Outage probability of dual-hop MIMO relay systems with direct links

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Abstract. In this paper we present approximations of the outage probability for an amplify-and-forward MIMO relaying system with three nodes, which employs multiple antennas at the nodes and orthogonal space-time block coding (OSTBC) transmission over a flat Rayleigh fading. In the amplify-and-forward relay, the incoming signal is decoupled, amplified and forwarded to the destination. Under assumption of availability of full channel state information at the relay and destination and availability of direct link between the source and destination we derived expression for approximation of the outage probability which is sufficiently accurate in the entire SNR range of practical interest. The results obtained by this approximation are compared with the the approximations for outage probability of the system without direct link to the destination.

Key words: Outage probability, Amplify-and-forward, MIMO relay system, Direct link

1 Introduction

Multiple-input multiple-output (MIMO) technology is becoming commonplace in the contemporary communication systems since it offers significant performance improvements in terms of their capacity and reliability, achieved through exploiting the multipath propagation in the wireless medium. Wireless systems with multiple antennas are currently used in local area networks (802.11n, 802.11ac) and in cellular systems such as LTE and LTE advanced. Partly motivated by the MIMO concept, a user cooperation has emerged as an additional breakthrough concept in wireless communications, called cooperative diversity, which has the potential to revolutionize the next generation communication systems by offering additional capacity and reliability improvements with small additional signal processing and cost [1].

In the literature (e.g. [2] - [7]) for the implementation of the MIMO AF relaying system a specific *amplify and forward* (AF) relaying scheme called Decouple-and-Forward (DCF) relaying is used that has been proposed by Lee et al. in [2]. DCF is linear processing technique by which the relay converts multiple spatial streams of the received OSTBC signal into a single spatial stream signal without symbol decoding.

In this paper we derived simple universal approximations for the outage probability of the amplify-and-forward MIMO relay system with direct link, consisted of a source, a decouple-and-forward half-duplex relay and a destination, each equipped with multiple antennas and utilizing an OSTBC transmission technique in a Rayleigh fading environment. The results are compared with the approximations of the outage probability of a dual hop decouple-and-forward relaying system that already exist in literature [4].

In [3]- [7] we analyzed the performance of the amplify-and-forward MIMO relay systems without direct link from the source to the destination. This analysis is applicable when the sent signal from the source to the destination is very weak or it does not exist at the destination. In such case the destination is reconstructing the sent signal by processing the signal sent by the relay. Hence we call such system a cascade amplify-and-forward MIMO relay system.

In [10] it is shown that the use of the relay which is helping the communication between the source and the destination might increase the capacity of the system. According [11] the total end-to-end instantaneous SNR of an amplify-and-forward relay system is a sum of two random variables: the SNR of the direct component from the source to the destination (γ_1) and the end-to-end SNR - γ_2 of the component that passes via the relay (Fig.1). We study the outage probability of this relay system under the assumption that γ_1 of the direct transmission branch is statistically independent from γ_2 of the relay transmission branch.

The remainder of this paper is organized as follows. Next section presents the system model. In Section 3 we derive the simple expressions for the probability density function (PDF) of the amplify-and-forward MIMO relay system with and without direct link to the destination and then derive the closed form expression for approximation of the outage probability of the amplify-and-forward MIMO relay system with direct link to the destination. Numerical results are presented in Section 4 and Section 5 concludes the article.

2 System Model

The system model is presented in Fig.1 where the upper branch of the system represents the cascade amplify-and-forward MIMO relay system, and the lower branch of the system represents the direct transmission link from the source to the destination. It consists of a source, an amplify-and-forward MIMO relay and a destination. Each of the three nodes are equipped with N antennas and utilize OSTBC transmission. The relay employs a decouple-and-forward transmission scheme in which if the additive channel noise is neglected, the estimate of the transmitted symbol at the relay can be mathematically expressed as product of the transmitted symbol and the sum of the squared modulus of the MIMO channel coefficients. After the relay decouples the OSTBC signal it re-encodes the decoupled symbols by usage of OSTBC, amplifies each of them separately and transmits them over the relay-destination hop.

