$\cos(x \cdot n)$ $\cos(\omega_c t)$ $\omega_c = 2\pi f = \frac{2\pi}{T_c}$ $T_c = 1.11\ \text{usec}$

$n = 0 - 2499$

$M = \text{lenght}(n) = 5000$

$\boxed{\omega_c = \omega_0}$

$D\left[0 : \cancel{1}_1 \cancel{4} \cdot 10^{-6}\right]$

$M = 5000$ $Dt = \frac{T_c}{4} = 2{,}77 \cdot 10^{-10} = \cancel{0}{,}28\ \text{us}$

$t = [0 : M] \cdot Dt$ $\cos\left(\frac{2\pi}{T_c} \cdot [0 : M] Dt\right) = \cos\left(\frac{2\pi}{\frac{4Dt}{2}}[0 : N]\right) = \cos(0{,}5\pi n)$

$$f_s = \frac{1}{Dt} = \frac{1}{T/4} = \frac{4}{T_c} = 4 \cdot f_c$$

$s(t) = I(t)\cos(\omega_c t) - Q(t)\sin(\omega_c t)$

$$s(t) = \cos(\omega_c t)\underbrace{\sum_{i=1}^{N} a_i \cos(\omega_0 t + \phi_i)}_{I(t)} - \sin(\omega_c t)\underbrace{\sum_{i=1}^{N} a_i \sin(\omega_0 t + \phi_i)}_{Q(t)}$$

$\boxed{s(t) = r(t)\cos(\omega_c t + \varphi)}$ $r(t) = \sqrt{I(t)^2 + Q^2(t)}$ $\varphi = \text{arctg}\frac{Q(t)}{I(t)}$

$I(t) = r(t) \cdot \cos\varphi$ $Q(t) = r(t) \cdot \sin\varphi$

$$\begin{bmatrix} a_1 a_1 \dots a_1 \\ a_2 a_2 \dots a_2 \\ a_N a_N \dots a_N \end{bmatrix}^T \overbrace{\begin{bmatrix} x_1 & x_1 & \dots & x_1 \\ x_2 & x_2 & & x_2 \\ \vdots \\ x_M & x_M & & x_M \end{bmatrix}}^{N}{}^T \cdot \left.\overbrace{\begin{bmatrix} \varphi_1 & \varphi_1 & \dots & \varphi_1 \\ \varphi_2 & \varphi_2 & \dots & \varphi_2 \\ \dots \end{bmatrix}}^{M}\right\} N \quad \varphi$$

$$M\left\{\begin{bmatrix} a_1 & a_2 & \dots & a_N \\ a_1 & a_2 & \dots & a_N \\ \dots \\ a_1 & a_2 & \dots & a_N \end{bmatrix}\right. \left.\underbrace{\begin{bmatrix} x_1 & x_2 & \dots & x_M \\ x_1 & x_2 & \dots & x_M \\ \dots \\ x_1 & x_2 & \dots & x_M \end{bmatrix}}_{M}\right\} N \cdot \varphi$$

$e^{jx} = \cos x + j\sin x$

$e^{-jx} = \cos x - j\sin x$

$$M\left\{\begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}\right. [a_1, a_2 \dots a_N]$$

$$\cos(\alpha) \cdot e^{j\beta} = \frac{1}{2}\left(e^{j\alpha} + e^{-j\alpha}\right) e^{j\beta} = \frac{1}{2}\left(e^{j\alpha+\beta} + e^{-j\alpha+\beta}\right) = \cos(\alpha + \beta)$$

$$N\left\{\begin{bmatrix} x_1 & x_2 & \dots & x_M \\ x_1 & x_2 & \dots & x_M \\ x_1 & x_2 & \dots & x_M \end{bmatrix}\right. \cdot \left.\begin{bmatrix} \varphi_1 & \varphi_2 & \dots & \varphi_N \\ \varphi_1 & \varphi_2 & \dots & \varphi_N \\ \varphi_1 & \varphi_2 & \dots & \varphi_N \end{bmatrix}\right\} M$$

$z = x + jy$

$z = \sqrt{x^2 + y^2} \cdot e^{j\,\text{arctg}\frac{y}{x}}$

$$\begin{bmatrix} \varphi_1 \\ \varphi_2 \\ \vdots \\ \varphi_N \end{bmatrix} [x_1\ x_2 \dots x_M] = \begin{bmatrix} x_1' & x_2' & \dots & x_M' \\ x_1'' & x_2'' & \dots & x_M'' \\ x_1^{(N)} & x_2^{(N)} & \dots & x_M^{(N)} \end{bmatrix}_{N \times M}$$

$n \cdot Dt = 1250 \cdot T = 1250 \cdot 1.11e{-9} = \underline{1,3875e{-6}}$

$5000\, Dt = 1{,}3875 \cdot 10^{-6}$ $\quad n \cdot \frac{T}{10} = 1250 \cdot T$ $\quad \boxed{n = 12500}$

$\cancel{f}d = \frac{v}{\lambda} \cdot \cos(\alpha) = \frac{v}{c} \cdot f_c \cos(\alpha) \Rightarrow w_d = 2\pi f_d = 2\pi f_c \cdot \frac{v}{c} \cdot \cos(\alpha)$

$s(t) = I(t) \cdot \cos(w_c t) - Q(t) \sin(w_c t)$

$s(t) \cdot \cos(w_c t) = I(t) \cdot \cos(w_c t) \cos(w_c t) - Q(t) \sin(w_c t) \cdot \cos(w_c t)$

$\cos\alpha \cdot \cos\alpha = \frac{1}{2}[\cos(2\alpha) + \cos(0)] = \frac{1}{2}[1 + \cos 2\alpha]$

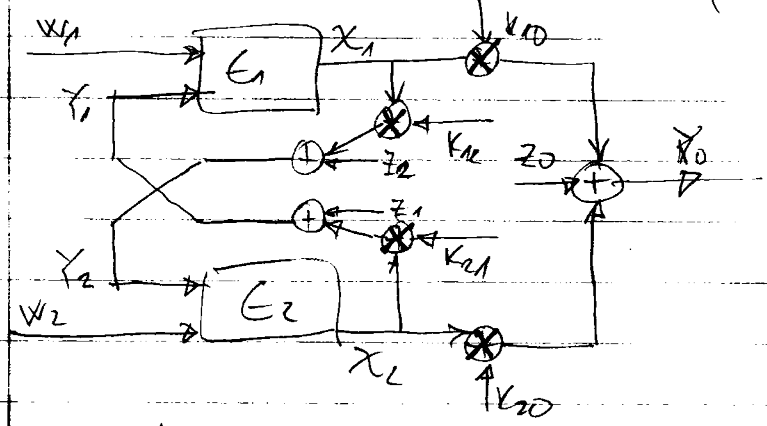
$\sin(\alpha) \cdot \sin(\alpha) = \frac{1}{2}[\cos(\alpha - \alpha) - \cos(2\alpha)] = \frac{1}{2}[1 - \cos 2\alpha]$

$\cos(2\alpha) = \cos^2\alpha - \sin^2\alpha \quad \to \frac{1}{2}[1 - \cos^2\alpha + \sin^2\alpha] = \sin^2\alpha$ ok

$s(t) \cdot \cos(w_c t) = \frac{1}{2} I(t) [1 + \cos 2w_c t] - Q(t) \frac{1}{2} [1 - \cos(2w_c t)] =$

$= \frac{1}{2}[I(t) - Q(t)] + \frac{1}{2}[I(t) + Q(t)] \cdot \cos(2w_c t)$ $\quad \boxed{\approx}$

• User Cooperation Diversity (TCOM)



Y_0 – Baseband signal received in BS

Y_1 – Baseband signal received in user 1

Y_2 – Baseband sig received in user 2

Z_i – Additive channel noise $i = 0,1,2$

• W_i $i = 1,2$ information to be send by the users

$Y_0(t) = K_{10} X_1(t) + K_{20} X_2(t) + Z_0(t)$ $\quad \boxed{K_{ij} \text{ – Fading Coefficients}}$

$Y_1(t) = K_{21} X_2(t) + Z_1(t)$

$Y_2(t) = K_{12} X_1(t) + Z_2(t)$

– ergodic $\quad \overline{f(t)} = \overline{f_\xi(t)}_{2\pi/T}$ $\quad \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} f(t)\, dt = \underline{\int_{-\infty}^{\infty} x f_\xi(t)\, dt}$

$\lim_{T \to \infty} \frac{1}{2\pi} \int_{-2\pi/T} F(jw)\, dw = \int_{-\infty}^{\infty} x f_\xi(t)\, dt$!?

• An Achievable Rate Region

$$\boxed{\begin{aligned} Y_0 &= K_{10} X_1 + K_{20} X_2 + Z_0 \\ Y_1 &= K_{21} X_2 + Z_1 \\ Y_2 &= K_{12} X_1 + Z_2 \end{aligned}}$$

$Z_0 \sim N(0, \Theta_0)$ $\quad Z_1 \sim N(0, \Theta_1)$

$Z_2 \sim N(0, \Theta_2)$ $\quad \Theta_1 = \Theta_2$

• KANALEN SISTEM BLOKOVI SO DOLZINA „n"

$j = 1, 2, \ldots n$

- SIGNALOT NA KORISNIKOT 1 VO MOMENTOT „j" e:

$$X_1(W_1, Y_1(j-1), Y_1(j-2), \ldots, Y_1(1))$$ W_1 PORAKA USER 1 → BS

$$X_2(W_2, Y_2(j-1), Y_2(j-2), \ldots, Y_2(1))$$

$$W_1 = \underbrace{W_{10}}_{BS} + \underbrace{W_{12}}_{user\,2}$$

COOPERATIVE INFORMATION TO BS

$$X_1 = X_{10} + X_{12} + U_1 \qquad P_1 = P_{10} + P_{12} + P_{U_1}$$

$$R_1 = R_{10} + R_{12} \qquad R_2 = R_{20} + R_{21}$$

$$R_{12} < E\left\{C\left(\frac{K_{12}^2 P_{12}}{K_{12}^2 P_{10} + \Theta_1}\right)\right\} \qquad R_{21} < E\left\{C\left(\frac{K_{21}^2 P_{21}}{K_{21}^2 P_{20} + \Theta_2}\right)\right\}$$

$$R_{10} < E\left\{C\left(\frac{K_{10}^2 P_{10}}{\Theta_0}\right)\right\} \qquad R_{20} < E\left\{C\left(\frac{K_{20}^2 P_{20}}{\Theta_0}\right)\right\}$$

$$R_{10} + R_{20} < E\left\{C\left(\frac{K_{10}^2 P_{10} + K_{20}^2 P_{20}}{\Theta_0}\right)\right\}$$

$$R_{10} + R_{20} + R_{12} + R_{21} < E\left\{C\left(\frac{K_{10}^2 P_1 + K_{20}^2 P_2 + 2K_{10}K_{20}\sqrt{P_{U_1} P_{U_2}}}{\Theta_0}\right)\right\}$$

$$\boxed{C(x) = \frac{1}{2}\log(1+x)}$$ → CAPACITY OF ADDITIVE GAUSSIAN NOISE CHANNEL
x - SNR

• PROBABILITY OF OUTAGE

P_{RX} - RECEIVED BY BS
P_{TX} - TRANSMITTED BY MS

$$P_{out} = P_r(R < r)$$ (r) - SUSTAINABILITY RATE

$$P_{RX} = P_{TX} - PL(d) \qquad PL(d) = B_1 + B_2 \log(d)$$

$$\log(d) = \frac{1}{B_2}(PL(d) - B_1) = \frac{1}{B_2}(P_{TX} - P_{RX} - B_1)$$

$$\log[d_{max}(P_{RX})] = \frac{1}{B_2}(P_{TX}^{max} - P_{RX} - B_1)$$

$$10\log\alpha = P_{RX}^{(1)} - P_{RX}^{(2)}$$

$$\log d_{max}^{(1)} - \log d_{max}^{(2)} = -\frac{1}{B_2}P_{RX}^{(1)} + \frac{1}{B_2}P_{RX}^{(2)} = \frac{1}{B_2}(P_{RX}^{(2)} - P_{RX}^{(1)}) = -\frac{10}{B_2}\log\alpha$$

$$\boxed{\frac{d_{max}^{(1)}}{d_{max}^{(2)}} = \alpha^{-\frac{10}{B_2}}}$$

$$B_1 = 17.3 \quad B_2 = 37.8 \quad f_c = 900MHz \quad h_{TE} = 50m$$

$$h_{RE} = 1m \; ; \; 10\log(G_T G_R) = 6dB$$

$$\boxed{P_R = \frac{P_T G_T G_R}{(4\pi)^2 d^2 \lambda^2}}$$

$$\boxed{\frac{d_{max}^{(1)}}{d_{max}^{(2)}} = \alpha^{-1/3{,}78}}$$

• KAPACITET OF NON-COOPERATIVE SCHEME

$$R_{10} + R_{20} < E\left[\frac{1}{2}\log\left(1 + (K_{10}^2 + K_{20}^2)\frac{P}{\Theta_0}\right)\right] = E\left[\frac{1}{2}\log(1 + A \cdot P)\right]$$

$$E[K_{i0}] \in [0, 100] \quad i = 1, 2$$

$$\boxed{R_{sum} \doteq \frac{1}{2}\log(1 + \mu A P)}$$

$R^c_{sum}(P') = \mu R^n_{sum}(P)$; $\mu \geq 1$, $P' = ?$ T.Š. $R^n_{sum}(P') = \mu R^n_{sum}(P)$

$\frac{1}{2}\log(1+\mu A P') = \mu \frac{1}{2}\log(1+\mu A P)$ $\quad 1+\mu A P' = (1+\mu A P)^\mu$

$\mu A P' = (1+\mu A P)^\mu - 1$ $\quad P' = \frac{(1+\mu A P)^\mu - 1}{\mu A}$ $\quad \frac{P'}{P} = \frac{(1+\mu A P)^\mu - 1}{\mu A P}$

$\frac{d^{(1)}_{max}}{d^{(2)}_{max}} = \left(\frac{(1+\mu A P)^\mu - 1}{\mu A P}\right)^{1/3{,}38}$ $\quad \boxed{\alpha = \frac{P}{P'}}$

$\Theta_0 = 1$; $r = 2$; $E[K_{10}] = E[K_{20}] = 0{,}63 \Rightarrow E[K_{10}^2] = E[K_{20}^2] = 0.5056$

$\bar{r} = \sigma\sqrt{\frac{\pi}{2}} = 1.25\sigma$ $\quad \sigma = \frac{\bar{r}}{1.25} = 0.504$

$E(r^2) = 2\sigma^2 = 2\frac{\bar{r}^2}{\frac{\pi}{2}} = \frac{4}{\pi}\cdot\bar{r}^2 = \frac{4}{\pi}(0.63)^2 = 0{,}505$

$A \cdot P = ?$ $\quad A = E(K_{10}^2 + K_{20}^2)/\Theta_0 = [E(K_{10}^2) + E(K_{20}^2)]/\Theta_0 = \frac{0{,}505+0{,}505}{1}$

$A = 1{,}01$ $\quad \boxed{A\cdot P = 2{,}02}$

$\mu = 0.8$ $\quad \mu \cdot A \cdot P = 1.616$

$\boxed{\frac{d^{(1)}_{max}}{d^{(2)}_{max}} = \left(\frac{2{,}62^\mu - 1}{1.62}\right)^{1/3{,}38}}$ $\quad \boxed{\text{Increase of coverage area} = \left(\frac{2.62^\mu - 1}{1.62}\right)^{\frac{2}{3.38}}}$

$p_{\xi\eta}(x,y) = p_\eta(y)\, p_{\xi\eta}(x/y)$ $\quad I(X,Y) = H(Y) - H(Y/X)$ – irrelevantna

$H(Y/X) = \overline{H(Y/x_i)} = \int_{-\infty}^{\infty} H(Y/x_i)\, p(x_i)\, dx\, dy = \int_{-\infty}^{\infty} p(x)\, dx \int_{-\infty}^{\infty} p(y/x)\, ld\frac{1}{p(y/x)}\, dy$

DISCRETEN KANAL: $H(Y/X) = \overline{H(Y/x_i)} = \sum_{i=1}^{m} P(x_i)\cdot H(Y/x_i) =$

$= \sum_{i=1}^{m} P(x_i) \sum_{j=1}^{r} P(y_j/x_i)\, ld\frac{1}{P(y_j/x_i)} = \sum_{i=1}^{m}\sum_{j=1}^{r} \underbrace{P(x_i)P(y_j/x_i)}_{P(x_i,y_j)}\, ld\frac{1}{P(y_j/x_i)}$

$H(X/y_j) = \sum_{i=1}^{m} P(x_i/y_j)\, ld\frac{1}{P(x_i/y_j)}$ $\quad j = 1,2,\dots r$

$H(X/Y) = \overline{H(X/y_j)} = \sum_{j=1}^{r} H(X/y_j)\, P(y_j) = \sum_{j=1}^{r} P(y_j) \sum_{i=1}^{m} P(x_i/y_j)\, ld(P(x_i/y_j))^{-1}$

$H(X,Y) = \sum_{i=1}^{m}\sum_{j=1}^{r} P(y_j)P(x_i/y_j)\, ld(P(x_i/y_j))^{-1} = \sum_{i=1}^{m}\sum_{j=1}^{r} P(x_i,y_j)\, ld\frac{1}{P(x_i/y_j)}$

$\boxed{C = \sigma(X,Y) \cdot \max_{p(x)}[I(X,Y)]}$ $\quad C = 2f_g \cdot \frac{1}{2}\, ld\left(1 + \frac{\sigma_X^2}{\sigma_N^2}\right)$

$$I(x,y) = H(y) - H(y/x) = H(y) - H(n)$$

$$H(n) = \int_{-\infty}^{\infty} p(n)\, ld \frac{1}{p(n)}\, dn$$

$$\max\left[I(x,y)\right] = \max_{p(x)}\left[H(y)\right] - H(n)$$

$$\max_{p(x)}\left[I(x,y)\right] = ld\sqrt{2\pi e y^2} - H(n)$$

$$\boxed{C = \frac{v(x,y)}{2} ld\left(1 + \frac{\sigma_x^2}{\sigma_N^2}\right)}$$

$$\Phi(x,y) = v(x,y)\; I(x,y)$$ — TRANSINFORMACIONI FLUKS.

$$\boxed{C_{BSC} = v(x,y)\left(1 - H(p)\right)}$$

$$H(p) = 1 \Rightarrow \boxed{C_{BSC} = 0}$$

$$H(p) = \sum_{i=1}^{2} P(s_i)\, ld \frac{1}{P(s_i)}$$

s_i	$P(s_i)$
1	p
0	$1-p$

$$H(p) = p\, ld\, p + (1-p)\, ld\, p$$

$$H'(p) = 0 \Rightarrow p = 0{,}5 \Rightarrow \underline{H(p) = 1}$$

- CDMA IMPLEMENTATION

COHERENCE TIME OF THE CHANNEL $= L$ $\frac{\text{BIT}}{\text{USER}}$

e.G $L = 3$

$b_j^{(i)}$ – user j's ith bit

$c_j(t)$ – –"– SPREADING CODE

$a_j = \sqrt{\frac{P_j}{T_S}}$; P_j – user j's POWER

T_S – SYMBOL PERIOD

$$X_1(t) = a_1 b_1^{(1)} c_1(t),\; a_1 b_1^{(2)} c_1(t),\; a_1 b_1^{(3)} c_1(t)$$

$$X_2(t) = \underbrace{a_2 b_2^{(1)} c_2(t)}_{\text{PERIOD } 1},\; \underbrace{a_2 b_2^{(2)} c_2(t)}_{\text{PERIOD } 2},\; \underbrace{a_2 b_2^{(3)} c_2(t)}_{\text{PERIOD } 3}$$

- $Y_0 = K_{10} x_1 + K_{20} x_2 + Z_0$; $Y_1 = K_{21} X_2 + Z_1$; $Y_2 = K_{12} X_1 + Z_2$

$$x_1(t) = a_{11} b_1^{(1)} c_1(t),\; a_{12} b_1^{(2)} c_1(t),\; a_{13} b_1^{(2)} c_1(t) + a_{14} \hat{b}_2^{(2)} c_2(t)$$

$$x_2(t) = a_{21} b_2^{(1)} c_2(t),\; a_{22} b_2^{(2)} c_2(t),\; a_{23} \hat{b}_1^{(2)} c_1(t) + a_{24} b_2^{(2)} c_2(t)$$

$\hat{b}_j^{(i)}$ → PARTNER'S ESTIMATE OF USER j's i-TH BIT

$$\Phi(x) = \frac{2}{\sqrt{\pi}} \int_0^x$$

- THROUGHPUT OF BPSK

$$\boxed{P(\varepsilon) = \operatorname{erfc} \frac{\rho}{\sqrt{2}}}$$

ZA DEFINICIJA $\operatorname{erfc}(t) = \frac{1}{\sqrt{\pi}} \int_x^{\infty} e^{-t^2} dt$

ZA DEFINICIJA: $\operatorname{erfc}(t) = 1 - \operatorname{erf}(t) = 1 - \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt =$

$$= \frac{2}{\sqrt{\pi}} \int_0^{\infty} e^{-t^2} dt - \frac{2}{\sqrt{\pi}} \int_0^{x} e^{-t^2} dt = \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-t^2} dt$$

$$P(\varepsilon) = \frac{1}{2}\left[1 - \Phi(x)\right] = \frac{1}{2}\left[1 - \operatorname{erf}(x)\right] = \frac{1}{2}\operatorname{erfc}(x) = \underline{\frac{1}{2}\operatorname{erfc}\frac{\rho}{\sqrt{2}} = \frac{1}{2}\operatorname{erfc}\frac{A}{\sqrt{2}\sigma_N}}$$

$$P(\varepsilon) = \frac{1}{2}\operatorname{erfc}\sqrt{\frac{SN}{2}}$$

- THROUGHPUT $\eta = (1-\gamma) \cdot C_{BSC}\left(Q\left(\sqrt{\frac{SNR_0}{1-\gamma}}\right)\right)$

$C_{BSC}(p)$ p – CROSSOVER PROBABILITY = PROBABILITY OF ERROR

$$Q\left(\frac{SNR_0}{1-\gamma}\right) = \operatorname{erfc}\left(\sqrt{\frac{SNR_0}{1-\gamma}}\right)$$

$2L_C \Rightarrow$ PERIODS FOR COOPERATION

$L-2L_C \Rightarrow$ PERIODS FOR USEFULL INFORMATION (NON-COOP INFORM)

$L_C = 0 \div \frac{L}{2}$ $\quad$ $\boxed{L_C = \frac{L}{2}} \Rightarrow$ THE USERS ARE FULLY COOPERATING

$L_n = L - 2L_C$

$$x_1(t) = \begin{cases} a_{11} b_1^{(i)} c_1(t) & i = 1,2,\dots L_n \\ a_{12} b_1^{(L_n+1+i)/2} c_1(t) & i = L_n+1, L_n+3, \dots L_1 - 1 \\ a_{13} b_1^{(L_n+i)/2} c_1(t) + a_{14} \hat{b}_2^{(L_n+i)/2} c_2(t) & i = L_n+2, L_n+4, \dots, L \end{cases}$$

$$x_2(t) = \begin{cases} a_{21} b_2^{(i)} c_2(t) & i = 1,2,\dots L_n \\ a_{22} b_2^{(L_n+1+i)/2} c_2(t) & i = L_n+1, L_n+3, \dots L_n - 1 \\ a_{23} \hat{b}_1^{(L_n+i)/2} c_1(t) + a_{24} b_2^{(L_n+1)/2} c_2(t) & i = L_n+2, L_n+4, \dots, L \end{cases}$$

- POWER CONSTRAINTS

$$P_1 = \frac{1}{L}\left(L_n a_{11}^2 + L_C (a_{12}^2 + a_{13}^2 + a_{14}^2)\right)$$

$$P_2 = \frac{1}{L}\left(L_n a_{21}^2 + L_C (a_{22}^2 + a_{23}^2 + a_{24}^2)\right)$$

$L_C = \frac{L}{2}$ $\quad$ $L_n = L - 2L_C = L - L = 0$

- PROOF OF THEOREM 1 (B BLOCKS WITH LENGTH n)

$$X_{10} = \sqrt{P_{10}}\, \tilde{X}_{10}\left(W_{10}(i), W_{12}(i-1), W_{21}(i-1)\right) \qquad i\text{ - BLOCK}$$

$$X_{12} = \sqrt{P_{12}}\, \tilde{X}_{12}\left(X_{12}(i), W_{12}(i-1), W_{21}(i-1)\right)$$

$$U_1 = \sqrt{P_{U1}}\, \tilde{U}\left(W_{12}(i-1), W_{21}(i-1)\right)$$

$(W_{12}(0), W_{21}(0)) = (0,0)$; $\quad Y_2 = K_{12}X_1 + Z_2$

$X_1 = X_{10} + X_{12} + U_1 \qquad X_{10} = f(W_{10}(1))$

$U_1 \sim (W_{12}(0), W_{21}(0))$

- RECONSTRUCTION OF $W_{12}(1)$ UNDER CONDITION

$$R_{12} < C\left(\frac{K_{12}^2 P_{12}}{K_{12}^2 P_{10} + \Theta_1}\right) \qquad C = \frac{1}{2}\operatorname{ld}\left(1 + \frac{\sigma_x^2}{\sigma_N^2}\right) = \frac{1}{2}\operatorname{ld}\left(1 + \frac{S}{N}\right)$$

$Y_0 = K_{10}X_1 + K_{20}X_2 + Z_0 \qquad X_1 = X_{10} + X_{12} + U_1$

$Y_1 = K_{21}X_2 + Z_1 \qquad X_2 = X_{20} + X_{21} + U_2$

$Y_2 = K_{12}X_1 + Z_2$

L_C - DEGREE OF COOPERATION

$\{a_{ij}\}$ - POWER ALLOCATION SCHEME

K_{ij} - FADING COEFFICIENTS

(1) ERROR RATE OF NONCOOPERATIVE PERIODS

$X_1 = a_{11} b_1 c_1 \qquad Y_0 = K_{10}X_1 + K_{20}X_2 + Z_0$

ESTIMATE OF FIRST USER BIT

$$\hat{b}_1 = \operatorname{SIGN}\left(\frac{1}{N_C} c_1^T Y_0\right) = \operatorname{sign}\left(K_{10} a_{11} b_1 + u_0\right)$$

$u_0 \sim N(0, \sigma_0^2/N_C) \qquad \sigma_0^2 = \frac{N_0}{2T_C} \qquad T_C$ - CHIP PERIOD

$N_0/2$ - SPECTRAL HIGHT OF $Z_0(t)$

N_C - CDMA SPREAD GAIN

$$\boxed{P_{e1} = Q\left(K_{10} a_{11} \frac{\sqrt{N_C}}{\sigma_0}\right)}$$

$x_1 = a_{12} b_1 c_1$; $Y_{①} = k_{12} x_1 + z_{①}$; $Y_0^{odd} = k_{10} x_1 + k_{20} x_2 + z_0^{odd}$

$\hat{b}_1 = \mathrm{sign}\left(\frac{1}{N_c} c_1^T Y_1\right)$ $\quad P_{e12} = Q\left(k_{12} a_{12} \frac{\sqrt{N_c}}{\sigma_1}\right)$ $\quad \sigma_1^2 = \frac{N_1}{2T_c}$

$\boxed{y_{odd} = \frac{1}{N_c} c_1^T Y_0^{odd}}$ $\quad \boxed{\begin{array}{l} x_1 = a_{13} \hat{b}_1 c_1(t) + a_{14} b_2 c_2(t) \\ x_2 = a_{23} \hat{b}_1 c_1(t) + a_{24} b_2 c_2(t) \end{array}}$

$Y_0^{even} = k_{10} x_1 + k_{20} x_2 + z_0^{even}$ $\quad y_{even} = \frac{1}{N_c} c_1^T Y_0^{even}$

$y_{odd} = k_{10} a_{12} b_1 + n_{odd}$

$y_{even} = k_{10} a_{13} \hat{b}_1 + k_{20} a_{23} \hat{b}_1 + n_{even}$

$$\left.(1-P_{e12}) A^{-1} e^{v_1^T y} + P_{e12} A e^{v_2^T y} \gtrless (1-P_{e12}) A^{-1} e^{-v_1^T y} + P_{e12} A e^{-v_2^T y}\right\} \text{DETECTOR}$$

$y = [y_{odd}\ y_{even}]^T \sqrt{\frac{N_c}{\sigma_0}}$ $\quad v_1 = [k_{10} a_{12} (k_{10} a_{13} + k_{20} a_{23})]^T \frac{\sqrt{N_c}}{\sigma_0}$

$v_2 = [k_{10} a_{12}\ (k_{10} a_{13} - k_{20} a_{23})]^T \frac{\sqrt{N_c}}{\sigma_0}$; $A = \exp\left(k_{10} k_{20} a_{13} a_{23} \frac{N_c}{\sigma_0^2}\right)$

$\boxed{\hat{b}_1 = \mathrm{sign}\left([k_{10} a_{12}\ \lambda (k_{10} a_{13} + k_{20} a_{23})]^T y\right)} \Rightarrow$ SUBOPTIMUM DETECTOR

$\lambda \in [0,1]$ – BS's CONFIDENCE

$P_{e2} = (1-P_{e12})\, Q\left(\frac{v_\lambda^T v_1}{\sqrt{v_\lambda^T v_\lambda}}\right) + P_{e12}\, Q\left(\frac{v_\lambda^T v_2}{\sqrt{v_\lambda^T v_\lambda}}\right)$ $\quad v_\lambda = [k_{10} a_{12}\ \lambda (k_{10} a_{13} + k_{20} a_{23})]^T$

· THROUGHPUT $\quad \eta_1(L_c, \{a_{ij}\}, \{k_{ij}\}) = \frac{1}{2}\left[L_n(1 - H(P_{e1})) + L_c(1 - H(P_{e2}))\right]$

$\eta^c_{sum}(P) = (1+\delta)\, \eta^n_{sum}(P)$ $\quad \delta \geq 0$ $\quad \eta^n_{sum}(P') = (1+\delta)\, \eta^n_{sum}(P)$ $\quad P' = ?$

Increase of coverage area $\doteq \left(\frac{P}{P'}\right)^{2/3.58}$

$\eta^n_{sum}(P) = \eta^n_1(P) + \eta^n_2(P) = 2\left(1 - \int_0^\infty H(Q(\alpha k))\, p_K(k)\, dk\right)$ $\quad p_K(k)$ – RAYLEIGH DISTRIBUTION WITH SAME MEAN AS k_{10} & k_{20}

$\alpha = \sqrt{\frac{P N_c}{\sigma_0^2}}$; $\boxed{\text{Increase in area coverage} \approx \mu\delta}$

$v = (1,-1)$ → chipping code $\quad$ vector∅ – vector

sender∅ $\quad$ code $(1,-1)$ $\quad$ data $(1,0,1,1)$

sender1 $\quad$ code $(1,1)$ $\quad$ data $(0,0,1,1)$ $\quad v\ -v\ v\ v$

encode∅ = vector∅ · data∅ = $(1,-1)\cdot(1,0,1,1) = ((1,-1),(-1,1),(1,-1),(1,-1))$

encode1 = vector1 · data1 = $(1,1)(0,0,1,1) = ((-1,-1),(-1,-1),(1,1),(1,1))$

→ orthogonal $\quad$ code0 · code1 $= 1\cdot 1 + 1\cdot(-1) = \emptyset$

encode∅ + encode1 = $(1,-1,1,1) + (-1,-1,1,1) = 0,-2,2,2$

$((0,-2),(-2,0),(2,0),(2,0))$ = PATTERN

PATTERN · VECTOR∅ = $((0,-2),(-2,0),(2,0),(2,0))(+1,-1) = ((0,+2),(-2,0),(2,0),(2,0))$

data = $(2,-2,2,2) = (1,\emptyset,1,1)$

N_C – TOTAL NUMBER OF SPREADING CODES
M_0 – ALL OTHERS SPREADING CODES USED BY OTHER USERS

$(1,1,1,1)\cdot(1,-1,-1,1) = 1-1-1+1 = 0$

$(1,1,-1,-1)\,(1,-1,-1,1) = 1-1+1-1 = 0$

f_s

$x_1(t) = a_{11}\,b_{11}\,c_{11}(t) + a_{12}\,b_{12}\,c_{12}(t) + a_{13}\,b_{13}\,c_{13}(t)$

$x_2(t) = a_{21}\,b_{21}\,c_{21}(t) + a_{22}\,b_{22}\,c_{22}(t) + a_{23}\,b_{23}\,c_{23}(t)$

b_{ji} bit, user; c_{ji} user, i-th code

$$\boxed{C = \frac{1}{\frac{1}{C_1}+\frac{1}{C_2}+\dots+\frac{1}{C_n}} = \frac{1}{\sum_{n=1}^{N}\frac{1}{C_n}} = \left(\sum_{n=1}^{N}\frac{1}{C_n}\right)^{-1}}$$

- **−1,59 dB** shannon limmit for minimal $\frac{E_b}{N_0}$

$\frac{1}{2}\,ld\left(1+\frac{E_b}{N_0}\right)$ $\quad \log\frac{E_b}{N_0} = -\frac{1{,}59}{10} \quad \frac{E_b}{N_0} = 10^{-0{,}159} = 0{,}693$

$$\frac{1}{2}\,ld\left(1+\frac{E_b}{N_0}\right) = \frac{0{,}76}{2} = 0{,}38$$

$v_s = \frac{1}{T_s} = f_s = 2f_g$

$$C = v_s\,\frac{1}{2}\,ld\left(1+\frac{E_b}{N_0}\right) = f_g\cdot ld\left(1+\frac{E_b}{N_0}\right) = 0{,}76\,f_g$$

$$\boxed{R_{max} = \frac{C}{f_g} = 0{,}76\ [\text{BIT/s/Hz}]} \rightarrow \text{NORMALIZED RATE}$$

$C = B\cdot ld\left(1+\frac{S}{N}\right) \qquad \frac{C}{B} = ld\left(1+\frac{S}{N}\right) \qquad \frac{S}{N} = 2^{\frac{C}{B}}-1$

$$\boxed{\underset{[B/s/Hz]}{\rho} = \frac{C}{B} = ld(1+SNR)} \qquad \rho \le ld(1+SNR)$$

$1+SNR \ge 2^{\rho} \qquad \boxed{SNR > 2^{\rho}-1}$

$SNR_{NORM} = \frac{SNR}{2^{\rho}-1}$ $\qquad$ SHANNON LIMMIT: $\boxed{SNR_{NORM} > 1}$ 0db

$\frac{E_b}{N_0} = \frac{E_s}{\rho N_0} = \frac{SNR}{\rho}$ $\qquad \boxed{\frac{E_b}{N_0} > \frac{2^{\rho}-1}{\rho}} \qquad \boxed{E_s = \rho\cdot E_b}$

$E_b = \frac{E_s}{\rho} \qquad \rho = 2 \qquad \frac{E_b}{N_0} = \frac{2^2-1}{2} = \frac{3}{2} = 1.76\,dB$

$\rho = 1 \qquad \frac{E_b}{N_0} = \frac{1}{1}\ (0\,db) \qquad \rho = 0$

$\cancel{\lim_{\rho\to 0}\frac{2^{\rho}-1}{\rho} = \frac{\rho\,2^{\rho-1}}{1}}$

$\cancel{\lim_{\rho\to 0}\frac{\rho}{2^{\rho}} = \frac{\rho}{2}}$

$(x^n)' = n x^{n-1}$ $\quad (2^x)' = (e^{x\ln 2})' = e^{x\ln 2} \cdot \ln(2)$

$$\lim_{\rho \to 0} \frac{(2^\rho - 1)'}{(\rho)'} = \frac{e^{\rho \ln 2} \ln(2)}{1}\Bigg|_{\rho \to 0} = \frac{1 \cdot \ln 2}{1} = \ln 2 = \cancel{0{,}693}$$

$2^x = e^{x \ln 2} \quad \ln 2^x = \ln e^{x\ln 2} \quad \ln 2^x = x \ln 2$

$\ln 2^x = \ln 2^x \quad 10\log \frac{\epsilon_B}{N_0} = 10 \log(\ln 2) = -1{,}59\ dB$

Primer: a) $C = 40\ kbps \quad B = 1\ MHz \quad S/N = ?$

$40 = 1000\ ld\left(1 + \frac{S}{N}\right) \quad 2^{\frac{4}{100}} = 1 + S/N \quad S/N = 1{,}028 - 1$

$\frac{S}{N} = 0{,}028 = -15{,}5\ dB \quad \boxed{\frac{S}{N} > -15{,}5\ dB}$

b) $C = 50\ kbps \quad \frac{S}{N} > 2^{\frac{5}{100}} - 1 = 1{,}035 - 1 = 0{,}035 = -14{,}5\ dB$

c) $B = 4\ kHz \quad S/N = 100 = 20\ dB$

$10\log(100) = 20$

$\boxed{C = 4\ ld(1 + 100) \quad C = 26.632\ bps \doteq 27\ kBps}$

$C \le 27\ kBps$

a) $\left(\frac{C}{B}\right) = \rho = 0{,}04\ bps/Hz$

- $\frac{\epsilon_B}{N_0} = 5\ dB \Rightarrow 10\log\frac{\epsilon_B}{N_0} = 5\ dB \quad \frac{\epsilon_B}{N_0} = 10^{0.5} = 3{,}16$

$\frac{\epsilon_B}{N_0} = \frac{2^\rho - 1}{\rho} \quad \rho = ld\left(1 + \frac{\rho \epsilon_B}{N_0}\right) \quad \frac{\epsilon_B}{N_0} = 3{,}16 \Rightarrow \rho = 3{,}65$

$10\log\left(\frac{\epsilon_B}{N_0}\right) = 0 \quad \rho = ld(1 + 1) = ? \quad \boxed{\frac{\epsilon_B}{N_0} = 10^0 = 1} \quad \rho = ld(2) = 1$

- $\frac{\epsilon_B}{N_0}\Big|_{dB} = 10 \quad \frac{\epsilon_B}{N_0} = 10^{\frac{10}{10}} = 10 \rightarrow \rho = 5{,}11$

$\rho = 2 \quad \boxed{\rho_5 = \frac{1}{\frac{1}{2} + \dots \frac{1}{2}} = \frac{4}{5 \cdot 2} = \frac{2}{5}}$

$\frac{\epsilon_B}{N_0} = \frac{2^\rho - 1}{\rho} \quad \rho = 2 \quad \rho_5 = 0.8 \quad \frac{\epsilon_B}{N_0} = 0{,}8$

$z = x + jy \quad p(x,y) = p(x)\, p(y) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-m_x)^2}{\sigma^2}} \cdot \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(y-m_y)^2}{\sigma^2}}$

$= \frac{1}{2\pi\sigma^2} \exp\left(x^2 - 2xm_x + m_x^2 - y^2 + 2m_y y + m_y^2\right)/2\sigma^2 = \frac{1}{2\pi\sigma^2} e^{-\frac{|z-m_z|^2}{2\sigma^2}}$

$\sigma^2 = E[(x - m_x)^2] = E[(y - m_y)^2] = \frac{1}{2} E[(z - m_z)^2]$

$|z - m_z|^2 = |x + jy - m_z|^2 = |(x - m_x) + j(y - m_y)|^2 = x^2 - 2xm_x + m_x^2 + y^2 - 2ym_y + m_y^2$

$m_z = m_x + jm_y$

$R\,[b/s]$, L bit packets

p_j - POWER TRANSMITTED BY USER j

$p_j \in [0, \infty]$ γ_j - SNR OF USER „j"

- NONCOHERENT FSK IN AWGN

$$\mu_j(p_j, \gamma_j) = \frac{R}{p_j}\left(1 - e^{-0.5\gamma_j}\right)^L$$

• OUTAGE PROBABILITY OF MULTIHOP TRANSMISSION OVER NAKAGAMI

N - CHANNELS / HOPS $n = 1, \ldots, N-1$

$$G_n^2 = \frac{1}{\alpha_n^2 + N_{0,n}}$$

α_n - FADING AMPLITUDE OF RECEIVING HOP

$N_{0,n}$ - POWER OF AWGN AT THE INPUT OF THE n-TH RELAY

- END TO END SNR

$$\gamma_{eq1} = \left[\prod_{n=1}^{N}\left(1 + \frac{1}{\gamma_n}\right) - 1\right]^{-1} \qquad \gamma_n = \frac{\alpha_n^2}{N_{0,n}}$$

$$\boxed{\frac{A^2}{G_n^2} = \frac{\epsilon}{n_0}}$$

$$\frac{1}{\gamma_{eq}} = \frac{1}{\gamma_1} + \frac{1}{\gamma_2} + \frac{1}{\gamma_3} + \frac{1}{\gamma_1\gamma_2} + \frac{1}{\gamma_1\gamma_3} + \frac{1}{\gamma_2\gamma_3}$$

UPPER BOUND: $\gamma_{eq} = \left[\sum_{n=1}^{N}\frac{1}{\gamma_n}\right]^{-1}$ $G_n^2 = \frac{1}{\alpha_n^2}$

$\gamma_{eq2} = \frac{\mu_H}{N}$ μ_H - HARMONIC MEAN OF THE INDIVIDUAL LINKS SNRs

$\gamma_{eq} < \gamma_{th}$ $P_{out} = P_r(\gamma_{eq} < \gamma_{th}) = P_r\left(\frac{1}{\gamma_{eq}} > \frac{1}{\gamma_{th}}\right) = 1 - \mathcal{L}^{-1}\left(\frac{M_{1/\gamma_{eq}}(s)}{s}\right)\Big|_{1/\gamma_{th}}$

$\mathcal{L}^{-1}$ - LAPLACE TRANSFORM

MGF - MOMENT GENERATING FUNCTION

$$P_{out} = P_r\left[\min(\gamma_1, \ldots, \gamma_n) < \gamma_{th}\right] = 1 - P_r\left[\gamma_1 > \gamma_{th}, \gamma_2 > \gamma_{th}, \ldots, \gamma_N > \gamma_{th}\right]$$

$$P_{out} = 1 - \prod_{n=1}^{N}\left(1 - \frac{\Gamma\left(m_n, \frac{m_n\gamma_{th}}{\bar{\gamma}_n}\right)}{\Gamma(m_n)}\right)$$

$\bar{\gamma}_n$ - AVERAGE SNR

$$M_{1/\gamma_n}(s) = \frac{2}{\Gamma(m)}\left(\frac{m_n s}{\bar{\gamma}_n}\right)^{m_n/2} K_{m_n}\left(2\sqrt{\frac{m_n s}{\bar{\gamma}_n}}\right)$$

$K_{m_n}(\,)$ - m_nth order modified Bessel function of second kind

• INCOMPLETE GAMMA FUNCTION

$$\gamma(\alpha, x) = \int_0^x e^{-t} t^{\alpha-1}\,dt \quad \left[\mathrm{Re}\{a\} > 0\right]$$

$$\Gamma(\alpha, x) = \int_x^\infty e^{-t} t^{\alpha-1}\,dt$$

- GAMMA FUNCTION (EULER'S INTEGRAL OF SECOND KIND)

$$\Gamma(z) = \int_0^\infty e^{-t} t^{z-1}\, dt \qquad Re(z) > 0$$

$$\Gamma(z+1) = z\,\Gamma(z) \qquad \Gamma(z+1) = \int_0^\infty e^{-t} t^{z}\, dt$$

$u = z+1$

$$\Gamma(u) = \int_0^\infty e^{-t} t^{u-1}\, dt \qquad u \equiv z \qquad \boxed{\Gamma(z) = \int_0^\infty e^{-t} t^{z-1}\, dt}$$

- NAKAGAMI FADING   D. ZWILLINGER

$$\boxed{p(x) = \frac{2m^m x^{2m-1}}{\Gamma(m)\,\Omega^m} \cdot e^{-\frac{m}{\Omega}x^2} \qquad x \geq 0,\ \Omega \geq 0,\ m \geq 0.5}$$

$m = \frac{E(x^2)}{VAR(x^2)}$ – FADING FIGURE OR SHAPE FACTOR $(x^2)' = 2x$

Ω – AVERAGE RECEIVED POWER

$$\boxed{\Gamma(z+1) = z\,\Gamma(z)}$$

$$\boxed{\Gamma(z) = \int_0^\infty \underbrace{e^{-t} t^{z-1}\, dt}_{u \cdot dv}} \qquad v = \int e^{-t} dt = -e^{-t} \qquad u = t^{z-1}$$

$$\Gamma(z) = \underbrace{u \cdot v}_{\emptyset} - \int v\, du = -e^{-t} t^{z-1} \Big|_0^\infty + \int_0^\infty e^{-t}\, d(t^{z-1}) =$$

$$= -\frac{t^{z-1}}{e^t}\Big|_0^\infty + \int_0^\infty e^{-t} (z-1) t^{z-2}\, dt = (z-1) \underbrace{\int_0^\infty e^{-t} t^{z-2}\, dt}_{\Gamma(z-1)}$$

$$\boxed{\Gamma(z) = (z-1)\,\Gamma(z-1)}$$

$$J = \begin{vmatrix} \frac{\partial z}{\partial r} & \frac{\partial z}{\partial \varphi} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \varphi} \end{vmatrix} = r$$

$$s(t) = (A + x(t)) \cos \omega_0 t - y(t) \sin \omega_0 t = z(t) \cos(\omega_0 t) - y(t) \sin \omega_0 t$$

$$s(t) = r(t) \cdot \cos(\omega_0 t + \varphi(t))$$

$$z = r \cdot \cos\varphi\,; \qquad y = r \cdot \sin\varphi \qquad r = \sqrt{z^2 + y^2} = \sqrt{(x+A)^2 + y^2}$$

$$tg\,\varphi = \frac{y}{z} \qquad \varphi = arctg\left(\frac{y}{z}\right)$$

$$p_{r\varphi}(r,\varphi) = |J| \cdot p(z,y) = \frac{r}{2\pi\sigma} e^{-\frac{(r\cos\varphi - A)^2}{2\sigma^2}} e^{-\frac{r^2 \sin^2\varphi}{2\sigma^2}} =$$

$$= \frac{r}{2\pi\sigma^2} e^{-\frac{r^2\cos^2\varphi - 2Ar\cos\varphi + A^2 + r^2\sin^2\varphi}{2\sigma^2}} = \frac{r}{2\pi\sigma} e^{-\frac{r^2+A^2}{2\sigma^2}} e^{+\frac{Ar\cos\varphi}{\sigma^2}}$$

$$p_r(r) = \int_0^{2\pi} p_{r\varphi}(r,\varphi)\, d\varphi = \frac{r}{\cancel{2\pi}\sigma^2} e^{-\frac{r^2+A^2}{2\sigma^2}} \int_0^{2\pi} e^{+\frac{Ar\cos\varphi}{\sigma^2}}\, d\varphi$$

$$p_r(r) = \frac{r}{\cancel{2\pi}\sigma^2} e^{-\frac{r^2+A^2}{2\sigma^2}} I_0\left(\frac{Ar}{\sigma^2}\right)$$

$$\boxed{p_Z(z) = 2m^m \frac{z^{(2m-1)}}{\Gamma(m)\,\Omega^m} e^{-\frac{mz^2}{\Omega}} \quad z \geq 0;\ \Omega \geq 0;\ m \geq 0{,}5} \qquad \Omega \equiv P_r$$

$\Omega = E(z^2)$ - AVERAGE RECEIVED POWER OR AVERAGE CNR

$m = \frac{E(z^2)}{Var(z^2)}$ - FADING FIGURE OR SHAPE FACTOR

$m = 1 \rightarrow$ RAYLEIGH FADING;

$m = \infty \rightarrow$ AWGN WITH NO FADING

$m = 0.5 \rightarrow$ ONE SIDED GAUSIAN DISTRIBUTION

RICIAN "K" FACTOR: $\boxed{K = \frac{m^2}{2\sigma^2}}$ $\quad m^2 = m_I^2 + m_Q^2$

$z = r\cos\varphi \qquad y = r\sin\varphi \qquad J = \begin{vmatrix} \frac{\partial z}{\partial r} & \frac{\partial z}{\partial \varphi} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \varphi} \end{vmatrix} = \begin{vmatrix} \cos\varphi & -r\sin\varphi \\ \sin\varphi & r\cos\varphi \end{vmatrix} =$

$= r\cos^2\varphi + r\sin^2\varphi = r$

$$\boxed{K = \frac{m^2 - m}{2m} - \frac{m^2 - m}{2}}$$

$\boxed{m = \frac{(K+1)^2}{2K+1}} \longrightarrow$ FOR THIS, "m" DISTRIBUTION BECOMES "RICIAN FADING"

- POWER DISTRIBUTION OF NAKAGAMI FADING

$\cancel{p(x) = m^3 x \frac{1}{\Gamma(m)} \Omega^m e^{-\frac{mx}{\Omega}}}$

- LEVEL CROSSING RATE

$N = \int_0^\infty r'\, p(r', r=R)\, dr'$

- AVERAGE FADE DURATION: $T = \frac{P(r \leq R)}{N}$

- POWER DISTRIBUTION (BY CHANGE OF VARIABLES)

$p_{z^2}(x) = 2\left(\frac{m}{\Omega}\right)^m \frac{(z^2)^m \cdot z^{-1}}{\Gamma(m)} e^{-\frac{mx}{\Omega}} = 2\left(\frac{m}{\Omega}\right)^m \frac{x^m \cdot x^{-\frac{1}{2}}}{\Gamma(m)} e^{-\frac{mx}{\Omega}}$

$x = z^2 \quad z = \sqrt{x}$

$p_{z^2}(x) = 2\left(\frac{m}{\Omega}\right)^m \frac{x^{m-\frac{1}{2}}}{\Gamma(m)} e^{-\frac{mx}{\Omega}} \qquad \Gamma(m) = \int_0^\infty e^{-t} t^{m-1} dt$

$E(z) = \int_{-\infty}^{\infty} z \cdot p(z)\, dz = \frac{2m^m}{\Gamma(m)\Omega^m} \underbrace{\int_0^\infty z^{2m-1} e^{-\frac{mz^2}{\Omega}} dz}_{I}$

$\frac{mz^2}{\Omega} = t \qquad \frac{2mz}{\Omega} dz = dt \qquad z = 0 \quad t = 0$

$z^2 = \frac{\Omega}{m} t \qquad\qquad z = \infty \quad t = \infty$

$$I = \int_0^\infty z^{2m-1}\cdot\frac{z}{z}\left(\frac{2m}{\Omega}\right)\cdot\frac{\Omega}{2m}\, e^{-t}\, dz = \frac{\Omega}{2m}\int_0^\infty z^{2m-2}e^{-t}\,dt$$

$$I = \frac{\Omega}{2m}\int_0^\infty (z^2)^{m-1}e^{-t}\,dt = \frac{\Omega}{2m}\int_0^\infty \left(\frac{\Omega}{m}t\right)^{m-1}e^{-t}\,dt =$$

$$= \frac{\Omega}{2m}\cdot\left(\frac{\Omega}{m}\right)^{m-1}\int_0^\infty t^{m-1}e^{-t}\,dt = \frac{1}{2}\left(\frac{\Omega}{m}\right)^m\Gamma(m)$$

$$E(z) = \frac{2m^m}{\Gamma(m)\,\Omega^m}\cdot\frac{1}{2}\left(\frac{\Omega}{m}\right)^m\Gamma(m) = \underline{1}$$

ISPUŠTIV DA MNOŽAM SO Z. OVA E DOKAZ $\int_0^\infty p(z)\,dz = 1$

$$E(z) = \underbrace{\frac{2m^m}{\Gamma(m)\,\Omega^m}}_{K}\int_0^\infty z\cdot z^{2m-1}\left(\frac{2m}{\Omega}\right)\frac{\Omega}{2m}\, e^{-t}\, dz = K\cdot\frac{\Omega}{2m}\int_0^\infty z^{2m-1}e^{-t}\,dt$$

$$E(z) = \frac{2m^m}{\Gamma(m)\,\Omega^m}\cdot\frac{\Omega}{2m}\int_0^\infty z^{2(m-\frac{1}{2})}e^{-t}\,dt = \frac{m^{m-1}}{\Gamma(m)\,\Omega^{m-1}}\left(\int_0^\infty t^{m-\frac{1}{2}}e^{-t}\,dt\right)\left(\frac{\Omega}{m}\right)^{m-\frac{1}{2}}$$

$$\mu - 1 = m - \frac{1}{2} \qquad \mu = m - \frac{1}{2} + 1 = m + \frac{1}{2}$$

$$E(z) = \frac{m^{m-1}}{\Gamma(m)\,\Omega^{m-1}}\left(\frac{\Omega}{m}\right)^{m-\frac{1}{2}}\underbrace{\int_0^\infty t^{\mu-1}e^{-t}\,dt}_{\Gamma(\mu)} \qquad \Gamma(\mu) = \Gamma\left(m+\frac{1}{2}\right)$$

$$E(z) = \frac{m^{m-1}}{\Gamma(m)\,\Omega^{m-1}}\,\Gamma\left(m+\frac{1}{2}\right)\cdot\left(\frac{\Omega}{m}\right)^{m-\frac{1}{2}} = \frac{\Gamma(m+\frac{1}{2})}{\Gamma(m)}\left(\frac{m}{\Omega}\right)^{m-1-m+\frac{1}{2}}$$

$$\boxed{E(z) = \frac{\Gamma(m+\frac{1}{2})}{\Gamma(m)}\cdot\left(\frac{\Omega}{m}\right)^{\frac{1}{2}}}$$

$$40\,dB = 10\log\gamma_{th} \Rightarrow \boxed{\gamma_{th} = 10^4 = 10.000}$$

$$P_{out} = 1 - \prod_{n=1}^{N}\left(1 - \frac{\Gamma\left(m, \frac{m\gamma_{th}}{\bar{\gamma}}\right)}{\Gamma(m)}\right)$$

- INCOMPLETE GAMMA ([illegible])

$$P(x,a) = \frac{1}{\Gamma(a)}\int_0^x e^{-t}t^{a-1}\,dt \qquad \Gamma(a)P(x,a) = \int_0^x e^{-t}t^{a-1}\,dt$$

$$\Gamma(a) = \int_0^\infty e^{-t}t^{a-1}\,dt \qquad \int_x^\infty e^{-t}t^{a-1}\,dt = \int_0^\infty e^{-t}t^{a-1}\,dt - \int_0^x e^{-t}t^{a-1}\,dt = \Gamma(a)\left(1 - P(x,a)\right)$$

• SHANKAR CONTINUATION

LINE 56 $\boxed{\text{raygen.m}}$

$$\boxed{m = \frac{[E(Y^2)]^2}{[E(Y^2 - \bar{Y}^2)]^2}}$$

NAKAGAMI PARAMETER

$\underline{Y = \mathrm{sqrt}(r)}$ $\quad p(r) = \frac{r}{\sigma^2} e^{-\frac{r^2}{2\sigma^2}}$

$$p(r) = \frac{r}{\sigma} e^{-\frac{r^2}{2\sigma}}$$

$$\bar{r} = \sqrt{\frac{\pi}{2}}\,\sigma$$

Y – RAYLEIGH DISTRIBUTED VARIABLE

$$\Omega = E(Y^2)$$

• OUTAGE PROBABILITY DUE TO FADING

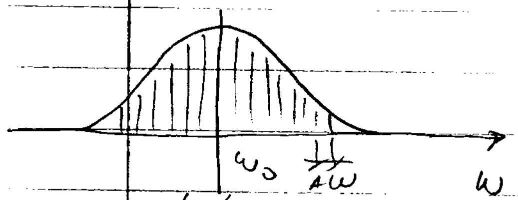
$\gamma_s \equiv SNR$ $\quad p_{\gamma_s}(\gamma)$ DISTRIBUTION OF SNR

$$P_{out} = p(\gamma_s < \gamma_0) = \int_0^{\gamma_0} p_{\gamma_s}(\gamma)\,d\gamma$$

DIGITIZED VOICE: $P_b = 10^{-3}$

BPSK RAYLEIGH FADING $\quad \gamma_b < 7\,dB \Rightarrow$ OUTAGE $\Rightarrow \underline{\gamma_0 = 7\,dB}$

$$n_0(t) = \sum_{k=-\infty}^{\infty} A \cdot \cos(k\Delta\omega t + \varphi_k(t)) \qquad p(\varphi) = \begin{cases} \frac{1}{2\pi} & |\varphi| \le \pi \\ 0 & |\varphi| > \pi \end{cases}$$



$$n_0(t) = \sum_{k=-\infty}^{\infty} A_k \cos(\omega_0 t + k\Delta\omega t + \varphi_k(t))$$

$$p(\varphi) = \begin{cases} \frac{1}{2\pi} & |\varphi| \le \pi \\ 0 & \text{OTHER} \end{cases}$$

$$n_0(t) = \underbrace{\left(\sum_{k=-\infty}^{\infty} A_k \cos(k\Delta\omega t + \varphi_k(t))\right)}_{x(t)} \cos\omega_0 t - \underbrace{\left(\sum_{k=-\infty}^{\infty} A_k \sin(k\Delta\omega t + \varphi_k(t))\right)}_{y(t)} \sin\omega_0 t$$

$$\boxed{n_0(t) = x(t)\cdot\cos(\omega_0 t) - y(t)\cdot\sin(\omega_0 t)}$$

BPSK $\quad P(\varepsilon) = \frac{1}{\hat{\rho}\sqrt{2\pi}} e^{-\hat{\rho}^2/2} \qquad \rho^2 = \frac{\hat{\rho}^2}{2} \qquad \rho^2 = \frac{A^2}{2\sigma_N^2}$

$$\hat{\rho}^2 = \frac{A^2}{\sigma_N^2}$$

$\boxed{\hat{\rho} = SNR}$

$$P_b = 10^{-3} = \frac{1}{\hat{\rho}\sqrt{2\pi}} e^{-\hat{\rho}^2/2} \Rightarrow \boxed{\rho = ?}$$

$\Rightarrow x = 3.15 = 4.93\,dB$ $\qquad$ ~~$10\log 4\cdot10^{-4} = 10\log 10^{-4} - 10\log 4$~~

$x[dB] = \hat{\rho}[dB] = 5\,dB$

$$\hat{\rho}^2 = SNR = \frac{A^2}{\sigma_N^2} = \frac{E}{N_0} = 10\log(3.15)^2 = 9.86\,dB$$

$$\rho^2 = \frac{\hat{\rho}^2}{2} = \frac{(A/\sqrt{2})^2}{\sigma_N^2} = \frac{A^2}{2\sigma_N^2} = 10\log\frac{3.15^2}{2} = \underline{\underline{6.86 \doteq 7\,dB}}$$

$\underline{\gamma_0 = 7\,dB}$

6.1.2 Error probability for BPSK

$$\boxed{Q(z)=\frac{1}{2}\,\mathrm{erfc}\left(\frac{z}{\sqrt{2}}\right)}$$

$$P_s = 2Q\left(\sqrt{\frac{A^2}{N_0}}\right) = 2Q\left(\sqrt{\frac{A_q^2}{2N_0}}\right) = 2Q\left(\sqrt{\frac{\gamma_s}{2}}\right)$$

$$Q(z) = P(x>z) = \int_z^\infty \frac{1}{\sqrt{2\pi}} e^{-x^2/2}\,dx \qquad \frac{x^2}{2}=t^2 \quad \frac{x}{\sqrt{2}}=t$$

$$dx = \sqrt{2}\,dt \qquad x=z \quad t=\frac{z}{\sqrt{2}}$$

$$Q(z) = \frac{1}{\sqrt{2\pi}}\,\frac{2}{\sqrt{\pi}}\cdot\frac{\sqrt{\pi}}{2}\int_{z/\sqrt{2}}^\infty e^{-t^2}\sqrt{2}\,dt = \frac{1}{2}\,\frac{2}{\sqrt{\pi}}\int_{z/\sqrt{2}}^\infty e^{-t^2}dt = \frac{1}{2}\,\mathrm{erfc}\left(\frac{z}{\sqrt{2}}\right)$$

$$P_s = \cancel{2}\cdot\frac{1}{\cancel{2}}\,\mathrm{erfc}\left(\frac{\sqrt{\gamma_s}}{2}\right) = \mathrm{erfc}\left(\frac{\sqrt{\gamma_s}}{2}\right) \qquad P_b \doteq \frac{P_s}{2}$$

$$P_b = Q\left(\sqrt{2\gamma_b}\right) = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\gamma_b}\right)$$

$$10^{-3} = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\gamma_b}\right) \qquad \boxed{\gamma_b = ?} \qquad \sqrt{\gamma_b} = 2{,}185$$

$$\gamma_b = 4{,}775 \qquad 10\log(\gamma_b) = 6{,}7899 \doteq 7\,dB$$

$$P_{out} = P(\gamma_s<\gamma_0) = \int_0^{\gamma_0} p_{\gamma_s}(\gamma)\,d\gamma \qquad \left(\frac{r}{\sigma}\right)^2 = \gamma$$

$$p_{\gamma_s}(\gamma) = \frac{r}{\sigma^2}\, e^{-\frac{r^2}{2\sigma^2}} \qquad \boxed{\bar{r} = \sqrt{\frac{\pi}{2}}\,\sigma} \qquad \boxed{\frac{\bar{r}}{\sigma\sqrt{2}} = \sqrt{\frac{\pi}{2}}\cdot\frac{1}{\sqrt{2}} = \frac{\sqrt{\pi}}{2}}$$

$$\boxed{\bar{\gamma} = \left(\frac{\bar{r}}{\sigma\sqrt{2}}\right)^2 = \frac{\pi}{2}}$$

$$p_{\gamma_s}(\gamma) = \frac{2}{r}\cdot\frac{r^2}{2\sigma^2}\, e^{-\gamma^2} = \frac{2}{r}\cdot\gamma\cdot e^{-\gamma^2}$$

$$\sigma = \sqrt{\frac{2}{\pi}}\,\bar{r} \qquad ???$$

$$P_{out} = \int_0^{\gamma_0} \frac{1}{\bar{\gamma}_s}\, e^{-\gamma_s/\bar{\gamma}_s}\,d\gamma_s = \frac{1}{\bar{\gamma}_s}\int_0^{\gamma_0} e^{-\frac{\gamma_s}{\bar{\gamma}_s}}\,d\gamma_s$$

$$P_{out} = -\frac{1}{\cancel{\bar{\gamma}_s}}\cdot\cancel{\bar{\gamma}_s}\, e^{-\frac{\gamma_s}{\bar{\gamma}_s}}\Big|_0^{\gamma_0} = 1 - e^{-\gamma_0/\bar{\gamma}_s} \qquad \boxed{P_{out} = 1 - e^{-\gamma_0/\bar{\gamma}_s}}$$

$$p(\gamma_s) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\gamma_s} \qquad \bar{\gamma}_s = \frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{\infty} \gamma_s\, e^{-\gamma_s}\,d\gamma_s$$

$$I = \int_{-\infty}^{\infty} x\,e^{-x}\,dx = \left|\begin{matrix} u = x & du = dx \\ v = \int e^{-x}dx = -e^{-x} & \end{matrix}\right| = -x e^{-x}\Big|_{-\infty}^{\infty} - \int_{-\infty}^{\infty} e^{-x}dx$$

$$= \left(e^{-x} - x e^{-x}\right)\Big|_{-\infty}^{\infty}$$

$$e^{-x}(1-x)\Big|_{-\infty}^{\infty} = \frac{1-x}{e^x}\Big|_{-\infty}^{\infty} = \emptyset - \frac{1+\infty}{e^{-\infty}} = 0 - \frac{\infty}{\emptyset} = -\infty\cdot\infty = -\infty$$

$$p(r) = \frac{r}{\sigma^2} e^{-\frac{r^2}{2\sigma^2}} = \frac{\sqrt{2}}{\sigma}\frac{r}{\sigma\sqrt{2}} e^{-\gamma_s} \qquad \gamma_s = \frac{r^2}{2\sigma^2} \qquad \boxed{\gamma_s = \left(\frac{r}{\sqrt{2}\sigma}\right)^2}$$

$$\boxed{p(\gamma) = \frac{\sqrt{2}}{\sigma}\sqrt{\gamma_s}\, e^{-\gamma_s}} \qquad \bar{\gamma}_s = \frac{\sqrt{2}}{\sigma}\int_{\emptyset}^{\infty} \gamma_s\sqrt{\gamma_s}\, e^{-\gamma_s} d\gamma_s$$

$$\bar{\gamma}_s = \frac{3}{4}\frac{\sqrt{2\pi}}{\sigma} \qquad \boxed{\bar{\gamma}_s = \frac{3\sqrt{2\pi}}{4\sigma}} \qquad \boxed{E(r^2) = 2\sigma^2}$$

$$\gamma_s = \frac{r^2}{2\sigma^2} \qquad E(\gamma_s) = \frac{E(r^2)}{2\sigma^2} = 1 \quad ??$$

$$p(r) = \frac{r}{\sigma^2} e^{-r^2/2\sigma^2} \qquad E(r^2) = \int_0^{\infty} r^2 p(r)\, dr = 2\sigma^2$$

- POWER DISTRIBUTION

$$r^2(t) = \left|\sqrt{I^2(t) + Q^2(t)}\right|^2 \qquad \boxed{P_r = 2\sigma^2}$$

$$p(r) = \frac{\left|\sqrt{I^2(t)+Q^2(t)}\right|}{\sigma^2} e^{-\frac{(I^2(t)+Q^2(t))}{2\sigma^2}}$$

$$p_{z^2}(x) = \frac{1}{P_r} e^{-x/P_r} = \frac{1}{2\sigma^2} e^{-\frac{x}{2\sigma^2}} \quad x > 0$$

$\boxed{Ex\ 3.2}$ RAYLEIGH FADING CHANNEL $\quad P_r = 20dB = 100W$

$$P(z^2 < 10dB) = \int_0^{10} \frac{1}{100} e^{-\frac{x}{100}} dx = 0{,}095$$

$$u(t) = x(t)\cdot\cos(\omega_0 t) - y(t)\cdot\sin(\omega_0 t) = \sqrt{x^2+y^2}\cdot\cos(\omega_0 t + \varphi)$$

$$x = r(t)\cdot\cos\varphi \qquad y = r(t)\cdot\sin\varphi$$

$$p_x(t) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2\sigma^2}} \qquad p_y(t) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{y^2}{2\sigma^2}}$$

$$p_{x,y}(x,y) = p(x)\cdot p(y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

$$p_{x^2}(u) = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-\frac{u}{2\sigma^2}} \qquad p_{y^2}(v) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{v}{2\sigma^2}}$$

$$p_{r\varphi}(r,\varphi) = |J|\cdot p_{u,v}(u,v) = |J|\cdot p(u)\cdot p(v)$$

$$u = x^2 \qquad \sqrt{u} = r(t)\cdot\cos\varphi \qquad \sqrt{v} = r(t)\cdot\sin\varphi$$

$$u = r^2(t)\cdot\cos^2\varphi \qquad v = r^2(t)\cdot\sin^2\varphi$$

$$J = \begin{vmatrix} \frac{\partial u}{\partial r} & \frac{\partial u}{\partial \varphi} \\ \frac{\partial v}{\partial r} & \frac{\partial v}{\partial \varphi} \end{vmatrix} = \begin{vmatrix} 2r\cdot\cos^2\varphi & -r^2 2\cos\varphi\cdot\sin\varphi \\ 2r\cdot\sin^2\varphi & r^2 2\sin\varphi\cdot\cos\varphi \end{vmatrix} =$$

$$= 2r^3\sin\varphi\cdot\cos^3\varphi + 2r^3\cos\varphi\cdot\sin^3\varphi = 2r^3\cdot\cos\varphi\sin\varphi\overbrace{(\cos^2\varphi+\sin^2\varphi)}^{1}$$

$$J = 2r^3\cos\varphi\sin\varphi$$

$$\begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \varphi} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \varphi} \end{vmatrix} = \begin{vmatrix} \cos\varphi & -r\cdot\sin\varphi \\ \sin\varphi & r\cos\varphi \end{vmatrix} = r$$

$$p_{r\varphi} = \cancel{2}r^3\cos\varphi\sin\varphi\cdot\frac{1}{\cancel{2}\pi\sigma^2}\cdot e^{-\frac{u+v}{2\sigma^2}} \quad (u+v = r^2)$$

$$p(r) = \frac{r^3}{\pi\sigma^2}e^{-\frac{r^2}{2\sigma^2}}\int_0^{2\pi}\cos\varphi\sin\varphi\,d\varphi = \frac{r^3}{\pi\sigma^2}e^{-\frac{r^2}{2\sigma^2}}\left(-\frac{\cos^2\varphi}{2}\right)\Big|_0^{2\pi} = \emptyset$$

$\boxed{Ex\,6.1}$ $P_b = ?$ $P_s = ?$ for QPSK $\gamma_b = 7\,dB$

EXACT $P_b, P_s = ?$

$P_b = Q(\sqrt{2\gamma_b})$ $\qquad \gamma_b = 7\,dB = 10^{\frac{7}{10}} = 10^{0.7} = 5{,}012$

$Q(z) = \frac{1}{2}\,erfc\left(\frac{z}{\sqrt{2}}\right)$; $\quad P_b = \frac{1}{2}erfc(\sqrt{\gamma_b}) = 0{,}00077265$

$P_s = 1-(1-Q(\sqrt{2\gamma_b}))^2 = \cancel{1}-\cancel{1}+2Q(\sqrt{2\gamma_b}) - Q^2(\sqrt{2\gamma_b}) \leq 2Q(\sqrt{2\gamma_b})$ $\quad Q^2 \to \emptyset$

$P_s = 1-\left(1-\frac{1}{2}erfc(\sqrt{\gamma_b})\right)^2 = 0{,}0015457$

$P_b' = \frac{P_s}{2} = 0{,}00077235$ $\quad \boxed{P_b - P_b' = 3\cdot 10^{-7}}$

nearest neighbor bound:

$P_s' = 2Q(\sqrt{\gamma_s/2}) = erfc\left(\frac{\sqrt{\gamma_s}}{(\sqrt{2})^2}\right) = erfc\left(\frac{\sqrt{2\gamma_b}}{2}\right) = erfc\left(\sqrt{\frac{\gamma_b}{2}}\right) = 0{,}01517$

$\boxed{P_s - P_s' \doteq -23{,}6\cdot 10^{-3}}$

$\frac{N_0}{2}$ - POWER SPECTRAL DENSITY OF NOISE $\qquad s(t) = Re\{u(t)e^{j2\pi f_c t}\}$

$r(t) = s(t) + n(t)$

TOTAL NOISE WITHIN $2B$: $\quad N = \frac{N_0}{2}\cdot 2B = N_0 B$

$SNR = \frac{P_r}{N_0 B}$ $\qquad E_b$ - SIGNAL ENERGY PER BIT

E_s - " - 1 SIGNAL

$P_r = \frac{E_s}{T_s}$ $\quad P_r = \frac{E_b}{T_b}$ $\quad SNR = \frac{E_s}{N_0\cdot B\cdot T_s} = \frac{E_b}{N_0\cdot B\cdot T_b}$

BINARY MODULATIONS: $\quad T_s = T_b \quad E_s = E_b$

$T_s = \frac{1}{B} \Rightarrow$ $\quad \boxed{SNR = \frac{E_s}{N_0}}$

$T_s = \frac{K}{B}$ $\quad \boxed{K\cdot SNR = \frac{E_s}{N_0}}$

$\gamma_s = \frac{E_s}{N_0}$ $\quad \gamma_b = \frac{E_b}{N_0}$

$\gamma_b \doteq \frac{\gamma_s}{\log_2 M}$ $\quad P_b \doteq \frac{P_s}{\log_2 M}$ $\quad \frac{1}{\sqrt{\pi}} \int_{+}^{\infty}$

КОХЕРЕНТНА ДЕМОД: $\quad P(\varepsilon) = \mathrm{erfc}\, \frac{\hat{\rho}}{2\sqrt{2}}$

ЗА ДЕФИНИЦИЈУ: $\quad \mathrm{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_{x}^{\infty} e^{-t^2} dt \Rightarrow \boxed{P(\varepsilon) = \frac{1}{2}\, \mathrm{erfc}\, \frac{\hat{\rho}}{2\sqrt{2}}}$

$\boxed{6.1.2}$

$$s_1(t) = A \cdot g(t) \cos(2\pi f_c t) \qquad a_n = 1$$
$$s_2(t) = -A\, g(t) \cos(2\pi f_c t) \qquad a_n = -1$$

$$P_b = Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right) \qquad d_{min} = |A - (-A)| = 2A$$

$$E_b = \int_0^{T_b} s_1^2(t)\, dt = \int_0^{T_b} A^2 \cos^2(\omega_c t)\, dt$$

$$\omega_c = 2\pi f_c = \frac{2\pi}{T_c} \qquad \cos^2\alpha = \cos(\alpha + \alpha) + \cos(\alpha - \alpha)$$

$\cos 2\alpha \qquad 1$

$$E_b = A^2 \int_0^{T_b} (1 + \cos(2\omega_c t))\, dt = A^2 T_b + \underbrace{A^2 \int_0^{T_b} \cos(2\omega_c t)\, dt}_{I}$$

$$I = A^2 \frac{1}{2\omega_c} \sin(\omega_c t)\Big|_0^{T_b} = \frac{A^2}{2\omega_c} \cdot \sin(\omega_c T_b) = \frac{A^2}{2\omega_c} \sin\left(\frac{2\pi}{T_c} T_b\right)$$

$$T_b = k \cdot T_c \qquad I = \frac{A^2}{2\omega_c} \cdot \underbrace{\sin(2\pi k)}_{0} = 0$$

$\boxed{E_b = A^2 \cdot T_b}$ $\qquad d_{min} = 2A \sim 2\sqrt{E_b}$

$$P_b = Q\left(\frac{2\sqrt{E_b}}{\sqrt{2N_0}}\right) = Q\left(\frac{2A}{\sqrt{2N_0}}\right) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = Q\left(\sqrt{2\gamma_b}\right)$$

$$P_b = \frac{1}{2}\, \mathrm{erfc}\left(\frac{\sqrt{2\gamma_b}}{\sqrt{2}}\right) = \frac{1}{2}\, \mathrm{erfc}\left(\sqrt{\gamma_b}\right)$$

ДТК: $\quad P(\varepsilon) = \frac{1}{2}\, \mathrm{erfc}\, \frac{\hat{\rho}}{\sqrt{2}} \qquad \hat{\rho}^2 = \frac{A^2}{\sigma_N^2} = \frac{E}{N_0} \qquad \hat{\rho} = \frac{A}{\sigma_N}$

$$\frac{\hat{\rho}}{\sqrt{2}} = \sqrt{\gamma_b} \qquad \boxed{\gamma_b = \frac{\hat{\rho}^2}{2} = \frac{A^2}{2\sigma_N^2}}$$

Flat Fading using Matlab for classroom instructions (continue)

- Rician fading

$$s(t) = \sum_{i=1}^{N} a_i \cos(w_c t + w_{di} t + \phi_i)$$

$$s(t) = \sum_{i=1}^{N} a_i \cos(w_c t + w_{di} t + \phi_i) + k_d \cos(w_c t + w_d t)$$

$$p(r) = \frac{r}{\sigma^2} \exp\left(-\frac{(r^2 + k_d^2)}{2\sigma^2}\right) I_0\left(\frac{r k_d}{\sigma^2}\right)$$

(a)

Qumulative Rician Dist. Function

$$F(r) = 1 - Q\left(\frac{k_d}{\sigma}, \frac{r}{\sigma}\right) \rightarrow \text{Marcum's Q function}$$

- Rician K factor: $K(dB) = 10 \log_{10}\left(\frac{k_d^2}{2\sigma^2}\right)$

$$25 \frac{m}{s} = 25 \cdot \frac{3600}{1000} \cdot \frac{1000 m}{3600 s} = 3.6 \cdot 25\ km/h = 90\ km/h$$

$$\boxed{\sigma^2 = \frac{\sigma_q^2 + \sigma_i^2}{2} \qquad \sigma = \sigma_{rice}}$$

$$\boxed{a = mean(i)}$$

$I = besseli(0, r.* \frac{a}{\sigma^2})$

$r = sqrt(env\_rice)$ $\quad$ $r = sqrt(r)$

$$p(r) = \frac{r}{\sigma^2} e^{-\frac{a^2 + r^2}{2\sigma^2}} \cdot I\left(\frac{a \cdot r}{\sigma^2}\right)$$

наместо со r^2 раб от со r

$$\boxed{m = \frac{\overline{r^2}^2}{\left(r^2 - \overline{r^2}\right)^2}}$$

$$f_d = \frac{v}{\lambda} \cdot \cos\alpha \qquad w_d = 2\pi \frac{v}{\frac{c}{f_c}} = 2\pi f_c \frac{v}{c}$$

Nakagami:

$$\boxed{p(x) = \frac{2 m^m x^{2m-1}}{\Gamma(m) \cdot \Omega^m} e^{-\frac{m x^2}{\Omega}}}$$

NAKAGAMI

$$\Gamma(m) = \int^{\infty} x^{m-1} e^{-x} dx$$

- **Marcum Q function** (ја има дефинирана во Matlab)

$$Q(a, b) = \int_b^{\infty} x e^{-\frac{x^2 + a^2}{2}} I_0(ax) dx$$

$$p(r) = \frac{r}{\sigma^2} e^{-\frac{A^2 + r^2}{2\sigma^2}} I_0\left(\frac{Ar}{\sigma^2}\right) \qquad \boxed{a = \frac{A}{\sigma} \quad x = \frac{r}{\sigma}}$$

$$\boxed{p(x) = \frac{1}{\sigma} \cdot x\, e^{-\frac{a^2 + x^2}{2}} I_0(a \cdot x)}$$

$$\boxed{\gamma = x^2}$$

$$P_R\left(\gamma < \gamma_0 \atop x < x_0\right) = \int_0^{\gamma_0} \frac{x}{\sigma} e^{-\frac{a^2 + x^2}{2}} I_0(ax) dx$$

$$P_R(x < x_0) = \frac{1}{\sigma} \int^{x_0} \frac{x}{\sigma} e^{-\frac{a^2 + x^2}{2}} I_0(ax) dx$$

$$P_R(x<x_0) = \int_0^{\infty} \frac{x}{\sigma} e^{-\frac{x^2+a^2}{2}} I_0(ax)\,dx - \int_{x_0}^{\infty} \frac{x}{\sigma} e^{-\frac{x^2+a^2}{2}} I_0(ax)\,dx$$

??

$$P_R(x<x_0) = 1 - \frac{1}{\sigma}\int_{x_0}^{\infty} x\, e^{-\frac{x^2+a^2}{2}} I_0(ax)\,dx = 1-\frac{1}{\sigma}Q(a,x)$$

$$P(r<r_0) = \int_0^{r_0} \frac{r}{\sigma^2} e^{-\frac{r^2+A^2}{2\sigma^2}} I_0\left(\frac{Ar}{\sigma^2}\right)dr = \left| \begin{array}{ll} a=\frac{A}{\sigma} & \\ x=\frac{r}{\sigma} & dx=\frac{dr}{\sigma} \\ r=r_0 & x=\frac{r_0}{\sigma}=x_0 \end{array}\right|$$

$$P(x<x_0) = \int_0^{\frac{r_0}{\sigma}} \frac{1}{\sigma} x \cdot e^{-\frac{x^2+a^2}{2}} I_0(a\cdot x)\,\sigma\cdot dx = \int_0^{x_0} x\cdot e^{-\frac{x^2+a^2}{2}} I_0(ax)\,dx$$

$$P(x<x_0) = 1 - \int_{x_0}^{\infty} x\cdot e^{-\frac{x^2+a^2}{2}} I_0(ax)\,dx \qquad \boxed{P(x<x_0) = 1-Q(a,x)}$$

$$p(r) = \frac{r}{\sigma^2} e^{-\frac{r^2+A^2}{2\sigma^2}} I_0\left(\frac{Ar}{\sigma^2}\right) \qquad x=\frac{r}{\sigma} \qquad J$$

$$\boxed{\int_0^{\infty} x\, e^{-\frac{x^2+a^2}{2}} I_0(ax)\,dx = 1} \qquad \begin{vmatrix} \frac{\partial x}{\partial r} & 0 \\ 0 & 1 \end{vmatrix} = \frac{1}{\sigma}$$

ТЕОРИЈА НА ИНФОРМАЦИИ

• ФУНКЦИОНАЛНА ТРАНСФОРМАЦИЈА НА ГУСТИНАТА НА ВЕРОЈАТНОСТ НА СЛУЧАЈНИ ПРОМЕНЛИВИ

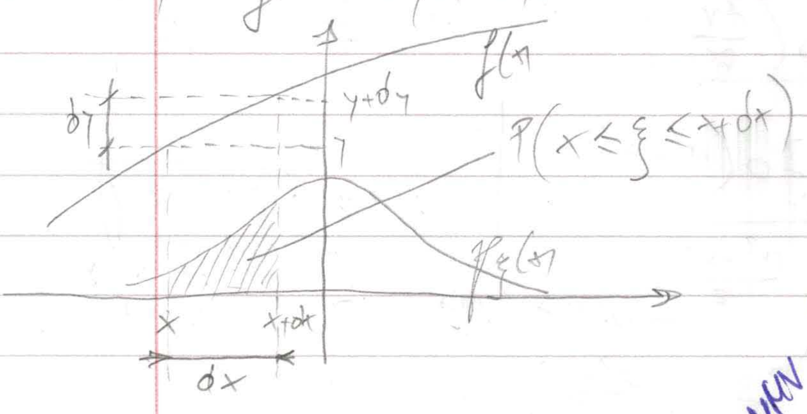
$\eta = f(\xi)$

Ако: $p_\xi(x)$, ξ е позната $\quad \xi \in (-\infty,\infty) \quad p_\eta(y) = ?$

Нека $\eta = f(\xi)$ $\quad y=f(x)$ е еднозначна

$P(y \le \eta \le y+dy) = p_\eta(y)\,dy \qquad dy>0$

$y < f(x) < y+dy$ ⊛ $\qquad$ ⊛ е исполнето за: $x<\xi<x+dx$



$$P(x\le\xi\le x+dx) = P(y\le\eta\le y+dy)$$

$$p_\xi(x)\,dx = p_\eta(y)\,dy$$

МНН

$$\boxed{p_\eta(y) = \left.\frac{p_\xi(x)}{\frac{dy}{dx}}\right|_{x=g(y)}}$$

$y=f(x)$

$x=g(x)$ - инверзна

$y = x^2 \quad x = \pm\sqrt{y}$

$$p_\eta(y) = \frac{p_\xi(x)}{\frac{dy}{dx}} = \left.\frac{p_\xi(x)}{2|x|}\right|_{x=\pm\sqrt{y}} = \frac{p_\xi(\sqrt{y}) + p_\xi(-\sqrt{y})}{2\sqrt{y}} \qquad y > 0$$

$y = x^2 \Rightarrow$ PARNA FUNKCIJA $\Rightarrow p_\xi(\sqrt{y}) = p_\xi(-\sqrt{y})$

$$p_y(y) = \frac{2 p_\xi(\sqrt{y})}{2\sqrt{y}} = \frac{p_\xi(\sqrt{y})}{\sqrt{y}}$$

- AKO FUNKCIJATA IMA n REALNI KORENI

$$\boxed{p_\eta(y) = \left.\frac{p_\xi(x_1)}{|f'(x_1)|}\right|_{x_1=g_1(y)} + \dots + \left.\frac{p_\xi(x_n)}{|f'(x_n)|}\right|_{x_n=g_n(y)}}$$

- n - SLUČAJNI PROMENLIVI: $\{\xi_1, \xi_2, \dots, \xi_n\}$

$\eta_1 = f(\xi_1, \dots, \xi_n) \qquad y_1 = f(x_1, x_2, \dots, x_n)$

$\eta_2 = f(\xi_1, \dots, \xi_n) \qquad y_2 = f(x_1, x_2, \dots, x_n)$

$$p_\xi(x_1, x_2, \dots, x_n)\, dx_1 dx_2 \dots dx_n = p_\eta(y_1, y_2, \dots, y_n)\, dy_1 dy_2 \dots dy_n$$

$$\boxed{dx_1\, dx_2 \dots dx_n = |J|\, dy_1\, dy_2 \dots dy_n}$$

$$|J| = \begin{vmatrix} \frac{\partial y_1}{\partial x_1} & \frac{\partial y_1}{\partial x_2} & \dots & \frac{\partial y_1}{\partial x_n} \\ \frac{\partial y_2}{\partial x_1} & \frac{\partial y_2}{\partial x_2} & \dots & \frac{\partial y_2}{\partial x_n} \\ \vdots & \vdots & & \vdots \\ \frac{\partial y_n}{\partial x_1} & \frac{\partial y_n}{\partial x_2} & \dots & \frac{\partial y_n}{\partial x_n} \end{vmatrix}$$

$$\boxed{p_\eta(y_1 y_2 \dots y_n) = \left.\frac{p_\xi(x_1, x_2 \dots x_n)}{|J|}\right|_{\substack{x_1 = g_1(y\dots) \\ x_2 = g_2(y\dots)}}}$$

MMV

RICIAN FADING — CHANGE OF VARIABLES

$$p(r) = \frac{r}{\sigma^2} e^{-\frac{A^2+r^2}{2\sigma^2}} I_0\left(\frac{Ar}{\sigma^2}\right) \qquad x = \frac{r}{\sigma} \qquad \boxed{\frac{dx}{dr} = \frac{1}{\sigma}}$$

$$p(x) = \left.\frac{p(r)}{\frac{dx}{dr}}\right|_{r=\sigma x} = \frac{1}{\frac{1}{\sigma}} \frac{\sigma x}{\sigma^2} e^{-\frac{A^2+\sigma^2 x^2}{2\sigma^2}} I_0\left(\frac{A\sigma x}{\sigma^2}\right)$$

$(a = \frac{A}{\sigma};\ A = a \cdot \sigma) \qquad p(x) = x\, e^{-\frac{\sigma^2 a^2 + \sigma^2 x^2}{2\sigma^2}} I_0\left(\frac{a\sigma \cdot x}{\sigma}\right)$

$$\boxed{p(x) = x \cdot e^{-\frac{a^2+x^2}{2}} I_0(a \cdot x)} \qquad x = \sqrt{\delta} \qquad \boxed{\delta - SNR}$$

MMV

(*) VIDI SHANKAR MANUSCRIPT

- Marcum's Q function (земи ⊛) (види за апроксимација во DTK)

$$P(x<x_0) = \int_0^{x_0} x\, e^{-\frac{a^2+x^2}{2}} I_0(ax)\,dx = \int_0^{\infty} x\, e^{-\frac{a^2+x^2}{2}} I_0(ax)\,dx - \int_{x_0}^{\infty} x\, e^{-\frac{a^2+x^2}{2}} I_0(ax)\,dx$$

$$P(x<x_0) = 1 - \int_{x_0}^{\infty} x\, e^{-\frac{a^2+x^2}{2}} I_0(ax)\,dx = 1 - Q(a, x_0)$$

voltage ratio threshold

$$\boxed{x_0 = \sqrt{\delta}}$$

δ има димензија на моќност!!! МНУ

(за има оваа функција во DTK – некохерентна демодулација на ДАМ)

- Power distribution & outage probability for Rayleigh fading

$$p(r) = \frac{r}{\sigma^2} e^{-\frac{r^2}{2\sigma^2}}$$

Rayleigh fading power distribution:

$$z = r^2 \qquad p(z) = \left.\frac{p(r)}{\frac{dz}{dr}}\right|_{r=\pm\sqrt{z}}$$

не постои за $r<0$!!

$$\boxed{\frac{dz}{dr} = 2r}$$

$$p(z) = \frac{1}{2r}\cdot\frac{\sqrt{z}}{\sigma^2}\cdot e^{-\frac{z}{2\sigma^2}} + \frac{1}{2r}\,\frac{-\sqrt{z}}{\sigma^2}\, e^{-\frac{z}{2\sigma^2}}$$

за $r=\sqrt{z}$ за $r=-\sqrt{z}$

затоа нема сумирања за вториот пат!!

$$p(z) = \frac{1}{2\sigma^2} e^{-\frac{z}{2\sigma^2}} + \frac{1}{2\sigma^2} e^{-\frac{z}{2\sigma^2}} = 2\,\frac{1}{2\sigma^2} e^{-\frac{z}{2\sigma^2}}$$

$$p(\delta) = 2\,\frac{1}{\delta} e^{-\frac{\delta}{\delta}}$$

$$p(z) = \left.\frac{p(r)}{\frac{dz}{dr}}\right|_{r=\sqrt{z}} = \frac{1}{2\sqrt{z}}\cdot\frac{\sqrt{z}}{\sigma^2} e^{-\frac{z}{2\sigma^2}} = \frac{1}{2\sigma^2} e^{-\frac{z}{2\sigma^2}}$$

$$E(r^2) = \int_0^{\infty} r^2 p(r)\,dr = 2\sigma^2 = P_r$$ — средна примена снага!!!

$$\boxed{p(z) = \frac{1}{P_r} e^{-\frac{z}{P_r}} = \frac{1}{2\sigma^2} e^{-\frac{z}{2\sigma^2}} \qquad z>0}$$

распределбата на моќноста во приемникот за скоро секогаш е експоненцијална распределба

- Ако наместо со снаги одиме со SNR ⇒

$$\boxed{p(\delta) = \frac{1}{\bar{\delta}} e^{-\delta/\bar{\delta}}} \qquad \boxed{P(\delta<\delta_0) = 1 - e^{-\delta_0/\bar{\delta}}}$$

види Sankar манускрипт

• NAKAGAMI DISTRIBUTION (CONTINUATION MATLAB CLASS (PROJECT.)

$$f(r) = \frac{2m^m \cdot r^{2m-1}}{\Gamma(m)\cdot \Omega^m} \cdot e^{-\frac{mr^2}{\Omega}} \quad r \geq 0 \quad m \geq \frac{1}{2}$$

$$m = \frac{E^2[r^2]}{E\{[r^2 - E(r^2)]^2\}} \qquad \Omega = E\{r^2\}$$

- CUMULATIVE DISTRIBUTION: $F(r) = P\left(\frac{mr^2}{\Omega}, m\right)$

$P(\,,\,)$ – INCOMPLETE GAMMA FUNCTION

$m = 1 \quad \Gamma(m) = 0! = 1$

$$f(r) = \frac{2 \cdot r}{\Omega} e^{-\frac{r^2}{\Omega}} = \Big/ \Omega = 2\sigma^2 = E(r^2) \Big/ = \frac{r}{\sigma^2} e^{-\frac{r^2}{2\sigma^2}}$$

CHI-SQUARE $\boxed{\chi^2 = \frac{(x - \text{expect}(x))^2}{\text{expect}(x)}}$ $\boxed{\sigma = \sum_{i=1}^{n} \frac{(x_i - \bar{x})^2}{n}}$

APPENDIX II CHI-SQUARE TEST

HIPOTESIS $F(x) = F_0(x)$ FOR SET $(m-1)$ POINTS a_i:

$H_0: F(a_i) = F_0(a_i), \quad 1 \leq i \leq m-1$

$H_1: F(a_i) \neq F_0(a_i), \quad$ some „i"

m – events (bins)

$A_i = \{a_{i-1} < x \leq a_i\}; \quad i = 1, \ldots, m$

$a_0 = -\infty \quad a_m = \infty$

S – SET OF OUTCOMES

k_i – SUCESS OF A_i = NUMBER OF SAMPLES x_i in (a_{i-1}, a_i)

H_0 HIPOTESIS: $P(A_i) = F(a_i) = F_0(a_{i-1}) = \underline{p_{0i}}$

$$\boxed{q = \sum_{i=1}^{m} \frac{(k_i - np_{0i})^2}{np_{0i}}}$$

MMV

n – TOTAL NUMBER OF SAMPLES OBSERVED

VO OVOJ SLUČAJ SE PRIFAĆA HIPOTEZATA

<u>ACCEPT</u> q IF $\boxed{q < \chi_{1-\alpha}(m-1)}$

$m = ?$
$\alpha = ?$

$$\boxed{\chi^2(\nu) = \frac{x^{\frac{\nu}{2}-1} e^{-\frac{x}{2}}}{2^{\frac{\nu}{2}}\, \Gamma\left(\frac{\nu}{2}\right)}}$$

E.G. $\nu = 2$ $\quad \chi^2(\nu) = \frac{e^{-\frac{x}{2}}}{2 \cdot \underbrace{\Gamma(1)}_{1}} = \frac{1}{2} e^{-\frac{x}{2}}$

$\min(y) = 0.0435 \quad \max(y) = 12.4655 \quad$ bins = 20

$\text{inter} = \max(y) - \min(y) = 12.4220 \quad \text{intstep} = \frac{\text{inter}}{20} = 0.6211$

$F_y(i) = \text{trapz}(x, f_y) \quad x = 0 : \text{intval}(i) \big/ (i \cdot 400) : \text{inval}(i)$

• LOGNORMAL DISTRIBUTION

$$a_i = \prod_{j=1}^{M_i} a_{ji} \qquad s = \sum_{i=1}^{N} a_i \cos(\omega_0 t + \omega_{d_i} t + \phi_i)$$

M_i – MULTIPLE REFLECTIONS PER PATH

LOGNORMAL DISTRIBUTION

$$\boxed{f(r) = \frac{1}{r\sigma\sqrt{2\pi}} e^{-\frac{(\ln r - \mu)^2}{2\sigma^2}} \qquad r \geq 0}$$

$\mu = \text{mean}(\log(r))$ $\ln$

$\sigma = \text{var}(\log(r))$

• RECEIVED POWER $\quad y(t) = I^2(t) + Q^2(t)$

$$\bar{r} = \sigma\sqrt{\frac{\pi}{2}} \qquad \sigma = \sqrt{\frac{2}{\pi}}\,\bar{r} \qquad \sigma^2 = b = \frac{2}{\pi}(\bar{r})^2$$

$$\bar{r} = \int_0^\infty r f(r)\,dr = \frac{1}{\sigma\sqrt{2\pi}} \int_0^\infty e^{-\frac{(\ln r - \mu)^2}{2\sigma^2}}\,dr$$

$\ln r + \mu = x \qquad \frac{1}{r}dr = dx \qquad x = 0 \quad r = e^{-\mu}$

$x = \infty \qquad \ln r = -\mu + \infty \qquad r = e^{+\infty} = \infty$

$$\bar{r} = \frac{1}{\sigma\sqrt{2\pi}} \int_{e^{-\mu}}^{\infty} r\, e^{-\frac{x^2}{2\sigma^2}}\,dx \qquad \boxed{r = e^{x-\mu}}$$

$$\bar{r} = \frac{1}{\sigma\sqrt{2\pi}} \int_{e^{-\mu}}^{\infty} e^{-\frac{x^2}{2\sigma^2} + x - \mu}\,dx = \frac{e^{-\mu - \frac{\sigma^2}{2}}}{\sigma\sqrt{2\pi}} \underbrace{\int_{e^{-\mu}}^{\infty} e^{-\frac{(x+\sigma^2)^2}{2\sigma^2}}\,dx}_{I}$$

$$\frac{x^2 + 2\sigma^2 x + \sigma^4}{2\sigma^2} - \mu - \frac{\sigma^2}{2} = \frac{(x+\sigma^2)^2}{2\sigma^2} - \mu - \frac{\sigma^2}{2}$$

$$I = \int_{e^{-\mu}}^{\infty} e^{-\left(\frac{x+\sigma^2}{\sqrt{2}\sigma}\right)^2} dx \qquad \frac{x+\sigma^2}{\sqrt{2}\sigma} = t \qquad \frac{dx}{\sigma\sqrt{2}} = dt \qquad dx = \sigma\sqrt{2}\,dt$$

$$x = e^{-\mu} \quad t = \frac{e^{-\mu}+\sigma^2}{\sqrt{2}\sigma} \qquad x = \infty \quad t = \infty$$

$$I = \int_{\frac{e^{-\mu}+\sigma^2}{\sqrt{2}\sigma}}^{\infty} e^{-t^2} \cdot \sigma\sqrt{2}\,dt = \sigma\sqrt{2} \cdot \frac{\sqrt{\pi}}{2} \cdot \frac{2}{\sqrt{\pi}} \int_{\frac{e^{-\mu}+\sigma^2}{\sqrt{2}\sigma}}^{\infty} e^{-t^2}\,dt \qquad \text{erfc}\left(\frac{e^{-\mu}+\sigma^2}{\sqrt{2}\sigma}\right)$$

$$\bar{r} = \frac{e^{-\mu - \frac{\sigma^2}{2}}}{\sigma\sqrt{2\pi}} \frac{\sqrt{\pi}}{\sqrt{2}} \sigma\, \text{erfc}\left(\frac{e^{-\mu}+\sigma^2}{\sqrt{2}\sigma}\right) = \frac{e^{-\mu - \frac{\sigma^2}{2}}}{2} \text{erfc}\left(\frac{e^{-\mu}+\sigma^2}{\sqrt{2}\sigma}\right)$$

$$\bar{r} = \int_0^\infty \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(\ln r-\mu)^2}{2\sigma^2}} dr = \left|\mu = \ln m\right| = \frac{1}{\sigma\sqrt{2\pi}} \int_0^\infty e^{-\frac{\ln^2(m\cdot r)}{2\sigma^2}} dr$$

$$t = \frac{\ln(m\cdot r)}{2\sigma^2} \qquad dt = \frac{1}{m\cdot r}\cdot m \frac{1}{2\sigma^2} dr \qquad dr = r \cdot 2\sigma^2 dt$$

$$r = \frac{1}{m} e^{t/2\sigma^2}$$

$$t = \frac{\ln^2(m r)}{2\sigma^2} \qquad dt = \frac{2\ln(m\cdot r)}{2\sigma^2}\cdot\frac{1}{mr}\cdot m\, dr$$

$$dt = \frac{\ln(m\cdot r)}{r\sigma^2} dr$$

$$\boxed{m^r = e^{r\ln m}} \qquad e^{-\ln^2(mr)} = e^{-\ln(mr)\cdot\ln(mr)} = \left[e^{\ln(mr)}\right]^{-\ln mr} = (mr)^{-\ln(mr)}$$

$$\frac{1}{\sigma\sqrt{2\pi}}\int_0^\infty (m\cdot r)^{-\frac{\ln(mr)}{2\sigma^2}} dr = \frac{m^{-\frac{\ln(mr)}{2\sigma^2}}}{\sigma\sqrt{2\pi}} \int_0^\infty r^{-\frac{\ln(mr)}{2\sigma^2}} dr \;?$$

- KODIRANI COOPERATIVNI (DA SO IZMEŠA SO SVOJATA INFORMACIJA)

KODIRANI KOOPERATIVNI KOMUNIKACII
- CHANEL EFFICIENCY
- DIVERSITY MULTIPLEXING TRADEOF

- SIMULACISKI VS KOMPJUTERSKI MODELI
- KOVARIANCE, KARHUNEN - LOAVE
- GAUSSIAN METHODS FOR NUMERICAL INTEGRATION
- BASE FUNCTIONS ??

