3=10$

$\quad 1 \cdot 3 + 2 \cdot 2 + 3 \cdot 1 + 0$

$C_{xy}(6) = x_1 \cdot \cancel{y_5}^{0} + x_2 \cdot y_4 + x_3 \cdot y_3 + x_4 \cdot y_2 = 6+6+4=16$

$\quad 2 \cdot 3 \quad 3 \cdot 2 \quad 4 \cdot 1$

$C_{xy}(7) = x_1 \cdot 0 + x_2 \cdot 0 + x_3 \cdot y_4 + x_4 \cdot y_3 = 3 \cdot 3 + 4 \cdot 2 = 17$

$C_{xy}(8) = x_1 \cdot 0 + x_2 \cdot 0 + x_3 \cdot 0 + x_4 \cdot y_4 = 12$

$x = [\overset{0}{1} \ \overset{1}{2} \ \overset{2}{3} \ \overset{3}{4}]$

$y = [0 \ 1 \ 2 \ 3]$

$$C_{xy}(k) = \sum_{n=0}^{3} \underbrace{x(n) \cdot y(k-n)}_{\text{KONVOLUCIJA}}$$

MMV

VAMA TREBA

$C_{xy}(0) = x_0 \cdot y(0) = 0$

$C_{xy}(1) = x_0 \cdot y_1 + x_1 \cdot y_0 = 1 \cdot 1 + 1 \cdot 0 = 1$

$C_{xy}(2) = x_0 \cdot y_2 + x_1 \cdot y_1 + x_2 \cdot y_0 = 1 \cdot 2 + 2 \cdot 1 + 3 \cdot 0 = 4$

$C_{xy}(3) = x_0 \cdot y_3 + x_1 \cdot y_2 + x_2 \cdot y_1 + x_3 \cdot y_0 = 1 \cdot 3 + 2 \cdot 2 + 3 \cdot 1 = 10$

$C_{xy}(4) = x_0 \cdot \cancel{y_4}^{0} + x_1 \cdot y_3 + x_2 \cdot y_2 + x_3 \cdot y_1 + \cancel{x_4 \cdot y_0}^{0} = 2 \cdot 3 + 3 \cdot 2 + 4 \cdot 1 = 16$

$C_{xy}(5) = x_0 \cdot 0 + x_1 \cdot 0 + x_2 \cdot y_3 + x_3 \cdot y_2 = 3 \cdot 3 + 4 \cdot 2 = 17$

$C_{xy}(6) = x_0 \cdot 0 + x_1 \cdot 0 + x_2 \cdot y_2 + x_3 \cdot y_3 = 4 \cdot 3 = 12$

$$\boxed{C_{xy} = [0, 1, 4, 10, 16, 17, 12]}$$

MMV!!!

- CORRELATION
  DSP USING MATLAB

$$V_{xy}(l) = \sum_{n=0}^{3} x(n)\, y(n-l)$$

$l = 0, 1, 2, 3$

$V_{xy}(0) = x(0) \cdot y(0) + x(1) \cdot y(1) + x(2) \cdot y(2) + x(3) \cdot y(3) = 2 + 6 + 12 = ⑳$

$\quad 3 \cdot 1 \quad 4 \cdot 2$

$V_{xy}(1) = x(0) \cdot y(-1) + x(1) \cdot y(0) + x(2) \cdot y(1) + x(3) \cdot y(2) = ⑪$

$V_{xy}(2) = x(0) \cdot y(-2) + x(1) \cdot y(-1) + x(2) \cdot y(0) + x(3) \cdot y(1) = ④$

$\quad 4 \cdot 1$

$V_{xy}(3) = x(3) \cdot y(0) = ⓪$

$$\boxed{R_{xy}(m) = \sum_{n=0}^{N-m-1} x_{n+m} y_n^*}$$

(DEFINICIJA KAKO VO MATLAB)

MMV
прочитај го
комнетарот!!!

$m = 0, \pm 1, \pm 2, \pm 3 ..$

$N-m-1 = 3-1=2$

$R_{xy}(0) = x_0 \cdot y_0 + x_1 \cdot y_1 + x_2 \cdot y_2 + x_3 y_3 = 2+6+12 = \textcircled{20}$

$R_{xy}(1) = x_1 \cdot y_0 + x_2 \cdot y_1 + x_3 \cdot y_2 = 3+8 = \textcircled{11}$

$R_{xy}(2) = x_2 \cdot y_0 + x_3 y_1 + \cancel{x_4} \cdot y_2 = \textcircled{4}$

$R_{xy}(3) = x_3 \cdot y_0 + \overset{\to 0}{\cancel{x_4}} y_1 \to 0 = \textcircled{0}$