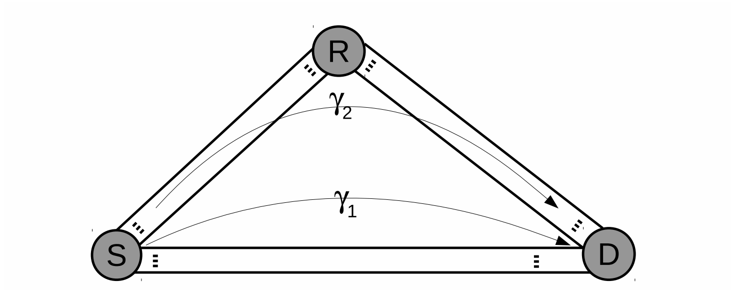

Fig. 1. Dual-hop amplify-and-forward MIMO relay system with direct link to the destination

We consider NxNxN relay configuration, where the source, the relay and the destination are each equipped with N antennas. We assume that there is no spatial correlation between the signals transmitted or received in different antennas. The amplify-and-forward relay applies variable-gain amplification of its input signal, which requires the instantaneous channel state information of the source-relay hop being available to the relay [12]. The destination is also assumed to have a full channel state information of the relay-destination and source-destination hops for coherent demodulation. The source-relay, relay-destination and source-destination hops are modeled as the independent MIMO Rayleigh channels with channel coefficients between the i-th transmit antenna and j-th receive antenna, considered as independent circularly-symmetric complex Gaussian random processes with zero mean and unit variance. Therefore, the squared envelope of signal transmitted over channel follows the exponentially decaying PDF [13] with unit mean squared values. The source transmits with power P. The utilized OSTBC codes are designated as three digit codes, NKL, where N is the number of antennas, K is the number of code symbols transmitted in a code block, and L is the number of required time slots for single codeword [8].

We assume that the relay branch of the system operates in half-duplex mode, divided in two phases (phase 1 and phase 2). The source transmits towards relay during phase 1, then relay transmits towards destination during phase 2. Since we assume that the source employs the OSTB encoding in the phase 1, group of K information symbols are transmitted over the N transmit antennas in L successive time slots. During the phase 2 the NxNxN system's relay decouples, amplifies, OSTB encodes and transmits the K received symbols to the destination.

We have focused on several practical OSTBC schemes, such as 222, 334 and 434, and established their respective approximate outage probabilities when applied in the considered system depicted on Fig.1 with codeword matrices picked according [8] and [9] and given with [3, eq.(32)], [4, eq.(15)] and [7, eq.(20)] with average power per symbol $E = P \cdot c,\ c = L/(KN)$. The received symbols in a single relay antenna for 222, 334 and 434 OSTB codes are decoupled according to [3, eq.(34)] i.e. [7, eq.(22)].

3 Outage probability analysis

The outage probability is defined as the probability that the instantaneous SNR falls below a predetermined threshold ratio γ_{th}. For the cascade amplify-and-forward MIMO relay channel the approximation of the outage probability is [4]:

$$P_{\text{out}} = F_{\Gamma}(\gamma)|_{\gamma=\gamma_{th}} \approx 1 - \sum_{n=0}^{m-1} \frac{(b+1)^n}{n!} \left(\frac{\gamma_{th}}{\overline{\gamma}}\right)^n \exp\left(-\frac{(b+1)\gamma_{th}}{\overline{\gamma}}\right), \quad (1)$$

where and $F(...)$ designates the Cumulative Distribution Function (CDF) approximation of the instantaneous end-to-end SNR which is obtained by taking the terms with $k = 0$ in [3, eq.(22)] i.e. [4, eq. (12)] and $\overline{\gamma}$ is the average SNR per symbol. This expression can be used for calculation of the outage probability for the Nx*1*xN system ($m = N$ and $b = c = L/(KN)$) and for the NxNxN system ($m = N^2$ and $b = 1$) where b is the power normalization factor. In this paper we focus on NxNxN system for which we select $b = 1$. The tight approximation of the PDF is given with [3, eq.(24)] and its accuracy is presented in the discussion of [3, sec.(5)] and depicted on [3, fig.(2) - fig.(7)]. The simplification of [3, eq.(24)] is necessary in order to simplify the mathematical analysis of the amplify-and-forward MIMO relay system with direct link to the destination. The accuracy of this approach of simplification is checked for the cascade amplify-and-forward MIMO relay system through the analysis of the approximation of error probability [7, eq. (19)] given in [7, sec.(5)] and the approximation of outage probability in [4, eq.(14)] presented in [4, sec.(5)].