- KARHUNEN - LOEVE THEOREM

$X(t)$ $t\in[a,b]$ $\qquad R(t,\tau)$ - COVARIANCE

$X(t)$ - SECOND ORDER RANDOM PROCESS

$E[X(t)] = \emptyset \quad t\in[a,b] \qquad E[X(t)X(t')] = R(t,\tau) \quad t,\tau\in[a,b]$

$$\mathrm{Cov}(X,Y) = E[(X-\mu)(Y-\nu)] = E[XY - \nu X - \mu Y + \mu\nu] =$$
$$E[XY] - \nu \underbrace{E[X]}_{\mu} - \mu \underbrace{E[Y]}_{\nu} + \mu\nu = E[XY] - \mu\nu$$

$$\sum_{i,j=1}^{n} c_i R(t_i, t_j) = E\left[\left(\sum_{i=1}^{n} c_i X(t_i)\right)^2\right] \geq \emptyset$$

g - REAL FUNCTION ON $[a,b]$ $\qquad \int_a^b g(t)\cdot X(t)\,dt$

$\Delta: a = t_0 < t_1 < \dots < t_n = b$ $\qquad |\Delta| = \max_{1 \le i \le n} |t_i - t_{i-1}|$

$$I(\Delta) = \sum_{k=1}^{n} g(t_k) X(t_k)(t_k - t_{k-1})$$

IF: $E\left[|I(\Delta) - I|^2\right] \to 0$ AS $|\Delta| \to 0$ THEN $g(t)X(t)$ IS INTEGRABLE OVER $[a,b]$

$$\boxed{I = \int_a^b g(t) X(t)\, dt}$$

Ω – PROBABILITY SPACE ON WICH $X(t)$ ARE DEFINED

Theorem 3.1 g CONTINOUS ON $[a,b]$ COVARIANCE FUNCTION $R(t,\tau)$ IS CONTINOUS OVER $[a,b]\times[a,b]$ $g(t)X(t)$ IS INTEGRABLE OVER $[a,b]$

$$\lim_{|\Delta|,|\Delta'| \to 0} E[I(\Delta) I(\Delta')] = \int_a^b \int_a^b g(t) g(\tau) R(t,\tau)\, dt\, d\tau$$

THEOREM 3.4 KARUHEN-LOEVE EXPANSION

$$\boxed{X(t) = \sum_{k=1}^{\infty} Z_k e_k(t) \qquad a \le t \le b}$$

e_k – EIGENFUNCTIONS OF INTEGRAL OPERATOR

$Z_k = \int_a^b X(t) e_k(t)\, dt$ $\qquad E[Z_k Z_j] = 0$ $\quad$ <u>ORTHOGONAL</u>

$$S = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \end{bmatrix} = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ x_1 & x_2 & x_3 & x_4 \\ x_1 & x_2 & x_3 & x_4 \end{bmatrix}$$

$x_1 x_1 x_1 x_2 x_2 x_2 x_3 x_3 x_3 x_4 x_4 x_4$

$a = [1\,1\,1\,1\;2\,2\,2\,2\;3\,3\,3\,3\;4\,4\,4\,4]$

$$A(i,j) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 \\ 3 & 3 & 3 & 3 \\ 4 & 4 & 4 & 4 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 4 & 0 & 0 \end{bmatrix} \qquad \boxed{A(:,2)}$$

$\frac{E_b}{N_0}$ – RATIO OF BIT ENERGY VS NOISE POWER SPECTRAL DENS.

$\frac{E_s}{N_0}$ – RATIO OF SYMBOL " VS – // –

$$\frac{E_s}{N_0} = \frac{E_b}{N_0}(dB) + 10\log_{10}(k)$$

$$10\log\frac{E_s}{N_0} = 10\log\frac{E_b}{N_0}\cdot k \qquad \frac{E_s}{N_0} = \frac{E_b}{N_0}\cdot k$$

k – INFORMATION BITS PER SYMBOL

• RELATIONSHIP BETWEEN E_s/N_0 AND SNR

$$\frac{E_s}{N_0}(dB) = 10\log_{10}\left(\frac{T_{SYM}}{T_{SAMP}}\right) + SNR(dB)$$ COMPLEX INPUT SIGNALS

$$\frac{E_s}{N_0}(dB) = 10\log_{10}\left(\frac{0.5\,T_{SYM}}{T_{SAMP}}\right) + SNR(dB)$$ REAL INPUTS

T_{SYM} - SIGNAL SYMBOL PERIOD T_{SAMP} - SIGNAL SAMPL. PERIOD

EXAMPLE: $T_{SYM} = 4T_{SAMP}$ COMPLEX

$$\frac{E_s}{N_0}(dB) = SNR + 10\log 4 = \underline{6dB + SNR}$$

COMPLEX:

$$\frac{E_s}{N_0} = 10\log\left(\frac{S\cdot T_{SYM}}{N\cdot\frac{1}{BN}}\right) = 10\log\left(\frac{S}{N}\right)\cdot(T_{SYM}\cdot B_N) = 10\log\frac{S}{N} + 10\log\frac{T_{SYM}}{T_{SAMP}}$$

$\frac{1}{T_S} = F_S$

$f_S \geq 2\cdot f_{max}$

REAL: $$\frac{E_s}{N_0} = 10\log\frac{S\cdot T_{SYM}}{\frac{N_0}{2}} ?$$

COMPLEX

$$SNR = \frac{P_r}{N_0\cdot B} = \frac{E_s}{N_0\cdot B\cdot T_s} \qquad \frac{E_s}{N_0} = B\cdot T_{SYM}\cdot SNR$$

$\frac{1}{T_{SAMP}}$

$$10\log\frac{E_s}{N_0} = 10\log\frac{T_{SYM}}{T_{SAMP}} + 10\log SNR$$

$W = B = \frac{F_S}{2} = \frac{1}{2T_{SAMP}}$

ZARADI NYQUIST-OVIOT KRITERIUM!! $\frac{1}{T_{SAMP}}$

REAL

$$SNR = \frac{P_r}{N_0\cdot\frac{B}{2}} = \frac{E_s}{T_s\cdot N_0\cdot\frac{B}{2}} \qquad \frac{E_s}{N_0} = T_s\cdot\frac{B}{2}\cdot SNR$$

$$\boxed{10\log\frac{E_s}{N_0} = 10\log\left(\frac{0.5\,T_s}{T_{SAMP}}\right) + 10\log SNR}$$

(**) PODOBRO E IZVEDUVANJETO NA STR.30 !!!

• BinaryAM_BER

$T_{SYM} = 100\cdot T_{SAMP}$ $\frac{T_{SYM}}{T_{SAMP}} = 100$ $\frac{E_s}{N_0} = 10\log(0.5)10^2 + SNR$

$$\frac{E_s}{N_0} = 20dB - 10\log 2 + SNR = 17dB + SNR$$

SER = berawgn(EbNo, 'pam', M) $\cdot$ ld(M)

$$\rho_{max} = \frac{A^2}{\sigma_N^2} = \frac{E}{N_0}$$ → MAKSIMALEN

SNR = EbNo + 3 + 10log10(k)

$$r(t) = U_0\, a_m\, x(t)\cdot\cos(\omega_0 t) \qquad a_m \in \{0, 1\}$$

$$s(t) = \sum_{m=-\infty}^{\infty} a_m x(t - nT_b) \qquad x(t) = \begin{cases} A_p & 0 \leq t \leq T_b \\ 0 & \text{OUT} \end{cases}$$

$$\boxed{y(t) = \frac{1}{2} x(t) \cdot \cos\omega_0 t \pm \frac{1}{2} \hat{x}(t) \sin\omega_0 t} \qquad \hat{x}(t) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{x(\tau)}{t-\tau} d\tau$$

HILBERTOVA TRANSFORMACIJA

- DFM $\quad a_m \in \{1, -1\}$

$$y(t) = U_m \cdot \cos(\omega_0 t + \varphi_{FM}(t)) \qquad \varphi_{FM}(t) = \int M_m(t)\, dt$$

$$y_{FM}(t) = U_m \cos(\omega_0 t + K_\omega \int M_m(t) dt)$$

$$\delta\Phi_{FM} = \frac{d\varphi}{dt} \Rightarrow \varphi_{FM} = \int \delta\Phi_{FM}\, dt = K_\omega \int M_m(t)\, dt \qquad \boxed{\delta\Phi_{FM} = K_\omega M_m(t)}$$

ΦM

$$\Phi_i = \Phi_0 + \delta\Phi_i \qquad \omega_i = \frac{d\Phi_i}{dt} = \omega_0 + K_\varphi U_m \frac{d u_m(t)}{dt}$$

$$\delta\Phi_i = K_\varphi M_m(t) \qquad = \omega_0 + \delta\omega_i = \omega_0 + K_\omega M_m(t)$$

$$\Phi_i = \int \omega_i\, dt = \omega_0 t + K_\omega \int M_m(t) dt \quad FM$$

$$y(t) = U_0 \cos[\omega_0 t + a_m \cdot \omega_d \int_0^t x(\omega) d\omega] \qquad \boxed{m = \frac{K_\omega \cdot U_m}{\omega_m}}$$

$$M_{FM}(t) = U_0 \cos[\omega_0 t + K_\omega \int U_m \cos\omega_m t\, dt] = U_0 \cos[\omega_0 t + \frac{K_\omega U_m}{\omega_m} \sin\omega_m t]$$

$$m = \frac{\omega_d}{\pi U} = \frac{2 f_d}{U} \qquad U \text{ - BRZINA DIGITA}$$

EXAMPLE: $\quad U_B = 2400 \frac{bit}{sec} \qquad f_N = \frac{1}{2T_B} = 1200\ Hz$

$$\boxed{B = 3 \cdot f_N = 3600\ Hz} \qquad \boxed{f_N = \frac{f_B}{2}} \qquad B = \frac{3 \cdot U_B}{2} \qquad \boxed{\frac{U_B}{B} = \frac{2}{3}} \frac{bit}{Hz}$$

DPM $\quad y(t) = U_0 \cos[\omega_0 t + \Phi a_m x(t)]$

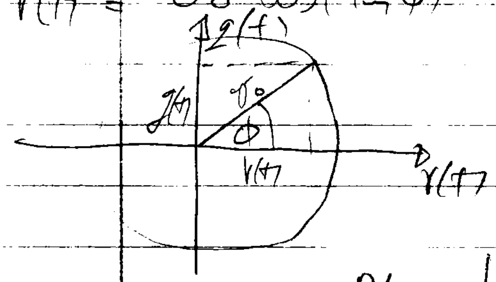
$$x(t) = 1 \quad y(t) = U_0 [\cos\omega_0 t \cdot \cos(a_m\Phi) - \sin(\omega_0 t) \sin(a_m\Phi)]$$

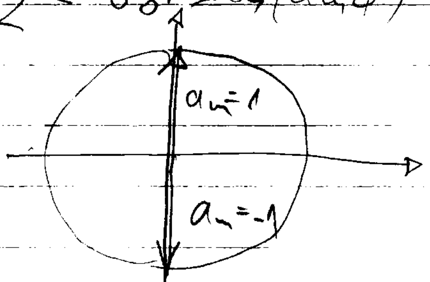
$$y(t) = r(t) \cos(\omega_0 t) - q(t) \cdot \sin(\omega_0 t) = Re\{(\underline{R} + \underline{Q}) e^{j\omega_0 t}\}$$

$$\underline{R} = r(t) \cdot e^{j0} \qquad \underline{Q} = q(t) \cdot e^{j\frac{\pi}{2}}$$

$$Re\{q(t) \cdot e^{j(\omega_0 t + \frac{\pi}{2})}\} = q(t) \cdot \cos(\omega_0 t + \frac{\pi}{2}) = -q(t) \sin(\omega_0 t)$$

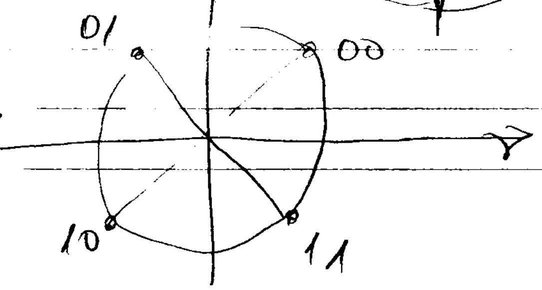
$$r(t) = U_0 \cos(a_m\Phi) \qquad q = U_0 \sin(a_m\Phi)$$





BINARNA DPM

- KVATERNARNA

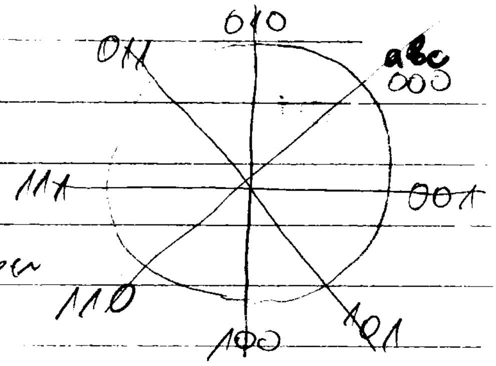


$$B = 4 \cdot f_N = 4 \cdot \frac{f_s}{2} = 4 \cdot \frac{1}{2T_s} = \left| T_s = 2 \cdot T_b \right| = \cancel{4} \cdot \frac{1}{\cancel{4}T_b} = \frac{1}{T_b} = v_b$$

$$\boxed{\frac{v}{B} = 1 \ \frac{bit}{Hz}}$$

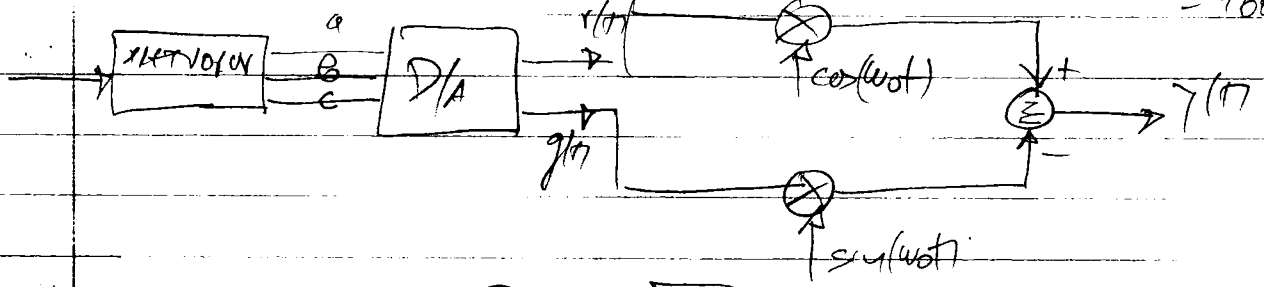
• OKTALNA MODULACIJA

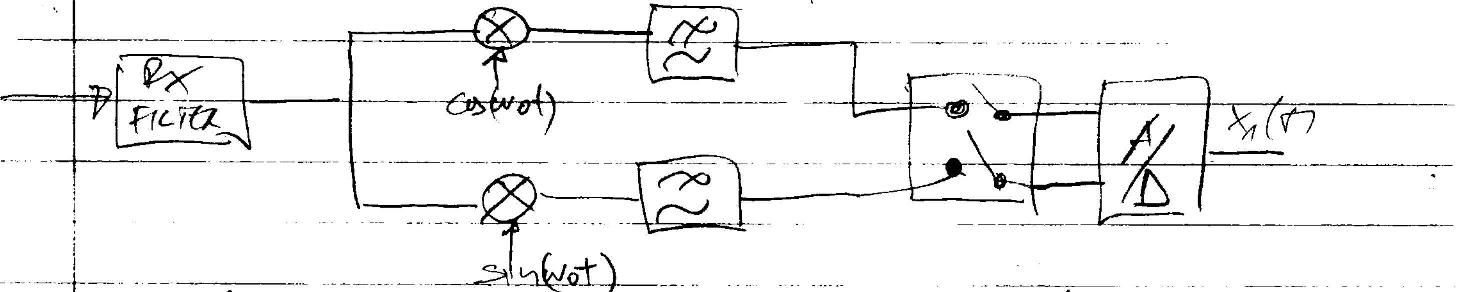
TRI BITA ZA DOBIVANJE EDEN



$$f_d = \frac{f_b}{ld M} \quad (4800)$$

$$f_d = \frac{4800}{ld 8} = 1600 \ \frac{dig}{sec} = 1600 \ bd$$



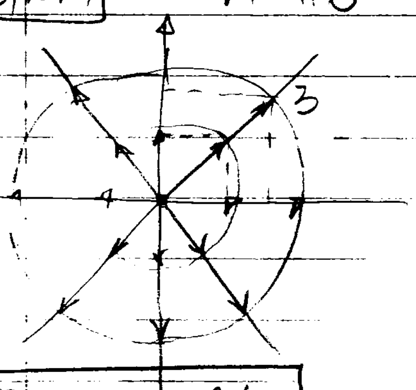


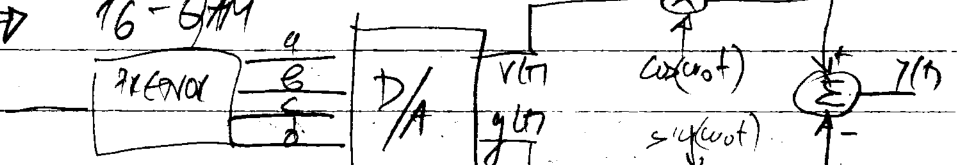
$v_b = 4800 \ \frac{bit}{sec}$

$$B = 3 f_N = 3 \cdot \frac{1}{2 \cdot T_d} = 3 \cdot \frac{1}{2 \cdot 3 T_b} = \frac{v_b}{2} = 2400 \ Hz$$

$$\boxed{\frac{v_b}{B} = 2 \ \frac{bit}{Hz}}$$

• QAM $M = 16 \Rightarrow$ 16-QAM





QAM-16 $v_b = 9600 \ \frac{bit}{sec}$

$$B = 4 \cdot f_N = 4 \cdot \frac{1}{2T_s} = \cancel{4} \frac{1}{2 \cdot \cancel{4} T_b} = \frac{v_b}{2}$$

$$B = \frac{9600}{2} = 4800 \ Hz$$

$$\boxed{\frac{v_b}{B} = 2 \ \frac{bit}{Hz}}$$

$$f_d = \frac{v_b}{ld M} = \frac{9600}{ld 16} = \frac{9600}{4} = 2400 \ bd$$

$\text{qammod}(x, M)$ $\quad M = 4$

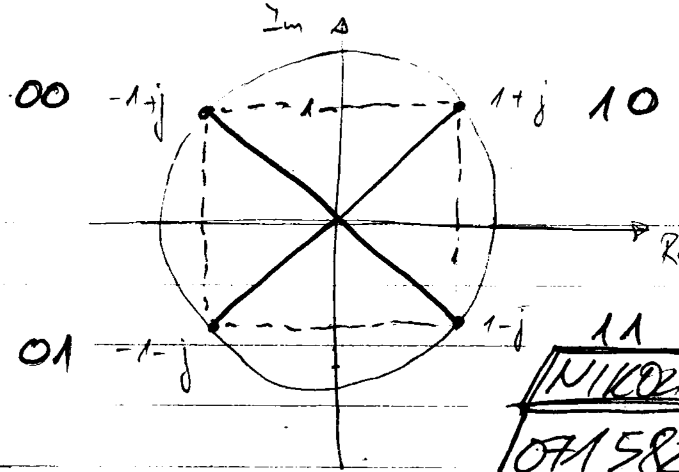
$\text{qammod}(1, 4) = -1-j$

$\text{qammod}(2, 4) = +1+j$

$\text{qammod}(3, 4) = 1-j$

$\text{qammod}(0, 4) = -1+j$

M - ALFABET SIZE



NIKOLINA

071582617

$s = [0, 1, 2, 3] = [00, 01, 10, 11]$

$\underline{T_s = 0.27\,\mu s}$ $\qquad T = 5000 \cdot T_s = 1350\,\mu s = 1.3\,ms$

$4\,\text{digit}/1.3\,ms = 2963$ baud $\qquad 8\,\text{bit}/1.3\,ms = \underline{5926\,\text{bps}}$

PM $\quad y = \cos(\omega_c t + \text{opt} \cdot x)$

$\text{opt} = \dfrac{\pi}{\max(\max(x))} \Rightarrow$ DEFAULT

$SNR = \dfrac{P_R}{N_0 B} = \dfrac{E_s}{T_s \cdot N_0 \cdot B}$ $\to$ ENERGY $\qquad \dfrac{E_s}{N_0} = SNR \cdot T_s \cdot B$ $\qquad \dfrac{f_{SAMP}}{2}$

$\boxed{B = \dfrac{f_{SAMP}}{2} = \dfrac{1}{2} \dfrac{1}{T_{SAMP}}}$ $\qquad \dfrac{E_s}{N_0} = SNR \cdot \dfrac{T_s}{2 \cdot T_{SAMP}}$

$E_s = K \cdot E_b \qquad K = ld(M) \Rightarrow$ DOLZINA NA KODOT

$\dfrac{E_b \cdot K}{N_0} = SNR \cdot \dfrac{0.5\,T_s}{T_{SAMP}} \quad / 10\log$

$10\log \dfrac{E_b}{N_0} + 10\log(K) = 10\log SNR + 10\log \dfrac{0.5\,T_s}{T_{SAMP}}$

$\boxed{10\log SNR = 10\log \dfrac{E_b}{N_0} + 10\log(K) - 10\log n_{SAMP}}$

n_{SAMP} - OVERSAMPLING RATE $\qquad f_c = 4 \cdot f_s \Rightarrow n_{SAMP} = 2$

$\boxed{n_{SAMP} = \dfrac{f_s}{2B}}$

MMV

$SER = \text{berawgn}(EbNo, \text{'qam'}, M) \cdot \log2(M)$ $\qquad \boxed{SER = \log2(M) \cdot BER}$

FSK $\quad M \cdot \text{freq\_step} <= fs$

1000

• PROAKIS, CONTEMPORARY COMM. SYSTEMS

⊙ DIGITAL TRANSMISSION VIA CARRIER MODULATION

• CARRIER - AM (ASK)

PAM: $s_m(t) = A_m g_T(t)$

$|f| < W$ $\qquad A_m = (2m - 1 - M) \cdot d \qquad m = 1, 2, \ldots M$

$M = 2 \qquad m = 1 \quad 2 - 1 - 2 = -1$

$\qquad m = 2 \quad 4 - 1 - 2 = 1$

$$\boxed{\text{DPM carrier}} \quad P(\epsilon) = \frac{1}{2} \operatorname{erfc}\left(\frac{\hat{\rho}}{\sqrt{2}}\right) = \frac{1}{2}\left(\frac{2}{\sqrt{\pi}} \int_{\hat{\rho}/\sqrt{2}}^{\infty} e^{-x^2} dx\right) = \left(1 - \operatorname{erf}\left(\frac{\hat{\rho}}{\sqrt{2}}\right)\right) \cdot \frac{1}{2}$$

$$= \frac{1}{2}\left(\frac{2}{\sqrt{\pi}} \underbrace{\int_0^{\infty} e^{-x^2} dx}_{\frac{\sqrt{\pi}}{2}} - \frac{2}{\sqrt{\pi}} \int_0^{\hat{\rho}/\sqrt{2}} e^{-x^2} dx\right) = \left(1 - \operatorname{erf}\left(\frac{\hat{\rho}}{\sqrt{2}}\right)\right)\frac{1}{2}$$

$$\boxed{\hat{\rho}^2 = \frac{A^2}{\sigma_N^2} \qquad \rho^2 = \frac{A^2}{2\sigma_N^2}} \quad \boxed{\rho^2 = \frac{\hat{\rho}^2}{2}} \quad \boxed{\rho = \frac{\hat{\rho}}{\sqrt{2}}}$$

$$P(\epsilon) = \frac{1}{2}\left(1 - \operatorname{erf}(\rho)\right) \qquad \boxed{\hat{\rho} = \sqrt{\frac{\epsilon_S}{N_0}}} \quad \rho = \sqrt{\frac{\epsilon_S}{2N_0}}$$

$$P(\epsilon) = \frac{1}{2} \operatorname{erfc}(\rho) \qquad \epsilon_S = K \cdot \ldots \qquad \rho = \sqrt{\frac{K}{2}} \cdot \sqrt{\frac{\epsilon_B}{N_0}} = \sqrt{\frac{K \cdot \epsilon_{BN_0}}{2}}$$

$$\rho = \sqrt{\frac{ld(M) \cdot \epsilon_{BN_0}}{2}} \qquad P(\epsilon) = \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{\epsilon_{BN_0}\, ld(M)}{2}}\right)$$

$$P(\epsilon) \doteq \frac{1}{\hat{\rho}\sqrt{2\pi}} e^{-\hat{\rho}^2/2} \qquad \hat{\rho} \gg 1 \qquad \hat{\rho} = \sqrt{2} \cdot \rho$$

$$P(\epsilon) = \frac{1}{2\rho\pi} e^{-\rho^2} = \frac{\sqrt{2}}{2\pi\sqrt{K \epsilon_{BN_0}}} \cdot e^{-\frac{K \cdot \epsilon_{BN_0}}{2}} = \frac{1}{\pi\sqrt{K \cdot \epsilon_{BN_0}}} e^{-\frac{K \epsilon_{BN_0}}{2}}$$

$$\boxed{Q(z) = \frac{1}{2} \operatorname{erfc}\left(\frac{z}{\sqrt{2}}\right)}$$

• BIPOLAR SIGNAL (BASEBAND) (MATCHED FILTER)

SKLAR $\quad P_b = Q\left(\sqrt{\frac{2\epsilon_b}{N_0}}\right) = \frac{1}{2} \operatorname{erfc}\left(\frac{1}{\sqrt{2}} \sqrt{\frac{2\epsilon_B}{N_0}}\right) = \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{\epsilon_B}{N_0}}\right)$

• UNIPOLAR SIGNAL (BASEBAND) (MATCHED FILTER)

SKLAR $\quad P_b = Q\left(\sqrt{\frac{\epsilon_b}{N_0}}\right) = \frac{1}{2} \operatorname{erfc}\left(\frac{1}{\sqrt{2}} \sqrt{\frac{\epsilon_B}{N_0}}\right) = \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{\epsilon_B}{2N_0}}\right)$?

$$\boxed{\text{ODTK}} \quad P(\epsilon)\Big|_{DAM} = \frac{1}{2} \operatorname{erfc} \frac{\hat{\rho}}{2\sqrt{2}} = \frac{1}{2} \operatorname{erfc} \frac{\rho}{2} = \frac{1}{2} \operatorname{erfc} \frac{1}{2}\sqrt{\frac{\epsilon_{BN_0}}{2}} = \frac{1}{2} \operatorname{erfc} \frac{1}{2}\sqrt{\frac{\epsilon_B}{2N_0}}$$

BIPOLAREN

$$\boxed{\sigma_N^2 = \frac{N_0}{2}}$$ - VARIANCE OF WHITE NOISE

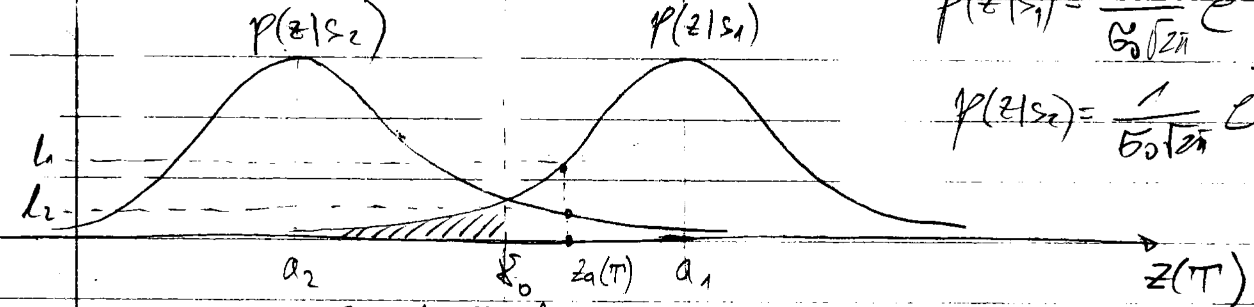
$$\hat{\rho}^2 = \frac{A^2}{\sigma_N^2} = \boxed{\rho^2 = \frac{A^2}{2\sigma_N^2} = \frac{\hat{\rho}^2}{2}} \qquad \hat{\rho}^2 = \frac{A^2}{\sigma_N^2} = \frac{E_s}{N_0} \quad \boxed{\hat{\rho} = \sqrt{\frac{E_s}{N_0}}}$$

$$\boxed{P_B = \int_x^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{u^2}{2}} du = Q(x)}$$ SKLAR DEFINITION

$$P_B = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-u^2/2} du = \left| \begin{matrix} t = \frac{u}{\sqrt{2}} & u = x \;\; t = \frac{x}{\sqrt{2}} \\ du = \sqrt{2}\,dt & u = \infty \;\; t = \infty \end{matrix} \right|$$

$$P_B = \frac{1}{\sqrt{2\pi}} \int_{x/\sqrt{2}} e^{-t^2} dt \cdot \sqrt{2} = \frac{1}{\sqrt{\pi}} \int_{x/\sqrt{2}}^{\infty} e^{-t^2} dt = \frac{1}{2} \operatorname{erfc}\left(\frac{x}{\sqrt{2}}\right)$$

$$\boxed{Q(x) = \frac{1}{2} \operatorname{erfc}\left(\frac{x}{\sqrt{2}}\right)} \qquad \boxed{x > 3 \quad Q(x) \approx \frac{1}{x\sqrt{2\pi}} e^{-\frac{x^2}{2}}}$$



$$p(z|s_1) = \frac{1}{\sigma_0\sqrt{2\pi}} e^{-\frac{(z-s_1)^2}{2\sigma_0^2}}$$

$$p(z|s_2) = \frac{1}{\sigma_0\sqrt{2\pi}} e^{-\frac{(z-s_2)^2}{2\sigma_0^2}}$$

SIGNAL POWER

$$\frac{E_b}{N_0} = \frac{S \cdot T_b}{N/W} = \frac{S \cdot T_b}{N} \cdot W \qquad \frac{S}{N} = \frac{E_b}{T_b \cdot N_0 \cdot W}$$ BANDWIDTH

DATA RATE

$$\boxed{\frac{E_b}{T_b} = \frac{S}{N} \cdot T_b \cdot W = SNR \cdot T_b W = \frac{S}{N} \cdot \frac{W}{R_b}}$$

BASEBAND

DTK VEROJATNOST NA GREŠKA NA M-EN SIGNAL

$$P(\varepsilon) = \frac{(M-1)}{M} \operatorname{erfc} \frac{d}{\sqrt{2}\sigma_N}$$

STANUVA ISTO SO SKLAR AKO ZEMEŠ $N_0 = \frac{N_0}{2}$

BPSK (DTK) $$P(\varepsilon) = \frac{1}{2} \operatorname{erfc} \frac{\hat{\rho}}{\sqrt{2}} = \frac{1}{2} \operatorname{erfc} \sqrt{\frac{E_s}{2N_0}}$$

BPSK (SKLAR) $$P(\varepsilon) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = \frac{1}{2} \operatorname{erfc}\left(\frac{1}{\sqrt{2}} \sqrt{\frac{2E_b}{N_0}}\right) = \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$$

$$b_0 = \frac{A_r}{\sigma_N}$$

UNIPOLAREN:

$$P(\varepsilon) = P(0) \frac{1}{2}\left[1 - \phi\left(\frac{b_0}{\sqrt{2}}\right)\right] + P(1) \frac{1}{2}\left[1 - \phi\left(\frac{\hat{\rho} - b_0}{\sqrt{2}}\right)\right]$$ 1/2 1/2

$$P(\varepsilon) = \frac{1}{4}\left[1 - \phi\left(\frac{A_r}{\sqrt{2}\sigma_N}\right)\right] + \frac{1}{4}\left[1 - \phi\left(\frac{A - A_r}{\sqrt{2}\sigma_N}\right)\right]$$

$$P(\varepsilon) = \left| A = \frac{A}{2} \right| = \frac{1}{4}\left[1-\phi\left(\frac{\rho}{2\sqrt{2}}\right)\right] + \frac{1}{4}\left[1-\phi\left(\frac{\rho}{2\sqrt{2}}\right)\right]$$

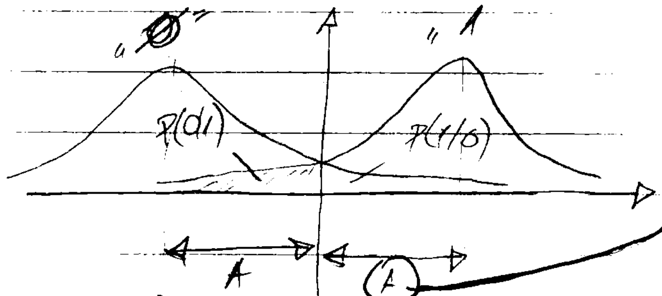
$$= \frac{1}{2}\left[1-\phi\left(\frac{\rho}{2\sqrt{2}}\right)\right] = \frac{1}{2}\,\mathrm{erfc}\left(\frac{\rho}{2\sqrt{2}}\right) = \frac{1}{2}\,\mathrm{erfc}\left(\frac{\rho}{2}\right)$$

$$P(\varepsilon) = \frac{1}{2}\,\mathrm{erfc}\left(\frac{1}{2}\sqrt{\frac{E_s}{2N_0}}\right)$$ SKLAR $$P_b = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$$

$$P_b = \frac{1}{2}\,\mathrm{erfc}\sqrt{\frac{E_b}{2N_0}}$$

$$\boxed{\rho = \sqrt{\frac{E_b}{N_0}}}$$

• BIPOLAR



VO DTK ZEMA $\frac{A}{2}$
PA ZATOA IZLEGUVA
$\frac{1}{2}\,\mathrm{erfc}\left(\frac{A}{2\sqrt{2}}\right)$

$$P(\varepsilon) = P(0)\cdot P(1/0) + P(1)\,P(0/1) \qquad P(0) = P(1) = \frac{1}{2}$$

$$P(1/0) = \int_0^\infty \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{(x+A)^2}{2\sigma^2}}\,dx \qquad \frac{x+A}{\sigma\sqrt{2}} = t \qquad dt = \frac{dx}{\sigma\sqrt{2}}$$

$x = 0 \quad t = \frac{A}{\sigma\sqrt{2}}$
$x = \infty \quad t = \infty$

$$P(1/0) = \frac{1}{\sigma\sqrt{2\pi}}\cdot\sigma\sqrt{2}\int_{A/\sigma\sqrt{2}}^{\infty} e^{-t^2}\,dt = \frac{1}{\sqrt{\pi}}\int_{A/\sigma\sqrt{2}}^{\infty} e^{-t^2}\,dt = \frac{1}{2}\,\mathrm{erfc}\left(\frac{A}{\sigma\sqrt{2}}\right)$$

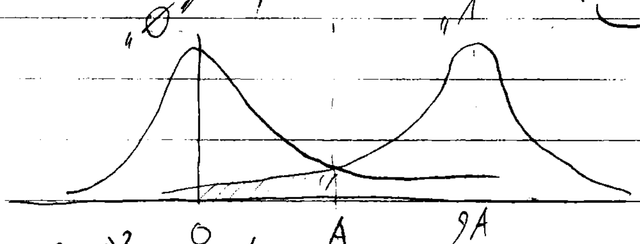
$$P(\varepsilon) = \frac{1}{2}\cdot\frac{1}{2}\,\mathrm{erfc}\left(\frac{A}{\sigma\sqrt{2}}\right) + \frac{1}{2}\cdot\frac{1}{2}\,\mathrm{erfc}\left(\frac{A}{\sigma\sqrt{2}}\right) = \frac{1}{2}\,\mathrm{erfc}\left(\frac{A}{\sigma\sqrt{2}}\right) = \frac{1}{2}\,\mathrm{erfc}\sqrt{\frac{E_b}{2N_0}}$$

DTK

• SKLAR: $$P_b = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = \frac{1}{2}\,\mathrm{erfc}\sqrt{\frac{E_b}{N_0}}$$

POVTORNO SE ~~RAZLIKUVAAT~~ SAMO
VO $\boxed{N_0 = \frac{N_0}{2}}$ OVA TREBA DA SE STAVI VO DTK

• UNIPOLAR

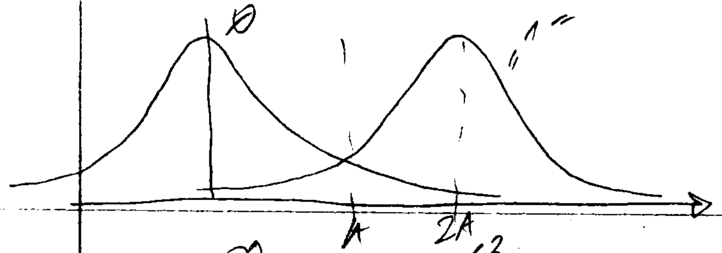


$$P(0/1) = \int_{-\infty}^{A} \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{(x-A)^2}{2\sigma^2}}\,dx \quad \Bigg\| \quad \frac{x-A}{\sigma\sqrt{2}} = t \qquad \frac{dx}{\sigma\sqrt{2}} = dt \qquad dx = \sigma\sqrt{2}\,dt$$

$x = -\infty \quad t = -\infty \qquad x = A \quad t = 0$

$$= \int_{-\infty}^{0} \frac{1}{\sigma\sqrt{2\pi}}\, e^{-t^2}\,dt = \left| \begin{matrix} u = -t & t = -\infty & u = \infty \\ t = 0 & u = 0 & \end{matrix} \right| = -\int_{+\infty}^{0} \frac{1}{\sigma\sqrt{2\pi}}\, e^{-u^2}\,du$$

$$P(0/1) = \frac{1}{\sigma\sqrt{2\pi}}\int_0^\infty e^{-u^2}\,du \qquad P(\varepsilon) = P(0)\cdot P(1/0) + P(1)\,P(0/1) \quad ?$$



$$P(1/0) = \int_A^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2}} dx \qquad \frac{x}{\sigma\sqrt{2}} = t \qquad dx = \sigma\sqrt{2}\, dt$$

$$x = A \quad t = \frac{A}{\sigma\sqrt{2}} \qquad x = \infty \quad t = \infty$$

$$P(1/0) = \sigma\sqrt{2} \int_{A/\sigma\sqrt{2}}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-t^2} dt = \frac{1}{\sqrt{\pi}} \Big[\underbrace{\int_0^{\infty} e^{-t^2} dt}_{\frac{\sqrt{\pi}}{2}} - \int_0^{A/\sigma\sqrt{2}} e^{-t^2} dt \Big] =$$

$$= \frac{1}{2} - \frac{1}{2}\frac{2}{\sqrt{\pi}} \int_0^{A/\sigma\sqrt{2}} e^{-t^2} dt = \frac{1}{2}\Big(1 - \underbrace{\frac{2}{\sqrt{\pi}} \int_0^{A/\sigma\sqrt{2}} e^{-t^2} dt}_{\mathrm{erf}(A/\sigma\sqrt{2})}\Big) = \frac{1}{2} \mathrm{erfc}\left(\frac{A}{\sigma\sqrt{2}}\right)$$

$$\hat{\rho} = \frac{A}{\sigma} = \sqrt{\frac{E_s}{N}} \qquad P(e) = \frac{1}{2} \cdot P(0/1) + \frac{1}{2} P(1/0) = \cancel{2}\frac{1}{\cancel{2}} P(1/0) = P(1/0)$$

$$P(e) = \frac{1}{2} \mathrm{erfc}\left(\frac{A}{\sigma\sqrt{2}}\right) = \frac{1}{2} \mathrm{erfc}\left(\frac{\rho}{\sqrt{2}}\right) = \frac{1}{2} \mathrm{erfc}\left(\sqrt{\frac{E_s}{2N}}\right)$$

SCLAR $\quad P_B = Q\left(\sqrt{\frac{E_b}{N_0}}\right) = \frac{1}{2} \mathrm{erfc}\left(\sqrt{\frac{E_b}{2N_0}}\right)$

•

$$\boxed{s_i(t) = \begin{cases} s_1(t) & \cancel{0 \le t \le T} \quad \text{binary "1"} \\ s_2(t) & \cancel{0 \le t \le T} \quad \text{-//- "0"} \end{cases}}$$

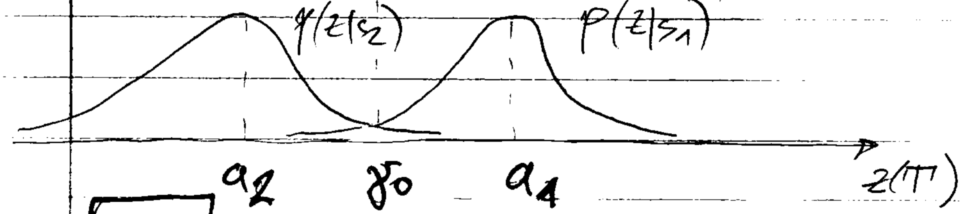
$$r(t) = s_i(t) * h(t) + n(t) \qquad i = 1, \ldots M$$

IDEAL SYSTEM: $\quad r(t) = s_i(t) + n(t)$

TEST STATISTIC $\quad \boxed{z(T) = a_i(T) + n_0(T) \qquad i = 1, 2}$

$$p(n_0) = \frac{1}{\sigma_0\sqrt{2\pi}} e^{-\frac{n_0^2}{2\sigma_0^2}} \qquad \sigma_0 \text{ - NOISE VARIANCE}$$

$$p(z|s_1) = \frac{1}{\sigma_0\sqrt{2\pi}} e^{-\frac{(n_0 - a_1)^2}{2\sigma_0^2}} \qquad p(z|s_2) = \frac{1}{\sigma_0\sqrt{2\pi}} e^{-\frac{(n_0 - a_2)^2}{2\sigma_0^2}}$$



$$\boxed{z(T) \underset{H_2}{\overset{H_1}{\gtrless}} \gamma}$$

~~H_1 TRUE if $z(T) > \gamma$~~ $\Rightarrow s_1(t)$ SENT

H_2 TRUE if $z(T) < \gamma \Rightarrow s_2(T)$ SENT

- **VECTOR VIEW OF SIGNAL WAVEFORMS**

N-DIMENSIONAL **ORTHOGONAL SPACE** $\{\phi_j(t)\}$ — SET OF LINEARLY INDEPENDENT

$$\boxed{\int_0^T \psi_j(t)\cdot\psi_k(t)\,dt = K_j\cdot\delta_{jk}} \qquad 0\le t\le T \qquad j,k=1,\dots,N$$

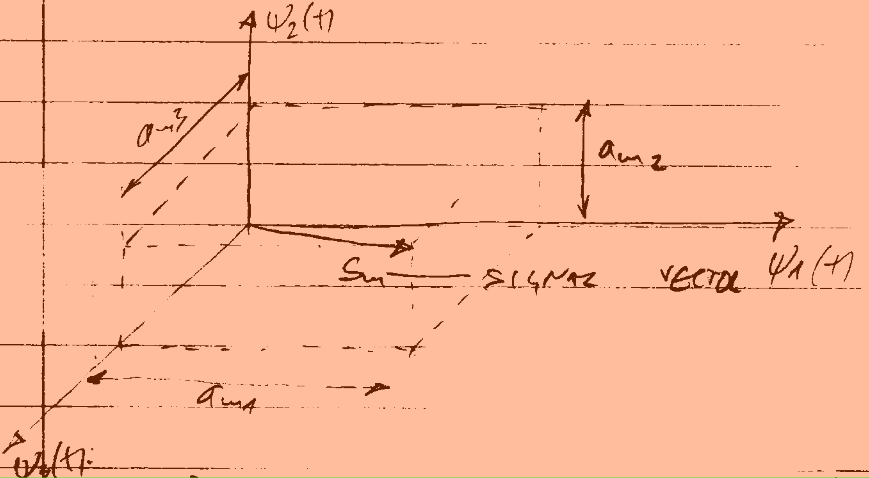
Ksi- базни функции

$$\delta_{jk} = \begin{cases} 1 & j=k \\ 0 & \text{otherwise} \end{cases}$$

KRONECKER DELTA FUNCTION

DOT PRODUCT

$v_1=(1,-1)$ $\quad v_2=(1,1)$ $\quad v_1\cdot v_2 = 1\cdot 1+(-1)\cdot 1 = \emptyset$ } ORTHOGONAL



$j=k$

$$\boxed{E_j = \int_0^T \psi_j^2\,dt = K_j}$$

→ DISIPACIJA NA ENERGIJA NA OPTOVARUVANJE OD 1Ω.

$\{s_i(t)\}$ — SET OF WAVEFORMS $\quad i=1,\dots,M$

$s_1(t) = a_{11}\psi_1(t) + a_{12}\psi_2(t) + \dots + a_{1N}\psi_N(t)$

$\vdots$

$s_M(t) = a_{M1}\psi_1(t) + a_{M2}\psi_2(t) + \dots + a_{MN}\psi_N(t)$

$$\boxed{s_i(t) = \sum_{j=1}^{N} a_{ij}\,\psi_j(t) \qquad i=1,\dots,M \quad N\le M}$$

$$\boxed{a_{ij} = \frac{1}{K_j}\int_0^T s_i(t)\psi_j(t)\,dt \qquad i=1,\dots,M \quad j=1,\dots,N \qquad 0\le t\le T}$$

$\{s_i\} = \{a_{i1}, a_{i2}, \dots, a_{iN}\}$ $\quad i=1,\dots,M$

if: $N=3$ $\qquad s_m(t) = a_{m1}\psi_1(t) + a_{m2}\psi_2(t) + a_{m3}\psi_3(t)$

$s_m(t)$ — POINT IN 3 DIMENSIONAL EUCLIDEAN SPACE WITH COORDINATES (a_{m1}, a_{m2}, a_{m3})

- **WAVEFORM ENERGY**

$$\frac{1}{\sqrt{2\pi}}\int_{\sqrt{\frac{2E_b}{N_0}}}^{\infty} e^{-x^2/2}\,dx = \left|\begin{array}{ll} t=\frac{x}{\sqrt2} & x=\sqrt{\frac{2E_b}{N_0}} \quad t=\sqrt{\frac{E_b}{N_0}} \\ dt=\frac{dx}{\sqrt2} & x=\infty \quad t=\infty \end{array}\right| =$$

$$= \frac{1}{\sqrt\pi}\int_{\sqrt{\frac{E_b}{N_0}}}^{\infty} e^{-t^2}\,dt = \frac12\,\mathrm{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right) = \frac12\,\mathrm{erfc}\left(\frac{\sqrt{\frac{2E_b}{N_0}}}{\sqrt2}\right)$$

$$= Q\left(\frac{2E_b}{N_0}\right)$$ SKLAR

$$Q(z) = \frac12\,\mathrm{erfc}\left(\frac{z}{\sqrt2}\right)$$

- ENERGY OF SIGNAL:

$$E_x^T = \int_{-T/2}^{+T/2} x^2(t)\,dt$$

- AVERAGE ENERGY DISSIPATED DURING "T"

$$P_x^T = \frac{1}{T} E_x^T = \frac{1}{T}\int_{-T/2}^{+T/2} x^2(t)\,dt$$

$$E_i = \int_0^T s_i^2(t)\,dt = \int_{0}^{T}\Big[\sum_j a_{ij}\,\psi_j(t)\Big]^2 dt = \int_0^T \sum_j a_{ij}\psi_j(t)\sum_k a_{ik}\psi_k(t)\,dt$$

$$= \sum_j\sum_k a_{ij}a_{ik}\int_0 \psi_j\,\psi_k\,dt = \sum_j\sum_k a_{ij}a_{ik}K_j\delta_{jk} = \sum_{j=1}^{N} a_{ij}^2\cdot K_j \qquad i=1,2,\dots M$$

ORTHOGONAL $\quad E_i = \sum_{j=1}^{N} a_{ij}^2 K_j \qquad i=1,2,\dots M$

ORTHONORMAL $K_j=1$ $\quad E_i = \sum_{j=1}^{N} a_{ij}^2 \qquad i=1,2,\dots M$

PARSEVAL THEOREM SPECIAL CASE!!

- GENERALIZED FOURIER TRANSFORMS

(ORTHOGONALITY STATEMENT)

$$\int_0^T \psi_j(t)\,\psi_k(t)\,dt = K_j\,\delta_{jk} \qquad 0\le t\le T \qquad j,k=1,\dots,N$$

$$\delta_{jk} = \begin{cases}1 & j=k\\ 0 & \text{otherwise}\end{cases}$$

ψ_j & ψ_k ARE ORTHOGONAL!!! IF FULFILLED

$$s_i(t) = \sum_{j=1}^{N} a_{ij}\psi_j(t) \qquad i=1,\dots M \qquad N\le M$$

(*) $$a_{ij} = \frac{1}{K_j}\int_0^T s_i(t)\,\psi_j(t)\,dt \qquad j=1,\dots N \quad i=1,\dots M \qquad 0\le t\le T$$

ORDINARY FOURIER $\{\psi_j(t)\}$ SET COMPRISE SINE & COSINE HARMONIC FUNCTIONS

$$f(t) = \sum_{n=1}^{\infty} a_n\cdot\cos(n\omega_0 t) + b_n\sin(n\omega_0 t) + \frac{a_0}{2}$$

$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\cdot\cos(n\omega_0 t)\,dt$$

$$b_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\cdot\sin(n\omega_0 t)\,dt$$

$$E_j = \int_0^T \psi_j^2(t)\,dt = K_j$$

$$\int_0^T \cos^2(n\omega_0 t)\,dt = K_j \qquad \int\cos^2(x)\,dx = \int\cos(x)\,d\sin(x) =$$

$$= \int\sqrt{1-\sin^2 x}\;d\sin(x) = \int\sqrt{1-u^2}\,du = \left|\begin{matrix}1-u^2=t & u=\sqrt{1-t}\\ -2u\,du = dt & du = -\frac{dt}{2\sqrt{1-t}}\end{matrix}\right|$$

$$2\cos^2(x) = \cos(x+x) + \cos(x-x) = \frac{1}{2}(1+\cos 2x)$$

$$\int \cos^2(x)\,dx = \frac{1}{2}\int (1+\cos 2x)\,dx = \frac{1}{2}x + \frac{1}{4}\int \cos(2x)\,d(2x)$$

$$= \frac{x}{2} + \frac{1}{4}\sin(2x)$$

$$\int_0^T \cos^2(n\omega_0 t)\,dt = \frac{1}{n\omega_0}\int_0^T \cos^2(n\omega_0 t)\,d(n\omega_0 t) = \frac{1}{n\omega_0}\left[\frac{n\omega_0 t}{2} + \frac{1}{4}\sin(2n\omega_0 t)\right]_0^T$$

$$= \frac{1}{n\omega_0}\left[\frac{n\omega_0 T}{2} + \frac{1}{4}\sin(2n\omega_0 T)\right] = \frac{T}{2} + \frac{1}{4n\omega_0}\sin\left(2n\frac{2\pi}{T}T\right)$$

$$= \frac{T}{2} + \frac{T}{8n\pi}\cancel{\sin(4n\pi)} = \frac{T}{2} \qquad \boxed{K_j = \frac{T}{2}}$$

$$\boxed{\begin{array}{l}\text{OD TUKA}\\ \text{PROIZLEGUVA}\\ g_T(t) = \sqrt{\frac{2}{T}}\end{array}}$$

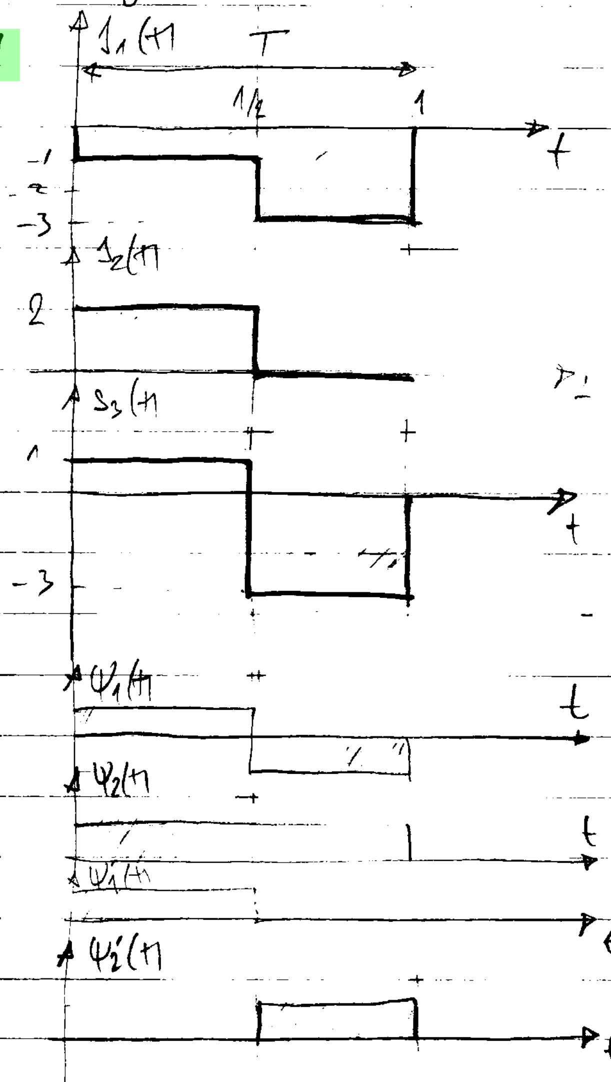
$$(*) \quad a_j = \frac{2}{T}\int_0^T f(t)\cdot\psi_j(t)\,dt \qquad j = 1, 2$$

$$\psi_1 = \cos(n\omega_0 t)$$

$$\psi_2 = \sin(n\omega_0 t)$$

$$a = a_1 = \frac{2}{T}\int_0^T f(t)\cdot\cos(n\omega_0 t)\,dt \qquad b = a_2 = \frac{2}{T}\int_0^T f(t)\cdot\sin(n\omega_0 t)\,dt$$

EA-3.1



(a) $$\int_0^T \psi_j(t)\psi_k(t)\,dt = K_j\cdot\delta_{jk} \qquad j,k = 1, 2, \ldots N$$

$$\int_0^T s_1(t)\cdot s_2(t)\,dt = ?$$

$$\left.\begin{array}{l}\int_0^T s_1 s_2\,dt = -T\\ \int_0^T s_1 s_3\,dt = +4T\\ \int_0^T s_2 s_3 = T\end{array}\right\} \neq \emptyset$$

NE SE ORTOGONALNI

waveform set $\{s_i(t)\}$ $i = 1,2,3$ is not orthogonal

(b) $$\int_0^T \psi_1(t)\,\psi_2(t)\,dt = \emptyset$$

$$\int_0^T \psi_1'(t)\,\psi_2'(t)\,dt = \emptyset$$

(c) $$s_i = \sum_{j=1}^{N} a_{ij}\psi_j \qquad i = 1, \ldots M$$

$j = 1, 2 \qquad \psi_1, \psi_2 \qquad N = 2 \qquad M = 3$

$$s_1 = a_{11}\psi_1 + a_{12}\psi_2 \qquad s_3 = a_{31}\psi_1 + a_{32}\psi_2$$

$$s_2 = a_{21}\psi_1 + a_{21}\psi_2$$

$$a_{ij} = \frac{1}{K_j}\int_0^T s_i(t)\cdot\psi_j(t)\,dt \qquad K_j = \int_0^T \psi_j^2(t)\,dt$$

$$K_1 = \int_0^T \psi_1^2(t)\,dt = \int_0^{T/2} 1^2\,dt + \int_{T/2}^T 1^2\,dt = \frac{T}{2} + \frac{T}{2} = T$$

$$K_2 = \int_0^T \psi_2^2(t)\,dt = T$$

$$a_{11} = \frac{1}{T}(-1)(1)\frac{T}{2} + \frac{1}{T}(3)\cdot(-1)\frac{T}{2} = -\frac{1}{2} + \frac{3}{2} = 1$$

$$\underline{\underline{a_{11}}} = \frac{1}{T}\int_0^T s_1(t)\cdot\psi_1(t)\,dt = \frac{1}{T}\int_0^{T/2}(-1)\cdot 1\,dt + \int_{T/2}^{T}(-3)\cdot(-1)\,dt$$

$$= \frac{1}{T}\left[-\frac{T}{2} + 3\left(T - \frac{T}{2}\right)\right] = \frac{1}{T}\left[-\frac{T}{2} + \frac{3T}{2}\right] = \frac{2T}{2}\cdot\frac{1}{T} = 1$$

$$\underline{\underline{a_{12}}} = \frac{1}{T}\int_0^T s_1(t)\,\psi_2(t)\,dt = \frac{1}{T}\int_0^{T/2}(-1)\,1\,dt + \frac{1}{T}\int_{T/2}^{T}(-3)\cdot 1\,dt =$$

$$= \frac{1}{T}(-1)\frac{T}{2} + \frac{1}{T}(-3)\cdot\frac{T}{2} = -\frac{1}{2} - \frac{3}{2} = -\frac{4}{2} = -2$$

$$a_{21} = \frac{1}{T}\int_0^T s_2(t)\,\psi_1(t)\,dt = \frac{1}{T}\int_0^{T/2} 2\cdot 1\,dt = \frac{1}{T}\cdot 2\cdot\frac{T}{2} = 1$$

$$a_{22} = \frac{1}{T}\int_0^T s_2(t)\,\psi_2(t) = \frac{1}{T}\int_0^{T/2} 2\cdot 1\,dt = 1$$

$$a_{31} = \frac{1}{T}\int_0^T s_3(t)\,\psi_1(t)\,dt = \frac{1}{T}\cdot\left(1\cdot 1\,\frac{T}{2} + (-3)(-1)\,\frac{T}{2}\right) = \frac{1}{2} + \frac{3}{2} = 2$$

$$a_{32} = \frac{1}{T}\int_0^T s_3(t)\,\psi_2(t)\,dt = \frac{1}{T}\cdot\left(1\cdot 1\,\frac{T}{2} + (-3)\cdot 1\,\frac{T}{2}\right) = \frac{1}{2} - \frac{3}{2} = -1$$

$$s_1 = a_{11}\psi_1 + a_{12}\psi_2 = \psi_1 - 2\psi_2$$

$$s_2 = a_{21}\psi_1 + a_{22}\psi_2 = \psi_1 + \psi_2$$

$$s_3 = a_{31}\psi_1 + a_{32}\psi_2 = 2\psi_1 - \psi_2$$

d)

$$K_1 = K_2 = \int_0^{T/2} 1\cdot dt = \frac{T}{2}$$

$$a_{11} = \frac{2}{T}(-1)\cdot(1)\,\frac{T}{2} = -1 \qquad a_{12} = \frac{2}{T}(-3)\cdot(1)\,\frac{T}{2} = -3$$

$$a_{21} = \frac{2}{T}(2)\cdot(1)\cdot\frac{T}{2} = 2 \qquad a_{22} = \frac{2}{T}\cdot 0 = 0$$

$$a_{31} = \frac{2}{T}(1)\cdot(1)\,\frac{T}{2} = 1 \qquad a_{32} = \frac{2}{T}(-3)\cdot(1)\,\frac{T}{2} = -3$$

$$s_1 = -\psi_1' - 3\psi_2' \qquad s_3 = \psi_1' - 3\psi_2'$$

$$s_2 = 2\psi_1'$$

- REPRESENTING WHITE NOISE WITH ORTHOGONAL WAVEFORMS

$n(t) = \hat{n}(t) + \tilde{n}(t) \qquad \hat{n}(t) = \sum_{j=1}^{N} n_j \psi_j(t) \qquad \tilde{n}(t) = n(t) - \hat{n}(t)$

$\hat{n}(t)$ – NOISE WITHIN SIGNAL SPACE (PROJECTION OF NOISE COMPONENTS ON THE SIGNAL COORDINATES ψ_j)

$$n(t) = \sum_{j=1}^{N} n_j \psi_j(t) + \tilde{n}(t)$$

$$n_j = \frac{1}{K_j} \int_0^T n(t) \cdot \psi_j(t)\, dt \quad \forall j \qquad \int_0^T \tilde{n}(t) \cdot \psi_j(t)\, dt = 0 \quad \forall j$$

$n \overset{def}{=} \hat{n}(t) \qquad \boxed{n = (n_1, n_2, \dots, n_N)}$

alex@[illegible].com.mk

- VARIANCE OF WHITE NOISE

$$\sigma^2 = Var[n(t)] = \int_{-\infty}^{\infty} \left(\frac{N_0}{2}\right) df = \frac{N_0}{2} \quad ? \; -$$

$$\sigma^2 = var(n_j) = E\left\{\left[\int_0^T n(t) \cdot \psi_j(t)\, dt\right]^2\right\} = \frac{N_0}{2} \qquad \boxed{\sigma^2 = \frac{N_0}{2}}$$

- The basic SNR PARAMETER FOR DIGITAL COM. SYSTEMS

$$\boxed{\frac{E_b}{N_0} = \frac{S \cdot T_b}{N/W}}$$

S – AVERAGE SIGNAL POWER

N – " NOISE " –

$R_b = 1/T_b$

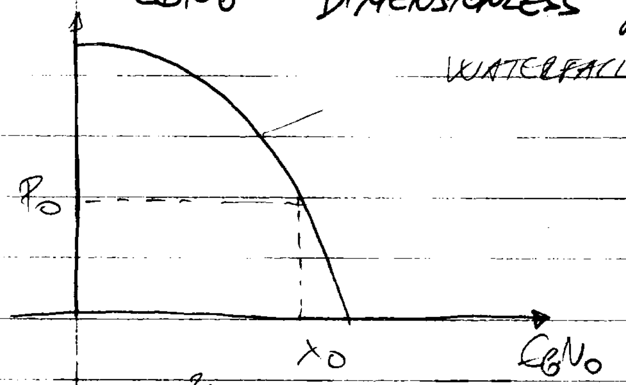
$$\frac{E_b}{N_0} = \frac{S/R_b}{N/W} \qquad \boxed{\frac{E_b}{N_0} = \frac{S}{N} \cdot \frac{W}{R_b}} \qquad \boxed{\frac{E_b}{N_0} = \frac{S}{N} \cdot \frac{T_b}{T_s}}$$

DRŽI SE DO OVAA DEFINICIJA $W = (1+r)\frac{R_s}{2}$

N_0 – NOISE POWER SPECTRAL DENSITY $\left[\frac{W}{Hz}\right]$ E_b – BIT ENERGY

$\frac{E_b}{N_0}$ – VERSION OF S/N NORMALIZED BY BIT RATE AND BANDWIDTH

E_bN_0 – DIMENSIONLESS figure



WATERFALL SHAPE

For $E_bN_0 \geq x_0 \quad P_B \leq P_0$

$$E_bN_0 = \frac{E_b}{N_0} = \frac{S}{N} \frac{W}{R}$$

ТУКА ЌЕ ~~ДРЖИ ВОДА~~ $2T_s$ ЗОШТО $W = \frac{1}{2T_s}$

ТУКА Е ВО MATLAB COM. TBX 4

$$\boxed{\frac{E_b}{N_0} = \frac{S}{N} \frac{T_b}{2T_s}}$$

Y – [illegible] МАЈОН

$$p(t) = \frac{v^2(t)}{R} \qquad p(t) = i^2(t) \cdot R \qquad R = 1\Omega \qquad p(t) = x^2(t)$$

$$E_x^T = \int_{-T/2}^{T/2} x^2(t)\, dt \qquad P_x^T = \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)\, dt$$

$x(t)$ – ENERGY SIGNAL if $0 < E_x < \infty$

$$E_x = \lim_{T \to \infty} \int_{-T/2}^{T/2} x^2(t)\, dt = \int_{-\infty}^{\infty} x^2(t)\, dt$$

• POWER SIGNALS $x(t)$ IS POWER SIGNAL IF IT HAS POWER $0 < P_x < \infty$

$$P_x = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} x^2(t)\,dt = \frac{1}{T}\int_{-\infty}^{\infty} x^2(t)\,dt$$

ENERGY SIGNAL HAS ZERO AVERAGE POWER

$$P_x = \frac{1}{T}\underbrace{\int_{-\infty}^{\infty} x^2(t)\,dt}_{\text{finite}} = \frac{K}{T \to \infty} = \emptyset$$

POWER SIGNAL HAS HAS INFINITE ENERGY

$$E_x = T\cdot \underbrace{\frac{1}{T}\int_{-T/2}^{T/2} x^2(t)\,dt}_{\text{finite}} = T\cdot K = \infty$$

$$E_sN_0 = \frac{E_s}{N_0} \qquad E_s = E_b\cdot K \qquad E_sN_0 = \frac{E_b\cdot K}{N_0} \qquad \frac{E_b}{N_0} = \frac{E_s}{K\cdot N_0}$$

$$\frac{S}{N} = \frac{E_s\cdot R}{N_0\cdot W} = \frac{K\cdot E_b}{N_0}\cdot\frac{R}{W} = \frac{K\cdot E_b}{N_0}\cdot\frac{T_s}{T_b}$$

$R \to 1/T_b$, $W \to 1/T_s$

PORTABILITY (?) $W = \frac{1}{2T_s}$

$$\frac{E_b}{N_0} = \frac{S}{N}\cdot\frac{T_b}{T_s}\cdot\frac{1}{K} \qquad \left.\frac{E_b}{N_0}\right|_{dB} = \left.\frac{S}{N}\right|_{dB} + 10\log(\text{VSAMP}) - 10\log(K)$$

$$\frac{E_b}{N_0} \Rightarrow \frac{\text{Joule}}{\text{Watt}/\text{Hz}} = \frac{\text{Joule}\cdot\text{Hz}}{\text{Watt}} = \frac{\text{Joule}/T}{\text{Watt}} = \frac{\text{Watt}}{\text{Watt}} = 1$$ NO DIMENSION

• DETECTION OF BINARY SIGNALS

• MAXIMUM LIKELIHOOD RECEIVER STRUCTURE

$$z(T) \underset{H_2}{\overset{H_1}{\gtrless}} \gamma \qquad \frac{p(z|s_1)}{p(z|s_2)} \underset{H_2}{\overset{H_1}{\gtrless}} \frac{P(s_2)}{P(s_1)}$$

$$\frac{e^{-\frac{(u_0-a_1)^2}{2\sigma_0^2}}}{e^{-\frac{(u_0-a_2)^2}{2\sigma_0^2}}} = e^{-\frac{(u_0-a_1)^2}{2\sigma_0^2}+\frac{(u_0-a_2)^2}{2\sigma_0^2}} = e^{-\frac{-2a_1u_0+a_1^2}{2\sigma_0^2}+\frac{-2u_0a_2+a_2^2}{2\sigma_0^2}}$$

$$= \exp\left(\frac{+2a_1u_0-a_1^2-2u_0a_2+a_2^2}{2\sigma_0^2}\right) = \exp\left(\frac{a_2^2+2u_0(a_1-a_2)-a_1^2}{2\sigma_0^2}\right)$$

$$\boxed{z(T) \underset{H_2}{\overset{H_1}{\gtrless}} \frac{a_1+a_2}{2} = \gamma_0}$$

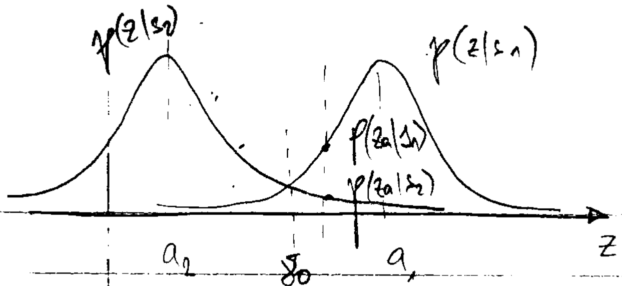
$$P(s_1) = P(s_2) \Rightarrow p(z|s_1) \gtrless p(z|s_2)$$

$$\exp\left(\frac{a_2^2+2u_0(a_1-a_2)-a_1^2}{2\sigma_0^2}\right) = 1 \qquad a_2^2+2u_0(a_1-a_2)-a_1^2 = \emptyset$$

$$u_0 = \frac{a_1^2-a_2^2}{2(a_1-a_2)}$$

$$\boxed{u_0 = \frac{\cancel{(a_1-a_2)}(a_1+a_2)}{2\cancel{(a_1-a_2)}} = \frac{a_1+a_2}{2}}$$

OPTIMAL THRESHOLD



$$p(z_a|s_1) > p(z_a|s_2)$$ MAXIMUM LIKELIHOOD DETECTOR

Оној сигнал што има максимална PDF во пресечната точка се смета дека е најверојатно пренесен

- Error Probability

$$P(e|s_1) = P(H_2|s_1) = \int_{-\infty}^{\gamma_0} p(z|s_1)\,dz$$

$$P(e|s_2) = P(H_1|s_2) = \int_{\gamma_0}^{\infty} p(z|s_2)\,dz$$

$$P_B = P(e|s_1)P(s_1) + P(e|s_2)P(s_2) = \sum_{i=1}^{2} P(e, s_i) = \sum_{i=1}^{2} P(s_i)P(e|s_i)$$

$$P_B = P(H_2|s_1)P(s_1) + P(H_1|s_2)P(s_2)$$

$$P(s_1) = P(s_2) = \frac{1}{2} \qquad P_B = \frac{1}{2}P(H_2|s_1) + \frac{1}{2}P(H_1|s_2)$$

$$P_B = \int_{\frac{a_1+a_2}{2}\,\gamma_0}^{\infty} p(z|s_2)\,dz = \int_{\frac{a_1+a_2}{2}}^{\infty} \frac{1}{\sigma_0\sqrt{2\pi}} e^{-\frac{(z-a_2)^2}{2\sigma_0^2}}\,dz$$

$$\frac{z-a_2}{\sqrt{2}\sigma_0} = u \qquad dz = \sqrt{2}\sigma_0\,du \qquad z = \frac{a_1+a_2}{2} \qquad u = \frac{\frac{a_1+a_2}{2} - \frac{2a_2}{2}}{\sqrt{2}\sigma_0} = \frac{a_1-a_2}{2\sqrt{2}\sigma_0}$$

$$P_B = \int_{\frac{a_1-a_2}{2\sqrt{2}\sigma_0}}^{\infty} \frac{1}{\sigma_0\sqrt{2\pi}} e^{-u^2}\sqrt{2}\sigma_0\,du = \frac{1}{2}\frac{2}{\sqrt{\pi}}\int_{\frac{a_1-a_2}{2\sqrt{2}\sigma_0}}^{\infty} e^{-u^2}\,du = \frac{1}{2}\,\mathrm{erfc}\left(\frac{a_1-a_2}{2\sqrt{2}\sigma_0}\right)$$

$$Q(z) = \frac{1}{2}\,\mathrm{erfc}\left(\frac{z}{\sqrt{2}}\right) \Rightarrow \qquad \boxed{P_B = Q\left(\frac{a_1-a_2}{2\sigma_0}\right)}$$

$$\boxed{Q(x) = \frac{1}{\sqrt{2\pi}}\int_{x}^{\infty} e^{-\frac{u^2}{2}}\,du} \qquad t = \frac{u}{\sqrt{2}} \quad dt = \frac{du}{\sqrt{2}} \quad du = \sqrt{2}\,dt$$

$$u = x \qquad t = \frac{x}{\sqrt{2}}$$

$$Q(x) = \frac{1}{\sqrt{2\pi}}\int_{x/\sqrt{2}}^{\infty} e^{-t^2}\sqrt{2}\,dt = \frac{1}{\sqrt{\pi}}\int_{x/\sqrt{2}}^{\infty} e^{-t^2}\,dt = \frac{1}{2}\,\mathrm{erfc}\left(\frac{x}{\sqrt{2}}\right)$$

$$\boxed{Q(x) \doteq \frac{1}{x\sqrt{2\pi}} e^{-x^2/2}}$$ GOOD APPROXIMATION FOR $x > 3$

- Matched Filter $$\left(\frac{S}{N}\right)_T = \frac{a_i^2}{\sigma_0^2}$$

$s(t)$ - INPUT SIGNAL

$$a_i(t) = \int_{-\infty}^{\infty} H(f)\cdot S(f)\cdot e^{j2\pi ft}\,dt$$

$$\sigma_0^2 = \frac{N_0}{2}\int_{-\infty}^{\infty} |H(f)|^2 df \qquad \left(\frac{S}{N}\right)_T = \frac{\left|\int_{-\infty}^{\infty} H(f) S(f) e^{j2\pi f t} df\right|^2}{\frac{N_0}{2}\int_{-\infty}^{\infty} |H(f)|^2 df}$$

Schwartz Inequality

$$\left|\int_{-\infty}^{\infty} f_1(x) f_2(x)\, dx\right|^2 \le \int_{-\infty}^{\infty} |f_1(x)|^2 dx \int_{-\infty}^{\infty} |f_2(x)|^2 dx$$

$H(f) = k \cdot S(f) \cdot e^{-j2\pi f T}$

$$\left|\int_{-\infty}^{\infty} H(f) S(f) e^{j2\pi f t}\right|^2 \le \int_{-\infty}^{\infty} |H(f)|^2 df \int_{-\infty}^{\infty} |S(f)|^2 df$$

$$\left(\frac{S}{N}\right)_T \le \frac{2}{N_0}\int_{-\infty}^{\infty} |S(f)|^2 df \qquad \boxed{\max\left(\frac{S}{N}\right)_T = \frac{2E}{N_0}} \circledast$$

$E = \int_{-\infty}^{\infty} |S(f)|^2 df$ – ENERGY OF THE INPUT SIGNAL $s(t)$

$\circledast$ VAŽI SAMO VO SLUČAJ NA OPTIMALEN FILTER T.E.

$$\boxed{H(f) = H_0(f) = k \cdot S^*(f) e^{-j2\pi f T}}$$

$$\boxed{h(t) = \begin{cases} k\, s(T-t) & 0 \le t \le T \\ 0 & \text{elsewere} \end{cases}} \qquad h(t) = \mathcal{F}^{-1}\{k\, S^*(f) e^{-j2\pi f T}\}$$

MATCHED FILTER

$$\mathcal{F}\{x(t-\tau)\} = X(j\omega) \cdot e^{-j\omega\tau}$$

$$\mathcal{F}\{x(t) \cdot e^{j\omega_0 t}\} = \int_{-\infty}^{\infty} x(t) e^{j\omega_0 t} e^{-j\omega t} dt = \int_{-\infty}^{\infty} x(t) e^{-j(\omega-\omega_0)t} dt = X(j\omega - \omega_0)$$

$$\mathcal{F}^{-1}\{X^*(j\omega)\} = \frac{1}{2\pi}\int X^*(j\omega) e^{j\omega t} d\omega = \frac{1}{2\pi}\int_{-\infty} \left(X(j\omega) e^{-j\omega t}\right)^* d\omega = \circledast$$

$$(a+jb)^* \cdot (c+jd) = \left[(a+jb)(c-jd)\right]^* = (ac - jad + jbc + bd)^*$$

$$= (ac+bd + j(bc-ad))^* = (ac+bd) - j(bc-ad) = (ac+bd) + j(ad-bc)$$

$$(a-jb)(c+jd) = ac + jad - jbc + bd = (ac+bd) + j(ad-bc)$$

$$\circledast = \left[\frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega) e^{-j\omega t} d\omega\right]^* = \left[x(-t)\right]^* = x(-t)$$

$z(T) = r(t) * h(t) = \int_{-\infty}^{\infty} r(\tau)\, h(t-\tau)\, d\tau$ $h(t) = \begin{cases} k\, s(T-t) & 0 \le t \le T \\ 0 & \text{OTHERWISE} \end{cases}$

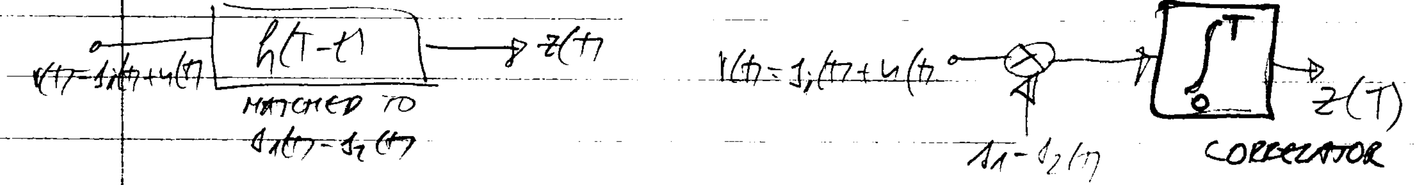
$h(t-\tau) = s[T-(t-\tau)]$ $z(t) = \int_{-\infty}^{\infty} r(\tau)\, s[T-(t-\tau)]\, d\tau$

$z(t) = \int_0^t r(\tau)\, s[T-t+\tau]\, d\tau$ $t=T$ $\boxed{z(T) = \int_0^T r(\tau)\, s(\tau)\, d\tau}$ CORRELATION

$s_i(t)$ $i = 1, 2, \dots M$

$\max[z_i(T)]$ $s_i(t)$ matches better to $r(t)$ than other $s_j(t)$, $j \neq i$

VO TO SLUČAJ SE ODLUČUVA DEKA E LANSIEN s_i



• OPTIMIZING ERROR PERFORMANCE

BINARY $\gamma_0 = \frac{a_1+a_2}{2}$ $P_B = Q\left[\frac{a_1-a_2}{2\sigma_0}\right] = \frac{1}{2}\,\mathrm{erfc}\left(\frac{a_1-a_2}{2\sqrt{2}\sigma_0}\right)$

$\max\left(\frac{a_1-a_2}{2\sigma_0}\right)$ $\frac{(a_1-a_2)^2}{\sigma_0^2}$ $\boxed{\sigma_0^2 = \frac{N_0}{2}}$

$\left(\frac{S}{N}\right)_T = \frac{(a_1-a_2)^2}{\sigma_0^2} = \frac{2E_d}{N_0}$ $\boxed{E_d = \int_0^T [s_1(t) - s_2(t)]^2\, dt}$ (*)

$\frac{a_1-a_2}{\sigma_0} = \sqrt{\frac{2E_d}{N_0}}$ $\boxed{P_B = Q\left[\frac{1}{2}\sqrt{\frac{2E_d}{N_0}}\right] = Q\left[\sqrt{\frac{E_d}{2N_0}}\right]}$

- TIME CROSS-CORRELATION COEFFICIENT BETWEEN SIGNALS s_1 & s_2

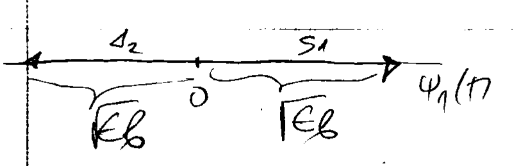
$\rho = \frac{1}{E_b}\int_0^T s_1(t)\, s_2(t)\, dt$ $\boxed{\rho = \cos\theta \quad -1 \le \rho \le 1}$ FOR VECTOR VIEW OF s_1 & s_2

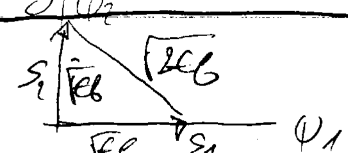
(*) $E_d = \int_0^T s_1^2(t)\,dt + \int_0^T s_2^2(t)\,dt - \int_0^T 2 s_1(t)\, s_2(t)\, dt$ $E_b = \int_0^T s_1^2(t)\,dt = \int_0^T s_2^2(t)\,dt = A^2 T$

$$\boxed{E_d = E_b + E_b - 2\rho E_b = 2E_b(1-\rho)}$$

$P_B = Q\left[\sqrt{\frac{E_b(1-\rho)}{N_0}}\right]$ $\rho = -1$ ANTIPODAL $\theta = 180°$
$\rho = 0$ NO-CORRELATION $\theta = 90°$

$\rho = 0 \Rightarrow$ ORTHOGONAL SIGNALS $\boxed{\int_0^T s_1(t) \cdot s_2(t)\, dt = 0}$





$d = \sqrt{E_b + E_b} = \sqrt{2E_b}$

• ANTIPODAL SIGNALS $\rho = -1$

$$\boxed{P_B = Q\left[\sqrt{\frac{E_b(1-(-1))}{N_0/2}}\right] = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)}$$

• ORTHOGONAL SIGNALS

$$\boxed{P_B = Q\left(\sqrt{\frac{E_b}{N_0}}\right)}$$

ALTERNATIVELY: $P_B = Q\left(\sqrt{\frac{E_d}{2N_0}}\right)$

ANTIPODAL – DISTANCE $2\sqrt{E_b} \Rightarrow E_d = 4E_b$ $\quad P_b = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$

ORTHOGONAL – DISTANCE $\sqrt{2E_b} \Rightarrow E_d = 2E_b$ $\quad P_b = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$

EXAMPLE 3.2 $s_1(t), s_2(t)$ + AWGN, BINARY COMM SYSTEM

$N_0 = 10^{-12}$ W/Hz $\qquad K = 1{,}38\cdot 10^{-23}$ J/K

$p_N(f) = k\cdot T$, $\qquad T = 20 + 273 = 293$ K

$p_N(f) = 1.38\cdot 10^{-23}\cdot 293 = 404\cdot 10^{-23} = 4{,}04\cdot 10^{-21}$ W

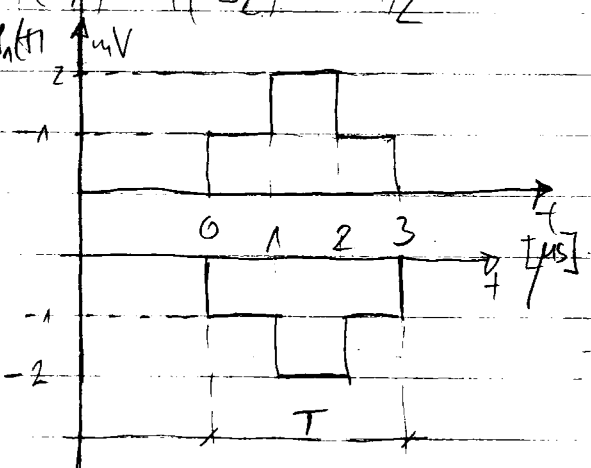
RASPOLOŽIVA ~~SREDNJA~~ SNAGA: $P_N = \int_{f_N}^{f_V} p_N(f)\,df = p_N(f)\cdot B$

$N_0 = p_N(f)\cdot B = 4\cdot 10^{-21}\cdot B$? $\quad B = \frac{10^{-12}}{4\cdot 10^{-21}} = 0{,}25\cdot 10^{9} = 250$ MHz

NE Є VAŽI OVO $\boxed{N_0 = k\cdot T}$ ČUDNO ZAŠTO Є TAKO MALA VRIJEDNOST.