TANJA MICEVA

DRAGANA JAKOVLEVSKA + NENAD

MO

$C_{xy}(k) = \sum_{n=-\infty}^{\infty} x(n) \cdot y(n-k) =$ ~~$\left| \begin{matrix} k-k=m \\ m=k+m \\ k\to\infty \\ m\to+\infty \end{matrix} \right| = \sum_{n=-\infty}^{+\infty} x(k-m) \cdot y($~~

$= \sum_{n=-\infty}^{\infty} x(n)\, y(-(k-n)) = \left| \begin{matrix} k-n=m \\ n=k-m \\ n\to\infty \\ m\to-\infty \end{matrix} \right| = \sum_{m=\infty}^{-\infty} x(k-m) \cdot y(-m)$

$= x(k) \circledast y(-k)$

$C_{xy}(k) = \mathrm{conv}(x(k), y(-k)) = \mathrm{conv}(x(k), \mathrm{fliplr}(y(k)))$.

$R_{xy}(-1) = x_{-1} \cdot y_0 + \overset{1}{x_0 \cdot y_1} + \overset{2\cdot 2=4}{x_1 y_2} + \overset{9}{x_2 y_3} = 14$

$R_{xy}(-2) = x_{-2} y_0 + x_{-1} y_1 + \overset{1\cdot 2}{x_0 y_2} + \overset{2\cdot 3}{x_1 y_3} = 8$

$R_{xy}(-3) = x_{-3} y_0 + x_{-2} y_1 + x_{-1} y_2 + x_0 y_3 = 3$

$$\boxed{R_{xy} = [\overset{-3}{3}, \overset{-2}{8}, \overset{-1}{14}, \overset{0}{20}, \overset{1}{11}, \overset{2}{4}, \overset{3}{0}]}.$$

OVA E POKAZ ZA MATEMATIČKATA PRETSTAVA NA KORELACIJATA

емпириски (xcorr)
1 2 3 4
0 1 2 3
0 1 2 3....

• ISTOTO VAŽI I ZA DEFINICIJATA (*) SAMO TREBA DA SE PUŠTI $l = 0, \pm 1, \pm 2, \pm 3$

$r_{xy}(-1) = x(0) \cdot y(1) + x(1) \cdot y(2) + x(2) \cdot y(3) = 1 + 4 + 9 = \textcircled{14}$

$r_{xy}(-2) = x(0) \cdot y(2) + x(1) \cdot y(3) + x(2) \cdot 0 = 2 + 6 + 0 = \textcircled{8}$

$r_{xy}(-3) = x(0) \cdot y(3) = 3 \cdot 1 = \textcircled{3}$

$$\boxed{r_{xy} = [\overset{-3}{3}, \overset{-2}{8}, \overset{-1}{14}, \overset{0}{20}, \overset{1}{11}, \overset{2}{4}, \overset{3}{0}]}$$

MMV

MATLAB IMPLEMENTED RAKERECEIVER\SUPPORTING\CROSSCORRE-LATION SIMPLEM

→ ZA NORMALNA SEKVENCA (PODOLGA I NERAJTEŠKA) MAKSIMUMOT NA KORELACIJATA TREBA DA SE DOBIE KADE SEKOJ ELEMENT OD X SE MNOŽI SO SAMIOT SEBE. ZA DA SE UVERIŠ TREBA DA ZA MATLAB SKRIPTATA SO X = [1,2,3,4,3,2,1] !!! MMV

⊙ DIFFERENCIAL PULSE CODE MODULATION (DPCM) – PROAKIS DIGCOM

(GO PROAPAM ZARADI LEVINGSTON-DURAN ALGOR.)

- PREDICT CURRENT SAMPLE FROM PREVIOUS „p" SAMPLES

$$\boxed{\hat{x}_n = \sum_{i=1}^{p} a_i x_{n-i}}$$

$\{a_i\}$ – PREDICTOR COEFFICIENTS

$\hat{x}_n$ – PREDICTED VALUE OF x_n

$$\varepsilon_p = E(e_n^2) = E\left[\left(x_n - \sum_{i=1}^{p} a_i x_{n-i}\right)^2\right] = E[x_n^2] - 2E\left[x_n \sum_{i=1}^{p} a_i x_{n-i}\right]$$

$$+ E\left[\left(\sum_{i=1}^{p} a_i x_{n-i}\right)^2\right] = E[x_n^2] - 2\sum_{i=1}^{p} a_i E[x_n \cdot x_{n-i}] +$$

$$+ \sum_{i=1}^{p}\sum_{j=1}^{p} a_i a_j E[x_{n-i} x_{n-j}]$$

- IF SOURCE IS (WIDE SENCE) STATIONARY (WSS)