For the derivation of the simplified PDF of cascade amplify-and-forward MIMO relay channel the second term in (1) may be express through Gamma and incomplete upper Gamma function [14, eq.(8.350.2)] by using of [14, eq.(8.352.2)]. If we use [15, eq.(6.5.3)] we obtain the following expression for the CDF of the cascade amplify-and-forward MIMO relay system:

$$F_{\Gamma}(\gamma) \approx 1 - \frac{\Gamma\left(m, \frac{(b+1)\gamma}{\overline{\gamma}}\right)}{\Gamma(m)} = \frac{\gamma\left(m, \frac{(b+1)\gamma}{\overline{\gamma}}\right)}{\Gamma(m)}\ , \quad (2)$$

where $\gamma(...)$ is lower incomplete Gamma function [14, eq. (8.350.1)]. CDF function given in (2) is CDF of the random variable that is following the gamma PDF with shape parameter m and scale parameter $\theta = \overline{\gamma}/(b+1)$, hence the instantaneous end-to-end SNR [3, eq.(8)] for cascade amplify-and-forward MIMO relay channel might be approximated by random variable which follows Gamma PDF:

$$f\left(\gamma\right)=\frac{1}{\theta^{m}\cdot\Gamma\left(m\right)}\;\gamma^{m-1}e^{-\frac{\gamma}{\theta}}=\frac{\left(b+1\right)^{m}}{\overline{\gamma}^{m}\cdot\Gamma\left(m\right)}\;\gamma^{m-1}e^{-\frac{\left(b+1\right)\gamma}{\overline{\gamma}}},\;\theta=\frac{\overline{\gamma}}{b+1}. \tag{3}$$

We assume that direct transmission branch is under the influence of Rayleigh fading, hence its instantaneous SNR follows Gama PDF, and the instantaneous SNR in the relay transmission branch is distributed by (3) in which we select $b=1$ since we consider the NxNxN system configuration:

$$f_{\Gamma_1}:=\frac{1}{\overline{\gamma}^{m}\cdot\Gamma\left(m\right)}\;\gamma^{m-1}e^{-\frac{\gamma}{\overline{\gamma}}},\;f_{\Gamma_2}:=\frac{2^{m}}{\overline{\gamma}^{m}\cdot\Gamma\left(m\right)}\;\gamma^{m-1}e^{-\frac{2\cdot\gamma}{\overline{\gamma}}}. \tag{4}$$

The resulting random variable is sum of two random variables each following Gamma PDF with different scale parameter:$\Gamma=\Gamma_1+\Gamma_2$. In arbitrary case the sum of n random variables which follow gamma PDFs with different shape and scale parameters is:

$$Y=X_1+X_2+...+X_n,\qquad f_i\left(x_i\right)=\frac{x_i^{\alpha_i}}{\theta_i^{\alpha_i}\cdot\Gamma\left(\alpha_i\right)}e^{-\frac{x_i}{\theta_i}}. \tag{5}$$

The random variable Y is distributed by following PDF [16]:

$$\begin{aligned}
&g\left(y\right)=C\cdot\sum_{k=0}^{\infty}\frac{\delta_k\cdot y^{\rho+k-1}}{\Gamma\left(\rho+k\right)\cdot\theta_l^{\rho+k}}e^{-\frac{y}{\theta_l}},\;\theta_l=\min_i\left(\theta_i\right),\;\rho=\sum_{i=1}^{n}\alpha_i,\\
&C=\prod_{i=1}^{n}\left(\frac{\theta_l}{\theta_i}\right)^{\alpha_i},\;\delta_{k+1}=\frac{1}{k+1}\cdot\sum_{i=1}^{k+1}i\cdot\gamma_i\cdot\delta_{k+1-i},\;k=0,1,2,\;\delta_0=1,\\
&\gamma_k=\frac{1}{k}\sum_{i=1}^{n}\alpha_i\left(1-\frac{\theta_l}{\theta_i}\right)^{k},\quad k=1,2,...
\end{aligned} \tag{6}$$

We simplify the parameters given in (6) for the case of two random variables:

$$\theta_l=\frac{\overline{\gamma}}{b+1},\;\rho=2\cdot m,\;C=\left(b+1\right)^{-m},\;\delta_i=\frac{\left(m\right)_i}{i!}\cdot\left(\frac{b}{b+1}\right)^{i}, \tag{7}$$

where $\left(...\right)_{...}$ represents pochhammer symbol. In case of NxNxN system ($b=1$) parameters in (7) are:

$$\theta_l=\frac{\overline{\gamma}}{2},\;\rho=2\cdot m,\;C=2^{-m},\;\delta_i=\frac{\left(m\right)_i}{i!}\cdot\left(\frac{1}{2}\right)^{i}. \tag{8}$$