Use VALUES OF RECEIVED VOLTAGE & TIME TO COMPUTE BER

$P(s_1) = P(s_2) = 1/2$



$$E_b = \int_0^T s_1^2(t)\,dt = \int_0^{T/3} 1\,dt + \int_{T/3}^{2T/3} 2^2\,dt + \int_{2T/3}^{T} 1\,dt$$

$$= 2\cdot\frac{T}{3} + 4\cdot\frac{T}{3} = 6\,\frac{T}{3}\,(mV)^2 \quad \text{(NA 1Ω TOVAR)}$$

$T = 3\mu s$

$$E_b = \frac{6\cdot 3\cdot 10^{-6}\cdot 10^{-6}}{3} = 6\cdot 10^{-12}\ W\cdot s$$

$E = P\cdot t \qquad P = \frac{dE}{dt}$

$$P_b = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right) = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{6\cdot 10^{-12}\,Ws}{10^{-12}\,W/Hz}}\right)$$

$$P_b = \frac{1}{2}\,\mathrm{erfc}(\sqrt{6}) = 0{,}00026 = 0{,}26\cdot 10^{-3}$$

$\frac{E_b}{N_0} = 6 \qquad 10\log\frac{E_b}{N_0} = 10\log 2 + 10\log 3 = 7{,}78$ dB $\quad \frac{E_b}{N_0}$ dB

$$\int_0^T s_1(t)\cdot s_2(t)\,dt = \int_0^{T/3} 1\cdot(-1)\,dt + \int_{T/3}^{2T/3} 2\cdot(-2)\,dt + \int_{2T/3}^{T} \underbrace{1(-1)}_{(mV)^2}\,dt =$$

$$= 10^{-6}\left(-\frac{T}{3} - 4\cdot\frac{T}{3} - \frac{T}{3}\right) = 10^{-6}\left(-\frac{6T}{3} = -2T\right) = 10^{-6}\left(-2\cdot 3\cdot 10^{-6}\right) = -6\cdot 10^{-12}\ (J)$$

$$\rho = \frac{1}{E_b}\cdot\int_0^T s_1 s_2\,dt = -\frac{6\cdot 10^{-12}}{6\cdot 10^{-12}} = -1$$ ⇒ ANTIPODAL ⇒ $P_B = Q\left(\frac{E_b}{N_0}\right)$ (*)

$$Q(x) \doteq \frac{1}{x\sqrt{2\pi}}\exp\left(\frac{-x^2}{2}\right) = \frac{1}{x\sqrt{2\pi}} e^{-x^2/2} =$$

$$\boxed{DTK}\ \frac{1}{2}\,\mathrm{erfc}\left(\frac{\rho}{2\sqrt{2}}\right) = \frac{1}{\rho}\sqrt{\frac{2}{\pi}}\,e^{-\rho^2/8} = \frac{1}{2x}\sqrt{\frac{2}{\pi}}\,e^{-\frac{x^2}{2}} = \frac{1}{2}\mathrm{erfc}\left(\frac{x}{\sqrt{2}}\right)$$

→ OD DTK DEFINICIJATA „ERFC"

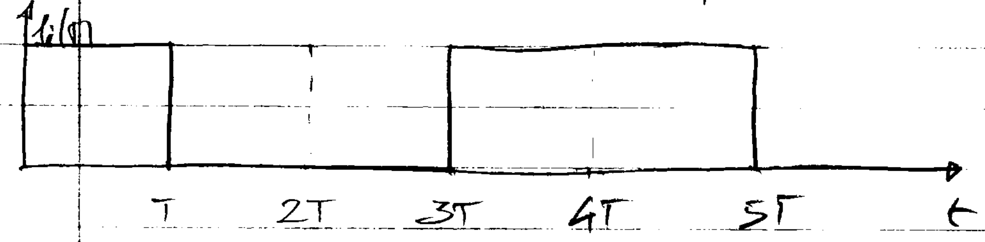
$$Q(x) = \frac{1}{2}\,\mathrm{erfc}\left(\frac{x}{\sqrt{2}}\right) = \left| x = \frac{\rho}{2}\right| = \frac{1}{x\sqrt{2\pi}} e^{-\frac{x^2}{2}} =$$

• Error Probability Performance of Binary Signaling

• UNIPOLAR

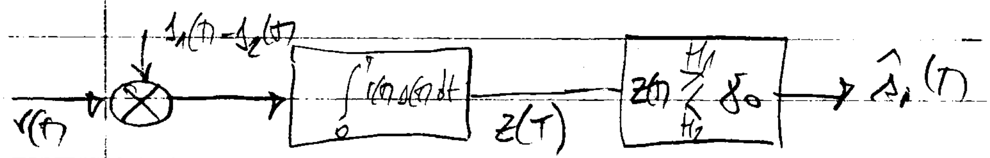
$s_1(t) = A \quad 0 \le t \le T$ binary „1"

$s_2(t) = 0 \quad 0 \le t \le T$ binary „2"



DO OVA SE DOAGJA OD ~~$z(T) = r(t) * h(t)$~~ ZA MATCHED FILTER

$$\boxed{z(T) = \int_0^T r(\tau)\cdot s(\tau)\,d\tau}$$



• $\boxed{r(t) = s_1(t) + n(t)}$

$$a_1(T) = E\{z(T)\,|\,s_1(t)\} = E\left\{\int_0^T (s_1(t) + n(t))\cdot s_1(t)\,dt\right\}$$

$$a_1(T) = E\left\{\int_0^T \left(A^2 + A\,n(t)\right)dt\right\} = A^2 T$$

$E\{z(T)\,|\,s_1(t)\}$ - EXPECTED VALUE OF $z(T)$ GIVEN $s_1(t)$ WAS SENT

$E\{n(t)\} = 0$

• $r(t) = s_2(t) + n(t) \Rightarrow a_2(T) = E\{z(T)\,|\,s_2(t)\} = 0$

$$\gamma_0 = \frac{a_1 + a_2}{2} = \frac{A^2T + 0}{2} = \frac{A^2 T}{2}\ ;\quad \boxed{z(T) \underset{s_2}{\overset{s_1}{\gtrless}} \frac{A^2T}{2}}$$

$$E_d = \int_0^T [s_1(t) - s_2(t)]^2 dt = \int_0^T A^2 dt = A^2 T$$

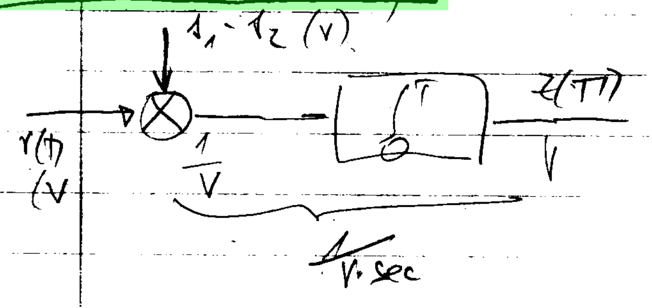
$$P_B = Q\left(\sqrt{\frac{E_d}{2N_0}}\right)$$ UNIVERZALNA!!!

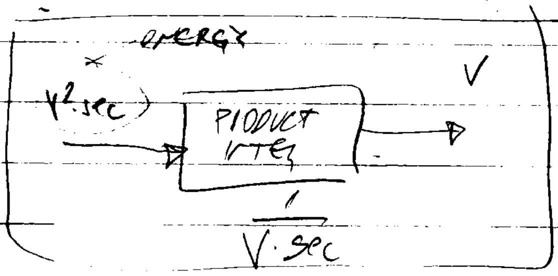
$$P_B = Q\left(\sqrt{\frac{A^2 T}{2N_0}}\right)$$

E_b – AVERAGE ENERGY PER BIT

$$E_{b1} = \int_0^T A^2 dt \qquad E_{b2} = \int_0^T 0^2 dt \quad \Rightarrow \quad E_b = \frac{E_{b1} + 0}{2} = \frac{A^2 T}{2}$$

$$P_b = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$$ UNIPOLAR → KAKO OSTAJE FORMULA ZA ORTOGONALNI SIG.





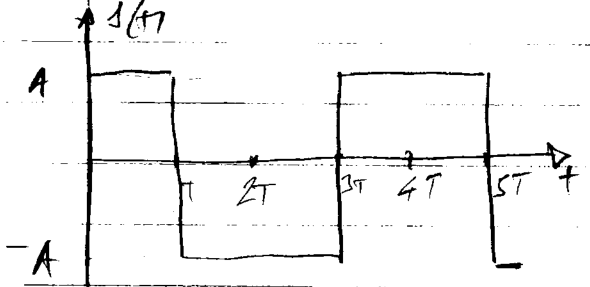
Q(z)=1/2*erf(z/sqrt(2))

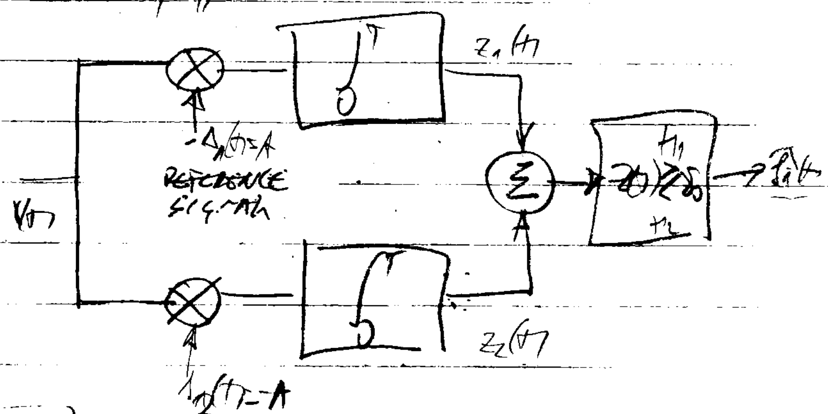
• BIPOLAR SIGNALING

-A:A

$s_1(t) = A \qquad 0 \le t \le T$ „1"

$s_2(t) = -A \qquad 0 \le t \le T$ „0"





MMV

(*) $$\frac{1}{2}\mathrm{erfc}\left(\frac{\frac{A}{2\sigma_N}}{\sqrt{2}}\right) = Q\left(\frac{A}{2\sigma_N}\right)$$

$$\frac{A^2}{\sigma_N^2} = \frac{E_b}{N_0/2} = \frac{2E_b}{N_0} \qquad \frac{A}{\sigma_N} = \sqrt{\frac{2E_b}{N_0}}$$

(*) $$= Q\left(\frac{1}{2}\sqrt{\frac{2E_b}{N_0}}\right) = Q\left(\sqrt{\frac{E_b}{2N_0}}\right)$$

$$\frac{1}{2}\mathrm{erfc}\left(\frac{A}{2\sqrt{2}\sigma_N}\right) = \frac{1}{2}\mathrm{erfc}\left(\frac{1}{2\sqrt{2}}\sqrt{\frac{2E_b}{N_0}}\right) = \frac{1}{2}\mathrm{erfc}\left(\frac{1}{2}\sqrt{\frac{E_b}{N_0}}\right)$$

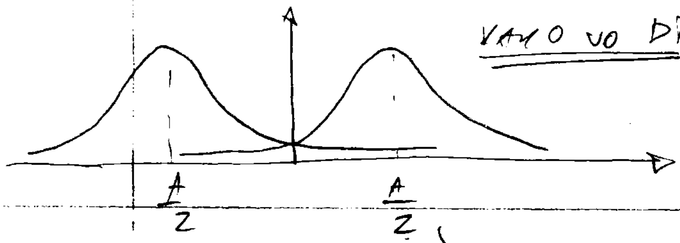
$$z(T) = z_1(T) - z_2(T)$$ ANTIPODAL $a_1 = -a_2 \Rightarrow \gamma_0 = 0$

$$\boxed{E_d = \int_0^T [s_1(t) - s_2(t)]^2 dt = \int_0^T (2A)^2 dt = 4A^2 T}$$ ENERGY OF DIFFERENCE SIGNAL — OVA E KLUČNO DA SE PRESMETA!!!

$$P_b = Q\left(\sqrt{\frac{E_d}{2N_0}}\right) = Q\left(\sqrt{\frac{4A^2T}{2N_0}}\right) = Q\left(\sqrt{\frac{2A^2T}{N_0}}\right) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

$$E_b = \frac{A^2T + A^2T}{2} = A^2 T$$

$$P_b = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = \frac{1}{2}\mathrm{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$$ BIPOLAR



VAŽNO VO DTK

$$P_b = Q\left(\sqrt{\frac{E_d}{2N_0}}\right)$$

$$E_d = \int_0^T \left(\frac{A}{2}+\frac{A}{2}\right)^2 dt = A^2 T$$

$$P_b = Q\left(\sqrt{\frac{E_d}{2N_0}}\right) = Q\left(\sqrt{\frac{A^2T}{2N_0}}\right) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

ODGOVARA NA UNIPOLARNIOT SIGNAL SO ISTA SREDNA BITSKA ENERGIJA ⊛

$$E_b = \frac{E_{b1}+E_{b2}}{2} = \frac{A^2T/4 + A^2T/4}{2} = \frac{A^2T}{4}$$

$$P(e) = \frac{1}{2}\,erfc\left(\sqrt{\frac{E_b}{2N_0}}\right) = \frac{1}{2}\,erfc\left(\sqrt{\frac{E_b}{2\cdot 2n_0}}\right) = \frac{1}{2}\,erfc\left(\frac{1}{2}\sqrt{\frac{E_b}{n_0}}\right)$$

$$P(\varepsilon) = \frac{1}{2}\,erfc\left(\frac{\sqrt{\rho}}{2}\right) \qquad \cancel{\frac{A^2}{8} = \frac{A^2}{\sigma_N^2} = \frac{A^2}{N_0/2} = \frac{2A^2}{N_0} = \frac{2E_b}{N_0}}$$

$\boxed{\text{DTK: UNIPOL } 0 \div A}$

$$P(\varepsilon) = \frac{1}{2}\,erfc\,\frac{A}{2\sqrt{2}\sigma_N} = \frac{1}{2}\,erfc\,\frac{\sqrt{E_b}}{2\sigma_N} = \frac{1}{2}\,erfc\,\frac{\sqrt{E_b}}{2\sqrt{\frac{N_0}{2}}}$$

$$E_b = \frac{A^2}{2} \qquad \frac{A}{\sqrt{2}} = \sqrt{E_b}$$

$$\sigma_N^2 = n_0 = \frac{N_0}{2} \qquad \boxed{\sigma_N = \sqrt{\frac{N_0}{2}}}$$

$$P(\varepsilon) = \frac{1}{2}\,erfc\sqrt{\frac{E_b}{2N_0}}$$

$$P(\varepsilon) = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$$ ODGOVARA SO SKLAR

SUŠTINATA NA USOGLASENOST SO SKLAR E DEFINICIJATA NA E_b!!!

$\boxed{\text{DTK BIPOLAREN } -\frac{A}{2} \div \frac{A}{2}}$

$$P(\varepsilon) = \frac{1}{2}\,erfc\,\frac{A}{2\sqrt{2}\sigma_N} \qquad E_b = \frac{\frac{A^2}{4}+\frac{A^2}{4}}{2} = \frac{A^2}{4}$$

$$\frac{A}{2} = \sqrt{E_b} \qquad P(\varepsilon) = \frac{1}{2}\,erfc\,\frac{\sqrt{E_b}}{\sqrt{2}\sigma_N} = \left|\sigma_N = \sqrt{\frac{N_0}{2}}\right| = \frac{1}{2}\,erfc\left(\sqrt{\frac{E_b}{N_0}}\right) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

ODGOVARA NA SKLAR

$\boxed{\text{NE ZABORAVAJ } n_0 = \frac{N_0}{2}}$

- SIGNALING DESCRIBED WITH BASIS FUNCTIONS

$$K_j = 1 \qquad E_j = \int_0^T \psi_j^2(t)\,dt = K_j$$

$$\int_0^T \psi_1^2(t)\,dt = 1 \qquad \psi_1^2(t) = \sqrt{\frac{1}{T}} \qquad \int_0^T \left(\sqrt{\frac{1}{T}}\right)^2 dt = \frac{1}{T}\cdot T = 1$$

UNIPOLAR: $s_1(t) = a_{11}\psi_1(t) = A\sqrt{T}\cdot\left(\sqrt{\frac{1}{T}}\right) = A$

$$a_{11} = \frac{1}{K_1}\int_0^T s_1(t)\,\psi_1(t)\,dt = \int_0^T A\cdot\sqrt{\frac{1}{T}}\,dt = A\cdot\sqrt{\frac{1}{T}}\cdot T = \underline{A\sqrt{T}}$$

$$s_2(t) = a_{21}\cdot\psi_1(t) = 0\cdot\sqrt{\frac{1}{T}} = 0$$

BIPOLAR: $s_1(t) = a_{11}\psi_1(t) = A\sqrt{T}\cdot\frac{1}{\sqrt{T}} = A$ $\quad s_2(t) = a_{21}\psi_1(t) = -A\sqrt{T}\cdot\frac{1}{\sqrt{T}} = -A$

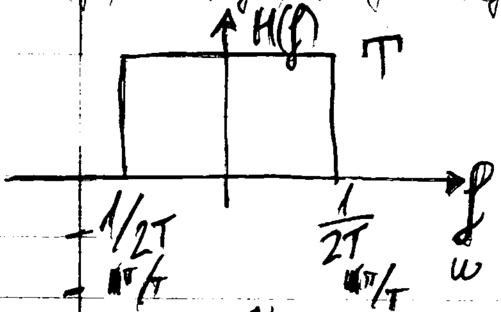
$$a_1(T) = E\{z(T)|s_1(t)\} = E\left\{\int_0^T \left(\frac{A}{\sqrt{T}} + \frac{n(t)}{\sqrt{T}}\right)dt\right\} = A\cdot\sqrt{T}$$

$$E_b = \frac{A^2 T + A^2 \cdot T}{2} = A^2 T \qquad A\sqrt{T} = \sqrt{E_b} \qquad a_1(T) = \sqrt{E_b}$$

FOR: $r(t) = s_2(t) + n(t) \qquad a_2 = -\sqrt{E_b}$

- **Intersymbol Interference** $\qquad f = \frac{\omega}{2\pi} \qquad$ ~~$\omega = 2\pi f$~~

$$H(f) = H_t(f)\cdot H_c(f)\, H_r(f)$$



$$f(t) = E \cdot \quad -\frac{\tau}{2} \le t \le \frac{\tau}{2}$$

$$F(j\omega) = E\cdot\tau\,\frac{\sin\frac{\omega\tau}{2}}{\frac{\omega\tau}{2}}$$

$$F(j\omega) = T\cdot\frac{1}{T}\,\frac{\sin \omega T}{\omega T}$$

$$h(t) = \frac{1}{2\pi}\int_{-1/2T}^{1/2T} T e^{j\omega t}\,d\omega = \frac{1}{2\pi}\,T\cdot\frac{e^{j\omega t}}{jt}\Bigg|_{-1/2T}^{1/2T} = \frac{1}{2\pi}\,\frac{T}{jt}\,e^{j2\pi f t}\Bigg|_{-\frac{1}{2T}}^{\frac{1}{2T}}$$

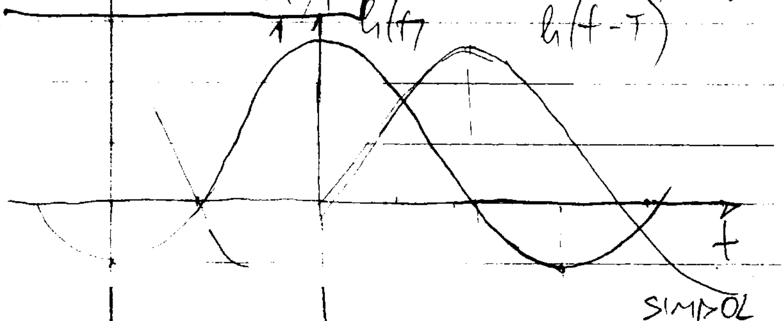
$$h(t) = \frac{1}{2\pi}\,\frac{T}{jt}\left(e^{j2\pi t\cdot\frac{1}{2T}} - e^{-j2\pi t\cdot\frac{1}{2T}}\right) = \frac{T}{\pi t}\cdot\frac{e^{j\pi t/T} - e^{-j\pi t/T}}{2j}$$

$$h(t) = \frac{\underbrace{\frac{1}{2j}\left(e^{+j\pi t/T} - e^{-j\pi t/T}\right)}_{\sin}}{\pi t/T} = \mathrm{sinc}(t/T)$$

$$h(t) = \frac{1}{2\pi}\int_{-\pi/T}^{\pi/T} T e^{j\omega t}\,d\omega = \frac{T}{2\pi}\,\frac{1}{jt}\,e^{j\omega t}\Bigg|_{-\pi/T}^{\pi/T} = \frac{T}{2\pi}\,\frac{1}{jt}\left[e^{jt\frac{\pi}{T}} - e^{-jt\frac{\pi}{T}}\right]$$

$$= \frac{T}{\pi t}\,\frac{e^{j\pi t/T} - e^{-j\pi t/T}}{2j} = \frac{T}{\pi t}\sin(\pi t/T) = \frac{\sin(\pi t/T)}{\pi t/T} = \mathrm{sinc}(t/T)$$

$$\boxed{h(t) = \frac{\sin(\pi t/T)}{\pi t/T} = \mathrm{sinc}(t/T)}$$



$$W = \frac{1}{2T} = \frac{R_s}{2}$$

$$2W = \frac{1}{T} = R_s$$

NYQUIST BANDWIDTH LIMIT

$\boxed{\text{SYMBOL-RATE PACKING} \quad 2\,\text{SYMBOL/Hz}}$

bandwidth efficiency $= \frac{R}{W}\left(\frac{b/s}{Hz}\right)$

PAM-64 $\quad M = 64 = 2^k \quad \overbrace{k = 6}^{\text{SIMBOL}}$ $\qquad B.E = \frac{2\,\text{symbols}}{Hz} = 12\ \text{BITS/s/Hz}$

RAISED-COSINE FILTER

268-HL

$$H(f)=\begin{cases}1 & |f|<2W_0-W\\ \cos^2\left(\frac{\pi}{4}\frac{|f|+W-2W_0}{W-W_0}\right) & 2W_0-W<|f|<W\\ \emptyset & |f|>W\end{cases}$$

f_s vo OTK

$\boxed{W_0=\frac{1}{2T}}$ $\quad R_s=\frac{1}{T}\quad$ $\boxed{W_0=\frac{R_s}{2}}$ → MINIMUM NYQUIST BANDWIDTH

OTK $\quad A(\omega)=k\begin{cases}\frac{1}{2}\left(1+\cos\frac{\pi\omega}{\omega_s}\right)=\cos^2\left(\frac{\pi\omega}{2\omega_s}\right) & |\omega|\le\omega_s\\ \emptyset & |\omega|>\omega_s\end{cases}$

$\cos\angle=\cos\left(\frac{\angle}{2}+\frac{\angle}{2}\right)=\cos\frac{\angle}{2}\cos\frac{\angle}{2}-\sin\frac{\angle}{2}\sin\frac{\angle}{2}=\cos^2\frac{\angle}{2}-\sin^2\frac{\angle}{2}=$

$=\cos^2\frac{\angle}{2}-1+\cos^2\frac{\angle}{2}=2\cos^2\frac{\angle}{2}-1\qquad\boxed{\frac{1}{2}(1+\cos\angle)=\cos^2\frac{\angle}{2}}$

W – ABSOLUTE BANDWIDTH $\qquad W_0=\frac{1}{2T}$

$W-W_0$ – EXCESS BANDWIDTH

$r=\frac{W-W_0}{W_0}$ – ROLLOFF FACTOR $\qquad 0\le r\le 1$

$W=\frac{R_s}{2}\qquad R_s=2W$

$r=\frac{W-W_0}{W_0}=1\qquad W-W_0=W_0\qquad \underline{W=2W_0}$

$H(f)=\cos^2\left(\frac{\pi}{4}\frac{f}{W_0}\right)=\cos^2\left(\frac{\pi}{4}\frac{\omega}{2\pi}\cdot 2T\right)=\cos^2\left(\frac{\omega T}{4}\right)$

OTK $\quad \omega_s=\frac{2\pi}{T}\quad T=\frac{2\pi}{\omega_s}\qquad H(\omega)=\cos^2\left(\frac{\omega\,2\pi}{4\,\omega_s}\right)=\cos^2\left(\frac{\pi\omega}{2\omega_s}\right)$

OVDE: $\omega_s=2\pi R_s\qquad H(f)=\cos^2\left(\frac{\pi}{2}\frac{2\pi f}{2\pi R_s}\right)=\cos^2\left(\frac{\pi}{2}\frac{f}{R_s}\right)$

$$\boxed{H=\frac{1}{2}\left(1+\cos\left(\frac{\pi f}{R_s}\right)\right)\qquad |f|<2W_0=\frac{1}{T}}$$

⇒ KAJ ĆE PODIGNAT KOSINUS KAKO VO OTK

$$h(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}H(j\omega)e^{j\omega t}d\omega=\frac{1}{2\pi}\int_{-\omega_s}^{\omega_s}\frac{1}{2}\left(1+\cos\left(\frac{\pi\omega}{\omega_s}\right)\right)e^{j\omega t}d\omega$$

$$h(t)=\frac{1}{2\pi}\frac{1}{2}\int_{-\omega_s}^{\omega_s}e^{j\omega t}d\omega+\frac{1}{2\pi}\frac{1}{2}\int_{-\omega_s}^{\omega_s}\cos\left(\frac{\pi\omega}{\omega_s}\right)e^{j\omega t}d\omega$$

$$h(t)=\frac{1}{2\pi}\frac{1}{2}\frac{e^{j\omega t}}{jt}\Bigg|_{-\omega_s}^{\omega_s}+\frac{1}{2\pi}\frac{1}{2}\frac{1}{2}\int_{-\omega_s}^{\omega_s}\left(e^{j\frac{\pi\omega}{\omega_s}}+e^{-j\frac{\pi\omega}{\omega_s}}\right)e^{j\omega t}d\omega$$

$$h(t) = \frac{1}{2\pi}\cdot\frac{1}{2jt}\left(e^{j\omega_s t} - e^{-j\omega_s t}\right) + \frac{1}{8\pi}\int_{-\omega_s}^{\omega_s} e^{j\omega\left(\frac{\pi}{2\omega_s}+t\right)} + e^{-j\omega\left(\frac{\pi}{2\omega_s}-t\right)}\, d\omega$$

$$h(t) = \frac{1}{2\pi}\,\frac{\sin\omega_s t}{t} + \frac{1}{8\pi}\underbrace{\left[\frac{1}{j\left(\frac{\pi}{2\omega_s}+t\right)}\, e^{j\omega\left(\frac{\pi}{2\omega_s}+t\right)}\Big|_{-\omega_s}^{\omega_s} - \frac{1}{j\left(\frac{\pi}{2\omega_s}-t\right)}\, e^{-j\omega\left(\frac{\pi}{2\omega_s}-t\right)}\Big|_{-\omega_s}^{\omega_s}\right]}_{\circledast}$$

$$\circledast = \frac{1}{4\pi}\left[\frac{1}{2j\left(\frac{\pi}{2\omega_s}+t\right)}\left(e^{j\omega_s\left(\frac{\pi}{2\omega_s}+t\right)} - e^{-j\omega_s\left(\frac{\pi}{2\omega_s}+t\right)}\right) - \frac{1}{2j\left(\frac{\pi}{2\omega_s}-t\right)}\left(e^{-j\omega_s\left(\frac{\pi}{2\omega_s}-t\right)} - e^{+j\omega_s\left(\frac{\pi}{2\omega_s}-t\right)}\right)\right]$$

$$e^{j\omega_s\left(\frac{\pi}{2\omega_s}+t\right)} = e^{j\frac{\pi}{2}+j\omega_s t} = j e^{+j\omega_s t} \qquad e^{-j\omega_s\left(\frac{\pi}{2\omega_s}+t\right)} = e^{-j\frac{\pi}{2}-j\omega_s t} = -j e^{-j\omega_s t}$$

$$e^{-j\omega_s\left(\frac{\pi}{2\omega_s}-t\right)} = e^{-j\frac{\pi}{2}+j\omega_s t} = -j e^{j\omega_s t} \qquad e^{j\omega_s\left(\frac{\pi}{2\omega_s}-t\right)} = e^{j\frac{\pi}{2}-j\omega_s t} = j e^{-j\omega_s t}$$

$$\circledast = \frac{1}{4\pi}\left[\frac{1}{2j\left(\frac{\pi}{2\omega_s}+t\right)}\left[j e^{j\omega_s t} + j e^{-j\omega_s t}\right] - \frac{1}{2j\left(\frac{\pi}{2\omega_s}-t\right)}\left[-j e^{j\omega_s t} - j e^{-j\omega_s t}\right]\right]$$

$$\circledast = \frac{1}{4\pi}\left[\frac{\cos\omega_s t}{\frac{\pi}{2\omega_s}+t} + \frac{\cos\omega_s t}{\frac{\pi}{2\omega_s}-t}\right] = \frac{\cos\omega_s t}{4\pi}\,\frac{\frac{\pi}{2\omega_s}}{\left(\frac{\pi}{2\omega_s}\right)^2-t^2} = \frac{\cos\omega_s t}{2\omega_s\left[\left(\frac{\pi}{2\omega_s}\right)^2-t^2\right]}$$

$$h(t) = \frac{\sin\omega_s t}{4\pi t} + \frac{\cos\omega_s t}{2\omega_s\left[\left(\frac{\pi}{2\omega_s}\right)^2-t^2\right]} = \frac{\omega_s\left[\left(\frac{\pi}{2\omega_s}\right)^2-t^2\right]\sin\omega_s t + 2\pi t\cos\omega_s t}{4\pi t\,\omega_s\left[\left(\frac{\pi}{2\omega_s}\right)^2-t^2\right]}$$

$$e^{j\omega_s\left(\frac{\pi}{\omega_s}+t\right)} = e^{j\pi}\cdot e^{j\omega_s t} \qquad e^{-j\omega_s\left(\frac{\pi}{\omega_s}+t\right)} = e^{-j\pi}\cdot e^{-j\omega_s t} = -1\cdot e^{-j\omega_s t}$$

$$e^{-j\omega_s\left(\frac{\pi}{\omega_s}-t\right)} = e^{-j\pi}\cdot e^{+j\omega_s t} \qquad e^{+j\omega_s\left(\frac{\pi}{\omega_s}-t\right)} = e^{+j\pi}\cdot e^{-j\omega_s t} = -e^{-j\omega_s t}$$

$$\circledast = \frac{1}{4\pi}\left[\frac{1}{2j\left(\frac{\pi}{\omega_s}+t\right)}\left(-e^{j\omega_s t} + e^{-j\omega_s t}\right) - \frac{1}{2j\left(\frac{\pi}{\omega_s}-t\right)}\left(-e^{j\omega_s t} + e^{-j\omega_s t}\right)\right]$$

$$\circledast = \frac{1}{4\pi}\left[\frac{-\sin(\omega_s t)}{\frac{\pi}{\omega_s}+t} + \frac{\sin(\omega_s t)}{\frac{\pi}{\omega_s}-t}\right] = \frac{\sin(\omega_s t)}{\frac{\cancel{4}\pi}{2}}\cdot\frac{\cancel{2}t}{\left(\frac{\pi}{\omega_s}\right)^2-t^2}$$

$$h(t) = \frac{1}{2\pi}\,\frac{\sin\omega_s t}{t} + \frac{t\cdot\sin\omega_s t}{2\pi\left[\left(\frac{\pi}{\omega_s}\right)^2-t^2\right]} = \frac{\sin\omega_s t}{2\pi}\,\frac{\left(\frac{\pi}{\omega_s}\right)^2\cancel{-t^2}\cancel{+t^2}}{t\left[\left(\frac{\pi}{\omega_s}\right)^2-t^2\right]}$$

$$h(t) = \frac{\sin(\omega_s t)}{2\pi}\,\frac{\frac{\pi^2}{\omega_s^2}}{t\left[\frac{\pi^2}{\omega_s^2}-t^2\right]} = \frac{\sin\omega_s t}{2\pi t}\,\frac{1}{\left(1-\frac{t^2\omega_s^2}{\pi^2}\right)}$$

$$\omega_s = 2\omega_c \qquad \boxed{h(t) = \frac{\omega_c}{\pi}\,\frac{\sin(2\omega_c t)}{2\omega_c\cdot t}\,\frac{1}{1-\left(\frac{2\omega_c t}{\pi}\right)^2} = \frac{1}{T}\,\frac{\sin(2\omega_c t)}{2\omega_c t}\,\frac{1}{1-\left(\frac{2\omega_c t}{\pi}\right)^2}}$$

so $\omega_s = \frac{2\pi}{T}$ $\quad \omega_c = \frac{\omega_s}{2} = \frac{\pi}{T}$

SKLAR

$$\boxed{h(t) = 2W_0 \cdot \text{sinc}(2W_0 t) \frac{\cos[2\pi(W-W_0)t]}{1-[4(W-W_0)t]^2}}$$

$$\boxed{W_0 \overset{\text{def}}{=} \frac{\omega_c}{2\pi}}$$

$W = 2W_0$

$W_0 = \frac{R_s}{2} = \frac{1}{2T}$

$$h(t) = 2\frac{\omega_c}{2\pi} \frac{\sin 2\pi W_0 t}{2\pi W_0 t} \frac{\cos 2\pi W_0 t}{1-[4 \cdot W_0 t]^2}$$

$$h(t) = \frac{1}{T} \frac{\sin \omega_c t}{\omega_c t} \frac{\cos \omega_c t}{1-\left(\frac{2\omega_c t}{\pi}\right)^2} = \frac{1}{T} \frac{\sin(2\omega_c t)}{2\omega_c t} \cdot \frac{1}{1-\left(\frac{2\omega_c t}{\pi}\right)^2}$$

ISTO KAKO OTK

$\sin(2\omega_c t) = \sin(\omega_c t + \omega_c t) = 2\cos\omega_c t \cdot \sin\omega_c t$ $\qquad \sin\omega_c t \cdot \cos\omega_c t = \frac{\sin(2\omega_c t)}{2}$

$r = \frac{W-W_0}{W_0}$

$r = 1 \Rightarrow W = 2W_0$ RISED COS

$r = \emptyset \Rightarrow W = W_0$ IDEAL NYQIST

$r = 0.5$ $\quad \frac{1}{2}W_0 = W - W_0 \quad W = 1.5\,W_0$

$$\boxed{H(f) = \frac{1}{2}\left(1 + \cos\left(\frac{\pi f}{R_s}\right)\right) \qquad |f| \leq 2W_0 = \frac{1}{T}}$$

$$\boxed{W = \frac{1}{2}(1+r)R_s} \qquad \boxed{r = \frac{W-W_0}{W_0}}$$

ASK & PSK $\qquad \boxed{W_{DSB} = (1+r)R_s}$ $\qquad$ BANDPASS TRANSMISSION

DOUBLE-SIDEBAND SIGNAL

Ex. 3.3 BANDWIDTH REQUIREMENTS

(a) MINIMUM REQUIRED BANDWIDTH FOR BASEBAND TRANSMISSION

PAM-4 $\qquad R_B = 2400$ bits/s $\quad r = 1$ (RAISED-COSINE)

$M = 4 = 2^k \quad k = 2 \qquad R_B = \text{ld}M \cdot R_s \qquad R_s = \frac{R_B}{\text{ld}M} = 1200$ bps

$T = \frac{1}{R_s} = \frac{1}{1200} = 0.83 \cdot 10^{-3}$ sec

$$\boxed{W = \frac{1}{2}(1+r) \cdot R_s = R_s = 1200\,\text{Hz}}$$ MMV

(b) DSB $\quad$ PAM-4 $\qquad W_{DSB} = (1+r)R_s = 2 \cdot R_s = 2400\,\text{Hz}$

Ex. 3.4 DIGITAL TELEPHONE CIRCUITS

3 kHz ANALOG TELEPHONE VOICE CHANNEL

$R_s = 8000$ samples/s $\qquad$ 256 LEVELS $\qquad M = 256 \quad k = \text{ld}256 = 8$

$T_B = \frac{1}{8000} = 125\,\mu s \qquad R_B = k \cdot R_s = 8 \cdot 8$ samp/s $= 64$ kbps

$W = \frac{1}{2}(1+r)R_s = \frac{1}{2}(1+1)R_s = 8\,\text{kHz} \quad r = 1$

$W = \frac{1}{2}(1+\emptyset)R_s = 4\,\text{kHz} \quad r = \emptyset$

$$\boxed{W \geq \frac{R_s}{2} = \text{ld}(M)\frac{R_s}{2}} = \frac{1}{2} \frac{8\,\text{bit}}{\text{sample}} \cdot \frac{8000\,\text{samples}}{\text{sec}} = 32\,\text{kbps} = 32\,\text{kHz}$$

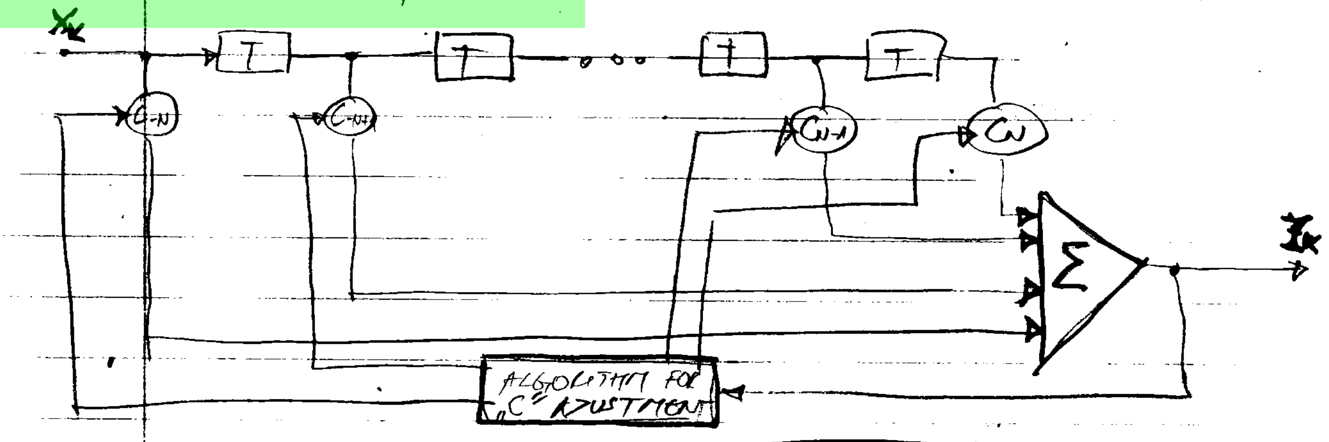
$W_{PCM} \doteq 8\,W_{ANALOG} = 32\,\text{kHz}$ (4 kHz)

PCM-30 $R_b = 2Mb/s$ $\boxed{W \geq \frac{R_b}{2} = 1MHz}$ (?) ova go nema vo knigata, no e logično $R_b = ld(M) \cdot R_s$

30 voice ch

Analog: $30 \cdot 4kHz = 120kHz$ 8 pati podobra spektralna iskoristenost !!!

• Transversal Equalizer



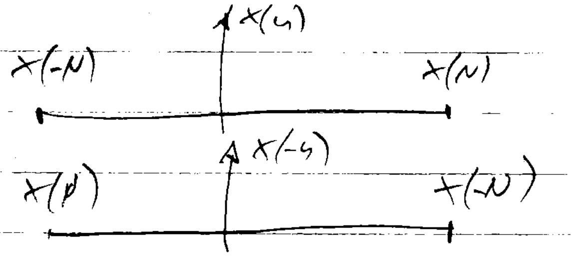
$$\boxed{z(k) = \sum_{n=-N}^{N} x(k-n) c_n \qquad k = -2N, \ldots, 2N}$$

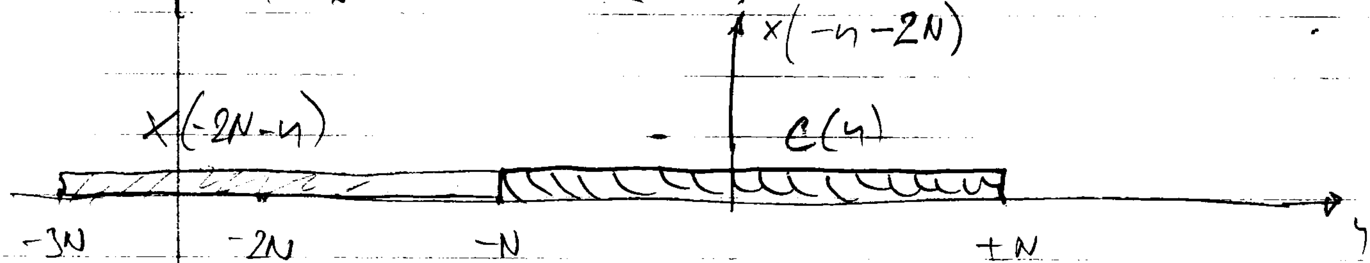
$$y(k) = \sum_{n=1}^{N} h(k-n) x(n) \qquad k = 1, \ldots, 2N-1$$ } definition of convolution

$Lenght(c_n) = N-(-N)+1 = 2N+1 = a$

$Lenght(z_k) = +2N+2N+1 = 4N+1 = 2a-1 = 4N+2-1 = 4N+1$

$$z = \begin{bmatrix} z(-2N) \\ \vdots \\ z(0) \\ \vdots \\ z(2N) \end{bmatrix} \qquad c = \begin{bmatrix} c_{-N} \\ \vdots \\ c_0 \\ \vdots \\ c_N \end{bmatrix}$$





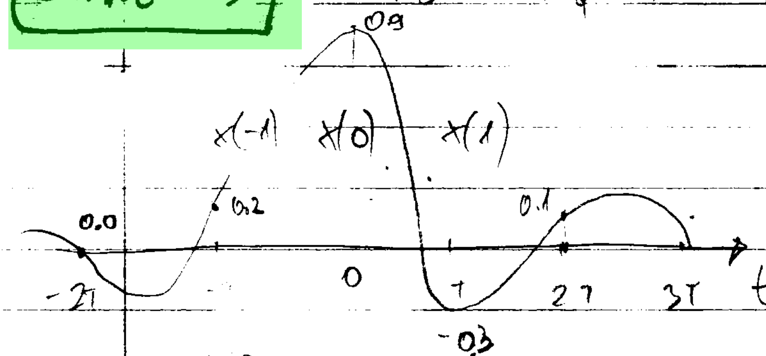
$$x = \begin{bmatrix} x(-N) & 0 & \ldots & 0 & 0 \\ x(-N+1), & x(-N) & \ldots & & \\ x(N), & x(N-1) & \ldots & x(-N+1), & x(-N) \\ 0 & 0 & \ldots & 0, & x(N) \end{bmatrix}$$

$\boxed{z = x \cdot c}$ $c = x^{-1} z$ ∄ for $\boxed{\text{square matrix } x^{a}}$

• ZERO-FORCING SOLUTION

$$z(k) = \begin{cases} 1 & k = 0 \\ 0 & k = \pm 1, \pm 2, \dots \in N \end{cases}$$

EXAMPLE 3.5. ZERO-FORCING THREE-TAP EQUALIZER



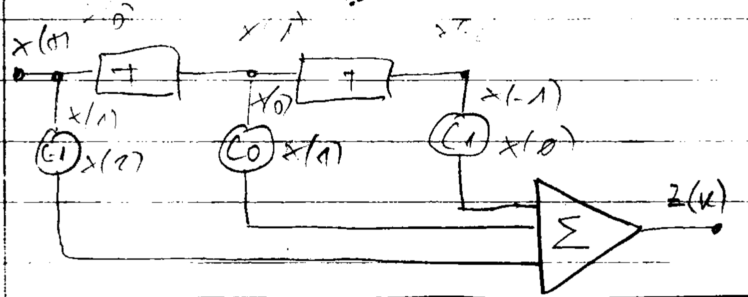
$\{c_{-1}, c_0, c_1\} = ?$

$\{z(k)\} = \{z(-1)=0, z(0)=1, z(1)=0\}$

$ISI = ?$ for $k = \pm 2, \pm 3;$

$N = 1$

$z = x \cdot c$



SAMO INC. RAZMISLUVAJTE

$$\begin{bmatrix} z_{-1} \\ z_0 \\ z_1 \end{bmatrix} = \begin{bmatrix} x(-1) & 0 & 0 \\ x(0) & x(-1) & 0 \\ x(1) & x(0) & x(-1) \end{bmatrix} \begin{bmatrix} c_{-1} \\ c_0 \\ c_1 \end{bmatrix}$$

LOGIKATA E MNOGO SLIČNA NA KONVOLUCIJA

SLEDI JA FORMULATA!!!

$$z_{-1} = c_{-1} x(-1)$$
$$z_0 = c_{-1} x(0) + c_0 x(-1)$$
$$z_1 = c_{-1} x(1) + c_0 x(0) + c_1 x(-1)$$

$$z_{-1} = c_{-1} \cdot 0.2$$
$$z_0 = c_{-1} \cdot 0.9 + c_0 \cdot 0.2$$
$$z_1 = c_{-1}(-0.3) + c_0 \cdot 0.9 + c_1 \cdot 0.2$$

$z_{-1} = 0 \Rightarrow \underline{c_{-1} = 0}$

$z_0 = 1 = \cancel{c_{-1}} \cdot 0.9 + c_0 \cdot 0.2 \Rightarrow \underline{c_0 = \frac{1}{0.2} = 5}$

$z_1 = 0 = \cancel{c_{-1}(-0.3)} + c_0 \cdot 0.9 + c_1 \cdot 0.2 \qquad 5 \cdot 0.9 + c_1 \cdot 0.2 = 0$

$c_1 = -\frac{4.5}{0.2} = \frac{45}{2} = \underline{\underline{22.5}}$

PRAVILNO REŠENIE

$$\begin{matrix} z_{-1} \\ z_0 \\ z_1 \end{matrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} x(0) & x(-1) & x(-2) \\ x(1) & x(0) & x(-1) \\ x(2) & x(1) & x(0) \end{bmatrix} \begin{bmatrix} c_{-1} \\ c_0 \\ c_1 \end{bmatrix}$$

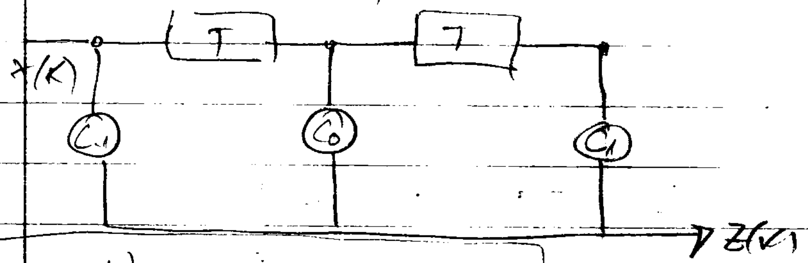
$$z_{-1} = c_{-1} x(0) + c_0 x(-1) + c_1 x(-2)$$
$$z_0 = c_{-1} x(1) + c_0 x(0) + c_1 x(-1)$$
$$z_1 = c_{-1} x(2) + c_0 x(1) + c_1 x(0)$$

$$\begin{bmatrix}0\\1\\0\end{bmatrix} = \begin{bmatrix}0,9 & 0,2 & 0\\ -0,3 & 0,9 & 0,2\\ 0,1 & -0,3 & 0,9\end{bmatrix}\begin{bmatrix}C_{-1}\\C_0\\C_1\end{bmatrix} \qquad \boxed{C = X^{-1} \cdot Z}$$

$$C = [-0,214;\ 0,9631;\ 0,3448;]$$

$$0 = 0,9\, C_{-1} + 0.2\, C_0$$
$$1 = -0,3\, C_{-1} + 0.9\, C_0 + 0.2\, C_1$$
$$0 = 0.1\, C_{-1} - 0.3\, C_0 + 0.9\, C_1$$

ISI = ? $\quad k = \pm 2;\ \pm 3$



$k = -1, 0, 1$

$$\boxed{z(k) = \sum_{n=-N}^{N} x(k-n)\, c(n) \qquad k = -2N \dots 2N}$$

$$\boxed{N=1 \qquad z(k) = \sum_{n=-1}^{1} x(k-n)\, c(n)}$$

$k = 2,3$ $\quad z(2) = \sum_{n=-1}^{1} x(2-n)\, c(n)$

$\quad z(3) = \sum_{n=-1}^{1} x(3-n)\, c(n)$

$k=-1 \quad z(-1) = x(0) \cdot c(-1) + x(-1)\, c(0) + x(-2)\, c(1) = 0$

$k=0 \quad z(0) = x(+1)\, c(-1) + x(0)\, c(0) + x(-1)\, c(1) = 1$

$k=1 \quad z(1) = x(2)\, c(-1) + x(1)\, c(0) + x(0)\, c(1) = 0$

$k=2 \quad z(2) = x(3)\, c(-1) + x(2)\, c(0) + x(1)\, c(1) = -0,0071$

$k=3 \quad z(3) = x(4)\, c(-1) + x(3)\, c(0) + x(2)\, c(1) = 0,0345$

$k=-2 \quad z(-2) = x(-1)\, c(-1) + x(-2)\, c(0) + x(-3)\, c(1) = -0,0428$

$k=-3 \quad z(-3) = x(-2)\, c(-1) + x(-3)\, c(0) + x(-4)\, c(1) = 0$

$$\boxed{X = \begin{array}{ccccccc} [\,0, & 0, & 0.2, & 0.9, & -0.3, & 0.1, & 0\,] \\ -3 & -2 & -1 & 0 & 1 & 2 & 3 \end{array}} \qquad x(-n) = \begin{array}{ccccccc} [\,0; & 0.1; & -0.3; & 0.9; & 0.2; & 0; & 0\,] \\ 3 & 2 & 1 & 0 & -1 & -2 & -3 \end{array}$$

$$\begin{bmatrix}Z_2\\Z_3\\Z_{-2}\\Z_{-3}\end{bmatrix} = \begin{bmatrix}X_3 & X_2 & X_1\\ X_4 & X_3 & X_2\\ X_{-1} & X_{-2} & X_{-3}\\ X_{-2} & X_{-3} & X_{-4}\end{bmatrix}\begin{bmatrix}C_1\\C_2\\C_3\end{bmatrix} = \begin{bmatrix}0 & 0.1 & -0.3\\ 0 & 0 & 0.1\\ 0.2 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}C_1\\C_2\\C_3\end{bmatrix}$$

 $\quad z = [0; -0,048; 0; 1; 0; -0.0071; 0.0345]$

• Minimum MSE Solution

MSE – mean square error

$\sigma^2 = \overline{\xi^2} - \bar{\xi}^2 = \overline{(\xi - \bar{\xi})^2}$ — VARIANCE

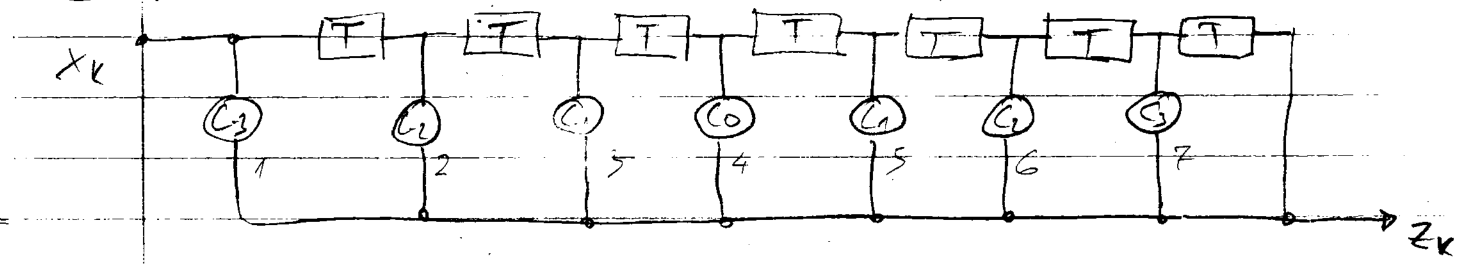
$z = x \cdot c \qquad x^T z = x^T x$

$R_{xz} = x^T z \qquad R_{xx} = x^T \cdot x \qquad \boxed{R_{xz} = R_{xx} \cdot c}$

$$\begin{bmatrix} x_3 & x_4 & x_5 \\ x_2 & x_3 & x_4 \\ x_1 & x_2 & x_3 \end{bmatrix} \cdot \begin{bmatrix} x_3 & x_2 & x_1 \\ x_4 & x_3 & x_2 \\ x_5 & x_4 & x_3 \end{bmatrix} = \begin{bmatrix} x_3^2+x_4^2+x_5^2; & x_3x_2+x_4x_3+x_5x_4; & x_3x_1+x_4x_2+x_5x_3 \\ x_2x_3+x_3x_4+x_4x_5; & x_2^2+x_3^2+x_4^2; & x_2x_1+x_3x_2+x_4x_3 \\ x_1x_3+x_2x_4+x_3x_5; & x_1x_2+x_2x_3+x_3x_4; & x_1^2+x_2^2+x_3^2 \end{bmatrix}$$

$$\boxed{MSE = \frac{\sum [x - est(x)]^2}{length(x)}} \qquad \boxed{c = R_{xx}^{-1} R_{xz}}$$

Example 3.6 A Minimum MSE 7-Tap Equalizer



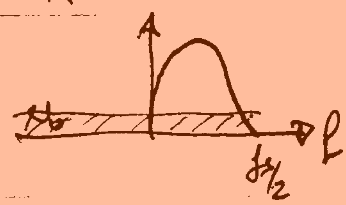
$x(k) = [0{,}0108; -0{,}0558; 0.1617; 1.0000; -0.1749; 0.0227; 0.0110]$

HMV $\qquad \boxed{T_{SIG} = T_{SYM}}$

$$\frac{S}{N}\bigg|_{SYMBOL} = \frac{E_S/T_{SIG}}{N_0 \cdot W} = \frac{E_S}{N_0} \cdot \frac{1}{W \cdot T_{SIG}} = \bigg| W = \frac{R_S}{2} = \frac{1}{2T_{SIG}} \bigg| = \frac{E_S}{N_0} \cdot \frac{1}{\frac{1}{2T_{SIG}} \cdot T_{SIG}} = \frac{2E_S}{N_0}$$

$$\frac{E_S}{N_0} = 0.5 \cdot \frac{S}{N} \qquad \bigg| \underline{E_S = K \cdot E_b} \bigg| \qquad \frac{E_b}{N_0} = \frac{S}{N} \cdot \frac{0.5}{K}$$

$$\cancel{\frac{E_b}{N_0}\bigg|_{dB} = 10\log\left(\frac{S}{N}\right) + 10\log 0.5 - 10\log K}$$



ALTERNAT 1:

$$\frac{S}{N}\bigg|_{SYMB} = \frac{E_S/T_{SIG}}{N_0 \cdot W} = \frac{E_S}{N_0} \cdot \frac{R_S}{W} \qquad \bigg| W = (1+r)\frac{R_S}{2} \bigg| = \frac{E_S}{N_0} \cdot \frac{2}{(1+r)}$$

ALTERNAT 2:

ZA REALNI SIGNALI

$$\frac{S}{N}\bigg|_{SYMB} = \frac{E_S/T_{SIG}}{N_0 \cdot W} = \frac{E_S/T_{SIG}}{N_0 \cdot \frac{f_S}{2}} = \frac{E_S/T_{SIG}}{N_0/2T_{SAMP}} = \frac{E_S}{N_0} \cdot \frac{2T_{SAMP}}{T_{SIG}}$$

$$\frac{E_S}{N_0} = \frac{S}{N} \cdot \frac{T_{SIG}}{2T_{SAMP}} = \frac{S}{N} \cdot \frac{0.5\,T_{SIG}}{T_{SAMP}} \qquad \frac{E_b}{N_0} = \frac{S}{N} \cdot \frac{0.5\,T_{SIG}}{K\,T_{SAMP}}$$

$$\boxed{\frac{E_b}{N_0}\bigg|_{dB} = 10\log\frac{S}{N} + 10\log\frac{0.5\,T_{SIG}}{T_{SAMP}} - 10\log K}$$

USAMP

VAZNO VO MATLAB !!!

• CONTEMPORARY COMMUNICATIONS

QAM $\quad u_m(t) = A_{mc}\, g_T(t)\cos(2\pi f_c t) + A_{ms}\, g_T(t)\sin(2\pi f_c t) \quad m = 1, 2, \ldots M$

CH. 7.1

$$\boxed{g_T(t) = \begin{cases} \sqrt{\frac{2}{T}} & 0 \le t \le T \\ \emptyset & \text{otherwise} \end{cases}}$$

IN ORDER TO HAVE UNIT SIGNAL ENERGY FOR $\psi(t)$ i.e. ORTHOGONALITY CONDITION

$u_m(t) = s_m \psi(t) \qquad \psi(t) = g_T(t) \cdot \cos(2\pi f_c t)$

$$\boxed{\int_{-\infty}^{\infty} \psi^2(t)\, dt = 1}$$

USLOV ZA ORTOGONALNOST

$$\int_0^T x^2 \cos^2(2\pi f_c t)\, dt = 1$$

$\cancel{\frac{1}{2\pi f_c} x \cdot \sin(2\pi f_c t)\Big|_0^T = 1 \qquad \frac{x}{2\pi f_c}\left[\sin\left(2\pi \frac{1}{T} T\right) - \sin(0)\right]}$

$\cancel{\int_{-T/2}^{T/2} x \cos(2\pi f_c t)\, dt = \frac{x}{2\pi f_c} \sin(2\pi f_c t)\Big|_{-T/2}^{T/2} \quad \frac{x}{2\pi f_c}\left[\sin\pi - \sin\pi\right]}$

$$(*) = \int_0^T \frac{x^2}{2}\left[1 + \cos(4\pi f_c t)\right] dt = \frac{x^2}{2} T + \frac{x^2}{2 \cdot 4\pi f_c} \cancel{\sin 4\pi f_c t}\Big|_0^T$$

$$\cos\left(\frac{\alpha}{2} + \frac{\alpha}{2}\right) = \cos^2\frac{\alpha}{2} - \sin^2\frac{\alpha}{2} = \cos^2\frac{\alpha}{2} - 1 + \cos^2\frac{\alpha}{2}$$

$$\cos^2\frac{\alpha}{2} = \frac{1}{2}(1 + \cos\alpha)$$

$$(*) = \frac{x^2}{2} T = 1 \qquad \boxed{x = \sqrt{\frac{2}{T}}}$$

$$\int_{-\infty}^{\infty} \psi^2(t)\, dt = \int_{-\infty}^{\infty} g_T^2(t) \cos(2\pi f_c t)\, dt = \frac{1}{2}\int_{-\infty}^{\infty} g_T^2(t)\, dt$$

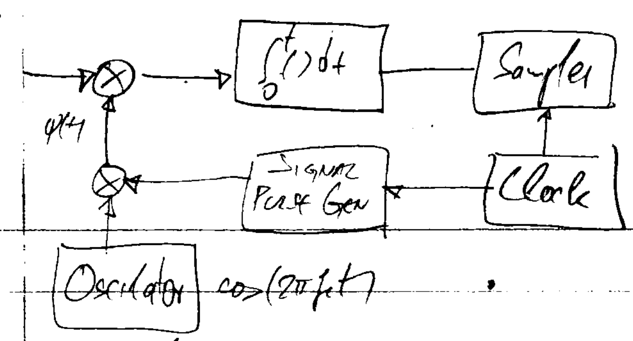
$$\boxed{\frac{1}{2}\int_{-\infty}^{\infty} g_T^2(t) \cos(2\pi f_c t)\, dt = 0 \qquad f_c \gg W} \qquad \boxed{\frac{1}{2}\int_{-\infty}^{\infty} g_T^2(t)\, dt = 1}$$

$r(t) = A_m\, g_T(t) \cos(2\pi f_c t) + n(t)$

$n(t) = n_c(t) \cos(2\pi f_c t) - n_s(t) \sin(2\pi f_c t)$

$$\boxed{\psi(t) = g_T(t) \cos(2\pi f_c t)}$$

$$\int_{-\infty}^{\infty} r(t) \cdot \psi(t)\, dt = A_m + n = s_m + n$$



$\psi(t) = g(t)\cos(2\pi f_c t)$

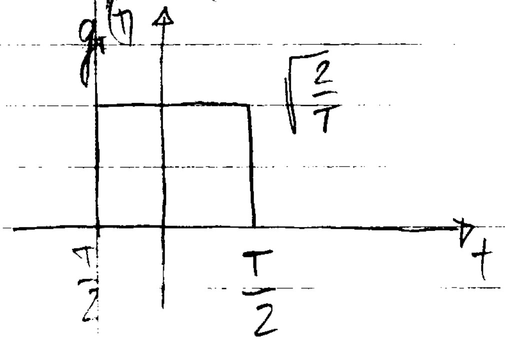
$$S_n(f) = \begin{cases} \frac{N_0}{2} & |f - f_c| < W \\ 0 & \text{otherwise} \end{cases}$$

$$\sigma_N^2 = \int_{-\infty}^{\infty} |\Psi(f)|^2 S_n(f)\, df \qquad \Psi(f) = \frac{1}{2}\left[G_T(f - f_c) + G_T(f + f_c)\right]$$

$$\sigma_N^2 = 2 \cdot \int_{f_c - W}^{f_c + W} \frac{N_0}{2} |G_T(f - f_c)|^2\, df$$

$$\boxed{W = \frac{R_s}{2}(1 + r) \overset{r=0}{=} \frac{1}{2T}}$$

$$g_T = \begin{cases} \sqrt{\frac{2}{T}} & 0 \le t \le T \\ 0 & \text{otherwise} \end{cases}$$



$$G_T(\omega) = \int_{-T/2}^{T/2} \underbrace{\sqrt{\frac{2}{T}}}_{\epsilon} e^{-j\omega t}\, dt$$

$$= \underbrace{\epsilon}_{\sqrt{\frac{2}{T}}} \cdot T \cdot \frac{\sin \omega T/2}{\omega T/2} \cdot \sqrt{\frac{2}{T}}\, T\, \frac{\sin \omega T/2}{\omega T/2}$$

$$G_T(\omega) = \frac{\epsilon}{-j\omega} e^{-j\omega t}\Big|_{-T/2}^{T/2} = -\frac{\epsilon}{j\omega}\left(e^{-j\omega T/2} - e^{+j\omega T/2}\right) = \frac{2\epsilon}{\omega} \sin\left(\frac{\omega T}{2}\right)$$

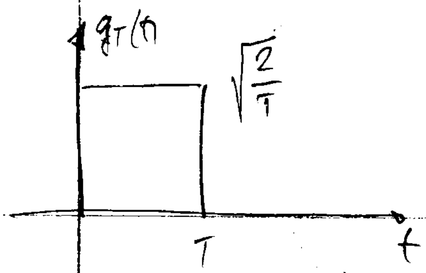
$$G_T(\omega) = \frac{\cancel{2}\epsilon}{\frac{\cancel{2}}{T}} \frac{\sin \omega T/2}{\omega T/2} = \epsilon \cdot T \frac{\sin\frac{\omega T}{2}}{\frac{\omega T}{2}} = \sqrt{\frac{2}{T}} \cdot T \frac{\sin\frac{\omega T}{2}}{\frac{\omega T}{2}}$$

$$\sigma_N^2 = 2 \int_{-W}^{W} \frac{N_0}{2} G_T^2(f)\, df = 2 \cdot \int_{-W}^{W} \frac{N_0}{\cancel{2}} \frac{\cancel{2}}{\cancel{T}} T^{\cancel{2}} \frac{\sin^2 \frac{\omega T}{2}}{\left(\frac{\omega T}{2}\right)^2}\, df$$

$$= 2 \int_{-W}^{W} N_0 \cdot T \frac{\sin^2 \frac{\cancel{2}\pi f T}{\cancel{2}}}{(\pi f T)^2}\, df = 2N_0 T \int_{-W}^{W} \frac{\sin^2(\pi f T)}{(\pi f T)^2}\, df = 2N_0 T \int_{-1/2T}^{1/2T} \frac{\sin^2(\pi f T)}{(\pi f T)^2}\, df$$

$$u = \pi f T \quad du = \pi T\, df \quad f = \frac{1}{2T} \quad u = \pi \cancel{T} \cdot \frac{1}{2\cancel{T}} = \frac{\pi}{2} \quad f = -\frac{1}{2T} \quad u = -\frac{\pi}{2}$$

$$\sigma_N^2 = \frac{2N_0 \cancel{T}}{\pi \cdot \cancel{T}} \int_{-\pi/2}^{\pi/2} \frac{\sin^2 u}{u^2}\, du = \frac{2N_0}{\pi} \int_{-\pi/2}^{\pi/2} \frac{\sin^2 u}{u^2}\, du = 1.54 N_0 \;?$$



$$G_T(j\omega) = \int_0^T \epsilon \cdot e^{-j\omega t}\,dt = \frac{\epsilon}{j\omega} e^{-j\omega t}\Big|_0^T$$

$$G_T(j\omega) = -\frac{\epsilon}{j\omega}\left(e^{-j\omega T} - 1\right)$$

$$G_T(j\omega) = -\frac{\epsilon \cdot e^{-j\omega T/2}}{j\omega}\left(e^{-j\omega T/2} - e^{j\omega T/2}\right) = \frac{2\epsilon e^{-j\frac{\omega T}{2}}}{\omega} \cdot \sin\frac{\omega T}{2}$$

$$G_T(j\omega) = \epsilon \cdot T e^{-j\omega T/2} \cdot \frac{\sin \omega T/2}{\omega T/2}$$

$$\mathcal{F}\left\{x\left(t \pm \frac{T}{2}\right)\right\} = X(j\omega) \cdot e^{\pm j\omega \frac{T}{2}}$$

PROAKIS ILLUSTRATIVE PROBLEM 7.1: ROLL OFF FACTOR $r = 0.5$

$r = \frac{W - W_0}{W_0}$ $\quad W_0 = \frac{f_s}{2}$ $\quad r = 0.5$ $\quad \frac{1}{2} W_0 = W - W_0$ $\quad \boxed{W = \frac{3W_0}{2}}$

$W = 1.5W_0$

$$H(f) = \begin{cases} 1 & |f| \leq 2W_0 - W \\ \sqrt{\cos \frac{2\pi}{4}\left(\frac{|f| + W - 2W_0}{W - W_0}\right)} & 2W_0 - W < |f| < W \\ 0 & |f| > W \end{cases}$$

$$H(f) = \begin{cases} 1 & |f| \leq 0.5W_0 \\ \cos \frac{\pi}{4} \frac{|f| - 0.5W_0}{0.5W_0} & 0.5W_0 < |f| < 1.5W_0 \\ 0 & |f| > 1.5W_0 \end{cases}$$

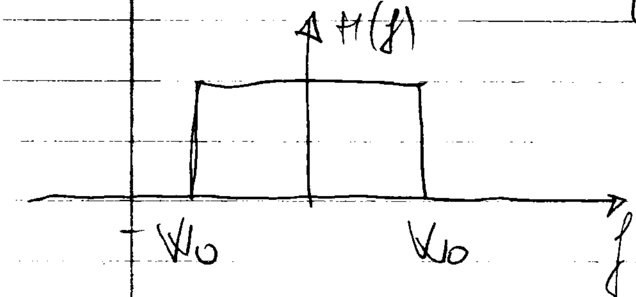
$$H(f) = \cos \frac{\pi}{2} \frac{|f| - 0.5W_0}{W_0} \qquad 0.5W_0 \leq |f| \leq 1.5W_0$$

RAISED COSINE IMPULSE RESPONSE

$$h(t) = 2W_0 \operatorname{sinc}(2W_0 t) \frac{\cos\left[2\pi (W - W_0) t\right]}{1 - \left[4(W - W_0)t\right]^2}$$

$$\boxed{h(t) = 2W_0 \operatorname{sinc}(2W_0 t) \frac{\cos(\pi W_0 t)}{1 - (2W_0 t)^2} \qquad r = 0.5}$$

$\boxed{r = 0}$ $\quad W = W_0 \quad H(f) = \begin{cases} 1 & |f| \leq W_0 \\ 0 & |f| > W_0 \end{cases}$



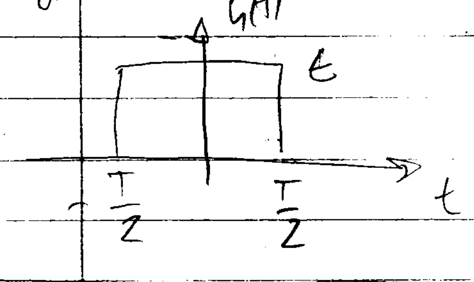
$f_s = 1e6\ Hz = 1MHz$

$$\boxed{W_0 = \frac{f_s}{2} = 500\,kHz}$$

$f = [-200:200]\,df$ $\quad df = 5\cdot 10^{+3}$ $\quad f = -10^{6} \div 10^{6}$ $\quad f_s = \frac{1}{T} = 10^{6}$

$t = [-200:200]\,dt$ $\quad dt = 5\cdot 10^{-9}$ $\quad T = 10^{-6}$

$f_s = 10^{6}$ $\quad W_0 = \frac{f_s}{2}$ $\quad \boxed{T_s = \frac{1}{f_s} = 10^{-6}}$



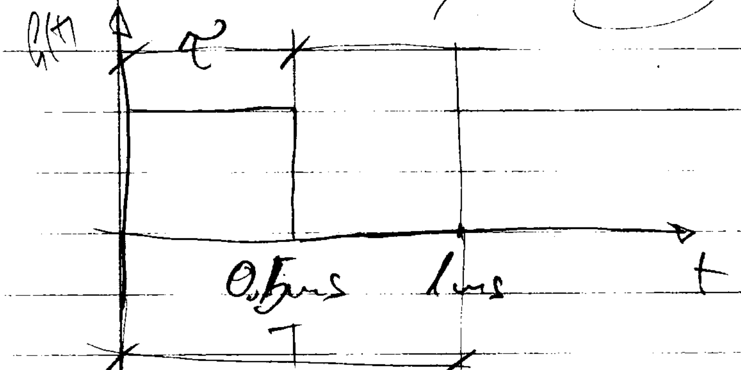
$H(j\omega) = \epsilon \cdot T \dfrac{\sin\frac{\omega T}{2}}{\frac{\omega T}{2}} = \epsilon T \dfrac{\sin \pi f T}{\pi f T}$

$H(j\omega) = \epsilon \cdot T \cdot \mathrm{sinc}(fT)$

$0.1\cdot 10^{-3} = T$ $\quad T = 0{,}1$ msec $\quad \frac{1}{T} = \frac{1}{10^{4}} = 10\,kHz$

$f = \left[-\frac{N}{2} : \frac{N}{2}\right] \cdot \frac{1}{N\cdot dt}$ $\quad f = [-200:200]\,MHz$

$N\cdot dt = 100 \cdot 10\mu s = 1ms = T$



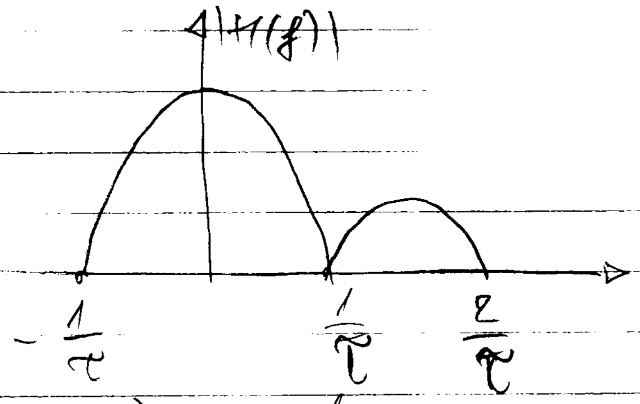
$H(j\omega) = \epsilon\cdot\tau \dfrac{\sin\left(\frac{\omega\tau}{2}\right)}{\frac{\omega\tau}{2}} = \epsilon\tau \dfrac{\sin(\pi f\tau)}{\pi f \tau}$

$= \epsilon\,\tau\,\sin^2(\pi f \tau)$

$f = \frac{1}{T} = 10^{3} = 1\,KHz$

$\frac{1}{\tau} = \frac{1}{0.5\cdot 10^{-3}} = 2\cdot 10^{+3} = 2\,kHz$

$0.2\cdot 10^{-3}$ $\quad \boxed{\frac{1}{0.2}\,10^{3} = 5\,kHz}$ nula



$T = 10\cdot 10^{-6} = 10^{-5}$

$\frac{1}{T} = 10^{5} = 100\,kHz$

$\cos(2\pi f_c t)$ $\quad f_c = 10^{3} f_s = 10^{3}\cdot 10^{6} = 10^{9} = 1\,GHz$

$t = 0 \div T = 0 \div 10^{-6}$ $\quad t = -\frac{T}{2} : \frac{T}{2} = -5\cdot 10^{-7} : 5\cdot 10^{-7}$

$f_c \cdot t = 10^{9}[0 : 10^{-6}] = [0 : 10^{3}]$ $\quad t f_c = 10^{9}[-5\cdot 10^{-7} : 5\cdot 10^{-7}] = [-5:5]\,10^{2}$

$\cos(2\pi f_c \cdot t) = \cos(2\pi 10^{9} N\cdot dt) = \cos(2\pi 10^{9}[-500:499]\,dt) \overset{10\cdot 10^{-3}}{=} \cos(2\pi[-500:499]$

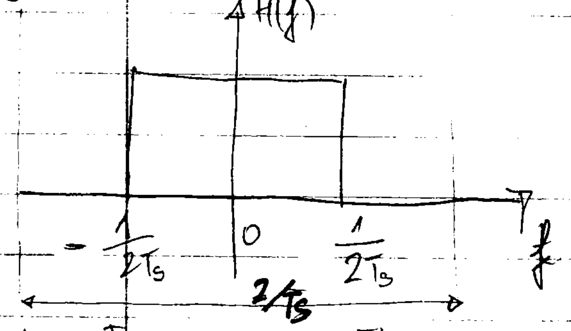
$f_c = 5\,GHz$ $\quad \cos\left(2\pi\cdot 5\cdot 10^{9}\left[-\frac{N}{2} : \frac{N}{2}-1\right]dt\right) = \cos\left(2\pi\cdot 5\cdot 10^{9}[-500:499]\,10\cdot 10^{-9}\right)$

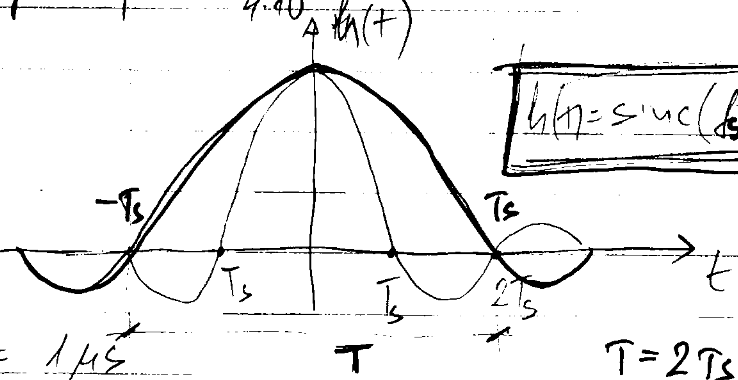
$\cos\left(2\pi\cdot 1\cdot 10^{9}\left[-\frac{N}{2}:\frac{N}{2}-1\right]5\cdot 10^{-10}\right) = \cos(\pi n\cdot 10^{1}\cdot 2/5) = \cos\left(\pi\left[-\frac{N}{2}:\frac{N}{2}-1\right]\right)$

$N = 400 \quad dt = 10 \cdot 10^{-9} \qquad t = \left[-\frac{N}{2} : \frac{N}{2} - 1\right] dt = [200 : 099]\, dt$

$T = 200 \cdot dt = 2\, 10^{3} \cdot 10^{-9} = 2 \cdot 10^{-6} = 2\,\mu s$

$f = [100 : 099] \cdot 10 \cdot 10^{3} \qquad df = \frac{1}{T} = \frac{1}{4 \cdot 10^{-6}} = 0{,}25 \cdot 10^{6}$





$\boxed{h(t) = \mathrm{sinc}(f_s t)}$

$t = [-1\,\mu s : 1\,\mu s] \qquad T_s = 1\,\mu s \qquad T = 2T_s$

$\mathrm{sinc}(f_s \cdot t) = \frac{\sin(\pi f_s t)}{\pi f_s t} = \mathrm{sinc}\left(10^{+6}\left[-10^{-6} : 10^{-6}\right]\right) = \mathrm{sinc}([-1 : 1])$

$t = \left[-\frac{N}{2} : \frac{N}{2} - 1\right] dt = [-100 : 99] \cdot 10 \cdot 10^{-9} = \left[-10^{3} \cdot 10^{-9} : 10^{3} \cdot 10^{-9}\right] = \left[10^{-6} : 10^{-6}\right]$