[I.E. SREDNATA VREDNOST $\xi \neq f(t)$ NITU $R_\xi(\tau) \neq f(t)$]

$$\boxed{\varepsilon_p = \phi(0) - 2\sum_{i=1}^{p} a_i \cdot \phi(i) + \sum_{i=1}^{p}\sum_{j=1}^{p} a_i a_j \phi(i-j)}$$

- MINIMISATION OF ε_p IN RESPECT TO PREDICTOR COEFFICIENT a_i RESULTS IN SET OF LINEAR EQUATIONS:

$$\sum_{i=1}^{p} a_i \phi(i-j) = \phi(j) \qquad j = 1, 2, \dots p$$

MOZE DA GO PREMESTAS DO Σ !!!

PROAKIS 3.5.10 ANALIZA

- CALCULATION i.e. ESTIMATION OF AUTOCORRELATION FUNCTION:

$$\hat{\phi}(n) = \frac{1}{N}\sum_{i=1}^{N-n} x_i x_{i+n} \qquad n = 0, 1, 2, \dots p$$

MMV

NORMALIZACIJA KOJA SE ZAVRA ZARADI P(x, x+j) VEROJATNOSTITE VO DEFINICIJATA VO TEORIJA NA INF.

$$x = [\overset{1}{1}, \overset{2}{2}, \overset{3}{3}, \overset{4}{4}]$$

$$y = [0, 1, 2, 3]$$

$N = 4$

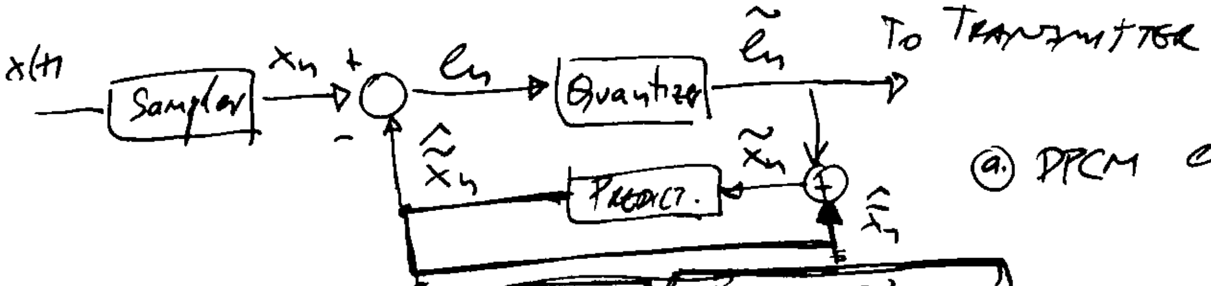
$$\boxed{\phi_{xy}(n) = \frac{1}{4}\sum_{i=1}^{4-n} x_i \cdot y_{i+n}}$$

$$\phi_{xy}(0) = \sum_{i=1}^{4} x_i \cdot y_i = x_1 \cdot y_1 + x_2 \cdot y_2 + x_3 \cdot y_3 + x_4 \cdot y_4 = 0+2+6+12 = 20$$

$$\phi_{xy}(1) = \sum_{i=1}^{4} x_i \cdot y_{i+1} = x_1 \cdot y_2 + x_2 \cdot y_3 + x_3 \cdot y_4 + 0 = 1+4+9 = 14$$

~~[illegible]~~

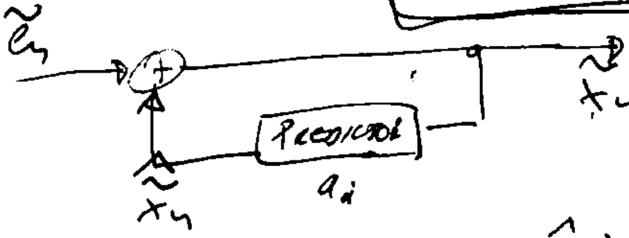
VNIMAVAJ OVA SE ODNESUVA NA AVTOCORRELACIJA!!!