If parameters from (8) are introduced in expression for $g(y)$ in (6) we obtain the PDF of the random variable Γ representing the instantaneous SNR in the destination of amplify-and-forward MIMO relay system with direct link:

$$f_{\Gamma}\left(\gamma\right)=\sum_{k=0}^{\infty}\frac{\left(m\right)_k}{k!}\cdot\frac{\gamma^{2m+k-1}2^{m}}{\Gamma\left(2m+k\right)\cdot\overline{\gamma}^{2m+k}}\cdot e^{-\frac{2\cdot\gamma}{\overline{\gamma}}}. \tag{9}$$

If (9) is introduced in $P_{\text{out}} = F_{\Gamma}(\gamma)|_{\gamma=\gamma_{th}}$ we obtain the outage probability of amplify-and-forward MIMO relay system with direct link to the destination:

$$P_{out} = 1 - \sum_{k=0}^{\infty} \frac{(m)_k}{k! \cdot 2^{m+k}} \cdot \frac{\Gamma\left(2m+k, \frac{2\cdot\gamma_{th}}{\overline{\gamma}}\right)}{\Gamma(2m+k)}. \tag{10}$$

4 Numerical results

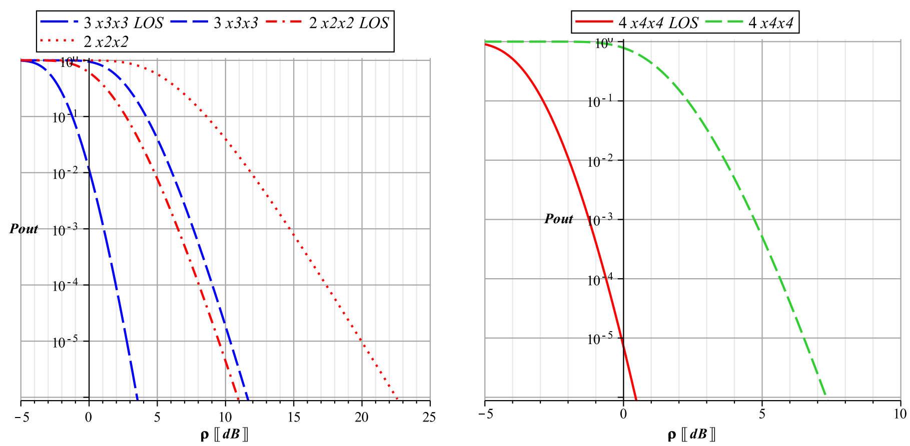

Fig. 2. Outage probability for NxNxN 222/334/434 OSTBC system with and without direct link to the destination for $\gamma_{th} = 5dB$

On Fig.2 we present the comparison of the outage probability for amplify-and-forward MIMO relay system with and without direct link to the destination for $\gamma_{th} = 5dB$. The curves on these figures are obtained by usage of the expressions (1) and (10) which are based on the approximation of the PDF of the cascade MIMO relay channel given with (3). From Fig.2 we conclude that the systems with direct link has much lower outage probability in comparison to the systems without direct link, as it could be expected from the information theoretic analysis given in [10]. For example, for the 2x2x2 system for $\rho = 10dB$ (where ρ is the average SNR per symbol per node) the outage probability of the system without direct link is $4 \cdot 10^{-2}$, and the outage probability of the same system with direct link is $4 \cdot 10^{-6}$ resulting in four order of magnitude improvement. Moreover, from Fig.2 it is obvious that the systems with direct link has greater diversity gain compared to the cascade systems. The difference in diversity gain is reduced by increase of the number of antennas.

5 Conclusion

In this paper we have analyzed the dual-hop relay system with multiple antennas at the source, the relay and the destination that utilize OSTBC and amplify-and-forward relaying schemes. We analyzed the outage probability of this system in case when direct link to the destination is available. For such relay system we have derived generalized closed form expressions for approximation of the outage probability (10). We compared the results obtained with this approximation with results from the literature for amplify-and-forward MIMO relay systems without direct link to the destination. The amplify-and-forward MIMO relay system with direct link to the destination shows significant outage probability improvement yielding to significant diversity gain.

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