$\frac{f_s}{2} = 0.05\ MHz \qquad \boxed{f_s = 0.1\ MHz} \qquad T_s = \frac{1}{0.1 \cdot 10^{6}} = 10 \cdot 10^{-6}$

$10 \cdot df = 10 \cdot 10 \cdot 10^{3} = 100\ kHz \qquad \frac{1}{T_s} = 100\ kHz \qquad T_s = \frac{1}{10^{5}} = 10^{-5}$

• Illustrative Problem 7.1: $\quad f_c = \frac{40}{T_s} = 40\, f_s$

e.g. $\quad f_s = 22.5\ MHz! \qquad f_c = 40 \cdot 22.5\ MHz = 900\ MHz$

$T_s = 44\ ns$

$\cos(2\pi f_c t) = \cos\left(2\pi\, 9 \cdot 10^{8} \cdot \left[-\frac{N}{2} : \frac{N}{2} - 1\right] \cdot 2.22 \cdot 10^{-\cancel{12}\,4}\right) = \cos\left(0.004\pi\left[-\frac{N}{2} : \frac{N}{2}\right]\right)$

$2\pi f_c \cdot t = 2\pi \cdot 900 \cdot 10^{6}\left[-\frac{N}{2} : \frac{N}{2} - 1\right] 22.2 \cdot 10^{-12} = 2 \cdot 2\pi \cdot 9 \cdot 10^{-4}\left[\frac{N}{2} : \frac{N}{2} - 1\right]$
$= 0.04\pi$

$t = [50 : 50] \cdot 25 \cdot 10^{-6} \qquad 25 \cdot 10^{3} \cdot 10^{+3} = \underline{25 \cdot 10^{3}}$

$f_s \cdot t = 0.5 \cdot 10^{6} \cdot [50 : 49]\, 20 \cdot 10^{-6} = 10 * [-50 : 49]$

$f_s = 500 \cdot 10^{3}\ Hz \qquad T_s = \frac{1}{f_s} = \frac{1}{0.5} \cdot 10^{-6} = 2 \cdot 10^{-6}$

$df = 50 \cdot 10^{3}$

$N \cdot dt = \frac{1}{df} = \frac{1}{50 \cdot 10^{3}} = \frac{1}{5 \cdot 10^{4}} = 0.2 \cdot 10^{-4} = 20 \cdot 10^{-6} = 20\,\mu s$

$T = N \cdot dt = 20\,\mu s \qquad dt = \frac{T}{N} = 0{,}2\,\mu s$

$f_s = 10 \cdot df = 500 \cdot 10^{3} = 0.5 \cdot 10^{6} \qquad T_s = \frac{1}{f_s} = \frac{1}{0.5} \cdot 10^{-6} = 2\,\mu s$

$\mathrm{sinc}(f_s t) = \qquad \left[-\frac{N}{2} : \frac{N}{2} - 1\right] dt = [-10 : 10]\, 10^{-6} = [-10 : 10]\,\mu s$

$t = [-\frac{N}{2} : \frac{N}{2}-1]\, dt = [-\frac{N}{2} : \frac{N}{2}-1] \cdot 0.5\cdot 10^{-6} = [-100:100]\,\mu s$

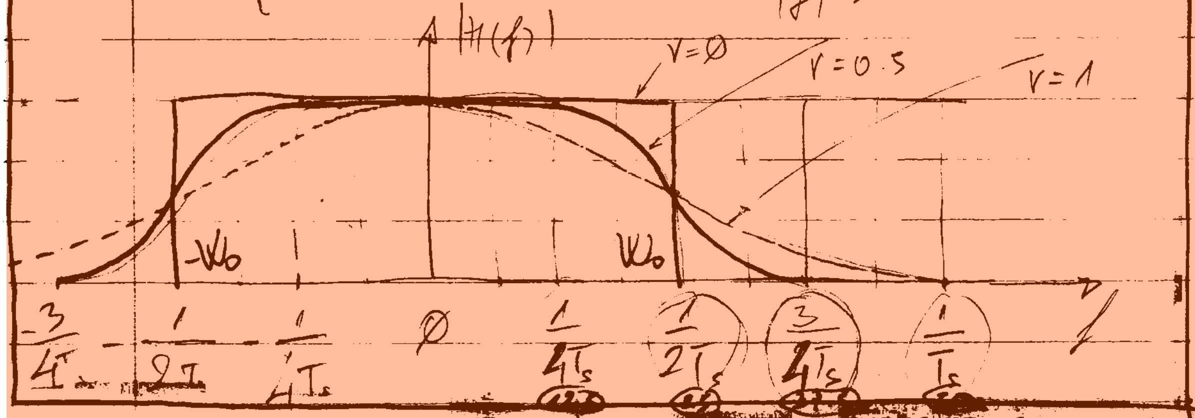
-200 400

$B = [-\frac{N}{2} : \frac{N}{2}-1]\, df = [-\frac{N}{2} : \frac{N}{2}-1]\cdot 5e^3 = [-200:200]\,5e^3 = [-1:1]$ MHz

$r = \frac{W-W_0}{W_0}$. $r = 0.5$ $\boxed{W = 1.5W_0}$

$W_0 = \frac{f_s}{2} = 0.5$ MHz $\frac{1}{2T_s} = 0.5$ MHz $t_s = 10^{-6} = 1\mu s$

$$H(f) = \begin{cases} 1 & |f| < 2W_0 - W \\ \cos^2\left(\frac{\pi}{4}\frac{|f|+W-2W_0}{W-W_0}\right) & 2W_0 - W < |f| < W \\ & |f| > W \end{cases}$$



$\boxed{MMV}$

$f_s = 1$ MHz $T_s = \frac{1}{f_s} = 10^{-6}$ $\frac{1}{2T_s} = 0.5\cdot 10^6$ Hz

$r = 0.5$ $H(f) = \cos^2\left(\frac{\pi}{4}\frac{|f|-0.5W_0}{0.5W_0}\right) = \cos^2\left(\pi \frac{|f|-0.5W_0}{2\cdot W_0}\right)$

$W_0 = \frac{f_s}{2} = 0.5\cdot 10^6$

$df = 50\cdot 10^3$ $f = [-\frac{N}{2} : \frac{N}{2}-1]\, df = [-50 : 49]\, df = [-50 : 49]\, 50\cdot 10^3$

~~$|f| < [0.25 : 0.25]$ MHz~~ $f = [-2.5 : 2.5]$ MHz

$[13 : 37]\, df = [650 : 1850]\cdot 10^3 = [0.65 : 1.85]$ MHz

$\frac{1}{T_s} = 50$ $\frac{1}{4T_s} = 12.5$ $df = \frac{1}{N dt}$ $N\,df = \frac{1}{T_s}$ $\boxed{df = \frac{10^6}{50} = 2\cdot 10^4}$

$r = \frac{W-W_0}{W_0}$ $rW_0 = W - W_0$ $(r+1)W_0 = W$

$H(f) = \cos^2\left(\frac{\pi}{4}\frac{|f| + (1+r)W_0 - 2W_0}{W_0(1+r) - W_0}\right) = \cos^2\left(\frac{\pi}{4}\frac{|f| + W_0 + rW_0 - 2W_0}{\cancel{W_0} + rW_0 - \cancel{W_0}}\right)$

$\boxed{H(f) = \cos^2\left(\frac{\pi}{4}\frac{|f| + rW_0 - W_0}{rW_0}\right) = \cos^2\left(\frac{\pi}{4}\frac{\frac{|f|}{W_0} + r - 1}{r}\right)}$ MMV

e.g. $r = 0.5$ $H(f) = \cos^2\left(\frac{\pi}{4}\frac{|f| - 0.5W_0}{0.5W_0}\right)$

$N_0 = \frac{N}{8} = \frac{256}{8} = 32$ $\quad r = 0.5$ $\quad df = 20\,kHz$ $\quad r \cdot N_0 = 16$ $\quad$ 071321786 KOLDEO

$f = [32-16 : 32+16]\, df = [16 : 48] \cdot 20 \cdot 10^3 = [320, 960]\, kHz$ $\quad$ 250 750

$$\boxed{N_0 = \frac{f_s}{2df}} \qquad N_0 = \frac{10^6}{20 \cdot 10^3} = \frac{10^3}{20} = 50$$

$$N_0 = \frac{f_s}{2df} : \quad f_s = 22.5e6 \quad df = 2.25e^6 \qquad T_s = \frac{1}{f_s} = 44.4\,10^{-9}$$

ILLUSTRATED PROBLEM 7.1. (PROAKIS SOLUTION)

$T = 1$; $dt = \frac{T}{200}$; $f_c = \frac{40}{T}$

dt - sampling interval

$t = -5T + dt : dt : 5*T$ % TIME AXIS

$N = length(t)$ $\quad N = 2000$

$$\cos\left(\frac{\alpha}{2}+\frac{\alpha}{2}\right) = \cos\frac{\alpha}{2}\cos\frac{\alpha}{2} - \sin\frac{\alpha}{2}\sin\frac{\alpha}{2}$$
$$\cos(\alpha) = \cos^2\frac{\alpha}{2} - 1 + \cos^2\frac{\alpha}{2} =$$
$$\cos(\alpha) = 2\cos^2\frac{\alpha}{2} - 1$$

OVA E OVIEN VISINOST ZCOS

$$\boxed{g_T = \mathrm{sinc}\left(\frac{t}{T}\right) \frac{\cos(\pi\alpha t/T)}{1 - \frac{4\alpha^2 t^2}{T^2}}}$$

SQUARE ROOT RAISED COSINE (PROAKIS)

$f = -\frac{0.5}{dt} : \frac{1}{dt(N-1)} : \frac{0.5}{dt}$

CONTINUE PP63

$$h(t) = 2W_0\, \mathrm{sinc}(2W_0 t) \frac{\cos[2\pi(W-W_0)t]}{1 - [4(W-W_0)t]^2}$$

$$r = \alpha = \frac{W - W_0}{W_0} \qquad W - W_0 = \alpha W_0 \qquad W = (1+\alpha) W_0 \qquad W_0 = \frac{1}{2T}$$

$$h(t) = 2W_0\, \mathrm{sinc}(2W_0 t) \frac{\cos 2\pi\alpha \cdot W_0 t}{1 - [4\alpha \cdot W_0 t]^2} = 2W_0\, \mathrm{sinc}(t/T) \frac{\cos(\pi\alpha t/T)}{1 - \left(\frac{4\alpha \cdot t}{2T}\right)^2}$$

$$\boxed{h(t) = \frac{1}{T}\, \mathrm{sinc}(t/T) \frac{\cos \pi\alpha t/T}{1 - \frac{4\alpha^2 t^2}{T^2}}} \quad \text{MMV}$$

$$\alpha \overset{def}{=} r$$

$$\boxed{h(t) = 2W_0\, \mathrm{sinc}(2W_0 t) \cdot \frac{\cos(2\pi r W_0 t)}{1 - 16 r^2 W_0^2 t^2}}$$

$\boxed{\alpha = 1}$ $\quad W = 2W_0$

$$H(f) = \begin{cases} \sqrt{\cos^2\left(\frac{\pi}{4}\frac{|f|}{W_0}\right)} & |f| < 2W_0 \\ 0 & |f| > 2W_0 \end{cases} = \begin{cases} \cos\left(\frac{\pi}{4}\frac{|f|}{W_0}\right) & |f| < 2W_0 \\ 0 & |f| > 2W_0 \end{cases}$$

$$h(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H(j\omega) e^{j\omega t} d\omega = \int_{-\infty}^{\infty} H(f) e^{j2\pi f t} df = \int_{-2W_0}^{2W_0} \cos\left(\frac{\pi}{4}\frac{|f|}{W_0}\right) e^{j2\pi f t} df =$$

$$= \int_{-2W_0}^{2W_0} \cos\left(\frac{\pi}{4}\frac{f}{W_0}\right) e^{j2\pi f t} df = \int_{-1/T}^{1/T} \cos\left(\frac{\pi}{2} f T\right) e^{j2\pi f t} df$$

$$h(t)=\frac{1}{2}\int_{-1/T}^{1/T}\left(e^{j\frac{\pi f T}{2}}+e^{-j\frac{\pi f T}{2}}\right)e^{j2\pi f t}df = \frac{1}{2}\int_{-1/T}^{1/T} e^{j\pi f(2t+\frac{T}{2})}df + \frac{1}{2}\int_{-1/T}^{1/T} e^{j\pi f(2t-\frac{T}{2})}df$$

$$h(t)=\frac{1}{2}\frac{1}{j\pi(2t+\frac{T}{2})}e^{j\pi f(2t+\frac{T}{2})}\Big|_{-1/T}^{1/T}+\frac{1}{2}\cdot\frac{e^{j\pi f(2t-\frac{T}{2})}}{j\pi(2t-\frac{T}{2})}\Big|_{-1/T}^{1/T}$$

$$h(t)=\frac{1}{\pi(2t+\frac{T}{2})}\frac{e^{j\frac{\pi}{T}(2t+\frac{T}{2})}-e^{-j\frac{\pi}{T}(2t+\frac{T}{2})}}{2j}+\frac{1}{\pi(2t-\frac{T}{2})}\frac{e^{j\frac{\pi}{T}(2t-\frac{T}{2})}-e^{-j\frac{\pi}{T}(2t-\frac{T}{2})}}{2j}$$

$\cos\left(\frac{2\pi t}{T}\right)$ $\quad -\cos\left(\frac{2\pi t}{T}\right)$

$$h(t)=\frac{1}{\pi(2t+\frac{T}{2})}\cancel{\sin}\left(\frac{2\pi t}{T}+\frac{\pi}{2}\right)+\frac{1}{\pi(2t-\frac{T}{2})}\cancel{\sin}\left(\frac{2\pi t}{T}-\frac{\pi}{2}\right)$$

$$h(t)=\cos\left(\frac{2\pi t}{T}\right)\left[\frac{2t-\frac{T}{2}-2t-\frac{T}{2}}{\pi(2t+\frac{T}{2})(2t-\frac{T}{2})}\right]=-\frac{T\cos(2\pi t/T)}{\pi(4t^2-\frac{T^2}{4})}$$

$$\boxed{h(t)=-\frac{T\cdot\cos(2\pi t/T)}{\pi\frac{T^2}{4}\left(\frac{4t^2 4}{T^2}-1\right)}=-\frac{\cos(2\pi t/T)}{\frac{\pi T}{4}\left(\frac{16t^2}{T^2}-1\right)}=\frac{4\cdot\cos(2\pi t/T)}{\pi T\left(1-\frac{16t^2}{T^2}\right)}}$$ (*)

IMPULSE RESPONSE FOR SQUARE-ROOT RAISED COSINE $\boxed{r=1}$

$r=0.5$ $\quad r=\frac{W-W_0}{W_0}$ $\quad 0.5W_0=W-W_0$ $\quad W=1.5W_0$

$$H(f)=\begin{cases}1 & |f|\le(1-r)W_0\\ \cos^2\left(\frac{\pi}{4}\frac{|f|-(1-r)W_0}{rW_0}\right) & (1-r)W_0<|f|<(1+r)W_0\\ \emptyset & |f|>(1+r)W_0\end{cases}$$

TRANSFER FUNCTION FOR RAISED COSINE

HHV

$W_0=\frac{1}{2T_s}$

$r=\frac{W-W_0}{W_0}$ $\quad r\cdot W_0=W-W_0;$ $\quad (r+1)W_0=W;$ $\quad \boxed{W=(1+r)W_0}$

$$2W_0-W=2W_0-(1+r)W_0=2W_0-W_0-rW_0=W_0(1-r)$$

$$\boxed{h(t)=2W_0\,\mathrm{sinc}(2W_0t)\frac{\cos(2\pi rW_0t)}{1-16r^2W_0^2t^2}}$$

IMPULSE RESPONSE FOR RAISED COSINE $\boxed{W_0=\frac{1}{2T_s}}$

$$h(t)=\int_{-2W_0}^{2W_0}\cos^2\left(\frac{\pi}{4}\frac{f-(1-r)W_0}{rW_0}\right)e^{j2\pi ft}df=2W_0\mathrm{sinc}(2W_0t)\frac{\cos(2\pi W_0t)}{1-16W_0^2t^2}$$

$\boxed{r=1}$

CONTINUE FROM PP.62

$t=-5T+dt:dt:5T$ $\quad T=1$ $\quad dt=\frac{T}{200}=5\cdot10^{-3}=5\,ms$ $\quad dt\cdot N=10$

$t=[-999:1000]\,dt$

$f=\left[-\frac{N}{2}+1:\frac{N}{2}\right]\frac{1}{dt\cdot N}=[-99.9:100]\,Hz$ $\quad \frac{1}{T}=1Hz=f_s$

$f_c=\frac{40}{T}=40Hz$ $\quad W_0=\frac{f_s}{2}=0.5f_s=\frac{1}{2T}$

$$g_T = \mathrm{sinc}\left(\frac{t}{T}\right) \cdot \frac{\cos(\pi\alpha t/T)}{1 - \frac{4\alpha^2 t^2}{T^2}} \qquad 1 - \frac{4\alpha^2 t^2}{T^2} = 0$$

$$\frac{4\alpha^2 t^2}{T^2} = 1 \qquad t^2 = \frac{T^2}{4\alpha^2} \qquad \boxed{t = \pm\frac{T}{2\alpha}} \qquad \boxed{|t| = \pm\frac{T}{2\alpha}}$$

$$\forall \emptyset$$

$$\lim_{t \to \frac{T}{2\alpha}} \sin\left(\frac{1}{2\alpha}\right) \frac{\cos\left(\frac{\pi}{2\alpha}\right)}{1 - \frac{4\alpha^2}{T^2}\cdot\frac{T^2}{4\alpha^2}} \neq 0$$

HO [illegible] SO [illegible]

- SQUARE-ROOTED RAISED COSINE IMPULSE RESPONSE

$$H(f) = \begin{cases} 1 & |f| < (1-r)W_0 \\ \cos\left(\frac{\pi}{4}\frac{|f| - (1-r)W_0}{rW_0}\right) & (1-r)W_0 \le |f| \le (1+r)W_0 \\ 0 & |f| > (1+r)W_0 \end{cases}$$

$$h(t) = \underbrace{\int_{-(1-r)W_0}^{(1-r)W_0} e^{j2\pi f t}\, df}_{\triangle} + \underbrace{2\int_{(1-r)W_0}^{(1+r)W_0} \cos\left(\frac{\pi}{4}\frac{f-(1-r)W_0}{r\underset{1/2T}{W_0}}\right) e^{j2\pi f t}\, df}_{I}$$

$$I_1 = \int_{+(1-r)W_0}^{(1+r)W_0} \cos\left(\frac{\pi}{\cancel{4}_2}\frac{f-(1-r)W_0}{r}\cdot 2T\right) e^{j2\pi f t}\, df \qquad f = r\mu + (1-r)W_0$$

$$\frac{f-(1-r)W_0}{r} = \mu \qquad \frac{df}{r} = d\mu \qquad \underline{df = r\cdot d\mu}$$

$$f = +(1-r)W_0 \qquad \mu = \emptyset$$

$$f = (1+r)W_0 \qquad \mu = \frac{W_0 + rW_0 - W_0 + rW_0}{r} = 2W_0 = \frac{1}{T}$$

$$I_1 = \int_0^{1/T} \cos\left(\frac{\pi}{2}\mu T\right) e^{j2\pi r\mu t} e^{j2\pi(1-r)W_0 t} d(r\mu) = \underbrace{e^{j2\pi(1-r)W_0 t}}_{=K_1} \underbrace{\int_0^{1/T} \cos\left(\frac{\pi}{2}\mu T\right) e^{j2\pi r\mu t} d(r\mu)}_{①}$$

$$I_1 = r\cdot e^{j2\pi(1-r)W_0 t} \int_0^{1/T} \cos\left(\frac{\pi}{2}\mu T\right) e^{j2\pi\mu t'} d\mu$$

$$I_2 = \int_{-(1+r)W_0}^{-(1-r)W_0} \cos\left(\frac{\pi}{2}\,\frac{-f-(1-r)W_0}{r}\cdot T\right) e^{j2\pi f t} df = \int_{-(1-r)W_0}^{-(1+r)W_0} \cos\left(\frac{\pi}{2}\,\frac{f+(1-r)W_0}{r}T\right) e^{j2\pi f t} df$$

$$\frac{f+(1-r)W_0}{r} = \mu; \qquad df = r\,d\mu; \qquad f = -(1-r)W_0 \qquad \mu = \emptyset; \qquad \boxed{f = r\mu - (1-r)W_0}$$

$$f = -(1+r)W_0 \qquad \mu = \frac{-W_0 - rW_0 + W_0 - rW_0}{r} = -\frac{2\cancel{r}W_0}{\cancel{r}} = -\frac{1}{T}$$

$$I_2 = \int_0^{1/T} \cos\left(\frac{\pi}{2}\mu T\right) e^{j2\pi r \mu t} \cdot e^{-j2\pi(1-r)W_0 t} \cdot r \cdot d\mu = \underbrace{r \cdot e^{-j2\pi(1-r)W_0 t}}_{K_2} \int_0^{1/T} \cos\left(\frac{\pi}{2}\mu T\right) e^{j2\pi\mu t'} d\mu$$

$$\frac{1}{2}\int_0^{1/T} \left(e^{j\frac{\pi\mu T}{2}} + e^{-j\frac{\pi\mu T}{2}}\right) e^{j2\pi\mu t'} d\mu = \frac{1}{2}\int_0^{1/T} e^{j\pi\mu\left(\frac{T}{2}+2t'\right)} d\mu + \frac{1}{2}\int_0^{1/T} e^{+j\pi\mu\left(t'-\frac{T}{2}\right)} d\mu$$

$$= \frac{1}{2}\,\frac{e^{j\frac{\pi}{T}\left(\frac{T}{2}+2t'\right)} - 1}{j\pi\left(2t'+\frac{T}{2}\right)} + \frac{1}{2}\,\frac{e^{j\frac{\pi}{T}\left(2t'-\frac{T}{2}\right)} - 1}{j\pi\left(2t'-\frac{T}{2}\right)} =$$

$$= \frac{1}{2}\,\frac{e^{j\frac{\pi}{2T}\left(\frac{T}{2}+2t'\right)}\left(e^{j\frac{\pi}{2T}\left(\frac{T}{2}+2t'\right)} - e^{-j\frac{\pi}{2T}\left(\frac{T}{2}+2t'\right)}\right)}{j\pi\left(2t'+\frac{T}{2}\right)} + \frac{e^{j\frac{\pi}{2T}\left(2t'-\frac{T}{2}\right)}\sin\left(\frac{\pi}{2T}\left(2t'-\frac{T}{2}\right)\right)}{\pi\left(2t'-\frac{T}{2}\right)}$$

$$= \frac{e^{j\frac{\pi t'}{T}} e^{j\frac{\pi}{4}} \sin\left(\frac{\pi t'}{T}+\frac{\pi}{4}\right)}{\pi\left(2t'+\frac{T}{2}\right)} + \frac{e^{j\frac{\pi t'}{T}} e^{-j\frac{\pi}{4}} \sin\left(\frac{\pi t'}{T}-\frac{\pi}{4}\right)}{\pi\left(2t'-\frac{T}{2}\right)}$$

$$= e^{j\frac{\pi t'}{T}}\ \frac{e^{j\frac{\pi}{4}}\left(2t'-\frac{T}{2}\right)\sin\left(\frac{\pi t'}{T}+\frac{\pi}{4}\right) + e^{-j\frac{\pi}{4}}\left(2t'+\frac{T}{2}\right)\sin\left(\frac{\pi t'}{T}-\frac{\pi}{4}\right)}{\pi\left(2t'+\frac{T}{2}\right)\left(2t'-\frac{T}{2}\right)} \quad (*)$$

$$(*) = e^{j\frac{\pi}{4}}\left(2t'-\frac{T}{2}\right)\left[\sin\frac{\pi t'}{T}\cdot\frac{\sqrt{2}}{2} + \cos\left(\frac{\pi t'}{T}\right)\cdot\frac{\sqrt{2}}{2}\right] + e^{-j\frac{\pi}{4}}\left(2t'+\frac{T}{2}\right)\left[\sin\frac{\pi t'}{T}\,\frac{\sqrt{2}}{2} - \cos\frac{\pi t'}{T}\,\frac{\sqrt{2}}{2}\right]$$

$$I_1 = 2\cdot K_1 \cdot \frac{\left(T\cdot e^{j\frac{2\pi t'}{T}} - 4jt'\right)}{\pi\left(T^2-16t'^2\right)} \qquad I_2 = \frac{2\cdot K_2\left(T\cdot e^{-j\frac{2\pi t'}{T}} + 4jt'\right)}{\pi\left(T^2-16t'^2\right)} \qquad K_1 = r\cdot e^{j\frac{\pi(1-r)t}{T}} \quad K_2 = r\cdot e^{-j\frac{\pi(1-r)t}{T}}$$

$$I_1 + I_2 = 2\cdot r\cdot\frac{e^{j\frac{\pi(1-r)t}{T}}\left(T e^{j\frac{2\pi r t}{T}} - 4jrt\right) + e^{-j\frac{\pi(1-r)t}{T}}\left(T e^{-j\frac{2\pi r t}{T}} + 4jrt\right)}{\pi\left(T^2-16r^2t^2\right)} \qquad 2\cdot r\cdot(*)$$

$$(*) = e^{j\frac{\pi t}{T}}\cdot \cancel{e^{-j\frac{\pi r t}{T}}}\cdot T e^{j\frac{2\pi r t}{T}} - 4jrt\cdot e^{j\frac{\pi t}{T}}\cdot e^{-j\frac{\pi r t}{T}} + e^{-j\frac{\pi t}{T}}\cdot \cancel{e^{+j\frac{\pi r t}{T}}}\, T e^{-j\frac{2\pi r t}{T}} + 4jrt\, e^{-j\frac{\pi t}{T}}\, e^{j\frac{\pi r t}{T}}$$

$$= T\cdot\left(e^{j\frac{\pi}{T}(1+r)t} + e^{-j\frac{\pi}{T}(1+r)t}\right) - 4jrt\left(+e^{j\frac{\pi(1-r)t}{T}} - e^{-j\frac{\pi(1-r)t}{T}}\right) =$$

$$= 2T\cos\frac{\pi}{T}(1+r)t + 8rt\sin\left(\frac{\pi}{T}(1-r)t\right)$$

$$\boxed{h(t) = (1) + 2\cdot r\,\frac{2\cdot T\cos\frac{\pi}{T}(1+r)t + 8rt\sin\left(\frac{\pi}{T}(1-r)t\right)}{\pi\left(T^2-16r^2t^2\right)}}$$

VIDI pg 63 (*)

$$r = 1 \qquad h(t) = \frac{4T\cos\left(\frac{2\pi}{T}t\right)}{\pi T^2\left(1-\frac{16r^2t^2}{T^2}\right)} = \frac{4\cos\left(\frac{2\pi}{T}t\right)}{\pi T\left(1-\frac{16r^2t^2}{T^2}\right)}$$

$$(1) = \int_{-(1-r)W_0}^{(1-r)W_0} e^{j2\pi f t} df = \left.\frac{e^{j2\pi f t}}{j2\pi t}\right|_{-(1-r)W_0}^{(1-r)W_0} = \frac{e^{j2\pi t(1-r)W_0} - e^{-j2\pi t(1-r)W_0}}{2j\pi t}$$

$$(1) = \frac{\sin(\pi(1-r)t/T)}{\pi t} = \frac{(1-r)}{T}\,\frac{\sin(\pi(1-r)t/T)}{\pi(1-r)t/T} = \frac{1-r}{T}\operatorname{sinc}\left((1-r)t/T\right)$$

$$\boxed{h(t) = \frac{1-r}{T}\,\mathrm{sinc}\left[(1-r)t/T\right] + \frac{4rT\cos(\pi(1+r)t/T) + 8rt\sin(\pi(1-r)t/T)}{\pi T\left(1-\frac{16r^2t^2}{T^2}\right)}}$$

IMPULSE RESPONSE SQUARE-ROOT RAISED COSINE

$$h(t) = \frac{\sin\pi(1-r)t/T}{\pi t} + \frac{4rT\cos(\pi(1+r)t/T) + 8rt\sin(\pi(1-r)t/T)}{\pi T\left(1-\frac{16r^2t^2}{T^2}\right)} =$$

$$= \frac{T\left(1-\frac{16r^2t^2}{T^2}\right)\sin(\pi(1-r)t/T) + 4rtT\cos(\pi(1+r)t/T) + 8rt^2\sin(\pi(1-r)t/T)}{t\pi T\left(1-\frac{16r^2t^2}{T^2}\right)}$$

$$h(t) = \frac{4rtT\cos(\pi(1+r)t/T) + \left(T - \frac{16r^2t^2}{T} + 8rt^2\right)\sin(\pi(1-r)t/T)}{t\pi T\left(1-\frac{16r^2t^2}{T^2}\right)}$$

$$\boxed{h(t) = \frac{4rtT\cos(\pi(1+r)t/T) + \frac{T^2-16r^2t^2+8Trt^2}{T}\sin(\pi(1-r)t/T)}{\pi tT\left(1-\frac{16r^2t^2}{T^2}\right)}}$$

ALTERNATIVE FORM

$-16r^2t^2 + 8Trt^2 = 8t^2r(-2r+T) = \underline{(T-2r)8t^2r}$

$x = [1, 2, 3, 4]$ circfold $(x) = [1, 4, 3, 2]$

4 3 2

$x_1 = [1, 2, 2]$ $x_2 = [1, 2, 3, 4]$

CONV

		1	2	3	4			
2	2	1						1
	2	2	1					4
		2	2	1				9
			2	2	1			14
				2	2	1		14
					2	2	1	8

CONV$(x_1, x_2) =$
1 4 9 14 14 8

CCONV

1	2	3	4	1	2	3	4	1	
2	2	1							
	2	2	1						15
		2	2	1					12
			2	2	1				9
				2	2	1			14

$y = $ CCONV (x_1, x_2) $N=4$
15, 12, 9, 14
fftshift (y)
9, 14, 15, 12

1	2	3	4	0	1	2	3	4	0	
		2	2	1						9
			2	2	1					4
				2	2	1				9
					2	2	1			14
						2	2	1		14

$y = $ CCONV (x_1, x_2) $N=5$
9 4 9 14 14

7.3. Carrier-Phase Modulation

$\theta_m = \frac{2\pi m}{M} \qquad m = 0, 1, \ldots M-1$

$$u_m(t) = A \cdot g_T(t) \cdot \cos\left(2\pi f_c t + \frac{2\pi m}{M}\right), \qquad m = 0, 1, \ldots, M-1$$

$$E_m = \int_{-\infty}^{\infty} u_m^2(t)\,dt = \int_{-\infty}^{\infty} A^2 g_T^2(t) \cdot \cos^2\left(2\pi f_c t + \frac{2\pi m}{M}\right) dt$$

$\cos^2(x) = \cos(x) \cdot \cos(x)$

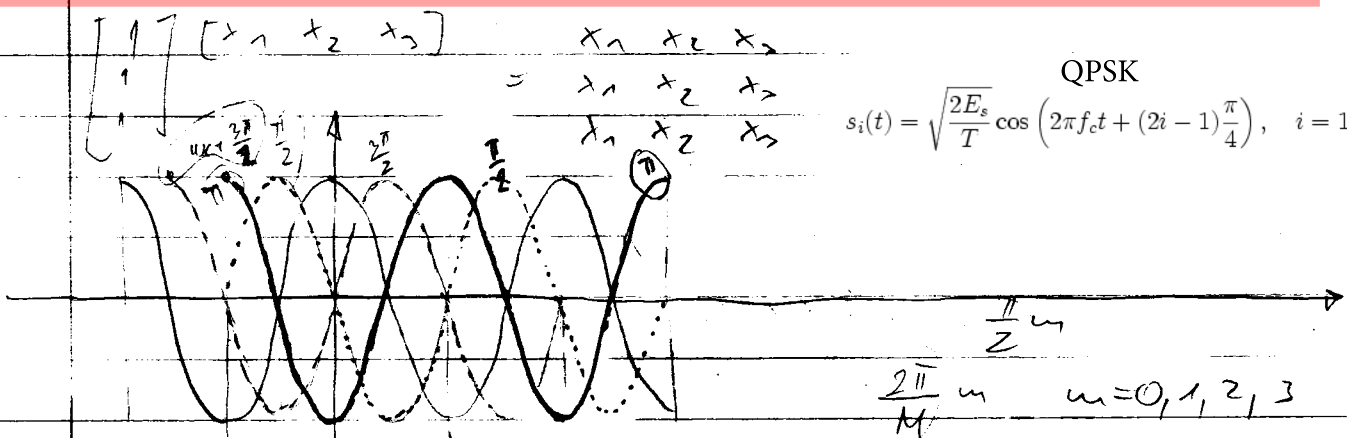
$\cos(x+x) = \cos^2(x) - \sin^2(x) = \cos^2(x) - 1 + \cos^2(x) \Rightarrow \qquad \cos^2(x) = \frac{1}{2}(1 + \cos(2x))$

$$E_m = \int_{-\infty}^{\infty} \frac{A^2}{2} g_T^2(t)\,dt + \underbrace{\int_{-\infty}^{\infty} \frac{A^2}{2} g_T^2(t) \cos\left(4\pi f_c t + \frac{4\pi m}{M}\right) dt}_{\to 0} = \frac{A^2}{2} \int_{-\infty}^{\infty} g_T^2(t)\,dt = E_s$$

$g_T(t) = \sqrt{\frac{2}{T}} \qquad 0 \le t < T \qquad$ FOR RECTANGULAR PULSE

$E_s = \frac{A^2}{\cancel{2}} \cdot \frac{\cancel{2}}{\cancel{T}} \cdot \cancel{T} \qquad \boxed{A = \sqrt{E_s}}$

$$u_m(t) = \sqrt{E_s}\sqrt{\frac{2}{T}} \cos\left(2\pi f_c t + \frac{2\pi m}{M}\right) = \sqrt{\frac{2E_s}{T}} \cos\left(2\pi f_c t + \frac{2\pi m}{M}\right)$$



QPSK

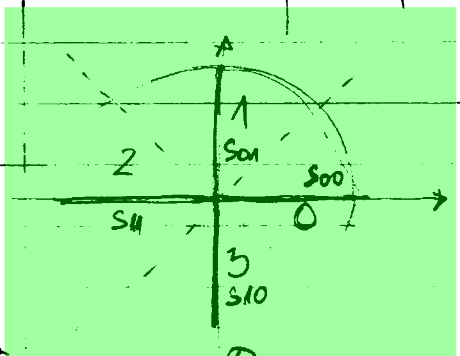
$$s_i(t) = \sqrt{\frac{2E_s}{T}} \cos\left(2\pi f_c t + (2i-1)\frac{\pi}{4}\right), \quad i = 1, 2, 3, 4.$$

$\frac{\pi}{2} m$

$\frac{2\pi}{M} m \qquad m = 0, 1, 2, 3$

$0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$

PSK-4



$$x = \left[\cos\left(\frac{2\pi m}{M}\right), \sin\left(\frac{2\pi m}{M}\right)\right]$$

$$x = [\underset{0}{1, 0};\ \underset{1}{0, 1};\ \underset{2}{-1, 0};\ \underset{3}{0, -1}]$$

- Illustrative Problem 7.2: $\boxed{M = 8}$
- Phase Demodulation and Detection

MMV

$$r(t) = u_m(t) + n(t) = u_m(t) + n_c(t)\cos(2\pi f_c t) - n_s(t)\sin(2\pi f_c t)$$

$$u_m(t) = g_T \sqrt{\frac{2E_s}{T}} \cos\left(2\pi f_c t + \frac{2\pi m}{M}\right) = \sqrt{\frac{2E_s}{T}} \cos\left(\frac{2\pi m}{M}\right) \cos(2\pi f_c t) \cdot g_T(t)$$

$$- \sqrt{\frac{2E_s}{T}} \sin\left(\frac{2\pi m}{M}\right) \cdot \sin(2\pi f_c t) \cdot g_T(t) = s_{mc} \cdot \psi_1 + s_{ms} \cdot \psi_2$$

$s_{mc} = \sqrt{\frac{2E_s}{T}} \cos\left(\frac{2\pi m}{M}\right) \qquad s_{ms} = +\sqrt{\frac{2E_s}{T}} \sin\left(\frac{2\pi m}{M}\right)$

$\psi_1(t) = g_T(t)\cdot\cos(2\pi f_c t) \qquad \psi_2(t) = -g_T(t)\sin(2\pi f_c t)$

$$\boxed{r(t) = s_{mc}\cdot\psi_1(t) + s_{ms}\,\psi_2(t) + n_c\cos(2\pi f_c t) - n_s\sin(2\pi f_c t)}$$

$$r = s_m + n = \left[\sqrt{E_s}\cos\frac{2\pi m}{M} + n_c \,,\; \sqrt{E_s}\sin\frac{2\pi m}{M} + n_s\right]$$

$$n_c = \frac{1}{2}\int_{-\infty}^{\infty} g_T(t)\cdot n_c(t)\,dt \qquad n_s = \frac{1}{2}\int_{-\infty}^{\infty} g_T(t)\cdot n_s(t)\,dt$$

$$\sigma_n = \int_{-\infty}^{\infty} |\psi(f)|^2 S_n(f)\,df \qquad \psi(t) = g_T(t)\cdot\cos(2\pi f_c t)$$

$$E(n_c) = E(n_s) = 0 \qquad E(n_c n_s) = 0$$

$$\int_{-\infty}^{\infty} r(t)\,\psi(t)\,dt = A_m + n = s_m + n \qquad \text{correlation}$$

$$\psi(t) = g_T(t)\cdot\cos(2\pi f_c t)$$

$$E(n_c^2) = E(n_s^2) = \frac{N_0}{2} = \sigma_N$$

- CORRELATION METRICS

$$C(r, s_m) = r\cdot s_m \qquad m = 0, 1, \ldots, M-1$$

$$r = (r_1, r_2) \qquad \boxed{\theta_r = \operatorname{arctg}\frac{r_2}{r_1}}$$

$$P(e) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \qquad \text{BPSK \& PSK-4}$$

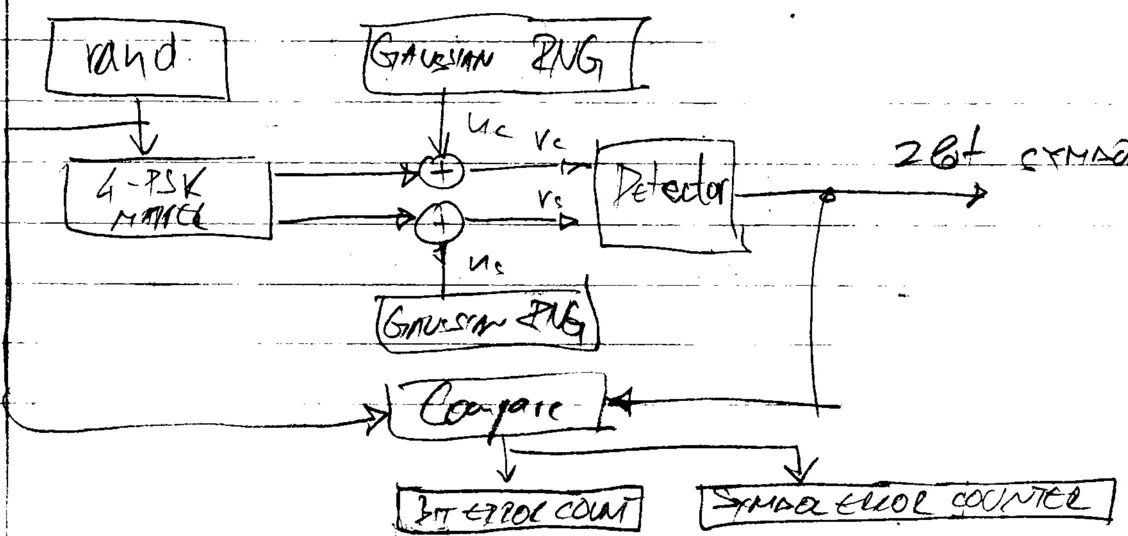
$$\boxed{k = \operatorname{ld} M}$$

- $$\boxed{P_M \doteq 2Q\left(\sqrt{\frac{2E_s}{N_0}}\cdot\sin\frac{\pi}{M}\right) = 2Q\left(\sqrt{\frac{2kE_b}{N_0}}\cdot\sin\frac{\pi}{M}\right)}$$

M-ARY PSK ERROR PROBABILITY

- GRAY CODING $\qquad \boxed{P_b \doteq \frac{1}{k}\cdot P_M}$

ILLUSTRATIVE PROBLEM 7.3 MONTE CARLO SIMULATION OF M=4 PSK COMMUNICATIONS SYSTEM



$$\sigma = \frac{1}{2}\sqrt{\frac{\epsilon}{SNR}} \qquad 4\sigma^2 = \frac{\epsilon}{SNR} \qquad \frac{S}{N} = \frac{E_s/T}{N_0 \cdot W} = \frac{E_s}{N_0}\cdot\frac{1}{T\cdot W}$$

$$W = \frac{1}{2T_{SAM}} = \frac{f_{SAM}}{2} \qquad \boxed{\frac{S}{N} = \frac{E_s}{N_0}\cdot\frac{2T_{SAM}}{T}} \qquad \boxed{R_s = \frac{1}{T}}$$

$$\boxed{\sigma^2 = \frac{N_0}{2}} \qquad SNR = \frac{E_s}{N_0} \qquad SNR = \frac{2E_s}{N_0} = \frac{E_s}{\sigma^2}$$

$$\sigma^2 = \frac{E_s}{SNR} \qquad \boxed{\sigma = \sqrt{\frac{E_s}{SNR}}} \qquad Q(z) = \frac{1}{2}\,erfc\left(\frac{z}{\sqrt{2}}\right)$$

$$\frac{S}{N} = \frac{E_s/T_{SIG}}{N_0\cdot W} = \frac{E_s/T_{SIG}}{N_0\cdot\frac{R_s}{2}} = \frac{E_s/\cancel{T_{SIG}}}{N_0/2\cancel{T_{SIG}}} = \frac{2E_s}{N_0}$$

$$\frac{N_0}{2} = \frac{E_s}{SNR} \qquad \sigma_N^2 = \frac{E_s}{SNR} \qquad \sigma_N = \sqrt{\frac{E_s}{SNR}}$$

$$\boxed{P(e) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = \frac{1}{2}\,erfc\left(\sqrt{\frac{E_b}{N_0}}\right)}$$

BINARY PSK

$SNR = 0:0.1:10$ [dB] $\qquad SNR = 10\log(snr)$

$snr = 10^{SNR/10} = 10^{0.1\,SNR}$

$$SNR = \frac{k\cdot E_b}{N_0}\cdot\frac{R_s}{W} = \left|W = \frac{R_s}{2}\right| = \frac{2kE_b}{N_0}$$

$$\frac{E_b}{N_0} = \frac{SNR}{2K} \qquad \frac{E_b}{N_0}\Big|_{dB} = 10\log(SNR) - 10\log 2 - 10\log k$$

$$E_bN_0 = 10^{0.1\,E_bN_0[dB]} \qquad e^{\ln 10^{0.1\,SNRind2(i)}} = 10^{0.1\,SNRind2(i)}$$

$$\boxed{\sigma = \frac{1}{2}\sqrt{\frac{\epsilon}{SNR}}} \qquad s_m = \left[\cos\frac{2\pi m}{M}\,,\ \sin\frac{2\pi m}{M}\right] \qquad M=4,\ m = 0,1,2,3$$

$s_0 = [1, 0]$; $s_1 = [0, 1]$; $s_2 = [-1, 0]$; $s_3 = [0, -1]$

$$r = s_m + n = \left[\cos\frac{2\pi m}{M} + n_c\,,\ \sin\frac{2\pi m}{M} + n_s\right]$$

$r = [1.1, 0.1] \qquad r\cdot s_0 = [1;0.1]\cdot[1;0] = 1 + \emptyset = 1$

$r\cdot s_1 = [1.1;0.1]\cdot[0,1] = 0 + 0.1 = 0.1$

$r\cdot s_2 = [1.1;0.1]\cdot[-1,0] = -1.1 + 0 = -1.1$

$r\cdot s_3 = [1.1;0.1]\,[0,-1] = 0 - 0.1 = -0.1$

• Differential Phase Modulation and Demodulation

$$s_{mc} = \sqrt{\frac{2E_s}{T}}\cos\left(\frac{2\pi m}{M}\right) \qquad s_{ms} = \sqrt{\frac{2E_s}{T}}\sin\left(\frac{2\pi m}{M}\right) \qquad r = s_m + n$$

$$r_1 = s_{mc} + n_c \qquad r_2 = s_{ms} + n_s$$

$$\boxed{\theta_r = \arctg\frac{r_2}{r_1}}$$

$r(t) = s_{ms}(t)\cdot\varphi_1(t) + s_{mc}(t)\cdot\varphi_2(t) + n_c\cos(2\pi f_c t) - n_s\sin(2\pi f_c t)$

$\varphi_1(t) = g_T(t)\cdot\cos(2\pi f_c t) \qquad \varphi_2(t) = -g_T(t)\cdot\sin(2\pi f_c t)$

$$\boxed{s_{ms}(t) = \sqrt{\frac{2E_s}{T}}\cos\left(\frac{2\pi m}{M}\right) \qquad s_{mc}(t) = \sqrt{\frac{2E_s}{T}}\sin\left(\frac{2\pi m}{M}\right)}$$

$r_k = \sqrt{E_s}\, e^{j(\theta_k-\varphi)} + n_k$ $\quad\varphi$ - carrier phase

$n_k = n_{kc} + j n_{ks}$ - noise

$r_{k-1} = \sqrt{E_s}\, e^{j(\theta_{k-1}-\varphi)} + n_{k-1}$

$r_k\cdot r_{k-1}^* = \left(\sqrt{E_s}\, e^{j(\theta_k-\varphi)} + n_k\right)\left(\sqrt{E_s}\, e^{-j\theta_{k-1}+j\varphi} + n_{k-1}^*\right) =$

$= E_s\, e^{j(\theta_k-\theta_{k-1})} + n_{k-1}^*\sqrt{E_s}\, e^{j(\theta_k-\varphi)} + n_k\sqrt{E_s}\, e^{-j\theta_{k-1}+j\varphi} + n_k n_{k-1}^*$

$\begin{bmatrix}0\\1\\0\end{bmatrix}\begin{bmatrix}1 & 0\end{bmatrix} = \begin{bmatrix}0 & 0\\1 & 0\\0 & 0\end{bmatrix} \qquad \begin{bmatrix}1\\0\end{bmatrix}\begin{bmatrix}0 & 1 & 0 & 0\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & 0\\0 & 0 & 0 & 0\end{bmatrix}$

$\begin{bmatrix}0\\1\end{bmatrix}\begin{bmatrix}1 & 1 & 1\end{bmatrix} \qquad \begin{bmatrix}1\\1\\1\end{bmatrix}\begin{bmatrix}0 & 1\end{bmatrix}$

$\begin{bmatrix}1 & 1\\2 & 2\\3 & 3\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\1 & 2 & 3\end{bmatrix} \qquad \begin{matrix}1 & 4 & 6\\4 & 8 & 12\\6 & 12 & 18\end{matrix}$

1 4 9

$N = 50\cdot10^3 \qquad \frac{1}{N} = \frac{1}{5\cdot10^4} = 0.2\cdot10^{-4} = 2\cdot10^{-5} = \underline{\underline{20\cdot10^{-6}}}$

PROAXIS!

theo err. prob $= Q\left(\sqrt{2SNR}\right) = \frac{1}{2}\,\mathrm{erfc}\left(\frac{\sqrt{2SNR}}{\sqrt{2}}\right) = \frac{1}{2}\,\mathrm{erfc}\sqrt{SNR}$

$\frac{S}{N} = \frac{E_s/T_{SIG}}{N_0\cdot W} = \frac{E_s/T_{SIG}}{N_0\cdot\frac{R_s}{2}} = \frac{E_s/T_{SIG}}{N_0\frac{1}{2T_{SIG}}} = \frac{2E_s}{N_0} = \frac{2\,\mathrm{ld}M\,E_b}{N_0}$

$M = 4 \quad - \quad \frac{S}{N} = 4\frac{E_b}{N_0} \qquad \sigma_N^2 = \frac{N_0}{2} \qquad \frac{N_0}{2} = \frac{2E_b}{SNR}$

$\sigma_N = \sqrt{\frac{2E_b}{SNR}} \qquad \sigma_N = \sqrt{\frac{E}{SNR}/2}$

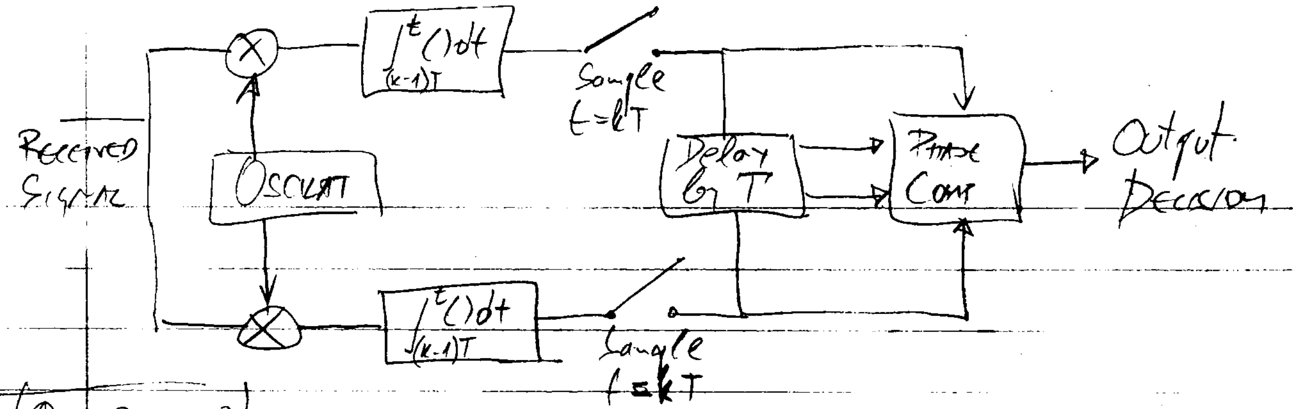
$r_k\cdot r_{k-1}^* = |n=\emptyset| = E_s e^{j(\theta_k-\theta_{k-1})}$

- PROBABILITY OF ERROR FOR DPSK

$$\boxed{P_b = \frac{1}{2}e^{-E_b/N_0}}$$

$P_b = Q\left[\sqrt{\frac{E_d}{2N_0}}\right] \qquad E_d = \int_0^T\left[s_1-s_2\right]^2 dt = \int_0^T 4A^2\,dt = 4A^2\cdot T$

$\int_0^T A^2 dt = A^2T = E_b \qquad E_d = 4E_b \qquad P_b = Q\left[\sqrt{\frac{2E_b}{N_0}}\right]$



• $\boxed{\theta_k - \theta_{k-1} = 0}$

$$r_k r_k^* = E_s + \sqrt{E_s}\left(n_k + n_{k-1}^*\right) + n_k n_{k-1}^* \qquad n_k \cdot n_{k-1}^* \to 0$$

$$x = \sqrt{E_s} + Re\left(n_k + n_{k-1}^*\right)$$

$$y = Im\left(n_k + n_{k-1}^*\right) \qquad \boxed{\sigma_N^2 = N_0} \qquad \boxed{\theta_k = \arctg \frac{y}{x}}$$

Ilustrative Problem 7.4 IMPLEMENT DIFFERENTIAL ENCODER FOR $M = 8$ DPSK

$$-87\,dBm = 10\log\frac{P}{10^{-3}} \qquad P = 10^{-3}\cdot 10^{-8,7} = 10^{-3}\cdot 2\cdot 10^{-9} = 2\cdot 10^{-12}$$

$$\boxed{P = 2\,pW}$$

x

y = mod(x, 4)

~~mapping = [0, 1, 3, 2, 7, 6, 4, 5]~~ TOCNO

mapping = [0, 1, 3, 2, 6, 7, 5, 4] VAŽA TREBA !!!

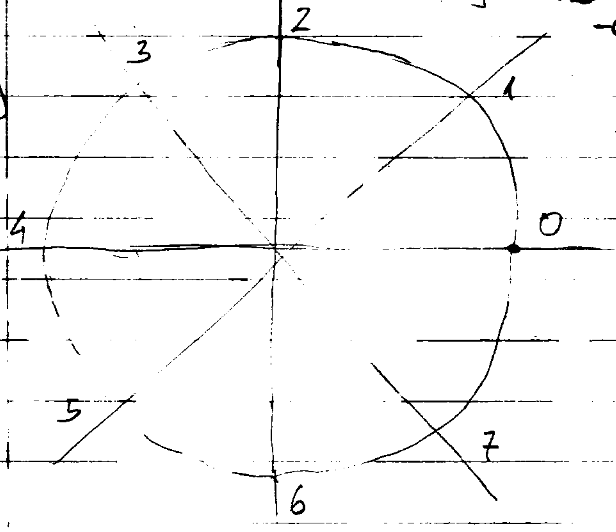
sequence = [0, 1, 0 | 0, 1, 1 | 0, 0, 1 | 1, 1, 1 | 1, 1, 0 | 0, 0, 0]

4 3 1 7 6 0

$$x = \left[\cos\frac{2\pi m}{M},\ \sin\frac{2\pi m}{M}\right]$$

[1, 0 ; 0.7, 0.7 ; 0 1 ; -0.7, 0.7 ; -1, 0 ; -0.7, -0.7 ; 0 -1 ; 0.7, -0.7]

$\boxed{M=8}$



$m_0 = [2, 3, 1, 7, 6, 0]$

$m_1 = [2, 5, 6, 5, 3, 3]$

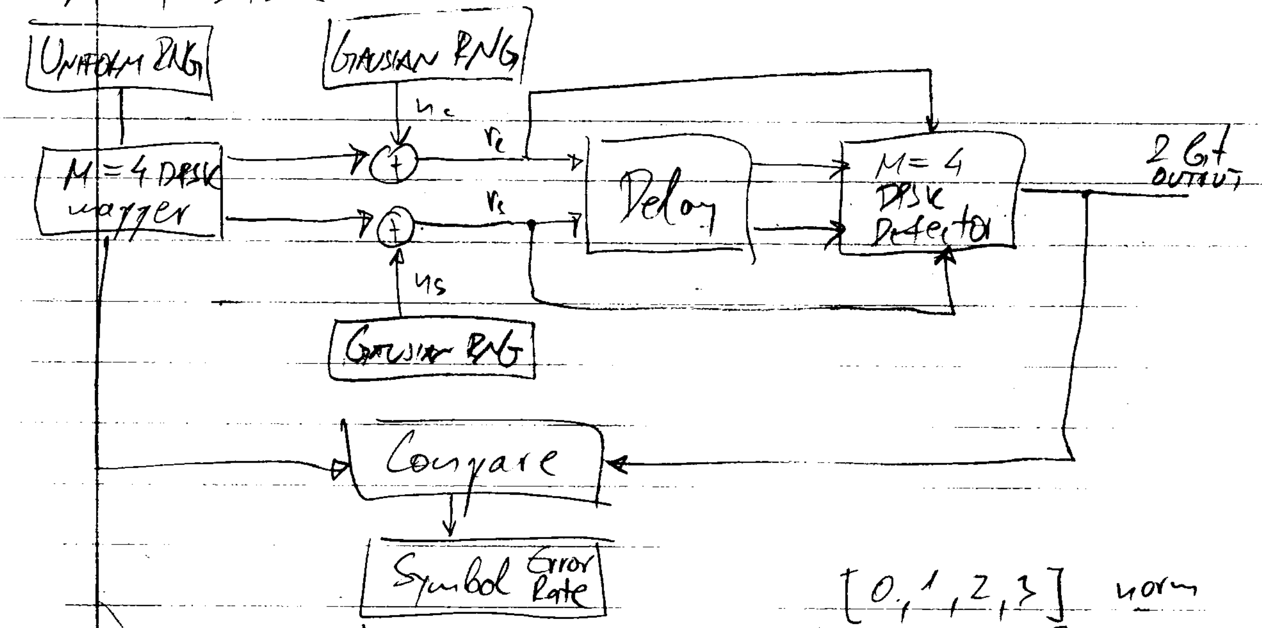
θ = mod (2 * π * mapping(index) + θ, 2π)

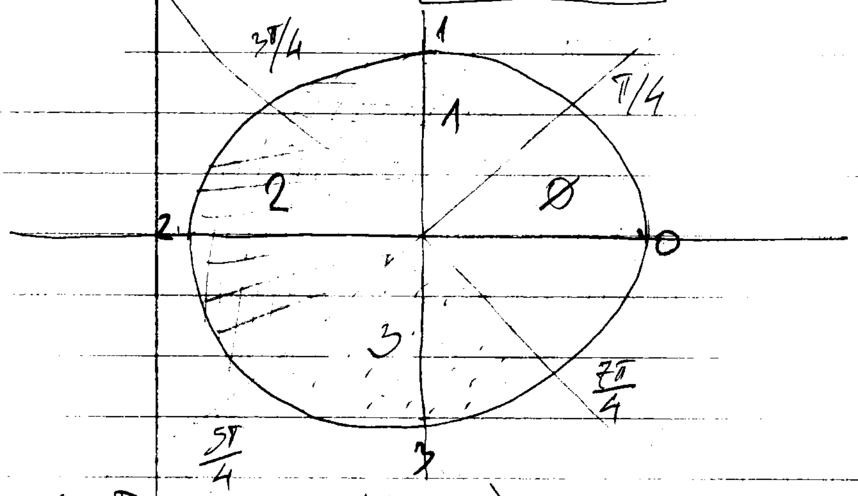
[2, 5, 6, 5, 3, 3]

$$r = \begin{bmatrix} 0 & 1 \\ -0.7 & -0.7 \\ 0 & -1 \\ -0.7 & -0.7 \\ -0.7 & 0.7 \\ -0.7 & 0.7 \end{bmatrix} \begin{matrix} 2 \\ 5 \\ 6 \\ 5 \\ 3 \\ 3 \end{matrix} \Bigg\} \text{ NO } \cancel{\text{GRAY}} \text{ CODING !!}$$

$$\begin{bmatrix} -0.7 & 0.7 \\ -0.7 & -0.7 \\ 0 & -1 \\ 0 & 1 \\ 0.7 & -0.7 \\ 0.7 & -0.7 \end{bmatrix} \begin{matrix} 3 \\ 5 \\ 6 \\ 2 \\ 7 \\ 7 \end{matrix} \Bigg\} \text{ WITH GRAY CODING}$$

7-5 Illustrative Problem Perform Monte Carlo simulation for $M=4$ DPSK





$[0,1,2,3]$ norm

$[0,1,3,2]$: gray

$$\begin{bmatrix}1\\1\\0\\0\\1\end{bmatrix}[1\ 1] = \begin{bmatrix}1&1\\1&1\\0&0\\0&0\\1&1\end{bmatrix}$$

$\boxed{\text{DPSK}}$ $P_b = 2Q\left(\sqrt{SNR}\right)$ $\qquad SNR = \frac{2\overset{2E_b}{E_s}}{N_0} = \frac{4E_b}{N_0} = \underline{\underline{4E_bN_0}}$

$\boxed{\text{PSK}}$: $P_e = Q\left(\sqrt{2SNR}\right) = Q\left(\sqrt{\frac{8E_b}{N_0}}\right)$

$$\frac{S}{N} = \frac{E_s/T_s}{N_0 \cdot W} = \frac{E_s/\cancel{T_s}}{N_0 \cdot \frac{1}{\cancel{T_s}}} \overset{T=1}{=} \frac{E_s}{N_0} \qquad SNR = \frac{E_s}{N_0}\ ??$$

$$N_0 = \frac{E_s}{SNR} \qquad \frac{N_0}{2} = \frac{E_s}{2\,SNR} \qquad \boxed{\sigma^2 = \frac{N_0}{2} = \frac{E_s}{2SNR}}$$

$$P_b = Q\left(\sqrt{\frac{E_d}{2N_0}}\right) \qquad E_d = \int_0^T [s_1 - s_2]^2\,dt$$

$$s_1 = A,\quad s_2 = -A \qquad E_d = \int_0^T 4A^2\,dt = 4A^2T$$

$$E_b = \int_0^T A^2\,dt = A^2 \cdot T \qquad \boxed{E_d = 4A^2T = 4E_b}$$

$$P_b = Q\left(\sqrt{\frac{4E_b}{2N_0}}\right) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$$

DPSK (ERROR PERFORMANCE SIGNAL) D830

$$\boxed{P_b = 2Q\left(\sqrt{\frac{2E_b}{N_0}}\right)\left[1 - Q\left(\sqrt{\frac{2E_b}{N_0}}\right)\right] \doteq 2Q\left(\sqrt{\frac{2E_b}{N_0}}\right)}$$

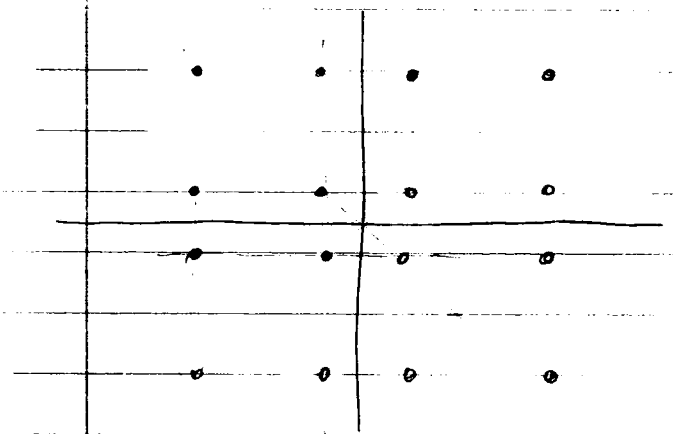
V0 ILLUSTRATIVE PROBLEM 7.5 PRODUCES ZA KONIČNI SLEDNJA FORMULA

$M=4$

$$P_b = 2Q\left(\sqrt{\frac{E_s}{N_0}}\right) = 2Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = \mathrm{erfc}\left(\sqrt{\frac{E_s}{2N_0}}\right)$$

7.4 QUADRATURE AMPLITUDE MODULATION

$$u_m(t) = A_{mc}\, g_T(t) \cos(2\pi f_c t) + A_{ms}\, g_T(t) \sin(2\pi f_c t) \qquad m = 1, 2, \ldots M$$



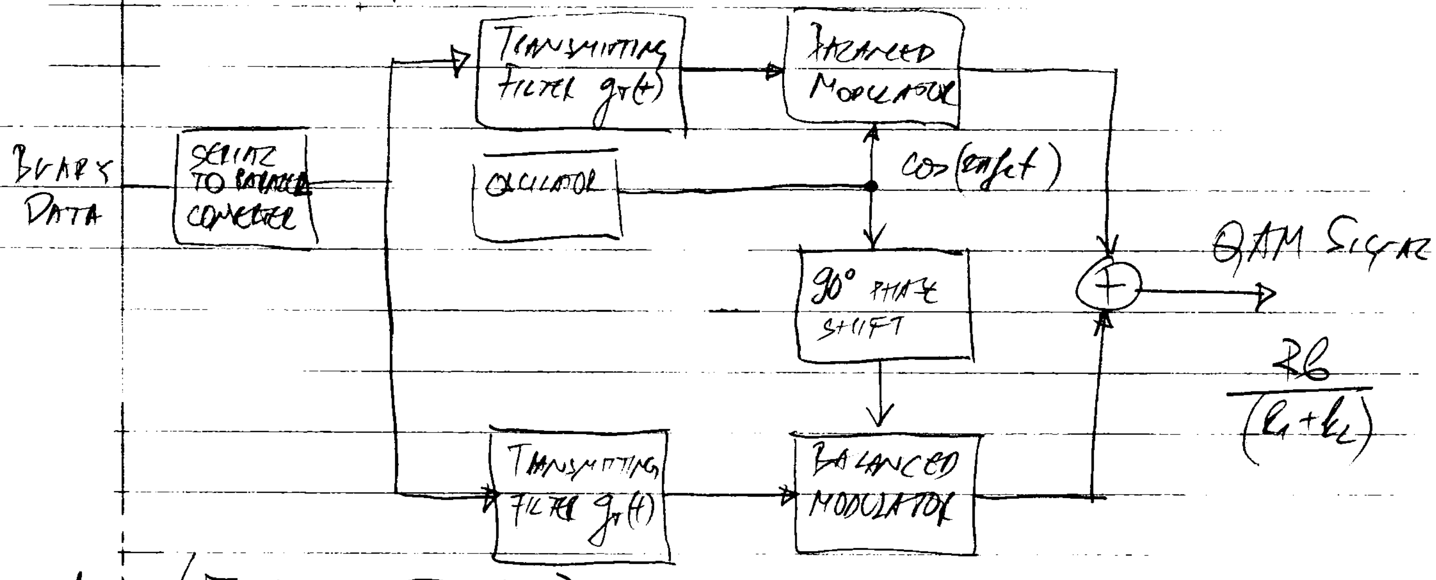
$$u_{mn}(t) = A_m\, g_T(t) \cos(2\pi f_c t + \theta_n)$$

$$m = 1, 2, \ldots M_1$$

$$n = 1, 2, \ldots M_2$$

$$M_1 = 2^{k_1} \qquad M_2 = 2^{k_2}$$

$$k_1 + k_2 = \mathrm{ld} M_1 + \mathrm{ld} M_2 = \mathrm{ld}(M_1 \cdot M_2)$$



$$s_m = \left(\sqrt{E_s} A_{mc}, \sqrt{E_s} A_{ms}\right) \qquad m = 1, 2, \ldots, M$$

$M=4$ QAM IDENTICAL TO $M=4$ PSK

7.4.1 DEMODULATION AND DETECTION OF QAM

$$r(t) = A_{mc}\, g_T(t) \cos(2\pi f_c t + \phi) + A_{ms}\, g_T(t) \sin(2\pi f_c t + \phi) + n(t)$$

$$n(t) = n_c(t) \cdot \cos(2\pi f_c t) - n_s(t) \cdot \sin(2\pi f_c t)$$

$$\psi_1(t) = g_T(t) \cos(2\pi f_c t + \phi) \qquad \psi_2(t) = g_T(t) \sin(2\pi f_c t + \phi)$$

$$r_1 = A_{mc} + n_c \cos(\phi) - n_s \sin(\phi)$$

$$r_2 = A_{ms} + n_c \sin(\phi) + n_s \cos(\phi)$$

$r(t) = A_{mc}\, g_T(t) \cdot \cos(2\pi f_c t + \phi) + A_{ms}\, g_T(t) \sin(2\pi f_c t + \phi) + n_c \cos(2\pi f_c t) - n_s \sin(2\pi f_c t)$

$\psi_1(t) = g_T(t) \cos(2\pi f_c t + \phi) \qquad \psi_2(t) = g_T(t) \cdot \sin(2\pi f_c t + \phi)$

$r_1(t) = A_{mc} \cos(2\pi f_c t + \phi) \cos(2\pi f_c t + \phi) + n_c \cos(2\pi f_c t) \cos(2\pi f_c t + \phi) - n_s \sin(2\pi f_c t) \cdot \cos(2\pi f_c t + \phi)$

$\sin\alpha \cdot \cos\alpha = \frac{1}{2}\left[\sin(\alpha+\alpha) + \sin(\alpha - \alpha)\right] = \frac{1}{2} \sin(2\alpha)$

$\cos\alpha \cos\alpha = \frac{1}{2}\left[\cos(\alpha+\alpha) + 1\right] = \frac{1}{2} + \frac{1}{2}\cos(2\alpha)$

$r_1(t) = \frac{1}{2} A_{mc} + \frac{n_c}{2}\left[\cos(\cancel{2\pi f_c t + 2\pi f_c t + \phi}) + \cos(\cancel{2\pi f_c t - 2\pi f_c t} - \phi)\right] -$

$- \frac{n_s}{2}\left[\sin(\cancel{2\pi f_c t + 2\pi f_c t + \phi}) + \sin(\cancel{2\pi f_c t - 2\pi f_c t} - \phi)\right]$

$$\boxed{r_1(t) = \frac{1}{2} A_{mc} + \frac{n_c}{2} \cos(\phi) + \frac{n_s}{2} \sin\phi}$$

$\sin(\alpha+\beta) = \sin\alpha \cdot \cos\beta + \sin\beta \cdot \cos\alpha$

$\sin(\alpha-\beta) = \sin\alpha \cdot \cos\beta - \sin\beta \cdot \cos\alpha$

$\sin\alpha \cdot \cos\beta = \frac{1}{2}\left[\sin(\alpha+\beta) + \sin(\alpha-\beta)\right]$ +

$\sin\alpha \cdot \sin\alpha = \sin^2\alpha = 1 - \cos^2\alpha = 1 - \cos\alpha \cdot \cos\alpha = 1 - \frac{1}{2} - \frac{1}{2}\cos 2\alpha$

$\sin\alpha \cdot \sin\alpha = \frac{1}{2} - \frac{1}{2}\cos(2\alpha)$

$r_2(t) = \frac{A_{ms}}{2} + n_c \cdot \underbrace{\cos(2\pi f_c t)}_{\beta} \underbrace{\sin(2\pi f_c t + \phi)}_{\alpha} - n_s \cdot \sin(2\pi f_c t) \sin(2\pi f_c t + \phi)$

$= \frac{A_{ms}}{2} + \frac{n_c}{2}\left[\sin(\cancel{2\pi f_c t + \phi + 2\pi f_c t}) + \sin(\cancel{2\pi f_c t} + \phi - \cancel{2\pi f_c t})\right] -$

$- \frac{n_s}{2}\left[\cos\phi\right] = \frac{A_{ms}}{2} + \frac{n_c}{2}\sin\phi - \frac{n_s}{2}\cos\phi$

$\sin(2\pi f_c t) \cdot \left[\sin(2\pi f_c t) \cos\phi + \cos(2\pi f_c t) \sin\phi\right] =$

$= \frac{1}{2}\cos\phi + \sin\phi\left(\cancel{\sin(2\pi f_c t) \cdot \cos(2\pi f_c t)}\right) = \frac{1}{2}\cos\phi$

$n_c = \frac{1}{2}\int_0^T n_c(t) \cdot g_T(t)\, dt \qquad n_s = \frac{1}{2}\int_0^T n_s(t) \cdot g_T(t)\, dt$

$r_1(t) = A_{mc} + n_c \cdot \cos\phi - n_s \sin(\phi) \qquad s_m = \left(\sqrt{E_s} A_{mc}, \sqrt{E_s} A_{ms}\right)$

$r_2(t) = A_{ss} + n_c \cdot \sin\phi + n_s \cos(\phi)$

$D(r, s_m) = |r - s_m|^2 \qquad m = 1, 2, \ldots, M$

$r = (r_1, r_2) \qquad s_m = \left(\sqrt{E_s} A_{mc}, \sqrt{E_s} A_{ms}\right)$

- PROBABILITY OF ERROR FOR QAM IN AWGN

$M_1 = 2^{k_1} \qquad M_2 = 2^{k_2} \qquad k_1 + k_2 = \mathrm{ld}\, M_1 + \mathrm{ld}\, M_2 = \mathrm{ld}(M_1 \cdot M_2)$

$k_1 = k_2 = k \qquad 2k = \mathrm{ld}(M^2) \qquad M^2 = 2^{2k}$

$\sqrt{M} = 2^{k/2}$ } SIGNAL POINTS PER CARRIER

CORRECT DECISION PROBABILITY

$$P_c = \left(1 - P_{\sqrt{M}}\right)^2$$

$$P_c = (1 - P_{\sqrt{M}})(1 - P_{\sqrt{M}})$$

$$P_M = \frac{2(M-1)}{M} Q\left(\sqrt{\frac{6(\mathrm{ld}M)\,\mathcal{E}_{avb}}{(M^2-1)N_0}}\right)$$

0752619941

$$P_{\sqrt{M}} = 2\left(1 - \frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{3\,\overbrace{\mathrm{ld}M\ \mathcal{E}_{avb}}^{\mathcal{E}_{av}}}{(M-1)N_0}}\right) = 2\left(1-\frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{3\,\mathcal{E}_{av}}{(M-1)N_0}}\right)$$

- PROBABILITY OF ERROR FOR M-ARY QAM:

$$\boxed{P_M = 1 - (1 - P_{\sqrt{M}})^2 \qquad P_{\sqrt{M}} = 2\left(1 - \frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{3}{M-1}\frac{\mathcal{E}_{av}}{N_0}}\right)}$$ k-even

- k - odd

$$P_M \le 1 - \left[1 - 2Q\left(\sqrt{\frac{3\mathcal{E}_{av}}{(M-1)N_0}}\right)\right]^2 \le 4Q\left(\sqrt{\frac{3k\,\mathcal{E}_{avb}}{(M-1)N_0}}\right)$$

- Ilustrative Problem 7.6

$$\frac{S}{N} = \frac{\mathcal{E}/T}{N_0 \frac{1}{T}} = \frac{(k)\,\mathcal{E}_b}{N_0} \qquad 10\log SNR = 10\log \mathcal{E}_b/N_0 + 10\log k$$

$M_1 = 8$ $\quad$ $M_2 = 2$

QAM - 16 $\quad$ $\boxed{m = 0:7}$ $\quad$ $\boxed{m = 0:1}$

$$\boxed{u_{mn}(t) = A_m g_T(t)\cos\left(2\pi f_c t + \frac{2\pi m}{M_1}\right)} \qquad A_n = \{1, 2\}_{n=0,1}$$

$$k_1 + k_2 = \mathrm{ld}M_1 + \mathrm{ld}M_2 = 3 + 1 = 4 = k$$

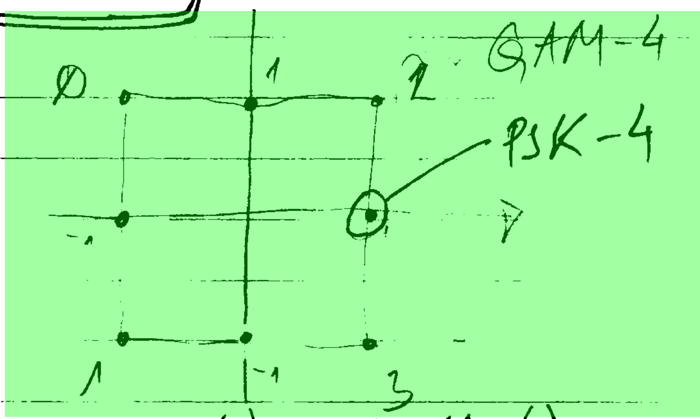
$$M = M_1 \cdot M_2 = 8 \cdot 2 = 16 \Rightarrow \text{QAM-16}$$

$$u_{mn}(t) = A_m g_T(t)\cos\left(\frac{2\pi n}{M_1}\right)\cos(2\pi f_c t) - A_m g_T(t)\sin\left(\frac{2\pi n}{M_1}\right)\cdot\sin(2\pi f_c t)$$

ILČE KRSTE $\quad$ ZATEGAČOT

QAM-4

$$\begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{bmatrix} -1 & 1 \\ -1 & -1 \\ 1 & 1 \\ 1 & -1 \end{bmatrix}$$



$$u_m(t) = A_{mc} g_T(t)\cos(2\pi f_c t + \phi) + A_{ms} g_T(t)\sin(2\pi f_c t + \phi)$$

$M = 4 \quad M_1 = 2 \quad M_2 = 2$

$A_{mc} = \{-1, 1\}$

$A_{ms} = \{-1, 1\}$

LOGICAL	A_{mc}	A_{ms}
0	-1	1
1	-1	-1
2	1	1
3	1	-1

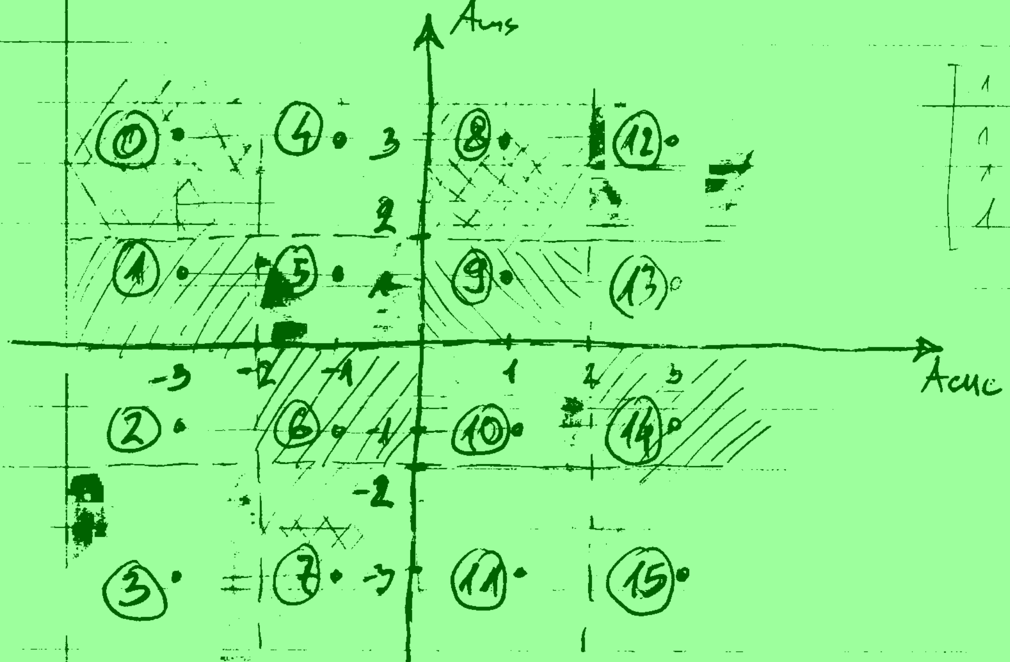
$M=8$ | $A_{nc}=\{-3,-1,1,3\}$ $\quad A_{ns}=\{-1,1\}$

$A_m=[-3,1;-3,-1;-1,1;-1,-1;1,+1;1,-1;3,1;3,-1]$

$$A'_{nc}=\begin{bmatrix}-3\\-1\\1\\3\end{bmatrix}\begin{bmatrix}1 & 1\end{bmatrix}=\begin{bmatrix}-3 & -3\\-1 & -1\\1 & 1\\3 & 3\end{bmatrix}$$

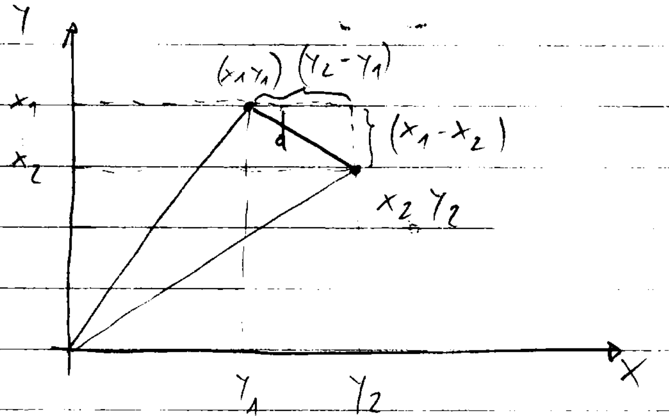
$$\begin{bmatrix}1\\-1\end{bmatrix}\begin{bmatrix}1 & 1\end{bmatrix}=\begin{bmatrix}1 & 1\\-1 & -1\end{bmatrix}$$

$M=32$ $\quad k=ld\,32=5$ $\quad k_1=3$ $\quad k_2=2$



$$\begin{bmatrix}1\\1\\1\\1\end{bmatrix}\begin{bmatrix}-3, 3\end{bmatrix}$$

$$X=\begin{bmatrix}-3 & -3 & -3 & -3 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 3 & 3 & 3 & 3\\3 & 1 & -1 & -3 & 3 & 1 & -1 & -3 & 3 & 1 & -1 & -3 & 3 & 1 & -1 & -3\end{bmatrix}$$



$$d^2=(x_1-x_2)^2+(y_2-y_1)^2$$

$$\begin{bmatrix}1\\2\\3\end{bmatrix}\begin{bmatrix}1 & 1 & 1\end{bmatrix}=\begin{bmatrix}1 & 1 & 1\\2 & 2 & 2\\3 & 3 & 3\end{bmatrix}\qquad\begin{bmatrix}1 & 1 & 1\end{bmatrix}\begin{bmatrix}1\\2\\3\end{bmatrix}$$

$$\begin{bmatrix}1\\1\\1\end{bmatrix}\begin{bmatrix}1 & 2 & 3\end{bmatrix}=\begin{bmatrix}1 & 2 & 3\\1 & 2 & 3\\1 & 2 & 3\end{bmatrix}$$

$$\frac{S}{N} = \frac{k \cdot E_b}{N_0} \qquad \sigma^2 = \frac{N_0}{2} \qquad N_0 = 2\sigma^2$$

$$snr = \frac{k \cdot E_b}{2\sigma^2} \qquad 2\sigma^2 = \frac{k \cdot E_b}{snr} \qquad \sigma^2 = \frac{k \cdot E_b}{2 \cdot snr} \; ?$$

$$\boxed{snr = \frac{E_b}{N_0}} \qquad \sigma^2 = \frac{N_0}{2} \qquad snr = \frac{E_b}{2\sigma^2} \qquad \boxed{\sigma^2 = \frac{E_b}{2\, snr}}$$

$$E_b = \frac{E_s}{ldM} = \frac{E_s}{k} \qquad \boxed{\sigma^2 = \frac{E_s}{2k \cdot snr}}$$

• QAM-16 $\quad k = 4$

$$\boxed{\sigma = \sqrt{\frac{E_s}{8 \cdot snr}}} \qquad \boxed{snr - \text{SNR PER BIT}}$$

$y =$ filter (B, A, x)

x – INPUT SIGNAL

y – output signal

A – filter feedback coefficients

B – filter feedforward "

$$y(n) = \sum_{k=0}^{M_n} B(k+1)\, x(n-k) - \sum_{k=1}^{N_n} A(k+1)\, y(n-k) \qquad n = 1, 2, \dots N_x$$

$N_x =$ length(x) $\quad N_n = \min\{N, n-1\}$ $\quad M_n = \min\{M, n-1\}$

$M+1 =$ lenght(B) $\quad N+1 =$ lenght(A)

DIGITAL SIGNAL PROCESSING:

DIFFERENCE EQUATION

$$\sum_{k=0}^{N} a_k\, y(n-k) = \sum_{m=0}^{M} b_m\, x(n-m) \qquad \forall n$$

$$y(n) = \sum_{m=0}^{M} b_m\, x(n-m) - \sum_{k=1}^{N} a_k\, y(n-k)$$

N ROOTS

$$y(n) = y_H(n) + y_P(n) \qquad y_H(n) = \sum_{k=1}^{N} c_k\, z_k^{\,n} \qquad z_k \quad k = 1, 2, \dots N$$

$$\sum_{0}^{N} a_k z_k^{\,k} = 0 \implies \text{CHARACTERISTIC EQUATION}$$

E. 2.9 $\quad \boxed{y(n) - y(n-1) + 0.9\, y(n-2) = x(n)}$

$a = [1, -1, 0.9]$ $\quad b = [1]$

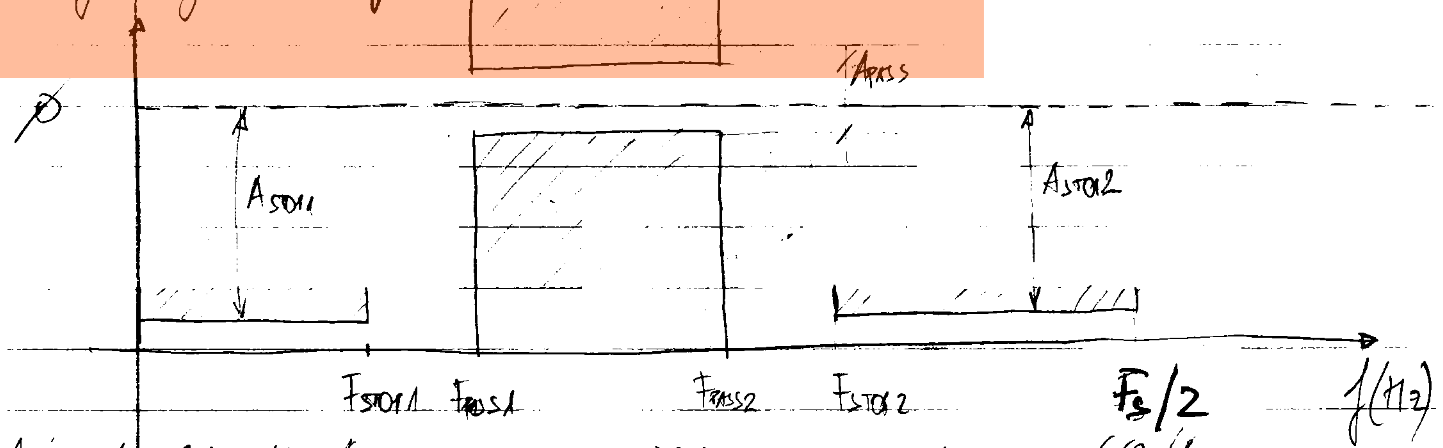
$y =$ filter(b, a, x)

$$\boxed{Y(z) = \frac{X(z)}{1 - z^{-1} + 0.9 \cdot z^{-2}}} \implies b \dots \implies a \dots$$

- SIMPLE LOW PASS FILTER
  B = [1, 1] → NUMERATOR
  A = [1] → DENOMINATOR

- **fdatool** - construct filter graphicaly
- fdesign **filterbuilder** = GRAPHICAL