ⓐ DPCM encoder

$e_n = x_n - \hat{\tilde{x}}_n$

$$\boxed{\hat{\tilde{x}}_n = \sum_{i=1}^{p} a_i \tilde{x}_{n-i}} \quad (**)$$



$\tilde{x}_n = \hat{\tilde{x}}_n + e_n$

$\boxed{q_n = \tilde{e}_n - e_n}$ ⇒ QUANTIZATION ERROR

$$q_n = \tilde{e}_n - e_n = \tilde{e}_n - (x_n - \hat{\tilde{x}}_n) = \tilde{x}_n - \cancel{\hat{\tilde{x}}_n} - x_n + \cancel{\hat{\tilde{x}}_n} = \tilde{x}_n - x_n$$

$\tilde{e}_n + \hat{\tilde{x}}_n = \tilde{x}_n \qquad \tilde{e}_n = \tilde{x}_n - \hat{\tilde{x}}_n$ ↗

$$\boxed{\tilde{x}_n = x_n + q_n}$$

- IMPROVMENT IN QUALITY OF (**):

$$\hat{\tilde{x}}_n = \sum_{i=1}^{p} a_i \tilde{x}_{n-1} + \sum_{i=1}^{q} b_i \tilde{e}_{n-i}$$

⊡ PROJECTS (DigCom): MODEL-BASED SOURCE CODING (GO IMA / VD DSP USING MATLAB)

LPC (Linear PREDICTIVE CODING)

$x_n,\ n = 0, 1, \ldots, N$ } SAMPLE SEQUENCE

$$\boxed{H(z) = \frac{G}{1 - \sum_{k=1}^{p} a_k z^{-k}}}$$

ALL-POLE (DISCRET TIME) FILTER

G – FILTER GAIN

- INPUT SEQUENCE OF THE FILTER
  $v_n \quad n = 0, 1, 2, \ldots$
- OUTPUT OF THE FILTER SATISFIES (SOURCE OUTPUT)

$$\boxed{x_n = \sum_{k=1}^{p} a_k x_{n-k} + G v_n} \qquad n = 0, 1, 2 \ldots$$

- THE ESTIMATE OF x_n

$$\hat{x}_n = \sum_{k=1}^{p} a_k x_{n-k} \qquad n > 0$$

$$e_n = x_n - \hat{x}_n = x_n - \sum_{k=1}^{p} a_k x_{n-k} = \underline{\underline{G \cdot v_n}}$$

- CHOSE FILTER COEFFICIENT IN ORDER TO MINIMIZE THE MSE

- SUPPOSE $\{v_n\}$ - WHITE-NOISE SEQUENCE

x_n - RANDOM SEQUENCE

$e_n = x_n - \tilde{x}_n$ — " —

$$\varepsilon_p = E(e_n^2) = E\left[\left(x_n - \sum_{k=1}^{p} a_k x_{n-k}\right)^2\right] =$$

$$= \phi(0) - 2\sum_{k=1}^{p} a_k \phi(k) + \sum_{k=1}^{p}\sum_{m=1}^{p} a_k a_m \phi(k-m)$$

$\phi(m)$ — AUTOCORRELATION OF x_n $\quad \underline{n = 0,1,\dots,N-1}$

$$E[(G\cdot v_n)^2] = G^2 E[v_n^2] = G^2 = E\left[\left(x_n - \sum_{k=1}^{p} a_k x_{n-k}\right)^2\right] = \varepsilon_p$$

$$\sum_{i=1}^{p} a_i \phi(i-j) = \phi(j)$$

OPTIMAL SOLUTION

$$\varepsilon_p = \phi(0) - 2\sum_{k=1}^{p} a_k \phi(k) + \sum_{k=1}^{p} a_k \phi(k) = \phi(0) - \sum_{k=1}^{p} a_k \phi(k)$$

$$a_{ii} = \frac{\phi(i) - \sum_{k=1}^{i-1} a_{i-1,k}\,\hat{\phi}(i-k)}{\varepsilon_{i-1}} \qquad i = 2,3,\dots p$$

$$a_{ik} = a_{i-1,k} - a_{ii}\,a_{i-1,i-k} \qquad 1 \le k \le i-1$$

$$\varepsilon_i = (1 - a_{ii}^2)\,\varepsilon_{i-1}$$

$$a_{11} = \frac{\hat{\phi}(1)}{\hat{\phi}(0)} \qquad \varepsilon_0 = \phi(0)$$

(!)

LEVINSON-DURBIN

$\boxed{i=2}$

$$a_{22} = \frac{\phi(2) - \sum_{k=1}^{2-1} a_{1k}\,\hat{\phi}(2-k)}{\varepsilon_1}$$

OD:
PROAKIS SOLUTION
MANUAL PROBLEM
3.38

$$a_{21} = a_{11} - a_{11}\cdot a_{22}$$

$$a_{11} = \frac{\hat{\phi}(1)}{\phi(0)} \qquad \varepsilon_0 = \hat{\phi}(0)$$

$$\varepsilon_1 = (1 - a_{11})^2\cdot\varepsilon_1$$

$$\varepsilon_2 = (1 - a_{22})^2\cdot\varepsilon_1$$