MAV



A_STOP1 = 60; % ATTENUATION IN THE FIRST STOPBAND = 60 dB
F_STOP1 = 8400; % EDGE OF THE STOPBAND
F_PASS1 = 10800; % EDGE OF THE PASSBAND
F_PASS2 = 15600; % CLOSING EDGE OF THE PASSBAND
F_STOP2 = 18000; % EDGE OF THE SECOND STOPBAND
A_STOP2 = 60; % ATTENUATION IN THE SECOND STOPBAND = 60 dB
A_PASS = 1; % AMOUNT OF RIPPLE

fvtool - FILTER VISUALIZATION TOOL

$\boxed{Y = \text{filter}(\text{FilterObj}, x)}$

$\boxed{[H, w] = \text{freqz}(Hd)}$ FREQUENCY RESPONSE OF THE FILTER

$f = 5 \cdot 10^3$ $T = \frac{1}{5 \cdot 10^3} = 0.2 \cdot 10^{-3} = 0.2$ msec

$dt = \frac{T}{10} = 0{,}02$ msec $t = 0 : 10T = 0 : 2$ msec

$F_s = \cancel{20}$ kHz $df = \frac{1}{N \cdot dt} = \frac{1}{\cancel{1000} \cdot 0{,}02 \cancel{10^{-3}}} = \frac{1}{0.02} = \frac{1}{2 \cdot 10^{-2}} = 50$

$F_0 = 5$ kHz $T_0 = \frac{1}{f_0}$ $dt = \frac{T_0}{10}$ $F_s = \frac{1}{dt} = \frac{10}{T_0} = 10 F_0$

$F_s = 50$ kHz

- FULL IMPLEMENTATION OF PASSBAND PSK

$$r(t) = M_m(t) + n(t) = g_r(t) \sqrt{\frac{2E_s}{T}} \cos\left(2\pi f_c t + \frac{2\pi m}{M}\right) + n(t)$$

$$= \sqrt{\frac{2E_s}{T}} \cos\left(\frac{2\pi m}{M}\right) \cos(2\pi f_c t) - \sqrt{\frac{2E_s}{T}} \sin\left(\frac{2\pi m}{M}\right) \underline{\sin(2\pi f_c t)} + n_c(t)\cos(2\pi f_c t) - n_s \underline{\sin(2\pi f_c t)}$$

$$v_1(t) = r(t) \cdot \cos(2\pi f_c t)$$

$\cos\alpha \cdot \cos\alpha = \frac{1}{2}\left[\cos(\alpha+\alpha) + \cos(\alpha-\alpha)\right] = \frac{1}{2}\left[1+\cos(2\alpha)\right]$

$\sin\alpha \cdot \cos\alpha = \frac{1}{2}\left[\sin(\alpha+\alpha) + \sin(\alpha-\alpha)\right] = \sin(2\alpha)$

$$v_1(t) = \left[\sqrt{\frac{2E_s}{T}}\cos\left(\frac{2\pi m}{M}\right) + n_c(t)\right]\frac{1}{2}\left(1+\cos(4\pi f_c t)\right) - \left[\sqrt{\frac{2E_s}{T}}\sin\left(\frac{2\pi m}{M}\right) + n_s(t)\right]\cdot\frac{1}{2}\sin(4\pi f_c t)$$

$$v_1(t) \rightarrow \boxed{\approx} \xrightarrow{z_1(t)} \frac{1}{2}\sqrt{\frac{2E_s}{T}}\cos\left(\frac{2\pi m}{M}\right) + \frac{1}{2}n_c(t)$$

$$v_2(t) = r(t)\cdot\sin(2\pi f_c t) = \left[\sqrt{\frac{2E_s}{T}}\cos\left(\frac{2\pi m}{M}\right)\cdot\cos(2\pi f_c t) - \sqrt{\frac{2E_s}{T}}\sin\left(\frac{2\pi m}{M}\right)\sin(2\pi f_c t) + n_c\cos(2\pi f_c t) - n_s\sin(2\pi f_c t)\right]\cdot\sin(2\pi f_c t)$$

$$= \frac{1}{2}\left[\sqrt{\frac{2E_s}{T}}\cos\left(\frac{2\pi m}{M}\right) + n_c\right]\sin(4\pi f_c t) - \frac{1}{2}\left[\sqrt{\frac{2E_s}{T}}\sin\frac{2\pi m}{M} + n_s\right]\cdot\left(1-\cos(2\alpha)\right)$$

$\sin^2\alpha = 1-\cos^2\alpha = 1 - \frac{1}{2} - \frac{1}{2}\cos(2\alpha) = \frac{1}{2}\left[1-\cos(2\alpha)\right]$

$$v_2(t) \rightarrow \boxed{\approx} \xrightarrow{z_2(t)} -\frac{1}{2}\sqrt{\frac{2E_s}{T}}\sin\left(\frac{2\pi m}{M}\right) - \frac{1}{2}n_s(t)$$

$$z = z_1(t) + z_2(t) = \frac{1}{2}\sqrt{\frac{2E_s}{T}}\cos\left(\frac{2\pi m}{M}\right) - \frac{1}{2}\sqrt{\frac{2E_s}{T}}\sin\left(\frac{2\pi m}{M}\right) + \frac{n_c(t)}{2} - \frac{n_s(t)}{2}$$

. PSK-M FULL IMPLEMENTATION

$T = 1e{-}9\ sec \quad dt = T/10 = 0.1\cdot10^{-9} = 10^{-10}\ sec \quad f_s = \frac{1}{dt} = 10\,GHz$

$f_c = 1/T = 10^9\ Hz = 1\,GHz \quad N = 10^5$

$f_b = 100\,kHz = 100\cdot10^3 = 10^5\ Hz$

- $K = 1000$ BROJ NA SIMBOLI ŠTO SE ISPRAĆAAT μs

~~ILUSTRACIJA~~ $t = (0 : dt : N\cdot T - dt) \quad dt = \frac{T}{10}$

$t = (0 : 0.1 : N-1)\,T \qquad t = 0 : 10^5\cdot 1e{-}9 = 0 : 10^{-4} = 0 : 100\cdot10^{-6}$

$\boxed{t = 0 : 100\,\mu s}$

$N = 10^5 \quad K = 1000 \qquad Y = \frac{N\cdot T}{K\cdot dt} = \frac{N\cdot T}{K\cdot\frac{T}{10}} = 10\cdot\frac{N}{K} = 10^3$

VKUPEN BROJ PRIMEROCI: $1e6 \qquad dt = \frac{100\cdot10^{-6}}{10^6} = 10^{-4}\cdot10^{-6} = 10^{-10} = 0.1\,ns$

BROJ NA SIMBOLI: 10^3

$T_s = \frac{100\cdot10^{-6}}{K} = \frac{100\cdot10^{-6}}{10^3} = 10^{-1}\cdot10^{-6} = 10^{-7} = 100\cdot10^{-9} = 100\ nsec$

ZA PERIOD OD $100\,\mu sec$ SE PRENESUVAAT 1000 SIMBOLI

$v_s = \frac{10^3}{100\cdot10^{-6}} = 10^{+1}\cdot10^6 = \boxed{10\ Mbps} \quad R_s = 10\,Mbps \quad W_c = \frac{R_s}{2} = 5\,MHz$

$dt = \frac{T_C}{10}$ $T_C = 10^{-9}$ $dt = 0.1\, 10^{-9} = 100\, 10^{-12} = 100\, ns$

$df = \frac{1}{N \cdot dt} = \frac{1}{\cancel{10^4} \cdot 0.1 \cdot 10^{-9}} = \frac{1}{0.1 \cdot 10^{-5}} = \frac{1}{10^{-6}} = 10^{6} = 1MHz$

$f = (0 : N-1)\, df = 0 : 10^4 \cdot 10^6 = 0 : 10^{10} = 0 : 10 \cdot 10^9 = 0 : 10GHz - 1$

$f = \left[-\frac{N}{2} : \frac{N}{2} - 1\right] df = \left[-5\,GHz : 5GHz - 1\right]$

$t = [0 : N-1]\, dt = [0 : 10^4 - 1]\, dt$ $\Delta t = 10^4 dt = 10^4 \cdot 0.1 \cdot 10^{-9} = 10^{-6} = 1\mu s$

$R_S = 10$ SIMB$/ 1\mu s = 10 \cdot 10^6 = 10\, Mbps$ $W_C = 5MHz = \frac{R_S}{2}$

$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ \cancel{1} & \cancel{2} & \cancel{3} \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix}$$

SNR PILEMA

* SNR PER BIT

$SNR = \frac{E_b}{N_0} = \frac{E_s}{k \cdot N_0}$ $\frac{E_s}{N_0} = SNR \cdot k$

* SNR PER SIMBOL

$SNR = \frac{E_s}{N_0} = \frac{k E_b}{N_0}$ $\frac{E_b}{N_0} = \frac{SNR}{k}$

$\left.\frac{E_b}{N_0}\right|_{dB} = SNR|_{dB} - 10 \log k$

$SNR_b = snr$

$\frac{E_b}{N_0} = \frac{E}{k \cdot N_0} = \frac{E}{k \cdot 2\sigma^2} = SNR_b$ $\boxed{\sigma^2 = \frac{E}{2k \cdot snr}}$

$M = 4$ $E_s = ld\, 4 = 2$

$\sigma = \frac{1}{2 snr}$ $\boxed{snr = \frac{E_b}{N_0}}$

AKO STAVIŠ $E = 1$ VAKO PLOAMIS TOGAJ

$\sigma^2 = \frac{1}{2k \cdot snr}$ $\boxed{snr = \frac{E_b}{k N_0} = \frac{E_s}{N_0}}$

SKLAR - BANDPASS MODULATION AND DEMODULATION

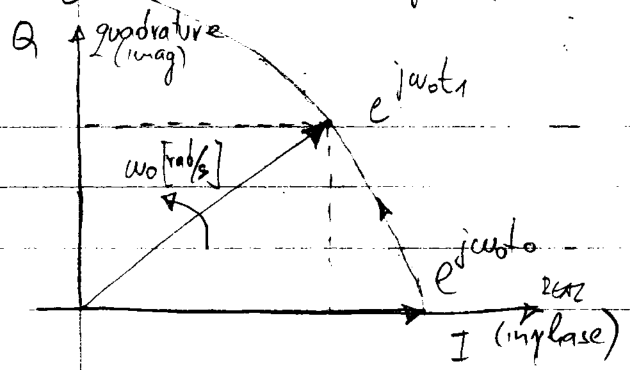
$s(t) = A(t) \cos \theta(t)$ $\theta(t) = \omega_0 t + \phi(t)$

$s(t) = A(t) \cdot \cos(\omega_0 t + \phi(t))$

- COMPLEX NOTATION OF SINUSOIDAL CARRIER

$e^{j\omega_0 t} = \cos(\omega_0 t) + j \sin(\omega_0 t)$

$$e^{j\omega_0 t} = \cos(\omega_0 t) + j\sin(\omega_0 t)$$



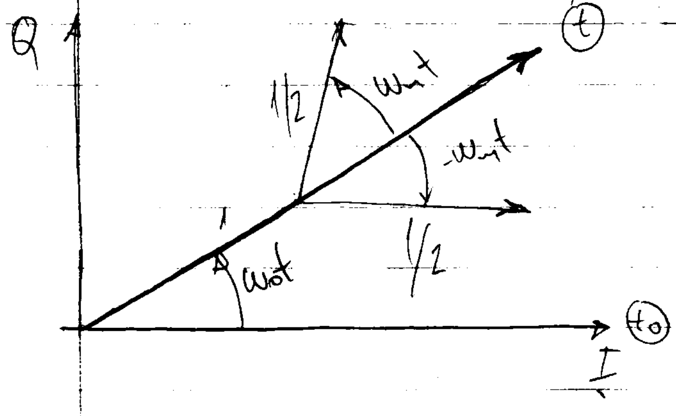
$$u_m(t) = \cos(\omega_m t)\cdot\cos(\omega_0 t)$$

$$\cos(\omega_m t) = \frac{1}{2}\left[e^{j\omega_m t} + e^{-j\omega_m t}\right]$$

$$\cancel{u_m(t) = \mathrm{Re}\left\{e^{j\omega_0 t}\left[1 + \frac{1}{2}\left[e^{j\omega_m t} + e^{-j\omega_m t}\right]\right]\right\}}$$

$$u_m(t) = \mathrm{Re}\left\{e^{j\omega_0 t} + \frac{1}{2}e^{j\omega_0 t + j\omega_m t} + \frac{1}{2}e^{j\omega_0 t - j\omega_m t}\right\}$$

$$= \cos(\omega_0 t) + \frac{1}{2}\cos(\omega_0 + \omega_m)t + \frac{1}{2}\cos(\omega_0 - \omega_m)t$$ KAM σημα



$$u_{FM}(t) = U_0\cos\left(\omega_0 t + k_\omega\int u_m(t)\,dt\right)$$

$$\delta\omega = \frac{d\varphi}{dt} \qquad \varphi = \int \delta\omega\, dt$$

$$\delta\omega = k_F \cdot u_m(t)$$

$$u_{FM}(t) = U_0\cos(\omega_0 t + \varphi)$$

$$s(t) = \mathrm{Re}\left\{e^{j\omega_0 t}\left(1 - \frac{m}{2}e^{-j\omega_m t} + \frac{m}{2}e^{j\omega_m t}\right)\right\}$$

$$u_m(t) = U_m\cos(\omega_m t)$$

$$u_{FM}(t) = U_0\cos\left(\omega_0 t + k_\omega\int U_m\cos(\omega_m t)\,dt\right) = U_0\cos\left(\omega_0 t + \frac{k_\omega U_m}{\omega_m}\sin\omega_m t\right)$$

$$u_{FM}(t) = U_0\cos\left(\omega_0 t + \frac{\Delta\omega_0}{\omega_m}\sin\omega_m t\right) \qquad \Delta\omega_0 = k_\omega U_m \qquad m = \frac{\Delta\omega_0}{\omega_m}$$

$$u_{FM}(t) = U_0\cos(\omega_0 t + m\sin\omega_m t)$$

$$\boxed{\cos(\alpha + m\sin\beta) = \sum_{n=-\infty}^{\infty} J_n(m)\cos(\alpha + n\beta)} \qquad \boxed{J_{-n}(m) = (-1)^n J_n(m)}$$

$$J_n(m) = F_n = \frac{1}{T}\int_{-T/2}^{T/2} U_0\cos(\omega_0 t + m\sin\omega_m t)\, e^{-jn\omega_0 t}\,dt$$

$$u_{FM}(t) = U_0 J_0(m)\cos(\omega_0 t) + U_0\sum_{n=1}^{\infty} J_n(m)\left\{\cos\left[(\omega_0 + n\omega_m)t + \frac{n\pi}{2}\right] + \cos\left[(\omega_0 - n\omega_m)t + \frac{n\pi}{2}\right]\right\}$$

$m \le 0.4$ οπόλις σε 2 κομπονεντι

$$u_{FM}(t) = U_0 J_0(m)\cdot\cos(\omega_0 t) - J_1(m) U_0\cdot\sin(\omega_0 + \omega_m)t - J_1(m) U_0\cdot\sin(\omega_0 - \omega_m)$$

$$\mathrm{Re}\left\{e^{j\omega_0 t} - \frac{m}{2}e^{j(\omega_0 - \omega_m)t} + \frac{m}{2}e^{j(\omega_0 + \omega_m)t}\right\} = \cos\omega_0 t - \frac{m}{2}\cos(\omega_0 - \omega_m t) + \frac{m}{2}\cos(\omega_0 + \omega_m t)$$

$$u_{FM}(t) = U_0\sum_{n=-1}^{1} J_n(m)\cos(\omega_0 t + n\cdot\omega_m t) =$$

$$= U_0 J_0(m)\cos(\omega_0 t) + U_0 J_{-1}(m)\cos(\omega_0 - \omega_m)t + U_0\cdot J_1(m)\cos(\omega_0 + \omega_m)t$$

$$= U_0 J_0(m)\cos(\omega_0 t) + U_0 J_1(m)\cos(\omega_0 + \omega_m)t - U_0 J_1(m)\cos(\omega_0 - \omega_m)t$$ 81

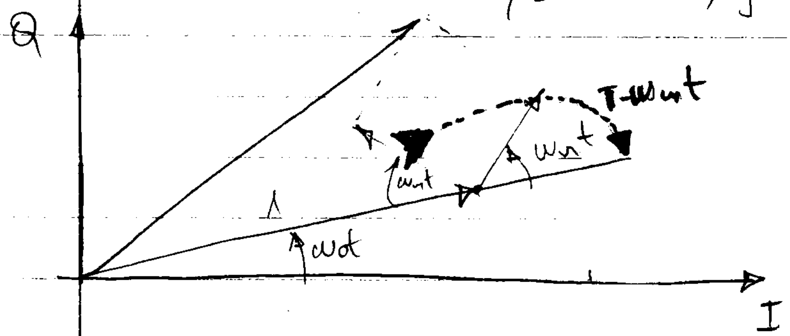
$J_n(m) = \frac{m^n}{2^n n!}$ ($m \ll 1$) $\quad n = 1 \quad \boxed{J_1(m) = \frac{m}{2}} \quad J_0 = 1$

$M_{FM}(t) = V_0 \cos\omega_0 t + \frac{V_0 m}{2}\cos(\omega_0+\omega_m)t - \frac{V_0 m}{2}\cos(\omega_0-\omega_m)t$

$V_0 = 1 \quad m = \beta$

- $M_{FM}(t) = \cos\omega_0 t + \frac{\beta}{2}\cos(\omega_0+\omega_m)t - \frac{\beta}{2}\cos(\omega_0-\omega_m)t$

$\mathrm{Re}\left\{e^{j\omega_0 t}\left(1 - \frac{\beta}{2}e^{-j\omega_m t} + \frac{\beta}{2}e^{j\omega_m t}\right)\right\} = \mathrm{Re}\left\{e^{j\omega_0 t}\left(1 + \frac{\beta}{2}e^{j(\pi-\omega_m t)} + \frac{\beta}{2}e^{j\omega_m t}\right)\right\}$



- PSK $\quad s_i(t) = \sqrt{\frac{2E}{T}}\cos(\omega_0 t + \phi_i(t)) \quad 0 \le t \le T, \; i = 1, \dots, M$

$\phi_i(t) = \frac{2\pi i}{M} \quad i = 1, 2, \dots, M$

- FSK $\quad s_i(t) = \sqrt{\frac{2E}{T}}\cos(\omega_i t + \phi) \quad 0 \le t \le T, \; i = 1, \dots, M$
- ASK $\quad s_i(t) = \sqrt{\frac{2E_i(t)}{T}}\cos(\omega_0 t + \phi) \quad 0 \le t \le T, \; i = 1, \dots, M$
- APK

$s_i(t) = \sqrt{\frac{2E_i(t)}{T}}\cos[\omega_0 t + \phi_i(t)] \quad 0 \le t \le T, \; i = 1, 2, \dots, M$

- WAVEFORM AMPLITUDE $\quad$ SREDNJA EFECTIVNA VREDNOST

$s(t) = A\cos(\omega t) = \sqrt{2}A_{rms}\cos(\omega t) = \sqrt{2A_{rms}^2}\cos(\omega t) = \sqrt{2P}\cos(\omega t)$

$\boxed{P = \frac{E}{T}} \quad \boxed{s(t) = \sqrt{\frac{2E}{T}}\cos(\omega t)}$

- DECISION REGION

$d(r, s_i) = \|r - s_i\| \qquad \|x\|$ NORM OR MAGNITUDE OF VECT. x

- CORRELATION RECEIVER

$r(t) = s(t) + n(t) \quad 0 \le t \le T, \; i = 1, \dots, M$

$\boxed{s_i(t) = \sum_{j=1}^{N} a_{ij}\psi_j(t) \quad i = 1, \dots, M, \; N \le M}$ $\quad$ REPRESENTING SIGNAL WITH BASIS FUNCTIONS

- BINARY DECISION TRESHOLD

$\boxed{z(T) = a_i(T) + n_0(T)} \quad i = 1, 2$

$p(z|s_1) = \frac{1}{\sigma_0\sqrt{2\pi}} e^{-\frac{(z-a_1)^2}{2\sigma_0^2}} \qquad p(z|s_2) = \frac{1}{\sigma_0\sqrt{2\pi}} e^{-\frac{(z-a_2)^2}{2\sigma_0^2}}$

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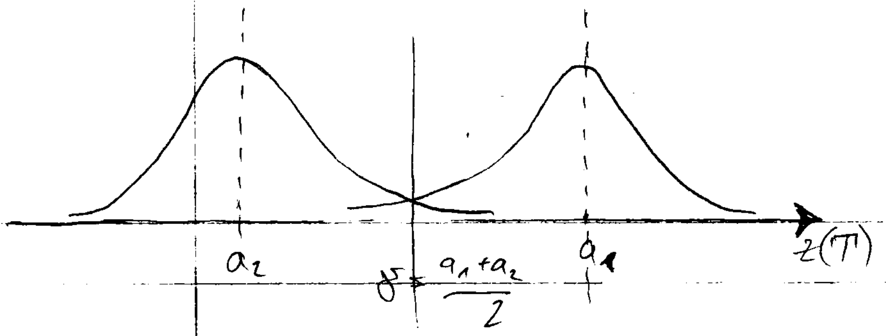


σ_0 - NOISE VARIANCE

$$z(T) \gtrless \frac{a_1+a_2}{2} = \gamma_0$$

- COHERENT DETECTION

$$s_1(t) = \sqrt{\frac{2E}{T}} \cos(\omega_0 t + \phi) \qquad 0 \le t \le T$$

$$s_2(t) = \sqrt{\frac{2E}{T}} \cos(\omega_0 t + \phi + \pi)$$

BPSK

$$s_2(t) = -\sqrt{\frac{2E}{T}} \cos(\omega_0 t + \phi)$$

$$\cos(\pi + \alpha) = \underbrace{\cos\pi \cdot \cos\alpha}_{} = -\cos\alpha \qquad \sin(\pi - \alpha) = \sin\pi \cdot \cos\alpha - \sin\alpha \cdot \cos\pi = \sin\alpha$$

$$s_i(t) = \sum_{j=1}^{N} a_{ij}\, \psi_j(t) \qquad i = 1, 2, \ldots M \qquad N \le M$$

$$a_{ij} = \frac{1}{K_j} \int_0^T s_i(t)\, \psi_j(t)\, dt \qquad K_j = 1$$

$$\boxed{\psi_1(t) = \sqrt{\frac{2}{T}} \cos(\omega_0 t)} \quad \text{for } 0 \le t \le T$$

T — SYMBOL PERIOD

$$s_1(t) = a_{11}\, \psi_1(t)$$
$$s_2(t) = a_{21}\, \psi_1(t)$$
$$s_1(t) = \sqrt{E}\, \psi_1(t)$$
$$s_2(t) = -\sqrt{E}\, \psi_1(t)$$

$$a_{11} = \int_0^T \sqrt{\frac{2E}{T}} \cos(\omega_0 t) \cdot \sqrt{\frac{2}{T}} \cos(\omega_0 t)\, dt = \frac{2}{T}\sqrt{E} \int_0^T \cos^2(\omega_0 t)\, dt$$

$$\cos^2(\alpha) = \frac{1}{2}\left[\cos(\alpha+\alpha) + \underbrace{\cos(\alpha-\alpha)}_{\cancel{0}\ 1}\right] = \frac{1}{2}(1 + \cos 2\alpha)$$

$$a_{11} = \frac{\cancel{2}\sqrt{E}}{T} \int_0^T \frac{1}{\cancel{2}} (1 + \cos(2\omega_0 t))\, dt = \frac{\sqrt{E}}{T} \cdot T = \sqrt{E}$$

$$E\left\{\int_0^T z_1 \cdot \psi_1(t)\, dt\right\}$$

$$E\{z_1 | s_1\} = E\left\{\int_0^T \left(\sqrt{E}\, \psi_1^2(t) + n(t)\, \psi_1(t)\right) dt\right\}$$

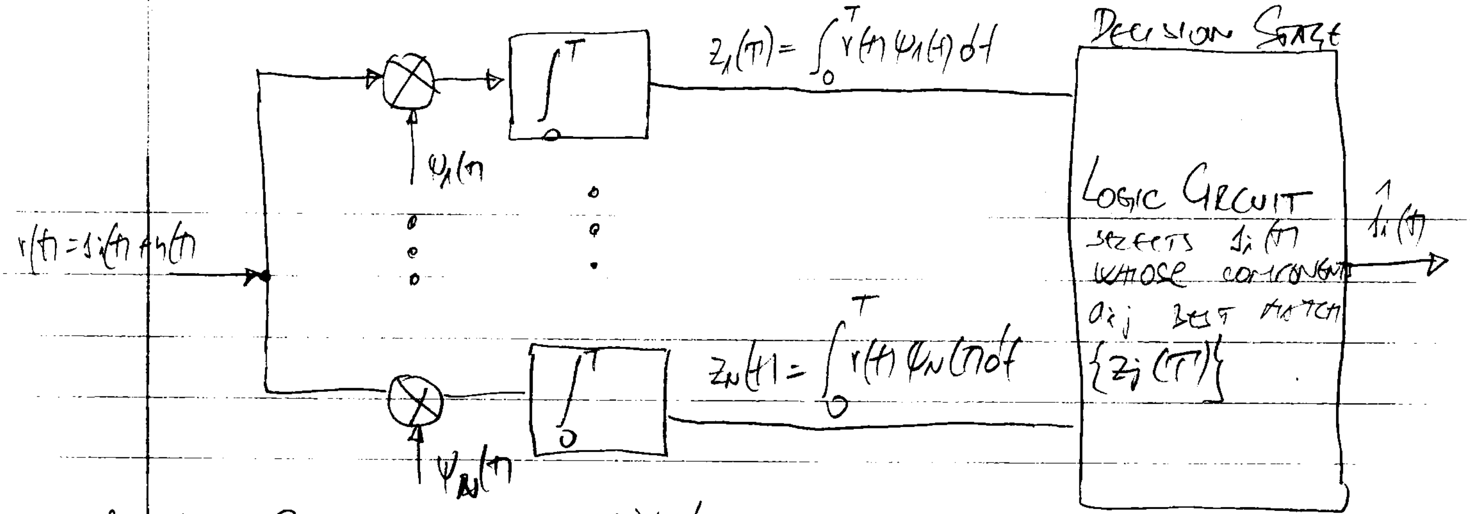
$$E\{z_2 | s_1\} = E\left\{\int_0^T \left(-\sqrt{E}\, \psi_1^2(t) + n(t)\, \psi_1(t)\right) dt\right\} \qquad E\{n(t)\} = 0$$

$$E\{z_1 | s_1\} = E\left\{\int_0^T \left(\sqrt{E} \frac{2}{T} \cos^2(\omega_0 t) + \underbrace{n(t) \sqrt{\frac{2}{T}} \cos(\omega_0 t)}_{\to 0}\right) dt\right\} = \sqrt{E}$$

$$E\{z_2 | s_1\} = E\left\{\int_0^T \left[-\sqrt{E} \frac{2}{T} \cos^2(\omega_0 t) + n(t) \sqrt{\frac{2}{T}} \cos(\omega_0 t)\right] dt\right\} = -\sqrt{E}$$

- SAMPLED MATCHED FILTER

$$H(f) = K \cdot S^*(f)\, e^{-j2\pi f T} \qquad h(t) = \mathcal{F}^{-1}\left\{K\, S^*(f)\, e^{-j2\pi f T}\right\}$$
$$= K \cdot s(T - t)$$



$$\mathcal{F}\{x(t \pm t_0)\} = X(j\omega)e^{\mp j\omega t_0}$$

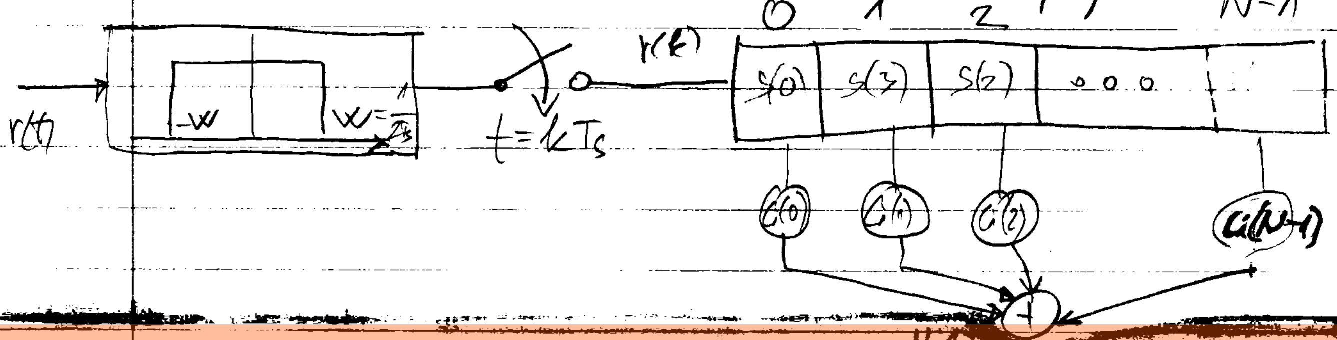
$$\mathcal{F}\{x(t)e^{\pm j\omega_0 t}\} = X\{j(\omega \mp \omega_0)\}$$

$$h(t) = \begin{cases} s(T-t) & 0 \le t \le T \\ 0 & \text{elsewhere} \end{cases}$$

$W = \frac{1}{2T_{sym}}$ T − SYMBOL RATE

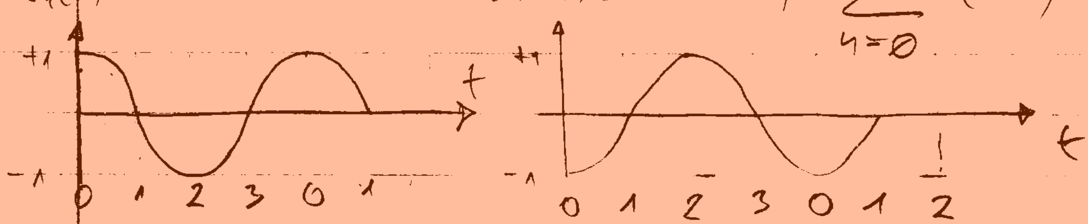
$f_s = 2W = \frac{1}{T_{sym}}$ ⇒ MINIMUM NYQUIST SAMPLING RATE

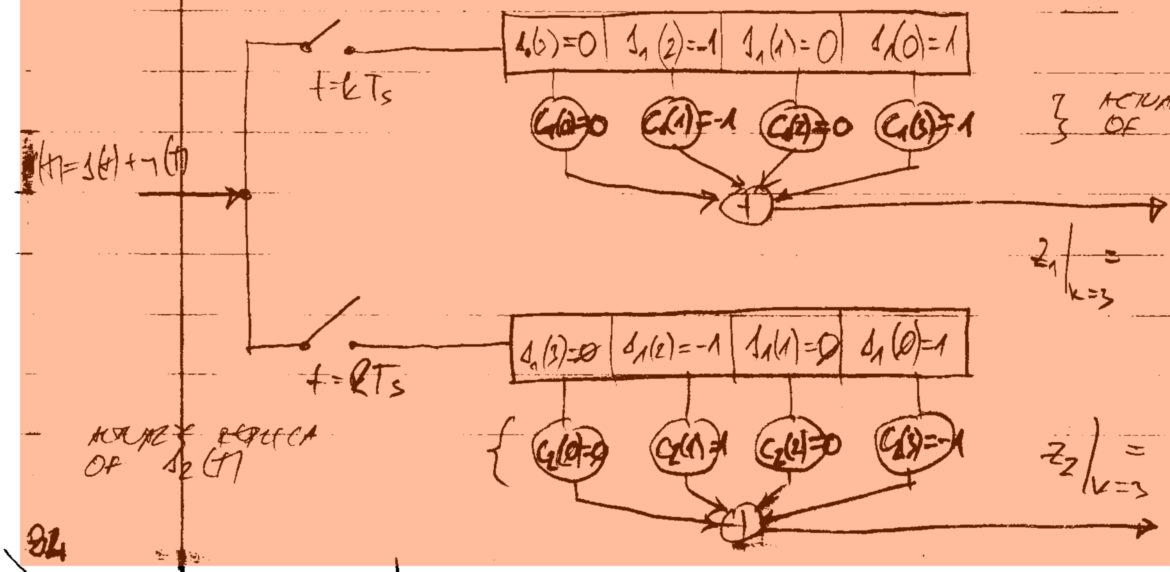
$T_{samp} \le T_{sym}$ (IN PRACTICE $T_{samp} \le \frac{T_{sym}}{4}$)



$$z(k) = \sum_{n=0}^{N-1} r(k-n)\,c(n) \qquad k = 0, 1, 2, \ldots$$

$s_1(t) = \cos(\omega_0 t)$ $s_2(t) = -\cos(\omega_0 t)$





ACTUALLY THIS IS REPLICA OF $s_1(t)$

$$z_1\Big|_{k=3} = \sum_{n=0}^{3} s_1(3-n)\,c_1(n) = 2$$

$$z_2\Big|_{k=3} = \sum_{n=0}^{3} s_1(3-n)\,c_2(n) = -2$$

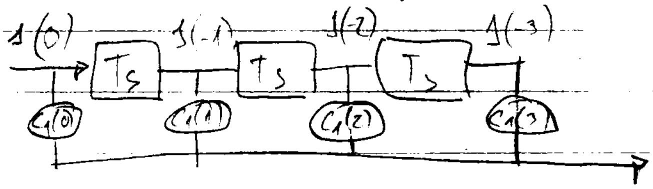
(070608208)

$$\begin{bmatrix} z(0)\\ z(1)\\ z(2)\\ z(3)\end{bmatrix} = \begin{bmatrix} s_1(0) & s_1(3) & s_1(2) & s_1(1)\\ s_1(1) & s_1(0) & s_1(3) & s_1(2)\\ s_1(2) & s_1(1) & s_1(0) & s_1(3)\\ s_1(3) & s_1(2) & s_1(1) & s_1(0)\end{bmatrix} \begin{bmatrix} c_1(0)\\ c_1(1)\\ c_1(2)\\ c_1(3)\end{bmatrix}$$

$$\boxed{z(k) = \sum_{n=0}^{N-1} s(k-n)\cdot c(n)} \quad \boxed{k=0,1,\dots \bmod(4)}$$

$$= \sum_{n=0}^{3} s(k-n)C(n) \qquad k=0,1,2,3$$

$$z(3) = \sum_{n=0}^{3} s(3-n)C(n)$$



$$z = s * c \qquad \boxed{C = S^{-1} * z}$$

$$\begin{array}{c|cccc} i & 0 & 1 & 2 & 3 \end{array}$$

$$s = [1, 0, -1, 0]$$

$$s\text{-flipLR} = [0, -1, 0, 1]$$

$$S = \begin{bmatrix} 1 & 0 & -1 & 0\\ 0 & 1 & 0 & -1\\ -1 & 0 & 1 & 0\\ 0 & -1 & 0 & 1\end{bmatrix}$$

- MINIMUM MSE SOLUTION

$$z = S * c \quad / S^T \qquad S^T\cdot z = S^T\cdot S\cdot c \qquad R_{xz} = R_{xx}\cdot c$$

$$c = R_{xx}^{-1}\cdot R_{xz}$$

$$k=0 \quad z(0) = \sum_{n=0}^{N-1} s(-n)c(n) = \overset{1}{s(0)}\,c(0) + \cancel{s(-1)}\,c(1) + \cancel{s(-2)}\,c(2) + s(-3)c(3)$$

$$S = \begin{bmatrix} 1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ -1 & 0 & 1 & 0\\ 0 & -1 & 0 & 1\end{bmatrix}$$

EQUALIZATION DEFINITION.

$$z(k) = \sum_{n=-N}^{N} x(k-n)c_n \qquad k = -2N, \dots, 2N \quad \Big\}\ 2n-1 \text{ NUMBER OF TAPS}$$

- ZNAČI VO SLUČAJ NA PRILAGODEN FILTER SE JAVI KONVOLUCIJA POMEĐU $s(t)$, $h(t)$

$$z_1(T) = \int_0^T s_1(t)\cdot h(t)\,dt = \int_0^T s_1(t)\cdot s_1(T-t)\,dt \Rightarrow \text{KONVOLUCIJA}$$

NO OVAA KONVOLUCIJA ĆE SVEDE NA KLASIČNA KORELACIJA ZOŠTO „C" KOEFICIENTITE ODGOVARAAT NA REALNATA OD SIGNALOT

$$z_1(T) = \int_0^T s_1(t)\cdot \psi_1(t)\,dt \Rightarrow \text{KORELACIJA}$$

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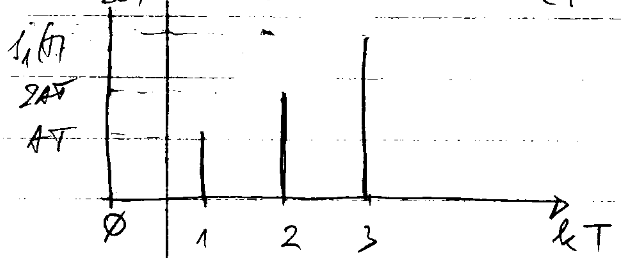
Jankulovski
Vasilevski
Dvodavm

Example 4.1 Sampled Matched Filter

$s_1(t) = At \qquad 0 \le t \le kT \qquad k = 0,1,2,3$

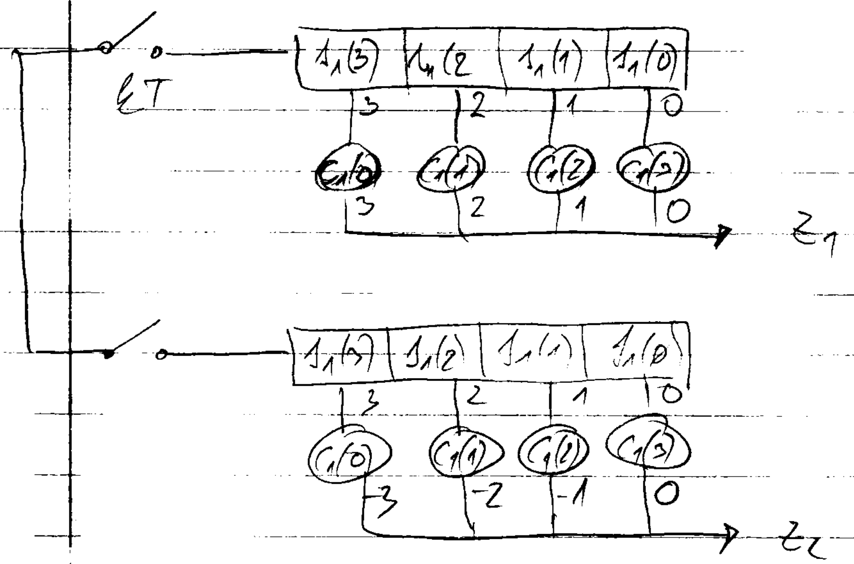
$s_2(t) = -A\cdot t \qquad 0 \le t \le kT \qquad k = 0,1,2,3$



$s_1(k) = [0,1,2,3]\cdot \frac{1}{AT}$

$s_1(-k) = [3,2,1,0]\cdot \frac{1}{AT}$

$$z_1(k) = \sum_{n=\emptyset}^{N-1} s_1(k-n)\cdot c_1(n) \qquad k = 0,1,2,3 \ldots \text{modulo } 4$$



$\boxed{3110066}$

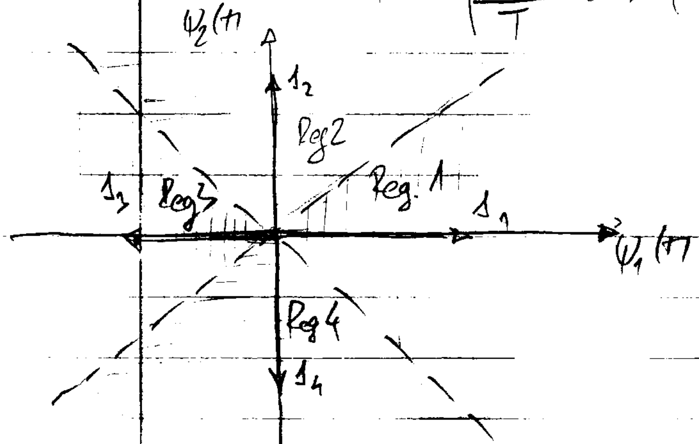
Vena Strujm.

DDI NUM

Boze Petrusevski

- Coherent detection of Multiple Phase-Shift Keying

MPSK $\quad s_i(t) = \sqrt{\frac{2E}{T}}\cos\left(\omega_0 t - \frac{2\pi i}{M}\right) \qquad 0 \le t \le T, \quad i = 1,\ldots,M$



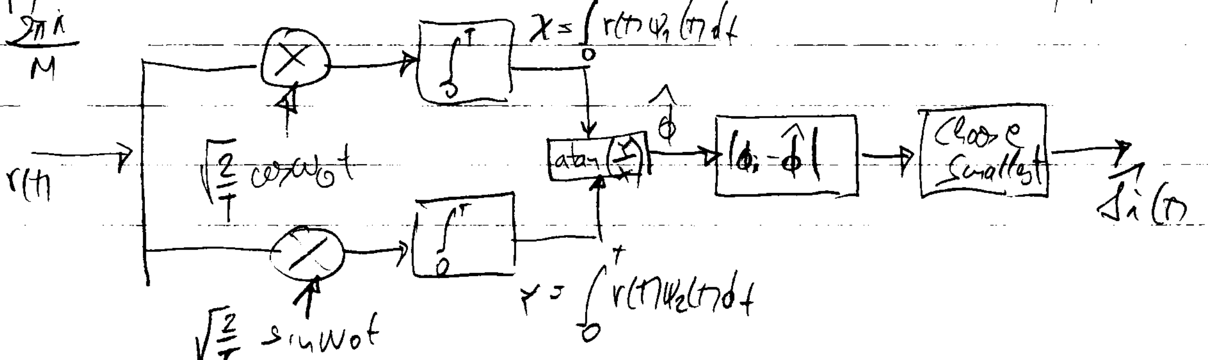
$\psi_1(t) = \sqrt{\frac{2}{T}}\cos(\omega_0 t)$

$\psi_2(t) = \sqrt{\frac{2}{T}}\sin(\omega_0 t)$

$s_i(t) = a_{i1}\psi_1(t) + a_{i2}\psi_2(t) =$
$= \sqrt{E}\cos\left(\frac{2\pi i}{M}\right)\cdot\psi_1(t) + \sqrt{E}\sin\left(\frac{2\pi i}{M}\right)\cdot\psi_2(t)$
$0 \le t \le T \qquad i = 1,2,\ldots M$

$r(t) = \sqrt{\frac{2E}{T}}\left(\cos\phi_i\cos\omega_0 t + \sin\phi_i\sin\omega_0 t\right) + n(t) \qquad 0 \le t \le T, \quad i = 1,2,\ldots M$

$\phi_i = \frac{2\pi i}{M}$



$$P_b = Q\left[\sqrt{\frac{E_d}{2N_0}}\right] \qquad E_d = \int_0^T [s_1(t) - s_2(t)]^2 dt = 4A^2 \cdot T$$

$\boxed{\text{PAM}}$ $\boxed{\text{BIPOLAR ANTIPODAL}}$

$$P_b = Q\left[\sqrt{\frac{4A^2T}{2N_0}}\right] = Q\left[\sqrt{\frac{2A^2T}{N_0}}\right] = Q\left[\sqrt{\frac{2E_b}{N_0}}\right]$$

$$E_b = \int_0^T s_1^2(t)dt = \int_0^T s_2^2(t)dt = A^2T$$

(EQUIVALENT TO BPSK)

$$P_{b\_BPSK} = \left| Q(z) = \frac{1}{2} erfc \frac{z}{\sqrt{2}} \right| = \frac{1}{2} erfc\left(\sqrt{\frac{E_b}{N_0}}\right)$$

$\boxed{2687804}$

$\boxed{\text{QPSK} = \text{PSK 4}}$ $\quad P_b = \frac{1}{2} erfc\left(\sqrt{\frac{E_b}{N_0}}\right)$

$$P_s = erfc\left(\sqrt{\frac{E_b}{2N_0}}\right) = erfc\left(0.707 \cdot \sqrt{\frac{E_b}{N_0}}\right)$$

K

$\boxed{2658300}$

$$F_s = \frac{1}{dt} \qquad F_c = \frac{1}{T_c}$$

$$\boxed{\begin{aligned} F_d &= R_s = 100MHz \\ F_c &= 1GHz \\ F_s &= 10GHz \end{aligned}}$$

$T_c = 10^{-9} = 1nsec \qquad F_c = 10^9 = 1GHz$

$dt = \frac{T_c}{10} = 10^{-10} = 0,1 nsec \quad F_s = 1/dt = 10^{10} = 10GHz$

$K = 10^4 \qquad N = 10^2 K = 10^6 = 1$ MILION PRIMEROV

$t = (0:N-1)dt = 10^6 \cdot 10^{-10} = 10^{-4} = \underline{0,1 msec} = \underline{10^5 \cdot T_c}$

$T_{SYM} = \frac{N\,dt}{K} = \frac{10^5 T_c}{K} = 10 \cdot T_c = \underline{10 nsec}$

$R_s = \frac{1}{T_{SYM}} = 100MHz \qquad df = \frac{1}{N\,dt} = \frac{10^{10}}{10^6} = 10^4$

$dt = \frac{T_c}{4} = 0,25 \cdot 10^{-9} = \underline{0,25 nsec} \qquad dF = 4GHz$

$T_s = 10T_c \qquad t = (0:N-1)dt = (0:10^6)dt = 0,25 \cdot 10^6 \cdot 10^{-9}$

$\Delta t = 0,25 \cdot 10^{-3} = \frac{1}{4} \cdot 10^{-9} \cdot 10^6 = 250 \cdot 10^3 \cdot T_c = 2,5 \cdot 10^5 T_c$

$N_1 = \frac{N}{25} = \frac{10^6}{25} = 4 \cdot 10^5 = 400.000$

$T_{SYM} = \frac{N \cdot dt}{K} = \underline{10 \cdot 10^{-9}} \quad 10 \cdot \cancel{10^{-9}} = \underline{0.25 \cdot \cancel{10^{-9}}} \cdot \frac{N}{K}$

$\frac{N}{K} = 40 \qquad N = 40 \cdot K \qquad K = 2,5 \cdot 10^4 \quad \underline{N = 10^6}$

$T_s = \frac{N\,dt}{K} = 40 \cdot 0.25 \cdot 10^{-9} = 10 \cdot 10^{-9}$

$N_{STOP} = 5000 \qquad df = 10.000 = 10\,kHz$

$N = 10^6 \Rightarrow \qquad \frac{N}{2} \cdot df = \frac{1}{2} 10^6 \, 10\,kHz = 5 \cdot 10^9 = 5\,GHz$

$\boxed{R_S = 100\,MHz} \qquad N_{STOP} = \frac{R_S}{2} \cdot \frac{1}{df} = \frac{50 \cdot 10^{\cancel{6}\,2}}{\cancel{10^4}} = 50 \cdot 10^2 = 5000$

$F_{STOP} = N_{STOP} \cdot df = 50\,MHz \qquad 3 \times N_{STOP} = 15000$

$3 \times F_{STOP} = 150\,MHz$

$R_B = Ld(M) \cdot R_S = 400\,MHz$

$u_m(t) = r_1(t) \cdot \cos(\omega_0 t) + r_2 \sin(\omega_0 t)$

$r_1 = A \cos\varphi \qquad tg\,\varphi = \frac{r_2}{r_1} \qquad \varphi = arctg \frac{r_2}{r_1}$

$r_2 = A \sin\varphi$

$u_m(t) = A \cdot \cos\varphi \cdot \cos(\omega_0 t) + A \cdot \sin\varphi \, \sin(\omega_0 t)$

$$\boxed{u_m(t) = A \cdot \cos(\omega_0 t - \varphi) = A \cdot \cos\left(\omega_0 t - arctg \frac{r_2}{r_1}\right)}$$

$r_1^2 + r_2^2 = A^2 (\cos^2\varphi + \sin^2\varphi) \qquad A = \sqrt{r_1^2 + r_2^2}$

$$s(t) = \sum_{i=1}^{N} a_i \cos(\omega_c t + \omega_{di} t + \phi_i) =$$

$$= \sum_{i=1}^{N} a_i \left[\cos(\omega_{di} t + \phi_i) \cos(\omega_c t) - \sin(\omega_{di} t + \phi_i) \sin(\omega_c t)\right]$$

$$= \left[\sum_{i=1}^{N} a_i \cos(\omega_{di} t + \phi_i)\right] \cos(\omega_c t) - \left[\sum_{i=1}^{N} a_i \sin(\omega_{di} t + \phi_i)\right] \cdot \sin(\omega_c t)$$

$s(t) = I(t) \cdot \cos(\omega_c t) - Q(t) \cdot \sin(\omega_c t)$

$I(t) = \sum_{i=1}^{N} a_i \cos(\omega_{di} t + \phi_i) \qquad I(t) = R(t) \cdot \cos[\varphi(t)]$

$Q(t) = \sum_{i=1}^{N} a_i \sin(\omega_{di} t + \phi_i) \qquad Q(t) = R(t) \cdot \sin[\varphi(t)]$

$$\boxed{\begin{array}{ll} s(t) = R(t) \cdot \cos(\omega_c t + \varphi(t)) & R^2(t) = I^2(t) + Q^2(t) \\ \varphi(t) = arctg \frac{Q(t)}{I(t)} & p(r) = \frac{r}{\sigma^2} \cdot e^{-\frac{r^2}{2\sigma^2}} \quad \text{RAYLEIGH} \end{array}}$$

WEIBUL DISTRIBUTION

$$p(x, \lambda, k) = \begin{cases} \frac{k}{\lambda} \left(\frac{x}{\lambda}\right)^{k-1} e^{-\left(\frac{x}{\lambda}\right)^k} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

$k > 0$ SHAPE PARAMETER

$\lambda > 0$ SCALE —"—

$R = \text{wblrnd}(A, B, m, n) = \text{wblrnd}(\lambda, k, m, n)$

$$p(x; A, B) = \begin{cases} \frac{B}{A}\left(\frac{x}{A}\right)^{B-1} e^{-(x/A)^B} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

VIDI GO BER ANALYSIS OF QAM ON FADING CHANNELS WITH TRANSMIT DIVERSITY

$$\bar{\tau} = \frac{\sum_k \tau_k P_k}{\sum_k P_k} \qquad \overline{\tau^2} = \frac{\sum_k \tau_k^2 P_k}{\sum_k P_k}$$

MEAN EXCESS DELAY

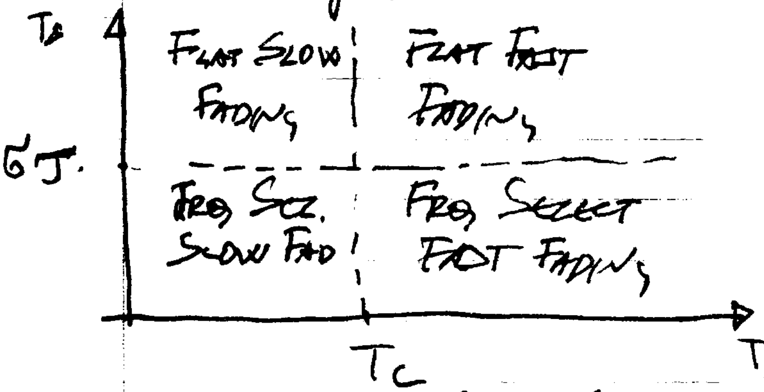
$$\sigma_\tau = \sqrt{\overline{\tau^2} - \bar{\tau}^2} \quad \text{RMS DELAY SPREAD}$$

$B_c \doteq \frac{1}{50\,\sigma_\tau}$ CORRELATION > 0.9 $\qquad B_c \doteq \frac{1}{5\,\sigma_\tau}$ CORRELATION > 0.5

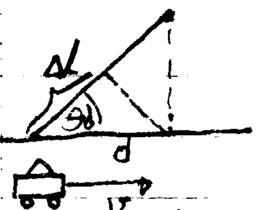
COHERENCE BANDWIDTH

(*) $T_C = \frac{1}{f_m}$ $\quad f_m$ (MAXIMUM DOPLER SHIFT) $f_m = \frac{v}{\lambda}$

(**) $T_C = \frac{9}{16\pi f_m}$ $\qquad$ (A) $T_C = \sqrt{\frac{9}{16\pi f_m^2}}$



- convolve signal to be transmitted with $\frac{1}{\sqrt{2}}(\text{randn}(N,1) + j * \text{randn}(N,2))$ I READ IT ON INTERNET !!!



$$\phi d = \frac{2\pi \Delta l}{\lambda} = \frac{2\pi\, d \cdot \cos\theta d}{\lambda} = \frac{2\pi v \cdot \Delta t \cdot \cos\theta d}{\lambda}$$

$$\omega d = \frac{\phi d}{\Delta t} = \frac{2\pi v \cdot \cos\theta d}{\lambda}$$

$$fd = \frac{\omega d}{2\pi} = \frac{v}{\lambda} \cdot \cos(\theta d) = f_m \cos(\theta d) \qquad f_m = \frac{v}{\lambda} = \frac{v\, f_c}{c}$$

$\lambda \doteq \frac{c}{f_c}$ $\qquad f_c = 900\,\text{MHz} \Rightarrow \lambda \doteq \left(\frac{10^9 \frac{1}{s}}{3\,10^8 \frac{m}{s}}\right)^{-1} = \frac{3}{10}\, m = 0.$

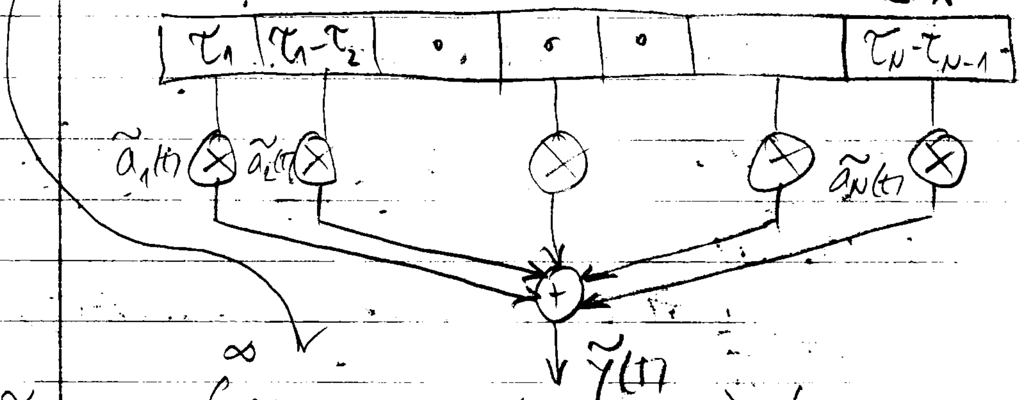
⊙ MATLAB, COMMUNICATION TOOLBOX, FADING CHANNELS:

• JERUCHIM'S DISCRETE MULTIPATH CHANNEL MODEL (SECTION 9.1.3.5.2)

$$c(\tau,t) = \sum_{k=1}^{K(t)} \tilde{a}_k(\tau_k(t),t)\,\delta(\tau-\tau_k(t)) \quad \Big\} \text{ IMPULSE RESPONSE}$$

$$\tilde{y}(t) = \sum_{k=1}^{K(t)} \tilde{a}_k(\tau_k,t)\,\tilde{s}(t-\tau_k(t)) \quad \Big\} \text{ OUTPUT SIGNAL}$$

$$c(\tau,t) = \sum_{k=1}^{K} \tilde{a}_k(t)\,\delta(\tau-\tau_k) \qquad \tilde{y}(t) = \sum_{k=1}^{K} a_k(t)\,s(t-\tau_k)$$

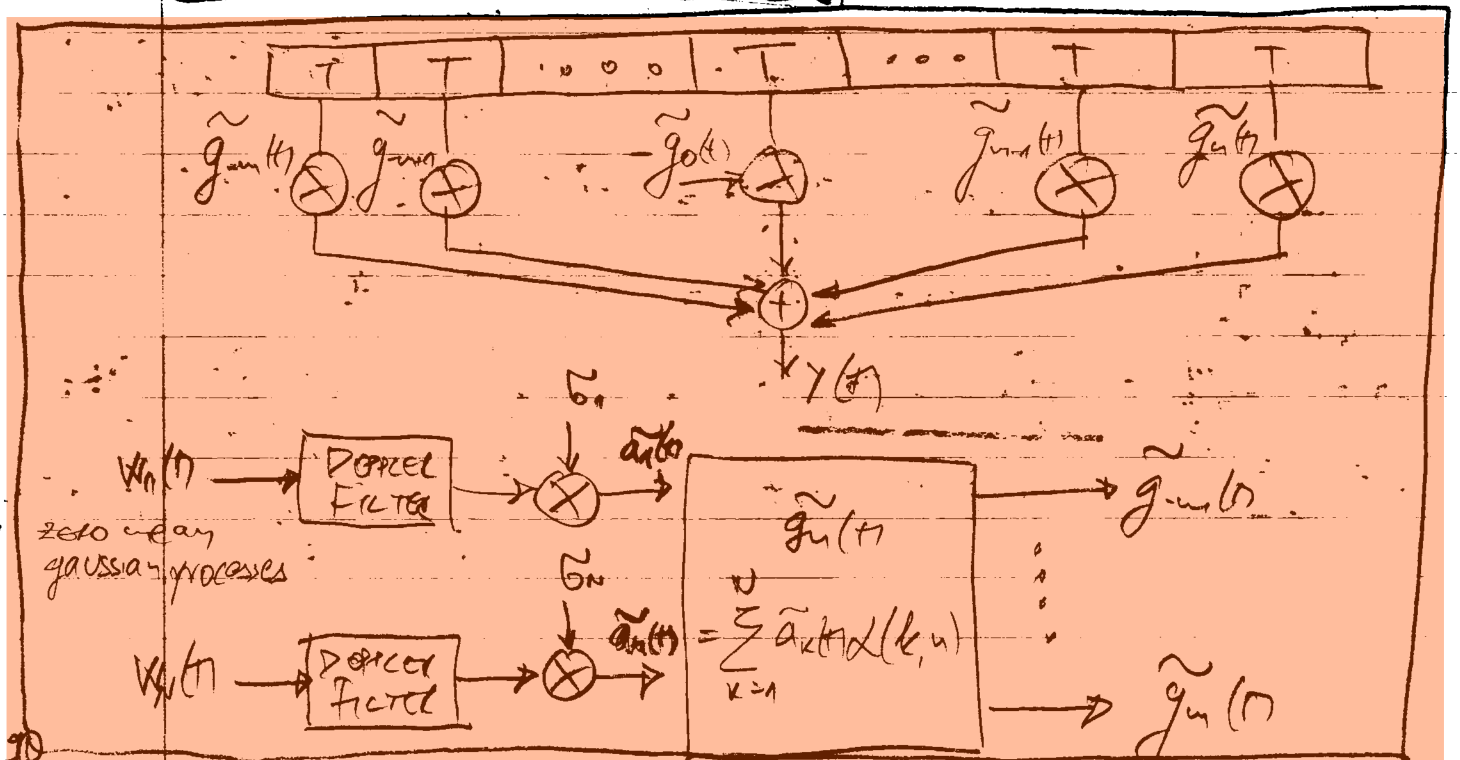


$$\tilde{g}_n(t) = \int_{-\infty}^{\infty} \tilde{c}(\tau,t)\,\mathrm{sinc}(B(\tau-nT))\,d\tau =$$

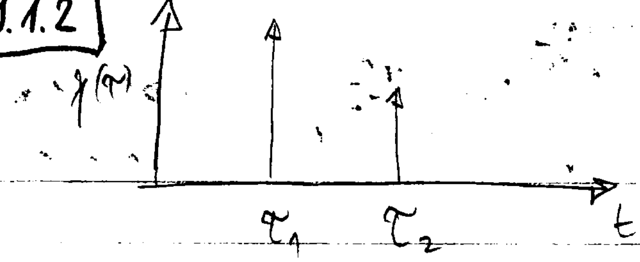
$$= \int_{-\infty}^{\infty}\left(\sum_{k=1}^{K} \tilde{a}_k(t)\,\delta(\tau-\tau_k)\right)\mathrm{sinc}(B(\tau-nT))\,d\tau = \sum_{k=1}^{K} a_k(t)\,\alpha(k,n) \qquad -N\le n\le N$$

$$T=\frac{1}{B} \qquad B=\frac{1}{T} \Rightarrow \boxed{\alpha(k,n)=\mathrm{sinc}\left(\frac{\tau_k}{T}-n\right)}$$

$$\boxed{\tilde{g}_n(t) = \sum_{k=1}^{K} a_k(t)\,\mathrm{sinc}\left(\frac{\tau_k}{T}-n\right)} \qquad -N\le n\le N$$



EXAMPLE 9.1.2



$$\Delta\tau = \frac{\tau_2 - \tau_1}{T}$$

$$T = \frac{1}{B}$$

$\left(\sigma_1 / \sigma_2\right)^2$ → RATIO OF RELATIVE POWERS IN TWO PATHS

$\Delta\tau \ll 1 \Rightarrow$ FREQUENCY - NONSELECTIVE

$\Delta\tau > 0.1 \Rightarrow$ FREQUENCY - SELECTIVE (ISI)

e.g.: $\Delta\tau = 0.75$

$$\begin{bmatrix} g_{-3}(t) \\ g_{-2}(t) \\ g_{-1}(t) \\ g_0(t) \\ g_1(t) \\ g_2(t) \\ g_3(t) \end{bmatrix} = \begin{bmatrix} \mathrm{sinc}(0+3) & \mathrm{sinc}(0.75+3) \\ \mathrm{sinc}(0+2) & \mathrm{sinc}(0.75+2) \\ \mathrm{sinc}(0+1) & \mathrm{sinc}(0.75+1) \\ \mathrm{sinc}(0) & \mathrm{sinc}(0.75+0) \\ \mathrm{sinc}(0-1) & \mathrm{sinc}(0.75-1) \\ \mathrm{sinc}(0-2) & \mathrm{sinc}(0.75-2) \\ \mathrm{sinc}(0-3) & \mathrm{sinc}(0.75-3) \end{bmatrix} \begin{bmatrix} \tilde{a}_1(t) \\ \tilde{a}_2(t) \end{bmatrix} = \begin{bmatrix} 0.0 & -0.060 \\ 0.0 & 0.082 \\ 0.0 & -0.129 \\ 1.0 & 0.300 \\ 0.0 & 0.900 \\ 0.0 & 0.180 \\ 0.0 & 0.100 \end{bmatrix} \begin{bmatrix} \tilde{a}_1(t) \\ \tilde{a}_2(t) \end{bmatrix}$$

MATLAB: s_i - SET OF SAMPLES AT INPUT OF CHANNEL

$$y_i = \sum_{n=-N_1}^{N_2} s_{i-n}\, g_n \qquad g_n = \sum_{k=1}^{K} a_k\, \mathrm{sinc}\left[\frac{\tau_k}{T_s} - n\right]$$

$$-N_1 \le n \le N_2$$

$$a_k = \sqrt{\Omega_k}\, z_k \qquad \Omega_k = E\left[|a_k|^2\right] \qquad \text{RAYLEIGH}$$

$$a_k = \sqrt{\Omega_k}\left[\frac{z_k\, e^{j2\pi f_{d,LOS,k} t + Q_{LOS,k}}}{\sqrt{K_{r,k}+1}} + \sqrt{\frac{K_{r,k}}{K_{r,k}+1}}\right] \qquad \text{RICIAN}$$

- MATLAB IMPLEMENTATION AS LINEAR FIR FILTER
  1. CREATE CHANNEL OBJECT
  2. ADJUST PROPERTIES OF THE OBJECT
  3. APPLY CHANNEL OBJECT TO YOUR SIGNAL BY USING FILTER FUNCTION

```
c1 = rayleighchan(1/100000, 130); % RAYLEIGH CHANNEL OBJECT
d = doppler.gaussian(0.1);
```

$$0.02 * \pi \cdot n = 0.02\pi \cdot (1{:}N)\frac{Dt}{Dt} = \frac{0.02}{Dt}\pi \cdot t = \frac{0.02}{0.01}\pi \cdot t = 2\pi \cdot t$$

$$\cos(0.02 * \pi \cdot n) = \left| f_c = 1\,\mathrm{Hz} \right| = \cos(2\pi f_c \cdot t)$$

• RICIAN „K" FACTOR

$$p(r) = \begin{cases} \frac{r}{\sigma^2} e^{-\frac{r^2+A^2}{2\sigma^2}} I_0\left(\frac{Ar}{\sigma^2}\right) & A \geq 0,\ r \geq 0 \\ \emptyset & r < \emptyset \end{cases}$$

$$K(dB) = 10 \log \frac{A^2}{2\sigma^2} \ dB$$

• $\tau_0 = 0$; $\tau_1 = 10^{-4}$; $\tau_2 = 2.1 \cdot 10^{-4}$ $a_0 = a_1 = a_2 = 1$

$F_s = 9600$ $T = \frac{1}{9600} = 1.0417 \cdot 10^{-4}$

$$s(t) = \sum_{i=0}^{N-1} \cos(\omega_s t + \omega_{d_i} \cdot t + \phi_i)$$

$$g_n = \sum_{k=1}^{3} a_k \operatorname{sinc}\left(\frac{\tau_k}{T} - n\right)$$

$\tau_1 = \emptyset$ $\tau_2 = 1.1 \cdot 10^{-4}$

$\tau_3 = 2.1 \cdot 10^{-4}$

$n = 1$

$$g_1 = \sum_{k=1}^{3} a_k \operatorname{sinc}\left(\frac{\tau_k}{T} - 1\right)$$

$$g_2 = \sum_{k=1}^{3} a_k \operatorname{sinc}\left(\frac{\tau_k}{T} - 2\right) \ ---$$

$$\begin{bmatrix} g_{-2} \\ g_{-1} \\ g_0 \\ g_1 \\ g_2 \end{bmatrix} = \begin{bmatrix} \operatorname{sinc}(0+2) & \operatorname{sinc}(10^{-4}+2) & \operatorname{sinc}(2.1 \cdot 10^{-4}+2) \\ \operatorname{sinc}(0+1) & \operatorname{sinc}(10^{-4}+1) & \operatorname{sinc}(2.1 \cdot 10^{-4}+1) \\ \operatorname{sinc}(0) & \operatorname{sinc}(10^{-4}) & \operatorname{sinc}(2.1 \cdot 10^{-4}) \\ \operatorname{sinc}(0-1) & \operatorname{sinc}(10^{-4}-1) & \operatorname{sinc}(2.1 \cdot 10^{-4}-1) \\ \operatorname{sinc}(0-2) & \operatorname{sinc}(10^{-4}-2) & \operatorname{sinc}(2.1 \cdot 10^{-4}-2) \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

$$\begin{bmatrix} g_{-2} \\ g_{-1} \\ g_0 \\ g_1 \\ g_2 \end{bmatrix} = \begin{bmatrix} 0{,}0000 & 0{,}0175 & 0{,}0040 \\ 0{,}0000 & -0{,}0204 & -0{,}0053 \\ 1{,}0000 & 0{,}0416 & 0{,}0079 \\ 0{,}0000 & -0{,}9974 & -0{,}0157 \\ -0{,}000 & -0{,}0384 & 0{,}9996 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

• AVERAGE PATH GAINS $-20dB \div 0dB$

$a_i \cdot e^{j\theta_i(t,\tau)}$ $\Delta\Phi_i = \frac{2\pi \cdot \Delta l}{\lambda} = \frac{2\pi \cdot d \cdot \cos\varphi}{\lambda}$

$\Delta\omega = \frac{\Delta\Phi_i}{\Delta t} = \frac{2\pi}{\lambda} \frac{d}{\Delta t} \cdot \cos\varphi = \frac{2\pi v}{\lambda} \cdot \cos\varphi$

$f_d = \frac{v}{\lambda} \cdot \cos\varphi$ $f_{d\,max} = \frac{v f_c}{c}$

$f_{d max} = 100Hz$ $\quad \theta_d = \Delta\omega_d \cdot \Delta t = 2\pi f_{dmax} \cdot \Delta t$

~~$f_s = 9600 \quad T_s = \frac{1}{9600} = 0{,}105 \cdot 10^{-3} = 0{,}105\,ms = 105\,\mu s$~~

~~$\theta_d = 2\pi \cdot 100 \cdot 0{,}105 \cdot 10^{-3} = 2\pi \cdot 0{,}105 \cdot 10^{-1} = 2\pi \cdot 0{,}0105 = 0{,}0660$ rad~~

~~2π rad $= 360°$ $\quad 1$ rad $= \frac{180°}{\pi}$ $\quad \theta_d = 0{,}0660 \cdot \frac{180°}{\pi} = 3{,}7815°$~~

$\cos(0{,}0660) + j\sin(0{,}0660)$

~~$f_{max} = \frac{f_s}{2}$ $\quad f_{max} = 4800$ $\quad \Delta t =$~~

$v = \frac{d}{\Delta t}$ $\quad v = 100\,km/h \Rightarrow d = 10 \quad \Delta t = 0{,}1$

$f_d = \frac{v}{\lambda}$ $\quad \theta_d = 2\pi\, 100 \cdot 0{,}1 = 2\pi \cdot 10$

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~~[illegible]~~
СТОЈАНОВСКИ
БОЖИДАР

$v = 1\,m/s \quad f_c = 2{,}4GHz \quad R_b = 10Mb/s$

$\lambda = \left(\frac{2400}{300}\right)^{-1} = \frac{3}{24} = \frac{1}{8} = 0{,}125\,m \quad f_d = \frac{v}{\lambda} = \frac{v}{c} \cdot f_c$

$f_d = \frac{1\,m/s}{3 \cdot 10^8\, m/s} \cdot 2.4 \cdot 10^9 = \frac{2.4}{0.3} = \frac{2.4}{3} \cdot 10 = \frac{24}{3} = 8\,Hz$

$T_d = \frac{1}{f_d} = \frac{1}{8} = 0{,}125 = 12{,}5 \cdot 10^{-3} = 125\,ms$

$T_b = \frac{1}{R_b} = \frac{1}{10} \cdot 10^{-6} = 0{,}1 \cdot 10^{-6}$

$\frac{T_d}{T_b} = \frac{12{,}5 \cdot 10^{-3}}{0{,}1 \cdot 10^{-6}} = 125\, 10^3 = 1.250 \cdot 10^6$

$\frac{T_d/100}{T_b} = \frac{0{,}125 \cdot 10^{-3}}{0{,}1\, 10^{-6}} = 1{,}25 \cdot 10 \cdot 10^{+3} = 12{,}5 \cdot 10^3 b$

$\boxed{T_b \ll 0.01\, T_d}$ INSIGNIFICANT CHANGE OF DOPPLER CHANNEL.

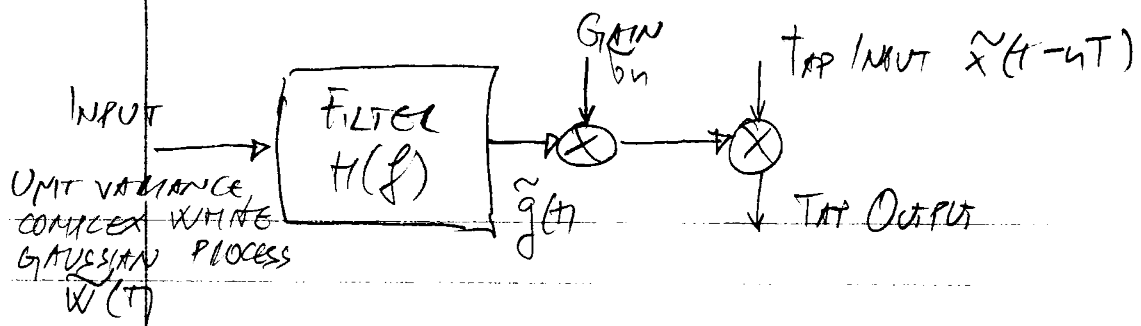
awgn IMPLEMENTATION IN MATLAB:

COMPLEX: Y = SQRT (imp * noisePower/2) (z(1:row) + j * z(row+1:end))

REAL: Y = sqrt (imp * noisePower) (randn(row))

- PAM + AWGN

$Y(t) = A(t) \cdot \cos(w_c t) + X(t) \cos w_c t - Y(t) \sin(w_c t)$



$\boxed{\text{THERMAL PASTE}}$

⊙ $H(f) = \sqrt{S(f)}$ THREE SPECTRA ARE SPECIFIED: FLAT, GAUSSIAN, AND JAKES SPECTRUM

a.) FLAT: $S_f(f) = A \quad |f| \le B \qquad H_f(f) = \sqrt{A} \quad |f| \le B$

b.) GAUSSIAN $S_G(f) = Ae^{-kf^2} \qquad H_G(f) = \sqrt{A}\, e^{-\frac{k}{2}f^2}$

$$h_g(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H_G(f)\, e^{j\omega t}\, d\omega = \frac{1}{\cancel{2\pi}}\int_{-\infty}^{\infty}\sqrt{A}\, e^{-\frac{k}{2}f^2} e^{j2\pi f t}\, df$$

$$h_g(t) = \sqrt{A}\int_{-\infty}^{\infty} e^{-\frac{k}{2}f^2} e^{j2\pi f t}\, df = \sqrt{\frac{2\pi A}{k}}\cdot e^{-\frac{2\pi^2 t^2}{k}}$$

c) JAKES SPECTRUM

$$S_J(f) = \frac{A}{\left[1 - (f/f_d)^2\right]^{1/2}}$$

f_d - MAX. DOPPLER FREQUENCY,

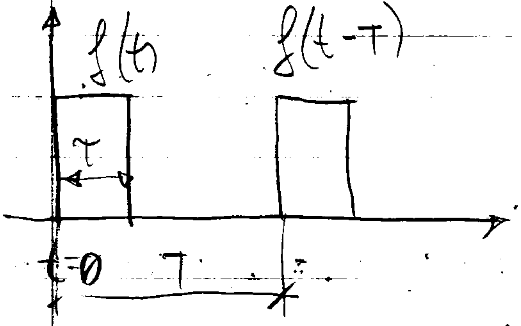
$$H_J(f) = \sqrt{S_J(f)} = \frac{A^{1/2}}{\left[1 - (f/f_d)^2\right]^{1/4}}$$

$$h_J(t) = \mathcal{F}^{-1}\{H_J(f)\} = \sqrt{A}\; 2^{1/4}\sqrt{\pi}\;\Gamma(3/4)\, f_d\, x^{-1/4} J_{1/4}(x)$$

$$\boxed{h_J(t) = \sqrt{A}\; 2.583\, f_d\, x^{-1/4} J_{1/4}(t)} \qquad x = 2\pi f_d |t|$$

⊙ SIMULATION OF COMM. SYSTEMS (CHAPTER 9)

$\tilde{h}(\tau, t) = \delta(t) + \tilde{c}(\tau, t)$



$h(t,\tau) = \Gamma[\delta(t-\tau)]$

LINEAR TIME-VARYING SYSTEM OPERATOR;

$$c(\tau, t) = \Gamma[\delta(t-(t-\tau))] = \Gamma[\delta(\tau)]$$

$$c(\tau, t) = h(t, t-\tau) = \Gamma[\delta(t-\tau)], \qquad h(t,\tau) = c(t-\tau, t)$$

$$c(t-\tau, t) = \Gamma[\delta(t-(\cancel{t}-\cancel{t}+\tau))] = \Gamma[\delta(t-\tau)]$$

$h(\tau,t) = s(t) * \tilde{c}(\tau,t)$

$S_R = S_T + G_T + G_R - L_p$ $\quad \boxed{L_p = \alpha + \beta \log_{10}(R)\ [dB]}$

$\alpha = -20\log\left(\frac{\pi\lambda}{4}\right) \qquad \beta = 20$

HATA-OKUMURA $\quad \beta = 44.9 + 6.55\log(h)$ ← URBAN ENVIRON.

$\boxed{\text{PG. 548}}$ $\quad \alpha = 69.55 + 26.16\log(f) - 13.82\log(h)$

$$\boxed{PL(d) = PL(d_0) + 10 \cdot \underbrace{n}_{\text{PATH LOSS EXPONENT}} \cdot \log\frac{d}{d_0}}$$

- COMPLEX LOWPASS-EQUIVALENT RESPONSE $\tilde{c}(\tau,t)$

$$y(t) = \sum_n a_n(t)\, s(t - \tau_n(t))$$

$s(t)$ – BANDPASS INPUT SIGNAL;
$a_n(t)$ – ATENUATION FACTOR FOR THE SIGNAL RECEIVED ON n-th PATH
$\tau_n(t)$ – CORRESPONDING PROPAGATION DELAY

$\boxed{s(t) = \mathrm{Re}\{\tilde{s}(t)\, e^{j2\pi f_c t}\}}$ $\quad \boxed{\text{MMV}}$

$$y(t) = \mathrm{Re}\left\{\left[\sum_n a_n(t) \cdot \tilde{s}(t-\tau_n(t)) \cdot e^{j2\pi f_c(t+\tau_n(t))}\right]\right\} =$$

$$= \mathrm{Re}\left\{\left[\sum_n a_n(t)\, e^{j2\pi f_c \tau_n(t)} \cdot \tilde{s}(t-\tau_n(t))\right] e^{j2\pi f_c t}\right\}$$

- COMPLEX ENVELOPE OF THE OUTPUT IS:

$$\tilde{y}(t) = \sum_n a_n(t)\, e^{j2\pi f_c \tau_n(t)} \cdot \tilde{s}(t-\tau_n(t)) = \sum_n \tilde{a}_n(\tau_n, t)\, \tilde{s}(t-\tau_n(t))$$

– LOWPASS-EQUIVALENT IMPULSE RESPONSE:

$$\boxed{\tilde{c}(\tau_n(t), t) = \sum_n \tilde{a}_n(\tau_n(t), t)\, \delta(\tau - \tau_n(t))}$$

– FOR DIFFUSE MULTIPATH CHANNEL

$$\tilde{y}(t) = \int_{-\infty}^{\infty} \tilde{a}(\tau,t)\, \tilde{s}(t-\tau)\, d\tau$$

$\tilde{a}(\tau,t)$ – COMPLEX ATENUATION OF SIGNAL COMPONENT AT DELAY „τ" AND TIME INSTANT „t"

$$\boxed{\tilde{c}(\tau,t) = \tilde{a}(\tau,t)\, e^{-j2\pi f_c \tau}}$$

$\tilde{c}(\tau,t)$ – MODELED AS GAUSSIAN PROCESS:

$$\tilde{c}(\tau,t) = \mathrm{Re}\{\tilde{c}(\tau,t)\} + j * \mathrm{Im}\{\tilde{c}(\tau,t)\}$$

• $R(\tau,t) = |\tilde{c}(\tau,t)|$ – ENVELOPE IS RAYLEIGH DISTRIB.

$f_R(r) = \frac{r}{\sigma^2} e^{-r^2/2\sigma^2}$ FOR $\text{Mean}\{\tilde{c}(\tau,t)\} = 0$

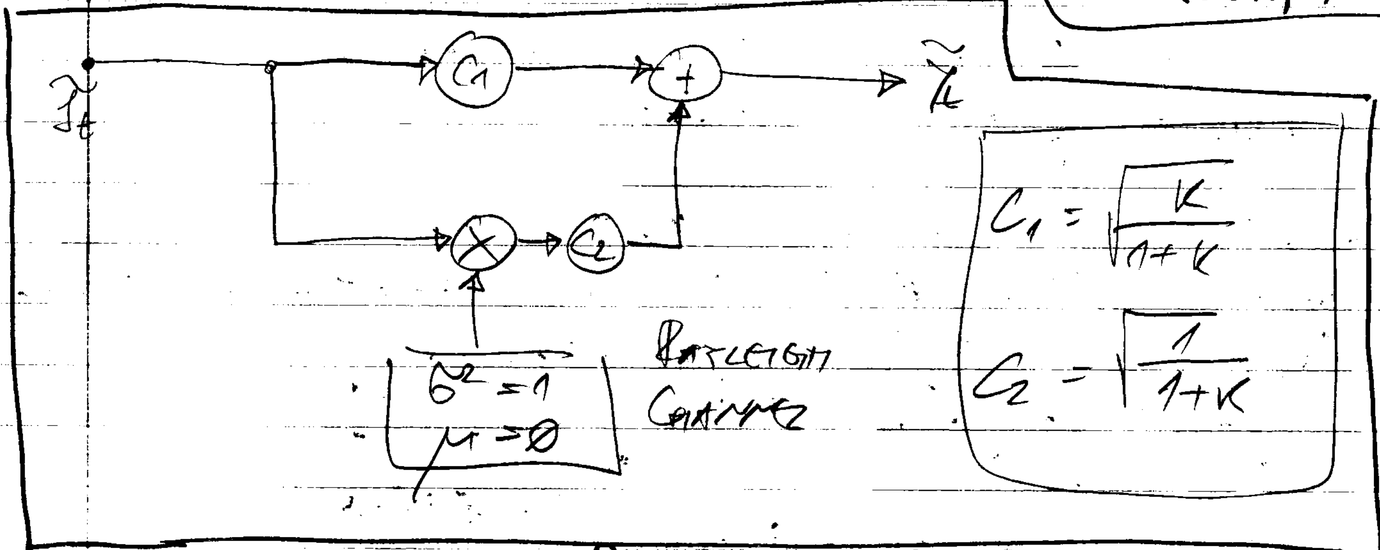
• LINE OF SITE $R(\tau,t) = |\tilde{c}(\tau,t)| \Rightarrow$ RICEAN DISTRIB.

$$f_R(r) = \frac{r}{\sigma^2} I_0\left[\frac{Ar}{\sigma^2}\right] e^{-(r^2+A^2)/2\sigma^2}$$

$A = \text{mean}\{\tilde{c}(\tau,t)\}$ $K = \frac{A^2}{\sigma^2}$

K>>1 SPECULAR

K<<1 ⇒ RAYLEIGH CH



SIMULATION MODEL OF RICEAN CHANNEL

◉ STATISTICAL CHARACTERIZATION: WSSUS MODEL

WSS – WIDE-SENSE STATIONARY RANDOM PROCESS

$$\boxed{R_{\tilde{c}}(\tau_1,\tau_2,\Delta t) = E[\tilde{c}^*(\tau_1,t)\,\tilde{c}(\tau_2,t+\Delta t)]}$$

$$\overline{\xi_1\xi_2} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x_1 x_2\, p_{\xi_1\xi_2}(x_1,x_2,\tau)\, dx_1 x_2 = R_{\xi\xi}(\tau) = R_\xi(\tau)$$

– ATENUATION AND DELAY ARE NOT CORELATED : US UNCORELATED SCATERING

$$R_{\tilde{c}}(\tau_1,\tau_2,\Delta t) = R_{\tilde{c}}(\tau_1,\Delta t)\,\delta(\tau_1-\tau_2)$$

WSS – WIDE-SENSE STATIONARY (STACIONARNOST VO POSIROK SMISOL) (VIDI SKRIPTA TEORIJA NA INF PR.29)

$\bar{\xi}$ – NE ZAVISI OD VREMETO T.E $\bar{\xi}$ = const

$R_{\xi\xi}(\tau) = R_\xi(\tau) = \overline{\xi_1\xi_2} \neq f(t)$

WSSUS – WSS UNCORELATED SCATERING – MODEL

$$\boxed{R_{\tilde{c}}(\tau_1,\tau_2,\Delta t) = R_{\tilde{c}}(\tau_1,\Delta t)\,\delta(\tau_1-\tau_2)}$$

$$R_{\tilde{c}}(\tau, \Delta t) = E\left[\tilde{c}^*(\tau, t)\,\tilde{c}(\tau, t+\Delta t)\right]$$ WSSUS

$$S(\tau, \nu) = \mathcal{F}_{\Delta t}\left[R_{\tilde{c}}(\tau, \Delta t)\right] = \int_{-\infty}^{\infty} R_{\tilde{c}}(\tau, \Delta t)\, e^{-j2\pi\nu\Delta t}\, d\Delta t$$

$S(\tau, \nu)$ - SCATTERING FUNCTION (RAPIDITY WITH which channel changes)

τ - DELAY VARIABLE ν - DOPPLER FRQ. VARIABLE

- POWER DELAY PROFILE i.e MULTIPATH INTENSITY PROFILE

$$p(\tau) = R_{\tilde{c}}(\tau, 0) = E|\tilde{c}(\tau, t)|^2$$

REPRESENTS AVERAGE RECEIVED POWER AS FUNCTION OF DELAY „τ"

$$p(\tau) = \int_{-\infty}^{\infty} S(\tau, \nu)\, d\nu$$

$$S(\nu) = \int_{-\infty}^{\infty} S(\tau, \nu)\, d\tau \Rightarrow \text{DOPPLER POWER SPECTRUM}$$

$$\hat{R}_f(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{\overline{|F_T(j\omega)|^2}}{2T}\, e^{j\omega\tau}\, d\omega$$

$$R_g(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \Phi_g(\omega)\, e^{j\omega\tau}\, d\omega \qquad \Phi_g(\omega) = \int_{-\infty}^{\infty} R_g(\tau)\cdot e^{-j\omega\tau}\, d\tau$$

$$R_{11}(\tau) = \int_{-\infty}^{\infty} f_1(t)\cdot f_1(t+\tau)\, dt = \int_{-\infty}^{\infty} f_1(t)\left[\frac{1}{2\pi}\int_{-\infty}^{\infty} F_1(j\omega)\, e^{j\omega t}\cdot e^{j\omega\tau}\, d\omega\right] dt$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} F_1(j\omega)\, e^{j\omega\tau} \underbrace{\left[\int_{-\infty}^{\infty} f_1(t)\, e^{j\omega t}\, dt\right]}_{F_1^*(j\omega)} d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} |F_1(j\omega)|^2 e^{j\omega\tau} d\omega$$

$$\boxed{R_{11}(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} |F_1(j\omega)|^2 e^{j\omega\tau}\, d\omega}$$

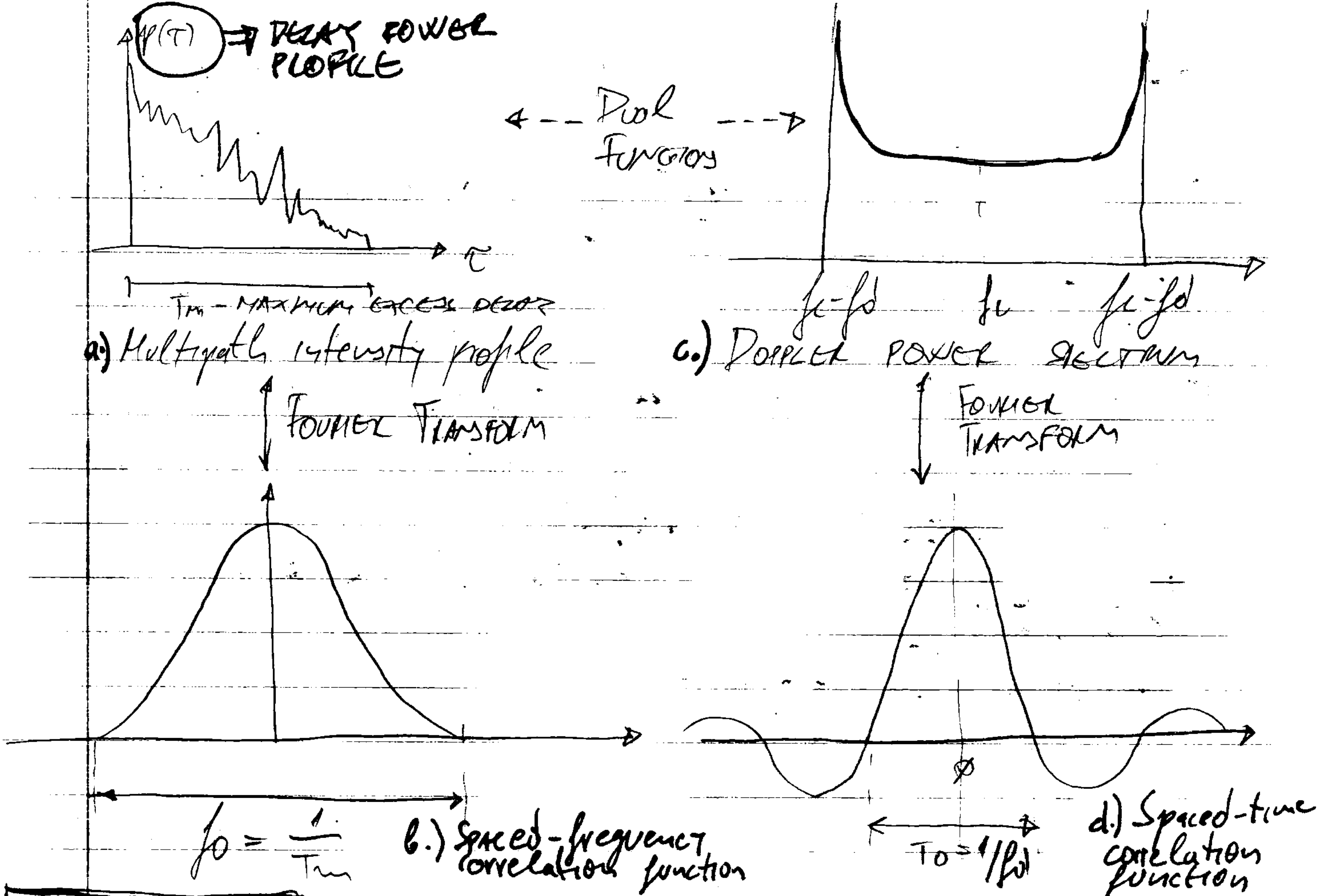
⊙ The DELAY POWER PROFILE

ⓐ $T_m > T_{SYM}$ → FRQ SELECTIVE FADING ≙ ISI

- RESOLVABLE MULTIPATH → COULD BE MITIGATED WITH RAKE RECEPTION

ⓑ $T_m \ll T_{SYM}$ → FLAT FADING

COUNTERMEASURE IS POWER CONTROL OR DIVERSITY



$\sigma_\tau < 0.1 \cdot T_{SYM}$ — IF TRUE ⇒ FLAT

MOST FREQUENTLY USED CRITERION FOR FRQ. SELECTIVITY I.E. FRQ. NON-SELECTIVITY
σ_τ – RMS DELAY SPREAD

$\tilde{y}(t) = \tilde{c}(t) \cdot \tilde{s}(t)$ FLAT CHANNEL

$\tilde{y}(t) = \tilde{c}(\tau, t) * \tilde{s}(t)$ – FRQ. SELECTIVE FADING

$$\sigma_\tau = \sqrt{\overline{\tau^2} - \bar{\tau}^2} \qquad \bar{\tau}^k = \frac{\int \tau^k p(\tau)\, d\tau}{\int p(\tau)\, d\tau}$$

① The SPACED-FREQUENCY CORRELATION FUNCTION

$P(\Delta f) = F(p(\tau))$

$f_0 \doteq \frac{1}{T_m}$ ⇒ COHERENCE BANDWIDTH

Frequency range where all frequency components amplitudes are correlated. That is, the spectral components in that range fade together.

$$f_0 = \frac{0.276}{\sigma_\tau} \doteq \frac{1}{5\sigma_\tau}$$

- $f_0 < B$ ⇒ FSF; B SIGNAL BANDWIDTH
  CHANNEL ACTS AS A FILTER ⇒ FSF OCCURS
- $f_0 >> B$ ⇒ FRQ NON-SELECTIVITY I.E. FLAT FADING

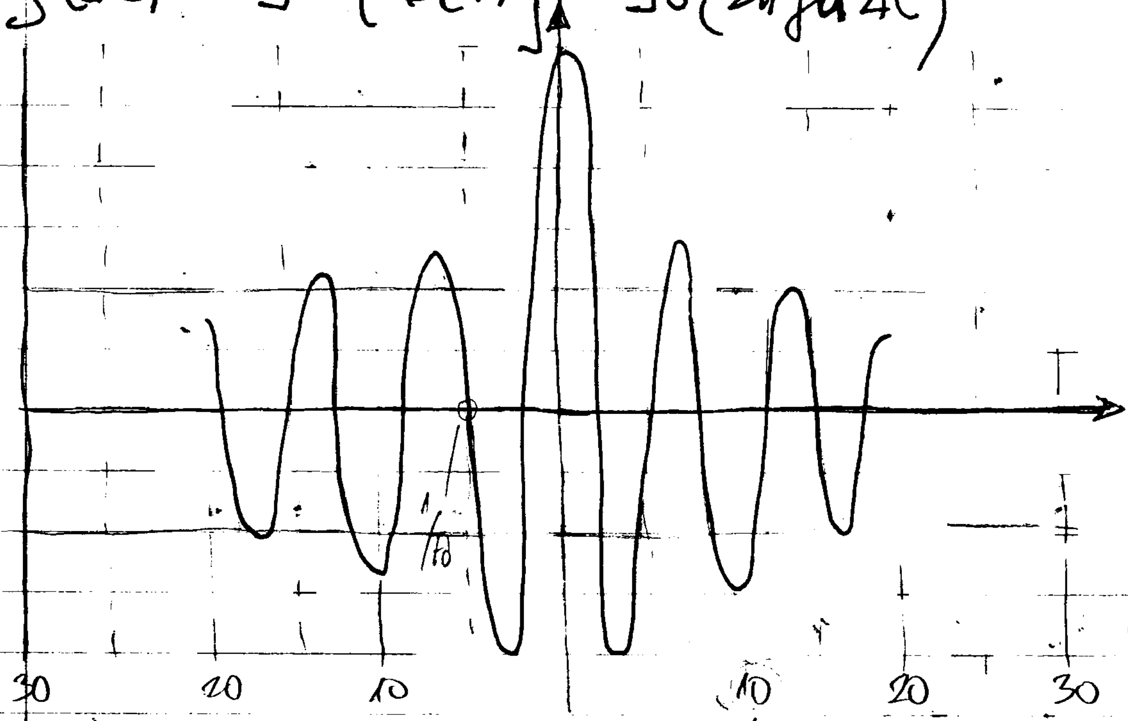
• The time-varying channel

$$S(\nu) = \frac{1}{\pi f_d \sqrt{1-\left(\frac{\nu}{f_d}\right)^2}} \qquad \nu \le f_d$$

$\boxed{\text{CF0013}}$

• Space-time correlation function $\rho(\Delta t)$

$$\rho(\Delta t) = \mathcal{F}^{-1}\{S(\nu)\} = J_0(2\pi f_d \Delta t)$$



Fd = 100 Hz

$30 = 2\pi Fd \cdot t_m \qquad t_m = \frac{30}{2\pi \cdot Fd} = \frac{30}{2\pi \cdot 100} = \frac{0.3}{2\pi} =$

$\delta f = \frac{f_m}{N} = \frac{100}{1000} = 0{,}1\ Hz$

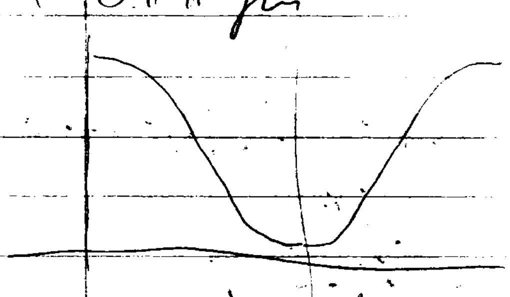
$\delta t = 1/(N\delta f) = \frac{1}{10^3 \cdot 0.1} = \frac{1}{10^2} = 0{,}01 = 10\ ms$

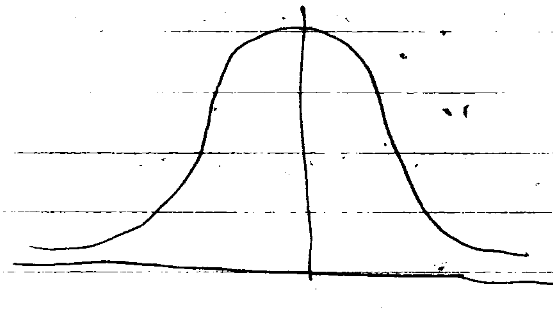
$$S(\nu) = \frac{1}{\pi f_d \sqrt{1-\left(\frac{\nu}{f_d}\right)^2}}$$

$S(0) = \frac{1}{\pi f_m} = 0.0032$

$S(0) = 0.1 = \frac{a}{\pi f_m}$

$a = 0.1 \cdot \pi \cdot f_m =$





$$S(\nu) = \frac{1}{\pi f_d \sqrt{1-\frac{\nu^2}{f_d^2}}}, \quad |\nu| \le f_d$$

$$\rho(\Delta t) = \mathcal{F}^{-1}\{S(\nu)\} = J_0(2\pi f_d \Delta t)$$

$\Delta t = t_2 - t_1$

- COHERENCE TIME T_0 IS EXPECTED TIME DURATION WITHIN WHICH THE TWO SIGNALS REMAIN CORRELATED.

- $\rho(\Delta t) = 1 \Rightarrow$ TIME INVARIANT CHANNEL.

• $T_0 < T_{SYM} \Rightarrow$ FAST FADING (SEVERLY DISTORTED SYMBOLS)

• $T_0 > T_{SYM} \Rightarrow$ SLOW –||–

$f_d = \frac{v}{\lambda} \qquad T_0 \sim \frac{1}{f_d} \qquad T_0 = \frac{\lambda/2}{v} = \frac{0.5}{f_d}$

$B \doteq \frac{1}{T_{SYM}}$

• $\frac{1}{f_d} < \frac{1}{B} \Rightarrow B < f_d$ i.e. $f_d > B \Rightarrow$ FAST FADING

• $f_d < B \Rightarrow$ SLOW FADING

• Diffuse multipath channel Model

$$\tilde{c}(\tau,t) = a(\tau,t)e^{-j2\pi f_c \tau} \qquad \tilde{y}(t) = \underbrace{\int_{-\infty}^{\infty} \tilde{s}(t-\tau)\tilde{c}(\tau,t)\,d\tau}_{\text{CONVOLUTION}}$$

LOWPASS EQUIVALENT SIGNAL

$$\boxed{s(t-\tau) = \sum_{n=-\infty}^{\infty} s(t-nT)\,\mathrm{sinc}(B(\tau-nT))}$$

REPRESENT ANALOG SIGNAL THROUGH ITS SAMPLES

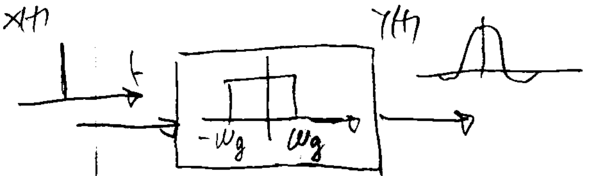
$T = \frac{1}{B} \Rightarrow$ SAMPLING PERIOD

$$\tilde{y}(t) = \int_{-\infty}^{\infty}\left[\sum_{n=-\infty}^{\infty} \tilde{s}(t-nT)\,\mathrm{sinc}(B(\tau-nT))\right]\tilde{c}(\tau,t)\,d\tau =$$

$$= \sum_{n=-\infty}^{\infty} \tilde{s}(t-nT)\int_{-\infty}^{\infty} \tilde{c}(\tau,t)\,\mathrm{sinc}(B(\tau-nT))\,d\tau$$

$$\tilde{y}(t) = \sum_{n=-\infty}^{\infty} \tilde{s}(t-nT)\,\tilde{g}_n(t) \qquad \tilde{g}_n(t) = \int_{-\infty}^{\infty} \tilde{c}(\tau,t)\,\mathrm{sinc}(B(\tau-nT))\,d\tau$$

$$\tilde{y}(t) = \sum_{n=0}^{N} \tilde{s}(t-nT)\,\tilde{g}_n(t) \qquad \tilde{g}_n(t) \simeq T\cdot\tilde{c}(nT,t)$$



$$X(j\omega) = 1$$

$$Y(j\omega) = H(j\omega)$$

$$\cancel{X(j\omega) \quad H(j\omega) \quad Y(j\omega)}$$

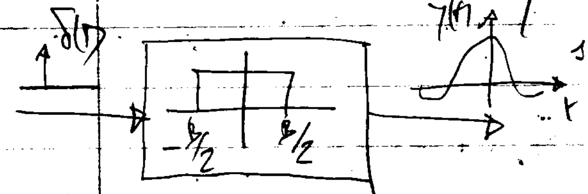
$$y(t) = \frac{1}{2\pi}\int_{-\omega_g}^{\omega_g} \cancel{H(j\omega)}^{\,1}\, e^{+j\omega t}\, d\omega$$

$$y(t) = \frac{1}{2\pi}\int_{-\omega_g}^{\omega_g} e^{j\omega t}\, d\omega = \frac{1}{2\pi}\,\frac{e^{j\omega t}}{jt}\Bigg|_{-\omega_g}^{\omega_g} = \frac{1}{2\pi}\,\frac{1}{jt}\left(e^{j\omega_g t} - e^{-j\omega_g t}\right)$$

$$\sin(x) = \frac{1}{2j}\left(e^{jx} - e^{-jx}\right)$$

$$y(t) = \frac{1}{\pi t}\cdot \sin(\omega_g t) = \frac{\sin(2\pi f_g t)}{\pi t}$$

$$y(t) = \mathrm{sinc}(2 f_g t) \qquad f_g = \frac{B}{2} \qquad \boxed{y(t) = \mathrm{sinc}(Bt)}$$



$$y(t) = \omega_g\,\frac{\sin(\omega_g t)}{\pi\omega_g t} = 2\pi f_g\cdot\frac{\sin(\omega_g t)}{\pi\omega_g t}$$

$$\boxed{y(t) = 2f_g\,\frac{\sin\omega_g t}{\omega_g t} \qquad A=1,\ t_0=0 \qquad E\tau=1}$$

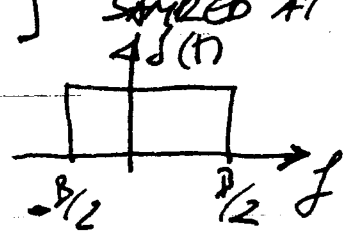
$$x_a(t) = \sum_{n=-\infty}^{\infty} x(n)\,\mathrm{sinc}\left[F_s(t - nT_s)\right]$$

$$s(\tau) = \sum_{n=-\infty}^{\infty} s(nT_s)\,\mathrm{sinc}\left[F_s(\tau - nT_s)\right]$$

$$s(t-\tau) = \sum_{n=-\infty}^{\infty} s(t-nT_s)\,\mathrm{sinc}\Big[F_s\underbrace{(\tau - nT_s)}_{\tilde{g}_n(t)}\Big]$$

$$\tilde{g}_n(t) = \int_{-\infty}^{\infty}\tilde{c}(\tau,t)\,\underbrace{\mathrm{sinc}(B(\tau-nT))}_{\text{IMPULSEN ODZIV NA NF FILTER}}\,d\tau \Bigg\}$$

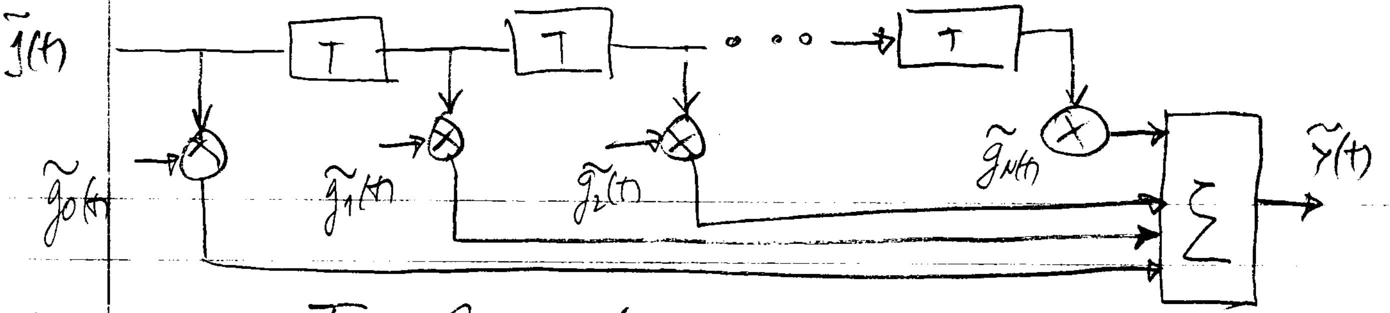
FILTERED VERSION OF THE CHANNEL IMPULSE RESPONSE SAMPLED AT $\underline{n*T}$



$$\boxed{\tilde{g}_n(t) \approx T\cdot\tilde{c}(nT,t)} \qquad T = 1/B$$

SAMPLING PERIOD

$$\boxed{y(t) = \sum_{n=0}^{N}\tilde{s}(t-nT)\,\tilde{g}_n(t)}$$



• STATISTICAL TAP GAIN MODELS 0007082659

$$g_n(t) = \int_{-\infty}^{\infty} \tilde{c}(\tau,t)\,\mathrm{sinc}(B(\tau - nT))\,d\tau$$

$$R_{kl}(\Delta t) = E\left[\tilde{g}_k(\tau,t)\,\tilde{g}_l^*(\tau,t+\Delta t)\right] =$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E\left[\tilde{c}(\tau,t)\,\tilde{c}^*(\mu,t+\Delta t)\right]\cdot \mathrm{sinc}(B(\tau-lT))\,\mathrm{sinc}(B(\mu-lT))\,d\tau\,d\mu$$

$$R_{\tilde{c}}(\tau_1,\tau_2,\Delta t) = E\left[\tilde{c}^*(\tau_1,t)\,\tilde{c}(\tau_2,t+\Delta t)\right]$$

$$R_{\tilde{c}}(\tau_1,\tau_2,\Delta t) = \underline{R_c(\tau_1,\Delta t)\,\delta(\tau_1-\tau_2)} \quad (*)$$

UNCORRELATED SCATTERING ASSUMPTION

$$R_{fg} = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} f(t)\cdot g(t+\tau)\,dt$$

$$R_{\xi\eta} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x\,y\cdot p_{\xi\eta}(x,y,\tau)\,dx\,dy$$

$$R_{kl} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} R_\tau(\tau,\Delta t)\,\underline{\delta(\tau-\mu)}\,\mathrm{sinc}(B(\tau-lT))\,\underline{\mathrm{sinc}(B(\mu-lT))}\,d\tau\,d\mu$$

$$I = \int_{-\infty}^{\infty} \delta(\tau-\mu)\,\mathrm{sinc}(B(\mu-lT))\,d\mu \qquad \tau-\mu = v \quad d\mu = -dv \quad \mu = \tau - v$$

$$I = -\int_{\infty}^{-\infty} \delta(v)\,\mathrm{sinc}(B(\tau-v-lT))\,dv \qquad \underline{\delta(\tau-\mu) = \delta(\mu-\tau)}$$

$$\int \delta(t-t_0)\,f(t)\,dt = f(t_0)$$

$$\int_{-\infty}^{\infty} \delta(\mu-\tau)\,\mathrm{sinc}(B(\mu-lT))\,d\mu = \mathrm{sinc}(B(\tau-lT))$$

$$R_{kl}(\Delta t) = \int_{-\infty}^{\infty} R_\tau(\tau,\Delta t)\,\mathrm{sinc}(B(\tau-kT))\,\mathrm{sinc}(B(\tau-lT))\,d\tau$$

a) UNCORRELATED TAP GAIN MODEL

$R_{kl}(\Delta t)=0 \quad k\neq l \Rightarrow$ TAP GAINS CAN BE CONSIDERED UNCORRELATED

$$\tilde{y}(t)=\sum_{n=0}^{N}\tilde{s}(t-nT)\,\tilde{g}(t)\doteq T\sum_{n=0}^{N}\tilde{s}(t-nT)\,\tilde{c}(nT,t) \qquad T=\frac{1}{B}$$

$$p(\tau)=R_{\tilde{c}}(\tau,0)=E\left|\tilde{c}(\tau,t)\right|^2 \qquad \tilde{g}(t)\doteq T\cdot E\left[\tilde{c}(nT,t)\right]$$

$$E\left[|\tilde{g}_n(t)|^2\right]=T^2E\left[|\tilde{c}(nT,t)|^2\right]=T^2p(nT)$$

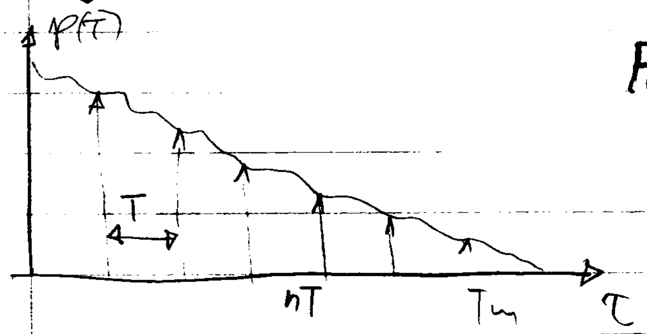


Figure: Sampled values of the power delay profile

$$\tilde{g}_n(t)=\underline{g_{n,r}(t)+j\,g_{n,i}} \quad \text{ZERO MEAN}$$

$$\sigma^2=\left(\overline{\xi^2}-\cancel{\bar{\xi}^2}\right)\cancel{\to 0}$$

$$E\left[|\tilde{g}_n(t)|^2\right]=2\sigma_n^2 \qquad |\tilde{g}_n(t)|^2=g_{n,r}^2+g_{n,i}^2$$

$$E\left[|\tilde{g}_n(t)|^2\right]=\underbrace{E\left[\tilde{g}_{n,r}^2\right]}_{\sigma_n^2}+\underbrace{E\left[\tilde{g}_{n,i}^2\right]}_{\sigma_n^2}=2\sigma_n^2$$

$$E\left[|\tilde{g}_n(t)|^2\right]=2\sigma_n^2=T^2p(nT)$$

b) Uncorrelated Tap Gain Model

COVARIANCE MATRIX

$$R(\Delta t)=\begin{bmatrix} R_{00}(\Delta t) & R_{01}(\Delta t) & \cdots & R_{0N}(\Delta t)\\ R_{10}(\Delta t) & R_{11}(\Delta t) & \cdots & R_{1N}(\Delta t)\\ \vdots & & & \\ R_{N0}(\Delta t) & & & R_{NN}(\Delta t)\end{bmatrix}$$

AVERAGE POWER OF TAP „n" $\quad 2\sigma_n^2=R_{nn}(0)$

$$C=\begin{bmatrix} C_{11} & C_{12} & C_{13}\\ C_{21} & C_{22} & C_{23}\\ C_{31} & C_{32} & C_{33}\end{bmatrix} \qquad A=\begin{bmatrix} -1 & 1 & 2\\ -2 & 3 & 1\\ 4 & 0 & 3\end{bmatrix}$$

$C_{12}=\mathrm{COV}\left[-1\ -2\ 4\quad 1\ 3\ 0\right]$

	COV:	COV:	COV:
	10.33	2.33	1.0
	SUM	SUM	SUM
	1	4	6
SUM/3	1/3	4/3	2

xc = bsxfun(@minus, x, mean(x)) $\qquad m=3 \qquad \boxed{m,n = size(x}$

$$xc = \begin{bmatrix} -1 & 1 & 2 \\ -2 & 3 & 1 \\ 4 & 0 & 3 \end{bmatrix}_{x} - \begin{bmatrix} 1/3 & 4/3 & 2 \\ 1/3 & 4/3 & 2 \\ 1/3 & 4/3 & 2 \end{bmatrix}_{\mu} = \begin{bmatrix} -1{,}33 & -0{,}33 & 0 \\ -2{,}33 & 1{,}66 & -1 \\ 3{,}66 & -1{,}33 & 1 \end{bmatrix}$$

$$x_c' = \begin{bmatrix} -1 & -2 & 4 \\ 1 & 3 & 0 \\ 2 & 1 & 3 \end{bmatrix}$$

$$xy = (xc' * xc)/(m-1) = \frac{1}{2}\begin{bmatrix} -1{,}33 & -2{,}33 & 3{,}66 \\ -0{,}33 & 1{,}66 & -1{,}33 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} -1{,}33 & -0{,}33 & 0 \\ -2{,}33 & 1{,}66 & -1 \\ 3{,}66 & -1{,}33 & 1 \end{bmatrix}$$

$$\cancel{\sigma^2 = \overline{(\xi^2 - \bar{\xi}^2)} = E[\xi^2 - \bar{\xi}^2] = E[x^2 - \mu^2]}$$

$$\sigma_x^2 = \overline{(\xi - \bar{\xi})^2} = E\left[(x-\mu)^2\right]$$

$$\sigma_{xy}^2 = E\left[(x-\mu)(y-\nu)\right]$$

$$x = [-1 \ -2 \ 4]$$

$$\sigma_{xx} = \overline{(x-\mu)^2} = \frac{(-1-0{,}33)^2 + (-2-0{,}33)^2 + (4-0{,}33)^2}{3} =$$

$$= \frac{1{,}777 + 5{,}444 + 13{,}448}{3} = \frac{20{,}66}{3} = 6{,}88$$

$$\sigma_{xx} = \sum_{n=1}^{N} \frac{(x_n-\mu)^2}{N-1} = \frac{22}{2} = 10{,}33$$

$\boxed{\text{lecclogic SDL rou 4.4}}$

$$\sigma_{xy} = \frac{(-1-0{,}33)(1-1{,}33) + (-2-0{,}33)(3-1{,}33) + (4-0{,}33)(0-1{,}33)}{}$$

$$= \frac{(-1{,}33)(-0{,}33) + (-2{,}33)(1{,}67) + (3{,}67)(-1{,}33)}{3} =$$

$$= \frac{1{,}33 \cdot 0{,}33 - 2{,}33 \cdot 1{,}67 - 3{,}67 \cdot 1{,}33}{3} = \frac{0{,}439 - 3{,}89 - 4{,}88}{3}$$

$$\sigma_{xy} = \frac{-8{,}33}{3} = -2{,}78 \qquad \sigma_{xy}' = \frac{-8{,}33}{2} = -4{,}166$$

$\sigma_{12} \qquad \sigma_{12}$

· SIMPLIFIED SCATTERING FUNCTION

$$S(\tau,\nu) = \psi(\tau)\, S(\nu)$$

$$p(\tau) = \int_{-\infty}^{\infty} S(\nu)\, d\nu = \psi(\tau) \int_{-\infty}^{\infty} S(\nu)\, d\nu = a \cdot \psi(\tau)$$

$$S(\tau,\nu) = p(\tau) \cdot S(\nu) \quad \Big/ \frac{1}{2\pi}\int_{-\infty}^{\infty} d\nu$$

$$S(\tau,\nu) = \int_{-\infty}^{\infty} R_{\tilde{z}}(\tau,\Delta t)\, e^{-j2\pi\nu\Delta t}\, d\Delta t \qquad R_{\tilde{z}}(\tau,\Delta t) = p(\tau)\cdot \rho(\Delta t)$$

$$\rho(\Delta t) = \mathcal{F}^{-1}(S(\nu)) = J_0(2\pi f_d \Delta t) \;\ldots$$

$$R_{mn}(\Delta t) = \rho(\Delta t) \int_{-\infty}^{\infty} p(\tau)\, \mathrm{sinc}(B(\tau - mT))\, \mathrm{sinc}(B(\tau - nT))\, d\tau$$

$$T = \frac{1}{B}$$

$$R_{mn}(\Delta t) = \rho(\Delta t) \int_{-\infty}^{\infty} p(\tau)\, \mathrm{sinc}(B\tau - m)\, \mathrm{sinc}(B\tau - n)\, d\tau$$

$$R(\Delta t) = R_0\, \rho(\Delta t)$$

$$\boxed{\tilde{g} = L \times Z} \qquad g = (\tilde{g}_0(t), \ldots, \tilde{g}_N(t))^T$$

$z = (z_0(t), \ldots, z_N(t))^T$ – COLUMN VECTOR OF INDEPENDENT STATIONARY COMPLEX GAUSSIAN PROCESSES

$$E[z_i(t_1)\, z_j(t_2)] = 0 \quad i \neq j \quad \text{ANY } t_1, t_2$$

$$E[z_n(t_1)\, z_n^*(t_2)] = \psi(\Delta t) \qquad \Delta t = t_1 - t_2 \text{ SAME FOR ALL } n = 0, 1, 2 \ldots N$$

$$E[\tilde{g}(t_1)\, \tilde{g}^{\dagger}(t_2)] = L\, \psi(\Delta t)\, I\, L^T = \psi(\Delta t)\, L \cdot L^T$$

† - complex conjugate TRANSPOSE

$$E[\tilde{g}(t_1)\, \tilde{g}^{\dagger}(t_2)] = E[L z\, L^T (z^*)^T] = \psi(\Delta t)\, L \cdot L^T$$

$$z \cdot (z^*)^T = K \cdot I = \psi(\Delta t) \cdot I$$

$$R_0\, \rho(\Delta t) = \psi(\Delta t)\, L L^T \quad \Rightarrow \quad R_0 = L L^T$$

$$\rho(\Delta t) = \psi(\Delta t)$$

$$L = \begin{bmatrix} l_{00} & l_{01} & \dots & l_{0N} \\ \emptyset & l_{11} & \dots & l_{1N} \\ \vdots & & & \\ \emptyset & l_{N1} & \dots & l_{NN} \end{bmatrix}$$

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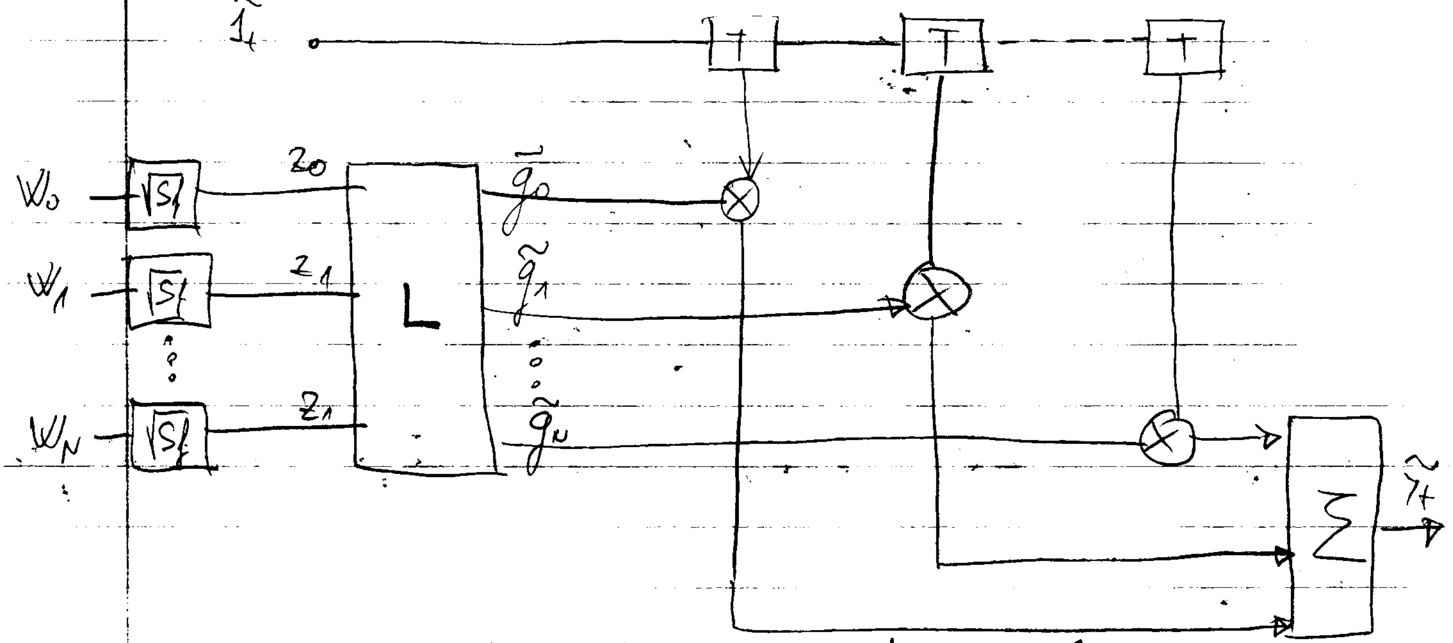


· Filtered Delay Power Profile and Dopler Spectrum

$$\tilde{C}_h(\tau,t) = \tilde{C}_M(\tau,t) * \tilde{h}(\tau)$$

$$\boxed{p(\tau) = R_{\tilde{c}}(\tau,0) = E|\tilde{c}(\tau,t)|^2}$$

MMV

$\boxed{\tilde{C}_M \text{ - MEASURED CHANNEL RESPONSE}}$

$$R_{\tilde{C}_h}(\tau,\Delta t) = E\left[\tilde{C}_h(\tau,t)\,\tilde{C}_h^*(\tau,t+\Delta t)\right]$$

$$R_{\tilde{C}_h}(\tau,\Delta t) = E\left[\int_{-\infty}^{\infty} c_M(s,t)\,h(\tau-s)\,ds \int_{-\infty}^{\infty} c_M^*(\mu,t+\Delta t)\,h^*(\tau-\mu)\,d\mu\right]$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E\left(c_M(s,t)\,c_M^*(\mu,t+\Delta t)\right) h(\tau-s)\,h^*(\tau-\mu)\,d\mu\,ds$$

$$\tilde{c}_M(\tau,t) \to \tilde{c}(\tau,t) \qquad (*) = E\left[\tilde{c}_M(s,t)\,\tilde{c}^*(\mu,t+\Delta t)\right]$$

UNCORRELATED SCATTERING ASSUMPTION

$$(*) \neq \emptyset \quad \text{IF} \quad \boxed{s = \mu}$$

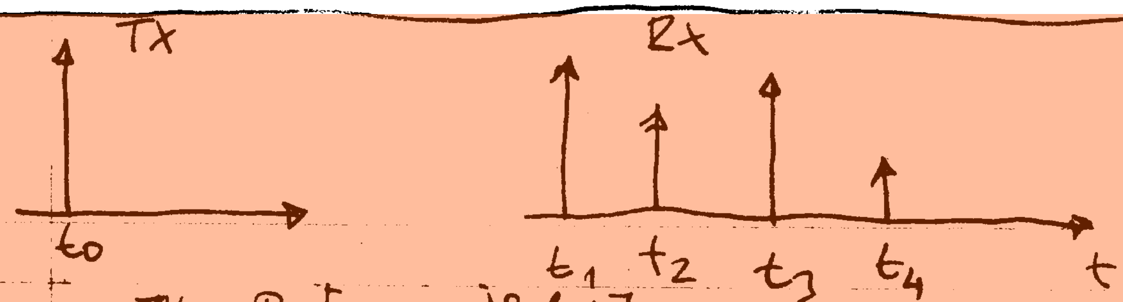
$$R_{\tilde{C}_h}(\tau,\Delta t) = R_{\tilde{c}}(\tau,\Delta t) * |\tilde{h}(\tau)|^2$$

$$\Delta t \to 0 \quad R_{\tilde{C}_h}(\tau,\Delta t) = R_{\tilde{c}}(\tau,0) * |\tilde{h}(\tau)|^2 = p(\tau) * |\tilde{h}(\tau)|^2$$

$$r_h(\tau) = p(\tau) * |\tilde{h}(\tau)|^2$$

$$\mathcal{F}\{h^*(t)\} = \int_{-\infty}^{\infty} h^*(t)\cdot e^{-j\omega t}\,dt = H(-j\omega)$$

$$H(-j\omega) = \int_{-\infty}^{\infty} h(t)\,e^{+j\omega t}\,dt = \left[\int_{-\infty}^{\infty} h^*(t)\,e^{-j\omega t}\,dt\right]^*$$



ISTO E KAKO VO JEDNOTNIM

$$x_T = \mathrm{Re}\left[x_{TB}\cdot e^{j2\pi f_c t}\right]$$

$$x_{RB} = \sum_{i=1}^{N} a_i \cdot x(t-\tau_i(t))$$

$$x_R = \mathrm{Re}\left[\sum_{i=1}^{N} a_i\, x(t-\tau_i(t))\cdot e^{j2\pi f_c (t-\tau_i(t))}\right]$$

$$x_R = \mathrm{Re}\left[e^{j2\pi f_c t}\sum_{i=1}^{N} a_i\, e^{j2\pi f_c \tau_i(t)}\cdot x(t-\tau_i(t))\right]$$

$$x_{RB} = \sum_{i=1}^{N} a_i\, e^{-j2\pi f_c \tau_i(t)}\, x(t-\tau_i(t)) = \sum_{i=1}^{N} a_i\, e^{-j\theta_i(t)}\, x(t-\tau_i(t))$$

$$\boxed{\theta_i(t) = 2\pi f_c \tau_i(t)} \qquad \boxed{h_B = \sum_{i=1}^{N} a_i(t)\, e^{-j\theta_i(t)}}$$

LOWPASS EQUIVALENT IMPULSE RESPON.

$$R_{11}(\tau) = \int_{-\infty}^{\infty} f(t)\cdot f(t+\tau)\,dt = \int_{-\infty}^{\infty} f(t)\left[\frac{1}{2\pi}\int_{-\infty}^{\infty} F(j\omega)\cdot e^{j\omega\tau}\, e^{+j\omega t}\,d\omega\right]dt$$

$$= \underbrace{\int_{-\infty}^{\infty} f(t)\, e^{+j\omega t}\,dt}_{F^*(j\omega)}\ \underbrace{\frac{1}{2\pi}\int_{-\infty}^{\infty} F(j\omega)\, e^{+j\omega\tau}\,d\omega} = \frac{1}{2\pi}\int_{-\infty}^{\infty} \underbrace{F(j\omega)F^*(j\omega)}_{|F(j\omega)|^2} e^{j\omega\tau}\,d\omega$$

$$R_{11}(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} |F(j\omega)|^2\, e^{j\omega\tau}\,d\omega$$

$$R_{11}(\tau) \xleftrightarrow{\mathcal{F}} |F(j\omega)|^2$$

$$f(t)\in R \Rightarrow \underline{F^*(j\omega)} = \int_{-\infty}^{\infty} \left(f(t)\, e^{-j\omega t}\right)^* dt$$

$$= \int_{-\infty}^{\infty} f(t)\, e^{+j\omega t}\,dt = \underline{F(-j\omega)}$$

$$p_{h}(\tau) = p(\tau) * |\tilde{h}(\tau)|^2 \qquad p_h(\tau) = R_{nn}(0) = 2\sigma_n^2$$

- FILTERED SCATTERING FUNCTION:

$$S_h(\tau,\nu) = \mathcal{F}[R_{C_h}(\tau,\Delta t)] \qquad S_h(\tau,\nu) = S(\tau,\nu) * |\tilde{h}(\tau)|^2$$

$$S(\tau,\nu) = p(\tau)\cdot S(\nu) \qquad S_h(\tau,\nu) = p(\tau)\cdot S(\nu) * |\tilde{h}(\tau)|^2$$

$$S_h(\tau,\nu) = p_h(\tau)\cdot S(\nu)$$

$$\tilde{c}_M(\tau,t) = c(\tau,t) * \tilde{h}_M(\tau)$$

$$\tilde{c}_h(\tau) = \tilde{c}_M(\tau,t) * h(\tau) = c(\tau,t) * h_M(\tau) * h(\tau)$$

$$\boxed{p_h(\tau) = p_M(\tau) * |\tilde{h}(\tau)|^2}$$

- MATLAB: SIMULATION OF MULTIPATH FADING CHANNELS – METHODOLOGY

$$y_i = \sum_{n=-N_1}^{N_2} s_{i-n}\, g_n \qquad g_n = \sum_{k=1}^{K} a_k \,\mathrm{sinc}\left[\frac{\tau_k}{T_s} - n\right]$$

$$-N_1 \le n \le N_2$$

g_n – TAP WEIGHTS

K – TOTAL NUMBER OF PATHS

$$a_k = \sqrt{\Omega_k}\, z_k \qquad \Omega_k = E[|a_k|^2] \qquad \boxed{a_k \text{ – COMPLEX PATH GAINS}}$$

RICIAN FADING:

$$a_k = \sqrt{\Omega_k}\left[\frac{z_k\, e^{j2\pi f_{d,LOS,k} t + \theta_{LOS,k}}}{\sqrt{K_{r,k}+1}} + \sqrt{\frac{K_{r,k}}{K_{r,k}+1}}\right]$$

EXAMPLE 9.1.1 DIFUSE MULTIPATH MODEL

$$p(\tau) = \frac{1}{T} e^{-0.4\tau/T} \qquad 0 \leq \tau \leq 4$$

TAP SPACING $T = 1$

1.) TAP GAIN FOR UNCORRELATED APROXIMATION

$|\tilde{g}_0| = 1.0 \quad |\tilde{g}_1| = 0.82 \quad |\tilde{g}_2| = 0.67 \quad |\tilde{g}_3| = 0.55 \quad |\tilde{g}_4| = 0.37$

$$E\left[|\tilde{g}_n(t)|^2\right] = 2\sigma_n^2 = T^2 p(nT)$$

$$|\tilde{g}_n(t)|^2 = 1 \cdot p(\tau) = 1 \cdot \frac{1}{1} \cdot e^{-0.4\tau}$$

$\tau = 0 \qquad g = \sqrt{1 \cdot \frac{1}{1} \cdot e^{-0.4 \cdot 0}} = 1$

$\tau = 1 \qquad g = \left(1 \cdot \frac{1}{1} \cdot e^{-0.4 \cdot 1}\right)^{1/2} = 0.8187 \doteq 0.82$

$\tau = 2 \qquad g = e^{-0.4 \cdot 2/2} = 0.6703 \doteq 0.67$

$\tau = 3 \qquad g = e^{-0.4 \cdot 3/2} = 0.5488 \doteq 0.55$

$\tau = 4 \qquad g = e^{-0.4 \cdot 4/2} = 0.4493 \doteq 0.45$

$$R_{mn}(\Delta t) = \rho(\Delta t) \int_{-\infty}^{\infty} p(\tau)\, \mathrm{sinc}(B\tau - m)\, \mathrm{sinc}(B\tau - n)\, d\tau$$

$$\tilde{g} = L \times Z$$

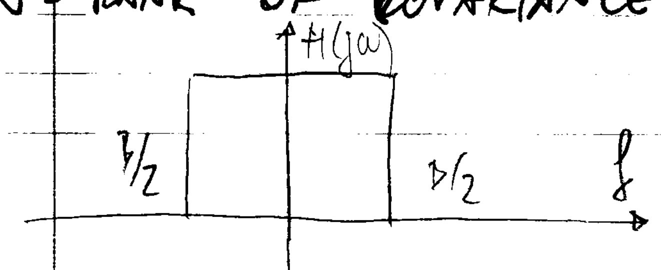
$$R_{mn}(0) = \sum_{k=0}^{K} p(k\Delta\tau)\, \mathrm{sinc}\left[\frac{k\Delta\tau}{T} - m\right] \mathrm{sinc}\left[\frac{k\Delta\tau}{T} - n\right] \Delta\tau$$

$$\int_a^b f(x)\, dx = \lim_{\Delta x \to 0} \left(\sum_{i=0}^{N-1} f(i\Delta x)\, \Delta x \right) \qquad \Delta x = \frac{b-a}{N}$$

$$\boxed{\int_a^b f(x)\, dx = \lim_{\Delta x \to 0} \sum_{i=1}^{n} f(x_i)\, \Delta x}$$

$K = 10 \div 20$ samples PER SYMBOL

N = RANK OF COVARIANCE MATRIX



$$h = \mathrm{sinc}(Bt)$$

$$p_h = p \cdot |h|^2 = p \cdot \mathrm{sinc}^2\left(\frac{t}{T}\right)$$

x	-6	-5	-4	-3	-2	-1	0	0,4	2	3	4	5	6	7	8
y	0	0	0,0025	0,05	0,0075	0,015	0,5	0,7	0,425	0,3	0,125	0,1	0,05	0	0

$$\begin{array}{r} 117,5 \\ 96,5 \\ \hline 21,0 \end{array}$$

$0,1 = 21$ $\quad \frac{0,1}{21} = 1 \quad 4,761\,e\text{-}3 : 1$

$1 = \frac{21}{0,1} = 210 \quad 1 : 210$

$\boxed{1 : 0,005}$ $\quad 2 = 25 \quad \frac{2}{25} = 1 \quad \boxed{0,08 : 1}$

x	y
0,4	0,7

$$R_{mn}(\theta) = \sum_{k=0}^{K} p(k\Delta t)\, \mathrm{sinc}\left[\frac{k\Delta\tau}{T} - m\right] \mathrm{sinc}\left[\frac{k\Delta\tau}{T} - n\right] \Delta\tau$$

$$p(\tau) = \frac{1}{T} e^{-0,4\tau/T} \qquad 0 \le \tau \le 4$$

$$x_{RB} = \sum_{i=1}^{N} a_i \cdot e^{-j2\pi f_c \tau_i(t)}\, x(t - \tau_i(t)) = \sum_{i=1}^{N} \underbrace{a_i}_{a_i} \cdot e^{-j\theta_i}\, \underbrace{x(t - \tau_i(t))}_{\tau_i}$$

$\theta_i = 2\pi f_c \tau_i(t)$

ch = rayleighchan(1/bitRate, 4, [0, 0.5/bitRate], [0, -10])

path gains : $\begin{bmatrix} -0,3662 + j\cdot 0.3682 \\ -0,1484 - j\cdot 0.0016 \end{bmatrix}$

bitRate = 50.000

$a_1^{dB} = 0$ dB $\qquad a_1^{dB} = 10 \log \frac{a_1}{1} \qquad a_1 = 10^{0.1 a_1^{dB}} = 1$

$a_2 = 10^{0.1 a_2^{dB}} = 10^{-1} = 0,1$

$\theta_1 = 2\pi \left(\frac{1}{\text{bitRate}}\right)^{-1} \tau_0 = 0$

$\theta_2 = 2\pi \cdot \left(\frac{1}{\text{bitRate}}\right)^{-1} \tau_1 = 2\pi \left(\frac{1}{5.10^4}\right)^{-1} \cdot \frac{0.5}{\text{bitRate}} = 2\pi\, \cancel{\text{bitRate}}\, \frac{0.5}{\cancel{\text{bitRate}}} = \pi$

9.1.3.5.2 Discrete Multipath Channel Model

IMPULSE RESPONSE

$$c(\tau, t) = \sum_{k=1}^{K(t)} \tilde{a}_k(\tau_k(t), t)\, \delta(\tau - \tau_k(t))$$

$$\tilde{y}(t) = \sum_{k=1}^{K(t)} \tilde{a}_k(\tau_k, t)\, s(t - \tau_k(t))$$

$$\tilde{c}(\tau, t) = \sum_{k=1}^{K} \tilde{\alpha}_k(t)\, \delta(\tau - \tau_k)$$

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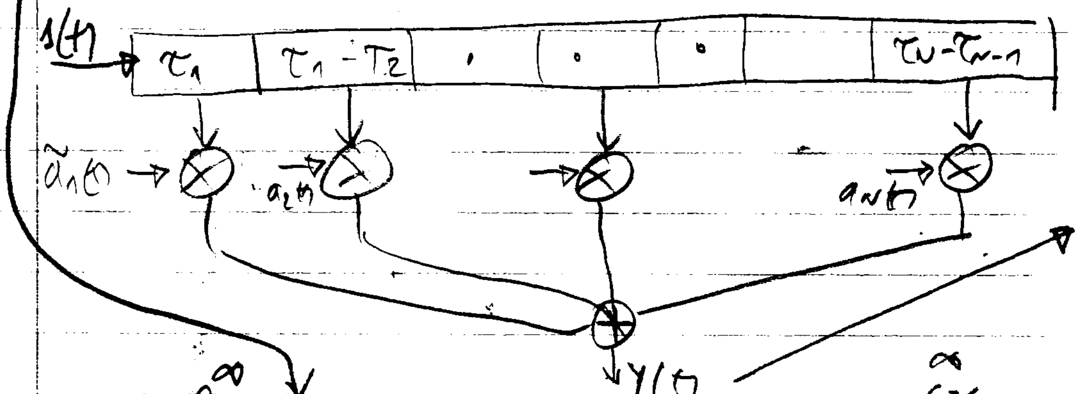
$$\tilde{y}(t) = \sum_{k=1}^{K} a_k\, s(t - \tau_k)$$



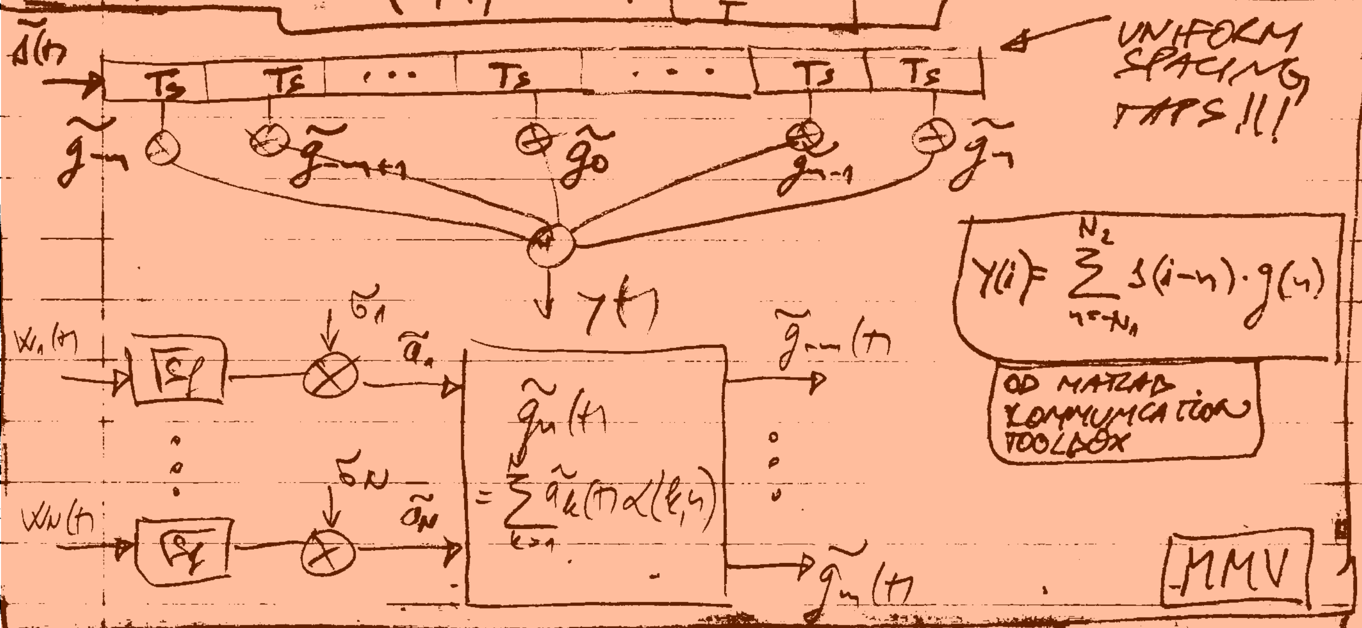
$\tilde{g}_n$ – BANDLIMITED ~~BEEN~~ TAP GAINS

$$\tilde{g}_n(t) = \int^{\infty} \tilde{c}(\tau, t)\, \mathrm{sinc}(B(\tau - nT))\, d\tau = \int \left(\sum_{k=1}^{K} \tilde{a}_k(t)\, \delta(\tau - \tau_k)\right) \mathrm{sinc}(B(\tau - nT))\, d\tau$$

MMV

$$\tilde{g}_n(t) = \sum_{k=1}^{K} \tilde{a}_k(t)\, \mathrm{sinc}(B(\tau_k - nT)) = \sum_{k=1}^{K} \tilde{a}_k(t)\, \alpha(k, n) \qquad -N \le n \le N$$

$$T = B^{-1} \qquad \alpha(k, n) = \mathrm{sinc}\left[\frac{\tau_k}{T} - n\right]$$

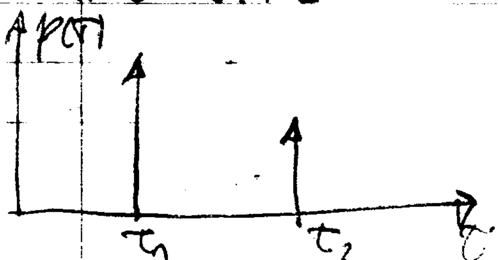


$$y(i) = \sum_{n=-N_1}^{N_2} s(i-n) \cdot g(n)$$

OD MATLAB COMMUNICATION TOOLBOX

MMV

EXAMPLE 9.1.2 DISCRETE MULTIPATH MODEL



$$\Delta\tau \rightarrow \frac{\tau_2 - \tau_1}{T} \qquad T = \frac{1}{B}$$

$$\text{RATIO OF RELATIVE POWERS} = \left(\frac{\sigma_1}{\sigma_2}\right)^2$$

$$y(n) = \sum_{k=0}^{K-1} h(k-n)\cdot x(n) \qquad n = 0 \ldots N-1$$

$x = [1, 2, 3, 4]$ $\quad N = 4$

$h = [1, 2, 2]$ $\quad K = 3$

$\boxed{\text{CONV}}$

1 2 3 4

2 2 1 →

$\text{length}(\text{conv}(x, h)) = N + K - 1 = 6$

$y(1) = 1$

$y(2) = 2 + 2 = 4$

$y(3) = 3 + 4 + 2 = 9$

$\boxed{\text{CCONV}}$ $N = 4$

~~1 2 3 4~~

1 2 2 →

1	2	2	0
1	4	3	2
2	1	4	3
3	2	1	4
4	3	2	1

~~$y(1) = 11$~~
~~$y(2) = 12$~~

$y(1) = 1 + 8 + 6 = 15$

$y(2) = 2 + 2 + 8 = 12$

$y(3) = 3 + 4 + 2 = 9$

$y(4) = 4 + 6 + 4 = 14$

~~$y(1) = 11$~~ ~~$y(2) = 16$~~

~~$y(3) = 2 + 3 + 8 = 13$~~

~~$y(4) = 2 + 4 + 4 = 10$~~

1	2	3	4
1	0	2	2 →
2	1	0	2
2	2	1	0
0	2	2	1

$y(1) = 1 + 6 + 8 = 15$

$y(2) = 2 + 2 + 8 = 12$

$y(3) = 3 + 4 + 2 = 9$

$y(4) = 4 + 6 + 4 = 14$

---

bit rate $= 50.000 = K$ $\qquad N = 2\cdot 5 + 1 = 11$

$M = K + N - 1 = 50.000 + 11 - 1 = 50.010$

$N + 1 - \left(\frac{N}{2} + 2\right) + 1 = N + 2 - \frac{N}{2} - 2 = \frac{N}{2}$

$dt = \frac{1}{N\cdot df} = \Big|\ df = \frac{f_m}{N} \qquad df = \frac{10^2}{10^4} = 10^{-2} = 0{,}01$

$N = 10.000$

$dt = \frac{1}{10^4\cdot 10^{-2}} = \frac{1}{10^2} = 0{,}01$ sec

$\boxed{3.85\cdot 10^{-7}\text{ sec PRVA MZA}}$

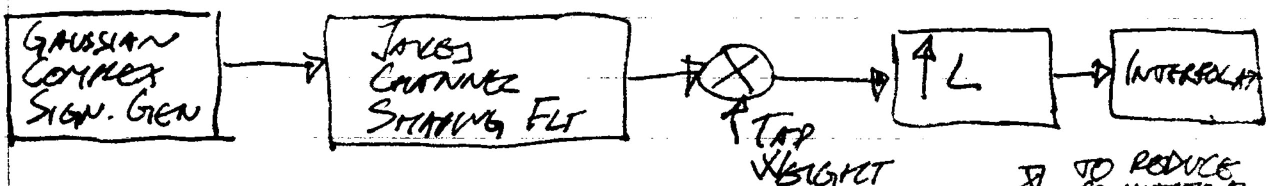
NA BESSELOVKA

9.1.3.5.3. GENERATION OF TAP-GAIN PROCESS

$$H_j(f) = \sqrt{S_j(f)} = \frac{A^{1/2}}{\left[1-\left(\frac{f}{fd}\right)^2\right]^{1/4}}$$

$$\boxed{h_j(t) = \sqrt{A}\cdot 2.583\, fd \cdot x^{-1/4} J_{1/4}(x) \qquad \boxed{x = 2\pi fd|t|}}$$

12.5 SIMULATION OF TAP-GAIN FUNCTIONS FOR RAYLEIGH FADING CHANNEL



TO REDUCE COMPUTATION COMPLEX.

TAP-GAIN FUNCTION IS SAMPLED AT LOWER RATE, THUS INTERPOLATION IS REQUIRED

- CDMA CHIP RATE $f_c = 1.2288\ Mc/s$

SAMPLING RATE : 8 SAMPLES / CHIP

$f_{s,sys} = 9.8304 \times 10^6$ samples/sec

$fd = fo\,\frac{v}{c}$ $\quad$ fo – CARRIER frequency

if: $v = 108\ km/h$ $\quad fd = 2\cdot 10^{9}\cdot \frac{108\cdot 10^3/3600}{3\cdot 10^8} = \frac{2\cdot 100\cdot 108}{3\cdot 36}$

$fo = 2GHz$ $\quad = 200\ Hz$

$6 \div 12$ SAMPLES/PERIOD of the highest frq of ANALOG SIG

$f_{s,t} = 10\, fd = 2000$ samples/sec

EXPANSION FACTOR: $\quad L = \left\lfloor \frac{f_{s,sys}}{f_{s,t}} \right\rfloor = 4195$

- TAP GAIN SIGNAL SAMPLING FREQUENCY

$f_{s,t} = \frac{f_{s,sys}}{L} = 2000{,}0814 \qquad T_{s,t} = \frac{1}{f_{s,t}} \doteq 5\cdot 10^{-4}\ sec$

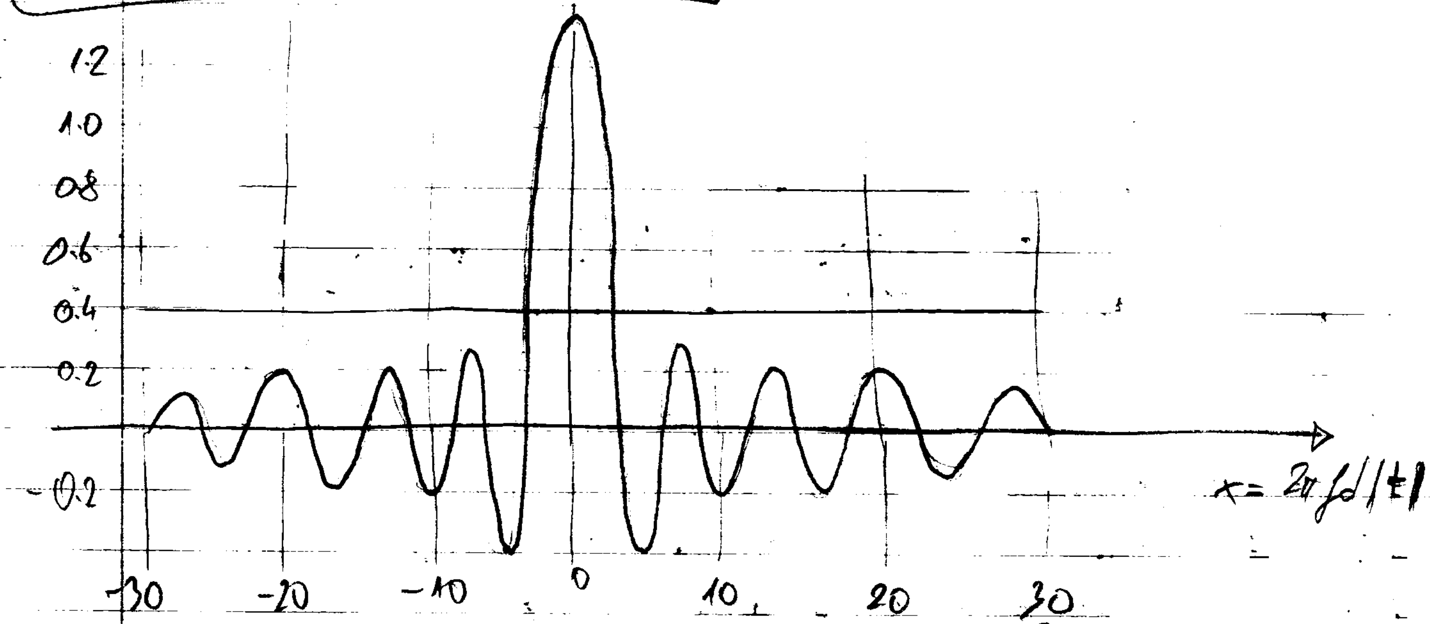
12A.3. Channel Shaping Filter

$h_j(t) = C \cdot f_d \, x^{-1/4} \cdot J_{1/4}(x)$

$x = 2\pi f_d |t|$

12A.4. FIR IMPLEMENTATION

$$\boxed{20 \log \frac{1.3}{0.1} = 22.3\,dB}$$



$x_{t_1} = -20$ $\quad x_{t_2} = 20$ $\quad T = \frac{x_{t_2} - x_{t_1}}{2\pi f_d} = \frac{\cancel{40}\,2}{\cancel{2}\pi \cdot 200} = \frac{2}{\frac{\cancel{20}}{10}\pi} = \frac{1}{10\pi}$

$\boxed{T = 3.18 \cdot 10^{-2}}$ $\quad M = \frac{T}{T_{s,t}} = \frac{T}{1/2000} = 2000 \cdot T = 63.66 = 64$ TAPS

Output of the shaping filter is: $\quad x(t) \; \boxed{A \cdot e^{-j\omega t_0}} \; A \cdot \delta(t - t_0)$

$$\boxed{y_j(n) = \sum_{m=0}^{M-1} h_j(m)\, x_q(n-m)}$$

$$\boxed{h_j(n) = h\left[n \frac{T}{M} - \frac{T}{2}\right]}$$

$\frac{T}{2} \Rightarrow$ LINEAR PHASE ~~[illegible]~~ IN FREQ RESPONCE $\boxed{e^{-j2\pi f T}}$

$h_{jw}(n) = h_j(n)\, W_H(n)$ — HAMMING WINDOW

$$W_H(n) = 0.54 + 0.46 \frac{\cos\left[2\pi\left[n - \frac{(M-1)}{2}\right]\right]}{M} \qquad 0 \le n \le M-1$$

12.4 CASE STUDY IV: PERFORMANCE EVALUATION OF A CDMA CELLULAR RADIO SYSTEM

6-bit WOR $\Rightarrow$ ONE 64 bit WALSH FUNCTIONS
THERE ARE 64 ORTOGONAL WALSH FUNCTION FORMING ONE SET OF WALSH FUNCTIONS.

$W^{(k)}(t) \Rightarrow$ 6 bit WORD

$W_k(t) \rightarrow$ CORESPONDING WALSH FUNCTION

PN SEQUENCE ~~...~~ PERIOD 2^{15} AT RATE 1.2288 Mc/s

- PROCESSING GAIN OF THE SYSTEM

$1{,}2288 \cdot 10^6 / 28.8 \cdot 10^3 = 42.67$ OR 16dB

$$S_T(t) = W_k(t)\left[PN_1(t) + j\,PN_2(t)\right] * h_T(t)$$

RISED SQRT COSINE FILTER

12.4.3.1.2 The Channel

PCS OUTDOR MODEL

τ_1 (μs)	τ_2 (μs)	τ_3 (μs)
0	1.5	14.5

Tap number	Tap strength $10\log(\rho_i)$
1	0
2	-3
3	-6

$f_d = 200\,Hz$

Jakes DOPLER SPECTRUM

- COMPLEX LOWPASS-EQUIVALENT IMPULSE RESPONSE:

$$\boxed{\tilde{c}(\tau, t) = \sum_{i=1}^{3} \rho_i(t)\, e^{j\theta_i(t)}\, \delta(t-\tau_i)}$$

$\rho(\Delta t) = \mathcal{F}^{-1}\left[S(\nu)\right]$ OVA NEMA VRAKA SO !?

12.4.3.1.3 The Receiver

$PN \cdot PN^* = 2$

$$S_R(t) = S_T(t) * \tilde{c}(\tau, t)$$

$$S_D(t,i) = S_R(t) * h_R(t)\left[PN_1(t-\tau_i) - j\,PN_2(t-\tau_i)\right] \quad i = 1,2,3$$

$$S_D(t,i) = \left[S_T(t) * \tilde{c}(\tau, t)\right] * h_R(t)\left[PN_1(t-\tau_i) - j\,PN_2(t-\tau_i)\right]$$

$$S_D(t,i) = 2\, W_k(t)\, \rho_i(t)\, e^{j\theta_i(t)}\, \delta(t-\tau_i) * h_R(t) * h_T(t) + N(t) \quad i = 1,2,3,$$

$$c_{ij} = \left| \int s_D(t,i)\, w_i(t-\tau_{max})\, dt \right|^2 =$$

$$= 4|\beta_i(t)|^2 \left| \int \left[w_k(t-\tau_{max}) * h_R(t) * h_T(t) + N(t) \right] w_j(t-\tau_{max})\, dt \right|^2$$

for $j = 1, 2, \dots, 64$

$$w_j = \max_j \left(\sum_{i=1}^{3} c_{ij} \right)$$

$$g_n = \sum_{k=1}^{K} a_k \cdot \mathrm{sinc}\left[\frac{\tau_k}{T_s} - n \right] \qquad -N_1 \le n \le N_2$$

$$y_i = \sum_{n=-N_1}^{N_2} s_{i-n}\, g_n \qquad i = 0, 1, 2, 3, 4$$

$s = [1\ 2\ 3\ 4\ 5]$ $\qquad -2 \le n \le 2$

$g = [1\ 2\ 2\ 1\ 2]$ $\qquad -2 \le n \le 2$

$-N_1 \qquad N_2$

$\boxed{-2\ \ -1\ \ 0\ \ 1\ \ 2}$

SUŠTINA ZA DOCNENJE NA TDL

$(-2)+(-2) \le i \le 2+2$

$-4 \le i \le 4$

vkupno $4+4+1 = \underline{9}$

$$y_i = \sum_{n=-2}^{2} s_{i-n}\, g_n$$

$$y_0 = \sum_{n=-2}^{2} s_{-n}\, g_n$$

n $\downarrow$ 1 2 2 1 2

$\longrightarrow$ 5 4 3 2 1

$$y_i = \sum_{n=-2}^{2} s(i-n)\, g(n) \qquad -4 \le i \le 4$$

$$s(-4-n) = s(-(n+4))$$

$g(n)$

n

1 2 1 1 2

$-4\ \ -3\ \ -2\ \ -1\ \ 0\ \ 1\ \ 2\ \ 3\ \ 4 \longrightarrow n$

5 4 3 2 1 $\quad s(-4-n)$

$y(-4) = 1$

$y(-3) = 1\cdot 2 + 1\cdot 2 = 4$

$y(-2) = 3\cdot 1 + 2\cdot 2 + 1\cdot 2 = 9$

$y(-1) = 4\cdot 1 + 3\cdot 2 + 2\cdot 2 + 1\cdot 1 =$

$= 4 + 6 + 4 + 1 =$

$= 15$

$f = [1\ 2\ 3\ 4\ 5] \qquad 0 \le n \le 4$

$g = [1\ 2\ 2\ 1\ 2] \qquad -2 \le n \le 2$

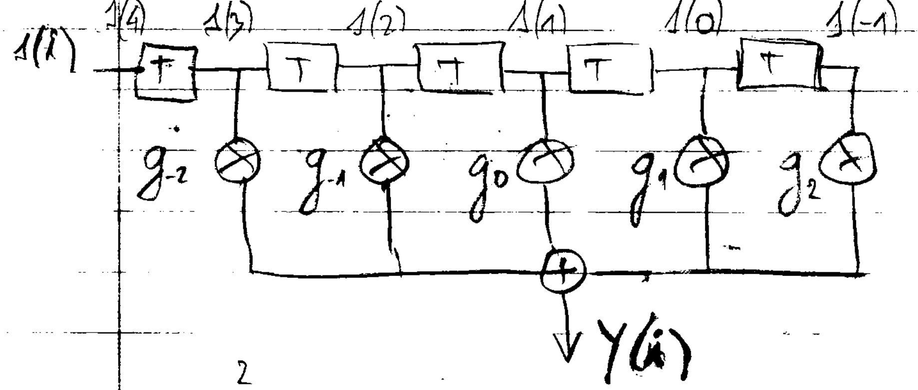
$|N_1| = 0 + (-2) = |-2| = 2$

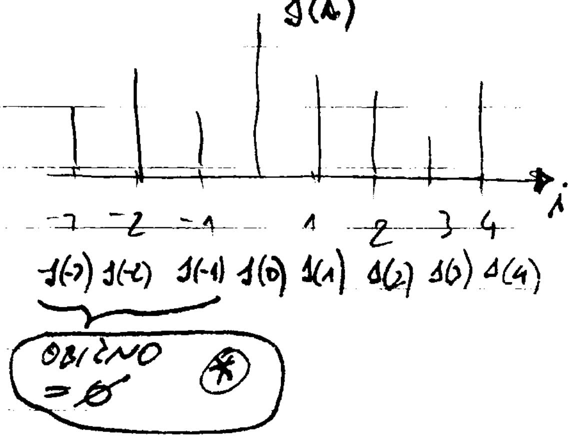
$N_2 = 6 \qquad N = N_2 + N_1 + 1$

$N = 9$

$$y(i) = \sum_{n=-N_1}^{N_2} f(i-n)\,g(n)$$

$$y(i) = \sum_{n=-2}^{6} f(i-n)\,g(n)$$





$$y(1) = \sum_{n=-2}^{2} f(1-n)\,g(n) = f(3)\cdot g(-2) + f(2)g(-1) + f(1)\cdot g(0) + f(0)g(1) + f(-1)\cdot g(2)$$

$$y(2) = \sum_{n=-2}^{2} f(2-n)\,g(n) = f(4)\cdot g(-2) + f(3)g(-1) + f(2)g(0) + f(1)g(1) + f(0)g(2)$$

$$y(0) = \sum_{n=-2}^{2} f(-n)\,g(n) = f(2)g(-2) + f(1)g(-1) + f(0)g(0) + f(-1)g(1) + f(-2)g(2)$$

$$y(-1) = \sum_{n=-2}^{2} f(-1-n)\,g(n) = f(1)g(-2) + f(0)g(-1) + \cancel{f(-1)}g(0) + \cancel{f(-2)}g(1) + \cancel{f(-3)}g(2)$$

$$y(-2) = \sum_{n=-2}^{2} f(-2-n)\,g(n) = f(0)g(-2) + \cancel{f(-1)}g(-1) + \cancel{f(-2)}g(0) + \cancel{f(-3)}g(1) + \cancel{f(-4)}g(2)$$

ZARADI OVIE ČLENOVI VO „WorkingWithDelays1.m" SE PRAVI TRUNCATION OD „N1+1" PA DO „END". N1 E VSUŠNOST DOCNENJE NA LINIJATA. AKO N1=0 NEMA DA IMA ČLENOVI SO NEGATIVEN INDEKS, A SO TOA NEMA DA IMA NITU DOCNENJE.

k								
			1	2	2			
1	4	3	2	1	0	0 →		
2		4	3	2	1	0		
3			4	3	2	1		
4				4	3	2	1	
5					4	3	2	1
6						4	3	2
7								

$$N_x + \cancel{N_y} - \cancel{1} + \cancel{N_y} + \cancel{1} = N_x$$

$$N_x + \cancel{N_y} - 1 - \cancel{N_y} = N_x - 1$$

$$a_k = \sqrt{\Omega_k}\, z_k$$

$$\Omega_k = E\left[|a_k|^2\right]$$

$g = [1\ 2\ 2]$

$s = [1\ 2\ 3\ 4\ 5]$

$$5 + 3 - 1 = 7$$

$$N_s + N_g - 1 = 7$$

K										
					1	2	2			
1	5	4	3	2	1	0	0			} I $(N_g - 1)$
2		5	4	3	2	1	0			
3			5	4	3	2	1			} II
4				5	4	3	2	1		
5					5	4	3	2	1	
6					0	5	4	3	2	1 } III
7					0	0	5	4	3	2 1

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• NAKAGAMI DISTRIBUTION

$$\boxed{p(y) = \frac{2 m^m x^{2m-1}}{\Gamma(m)\,\Omega^m}\, e^{-\frac{m x^2}{\Omega}} \qquad m = \frac{E^2[y^2]}{E\left[\left(y^2 - \overline{y^2}\right)^2\right]} \qquad \Omega = \overline{y^2}}$$

$$\begin{bmatrix} \Omega_1 \\ \Omega_2 \\ \Omega_3 \\ \Omega_4 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 1 & 1 & \cdots \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\sum |a_k|^2 = 1$$

$$df = \frac{f_{max}}{20 \cdot N} = \frac{5 \cdot 10^3}{}$$

$$0.05 = \frac{50 \cdot 10^3}{20 \cdot N} = \frac{\cancel{50 \cdot 10^3}}{20 \cdot \cancel{50 \cdot 10^3}} = \frac{1}{2} \cdot 10^{-1} = 0.5 \cdot 10^{-1} = 5 \cdot 10^{-2} = \underline{\underline{0.05}}$$

$$0.05 \cdot 50 \cdot 10^3 = 5 \cdot 10^{-2} \cdot 50 \cdot 10^3 = 5 \cdot 500 = \underline{\underline{2500}}$$

$\sigma^2 = \overline{(\xi-\bar{\xi})^2} = \sum_{n=1}^{N} \frac{(\xi-\bar{\xi})^2}{N}$ $\quad \xi = \xi_1, \xi_2, \dots \xi_N$

$\sum_{n=1}^{N} |a_k|^2 = 1$ $\quad a_1, a_2, \dots a_N$

$P_{vk} = \sum_{n=1}^{N} a_k$ $\quad \frac{a_1}{P_{vk}}, \frac{a_2}{P_{vk}}, \dots \frac{a_N}{P_{vk}}$

$\sum_{n=1}^{N} |a_k'|^2 = \frac{|a_1|^2}{P_{vk}^2} + \frac{|a_2|^2}{P_{vk}^2} + \dots + \frac{|a_N|^2}{P_{vk}^2} = \frac{P_{vk}}{P_{vk}^2} = \frac{1}{P_{vk}}$

~~ZATO~~: $a_k' = \left\{ \frac{a_1}{\sqrt{P_{vk}}}, \frac{a_2}{\sqrt{P_{vk}}}, \dots \frac{a_N}{\sqrt{P_{vk}}} \right\}$

$P_{vk}' = \sum_{k=1}^{N} |a_k'|^2 = \left(\frac{|a_1|}{\sqrt{P_{vk}}}\right)^2 + \dots \left(\frac{|a_N|}{\sqrt{P_{vk}}}\right)^2 = \frac{P_{vk}}{P_{vk}} = 1$

BitRate = 50000 = 50k

$T_s = \frac{1}{50k} = \frac{1}{5} \cdot 10^{-4} = 0.2 \cdot 10^{-4} = 2 \cdot 10^{-5}$ sec

$df = \frac{f_s/2 + f_s/2}{N} = f_s/N$

$N - delay + 1 + delay - 1$

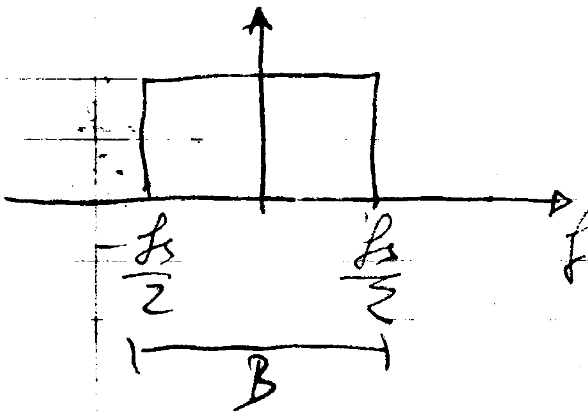
$(N + \cancel{delay - 1}) - \cancel{delay} + \cancel{1}$

$$\left.\begin{aligned} x \quad & (-N_{x1} : N_{x2}) \\ y \quad & (-N_{y1} : N_{y2}) \end{aligned}\right\} \text{conv} \quad \underbrace{-N_{x1} - N_{y1} : N_{x2} + N_{y2}}_{z}$$

$x \quad (-2 : 2) \quad N_x = 5 \quad count(conv(x,y)) = N_x + N_y - 1$

$y \quad (-2 : 2) \quad N_y = 5 \quad = 9$

$\boxed{z\ (-4 : 4)}$



$\boxed{B = f_s = \frac{1}{T_s}}$

$\boxed{df = \frac{B}{N} = \frac{1}{T_s \cdot N}}$

$P_{k1} = \text{mean}(\text{abs}(a_{k1}).\hat{}2)$

$P_{k2} = \text{mean}(\text{abs}(a_{k2}).\hat{}2)$

$P_{k3} = \text{mean}(\text{abs}(a_{k3}).\hat{}2)$

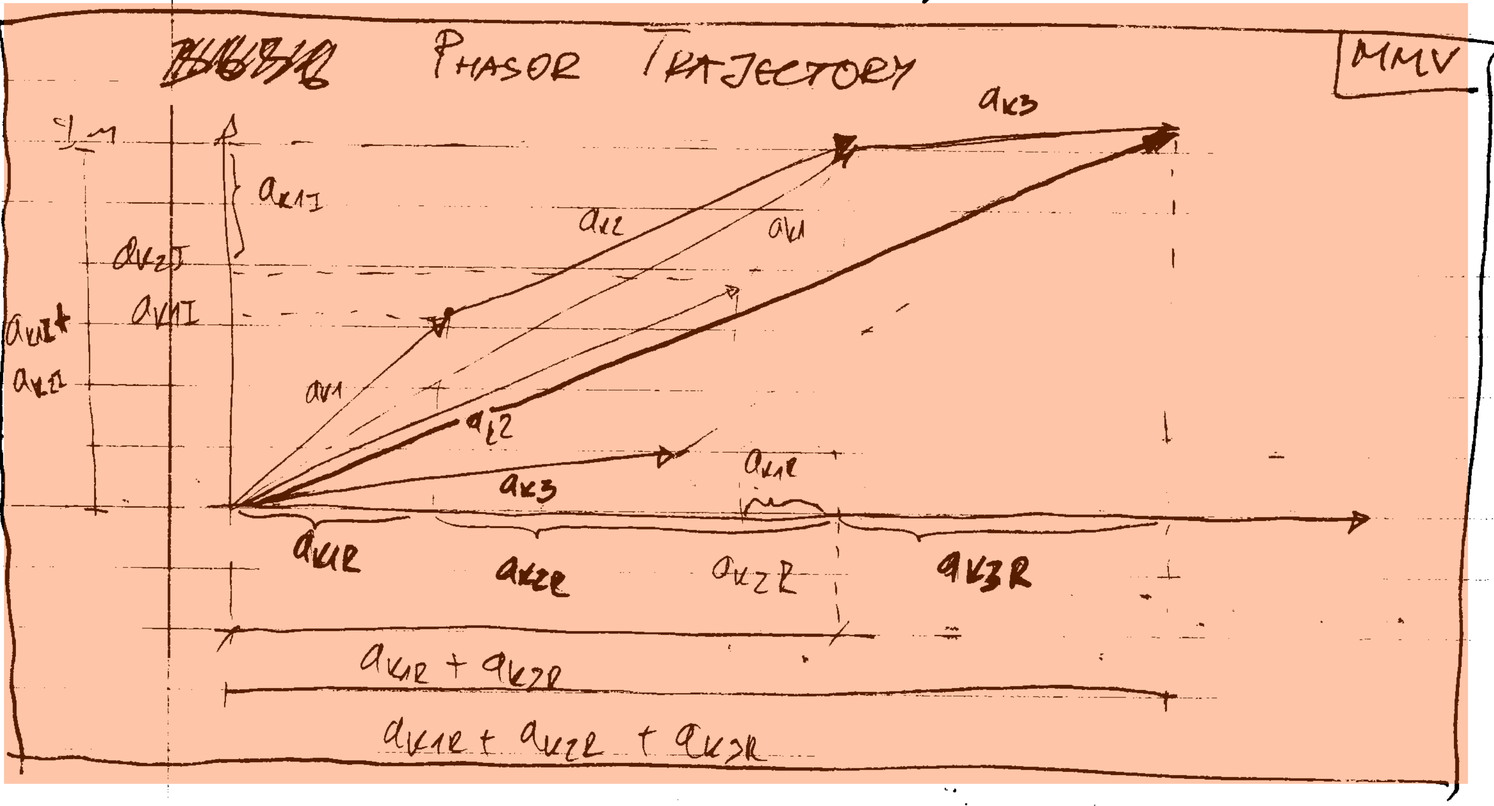
coefSets

0.07, 0.21, 0.14

0.17, 0.51, 0.34

$P_{k1} + P_{k2} + P_{k3} = X$

$$\frac{P_{k1} + P_{k2} + P_{k3}}{X} = 1$$



JERUCHIM: 12A.3. CHANNEL SHAPING FILTER

$$h_j(t) = C f_d\, x^{-1/4} J_{1/4}(x)$$

- TRUNCATE THE TAILS OF $|h_j(t)|$ SO ↕ 20dB from MAXIMUM VALUE

$x = 2\pi f_d |t| \Rightarrow \quad x_{t_1} = -20 \quad x_{t_2} = 20$

for: $f_d = 200\text{Hz} \rightarrow 2\pi f_d = 1.2566 \cdot 10^3$

$$T = \frac{x_{t_2} - x_{t_1}}{2\pi f_d} = \frac{40}{1.2566 \cdot 10^3} = 3.18 \cdot 10^{-2}$$

$$\frac{T}{T_{s,t}} = \Big| \; T_{st} = \frac{1}{10 f_d} = \frac{1}{2000} = 0.5 \cdot 10^{-3} \Big| = \frac{3.18 \cdot 10^{-2}}{0.5 \cdot 10^{-3}}$$

$$\left[\frac{T}{T_{s,t}}\right] = \frac{3.18}{0.5 \cdot 10^{-1}} = \frac{3.18}{0.5} \cdot 10 = \frac{31.8}{0.5} \doteq 64$$

$$h_j(n) = h\left[n\frac{T}{M} - \frac{T}{2}\right]$$

$$w_H(n) = 0.54 + 0.46 \frac{\cos\left[2\pi\left(n - \frac{M-1}{2}\right)\right]}{M}, \quad \underline{0 \le n \le M-1}$$

⊛ N_1 · 1 2 3 4 5

N 1 2 3 4 5 6 7 8 9 10

$$\text{step} = \frac{N_1}{N} = \frac{5}{10} = 0.5$$

interp1(1:N1, aK1, 1:step:N1)

1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5, 5, 5.5, 6, 6.5, 7, 7.5, 8, 8.5, 9, 9.5, 10

1+ M.step

$$S = a_1 + a_2 + a_3 + \cdots + a_n$$

$$a_2 = a_1 + d \qquad a_3 = a_2 + d = a_1 + 2d \quad \ldots \quad a_n = a_1 + (n-1)d$$

$$a_n = 10 \qquad \underline{\underline{n = ?}} \quad d = ? \quad : \quad n = M = 20$$

$$a_N = a_1 + (M-1)d \qquad \boxed{d = \frac{a_M - a_1}{M-1}}$$

$$* \quad d = \frac{\overset{a_M = 10}{10} - 1}{20-1} = \frac{9}{19} = 0{,}4737$$

КУРЗОВ
1€ = 61,45
1€ = 61,8 MKD

ФАКТУРА	STD ZADOLZUVANJE	KONVERZIJA
78173,00	1273,18 €	61.400
6490,00	105,70 €	61.400
17240,00	280,78 €	61.400
16350,00	266,29 €	61.400
118.253,00 MKD	1925,95 €	
: 61,45 MKD		
1.924,00 €		

$$\boxed{w(n) = 0.54 + 0.46 \cos\left(\frac{2\pi n}{N}\right) \quad -\frac{N-1}{2} \le n \le \frac{N-1}{2}}$$

$$w(n) = 0.54 + 0.46 \cos\left(\frac{2\pi}{N}\left(n - \frac{N}{2}\right)\right) = 0.54 + 0.46 \cos\left(\frac{2\pi n}{N} - \pi\right)$$

$$w(n) = 0.54 + 0.46 \cos\left(\pi - \frac{2\pi n}{N}\right) = 0.54 - 0.46 \cos\left(\frac{2\pi n}{N}\right)$$

$$\cos(\pi - \alpha) = \cos\pi \cdot \cos\alpha + \underbrace{\sin\pi}_{0} \cdot \sin\alpha = -\cos\alpha$$

RAISED COSINE

$$\boxed{H(f) = \frac{1}{2}\left[1 + \cos\left(\frac{\pi f}{R_s}\right)\right] \qquad |f| \leq R_s = \frac{1}{T_s}}$$

HAMMING WINDOW – RAISED COSINE WITH SPECIFIC COEFFICIENTS : $\boxed{\text{hamming.m}}$

$$\boxed{W(n) = 0.54 - 0.46 \cos\left(\frac{2\pi n}{N}\right)} \qquad \boxed{n = 1, 2, \ldots N}$$

$$n = \frac{N}{2} \Rightarrow W(n) = 0.54 + 0.46 = 1$$

- MATLAB IMPLEMENTATION ANALYSIS

FUNCTION: setrates(R, T_s, f_c) cutoff frq. (i.e. f_d)

VO RAZCHAN STAVAM T_s = 1e-5 NO VO setrates T_s = 1e-3 !? ZOSTO?

OUTPUT SAMPLE PERIOD } OVA SO CUVA VO PRIVATE ~~DATA~~ !!

VO filtGaussian

$$\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt \qquad \Gamma(x+1) = x\Gamma(x)$$

$$x! = \Gamma(x+1)$$

$\boxed{\text{jakesir.m}}$

PICK VALUE OF JAKES FUNCTION

$\nu = \frac{1}{4}$

$|x| = 2\pi f_d |t|$ $\qquad J_p = \dfrac{\left(\frac{1}{2}\right)^\nu}{\text{gamma}(\nu+1)}$

J = Re{ |x|^(-ν) . besselj (ν, |x|) }

J (isnan(J)) = Jp ; LJ = lenght(J)

hjw = J . hamming(LJ);

h = hjw./sqrt(sum(abs(hjw).^2))

$$\boxed{t_{max} = \frac{50}{2\pi f_d}} \qquad f_d = 100 \quad t_{max} = \frac{\cancel{50}}{2\pi \cdot \cancel{100}_{2}} = \frac{1}{4\pi}$$

$t_{max} = 0.0796$

$T_s = 10^{-3}$ $\qquad \boxed{t = -t_{max} : T_s : t_{max}}$ $\qquad \boxed{\text{jakes.m}}$

- PATH GAIN CHANGES INSIGNIFICANTLY OVER PERIOD $\boxed{1/(100 f_d)}$.

fgTs - FILTER GAUSSIAN SAMPLING ~~FRE~~ PERIOD

$$fgT_s = \frac{1}{N \cdot f_c} \qquad \boxed{f_c = f_d}$$

N - OVERSAMPLING FACTOR (N=10)

KAKO ŠTO E DEFINIRANO VO JERUCHIM

if $f_c = 0$ then : $\boxed{fgT_s = T_s}$

```
Ks1 : POLYPHASE INTERPOLATION FACTOR (Ks1min=10  Ks1max=20)
Ks2 : LINEAR          -//-            -//-
d = N·Ts·fc   #{N=10; Ts=1e-5; fc=100 => d= 0.0100}
                      INPUT SAMPLING
if d<=1
   Ks = floor(1/d)   # {Ks=100}
   if Ks <= Ks1max
      Ks1 = Ks
      Ks2 = 1
   else
      Ks1 = Ks1min              % {Ks1 =10}
      Ks2 = round(Ks/Ks1)       % {Ks2= 10}
      Ks = Ks1·Ks2              % {Ks =100}
      Kl= [Ks Ks1 Ks2]                          RECOMPUTED FOR OUTPUT
      N = 1/(Kl(1)*Ts*fc);      % {N=10}
   end
else
   Kl =[1 1 1]; N = NaN;
end
```

intfiltgaussian_intfactor.m

$N + \cancel{Nd} \cancel{-1} - \cancel{Nd} \cancel{+1} = N$

$t_{max} = 0.0663$

$fgT_s = 8.00\,e{-4}$

$fgT_s = \frac{1}{10 \cdot 120} =$

$APGdB = [0\ -3\ -3]$ $\qquad APG = 10^{0.1 APGdB}$

channel.interpfilter

channel.rayleighfading.interpfilter

@intfiltgaussian\generateoutput } TUKA GO GENERIRA z

SECTION 3.5 FOR POLYPHASE FILTERING } JERUCHIM

Ifggen() interpolating-filtered Gaussian source

• generate output function
h.FiltGaussian.PrivateData.NumChannels
h.MaxBlockLength = 1000
~~h.BlockLength~~

OVIE SVOJSTVA GI KORISTI generateoutputs FUNKCIJATA

• generateblock(h, N)
h.CutoffFrequency
h.FiltGaussian
h.InterpFilter
h.UseCMEX
h.FiltGaussian.LastOutputs(:, end)

filter (MP) → filterblock → generateoutputs → generateblock

MP → MULTIPATH FILTER

AKO h.UseCMEX = 1 TOGAŠ SE POVIKUVA
channel.interpfilt filter(h,s,N) FUNKCIJATA

Used OBJECTS	REDOSLED NA POVIKUVANJE
@baseclass	
@basefiltgaussian	(8) (6)
@filtgaussian	(5)
@interpfilter	(8)(6)
@intfiltgaussian	(4)
@rayleigh	(1)
@rayleighfading	(3)
@sigstatistics	(6)
@multipath	(2)
@sigresponse	(7)
@channelfilter	(9)
@slidebuffer	(10)
@buffer	(11)

channel.rayleighfading

OutputSamplePeriod: 1e-05
CutoffFrequency: 100
NumChannels: 3
MaxBlockLenght: 1000
TargetFGOversampleFactor: 10
FiltGaussian: [1x1]
InterpFilter: [1x1]
CutoffFrequencyFactor: 1
MaxDopplerShift: 0

channel.filtgaussian

ImpulseResponse: [1x160]
NumChannels: 3
LastOutputs: ∅
State: ∅
WGNState: ∅
QuasiStatic: ∅
OutputSamplePeriod: 1.0e-3
CutoffFrequency: 100
OversamplingFactor: 10
ImpulseResponseFcn: {[1x1]}
DopplerSpectrum [0x1] ∅
TimeDomain [1x160]
NumFrequencies: 1024
Autocorrelation: [1x1]
PowerSpectrum: [1x1]
Statistics: [1x1]

• STAVI DEBUG COMAND-I VO ORIGINALNITE MATLAB FAJLOVI ZA IMPLEMENTACIJA NA rayChan ..

$y = \sin(0.2\pi n)$ $n = 1:10$ $N = 10$

$D = [\mathrm{diff}(y), 0]$

$y = [0.58779, 0.95106, 0.95106, 0.58779, \ldots, -0.58779, -2.5 \cdot 10^{-16}]$

$D = [y(2)-y(1), y(3)-y(2), \ldots] = [0.36327, 1.102e-16 \ldots]$

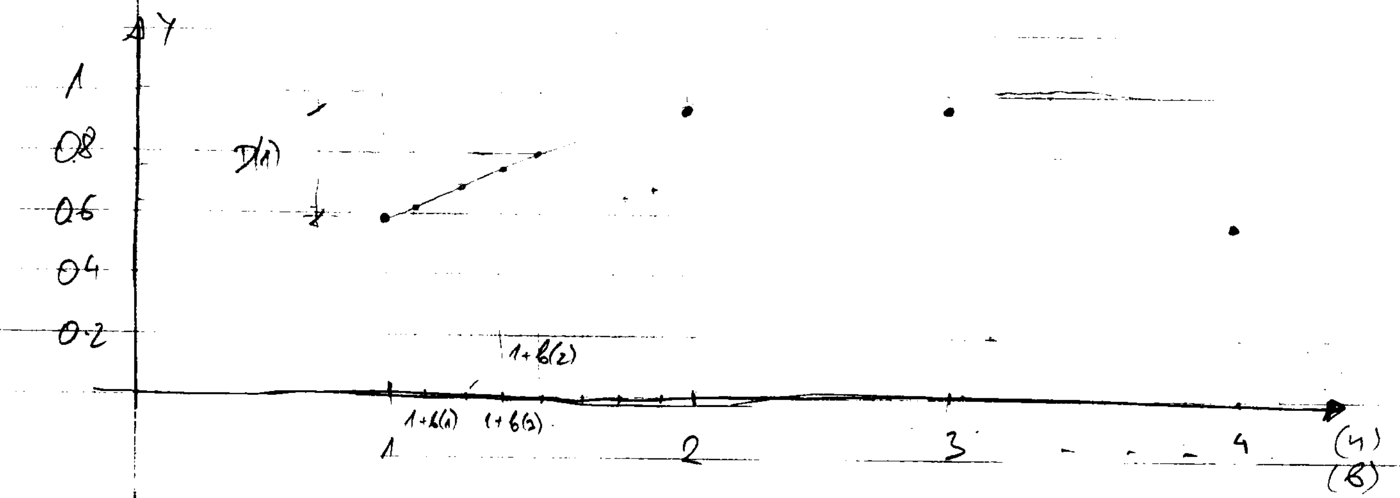
$i = 1:100$

$b = \frac{i-1}{N} = \frac{0:99}{N_{=10}} = [0; 0.1; 0.2; 0.3 \ldots 9,9]$

$k = floor(b) = [10\times 0; 10\times 1; 10\times 2; \ldots 10\times 8; 10\times 9] = [0,0,0\ldots 0; 1,1,\ldots 1; 2,2 \ldots 2 \ldots]$

$n = k+1 = [10\times 1, 10\times 2, 10\times 3 \ldots 10\times 10] = [1,1,\ldots 1; 2,2,\ldots 2; \ldots 10,10,\ldots 10]$

$z = y(n) + (b-k)\cdot D(n)$



$Y(1) = \sin(0.2\cdot\pi\cdot 1) = \sin(0.2\pi) = 0.58779.$

$Y(2) = \sin(0.2\pi\cdot 2) = 0.95106$

$Y(3) = \sin(0.2\pi\cdot 3) = 0.95106$

$D = [0.36327;\ 1.1102e-16;\ -0.36327;\ -0.58779;\ \ldots\ 0]$

$z(1) = Y(1) + (b(1) - k(1))\,D(1) = 0.58779 + (0-0)\cdot D(1) = 0.58779$

$z(2) = Y(1) + (b(2) - k(2))\,D(1) = 0.58779 + 0.1 \cdot 0.36327 = 0.62411$

$$\boxed{f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \frac{d}{dx} f(x)}$$

1.	@rayleigh	GO KORISTI CONSTRUCT-OT OD @multipath.construct
2.	@multipath	TUKA GO KREIRA channel.rayleigh
3.	@rayleighfading	OVOJ GO KORISTI T.E POVIKUVA @intfiltgaussian.construct
4.	@intfiltgaussian	TUKA SE FUNKCIITE: generateblock i setrates
5.	@filtgaussian	VO INITIALIZE.m GO SETIRA DOPLEROVIOT FILTER SO POVIK NA jakes.m GO POVIKUVA @basefiltgaussian.basefiltgaussian_reset
6.	@sigstatistics	ISTO POSEBNO GO POVIKUVA SIG RESPONSE
7.	@sigresponse	ONDE GI SETIRA SIG.VALUES I SIG.DOMAIN, NE E VAZNO!!
8 a.	@basefiltgaussian	TUKA SE NAOGJA basefiltgaussian_reset STO GO SETIRA VO channel.filtgaussian State GAUSOVIOT ANSAMBL OD ...
8 b.	@interpfilter	GO KREIRA INTERPOLACISKIOT FILTER OBJEKT. VAZNI SVOJSTVA SE: POLYPHASE INTERP FACTOR, LINEAR INTERP FACTOR, FILTER LAST OUTPUTS (intfiltgaussian interpfact)
9.	@channelfilter	TUKA GO PRESMETUVA „α" I DOLZINATA NA TDL
10.	@slidebuffer	GO POVIKUVA @buffer\construct.m
11.	@buffer	

@channelfilter.initialize $\boxed{\text{computetapindices}}$

$erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-x^2} dx$

err = 0.1

$c = \frac{1}{\pi \cdot err} = 3.1831$

$erfc = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-t^2} dt = \frac{2}{\sqrt{\pi}} \left[\int_0^\infty e^{-t^2} dt - \int_0^x e^{-t^2} dx \right] = 1 - erf(x)$

$\int_0^\infty e^{-t^2} = \frac{\sqrt{\pi}}{2}$

$tRatio = \frac{pd.PathDelays}{pd.InputSamplePeriod} \quad \frac{[0\ 2e{-5}\ 4e{-5}]}{1e{-5}} = [0\ 2\ 4]$

$\tilde{g}_n(\tau) = \sum_{k=1}^{K} \tilde{a}_k(t) \cdot sinc\left(\frac{\tau_k}{T} - n\right) \quad N_1 \leq n \leq N_2$

$N_1, N_2 = ?$ $\quad$ tapidx = $N_1 : N_2$

$\boxed{\text{tapidx} = \min(\text{floor}(\min(tRatio) - c), 0) : \text{ceil}(\max(tRatio)+c);}$

tapidx = [-4:8]

$\underbrace{\begin{bmatrix} 0 \\ 2 \\ 4 \end{bmatrix}}_{x1} \quad \underbrace{[-4\ -3\ -2\ \ldots\ 6\ 7\ 8]}_{x2}$

rowcol Subtract

y = repmat(x1, size(x2)) - repmat(x2, size(x1))

$$y = \begin{bmatrix} 0 & 0 & -- & 0 \\ 2 & 2 & -- & 2 \\ 4 & 4 & -- & 4 \end{bmatrix} - \begin{bmatrix} -4 & -3 & -2 & \ldots & 6 & 7 & 8 \\ -4 & -3 & -2 & \ldots & 6 & 7 & 8 \\ -4 & -3 & -2 & \ldots & 6 & 7 & 8 \end{bmatrix}$$

A = sinc(y) - PRECOMPUTE α MATRIX

maxA = [3e-17, 3e-17, 3e-17, 3e-17, 1, 3e-17, 1, 3e-17, 1, 3e-17, 3e-17, 3e-17, 3e-17]

err2 = 0.01

significantA = maxA > err2 * $\overbrace{\max(\max(A))}^{1}$

maxA > 0.01

= [0 0 0 0 1 0 1 0 1 0 0 0 0]

cumsum(sigA) = [0 0 0 0 1 1 2 2 3 3 3 3 3]

cumsum(sigA)==1 ans = [0 0 0 0 1 1 0 0 0 0 0 0 0]

b = find(ans, 1) = 5 $\quad$ t_1 = min(tapidx(b), 0) = min[0, 0] = 0

$x = fliplr(sigA) = [0 0 0 0 1 0 1 0 1 0 0 0 0]$
$x_1 = cumsum(x) = [0 0 0 0 1 1 2 2 3 3 3 3 3]$
$x_2 = fliplr(x_1) = [3 3 3 3 3 2 2 1 1 0 0 0 0]$
$x_2 == 1 \quad x_3 = [0 0 0 0 0 0 0 1 1 0 0 0 0]$

$$\begin{array}{r} 107 \\ -54 \\ \hline 43 \end{array}$$

find$(x_3, 1, last) = 9 \quad t_2 = tapidx(9) = 4$
tapidx $= t_1 : t_2 = 0\ 1\ 2\ 3\ 4$

h.tapIndicesSmooth $= (t_1 - 3 : $ h.TapIndicesSmoothStep $: t_2 + 3)$ 0.1

$25/2 = 12.5 \quad floor(12.5) = 12 \quad -12 \div 0 \div 12$

$$\begin{array}{r} 12 \\ 69 \\ \hline 23 \end{array} \quad 23/2 = 11.5 = 11$$

$$\frac{218\,9\cancel{0}}{10\cancel{0}} = 218.09$$

- h.TapIndices $= [0\ 1\ 2\ 3\ 4]$

TG = h.tapGains
TG.Domain = tapidx * Ts = [0 1e-5 2e-5 3e-5 4e-5]
TG.Values = zeros(size(tapidx));
ntaps = length(tapidx)

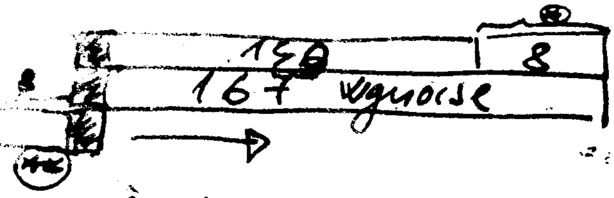
h.AlphaMatrix = sinc(Y)

$$Y = \begin{bmatrix} 0 & 0 & \vdots & 0 \\ 2 & 2 & \vdots & 2 \\ 4 & 4 & \vdots & 4 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 & 4 \end{bmatrix} = \begin{bmatrix} 0 & -1 & -2 & -3 & -4 \\ 2 & 1 & 0 & -1 & -2 \\ 4 & 3 & 2 & 1 & 0 \end{bmatrix}$$

h.AlphaMatrixSmooth = sinc(rowcolsubstract(tau/Ts, h.TapIndicesSmooth))

@multipath\initialize.m → reset.m → @intfiltgaussian\reset

(*) → РЕСЕТИРАЈ ГО channel.rayleighfading ОБЈЕКТОТ
→ @filtgaussian\reset.m → basefiltgaussian_reset.m
Nonof LastOutputs МИ СЕ ~~РАЗЛИКУВАЛО~~ ОД ПРЕТХОДНИТЕ!?
→ reset(h.Statistics(i)) $i = 1 : length(h.CutoffFreq)$
СЕГА ГО СЕТИРА h.SigStatistics ОБЈЕКТОТ



AKO GO STAVIS <1 OVERSAMLING

Lh=160

$$\boxed{\Delta f = \frac{1}{N\,dt}}$$

length (yrow) = **326**

4+4-1 = 7

```
    1 2 3 4
4 3 2 1
  4 3 2 1 4
    4 3 2 1
      4 3 2 1   } 4
      ....      } 3
```

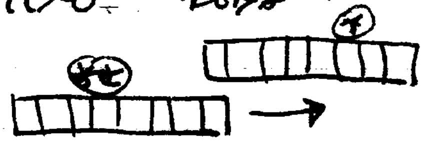
327
160

167

y(n,:) = yrow (Lh : end - Lh + 1) = yrow(160 : 326 - 160 + 1)

= yrow(160 : 167)

PLANIRANO TREBA



NE MOZE VAKA! TOA SI DADO KAKO DA PRAVIS CIRCONV

167 - 159 = 8   wgnoise (:, end-Ls+1 : end) = wgnoise(8:167)

. f. FilterInputState - NE ZNAM KADE GO SETIRA

3x8

@ mpfregaxes\newchannel   FREKVENTEN ODZIV NA RAYLEIGH KANAL!!!

$$f_{max} = \frac{1}{2 \cdot T_s} = \frac{f_{SAMP}}{2} = \frac{1}{2 \cdot 10^{-5}} = 0.5 \cdot 10^{+5} = 5 \cdot 10^4 = 50\,kHz$$

tauMax = max(tau) = max([0, 2e-5, 4e-5]) = 4e-5 = 0.4 µs

if (tauMax == 0)
    fs = fmax/10 ;   # SE PRESMETUVA OD fSAMP
else   # OVA E VSUSNOST df

$$f_s = \min\left(\underbrace{\frac{0.1}{tauMax}}_{2500},\ \underbrace{\frac{f_{max}}{100}}_{500}\right) = 500$$

$$\boxed{f = -f_{max} + f_s : f_s : f_{max}} = (-49.5 : 0.5 : 50.0)\ kHz$$

$$H = z * e^{-j 2 \cdot \pi \cdot \tau' * f} = z * \exp(-j * 2 * \pi * tau.' * f)$$

• DTFT $X[jk] = \sum_{n=-\infty}^{\infty} x(n) \cdot e^{-j\frac{2\pi}{} \cdot \frac{n}{N} k} = \sum_{n=-\infty}^{\infty} x(n) e^{-j\frac{2\pi}{N} n \cdot k}$

$X(jk) = \sum_{n=-\infty}^{\infty} x(n) W^{nk}$ $\quad | W = e^{-j\frac{2\pi}{N}} | \quad k = -\infty : \infty$

• DFT $X(jk) = \sum_{n=-N}^{N} x(n) e^{-j\frac{2\pi}{N} nk}$ $\quad k = -N : N$

$X(k) = \sum_{n=-N}^{N} x(n) W^{nk}$ $\quad k = -N : N$

$k = 1$

$X(1) = x(-N) \cdot W^{-N} + \dots x(0) + \dots X(N) \cdot W^{N-1} + x(N) \cdot W^{N}$

$X(2) = x(-N) W^{-2N} + \dots X(0) + \dots x(N-1) W^{2(N-1)} + x(n) \cdot W^{2N}$

$X(N) = x(-N) W^{-N^2} + \dots x(0) + \dots x(N-1) W^{N(N-1)} + x(N) W^{N^2}$

EXAMPLE: (RECALL)

$N = 4$ $\quad x = [1, 2, 3, 4]$ $\quad$ ~~n=0:N-1~~ $n = 1 : N$; ~~k=0:N-1~~ $k = 1 : N$

$X(k) = \sum_{n=\cancel{1}}^{N} x(n) W^{nk} = \sum_{n=0}^{N-1} [1, 2, 3, 4] W^{nk}$

$\cancel{X(1) = 1 \cdot W^{0} + 2 \cdot W^{1} + \dots 4 W^{3}}$

$\cancel{X(2) = 1 \cdot W^{0} + 2 W^{2} + \dots 4 W}$

$\boxed{\text{SCRATCHE: } n = 0 : N-1,\ k = 0 : N-1}$

~~X(0)=~~

$X(1) = 1 \cdot W + 2 \cdot W^2 + 3 \cdot W^3 + 4 \cdot W^4$

$X(2) = 1 \cdot W^2 + 2 \cdot W^4 + 3 \cdot W^6 + 4 \cdot W^8$

$X(4) = 1 \cdot W^4 + 2 \cdot W^8 + 3 \cdot W^{12} + 4 \cdot W^{16}$

$X = x * \exp\left(-j \frac{2\pi}{N} * k \cdot n\right)$

$X(k) = [1\ 2\ 3\ 4] \exp\left(-j \frac{2\pi}{N} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} [1\ 2\ 3\ 4]\right)$

$X(1) = 1 \cdot \exp\left(-j \frac{2\pi}{N}\right) + 2 \cdot \exp\left(-j \frac{2\pi}{N} \cdot 2\right) + \dots + 4 \exp\left(-j \frac{2\pi}{N} \cdot 4\right)$

$X(2) = \dots$

Proakis Definition: $X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N} nk}$

$X(k) = \sum_{n=0}^{N-1} x(n) W_N^{nk}$ $\quad W_N = e^{-j\frac{2\pi}{N}}$ $\quad k = 0 : N-1$

$f = -f_{max} + f_s : f_s : f_{max} = -49{,}5k : 0.5k : 50k$

$H = z * \underbrace{\exp(-j * 2 * \pi * tau.' * f)}_{W_N^{nk}}$

$size(z) = 500 \times 3 \qquad size(f) = 1 \times 200$

$W_N^{nk} = \exp\left(-j * 2 * \pi * \begin{bmatrix} 0 \\ 2e-5 \\ 4e-5 \end{bmatrix} * [-49{,}5k : 0.5k : 50k]\right)$

$\boxed{\begin{matrix} k = 200 \\ n = 3 \end{matrix}}$

$size(W_N^{nk}) = 3 \times 200$

$$H = z * W_N^{nk} = \begin{bmatrix} z_{11} & z_{12} & z_{13} \\ z_{21} & z_{22} & z_{23} \\ z_{500,1} & z_{500,2} & z_{500,3} \end{bmatrix} \begin{bmatrix} W_N^{11} & W_N^{12} & \dots & W_N^{1\,200} \\ W_N^{21} & W_N^{22} & \dots & W_N^{2\,200} \\ W_N^{31} & W_N^{32} & & W_N^{3\,200} \end{bmatrix}$$

$size(H) = 500 \times 200$

$$x(k) = \sum_{n=0}^{N-1} x(n) \exp(j * 2 * \pi * k * n / N)$$

$\boxed{W_N^{nk}}$

ZNAČI ZA SEKOE $z(i,:)$ SE PRESMETUVA DFT, ŠTO E LOGIČNO. IMPULSNIOT ODZIV A SO TOA I FREKVENTNIOT ODZIV KAJ REJLIEV FEDING SE MENUVAAT SO TEK NA VREME.

- NE MI E JASNO ZOŠTO PRAVI DFT OD PATH GAIN A NE OD TAPGAINS.

- $x(t) = R\{x_b(t) e^{j2\pi f_c t}\}$, x_b – baseband

$r(t) = \sum_n a_n(t) \cdot x(t - \tau_n(t))$

$r(t) = R\{\sum_n a_n(t) \cdot x_b(t - \tau_n(t)) e^{j2\pi f_c t} \cdot e^{-j2\pi f_c \tau_n(t)}\}$

$r_b(t) = \sum_n a_n(t) \cdot x_b(t - \tau_n(t)) e^{-j2\pi f_c \tau_n(t)}$

$r_b(t) = \sum_n a_n(t) \cdot e^{-j\theta_n(t)} x_b(t - \tau_n(t))$ $\quad \theta_n(t)$ – PHASE OF THE n-TH PATH

IMPULSE RESPONSE: $h_b = \sum_n a_n(t) e^{-j\theta_n(t)}$

$500 \cdot 10^{-5} = 5 \cdot 10^{-3}$ sec   FRAMES * T_s

Az: -8   El: 46

~~10 step~~ $a_n = a_1 + (n-1)d$

$a_1 = 1$ $a_n = 1 + 10 \cdot 49{,}9 =$

~~a_n~~ $= 500$ $d = 49{,}9$ $= 1 + 499 = 500$

$\boxed{n = 10}$ $a_n - a_1 = (n-1)d$ ⇒ $d = \dfrac{a_n - a_1}{n-1}$

$d = \dfrac{500 - 1}{10 - 1}$

$\left.\begin{matrix} 0.98, & 0.97, & 0.97 \\ 0.35, & 0.2, & 0.33 \end{matrix}\right\}$ LLA

$\left.\begin{matrix} 0.96, & 0.98, & 0.99 \\ 0.15, & 0.23, & 0.37 \end{matrix}\right\}$ SLA

→ BANDLIMITED
→ ∞ BW

z - PATH GAINS
g - TXT GAINS

- TOTAL ENERGY OF COMPONENTS

(A) E1 = 10*log10(sum(abs(z).^2, 2)   500×1

- ENERGY OF BANDLIMITED IMPULSE RESPONSE

(B) E2 = 10*log10(abs(g).^2,2)   500×1

- NARROWBAND ENERGY (ENERGY OF PHASOR SUM)

(C) E3 = 10*log10(abs(sum(z,2))).^2   500×1

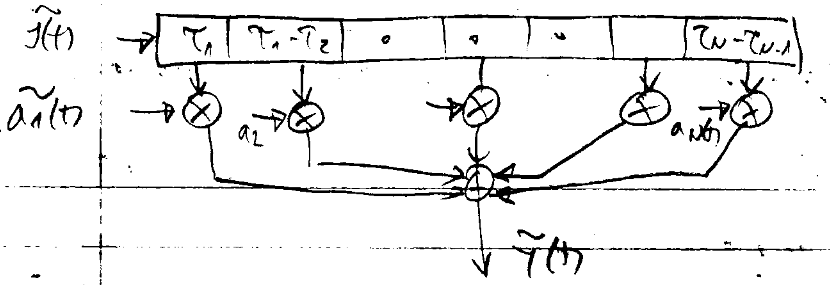
E = [E1, E2, E3] $\underline{\underline{500 \times 3}}$

$z_1 = a_1 + j b_1$; $z_2 = a_2 + j b_2$; $z_3 = a_3 + j b_3$

(A) $|z_1|^2 + |z_2|^2 + |z_3|^2 = a_1^2 + b_1^2 + a_2^2 + b_2^2 + c_1^2 + c_2^2$

(C) $|z_1 + z_2 + z_3|^2 = \left| (a_1 + a_2 + a_3) + j(b_1 + b_2 + b_3) \right|^2 =$

$= (a_1 + a_2 + a_3)^2 + (b_1 + b_2 + b_3)^2$



$$\tilde{c}(\tau,t) = \sum_{k=1}^{K} \tilde{a}_k(t)\,\delta(\tau-\tau_k)$$

$$\tilde{y}(t) = \sum_{k=1}^{K} \tilde{a}_k(t)\,\tilde{s}(t-\tau_k)$$

LOWPASS EQUIVALENT OUTPUT

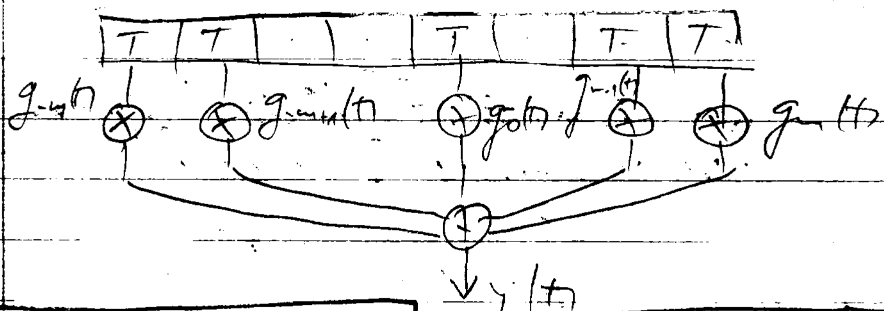
$$\underline{\underline{\tilde{g}_n(t)}} = \int_{-\infty}^{\infty} \tilde{c}(\tau,t)\,\mathrm{sinc}(B(\tau-nT))\,d\tau = \int_{-\infty}^{\infty}\left[\sum_{k=1}^{K} \tilde{a}_k(t)\,\delta(\tau-\tau_k)\right]\mathrm{sinc}\ldots$$

$$= \sum_{k=1}^{K} \tilde{a}_k(t) \int_{-\infty}^{\infty} \delta(\tau-\tau_k)\,\mathrm{sinc}(B(\tau-nT))\,d\tau =$$

$$= \sum_{k=1}^{K} \underline{\underline{\tilde{a}_k(t)\,\mathrm{sinc}(B(\tau_k-nT))}} = \sum_{k=1}^{K} \tilde{a}_k(t)\,\alpha(k,n) \qquad -N \le n \le N$$

$$\alpha(k,n) = \mathrm{sinc}\left(\frac{1}{T}(\tau_k - nT)\right) = \mathrm{sinc}\left(\frac{\tau_k}{T} - n\right)$$

- WITH BANDLIMITING OF THE CHANNEL WE GET UNIFORMLY SPACED TAPPED DELAY LINE.



MMV

ZNAČI a_k - PATH GAINS GO DEFINIRAAT ODZIVOT NA KANALOT BEZ PROPUŠTANJE NA ISTOT NIZ NF FILTER. AKO KANALOT ~~SE~~ PROPUŠTI NIZ NF FILTER SE DOBIVAAT g_n - TAP GAINS.

filtergaussian.PowerSpectrum → Domain
→ Values
channel.mydopplerrates.NewChannelData

START	END
262208	312581791
	312319584

• PRESMETKA NA AVTOKORELACIJATA I SPEKTAROT NA SNAGA: $t_{max} = \frac{50}{2\pi f_d}$; $t = t_{min} : t_{max}$ → ZACUVA NA MNOGU KONZERVATIVNA VARIJANTA OD COH. TIME. length(t) = 160 (EXAMPLE)

$f_s = \frac{1}{dt}$ $dt = t(2) - t(1)$ $t_{max} = t(end)$

$f = linspace(-fs/2, fs/2, Nf)$

$dt = 0.001$ (EXAMPLE) $f_s = \frac{1}{dt} = 10^3$

$\boxed{dt = \frac{1}{N_i f_d}}$ N_i - OVERSAMPLING

$\boxed{f = -500 : df : 500}$ $df = ?$ $Nf = 1024$

$f_2 = f_1 + df$ $f_N = f_1 + (Nf-1)df$

$df = \frac{f_N - f_1}{Nf-1} = \frac{500+500}{1023} = \frac{1000}{1023}$

MAV

$S_j =$ fftshift(abs(dt*fft(h, Nf)).^2);

$$\boxed{S_j(k) = \left| dt \cdot \sum_{n=0}^{N-1} h(n) \, W_{N_f}^{nk} \right|^2}$$

POWER SPECTRUM

$h \equiv \tilde{c}(\tau, t)$

WSSUS MODEL: $\boxed{R_{\tilde{c}}(\tau, \Delta t) \equiv E[\tilde{c}^*(\tau, t)\tilde{c}(\tau, t+\Delta t)]}$

AUTOCORRELATION FUNCTION

$$\boxed{S(\tau, \nu) = F_{\Delta t}[R_{\tilde{c}}(\tau, \Delta t)] = \int_{-\infty}^{\infty} R_{\tilde{c}}(\tau, \Delta t) e^{-j2\pi\nu\Delta t} d\Delta t}$$

- AUTOCORRELATION FUNCTION

h2 = ifft(fftshift(Sj))./(dt^2)

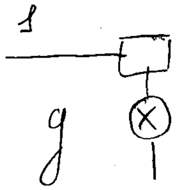
ac = real(h_2(1:Ltdiff))

tSmooth = -3 : 7 step = 0.1

ii = (tSmooth > t(end) | tSmooth < t(1)) & tSmooth - floor(tSmooth) < eps

$T_c = \frac{9}{16\pi f_d} = \frac{9}{16 \cdot \pi \cdot 100} = 0.0018 = \underline{1{,}8 \text{ msec}}$

- OD IRWatterfall NAVISTINA SE GLEDA DEKA VO T_c (COHERENCE TIME) IMPULSNIOT ODZIV NEZNAČITELNO SE MENUVA



$$y(n) = \sum_{k=1}^{K} g(k) \cdot s(k-n)$$

$\boxed{3290053}$

$$(1+j)(2+j\cdot 2) = \cancel{2} + j2 + 2j + \cancel{j2} = 4j$$

$$f_d = \frac{1}{2 \cdot f_{max}} = \frac{T_{sym}}{2}$$

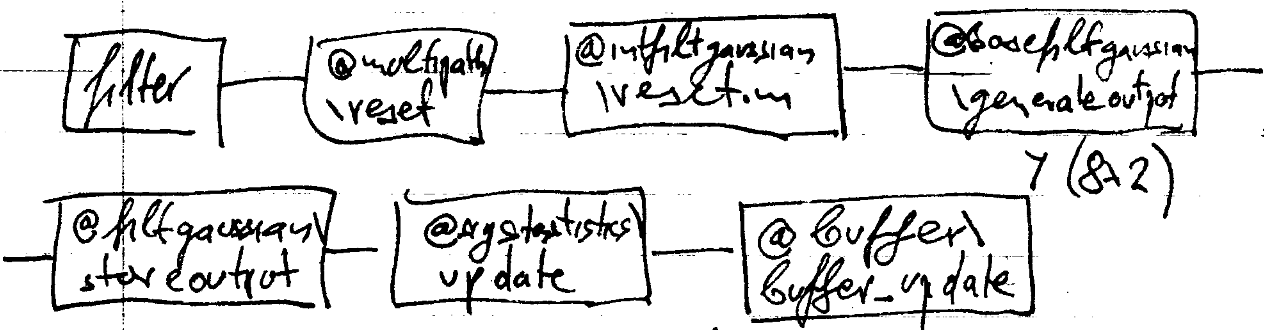
• CREATE FAST FADING CHANNEL

$T_s = 10^{-4} = 0,1$ m sec

$$T_c = \sqrt{\frac{9}{16\pi f_d^2}} =$$

$$\frac{1}{10 \cdot T_s} = \frac{10^3}{10} = 100$$

• DOPPLER SPECTRUM MEASUREMENT



2000-8 = 1992   1992/2 = 996

18
 19
 18
  12

397
199
198

wgn(=) 167   dpir (=) 160   fd=50
wgn(=) 199   dpir (=) 197   fd=100

199 : 397-199+1 = 199 : 198

[0.85, 0.16, 0]   RED
1, 0.6, 078   MAG
0.48, 0.06, 0.89   BLUE
0.08 0.17 0.55   D.BLUE
0.17 0.51 0.24   D.GREEN

$APG = [APG1 \quad APG2 \quad APG3]$ GOLEMA BUKVA (=) dB

$$apg1 = 10^{0.05\,APG1} \qquad APG1 = 20 \cdot \log(apg1)$$

$$APG\_AVG = \frac{APG1 + APG2 + APG3}{3} = \frac{20\log(apg1 \cdot apg2 \cdot apg3)}{3}$$

$$apg\_AVG = 10^{0.05 \cdot \frac{APG1 + APG2 + APG3}{3}}$$

$$APG = [0 \quad 0 \quad 0]$$
$$apg = [1 \quad 1 \quad 1]$$

NORMALIZE : $apg = [1 \quad 1 \quad 1] / sum([1 \quad 1 \quad 1])$

POEN~~TATA~~ E VKUPNATA SNAGA

VKUP~~NATA~~ SNAGA E :

$$\frac{apg.^2}{sum(apg^2)} = \frac{[1^2 + 1^2 + 1^2]}{sum(apg.^2)} = app\_n$$ AVERAGE PATH POWER NORMALIZED

$$\boxed{app\_n = \frac{apg.^2}{sum(apg.^2)}}$$ APATH POWER NORM.

AV. PATH GAIN NORM.

$$apg\_n = \sqrt{app\_n}$$

$$\boxed{apg\_n = \frac{apg}{\sqrt{sum(apg.^2)}}}$$

0.70669
0.5003
0.5003

MMV

[MICHAEL JERUCHIM (POVTORUVANJE)]

$$\tilde{h}(\tau, t) = s(t) \times \tilde{c}(\tau, t)$$

$s(t)$ – SHADOW FADING

$\tilde{c}(\tau, t)$ – MULTIPATH COMPONENT

$\tilde{c}(\tau, t)$ – RESPONSE MEASURED AT TIME t TO THE UNIT IMPULSE APPLIED AT TIME $t - \tau$

$$\tilde{c}(\tau, t) = h(t, t-\tau) \qquad h(t, \tau) = c(t - \tau, t)$$

$S_r = S_t + G_t + G_r - L_p$

$L_p = \alpha + \beta \log_{10}(t)$ [dB]

$\beta = 44.9 + 6.55 \log_{10}(h)$

hata-okamura

$\alpha = f(f, h)$

$x(t) = r(t) \cdot \cos[2\pi f_c t + \phi(t)] = \mathrm{Re}\{\underbrace{r(t) \cdot e^{j\phi(t)}} \cdot e^{j 2\pi f_c t}\}$

$= \tilde{x}(t) \cdot \mathrm{Re}\{e^{j 2\pi f_c t}\}$ $\quad \boxed{\tilde{x}(t) = r(t) \cdot e^{j\phi(t)}}$

$\tilde{x}$ - complex lowpass equivalent i.e complex enverope of the signal.

- DISCRETE MULTIPATH CHANNEL

$\boxed{y(t) = \sum_n a_n(t)\, s(t - \tau_n(t))}$ $\quad s$ - BANDPASS ~~RESPONSE~~ ~~PHASE~~ INPUT SIGNAL

a_n - ATENUATION FACTOR FOR THE SIGNAL ON n-TH PATH

τ_n - CORESPONDING PROPAGATION DELAY

ZAMJENA

$s(t) = \mathrm{Re}\{\tilde{s}(t) \cdot e^{j 2\pi f_c t}\}$

$y(t) = \mathrm{Re}\{[\sum_n a_n(t)\, e^{-j 2\pi f_c \tau_n(t)} \tilde{s}(t - \tau_n(t))] e^{j 2\pi f_c t}\}$

COMPLEX ENVELOPE OF THE OUTPUT

$$\boxed{\tilde{y}(t) = \sum_n \underbrace{a_n(t)\, e^{-j 2\pi f_c \tau_n t}}_{\tilde{a}_n(\tau_n, t)} \tilde{s}(t - \tau_n(t)) = \sum_n \tilde{a}_n(\tau_n, t)\, \tilde{s}(t - \tau_n(t))}$$

$$\boxed{\tilde{c}(\tau_n(t), t) = \sum_n \tilde{a}_n(\tau_n, t)\, \delta(\tau - \tau_n(t))}$$

- DIFUZE MULTIPATH CHANNEL

$$\tilde{y}(t) = \int_{-\infty}^{\infty} \tilde{a}(\tau, t)\, \tilde{s}(t - \tau)\, d\tau$$

$$\boxed{\tilde{c}(\tau, t) = \int_{-\infty}^{\infty} \tilde{a}(\tau, t) \cdot \delta(t - \tau)\, d\tau = \tilde{a}(\tau, t) = a(t) \cdot e^{-j 2\pi f_c \tau}}$$

$f_d = \frac{v}{\lambda} = \frac{60\ m/s}{0{,}33\ m} = \underline{\underline{180\ Hz}}$ $\quad f_d = 1\ Hz = \frac{v}{0.33}$

$v = 0.33\ \frac{m}{s} = 0.33 \cdot \frac{3600}{1000} = 0.33 \cdot 3.6 = 1.2\ km/h$

• STATISTICAL CHARACTERIZATION OF TIME-VARIANT BEHAVIOUR

$$c(\tau_n,t) = \sum_n \tilde{a}_n(\tau_n,t)\,\delta(\tau-\tau_n(t))$$

$$\tilde{a}_n(\tau_n,t) = a_n(\tau)\cdot e^{-j2\pi f_c\tau_n}$$

$$y(t) = \sum_n a_n(t)\cdot s(t-\tau_n(t))$$

$s(t) = Re[\tilde{s}(t)\cdot e^{j2\pi f_c t}]$

$$s(t-\tau_n(t)) = Re\left[\tilde{s}(t-\tau_n t)\, e^{-j2\pi f_c\tau_n}\cdot e^{-j2\pi f_c t}\right]$$

$$y(t) = Re\left[\sum_n \underbrace{a_n(t)\,\tilde{s}(t-\tau_n(t))\,e^{-j2\pi f_c\tau_n}}_{\tilde{a}_n(\tau_n,t)}\, e^{-j2\pi f_c t}\right]$$

$$y(t) = Re\left[\underbrace{\sum_n \tilde{a}_n(\tau_n,t)\,\tilde{s}(t-\tau_n(t))}_{\tilde{y}(t)}\, e^{-j2\pi f_c t}\right]$$

$$\tilde{y}(t) = \sum_n \tilde{a}_n(\tau_n,t)\,\tilde{s}(t-\tau_n(t))$$ ⇒ OUTPUT SIGNAL COMPLEX ENVELOPE

$$c(\tau_n,t) = \sum_n \tilde{a}_n(\tau_n,t)\,\delta(t-\tau_n(t))$$ ⇒ COMPLEX ENVELOPE OF THE IMPULSE RESPONSE !!!

MMV

$$q(t) = \int_{-\infty}^{\infty} \tilde{a}(t,\tau)\,\delta(t-\tau)\,d\tau = \int_{-\infty}^{\infty} a(\tau)\cdot e^{-j2\pi f_c\tau}\,\delta(t-\tau)\,d\tau$$

$$q(t) = a(t)\, e^{-j2\pi f_c t} \qquad c(t,\tau) = a(t)\,e^{-j2\pi f_c\tau}$$

$$y(t) = \int_{-\infty}^{\infty} \tilde{a}(t,\tau)\, s(t-\tau)\,d\tau$$

$$y(\tau) = \int_{-\infty}^{\infty} h(t)\,x(\tau-t)\,dt$$

$R(\tau,t) = |c(\tau,t)|$ → ENVELOPE OF COMPLEX GAUSSIAN IS RAYLEIGH PROCESS

$$f_R(r) = \frac{r}{\sigma^2} e^{-\frac{r^2}{2\sigma^2}} \quad \text{IF } Mean[c(\tau,t)] = \emptyset$$

070383529
SROE DAMJAN.

CVETA SPASEVSKA
PODEDELNIK

SEKTOR
ZA FIN

- IF $Mean[c(\tau,t)] \neq \emptyset$

$$f_R(r) = \frac{r}{\sigma^2} I_0\left[\frac{Ar}{\sigma^2}\right] e^{-\frac{r^2+A^2}{2\sigma^2}}$$ } RICEAN DISTRIBUTION

I_0 - ZERO ORDER MODIFIED ~~BESSEL~~ FUNCTION OF FIRST KIND

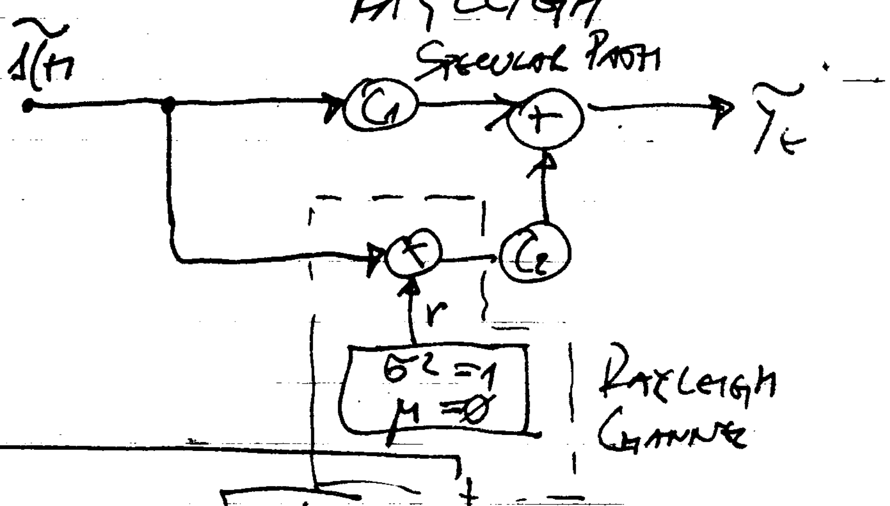
$A \to Mean[c(\tau,t)]$ $\quad K = \frac{A^2}{\sigma^2}$

$K \gg 1 \Rightarrow$ KANAZOT E TOPIRECTEN (SPECULAR)

$K \ll 1 \Rightarrow$ CHANEL TENDS TO BE RAYLEIGH

287 × 176
297 × 210
10 × 46
103,5% | 119,3%

298,5 209,4



$$C_1 = \sqrt{\frac{K}{1+K}} \qquad C_2 = \sqrt{\frac{1}{1+K}}$$

MMV

OD RAYLEIG KANAL SO OVIE KOEFICIENTI PREODAS VO RICIAN.

$$\tilde{y}(t) = \tilde{s}(t) \cdot C_1 + \tilde{s}(t)\, r \cdot C_2 = \tilde{s}(t)(C_1 + C_2 \cdot r)$$

$$r = [rand(N,1) + j\, rand(N,1)] / \sqrt{2}$$

$$\tilde{Y}(t) = \tilde{s}(t)\left[\sqrt{\frac{K}{1+K}} + \sqrt{\frac{1}{1+K}} \cdot \frac{1}{\sqrt{2}} \cdot r\right]$$

$$= \tilde{s}(t)\left[\frac{1}{2}\sqrt{\frac{K}{1+K}} + \frac{1}{2}\sqrt{\frac{K}{1+K}} + \frac{1}{\sqrt{2}}\frac{1}{\sqrt{1+K}} Re(r) + j\frac{1}{\sqrt{2}}\frac{1}{\sqrt{1+K}} I_m(r)\right]$$

$$= \tilde{s}(t)\left[s \cdot Re(r) + mu + j(s \cdot I_m(r) + mu)\right] \quad ??? \quad s = \frac{1}{\sqrt{2(1+K)}} \quad mu = ?$$

• The WSSUS Model

– Autocorelation function

$$R_{\tilde{c}}(\tau_1,\tau_2,\Delta t) = E\left[\tilde{c}^*(\tau_1,t)\,\tilde{c}(\tau_2,t+\Delta t)\right]$$

$$\overline{f(t)\,f(t+\tau)} = \int_{-\infty}^{\infty} f(t)\,f(t+\tau)\,dt = \lim_{T\to\infty}\int_{-T} f(t)\,f(t+\tau)\,dt =$$

$$= R_{ff}(\tau) = R_f(\tau)$$

$$\overline{\xi_1 \xi_2} = \iint x_1 x_2\, p_{\xi_1\xi_2}(x_1,x_2,\tau)\,dx_1\,dx_2 = R_{\xi\xi}(\tau) = R_\xi(\tau)$$

ERGODIČEN: $R_f(\tau) = R_\xi(\tau)$

$$\lim_{T\to\infty}\frac{1}{T}\int_{-T}^{T} f(t)\,dt = \underbrace{\int_{-\infty}^{\infty} x\,p_\xi(x)\,dx}_{\overline{f(t)}} \qquad \overline{f(t)} = \overline{f(t)}$$

VO ŠIROK SMISOL: $R_\xi(\tau) = R_f(\tau) \neq f(t)$ → FUNKCIJA OD VREME

$$R_{\tilde{c}}(\tau,\Delta t) \equiv E\left[\tilde{c}^*(\tau,t)\,c(\tau,t+\Delta t)\right]$$

– Uncorrelated scattering:

$$R_{\tilde{c}}(\tau_1,\tau_2,\Delta t) = R_{\tilde{c}}(\tau_1,\Delta t)\,\delta(t_1-t_2)$$

$$S(\tau,\nu) = F_{\Delta t}\left[R_{\tilde{c}}(\tau,\Delta t)\right] = \int_{-\infty}^{\infty} R_{\tilde{c}}(\tau,\Delta t)\,e^{-j2\pi\nu\Delta t}\,d\Delta t$$

$$R_{11}(\tau) = \int_{-\infty}^{\infty} f(t)\cdot f(t+\tau)\,dt = \int_{-\infty}^{\infty}\left[\frac{1}{2\pi}\int_{-\infty}^{\infty} F(j\omega)\,e^{j\omega t}\,d\omega\cdot\right.$$

$$\left.\cdot\frac{1}{2\pi}\int_{-\infty}^{\infty} F(j\omega)\cdot e^{j\omega\tau}\,e^{j\omega t}\,d\omega\right]dt$$

$$= \int_{-\infty}^{\infty} f(t)\left[\frac{1}{2\pi}\int_{-\infty}^{\infty} F(j\omega)\,e^{j\omega\tau}\,e^{j\omega t}\,d\omega\right]dt$$

$$= \int_{-\infty}^{\infty} \frac{1}{2\pi} F(j\omega)\, e^{j\omega\tau} \Big[\underbrace{\int_{-\infty}^{\infty} f(t)\, e^{j\omega t}\, dt}_{F^*(j\omega)} \Big] d\omega$$

$$R_{11}(\tau) = \int_{-\infty}^{\infty} f(t) f(t+\tau)\, dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |F(j\omega)|^2 e^{j\omega\tau}\, d\omega$$

FUR. TRANSF. PAR.

$$\boxed{R_{11}(0) = \frac{1}{2\pi} \int_{-\infty}^{\infty} |F(j\omega)|^2\, d\omega}$$

$$f_T(t) = \begin{cases} f(t) & |t| \le T \\ 0 & \text{otherwise} \end{cases}$$

$$F_T(j\omega) = \int_{-\infty}^{\infty} f_T(t) \cdot e^{-j\omega t}\, dt = \int_{-T}^{T} f_T(t)\, e^{-j\omega t}\, dt$$

$$\int_{-\infty}^{\infty} f_T(t) f_T(t+\tau)\, dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |F_T^*(j\omega)|^2 e^{j\omega\tau}\, d\omega$$

$$\frac{1}{2T} \int_{-T}^{T} f_T(t) f_T(t+\tau)\, dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{|F_T(j\omega)|^2}{2T} e^{j\omega\tau}\, d\omega$$

$$R_T(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{|F_T(j\omega)|^2}{2T} e^{j\omega\tau}\, d\omega \qquad \boxed{\frac{1}{2\pi} \int_{-\infty}^{\infty} \phi(j\omega) e^{j\omega\tau}\, d\omega = R_c(\tau)}$$

$$\phi(j\omega) = \lim_{T \to \infty} \frac{|F_T(j\omega)|^2}{2T}$$

$$R_c(\tau) = R_f(\tau) = \lim_{T \to \infty} R_T(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \phi(j\omega) e^{j\omega\tau}$$

$$R_{\tilde{c}}(\tau, \Delta t) = E\left[\tilde{c}^*(\tau, t) \cdot \tilde{c}(\tau, t+\Delta t) \right]$$

$$R_{\tilde{c}}(\tau, 0) = E\left[\tilde{c}^*(\tau, t) \cdot \tilde{c}(\tau, t) \right] = E\left[|c(\tau, t)|^2 \right]$$

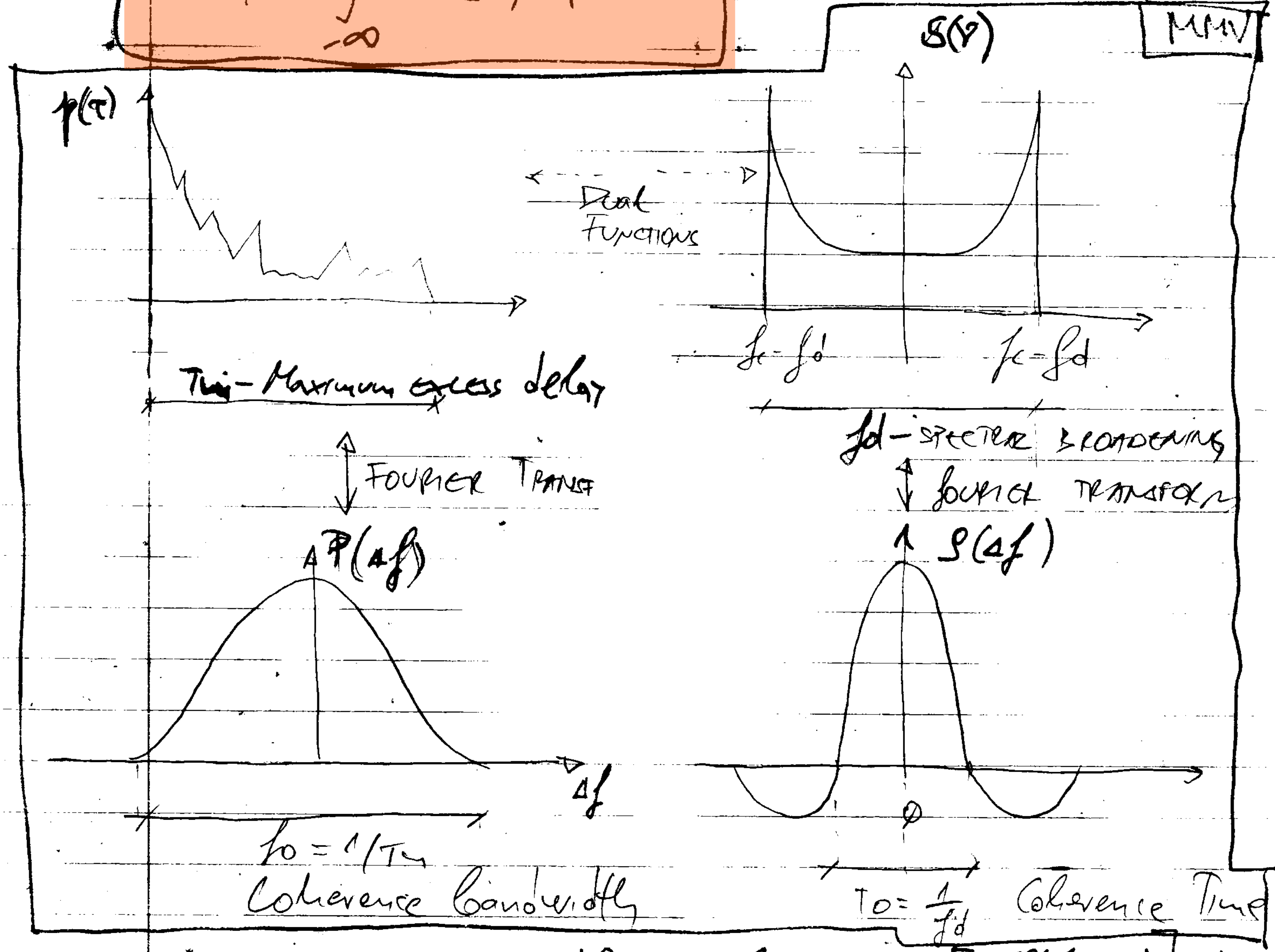
• delay power profile

$$p(\tau) = R_{\tilde c}(\tau, 0) = E\left[|\tilde c(\tau,t)|^2\right]$$

$$\boxed{p(\tau) = \int_{-\infty}^{\infty} S(\tau,\nu)\, d\nu}$$

• Doppler Power Spectrum

$$\boxed{S(\nu) = \int_{-\infty}^{\infty} S(\tau,\nu)\, d\tau}$$



$S = |dt \cdot fft(obj.dpw)|^2$ $\quad$ obj.ac = Real[ifft(obj.s)] $/dt^2$

$$S(\tau,\nu) = F_{\Delta t}\left[R_{\tilde c}(\tau,\Delta t)\right] = \int_{-\infty}^{\infty} R_{\tilde c}(\tau,\Delta t)\, e^{-j2\pi\nu\Delta t}\, d(\Delta t)$$

$$R_c(\tau,\Delta t) = E\left[\tilde c^*(\tau,t)\cdot \tilde c(\tau,t+\Delta t)\right]$$

$$R_c(\tau,0) = E\left[\tilde c^*(\tau,t)\cdot \tilde c(\tau,t+\cancel{\Delta t})\right] = \underline{E\left[|\tilde c(\tau,t)|^2\right]}$$

⊙ USTE ODMAS: STATISTICAL CHARACTERIZATION: THE WSSUS MODEL

• WSS

$$R_{\tilde{c}}(\tau_1,\tau_2,\Delta t)= E\left[\tilde{c}^*(\tau_1,t)\tilde{c}(\tau_2,t+\Delta t)\right]$$

US – UNCORRELATED SCATTERING

– NORMALIZIRAN IMPULSEN ODZIV ZA JAKES KANAL

$$\boxed{h_j(\tau)= x^{-1/4} J_{1/4}(x)} \qquad x = 2\pi f_d |t|$$

– AVTOKORELACIONATA FUNKCIJA E: (NORMALIZIRANATA)

$$\boxed{ac_j(\tau) = J_0(x)}$$

$$R_{\tilde{c}}(\tau_1,\tau_2,\Delta t)= R_{\tilde{c}}(\tau_1,\Delta t)\,\delta(\tau_1-\tau_2)$$

• WSSUS

$$R_{\tilde{c}}(\tau,\Delta t)\equiv E\left[\tilde{c}^*(\tau,t)\,\tilde{c}(\tau,t+\Delta t)\right]$$

$$S(\tau,\nu)= \mathcal{F}_{\Delta t}\left[R_{\tilde{c}}(\tau,\Delta t)\right]= \int_{-\infty}^{\infty} R_{\tilde{c}}(\tau,\Delta t)\, e^{-j2\pi\nu\Delta t}\, d\Delta t$$

NNV

• DELAY PROFILE

$$\boxed{\varphi(\tau) = R_{\tilde{c}}(\tau,0) = E\left[|\tilde{c}(\tau,t)|^2\right]}$$

GO SVAKAM KAKO USREDNUVANJE NA (PATH GAINS)^2 PO VREME

• $$S(\nu)= \int_{-\infty}^{\infty} S(\tau,\nu)\, d\tau$$

⊙ THE DELAY POWER PROFILE

$T_m > T_{SYM} \Rightarrow$ FREQ SELECTIVE FADING (ISI)

• ISI distortion can be mitigated by rake reception !!!

$$f_s \geq 2 f_{max} \qquad \frac{1}{T_s} = \frac{2}{T_{SYM}} \qquad \boxed{T_{SYM} = 2T_s}$$

• $T_{SYM} \gg T_m \Rightarrow$ FLAT FADING } COPE WITH POWER CONTROL & DIVERSITY

$T_{SYM} > 10\sigma_\tau$ σ_τ – RMS DELAY SPREAD

$$\tilde{r}(t) = \tilde{c}(t)\tilde{s}(t) \quad \text{– FLAT CHANNEL}$$

$$\tilde{r}(t) = \tilde{c}(t) * \tilde{s}(t) = \tilde{c}(\tau,t) * \tilde{s}(t)$$

KONVOLUTION IN τ DOMAIN FOR FREQ SEL. FADING

$$\sigma_\tau = \sqrt{\overline{\tau^2} - \bar{\tau}^2} \qquad \overline{\tau^k} = \frac{\int \tau^k \varphi(\tau)\, d\tau}{\int \varphi(\tau)\, d\tau}$$

$$-10\,dB = 10 \log P(\tau) \qquad P(\tau) = 10^{-1} = 0.1\,W$$

$$\boxed{\overline{\tau^k} = \frac{\sum_n \tau_n^k\, p(\tau_n)}{\sum_n p(\tau_n)}}$$

FLAT	FS
$T_m < T_s$	$T_m > T_s$
FAST	SLOW
$T_c < T_s$	$T_c > T_s$

$$\boxed{\sigma_\tau = \sqrt{\overline{\tau^2} - \bar{\tau}^2}}$$

$apg = 10^{0.1 APG}$ $\quad app = apg.^2 \quad app = 10^{0.1 APG}$

$APP = 10 \log(app) = 10 \log 10^{0.1 APG} = \underline{APG}$

⊙ SPACED-FREQUENCY CORRELATION FUNCTION

$f_0 \approx \frac{1}{T_m}$ $\quad$ f_0 - COHERENCE BANDWIDTH
T_m - MAXIMUM EXCESS TIME

- JAKES MODEL:

$\boxed{f_0 \doteq \frac{1}{5\sigma_\tau}}$ COHERENCE BANDWIDTH

$\boxed{S = |fft(apg.dyir.^2).dt|^2}$ MATLAB IMPLEMENTATION

⊙ TIME-VARYING CHANNEL

TIME VARIATION OF THE CHANNEL IS CHARACTERIZED BY DOPPLER POWER SPECTRUM

$$S(\nu) = \frac{1}{\pi f_d \sqrt{1-\left(\frac{\nu}{f_d}\right)^2}} \quad |\nu| \le f_d$$

• SPACED-TIME CORRELATION FUNCTION

$$\rho(\Delta t) = F^{-1}(S(\nu)) = J_0(2\pi f_d \Delta t)$$

• TIME-INVARIANT CHANNEL $\rho(\Delta t) = 1$

$\boxed{T_c = \frac{9}{\sqrt{4\pi f_d^2}}}$ $\quad$ $T_{sym} < T_c$ SLOW FADING
$T_{sym} > T_c$ FAST FADING

• DOPPLER POWER SPECTRUM $\quad \boxed{f_d = \frac{v}{\lambda}}$

$T_0 \sim \frac{1}{f_d}$ $\quad$ $T_0 = \frac{\lambda/2}{v} = \frac{0.5}{f_d}$

$100 \cdot 10^{-4} = 10^{-2}$ } 00 Power Of faded Signal $\quad$ $T_0 = \frac{0.5}{100} = 0.5 \cdot 10^{-2}$

⊙ STRUCTURAL MODELS FOR MULTIPATH FADING CHANNELS

• DIFFUSE MULTIPATH STRUCTURAL MODEL

$$c(\tau,t) = a(\tau,t)\, e^{-j2\pi f_c \tau}$$

$$\tilde{r}(t) = \int_{-\infty}^{\infty} \tilde{a}(\tau,t)\, \tilde{s}(t-\tau)\, d\tau \quad \ldots\ldots \; 9.1.8$$

$$\tilde{a}(\tau,t) \equiv \tilde{c}(\tau,t)$$

$$\tilde{r}(t) = \int_{-\infty}^{\infty} \tilde{c}(\tau,t)\cdot \tilde{s}(t-\tau)\, d\tau$$ KONVOLUCIJA VO τ DOMEN !!!

$$y(\tau) = \sum_{t=-\infty}^{\infty} h(\tau-t)\, x(t) \qquad -\infty \le \tau \le \infty$$

$$y(k) = \sum_{n=0}^{N-1} h(k-n)\, x(n)$$ $\underline{k = 0 \ldots N-1}$ KONVOLUCIJA VO k-DOMEIN

$$y(\tau) = \int_{-\infty}^{\infty} h(\tau-t)\, x(t)\, dt$$ KONVOLUCIJA VO t-DOMEN

$$s(t-\tau) = \sum_{n=-\infty}^{\infty} \tilde{s}(t-nT)\, \mathrm{sinc}\big(B(\tau-nT)\big)$$

$$\tilde{r}(t) = \int_{-\infty}^{\infty} \tilde{c}(\tau,t) \left[\sum_{n=-\infty}^{\infty} \tilde{s}(t-nT)\, \mathrm{sinc}\big(B(\tau-nT)\big) \right] d\tau$$

$$\tilde{r}(t) = \sum_{n=-\infty}^{\infty} \tilde{s}(t-nT) \underbrace{\int_{-\infty}^{\infty} \tilde{c}(\tau,t)\, \mathrm{sinc}\big(B(\tau-nT)\big)\, d\tau}_{g_n(t)}$$

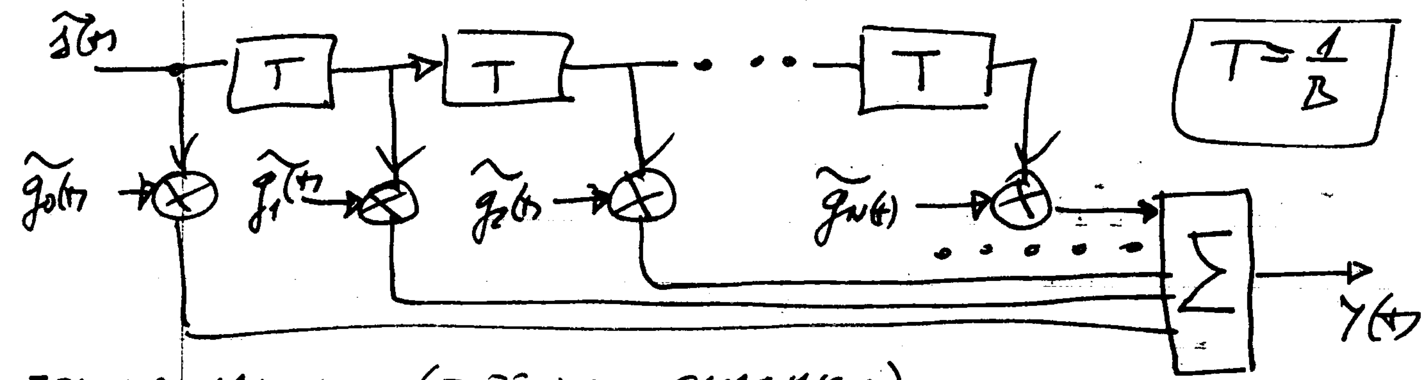
$$\tilde{r}(t) = \sum_{n=-\infty}^{\infty} \tilde{s}(t-nT)\, \tilde{g}_n(t)$$ MMV

$$\tilde{g}_n(t) = \int_{-\infty}^{\infty} \tilde{c}(\tau,t)\, \mathrm{sinc}\big(B(\tau-nT)\big)\, d\tau$$

$$\tilde{r}(t) = \sum_{n=0}^{N} \tilde{s}(t-nT)\, \tilde{g}_n(t)$$

• IF $\tilde{c}(\tau,t)$ IS SMOOTH OVER T THEN

$$g_n(t) \approx T\cdot \tilde{c}(nT,t) \qquad \boxed{T = \frac{1}{B}}$$



POVTORUVANJE: (DIFFUSE CHANNEL)

$$y(t) = \int_{-\infty}^{\infty} \tilde{a}(\tau,t)\cdot \tilde{s}(t-\tau)\,d\tau$$

$$\tilde{a}(\tau,t) = a(\tau,t)\cdot e^{-j2\pi f_c \tau} \equiv \tilde{c}(\tau,t)$$

$$y(t) = \int_{-\infty}^{\infty} \tilde{c}(\tau,t)\cdot \tilde{s}(t-\tau)\,d\tau$$

- NF filtered $\tilde{s}(t-\tau)$ AND SAMPLE AT $n\cdot T_s$

$$\tilde{s}(t-\tau) = \sum_{n=0}^{N-1} s(t-nT_s)\cdot \mathrm{sinc}(B(\tau-nT_s))$$

$$y(t) = \int_{-\infty}^{\infty} \tilde{c}(\tau,t)\cdot\left[\sum_{n=0}^{N-1} s(t-nT_s)\,\mathrm{sinc}(B(\tau-nT_s))\right]d\tau$$

$$y(t) = \sum_{n=0}^{N-1} s(t-nT_s) \underbrace{\int_{-\infty}^{\infty} \tilde{c}(\tau,t)\cdot \mathrm{sinc}(B(\tau-nT_s))\,d\tau}_{\tilde{g}_n(t)}$$

$$\boxed{y(t) = \sum_{n=0}^{N} \tilde{s}(t-nT_s)\,\tilde{g}_n(t)}$$

- STATISTICAL TAP GAIN MODELS

$$\tilde{g}_n(t) = \int_{-\infty}^{\infty} \tilde{c}(\tau,t)\cdot \mathrm{sinc}(B(\tau-nT_s))\,d\tau \qquad 9.1.32b$$

- TAP GAINS CROSS-COVARIANCES:

$$R_{kl}(\Delta t) = E\left[g_k(\tau,t)\, g_l^*(\tau,t+\Delta t)\right] =$$

$$= E\left[\int_{-\infty}^{\infty} \tilde{c}(\tau,t)\,\mathrm{sinc}(B(\tau-kT))\,d\tau \int_{-\infty}^{\infty} \tilde{c}^*(\mu,t+\Delta t)\,\mathrm{sinc}(B(\mu-lT))\,d\mu\right]$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E\left[c(\tau,t)\,\tilde{c}^*(\mu,t+\Delta t)\right] \mathrm{sinc}(B(\tau-kT))\,\mathrm{sinc}(B(\mu-lT))\,d\tau\, d\mu$$

$$R_{\tilde c}(\tau_1,\tau_2,\Delta t) = R_{\tilde c}(\tau_1,\Delta t)\,\delta(\tau_1-\tau_2)$$

VO OVOJ SLUČAJ:

$$R_{\tilde c}(\tau,\mu,\Delta t) = R_c(\tau,\Delta t)\,\delta(\tau-\mu)$$

$$R_{kl}(\Delta t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} R_{\tilde c}(\tau,\Delta t)\,\delta(\tau-\mu)\,\underline{\mathrm{sinc}(B(\tau-kT))}\,\mathrm{sinc}(B(\mu-lT))\,d\mu\,d\tau$$

$$= \int_{-\infty}^{\infty} R_{\tilde c}(\tau,\Delta t)\,\mathrm{sinc}(B(\tau-kT))\,\underbrace{\left(\int_{-\infty}^{\infty}\delta(\tau-\mu)\,\mathrm{sinc}(B(\mu-lT))\,d\mu\right)}_{\mathrm{sinc}(B(\tau-lT))}\,d\tau$$

$$\boxed{R_{kl}(\Delta t) = \int_{-\infty}^{\infty} R_c(\tau,\Delta t)\,\mathrm{sinc}(B(\tau-kT))\,\mathrm{sinc}(B(\tau-lT))\,d\tau}$$

$\underset{k\neq l}{R_{kl}(\Delta t)} = 0$ UNCORRELATED TAP GAINS

$$\boxed{R_{11}(\tau) = \int_{-\infty}^{\infty} f(t)\cdot f(t+\tau)\,dt}$$

OTK DEFINICIJA NA AK FUNKCIJA

$$\tilde y(t) = \sum_{n=0}^{N}\tilde s(t-nT)\,\tilde g_n(t) = T\sum_{n=0}^{N}\tilde s(t-nT)\,\tilde c(nT,t)$$

$$p(\tau) = R_c(\tau,0) = E|\tilde c(\tau,t)|^2$$

$$E[|\tilde g_n(t)|^2] = T^2 E[|\tilde c(\tau,t)|^2] = T^2 p(nT)$$

$$\tilde g_n(t) = g_{n,r}(t) + j\,g_{n,i}(t)$$

$$E[|\tilde g_n(t)|^2] = E[g_{n,r}^2 + g_{n,i}^2] = E[g_{n,r}^2] + E[g_{n,i}^2] =$$

$$2\sigma_n^2 = T^2 p(nT)$$

- CORRELATED TAP-GAIN MODELS

$$R(\Delta t) = \begin{bmatrix} R_{00}(\Delta t) & R_{01}(\Delta t) & \dots & R_{0N}(\Delta t) \\ R_{10}(\Delta t) & R_{11}(\Delta t) & \dots & R_{1N}(\Delta t) \\ \vdots & & & \\ R_{N0}(\Delta t) & R_{N1}(\Delta t) & \dots & R_{NN}(\Delta t) \end{bmatrix}$$

$\cancel{3.87} \times 4.13$
$15.98\ m^2$
$3.30 \times \cancel{4.13}$
$\times 3.87$
$12.8\ m^2$

$R_{mm}(0) = 2\sigma_m^2 \Rightarrow$ AVERAGE POWER OF TAP "m"

$$R_T(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{|F(j\omega)|^2}{2T} e^{j\omega\tau} d\tau = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Phi(j\omega) e^{-j\omega\tau} d\tau$$

$$R_T(0) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Phi(j\omega) d\tau \rightarrow \underline{\text{UKUPNA SREDNJA SNAGA}}$$

- SIMPLIFIED SCATTERING FUNCTION

- ASSUMPTION THE SHAPE OF $S(\tau,\nu)$ IS INDEPENDENT OF τ.

$$S(\tau,\nu) = \psi(\tau) S(\nu) \qquad p(\tau) = \int_{-\infty}^{\infty} S(\tau,\nu) d\nu$$

$$p(\tau) = \psi(\tau) \int_{-\infty}^{\infty} S(\nu) d\nu = a \cdot \psi(\tau)$$

$$S(\tau,\nu) = p(\tau) \cdot S(\nu) \quad / \frac{1}{2\pi} \int e^{j2\pi\nu\Delta t} d\nu$$

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} S(\tau,\nu) e^{+j2\pi\nu\Delta t} d\nu = p(\tau) \frac{1}{2\pi} \int_{-\infty}^{\infty} S(\nu) e^{j2\pi\nu\Delta t} d\nu$$

$$\boxed{R_\tau(\tau,\Delta t) = p(\tau) \cdot \rho(\Delta t) \qquad p(\tau) \sim \text{APG}}$$

$$R_{kl}(\Delta t) = \int_{-\infty}^{\infty} R_\tau(\tau,\Delta t)\, \text{sinc}(B(\tau - kT))\, \text{sinc}(B(\tau - lT))\, d\tau$$

$$R_{mn}(\Delta t) = \rho(\Delta t) \int_{-\infty}^{\infty} p(\tau)\, \text{sinc}(B(\tau - mT))\, \text{sinc}(B(\tau - nT))\, d\tau$$

$$R_{mn}(\Delta t) = \rho(\Delta t) \int_{-\infty}^{\infty} p(\tau)\, \text{sinc}(B\tau - m)\, \text{sinc}(B\tau - n)\, d\tau$$

$$R(\Delta t) = R_0 \cdot \rho(\Delta t)$$

$$\tilde{\rho} = L \times Z$$

$\tilde{g} = (\tilde{g}_0(t), \ldots, \tilde{g}_N(t))^T \quad Z = (z_0(t), \ldots, z_N(t))^T$

$\tilde{g}$ – TAP-GAIN VECTOR

Z – INDEPENDENT STATIONARY COMPLEX GAUSSIAN PROCESSES

$E[z_i(t_1)\, z_j(t_2)] = 0$ FOR $i \neq j$ AND ANY t_1, t_2

COVARIANCE of $z_n(t)$:

$$\boxed{E[z_n(t_1) z_n^*(t_2)] = \psi(\Delta t)}$$

$$E[(X-\tau)(Y-\mu)] = E[XY - \mu X - \tau Y + \mu\tau]$$

$$= E[XY] - \tau\mu - \cancel{\tau\mu} + \cancel{\mu\tau} = E[XY] - \mu\tau$$

$$\sigma^2 = \overline{(\xi - \bar{\xi})^2} = \overline{\xi^2} - \bar{\xi}^2$$

$\Delta t = t_1 - t_2$ IS SAME FOR ALL $n = 0, 1, \ldots, N$

$$A = \begin{bmatrix} -1 & -2 \\ 1 & 3 \\ 2 & 1 \end{bmatrix} \quad \mathrm{cov}(A) = \begin{bmatrix} 2.333 & 2.8333 \\ 2.833 & 6.3333 \end{bmatrix}$$

$$\mathrm{cov} = E[(X-\mu)(Y-\eta)] = E[X \cdot Y] - \mu\eta$$

mean $(A(:,1)) = 0.66667$

mean $(A(:,2)) = 0.66667$

GENERATION OF CORRELATED TAP GAINS!

$\boxed{\tilde{g} = L \times Z\ ;}$ $\quad E[z_n(t_1) z_n^*(t_2)] = \psi(\Delta t)$

$$E[\tilde{g}(t_1)\tilde{g}^\dagger(t_2)] = L\psi(\Delta t) I \cdot L^T = \psi(\Delta t) L \cdot L^T$$

$$E[L \times Z \cdot L^T \cdot Z^*] = E[Z \cdot Z^*] \cdot L L^\dagger = \psi(\Delta t) L \cdot L^T$$

$R(\Delta t) = R_0 \rho(\Delta t) \quad R_0\, \rho(\Delta t) = \psi(\Delta t) L \cdot L^T \Rightarrow$

$\psi(\Delta t) = \rho(\Delta t) \Rightarrow \quad R_0 = L \cdot L^T$

$$L = \begin{bmatrix} l_{00} & l_{01} & \cdots & l_{0N} \\ 0 & l_{11} & \cdots & l_{1N} \\ \vdots & & & \\ 0 & \cancel{l_{N1}} & \cdots & l_{NN} \end{bmatrix}$$

Cholesky Factorization

612.50

$$E\left[z_n(t_1)\, z_n^*(t_2)\right] = \psi(\Delta t)$$

$$\tilde{c}_h(\tau,t) = \tilde{c}_M(\tau,t) * \tilde{h}(\tau)$$

$$R_{\tilde{c}_h}(\tau,\Delta t) = E\left[\tilde{c}_h(\tau,t)\cdot \tilde{c}_h^*(\tau,t+\Delta t)\right]$$

$$R_{\tilde{c}}(\tau,\Delta t) = E\left[\tilde{c}(\tau,t)\cdot \tilde{c}^*(\tau,t+\Delta t)\right]$$

SAMO DA VIDAM KAKO SE MENUVA SO PROMENA NA Δt

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 0 \\ 3 & 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix} + \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} 2 & 3 & 4 \\ 2 & 3 & 4 \\ 2 & 3 & 4 \end{bmatrix}$$

$$R_{\tilde{c}_h}(\tau,\Delta t) = E\left[\tilde{c}_M(\tau,t) * \tilde{h}(\tau) \cdot \tilde{c}_M^*(\tau,t+\Delta t) * \tilde{h}^*(\tau)\right]$$

$$= E\left[\int_{-\infty}^{\infty} c_M(s,t)\, h(\tau-s)\, ds \int_{-\infty}^{\infty} c_M^*(\mu,t+\Delta t)\, h^*(\tau-\mu)\, d\mu\right] =$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E\left[c_M(s,t)\, c_M^*(\mu,t+\Delta t)\right] h(\tau-s)\, h^*(\tau-\mu)\, ds\, d\mu$$

for $M \to \infty \Rightarrow c_M(\tau,t) \to c(\tau,t)$

$E\left[c(s,t)\, c^*(\mu,t+\Delta t)\right] \neq 0$ IF $s=\mu=\tau$

DUE TO UNCORRELATED SCATTERING

$$R_{\tilde{c}_h}(\tau,\Delta t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} R_{\tilde{c}}(\tau,t)\cdot h(\tau-s)\, h^*(\tau-\mu)\, ds\, d\mu$$

$$R_{\tilde{c}_h}(\tau,\Delta t) = R_{\tilde{c}}(\tau,t) * |h(\tau)|^2$$

$$\Delta t = 0 \Rightarrow \boxed{p(\tau) = R_c(\tau,0)}$$

$$p_h(\tau) = p(\tau) * |h(\tau)|^2$$

$$S_g(\tau,\nu) = F_{\Delta t}\left[R_{C_g}(\tau,\Delta t)\right]$$

$$S_g(\tau,\nu) = S(\tau,\nu) * |\tilde{g}(\tau)|^2$$

FOR SIMPLIFIED SCATTERING FUNCTION

$$S_g(\tau,\nu) = S(\nu)\cdot p(\tau) * |\tilde{g}(\tau)|^2 = S(\nu)\cdot p_g(\tau)$$

$$\tilde{C}_M(\tau,t) = \tilde{c}(\tau,t) * l_M(\tau)$$

$$\tilde{C}_g(\tau,t) = \tilde{C}_M(\tau,t) * \tilde{g}(\tau) = \tilde{c}(\tau,t) * \tilde{l}_M(\tau) * \tilde{g}(\tau)$$

$$p_g(\tau) = p(\tau) * |\tilde{l}_M(\tau) * g(\tau)|^2 = \underbrace{p(\tau) * |\tilde{l}_M(\tau)|^2}_{p_M(\tau)} * |g(\tau)|^2$$

$$\boxed{p_g(\tau) = p_M(\tau) * |g(\tau)|^2}$$

EXAMPLE 9.1.1

$$\boxed{p(\tau) = \frac{1}{T} e^{-0.4\tau/T} \quad 0<\tau\le 4}$$

EXPONENTIAL DIFFUSE CHANNEL MODEL

TAP SPACING $T=1$

1. MAGNITUDES OF THE TAP-GAIN PROCESSES UNCORRELATED APPROXIMATION

$|\tilde{g}_0| = 1.0 \quad |\tilde{g}_1| = 0.8 \quad |\tilde{g}_2| = 0.67 \quad |\tilde{g}_3| = 0.55$

$|\tilde{g}_4| = 0.37$

2. CORRELATED MAGNITUDES OF THE TAP GAINS

$\Delta t = 0$ Riemann Sum

$$R_{mn}(0) = \sum_{k=0}^{K} p(k\Delta\tau)\,\mathrm{sinc}\left[\frac{k\Delta\tau}{T} - m\right]\mathrm{sinc}\left[\frac{k\Delta\tau}{T} - n\right]\Delta\tau$$

$$R_{mn}(\Delta t) = \rho(\Delta t)\int_{-\infty}^{\infty} p(\tau)\,\mathrm{sinc}(B\tau - m)\,\mathrm{sinc}(B\tau - n)\,d\tau$$

K IS USUALLY CHOSEN SO THAT $p(\tau)$ IS SAMPLED 10-20 SAMPLES PER SYMBOL

- $N=8$ $\quad m = 1:8$

$\quad n = 1:8$

$R_0 = R_{mn}(0)$ $\quad \boxed{L = \mathrm{chol}(R_0)}$

- CORRELATED TAP GAINS ARE GENERATED BY

$$\boxed{\tilde{g} = L \times z}$$

$$B_T = B_S + f_D$$

B_S - INPUT BANDWIDTH

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jevuchim yick : $r(0.6904)$ $x(0.6316)$
js - yick : $r(2.212)$ $x(0.6316)$

$2.212/0.6904 = 3.2$

$$g_n(t) = \int_{-\infty}^{\infty} c(\tau,t)\,\mathrm{sinc}\big(B(\tau-nT)\big)\,d\tau$$

$$\boxed{R_{kl}(\Delta t) = \int_{-\infty}^{\infty} R_{\tilde c}(\tau,\Delta t)\,\mathrm{sinc}\big(B(\tau-kT)\big)\,\mathrm{sinc}\big(B(\tau-lT)\big)\,d\tau}$$

$$R_{\tilde c}(\tau,\Delta t) = E\left[\tilde c(\tau,\Delta t)\,\tilde c(\tau,t+\Delta t)\right]$$

$$E\left[|\tilde g_n(t)|^2\right] = T^2 p(nT) = T^2 E\left[|\tilde c(nT,t)|^2\right]$$

$$S(\tau,\nu) = p(\tau)\cdot S(\nu) \quad \mathcal{F}^{-1}[\,\cdot\,]$$

$$R_{\tilde c}(\tau,\Delta t) = p(\tau)\,\rho(\Delta t)$$

$$R(\Delta t) \equiv R_{mn}(\Delta t) = \rho(\Delta t)\underbrace{\int p(\tau)\,\mathrm{sinc}\big(B(\tau-kT)\big)\,\mathrm{sinc}\big(B(\tau-lT)\big)\,d\tau}_{\equiv R_0}$$

$$\boxed{R(\Delta t) = \rho(\Delta t)\cdot R_0}$$

$$\boxed{\tilde g(t) = L \times Z}$$

- Discrete Multipath Model

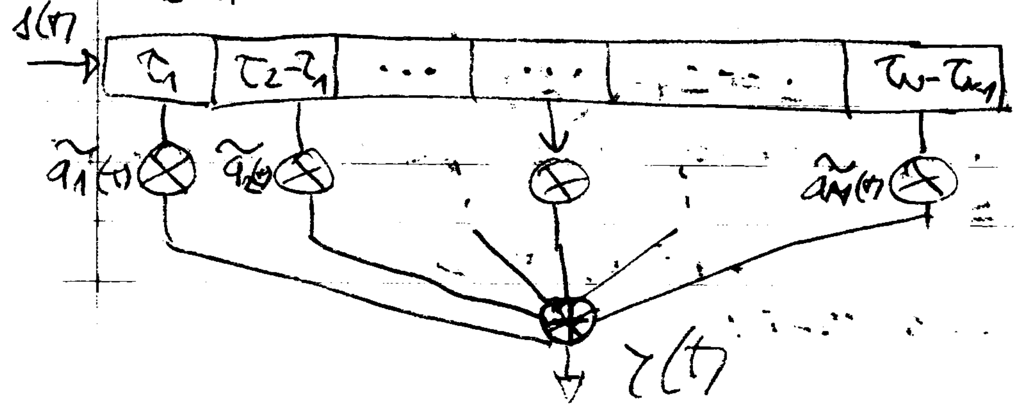
$$\tilde c(\tau,t) = \sum_{k=1}^{K(t)} \tilde a_k(\tau_k(t),t)\,\delta(\tau-\tau_k(t))$$

$$r(t) = \sum_{k=1}^{K(t)} \tilde a_k(\tau_k(t),t)\cdot \tilde s(t-\tau_k(t))$$

- ASSUMPTION OF CONSTANT NUMBER OF DISCRETE COMP.

- $$\tilde c(\tau,t) = \sum_{k=1}^{K} a_k(\tau_k,t)\,\delta(\tau-\tau_k)$$

$$r(t) = \sum_{k=1}^{K} \tilde a_k(\tau_k,t)\cdot \tilde s(t-\tau_k)$$



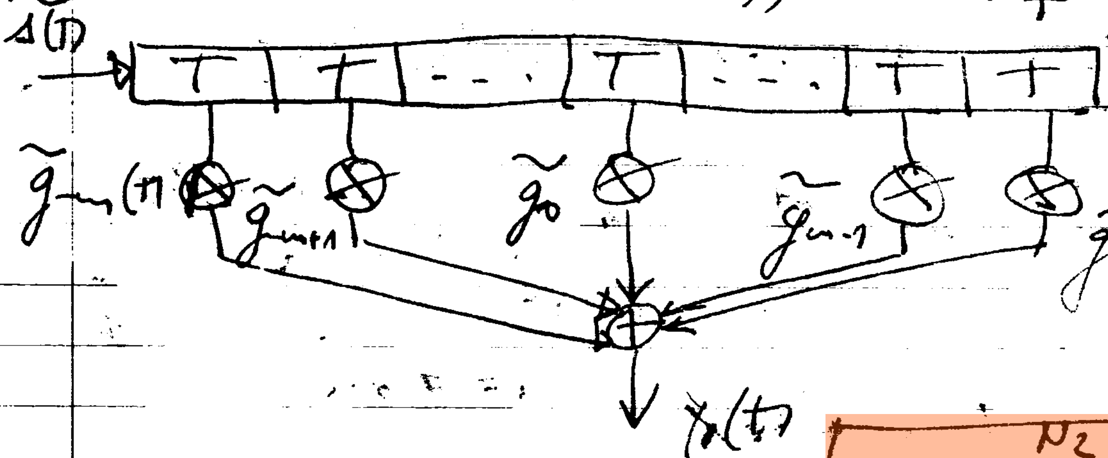
$$\tilde g_n(t) = \int_{-\infty}^{\infty} \tilde c(\tau,t)\,\mathrm{sinc}(B(\tau - nT))\,d\tau$$

$$\tilde c(\tau,t) = \sum_{k=1}^{K} \tilde a_k(t)\,\delta(\tau-\tau_k)$$

$$\tilde g_n(t) = \sum_{k=1}^{K} \int_{-\infty}^{\infty} \tilde a_k(t)\,\delta(\tau-\tau_k)\,\mathrm{sinc}(B(\tau-nT))\,d\tau$$

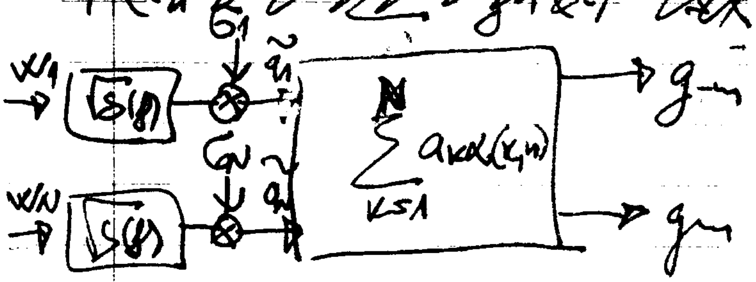
$$\tilde g_n(t) = \sum_{k=1}^{K} \tilde a_k(t)\,\mathrm{sinc}(B(\tau_k-nT)) = \sum_{k=1}^{K} \tilde a_k(t)\cdot\alpha(k,n) \qquad -N\le n\le N$$

$$\alpha(k,n) = \mathrm{sinc}(B(\tau_k-nT)) = \mathrm{sinc}\left(\frac{\tau_k}{T}-nT\right)$$

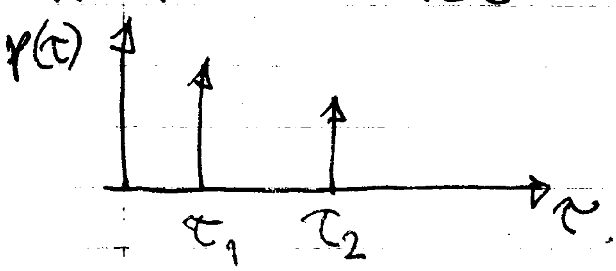


MATLAB USER GUIDE

$$\boxed{y(i) = \sum_{n=-N_1}^{N_2} s(i-n)\cdot g(n)}$$



Ex. 9.1.2 Discrete Multipath Model $\left(T=\frac{1}{B}\right)$

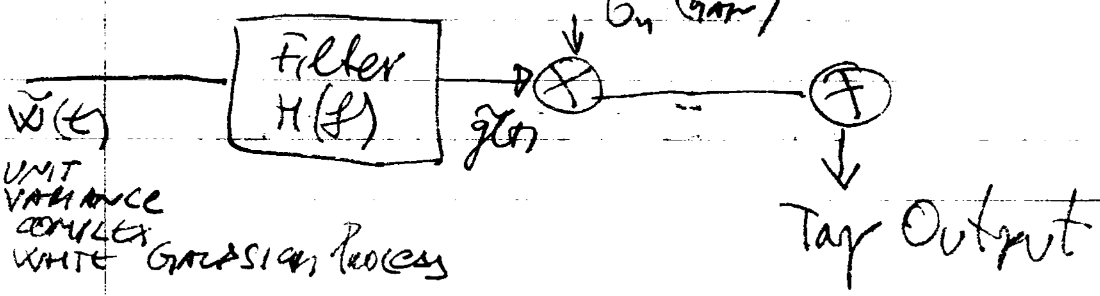


$\Delta\tau = \dfrac{\tau_2-\tau_1}{T} \Rightarrow$ NORMALIZED DELAY SPREAD

$\Delta\tau \ll 1$ FREQ. NON SELECTIVE

$\Delta\tau > 0.1$ FREQ. SELECTIVE

LETS ASSUME $\Delta\tau = 0.75$



$$S_J(f) = \frac{A}{\left[1-\left(\frac{f}{f_d}\right)^2\right]^{1/2}} \quad H_J = \sqrt{S_J} = \frac{\sqrt{A}}{\left[1-\left(\frac{f}{f_d}\right)^2\right]^{1/4}}$$

$$h_J(t) = F^{-1}\left[H_J(f)\right] = \sqrt{A \cdot \pi}\ \sqrt[4]{2}\ \Gamma(3/4)\, f_d\, x^{-1/4} J_{1/4}(x)$$

$$h_J(t) = \sqrt{A}\ 2.583\, f_d\, x^{-1/4} J_{1/4}(x)$$

$$I = \mathcal{F}^{-1}\{S_J(f)\} = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{A}{\sqrt{1-\left(\frac{f}{f_d}\right)^2}}\, e^{j2\pi f t}\, df \qquad \boxed{|f| \le f_d}$$

$$\int_{-1}^{1} \frac{e^{2x} dx}{\sqrt{1-x^2}} = \pi I_0(2)$$

MATEMATIKA GO REŠAVA OVOJ INTEGRAL

$$\boxed{x = \frac{f}{f_d} \Rightarrow \quad dx = \frac{df}{f_d};\ f = f_d \cdot x} \quad I = \frac{f_d}{2\pi}\int_{-\infty}^{\infty} \frac{1}{\sqrt{1-x^2}}\, e^{j\frac{2\pi f_d x t}{a}}\, \frac{df}{\cancel{f_d}}$$

$$\boxed{a = 2\pi f_d \cdot t}$$

$$I = \frac{f_d}{2\pi}\int_{-1}^{1} \frac{e^{ja\cdot x}}{\sqrt{1-x^2}}\, df = \frac{f_d}{2\pi}\cdot \pi \text{BesselJ}[0,a]$$

$$\boxed{I = \frac{f_d}{2} J_0(2\pi f_d t)}$$

VARIJATORI

$$y(d,t) = x(t) \otimes h(d,t) = \int^{\infty} x(\tau)\cdot h(d, t-\tau)\, d\tau$$

$$y(d,t) = \int_{-\infty}^{t} x(\tau)\, h(d, t-\tau)\, d\tau \qquad d = v\cdot t$$

$$y(v\cdot t, t) = \int_{-\infty}^{t} x(\tau)\cdot h(v\cdot t, t-\tau)\, d\tau$$

$$y(t) = \int_{-\infty}^{t} x(\tau)\ h(v\cdot t, t-\tau)\, d\tau$$

$$\boxed{y(t) = \int_{-\infty}^{\infty} x(\tau)\ h(t,\tau)\, d\tau} \quad y(t) = x(\tau) \otimes h(t,\tau)$$

$h(t,\tau) = \mathrm{Re}\{h_b(t,\tau)\, e^{j\omega_c t}\}$

$x(t) \to h(t,\tau) \to y(t) = x(t) \otimes h(t) = \mathrm{Re}\{r(t)\, e^{j\omega_c t}\}$

$c(t) \to h_b(t,\tau) \to r(t) = \frac{1}{2} c(t) \otimes h_b(t,\tau)$

$x(t) = \mathrm{Re}\{c(t) \cdot e^{j2\pi f_c t}\}$

$y(t) = \mathrm{Re}\{r(t)\, e^{j2\pi f_c t}\}$

AVERAGE POWER OF BANDPASS SIGNAL $\overline{x^2(t)}$ IS EQUAL TO $\frac{1}{2}\overline{|c(t)|^2}$.

$$h_b(t,\tau) = \sum_{i=0}^{N-1} \underbrace{a_i(t,\tau)\exp[j(2\pi f_c \tau_i(t) + \phi_i(t,\tau))]}_{\tilde{a}_i(t,\tau)\ \Rightarrow\ \text{JEDNIM SINTAKSA}}\,\delta(\tau-\tau_i(t))$$

- time-invariant impulse response OR WIDE SENSE STATIONARY

$$\boxed{h_b(\tau) = \sum_{i=0}^{N-1} a_i \exp(-j\theta_i)\,\delta(\tau-\tau_i)}$$

$p(t) \approx \delta(t-\tau) \Rightarrow$ PROBING IMPULSE FOR SOUNDING THE CHANNEL

- POWER DELAY PROFILE

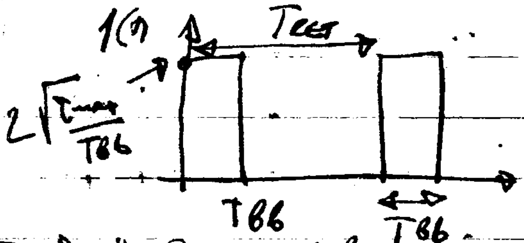
$P(t,\tau) = k\,|h_b(t,\tau)|^2$

JEDNOM:

$$p(\tau) = \int_{-\infty}^{\infty} S(\tau,\nu)\,d\nu \qquad S(\nu) = \int_{-\infty}^{\infty} S(\tau,\nu)\,d\tau$$

⊙ RELATIONSHIP BETWEEN BANDWIDTH AND RECEIVED POWER

EX: $x(t) = \mathrm{Re}\{p(t)\, e^{j2\pi f_c t}\}$



$p(t) = 2\sqrt{\frac{\tau_{max}}{T_{bb}}} \quad 0 \le t \le T_{bb} \rightarrow$ WIDEBAND PULSE

$$r(t) = \sum_{i=0}^{N-1} a_i \exp(-j\theta_i)\, p(t-\tau_i) = \sum_{i=0}^{N-1} a_i \exp(j\theta_i) \cdot \sqrt{\frac{\tau_{max}}{T_{bb}}}$$

$$\cdot\ \mathrm{rect}\left[t - \frac{T_{bb}}{2} - \tau_i\right]$$

$$|r(t_0)|^2 = \frac{1}{T_{max}} \int_0^{T_{max}} r(t) * r^*(t)\, dt = \frac{1}{T_{max}} \int_0^{T_{max}} \frac{1}{4} \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} a_i a_j e^{-j\theta_i} e^{+j\theta_j} \cdot p(t-\tau_i) p(t-\tau_j)$$

$$|r(t_0)|^2 = \frac{1}{T_{max}} \frac{1}{4} \int_0^{T_{max}} \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} a_i a_j \exp(-j(\theta_i - \theta_j)) p(t-\tau_i) p(t-\tau_j)\, dt$$

$$= \frac{1}{T_{max}} \frac{1}{4} \int_0^{T_{max}} \sum_{k=0}^{N-1} a_k^2(t_0)\, p^2(t-\tau_k)\, dt =$$

$$= \frac{1}{\cancel{T_{max}}} \sum_{k=0}^{N-1} a_k^2(t_0) \frac{1}{\cancel{4}} \int_0^{T_{max}} \cancel{4} \frac{\cancel{T_{max}}}{T_{bb}} \mathrm{rect}^2\left[t - \frac{T_{bb}}{2} - \tau_k\right] dt$$

$$|r(t_0)|^2 = \sum_{k=0}^{N-1} a_k^2(t_0) \cdot \frac{1}{\cancel{T_{bb}}} \cancel{\int_0^{T_{max}} dt}\ \cancel{T_{bb}} = \sum_{k=0}^{N-1} a_k^2(t_0)$$

$$\boxed{|r(t_0)|^2 = \sum_{k=0}^{N-1} a_k^2(t_0)}$$

$$E_{a,\theta}[P_{WB}] = E_{a,\theta}\left[\sum_{i=0}^{N-1} |a_i e^{j\theta_i}|^2\right] = \sum_{i=0}^{N-1} \overline{a_i^2}$$

PRVO SI KREVA NA KVADRAT A SI SOBIRA.

~~SE SUMIRA VO TAU DOMEN??? BI TREBALO VO t DOMEN~~

PRVO SI SUMIRA VO TAU PA SI KREVA VO t

PRVO SI SOBIRA KOMPONENTITE PA SI KREVA NA KVADRAT.

$\boxed{CW}$ (Continuous Wave Signal)

COMPLEX ENVELOPE $\boxed{C(t) = 2}$

$$r(t) = \sum_{i=0}^{N-1} a_i \exp(j\theta_i(t,\tau))$$

$$|r(t)|^2 = \left| \sum_{i=0}^{N-1} a_i \exp(j\theta_i(t,\tau)) \right|^2$$

$$E_{a,\theta}[P_{CW}] = E_{a,\theta}\left[\left| \sum_{i=0}^{N-1} a_i \exp(j\theta_i) \right|^2 \right] =$$

$$= \left[a_0 e^{j\theta_0} + a_1 e^{j\theta_1} + \dots + a_{N-1} e^{j\theta_{N-1}}\right]\left[a_0 e^{-j\theta_0} + a_1 e^{-j\theta_1} + \dots + a_{N-1} e^{-j\theta_{N-1}}\right]$$

$$= \sum_{i=0}^{N-1} \overline{a_i^2} + 2 \sum_{i=0}^{N-1} \sum_{i,j \neq i}^{N} r_{ij} \overline{\cos(\theta_i - \theta_j)}$$

$$2 a_0 e^{j\theta_0} a_1 e^{-j\theta_1} + 2 a_0 e^{j\theta_0} a_2 e^{-j\theta_2} \dots$$

r_{ij} – PATH AMPLITUDE CORRELATION COEFFICIENT

$$r_{ij} = E_a[a_i a_j]$$

$$E_{a,\theta}[P_{CW}] = \sum_{i=0} \overline{a_i^2} + 2 \sum_{i=0}^{N-1} \sum_{i,j \neq i}^{N} r_{ij} \overline{\cos(\theta_i - \theta_j)}$$

Example 4.2 $\Delta\tau_{max} = 100\mu s$ – URBAN RADIO CHANNELS
$\Delta t_{max} = 4\mu s$ – MICROCELLULAR CHANNELS

(a.) $\Delta\tau = ?$ $N = 64$ BINS

(b.) MAXIMUM BANDWIDTH FOR THE TWO MODELS

(a.)
$$\Delta\tau_1 = \frac{\tau_{max1}}{64} = \frac{100\mu s}{64} = 1.5625\ \mu s$$
$$\Delta\tau_2 = \frac{\tau_{max2}}{64} = \frac{4\mu s}{64} = 62.5e{-9} = 62.5\ \text{nsec}$$

(b.)
$$B_1 = \frac{1}{2\Delta\tau} = \frac{1}{2\ 1.5625e{-6}} = 32\text{kHz}$$
$$B_2 = \frac{1}{2\Delta\tau} = \frac{1}{2\ 625e{-9}} = 8\text{MHz}$$

$$\tau_{max} = \tau_N = 500\mu s \qquad \Delta\tau = \frac{(500e{-9})}{64} = 7.8125\ \text{nsec}$$
$$B = \frac{1}{2\Delta\tau} = \frac{1}{2 \cdot 7.8125e{-9}} = 64\text{MHz}$$

Example 4.3 $v = 10\text{m/s}$; $f_c = 1\text{GHz}$; TWO COMPONENTS
1st COMPONENT $\tau = 0$, PHASE $0°$, POWER -70dBm
2nd COMPONENT $\tau = 1\mu s$, PHASE $0°$, POWER 3dB WEAKER
- NARROWBAND INSTANTENOUS POWER AT: $\boxed{0.1\sec = \Delta t \quad t = 0.05\sec}$
- WIDEBAND –"– POWER = ?

OVA $\in$ VSW-OOT
τ "
"VIDI vicianchan(·)
@multipath)scalepathgain

$$\lambda = \frac{c}{f_c} = \frac{300}{1000} = \frac{3}{10} = 0.3\text{ m}$$

$$E_{a,\theta}[P_{WB}] = E\left[\left|\sum_{i=0}^{N-1} a_i e^{+j\theta_i}\right|^2\right]$$

$$E_{a,\theta}[P_{WB}] = E_{a,\theta}\left[\sum_{i=0}^{N-1} a_i^2\right] = \sum_{i=0}^{N-1}|a_i|^2$$

$$P_{CW} = \left|\sum_{i=0}^{N-1} a_i e^{j\theta_i}\right|^2 = ?$$

$$a_1 = A_1 \cdot e^{j0} = A_1 \quad 10\log(A_1)/1\text{mW} = \underline{-70\text{dBm}}$$

$$A_1^2 = 1\text{mW} \cdot 10^{-7} = 10^{-3} \cdot 10^{-7} = 10^{-10} = 0.1 \cdot 10^{-9} = 0.1\ \text{nW} = 100\ \text{pW}$$
$$A_2^2 = 1\text{mW} \cdot 10^{-7.3} = 10^{-3} \cdot 5 \cdot 10^{-8} = 5 \cdot 10^{-11} = 50 \cdot 10^{-12} = 50 = 50\ \text{pW}$$

~~$P_{CW} = |100\text{pW} + 50\text{pW}|^2 = 150\text{pW} = (150 \cdot 10^{-12})^2 = 225 \cdot 10^{-20}$~~

$$P_{WB} = |100 \cdot 10^{-12}|^2 + |50 \cdot 10^{-12}|^2 = 10^{-20} + 0.25 \cdot 10^{-20} = 1.25 \cdot 10^{-20}$$

1^{st} comp → • ← 2^{nd} comp

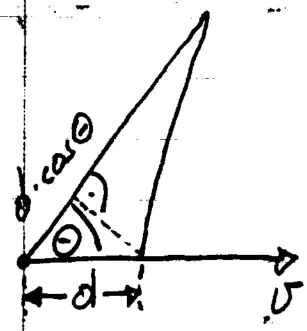
$v = 10$ m/sec

$\tau_1 = 0$	-70 dBm
$\tau_2 = 1\mu s$	-73 dBm

$$h_b(t,\tau) = \sum_{i=0}^{N-1} a_i \exp\left(j(2\pi f_c \tau + \theta_i)\right) \delta(\tau - \tau_i)$$

$i = 0, 1$ $\quad \varphi_i = 2\pi f_c \tau_i + \theta_i$

VIDI STANDARD MANUSCRIPT 99.6.



$$\varphi = \frac{d\cos\Theta}{\lambda} \cdot 2\pi$$

$$\omega_d \cdot t = \frac{d\cos\Theta}{\lambda} \cdot 2\pi$$

$$2\pi f_d = 2\pi \frac{d}{t} \cdot \frac{1}{\lambda} \cos\Theta$$

$$f_d = \frac{v}{\lambda} \cdot \cos\Theta \qquad f_{d\,max} = \frac{v}{\lambda} = \frac{10\ m/s}{0.3} = 33{,}33\ Hz$$

$$\theta_2 = 2\pi f_d \cdot \tau_2 = 2\pi \cdot 33.33 \cdot 1e{-6} = 0.21 \cdot 10^{-3}\ rad$$

$$2\pi f_c \tau_i = 2\pi \cdot 1e^9 \cdot 1e^{-6} = 2\pi \cdot 1000 \qquad e^{j\,2\pi \cdot 1000} = 1^{1000} = 1$$

$$2\pi\ rad = 360^\circ \qquad 1\ rad = \frac{360}{2\pi} = \frac{180}{\pi} \qquad 1^\circ = \frac{2\pi}{360}$$

$$\theta_2 = 0.21 \cdot 10^{-3} \cdot \frac{180}{\pi} = 0{,}012^\circ \quad ??$$

$t=1$

$$\theta_i = \frac{d}{\lambda} \cdot 2\pi = 2\pi \frac{v \cdot t}{\lambda} = 2\pi \frac{10\ m/s \cdot 0.1}{0.3\ m} = 2\pi \cdot 3.33 = 20.94$$

$$\boxed{t = 0:0.1:0.5}$$

$\theta_i = 2\pi f_{d\,max}\, t$

$$\theta_i = 20.94\ rad = 2.09\ rad$$
$$= 2.09 \cdot \frac{180}{\pi} = 120^\circ$$

$$\frac{20.94}{2\pi} = 3{,}33\ rad \qquad 0.33 \cdot 2\pi = 2.0735\ rad \cdot \frac{180}{\pi} = 118.8^\circ \approx 120^\circ$$

$$\theta_0 = 0^\circ;\ \theta_1 = 120^\circ;\ \theta_2 = -120^\circ$$

$$\theta_2 = 2\pi \cdot \frac{10\ m/s \cdot 0.2}{0.3} = 40.94\ rad = 0.5158 \cdot 2\pi\ rad = 3.241\ rad$$

$t=0$

$$|r(t)|^2 = \left|\sum_{i=0}^{N-1} a_i \cdot e^{j\theta_i(t,\tau)}\right|^2 = \left|\sqrt{100pW}\, e^{+j0} + \sqrt{50pW}\, e^{j0}\right|$$

$$= \left|\sqrt{10^2 \cdot 10^{-12}} + \sqrt{5 \cdot 10 \cdot 10^{-12}}\right|^2 = \left|\sqrt{10^{-10}} + \sqrt{5} \cdot \sqrt{10^{-11}}\right|^2$$

$$= \left|10^{-5} + \sqrt{5} \cdot 10^{-5.5}\right|^2 = \left|10^{-5} + 0.71 \cdot 10^{-5}\right|^2 = \left|1.71 \cdot 10^{-5}\right|^2 = 291.41\ pW$$

$\boxed{t=1}$ $\theta_1 = 120^\circ$ $\theta_2 = -120$

$$|r(t)|^2 = \left| A_1 e^{+j\theta_1} + A_2 e^{j\theta_2} \right|^2 = \left| \sqrt{100pW}\, e^{j2.09} + \sqrt{50pW}\, e^{-j2.09} \right|$$

$$= 7.93 \cdot 10^{-11} = 79.3\, 10^{-12} = 79{,}3\ pW$$

$\boxed{t=2}$ $$\theta_i = 2\pi \frac{d}{\lambda} = 2\pi \frac{v \cdot t}{\lambda} = 2\pi \frac{10 \cdot 0.2}{0.3} = \cancel{\ldots} 41.88_{rad}$$

$$\cancel{\ldots} = 0.66 \cdot 2\pi = 4.2\ rad \cdot \frac{180}{\pi} = \underline{240^\circ}$$

$$|r(t)|^2 = \left| \sqrt{100pW}\, e^{j4.2} + \sqrt{50}\, e^{-j4.2} \right| = 79{,}3\ pW$$

$\boxed{t=3}$ $\theta = 360$ $|r(t)|^2 = 291.42\ pW$

$\boxed{t=4}$ $\theta = 120$ $|r(t)|^2 = 79{,}3\ pW$

$\boxed{t=5}$ $\theta = 240$ $|r(t)|^2 = 79{,}3\ pW$

SREDNJA SNAGA E: $$\frac{2 \cdot 291.42 + 4 \cdot 79{,}3}{6} = 150\ pW$$

$$P_{WB} = \sum_{\lambda=0}^{N-1} |a_\lambda|^2 = \left|\sqrt{100pW}\right|^2 + \left|\sqrt{50pW}\right|^2 = 150\ pW$$

- OUTAGE PROBABILITY OF MULTIHOP TRANSMISSION OVER NAKAGAMI FADING CHANNELS

- IN ORDER TO SATISFY THE AVERAGE POWER CONSTRAINT

$$\boxed{G_n^2 = \frac{1}{a_n^2 + N_{0,n}} \qquad n = 1, \ldots, N-1}$$

a_n - FADING AMPLITUDE OF PRECEDING HOP

$N_{0,n}$ - POWER OF THE ADDITIVE GAUSSIAN NOISE

$$\gamma_{eq1} = \left[\prod_{n=1}^{N} \left(1 + \frac{1}{\gamma_n}\right) - 1 \right]^{-1} \qquad \gamma_n = \frac{a_n^2}{N_{0,n}}$$

γ_n - SNR OF THE nth HOP.

$$\frac{1}{\gamma_{eq1}} = \frac{1}{\gamma_1} + \frac{1}{\gamma_2} + \frac{1}{\gamma_3} + \frac{1}{\gamma_1\gamma_2} + \frac{1}{\gamma_1\gamma_3} + \frac{1}{\gamma_2\gamma_3}$$

- UPPER BOUND

$$\gamma_{eq2} = \left[\sum_{n=1}^{N} \frac{1}{\gamma_n} \right]^{-1}$$

$$\frac{1}{\gamma_{eq_1}} = \frac{1}{\gamma_1} + \frac{1}{\gamma_2} + \frac{1}{\gamma_3} + \frac{1}{\gamma_1\gamma_2} + \frac{1}{\gamma_1\gamma_3} + \frac{1}{\gamma_2\gamma_3}$$

$$\gamma_{eq_1} = \frac{1}{\frac{1}{\gamma_1} + \frac{1}{\gamma_2} + \frac{1}{\gamma_3} + \frac{1}{\gamma_1\gamma_2} + \frac{1}{\gamma_1\gamma_3} + \frac{1}{\gamma_2\gamma_3}} =$$

$$= \frac{1}{\frac{\gamma_2\gamma_3 + \gamma_1\gamma_3 + \gamma_1\gamma_2 + \gamma_3 + \gamma_2 + \gamma_1}{\gamma_1\gamma_2\gamma_3}} =$$

$$\boxed{\gamma_{eq_1} = \frac{\gamma_1\gamma_2\gamma_3}{\gamma_2\gamma_3 + \gamma_1\gamma_3 + \gamma_1\gamma_2 + \gamma_3 + \gamma_2 + \gamma_1}}$$

UPPER BOUND

$$\frac{1}{\gamma_i\gamma_j} \to \emptyset \qquad \boxed{\gamma_i \cdot \gamma_j \to \infty}$$

$$\boxed{\gamma_{eq_2} = \frac{1}{\frac{1}{\gamma_1} + \frac{1}{\gamma_2} + \frac{1}{\gamma_3}} = \left[\sum_n \frac{1}{\gamma_n}\right]^{-1}}$$

RELAY GAIN IS SET TO: $\boxed{G_n^2 = \frac{1}{a_n^2}}$

$$\gamma_{eq_2} = \frac{\mu_H}{N} \qquad \mu_H \text{ – HARMONIC MEAN}$$

- HARMONIC MEAN

$$\boxed{\mu_H = \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \dots \frac{1}{x_n}} = \frac{n}{\sum_{i=1}^{n} \frac{1}{x_i}}} \Rightarrow \gamma_{eq_2} = \frac{\mu_H}{N}$$

$$P_{out} = P_r(\gamma_{eq} < \gamma_{th}) = P_r\left(\frac{1}{\gamma_{eq}} > \frac{1}{\gamma_{th}}\right) = 1 - \mathcal{L}^{-1}\left(\frac{M_{1/\gamma_{eq}}(s)}{s}\right)\Bigg|_{1/\gamma_{th}}$$

$\mathcal{L}^{-1}(\cdot)$ – INVERSE LAPLASE TRANSFORM

$M_{1/\gamma_{eq}}(\cdot)$ – MFG Moment Generating Function

– MOMENT GENERATING FUNCTION OF RANDOM VARIABLE X:

$$\boxed{M_X(t) := E(e^{tX}) \quad t \in \mathbb{R}}$$

- REGENERATIVE SYSTEM

$$P_{out} = P_r[\min(\gamma_1, \dots \gamma_N) \leq \gamma_{th}] = 1 - P_r[\gamma_1 > \gamma_{th}, \gamma_2 > \gamma_{th}; \gamma_N > \gamma_{th}]$$

$$\boxed{P_{out} = 1 - \prod \frac{\Gamma\left(m_n, \frac{m_n \gamma_{th}}{\bar{\gamma}_n}\right)}{\Gamma(m_n)}}$$

$\bar{\gamma}_n \to$ AVERAGE SNR ON n-th HOP

$\Gamma(\cdot,\cdot)$ – incomplete GAMMA

$\Gamma(\cdot)$ – GAMMA FUNCTION

297 x 210
210 x 148
1414 1427
141,4 141,9

$$\boxed{p(z) = \frac{2m^m z^{2m-1}}{\Gamma(m)\,\Omega^m} e^{-\frac{m z^2}{\Omega}} \quad z \geq 0,\ \Omega \geq 0,\ m \geq 0.5}$$

N3P11

$$\Gamma(\alpha) = \int_0^\infty e^{-t} t^{\alpha-1} dt \qquad \Gamma(n+1) = n! \qquad \boxed{\Gamma(z+1) = z\,\Gamma(z)}$$

$$\boxed{m = \frac{E^2(z^2)}{var(z^2)} \qquad \Omega = E(z^2)}$$

$m=1$ NAKAGAMI $\rightarrow$ RAYLEIGH

$$p(z) = \frac{2 \cdot z}{\Omega} e^{-\frac{z^2}{\Omega}} \qquad \frac{\Omega}{2} \equiv \sigma^2 \qquad \boxed{\Omega = 2\sigma^2}$$

$$\Gamma(1) = \int_0^\infty e^{-t} dt = -e^{-t}\Big|_0^\infty = -\left(\frac{1}{\infty} - 1\right) = 1$$

$$p(z) = \frac{\cancel{2} z}{\cancel{2}\sigma^2} e^{-\frac{z^2}{2\sigma^2}} = \frac{z}{\sigma^2} e^{-\frac{z^2}{\sigma^2}}$$

$$p(x) = \frac{2m^m x^{2m-1}}{\Gamma(x)\Omega^m} e^{-\frac{m x^2}{\Omega}}$$

$$E(x^2) = \int_0^\infty x^2 p(x) dx = \frac{\Omega}{\Gamma(m) m} \Gamma(m+1) \qquad \Gamma(m+1) = m \cdot \Gamma(m)$$

$$E(x^2) = \Omega \qquad E(x) = \frac{\Gamma(m+\frac{1}{2})}{\Gamma(m)} \left(\frac{\Omega}{m}\right)^{1/2}$$

$$\sigma^2 = E(x^2) - E^2(x) = \Omega - \frac{\Omega}{m} \frac{\Gamma^2(m+\frac{1}{2})}{\Gamma^2(m)}$$

$$Q(x) = \int_0^{x} p(t) dt = \frac{1}{\Gamma(m)} \int_0^{\frac{m x^2}{\Omega}} y^{m-1} e^{-y} dy = \frac{1}{\Gamma(m)} \gamma\left(m, \frac{m x^2}{\Omega}\right)$$

$\gamma(\cdot,\cdot)$ – INCOMPLETE GAMA FUNCTION 8.350.1 G.R.

$$\boxed{\Gamma(\alpha, x) = \int_x^\infty e^{-t} t^{\alpha-1} dt}$$ – INCOMPLETE GAMA FUNCTION 8.350.2 G.R.

$$P_{out} = 1 - P_r[\gamma_1 > \gamma_{th}, \gamma_2 > \gamma_{th}, \dots \gamma_N > \gamma_{th}]$$

$$p(x) = \frac{2m^m x^{2m-1}}{\Gamma(m)\Omega} e^{-\frac{m x^2}{\Omega}} \qquad p(\gamma) = \frac{2m^m \gamma^{2m-1}}{\Gamma(m)\Omega^m} e^{-\frac{m\gamma^2}{\Omega}}$$

$$P(\gamma > \gamma_{th}) = \int_{\gamma_{th}}^{\infty} \frac{2m^m \gamma^{2m-1}}{\Gamma(m)\Omega^m} e^{-\frac{m\gamma^2}{\Omega}} d\gamma$$

$$\boxed{\Gamma(\alpha, x) = \int_x^\infty e^{-t} t^{\alpha-1} dt}$$

$$\boxed{\frac{m\gamma^2}{\Omega} = t; \quad \frac{2m\gamma d\gamma}{\Omega} = dt; \quad d\gamma = \frac{\Omega}{2m\gamma}dt; \quad \gamma \to \infty,\ t \to \infty; \quad \gamma = \gamma_{th},\ t = \frac{m\gamma_{th}^2}{\Omega}}$$

$$P(\gamma > \gamma_{th}) = \int_{\frac{m\gamma_{th}^2}{\Omega}}^{\infty} \frac{\cancel{2}m^m \gamma^{2m-1}}{\Gamma(m)\Omega^m} e^{-\frac{m\gamma^2}{\Omega}} \frac{\Omega}{\cancel{2}\gamma m} dt =$$

$$= \frac{1}{\Gamma(m)} \int_{\frac{m\gamma_{th}^2}{\Omega}}^{\infty} \frac{m^{m-1}\gamma^{2m-2}}{\Omega^{m-1}} e^{-t} dt = \frac{1}{\Gamma(m)} \int_{\frac{m\gamma_{th}^2}{\Omega}}^{\infty} \left(\frac{m\gamma^2}{\Omega}\right)^{m-1} e^{-t} dt$$

$$P(\gamma > \gamma_{th}) = \frac{1}{\Gamma(m)} \int_{\frac{m\gamma_{th}^2}{\Omega}}^{\infty} t^{m-1} e^{-t} dt = \frac{\Gamma\left(m, \frac{m\gamma_{th}^2}{\Omega}\right)}{\Gamma(m)}$$

$$\boxed{P_{out} = 1 - \prod_{n=1}^{N} \frac{\Gamma\left(m_n, \frac{m_n\gamma_{th}^2}{\Omega}\right)}{\Gamma(m_n)}}$$

• SHANKAR

$$m = \frac{\epsilon^2[\gamma^2]}{\epsilon\left\{[\gamma^2 - \epsilon[\gamma^2]]^2\right\}} \qquad \Omega = \epsilon[\gamma^2]$$

$$p_{NAK} = \frac{2m^m \gamma^{m-1}}{\Gamma(m)\Omega^{m}} e^{-\frac{m\gamma^2}{\Omega}}$$

$$\sigma^2 = \overline{(\xi - \bar{\xi})^2} \Rightarrow \xi \equiv \gamma^2 \Rightarrow \sigma^2 = \overline{(\gamma^2 - \overline{\gamma^2})^2} =$$

$$= E\left\{[\gamma^2 - \epsilon(\gamma^2)]^2\right\} = \mathrm{var}(\gamma^2)$$

$$\boxed{m = \frac{\mathcal{E}^2(y^2)}{VAR(y^2)}} \; ; \qquad \Omega = \mathcal{E}(y^2) \; ;$$

NAKA. PARAMETERS

$m = 1$ NAKAGAMI $\Rightarrow$ RAYLEIGH

$m = \frac{(K+1)^2}{2K+1}$ NAKAGAMI $\Rightarrow$ RICIAN

$$\boxed{\hat{\rho}^2 = \frac{A^2}{\sigma_N^2} = \frac{\mathcal{E}}{N_0/2} = \frac{2\mathcal{E}}{N_0}} \qquad \hat{\rho} = \frac{A}{\sigma_N}$$

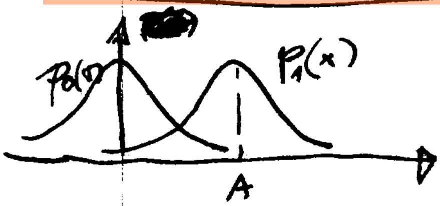
- BASEBAND UNIPOLAR $(0 \div A)$

$$P(e) = \frac{1}{2}\,\mathrm{erfc}\,\frac{A}{2\sqrt{2}\,\sigma_N} = \frac{1}{2}\,\mathrm{erfc}\,\frac{\rho}{\sqrt{2}}$$

$$\rho^2 = \frac{\hat{\rho}^2}{2} = \frac{A^2}{2\sigma_N^2} = \frac{\mathcal{E}}{N_0} \qquad \rho = \sqrt{\frac{\mathcal{E}}{N_0}}$$

$$P(\mathcal{E}) = \frac{1}{2}\,\mathrm{erfc}\sqrt{\frac{\mathcal{E}}{2N_0}}$$

$$\boxed{Q(x) = \frac{1}{2}\,\mathrm{erfc}\,\frac{x}{\sqrt{2}}} \qquad \boxed{P(\mathcal{E}) = Q\left(\sqrt{\frac{\mathcal{E}}{N_0}}\right)}$$



$$\mathcal{E}_b = \frac{A^2T + 0^2}{2} = \frac{A^2}{2} \qquad \frac{A}{\sqrt{2}} = \sqrt{\mathcal{E}_b} \qquad \sigma_N^2 = \frac{N_0}{2}$$

$$P(e) = \frac{1}{2}\,\mathrm{erfc}\,\frac{A}{2\sqrt{2}\,\sigma_N} = \frac{1}{2}\,\mathrm{erfc}\,\frac{\sqrt{\mathcal{E}_b}}{2\sigma_N} = \frac{1}{2}\,\mathrm{erfc}\,\frac{\sqrt{\mathcal{E}_b}}{\frac{2}{\sqrt{2}}\sqrt{\frac{N_0}{2}}}$$

$$\boxed{P(\mathcal{E}) = \frac{1}{2}\,\mathrm{erfc}\sqrt{\frac{\mathcal{E}_b}{2N_0}}} = \frac{1}{2}\,Q\left(\sqrt{\frac{\mathcal{E}_b}{N_0}}\right)$$

- BASEBAND BIPOLAR $\left(-\frac{A}{2} : \frac{A}{2}\right)$

$$P(\mathcal{E}) = \mathrm{erfc}\left(\frac{A}{2\sqrt{2}\,\sigma_N}\right) \qquad \left(\frac{A}{2}\right)^2 = \mathcal{E}_b \qquad \frac{A}{2} = \sqrt{\mathcal{E}_b}$$

$$\mathcal{E}_b = \frac{\left(\frac{A}{2}\right)^2 + \left(\frac{A}{2}\right)^2}{2} = \frac{A^2}{4} \; ; \; P(\mathcal{E}) = \frac{1}{2}\,\mathrm{erfc}\,\frac{\sqrt{\mathcal{E}_b}}{\sqrt{2}\sqrt{\frac{N_0}{8}}} = \frac{1}{2}\,\mathrm{erfc}\sqrt{\frac{\mathcal{E}_b}{N_0}}$$

$$\mathcal{E}_b = Q\left(\sqrt{\frac{2\mathcal{E}_b}{N_0}}\right) \qquad \frac{1}{2}\,\mathrm{erfc}\,\frac{\sqrt{2}\cdot\sqrt{\frac{\mathcal{E}_b}{N_0}}}{\sqrt{2}} = Q\left(\sqrt{\frac{2\mathcal{E}_b}{N_0}}\right)$$

BASEBAND BIPOLAR $(-A:A)$ A

$$\underbrace{P(\varepsilon) = \frac{1}{2}\,\mathrm{erfc}\,\frac{A}{\cancel{2}\sqrt{2}\sigma_N}}_{\text{DTR FORMULA}} = \frac{1}{2}\,\mathrm{erfc}\,\frac{A}{\sqrt{2}\sigma_N}$$

$$E_b = \frac{A^2+A^2}{2} = A^2 \; ; \; A = \sqrt{E_b} \; ; \; P(e) = \frac{1}{2}\mathrm{erfc}\,\frac{\sqrt{E_b}}{\cancel{2}\sqrt{2}\sqrt{\frac{N_0}{2}}}$$

$$P(e) = \frac{1}{2}\,\mathrm{erfc}\left(\frac{\sqrt{\frac{E_b}{N_0}}\sqrt{2}}{\sqrt{2}}\right) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

$$P(\varepsilon) = Q\left(\sqrt{\frac{E_d}{2N_0}}\right) \qquad E_d = \int_0^T [s_1 - s_2]^2\,dt = 4A^2T$$

$$P(e) = Q\left(\sqrt{\frac{4A^2T}{2N_0}}\right) = Q\left(\sqrt{\frac{2A^2T}{N_0}}\right)$$

$$\boxed{E_b = \frac{A^2+A^2}{2}T = A^2T} \qquad P(e) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

$$\boxed{P(e) = \frac{1}{2}\,\mathrm{erfc}\,\sqrt{\frac{E_b}{N_0}}}$$

NAKAGAMI:

$$p(r) = \frac{2m^m r^{2m-1}}{\Gamma(m)\,\Omega^m}\, e^{-\frac{mr^2}{\Omega}}$$

$$E(r) = \frac{\Gamma(m+1/2)}{\Gamma(m)}\left(\frac{\Omega}{m}\right)^{1/2} \; ; \; E(r^2) = \Omega$$

$$\sigma^2 = E(r^2) - E(r)^2 = \Omega\left(1 - \frac{1}{m}\left(\frac{\Gamma(m+\frac{1}{2})}{\Gamma(m)}\right)^2\right)$$

RAYLEIGH:

$$f_R(r) = \frac{r}{b}\exp\left(-\frac{r^2}{2b}\right) \quad r \geq 0 \qquad (\sigma^2 \stackrel{\text{def}}{=} b)$$

$$\bar{R} = \sigma\sqrt{\frac{\pi}{2}} = 1.2533\sqrt{b}$$

$$\overline{R^2} = 2\sigma^2 = 2b$$

$$\sigma_r^2 = \overline{R^2} - \bar{R}^2 = \sigma^2(2 - \pi/2) = \underline{0.4292\,b}$$

RICEAN:

$$f(x) = \frac{x}{\sigma^2} e^{-\frac{x^2+\nu^2}{2\sigma^2}} I_0\left(\frac{x\nu}{\sigma^2}\right)$$

$$E(x) = \sigma \sqrt{\frac{\pi}{2}}\, L_{1/2}\left(-\frac{\nu^2}{2\sigma^2}\right)$$

$$\mathrm{var}(x) = 2\sigma^2 + \nu^2 - \frac{\pi\sigma^2}{2} L_{1/2}^2\left(\frac{-\nu^2}{2\sigma^2}\right)$$

$L_\nu(x)$ – Laguerre polynomial

$\nu = 1/2$ $\qquad L_{1/2}(x) = e^{x/2}\left[(1-x) I_0\left(\frac{-x}{2}\right) - x I_1\left(\frac{-x}{2}\right)\right]$

• RELATIONSHIP BETWEEN RICE FACTOR AND NAKAGAMI SHAPE FACTOR

$$\boxed{K = \frac{\sqrt{m^2-m}}{m-\sqrt{m^2-m}}}$$

$$\boxed{m = \frac{(K+1)^2}{2K+1}}$$

ZA OVAA VREDNOST NA m NAKAGAMI PREODA VO RICIAN

ZA $1 \le m < \infty$

NAKAGAMI FEDINGOT E APROKSIMATIVNO ~~RA~~ SO V FEDING SO VAKVO „K"!

$\boxed{m = 2.7778}$

randg – GENERATE GAMMA RANDOM NUMBERS

NAKAGAMMI:

$$p(x) = \frac{2m^m x^{2m-1}}{\Gamma(m)\,\Omega^m} e^{-\frac{mx}{\Omega}}$$

• GAMMA DISTRIBUTION

$$f(x|a,b) = \frac{1}{b^a \Gamma(a)} x^{a-1} e^{-\frac{x}{b}}$$

$$p(x) = \sqrt{f(x)} = \frac{1}{b^{a/2}\sqrt{\Gamma(a)}} x^{\frac{a-1}{2}} e^{-\frac{x}{2b}}$$

$m(2K+1) = (K+1)^2 = K^2 + 2K + 1$

$K^2 + 2K + 1 - m(2K+1) = 0; \quad K^2 + 2K - m\cdot 2K + 1 - m = 0$

$K^2 + 2K(1-m) + (1-m) = 0$

$$K_{1,2} = \frac{-2(1-m) \pm \sqrt{4(1-m)^2 - 4(1-m)}}{2}$$

D. Middleton

$$p_N(x) = \frac{2m^m x^{2m-1}}{\Gamma(m)\cdot\Omega^m} e^{-\frac{mx}{\Omega}}$$

$$p_G(x) = \frac{1}{b^a \Gamma(a)} x^{a-1} e^{-\frac{x}{b}}$$

$$p_N(x) = 2m^m \cdot x^{m} \underbrace{\frac{x^{m-1}}{\Gamma(m)\cdot\Omega^m} e^{-\frac{x}{\Omega}}}_{p_G(x)} \cdot e^{-\frac{(m-1)x}{b}}$$

$$p_N(x) = 2m^m \cdot x^m \cdot e^{-\frac{(m-1)x}{b}} \underbrace{\frac{x^{m-1}}{\Gamma(m)\Omega^m} e^{-\frac{x}{\Omega}}}_{p_G(x)}$$

$$\frac{mx}{\Omega} = -\frac{(m-1)x}{\Omega} - \frac{x}{\Omega} = \frac{-mx + x - x}{\Omega} = -\frac{mx}{\Omega}$$

$$p_N(x) = 2m^m \cdot x^m e^{-\frac{(m-1)x}{\Omega}} \cdot p_G(x)$$

PROBAJ SO IZRAZOT ZA FUNKCIONALNA TRANSFORMACIJA NA GUSTINATA NA VEROJATNOST !!

$$p_\eta(y) = \frac{p_\xi(x)}{\frac{dy}{dx}} \Bigg|_{x=g(y)}$$

VIDI YP 20 N3

$$\Gamma(m+1) = m\cdot\Gamma(m)$$

$$\Gamma(m) = \int_0^\infty x^{m-1} e^{-x}\,dx$$

RAYLEIGH:

$$p_{ray}(x) = \frac{x}{\sigma^2} e^{-\frac{x^2}{2\sigma^2}} = \frac{x}{b} e^{-\frac{x^2}{2b}}$$

RICE:

$$p_{rice}(x) = \frac{x}{\sigma_N^2} e^{-\frac{x^2+A^2}{2\sigma_N^2}} I_0\left(\frac{A\cdot x}{\sigma_N^2}\right)$$

SLIKA ZEBUCHIN !!

(*) $A \cdot \left(\frac{K}{1+K} + \frac{1}{\sqrt{2}}\left(\text{randn}() * \frac{1}{\sqrt{1+K}} + j\,\text{randn}()\frac{1}{\sqrt{1+K}}\right)\right)$

$H = \text{sqrt}\left(\frac{K}{K+1}\right) * H\_d + \text{sqrt}\left(\frac{1}{K+1}\right) * H\_s$

$K = \frac{Pd}{Ps}$; $Pd + Ps = 1$

$H\_s = (\text{randn}(M,N) + j\,\text{randn}(M,N)) / \text{sqrt}(2)$

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IMPLEMENTACIJA MODA OD MATLAB:

$$\mu = \sqrt{\frac{K}{2(K+1)}} \qquad s = \frac{1}{\sqrt{2(K+1)}}$$

$$fad = \frac{1}{\sqrt{2(K+1)}} \operatorname{randn}(\cdot) + \sqrt{\frac{K}{2(K+1)}} + j\left(\frac{1}{\sqrt{2(K+1)}} \operatorname{randn}(\cdot) + \sqrt{\frac{K}{2(K+1)}}\right)$$

(**)

OVA E VIŠOK VO SPORED. DA SO TEKUCHIV SLIKATA

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$$H = \underbrace{A\cdot\sqrt{\frac{K}{K+1}} + \sqrt{\frac{1}{2(K+1)}}\left(\operatorname{randn}(\cdot) + j \operatorname{randn}(\cdot)\right)}_{\text{ODGOVARA NA SLIKATA OD TEKUČIM}}$$

PRI VAKVO A
$$\boxed{K = \frac{abs(a)^2}{2\sigma^2}}$$

K - VO DECIBELI !!!

SEGA ZA DA SITE TU SE VO SOGLASNOST

$$A = a(1+j)$$

$$H = \operatorname{randn}(\cdot)\sqrt{\frac{1}{2(K+1)}} + a\sqrt{\frac{K}{K+1}} + j\left(\operatorname{randn}(\cdot)\frac{1}{\sqrt{2(K+1)}} + a\sqrt{\frac{K}{K+1}}\right)$$

ŠTO E IDENTIČNO NA (*) I (**)

$$C_1 = \sqrt{\frac{K}{K+1}} \qquad C_2 = \frac{1}{\sqrt{K+1}}$$

$$H = \left(\frac{C_2}{\sqrt{2}}\operatorname{randn}(\cdot) + a\cdot C_1\right) + j\left(\frac{C_2}{\sqrt{2}}\operatorname{randn}(\cdot) + aC_1\right)$$

$$P_d + P_s = 1$$

$$\text{Bessel } I\left(0, \frac{Ax}{\sigma^2}\right) = I_0\left(\frac{Ax}{\sigma^2}\right) = \frac{1}{2\pi}\int_0^{2\pi} e^{\frac{Ax}{\sigma^2}\cos\varphi}\, d\varphi$$

RAISON

$$p(r) = \frac{r}{\sigma^2} e^{-\frac{r^2+r_s^2}{2\sigma^2}} I_0\left(\frac{r r_s}{\sigma^2}\right)$$

$$p_r(r) = \frac{2r\,10^{K/10}}{r_s^2}\exp\left(-\frac{10^{K/10}}{r_s^2}(r^2+r_s^2)\right) I_0\left(\frac{2r\,10^{K/10}}{r_s}\right)$$

$$K(dB) = 10\log\left(\frac{r_s^2}{2\sigma^2}\right)$$

$$p_{ric}(x) = \frac{x}{\sigma^2} e^{-\frac{x^2+a^2}{2\sigma^2}} I_0\left(\frac{ax}{\sigma^2}\right)$$

$$K = \frac{a^2}{2\sigma^2} \Rightarrow p(x) = \frac{2\cdot x}{a^2} \underbrace{\frac{a^2}{2\sigma^2}}_{K} e^{-\frac{x^2+a^2}{a^2}\cdot K} I_0\left(\frac{a\cdot x}{a^2}\cdot 2K\right)$$

$$\boxed{\sigma^2 = \frac{a^2}{2K}}$$

$$\boxed{p(x) = \frac{2x\cdot K}{a^2} e^{-(x^2+a^2)\frac{K}{a^2}} \cdot I_0\left(\frac{2Kx}{a}\right)}$$

RICIAN IMPLEMENTATION MATLAB:

$z_1 = z_1 \cdot \sqrt{K}$; $z_1 = \exp(j 2\pi \varepsilon_1)$ — FAZATA NA PRESRETNIOT PAT

$$\underline{z(1,:) = (z(1,:) + z_1)/\mathrm{sqrt}(K+1)}$$

$z_1 = 1$

$$z(1,:) = (a + jb + \sqrt{K})/\mathrm{sqrt}(K+1)$$

$$y(t) = \sum_{k=1}^{N} a_k(\tau,t) \cdot x(t-\tau_k)$$

$$x(t) = \mathrm{Re}\{\tilde{x}_b(t)\cdot e^{+j\omega_c t}\}$$

$$x(t-\tau_k) = \mathrm{Re}\{\tilde{x}_b(t-\tau_k) e^{j2\pi f_c t} \cdot e^{-j2\pi f_c \tau_k}\}$$

$$x(t-\tau_k) = \mathrm{Re}\{x_b(t-\tau_k)\cdot e^{-j2\pi f_c \tau_k} e^{-j2\pi f_c t}\}$$

$$y(t) = \sum_{k=1}^{N} a_k(\tau,t)\,\mathrm{Re}\{x_b(t-\tau_k) e^{-j2\pi f_c \tau_k} e^{-j2\pi f_c t}\}$$

$$= \mathrm{Re}\left\{\sum_{k=1}^{N} \underbrace{a_k(\tau,t)\, e^{-j2\pi f_c \tau_k}}_{\tilde{a}_k(\tau,t)} x_b(t-\tau_k)\, e^{-j2\pi f_c t}\right\}$$

$$\tilde{Y} = \sum_{k=1}^{N} \tilde{a}_k(\tau,t)\, x_b(t-\tau_k) = \sum_{k=1}^{N} \tilde{a}_k(\tau,t)\, \tilde{x}(t-\tau_k)$$

$x_b = \tilde{x}$

$$\tilde{c}(\tau, t) =: \sum_{k=1}^{\infty} \tilde{a}_k(\tau, t)\,\delta(\tau-\tau_k)$$

$$h_b(t) = \sum_{k=1}^{\infty} a_k \cdot \exp\left(j(2\pi f_c \tau_k + \theta_k)\right)$$

$$\boxed{\theta_k = 2\pi f_d \tau_k} = \sum_{k=1}^{\infty} a_k \exp\left[j 2\pi (f_c + f_d)\tau_k\right]$$

$\theta = f_{d_1} \cdot T_s$ — DOPPLER SHIFT FOR FIRST COMPONENT; T_s — SAMPLING PERIOD

$90^\circ \cdot \frac{\pi}{180} = \frac{\pi}{2}$ rad $\qquad 360^\circ = 2\pi$ rad

$1 \text{ rad} = \frac{360^\circ}{2\pi} = \frac{180^\circ}{\pi}$

frmSize

$\theta_1 = [\theta_{init}, \text{repmat}(\theta, [1, N])]$

$\theta_1 = \text{cumsum}(\theta_1)$

$z_1 = \exp(j 2\pi \theta_1)$

NE POTPU ZA NEGATIVNI KOR. NA r

MMV

RAYLEIGH POWER DISTRIBUTION

$p(r) = \frac{r}{\sigma^2} e^{-\frac{r^2}{2\sigma^2}} \qquad x = r^2 \qquad r = \pm\sqrt{x}$

$$p(x) = \frac{p(r)}{\frac{dx}{dr}}\Bigg|_{r=f(x)} = \frac{\frac{r}{\sigma^2} e^{-\frac{r^2}{2\sigma^2}}}{2r}\Bigg|_{r=\pm\sqrt{x}} = \frac{\frac{r}{\sigma^2} e^{-\frac{r^2}{2\sigma^2}}}{2\sqrt{x}}$$

$$= \frac{\cancel{\sqrt{x}}\, e^{-x/2\sigma^2}}{2\sigma^2 \cancel{\sqrt{x}}} = \frac{e^{-\frac{x}{2\sigma^2}}}{2\sigma^2}$$

$x \equiv \gamma$ DIMENSION OF POWER

$E(r^2) = \int_0^{\infty} r^2 p(r)\,dr = 2\sigma^2 \Rightarrow$ VKUPNA SREDNA PRIEMNA SNAGA (NA SVETOT). VIDI: PowRayleighSingleHop.m

$$\boxed{p(x) = \frac{e^{-\frac{x}{P_r}}}{P_r} = \frac{e^{-\frac{x}{\bar{\gamma}}}}{\bar{\gamma}} = \frac{e^{-\frac{\gamma}{\bar{\gamma}}}}{\bar{\gamma}}}$$

FUNKCIJA NA GUSTINA NA VEROJATNOST NA SNAGATA NA SIGNALOT

• RICIAN FADING

$$p(r) = \frac{r}{\sigma^2} e^{-\frac{r^2+A^2}{2\sigma^2}} I_0\left(\frac{A\cdot r}{\sigma^2}\right) \qquad \boxed{a = \frac{A}{\sigma}} \quad \boxed{z = \frac{r}{\sigma}} \quad \frac{dz}{dr} = \frac{1}{\sigma}$$

$$r = \sigma z$$

$$p(z) = \frac{p(r)}{\frac{dz}{dr}}\Bigg|_{r=f(z)} = \sigma \frac{r}{\sigma^2} e^{-\frac{z^2+A^2/\sigma^2}{2}} I_0\left(\frac{A}{\sigma} z\right)$$

$$p(z) = z\, e^{-\frac{z^2+a^2}{2}} I_0(a,z) \qquad z = \sqrt{\gamma} \text{ — POWER}$$

$$p(z<z_0) = \int_0^{z_0} z\, e^{-\frac{z^2+a^2}{2}} I_0(a,z)\, dz =$$

$$= \underbrace{\int_0^{\infty} z\, e^{-\frac{z^2+a^2}{2}} I_0(a,z)\, dz}_{1} - \int_{z_0}^{\infty} z\, e^{-\frac{z^2+a^2}{2}} I_0(a,z)\, dz$$

$$p(z<z_0) = 1 - \int_{z_0}^{\infty} z\, e^{-\frac{z^2+a^2}{2}} I_0(a,z)\, dz = 1 - Q(a,z_0)$$

z_0 — VOLTAGE RATIO THRESHOLD

$Q(a,z_0)$ — MARCUM'S Q FUNCTION

ŠTOM ZA IMAŠ ZA NAPON MOŽEŠ DA GO KONVERTIRAŠ PRAGOT ZA SNAGA.

• RAYLEIGH OUTAGE PROBABILITY

$$P(\gamma<\gamma_0) = \int_0^{\gamma_0} \frac{1}{\bar\gamma} e^{-\frac{\gamma}{\bar\gamma}} d\gamma = \int_0^{\gamma_0/\bar\gamma} e^{-\frac{\gamma}{\bar\gamma}} d\left(\frac{\gamma}{\bar\gamma}\right)$$

$$\mu = \frac{\gamma}{\bar\gamma} \qquad d\mu = \frac{d\gamma}{\bar\gamma} \qquad \gamma = \gamma_0 \quad \mu = \frac{\gamma_0}{\bar\gamma}$$

$$P(\gamma<\gamma_0) = -e^{-\frac{\gamma}{\bar\gamma}}\Big|_0^{\gamma_0/\bar\gamma} = -e^{-\gamma_0/\bar\gamma} + e^0 = 1 - e^{-\gamma_0/\bar\gamma}$$

$$\int e^{-x} dx = \left| \begin{array}{l} y=-x \\ dy=-dx \end{array} \right| = -e^{-x}$$

$n = 1, \dots, N-1$ OUTAGE PROBABILITY OF MULTIHOP TRANSM. OVER NAKAGAMI ∃ RELAYS

$$G_n^2 = \frac{1}{\alpha_n^2 + N_{0,n}} \Big\} \text{ GAIN OF THE } n\text{-th relay}$$

α_n - FADING AMPLITUDE OF THE PREVIOUS HOP

N_{0n} - Gaussian noise at input of the n-th relay

END-END SNR

$$\gamma_{eq1} = \left[\prod_{n=1}^{N} \left(1 + \frac{1}{\gamma_n}\right) - 1 \right]^{-1}$$

$$\boxed{\gamma_n = \frac{\alpha_n^2}{N_{0n}}} \Rightarrow \text{SNR of } n\text{-th hop}$$

$$\frac{1}{\gamma_{eq1}} = \frac{1}{\gamma_1} + \frac{1}{\gamma_2} + \frac{1}{\gamma_3} + \frac{1}{\gamma_1\gamma_2} + \frac{1}{\gamma_1\gamma_3} + \frac{1}{\gamma_2\gamma_3}$$

UPPER BOUND:

$$\gamma_{eq2} = \left[\sum_{n=1}^{N} \frac{1}{\gamma_n} \right]^{-1} \quad \text{VIDI pg. 160}$$

SNR OF N HOP SYSTEM WITH GAIN

$$\boxed{G_n^2 = \frac{1}{\alpha_n^2}}$$

$$\gamma_{eq2} = \frac{\mu_H}{N} \qquad \mu_H = \frac{N}{\sum_{n=1}^{N} \frac{1}{\gamma_n}}$$

• NON REGENERATIVE SYSTEM

$$P_{out} = P_r(\gamma_{eq} < \gamma_{th}) = P_r\left(\frac{1}{\gamma_{eq}} > \frac{1}{\gamma_{th}}\right) =$$

$$= 1 - L^{-1}\left(\frac{M_{1/\gamma_{eq}}(s)}{s} \right) \Bigg|_{1/\gamma_{th}}$$

$L^{-1}(\cdot)$ - INVERSE LAPLACE TRANSFORM

$M_{1/\gamma_{eq}}(\cdot)$ - MOMENT GENERATING FUNCTION

$$M_X(t) = E[e^{tX}] = \int_{-\infty}^{\infty} e^{tx} p(x)\, dx, \quad t \in \mathbb{R}$$

X - RAND. VARIABLE

$$M_{1/\gamma_{eq}}(s) = \int_{-\infty}^{\infty} e^{\frac{s}{\gamma_{eq}}} p\left(\frac{1}{\gamma_{eq}}\right) d\left(\frac{1}{\gamma_{eq}}\right)$$

$$\left(\frac{1}{x}\right)' = (x^{-1})' = -1 \cdot x^{-1-1} = -\frac{1}{x^2}$$

- REGENERATIVE SYSTEM

$$P_{OUT} = P_r[\, Min(\gamma_1, \dots, \gamma_N) < \gamma_{th}] =$$

$$= 1 - P_r[\gamma_1 > \gamma_{th}, \gamma_2 > \gamma_{th}, \dots, \gamma_n > \gamma_{th}]$$

$$p(x) = \frac{2m^m \cdot x^{2m-1}}{\Gamma(m)\,\Omega^m} e^{-\frac{m\cdot x^2}{\Omega}}$$

$$P_{out} = 1 - \prod_{n=1}^{N}\left(1 - \frac{\Gamma\left(m_n, \frac{m_n \gamma_{th}}{\bar{\gamma}_n}\right)}{\Gamma(m_n)}\right)$$

$$\Gamma(\alpha) = \int_0^\infty t^{\alpha-1} e^{-t}\, dt \qquad \Gamma(\alpha+1) = \alpha \cdot \Gamma(\alpha)$$

$$\Gamma(\alpha, x) = \int_x^\infty t^{\alpha-1} e^{-t}\, dt$$

$$p(x) = \frac{2m^m x^{2m-1}}{\Gamma(m)\Omega^m} e^{-m\left(\frac{x}{\Omega}\right)^2}$$

$$\gamma = \frac{x}{\Omega} \qquad p(\gamma) = \frac{p(x)}{\frac{d\gamma}{dx}}\Bigg|_{x = \Omega\cdot\gamma}$$

$$p(\gamma) = \frac{2m^m \cdot (\Omega\cdot\gamma)^{2m-1}}{\Gamma(m)\cdot\Omega^m} e^{-m\gamma^2} \cdot \frac{1}{\frac{1}{\Omega}} =$$

$$= \frac{2m^m\, \Omega^{\cancel{2m-1}\, m}\, \gamma^{2m-1}}{\Gamma(m)\cdot \cancel{\Omega^{m-1}}} e^{-m\gamma^2} \qquad 2m - 1 - m + 1 = m$$

$$P(\gamma > \gamma_{th}) = \int_{\gamma_{th}}^{\infty} p(\gamma)\, d\gamma = \int_{\gamma_{th}}^{\infty} \frac{2m^m \cdot \Omega^m \gamma^{2m-1}}{\Gamma(m)} e^{-m\gamma^2} d\gamma$$

$$\gamma = \left(\frac{x}{\Omega}\right)^2 = \frac{x^2}{\Omega^2} \qquad d\gamma = \frac{2x}{\Omega^2} dx$$

$x = \pm\Omega\sqrt{\gamma}$ → NE POSTOI ZA NEG. VREDNOSTI

$$p(\gamma) = \frac{2m^m (x^2)^m \cdot x^{-1}}{\Gamma(m)\cdot\Omega^m} e^{-m\cdot\gamma} \frac{1}{\frac{2x}{\Omega^2}}\Bigg|_{x = \Omega\sqrt{\gamma}}$$

$$p(\gamma) = \frac{2m^m \left(\frac{x^2}{\Omega^2}\right)^m \cdot \Omega^{2m} \cdot \frac{1}{x}}{\Gamma(m) \cdot \Omega^m} e^{-m\cdot\gamma} \frac{\Omega^2}{2x} =$$

$$= \frac{2m^m \gamma^m \cdot \Omega^{\cancel{2m}}}{\Gamma(m) \cancel{\Omega^m}} e^{-m\cdot\gamma} \frac{\Omega^2}{2x^2} = \frac{\cancel{2}m^m \gamma^m \cdot \cancel{\Omega^{2m}}}{\Gamma(m)} e^{-m\gamma} \frac{1}{\cancel{2}\gamma}$$

$$p(\gamma) = \frac{m^m \cdot \gamma^{m-1} \Omega^m}{\Gamma(m)} e^{-m\gamma}$$

PROVERKA:

$$p(\gamma) = \frac{\cancel{2}m^m \cdot (\Omega^2 \cdot \gamma)^m}{\Gamma(m) \cdot \Omega^m} e^{-m\cdot\gamma} \cdot \frac{\cancel{\Omega^2}}{\cancel{2}x^2} =$$

$$= \frac{m^m \cdot \Omega^{2m} \gamma^m}{\Gamma(m) \cdot \cancel{\Omega^m}} e^{-m\cdot\gamma} \frac{1}{\gamma} = \frac{m^m \cdot \Omega^m \cdot \gamma^{m-1}}{\Gamma(m)} e^{-m\gamma}$$

$$p(\gamma) = \frac{m^m \cdot \Omega^m \cdot \gamma^{m-1}}{\Gamma(m)} e^{-m\gamma}$$

$$p(\gamma > \gamma_{th}) = \int_{\gamma_{th}}^{\infty} p(\gamma)\, d\gamma = \int_{\gamma_{th}}^{\infty} \frac{m^m \cdot \Omega^m \cdot \gamma^{m-1}}{\Gamma(m)} e^{-m\gamma} \cdot d\gamma$$

$$P(\gamma > \gamma_{th}) = \frac{m^m \cdot \Omega^m}{\Gamma(m)} \int_{\gamma_{th}}^{\infty} \gamma^{m-1} e^{-m\gamma} d\gamma$$

$$t = m\cdot\gamma \qquad d\gamma = \frac{dt}{m} \qquad \gamma = \gamma_{th} \quad t = m\cdot\gamma_{th}$$
$$\gamma = \frac{t}{m} \qquad \gamma \to \infty \quad t \to \infty$$

$$P(\gamma > \gamma_{th}) = \frac{\cancel{m^m}\, \Omega^m}{\Gamma(m)} \int_{m\cdot\gamma_{th}}^{\infty} \frac{t^{m-1}}{\cancel{m^{m-1}}} e^{-t} \frac{dt}{\cancel{m}}$$

$$\boxed{P(\gamma > \gamma_{th}) = \frac{\Omega^m \cdot \Gamma(m, m\gamma_{th})}{\Gamma(m)}}$$

$\Omega^m \Rightarrow$ OVA E VIŠOK VO POGLED NA IZLEZOT OD ALOUTI.

$$\boxed{\begin{aligned} P_{OUT} &= 1 - P[\gamma_1 > \gamma_{th}, \gamma_2 > \gamma_{th}, \ldots, \gamma_N > \gamma_{th}] = \\ &= 1 - \Omega^{N\cdot m} \prod_{n=1}^{N} \frac{\Gamma(m_n, m_n \gamma_{th_n})}{\Gamma(m_n)} \end{aligned}}$$

$$p(\gamma) = \frac{m^m \Omega^m \cdot \gamma^{m-1}}{\Gamma(m)} e^{-m\gamma} \quad (*)$$

$$P(\gamma > \gamma_{th}) = \int_{\gamma_{th}}^{\infty} \frac{m^m \Omega^m \gamma^{m-1}}{\Gamma(m)} e^{-m\gamma} \cdot d\gamma =$$

$$= \frac{m^m \Omega^m}{\Gamma(m)} \int_{\gamma_{th}}^{\infty} \gamma^{m-1} e^{-m\gamma} d\gamma = \left| \begin{matrix} t = m\gamma & dt = m \cdot d\gamma \\ d\gamma = \frac{dt}{m} & \gamma = \frac{t}{m} \\ \gamma = \gamma_{th} & t = m \cdot \gamma_{th} \end{matrix} \right| =$$

$$= \frac{\cancel{m^m} \Omega^m}{\Gamma(m)} \int_{m\gamma_{th}}^{\infty} \frac{t^{m-1}}{\cancel{m^{m-1}}} e^{-t} \frac{dt}{\cancel{m}} = \frac{\Omega^m}{\Gamma(m)} \Gamma(m, m\gamma_{th})$$

(B) MGF OF $\frac{1}{\gamma_{eq}}$

$$M_x(t) = E[e^{tx}] \quad t \in \mathbb{R}$$

$$\cancel{K_\nu(z) = \int_0^\infty e^{-z\,ch\,t} ch\,\nu t\,dt \quad |\arg z| < \frac{\pi}{2}, \ Re\,z = 0}$$

Gradshtayn 3.471.9

$$\int_0^\infty x^{\nu-1} e^{-\frac{\beta}{x} - \gamma x} dx = 2\left(\frac{\beta}{\gamma}\right)^{\frac{\nu}{2}} K_\nu(2\sqrt{\beta\gamma}) \quad \begin{matrix} Re\,\beta > 0 \\ Re\,\gamma > 0 \end{matrix} \quad (A)$$

$$\cancel{M_{\frac{1}{\gamma}}(s) = E[e^{s/\gamma}] = \int_0^\infty e^{t/\gamma} \frac{m^m \gamma^{m-1} \Omega^m}{\Gamma(m)} e^{-m\gamma} d\gamma}$$

$$M_{\frac{1}{\gamma}}(s) = \int_0^\infty \frac{m^m \Omega^m}{\Gamma(m)} s^{m-1} \cdot e^{-ms} \cdot e^{\frac{s}{\gamma}} ds =$$

$$M_{\frac{1}{\gamma}}(s) = \frac{m^m \Omega^m}{\Gamma(m)} \int_0^\infty s^{m-1} e^{-ms + \frac{s}{\gamma}} ds$$

$\varphi\left(\frac{1}{\gamma}\right) = ?$ $\qquad \varphi(x) = \frac{2 m^m x^{2m-1}}{\Gamma(m)\,\Omega^m} \cdot e^{-\frac{m x^2}{\Omega}}$

$$(*) \rightarrow p(z) = \frac{m^m \Omega^m z^{m-1}}{\Gamma(m)} e^{-mz} \qquad \boxed{\cancel{\gamma} = \frac{1}{\cancel{z}}} \qquad \frac{d\gamma}{dz} = -\frac{1}{z^2}$$

$$\cancel{dz = -\gamma^{-2} d\gamma \quad d\gamma = -z^{-2} dz} \qquad \boxed{\gamma = \frac{1}{z}}$$

$$p(\gamma) = \left.\frac{p(z)}{\frac{d\gamma}{dz}}\right|_{z=\frac{1}{\gamma}} = \frac{m^m \Omega^m \left(\frac{1}{\gamma}\right)^{m-1} e^{-\frac{m}{\gamma}}}{-1/z^2 \; \Gamma(m)}$$

$$p(\gamma) = \frac{m^m \Omega^m}{-\gamma^{m-1}\cdot \gamma^2 \,\Gamma(m)} \cdot e^{-\frac{m}{\gamma}} = -\frac{m^m \Omega^m}{\Gamma(m)\cdot \boxed{\gamma^{m+1}}} e^{-\frac{m}{\gamma}}$$

$$M_{1/\gamma}(s) = -\frac{m^m \Omega^m}{\Gamma(m)} \int_0^\infty e^{\frac{s}{\gamma}} \gamma^{-m-1} e^{-\frac{m}{\gamma}} d\gamma$$

$$M_{\frac{1}{\gamma}}(s) = E\left[e^{\frac{s}{\gamma}}\right] = \int_{-\infty}^{\infty} e^{\frac{s}{\gamma}} \underline{p(\gamma)}\, d\gamma = \quad \leftarrow (*)$$

$$= \int_0^\infty e^{\frac{s}{\gamma}} \frac{m^m \Omega^m \gamma^{m-1}}{\Gamma(m)} e^{-m\gamma} d\gamma = \frac{m^m \Omega^m}{\Gamma(m)} \int_0^\infty \gamma^{m-1} e^{\frac{s}{\gamma} - m\gamma} d\gamma$$

GO ISUSTAN ①

$$= \frac{m^m \Omega^m}{\Gamma(m)} \cdot 2 \left(\frac{\Theta s}{m}\right)^{\frac{m}{2}} K_m\left(2\sqrt{\Theta s \cdot m}\right) \Rightarrow$$

$$M_{\frac{1}{\gamma}}(s) = \frac{2\, m^{m-\frac{m}{2}}}{\Gamma(m)} \cdot s^{\frac{m}{2}} K_m\left(2\sqrt{s\cdot m}\right)$$

$$\boxed{M_{\frac{1}{\gamma}}(s) = \frac{2}{\Gamma(m)} \cdot (m\cdot s)^{\frac{m}{2}} K_m\left(2\sqrt{s\cdot m}\right)}$$

OK!!

$$\boxed{\bar{\gamma} = \int_0^\infty \gamma\, p_\gamma(\gamma)\, d\gamma} \Rightarrow$$

AVERAGE SNR

ALOUINI'S BOOK pp. 4

$$p(x) = \frac{2m^m \cdot x^{2m-1}}{\Gamma(m)\cdot \Omega^m} \cdot e^{-\frac{m x^2}{\Omega}}$$

$x \rightarrow$ SIGNAL

$\Omega = E(x^2) \rightarrow$ AVERAGE RECEIVED POWER

- VOVEDUVAM PROMENLIVA „n" - NOISE

$$p(x) = \frac{2m^m \cdot \frac{x^{2m-1}}{n^{2m-1}} \cdot n^{2m-1}}{\Gamma(m)\cdot \Omega^m} \cdot e^{-m\frac{\frac{x^2}{n^2}}{\frac{\Omega}{n^2}}} =$$

$$p(x) = \frac{2m^m \left(\frac{x}{n}\right)^{2m} \left(\frac{x}{n}\right)^{-1}}{\Gamma(m)\cdot \frac{\Omega^m}{(n^2)^m}\cdot n} e^{-\frac{m\left(\frac{x}{n}\right)^2}{\frac{\Omega}{n^2}}}$$

$$\left(\frac{\Omega}{n^2}\right) = \bar{\gamma} = \int_0^\infty \frac{x^2}{n^2} p(x)\, dx = \frac{1}{n^2} \underbrace{\int_0^\infty x^2 p(x)\, dx}_{\Omega} \quad \text{const}$$

$E(x^2) = \Omega$

$$p(x) = \frac{2m^m \left(\frac{x^2}{n^2}\right)^{m} \left(\frac{x}{n}\right)^{-1}}{\Gamma(m)\cdot \bar{\gamma}^m \cdot n} e^{-\frac{m\left(\frac{x}{n}\right)^2}{\bar{\gamma}}} \qquad \left(\frac{x}{n}\right)^{-1} = \frac{1}{\sqrt{\gamma}}$$

$\gamma = \frac{x^2}{n^2} \qquad d\gamma = \frac{2x}{n^2} dx \qquad x = \pm n\sqrt{\gamma}$ POSTOI

$$p(\gamma) = \frac{\cancel{2} m^m \cdot \gamma^{m} \frac{1}{\sqrt{\gamma}}}{\Gamma(m)\cdot \bar{\gamma}^m \cdot \cancel{n}} e^{-\frac{m\gamma}{\bar{\gamma}}} \cdot \frac{1}{\frac{\cancel{2}\, n\sqrt{\gamma}}{n^2}}$$

$$p(\gamma) = \frac{m^m \gamma^{m}}{\Gamma(m)\cdot \bar{\gamma}^m (\sqrt{\gamma})^2} \cdot e^{-\frac{m\gamma}{\bar{\gamma}}} = \frac{m^m \gamma^{m-1}}{\Gamma(m)\cdot\bar{\gamma}^m} e^{-\frac{m\gamma}{\bar{\gamma}}}$$

$\gamma = x^2 \qquad \frac{d\gamma}{dx} = 2x \qquad x = \pm\sqrt{\gamma}$

SKRATENA VARIJANTA

$$p(\gamma) = \frac{2m^m \cdot x^{2m-1}}{\Gamma(m)\cdot \Omega^m} \cdot e^{-\frac{m\gamma}{\Omega}} \frac{1}{2\sqrt{\gamma}} =$$

$$= \frac{\cancel{2} m^m \gamma^m \cdot (\sqrt{\gamma})^{-1}}{\Gamma(m)\cdot \Omega^m} \cdot e^{-\frac{m\gamma}{\Omega}} \cdot \frac{1}{\cancel{2}\sqrt{\gamma}} = \frac{m^m \gamma^{m-1}}{\Gamma(m)\cdot \Omega^m} e^{-\frac{m\gamma}{\Omega}}$$

PLOVERKA:

$$p(x) = \frac{2\cdot m^m \cdot x^{2m-1}}{\Gamma(m)\cdot \Omega^m}\cdot e^{-\frac{m x^2}{\Omega}}$$

$$\gamma = \frac{x^2}{n^2} \qquad \frac{d\gamma}{dx} = \frac{2x}{n^2} \qquad \boxed{x = n\sqrt{\gamma}}$$

$$p(\gamma) = \frac{2\cdot m^m \cdot (n\sqrt{\gamma})^{2m}\cdot (n\sqrt{\gamma})^{-1}}{\Gamma(m)\cdot \Omega^m}\cdot e^{-\frac{m\cdot n^2\gamma}{\Omega}} \cdot \frac{1}{\frac{2\cdot n\sqrt{\gamma}}{n^2}}$$

$$= \frac{2\cdot m^m \cdot n^{2m}\gamma^m}{\Gamma(m)\Omega^m \cdot n\sqrt{\gamma}\cdot \frac{2\sqrt{\gamma}}{n}}\, e^{-\frac{m\gamma}{\frac{\Omega}{n^2}}} =$$

$$\boxed{MMV}$$

$$= \frac{m^m\cdot \gamma^{m-1}}{\Gamma(m)\left(\frac{\Omega}{n^2}\right)^m}\cdot e^{-\frac{m\gamma}{\bar{\gamma}}}$$

NAKAGAMI DISTRIBUTION OF SNR PER SYMBOL

$$\boxed{p(\gamma) = \frac{m^m\, \gamma^{m-1}}{\Gamma(m)\,\bar{\gamma}^m}\, e^{-\frac{m\gamma}{\bar{\gamma}}} \quad \boxed{MMV}}$$

<u>MMV</u>

$$P(\gamma > \gamma_{th}) = \int_{\gamma_{th}}^{\infty} \frac{m^m \gamma^{m-1}}{\Gamma(m)\bar{\gamma}^m}\, e^{-\frac{m\gamma}{\bar{\gamma}}}\, d\gamma =$$

$$= \frac{m^m}{\Gamma(m)\,\bar{\gamma}^m}\int_{\gamma_{th}}^{\infty} \gamma^{m-1} e^{-\frac{m\gamma}{\bar{\gamma}}}\, d\gamma = \left|\begin{array}{l} t = \frac{m\gamma}{\bar{\gamma}} \quad \gamma = \frac{\bar{\gamma}t}{m} \\ dt = \frac{m}{\bar{\gamma}}\, d\gamma \\ \gamma \to \gamma_{th} \\ t = \frac{m\gamma_{th}}{\bar{\gamma}} \end{array}\right| =$$

$$= \frac{\cancel{m^m}}{\Gamma(m)\bar{\gamma}^m}\int_{\frac{m\gamma_{th}}{\bar{\gamma}}}^{\infty} \frac{(\bar{\gamma}t)^{m-1}}{\cancel{m^{m-1}}}\cdot e^{-t}\, \frac{\bar{\gamma}}{\cancel{m}}\, dt = \frac{\cancel{\bar{\gamma}^m}}{\Gamma(m)\cancel{\bar{\gamma}^m}}\int_{\frac{m\gamma_{th}}{\bar{\gamma}}} t^{m-1} e^{-t}\, dt$$

$$\boxed{P(\gamma > \gamma_{th}) = \frac{\Gamma\left(m, \frac{m\gamma_{th}}{\bar{\gamma}}\right)}{\Gamma(m)} \quad \boxed{MMV}}$$

DOKAZANO!!

$$\boxed{K = ld\, M}$$

$$W = \cancel{\ldots}\cdot f_g$$

$$E_s/N_0 = \frac{P_s\cdot T_{SYM}}{N_0\cdot W\cdot T_{SYM}} = \frac{P_s\cdot T_b\cdot K}{N_0\cdot \frac{f_s}{2}\cdot T_{SYM}}$$

$$f_{max} = \frac{f_s}{2} = f_g$$

$$E_sN_0 = \frac{P_s \cdot T_{SYM}}{N_0 \cdot W \cdot T_{SYM}} = \frac{P_s \cdot T_B \cdot \log M}{N_0 \cdot \frac{f_s}{2} T_{SYM}} = \frac{E_b \cancel{\log M} \to K}{N_0 \cdot \frac{f_s}{2} \cdot T_{SYM}}$$

$$\rho^2 = \frac{A^2}{\sigma_N^2} = \frac{E_b}{N_0/2} \Big\} \text{DTK}$$

$$E_sN_0|_{dB} = 10 \log \frac{E_b}{N_0} + 10\log(K) + 10\log \frac{1}{\frac{T_{SYM}}{2T_{SAMP}}}$$

$$E_sN_0|_{dB} = E_bN_0|_{dB} + 10\log K - 10\log \frac{T_{SYM}}{2T_{SAMP}}$$

$$\boxed{E_bN_0|_{dB} = E_sN_0|_{dB} + 10\log \frac{T_{SYM}}{2 \cdot T_{SAMP}} - 10\log K}$$

$$SNR|_{SYM} = \frac{P_s}{N_0 \cdot W} = \frac{E_{SYM}/T_{SYM}}{N_0/2 T_{SAM}} = \frac{E_{SYM}}{N_0} \cdot \frac{2 \cdot T_{SAM}}{T_{SYM}}$$

$$\boxed{SNR|_{SYM} = \frac{E_{SYM}}{N_0} \frac{2 \cdot T_{SAM}}{T_{SYM}}}$$

$$SNR\_dB|_{SYM} = \underbrace{10\log \frac{E_{SYM}}{N_0}}_{E_sN_0} + 10\log \frac{2T_{SAMP}}{T_{SYM}} =$$

$\cancel{10 \log K}$

$$SNR|_{SYM} = \frac{E_b \cdot \log M / T_{SYM}}{N_0/2 T_{SAM}} = \frac{E_b \cdot K}{N_0} \frac{2 \cdot T_{SAM}}{T_{SYM}}$$

$$SNR^{dB}|_{SYM} = 10\log \frac{E_b}{N_0} + 10\log K - 10\log \frac{T_{SYM}}{2 \cdot T_{SAMP}}$$

$$\boxed{E_bN_0\text{-}dB = SNR\_dB|_{SYM} - 10\log K + 10\log \frac{T_{SYM}}{2 \cdot T_{SAM}}}$$

$$SNR|_{bit} = \frac{E_b/T_b}{N_0/2 T_{SAMP}} \qquad SNR_{dB}|_{bit} = E_bN_0 - 10\log \frac{T_b}{2 \cdot T_{SAMP}}$$

~~USTE EDMAS~~:

$$SNR|_{SYM} = \frac{E_s/T_{SYM}}{N_0 \cdot W} = \left| W = \frac{f_s}{2} \right| = \frac{E_s/T_{SYM}}{N_0 \cdot \frac{1}{2T_{SAM}}}$$

$$SNR|_{SYM} = \frac{E_s}{N_0} \cdot \frac{2T_{SAM}}{T_{SYM}} = \frac{K \cdot E_b}{N_0} \frac{2T_{SAM}}{T_{SYM}}$$

$$\frac{E_b}{N_0} = \frac{S}{N}\Big|_{SYM} \cdot \frac{T_{SYM}}{2T_{SAM}} \cdot \frac{1}{K} \to E_bN_0|_{dB} = SNR|_{dB} + 10\log \frac{0.5\,T_{SYM}}{T_{SAM}} + 10\log \dots$$

22750
370 ×61.5
14/09

XENCRYPT CONTROL
PERSITS SOFTWARE

JOCO - SODA
(246 + 145) × 2 782

DNEVNA
(125 + 125) × 4 1000
(227 + 105) × 4 1328

SVEZE IVAN
(247 + 144) × 2 782

3892 cm
38.92 m

ADVOKAT ZA VODENJE USTAVNA POSTAPKA:
STANISLAV (STANOE) FILIPOV

ADVOKAT DRAGI (BRATUČED NA IVAN)
075 402 406

179

$SNR = e^{0.1\,SNR_{dB} \ln 10} = 10^{0.1\,SNR_{dB}}$

IVAN STOŠIĆ

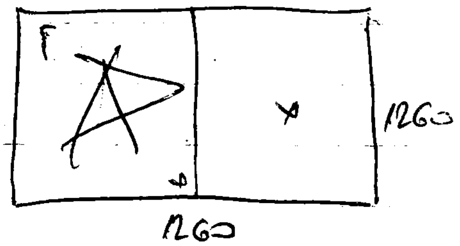
072236448

www.pazar3.com.mk

$40075\ km/24h = \frac{40075}{24 \cdot 3600} = \frac{0.4638\ km}{sec} = 463{,}8\ \frac{m}{s}$

3091050 LSDN

1260 x 1260



5x1260
3x1260

101
35
25
160 9...

t-pe 13/09

VODA Kozze	✓	727,00 MKD	474 + 254,50 = 728,50
EVN Kozze	✓		
TMK	✓		
MKT	✓		
EVN Ohrid		1.242,50 MKD	
EVN Nerezi	2,3,4	5 — 249,00 MKD	

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