Approximate Performance Analysis of Dual-hop Decouple-and-Forward MIMO Relaying

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***Abstract*—In this paper we present two approximations of the error performance over the entire signal-to-noise ratio (SNR) range of practical interest for an amplify and forward (AF) MIMO relaying system, which employs orthogonal space-time block coding (OSTBC) transmission in Rayleigh fading. The AF relay decouples the incoming signal, re-encodes the decoupled symbols by usage of OSTBC, amplifies and transmits them over the relay-destination hop. Monte Carlo simulations and exact numerical analysis show tight correspondence of these two approximations with the actual error performance of the considered system.**

***Index Terms*—MIMO relay, OSTBC, Dual-hop relay systems, Bit error performance, Rayleigh fading.**

I. Introduction

Partly motivated by the MIMO concept, a user cooperation has recently emerged as an additional breakthrough concept in wireless communications, called cooperative diversity, which has the potential to revolutionize the next generation communication systems by offering additional capacity and reliability improvements with small additional signal processing and cost [1]. Some cooperative (i.e. relay) techniques are already part of the standard for LTE-Advanced [2]. The neighbouring wireless nodes (also called relays or partners) assist each other's communication process by dedicating some of their resources to transmit part (or all) of the partners' information. The distributed nature of such a communication process provides a unique opportunity for cooperation, distributed signal processing and of gaining the same advantages as those found in the MIMO systems. The combination of MIMO and cooperative relaying schemes merge the benefits of MIMO system's diversity and multiplexing gains with benefits of relaying for overcoming shadowing, reducing unnecessary high transmission power and alleviating radio frequency interference to the neighbouring nodes.

In this paper we will focus on the MIMO AF relaying systems. There is abundance of research papers related to MIMO AF relaying systems. Some focus on single-antenna relays [3]- [6] and some on multiple-antenna relays [7]- [13]. The paper [3] provide approximate and asymptotic bit error probability (BEP) analysis for distributed space time cooperative systems with a single-antenna source, two single-antenna relays, and a single-antenna destination. A BEP analysis for a dual-hop MIMO systems using OSTBC transmissions with a multiple-antenna source, a single/multiple-antenna destination and a single-antenna relay are conducted in [4] and [5]. The paper [6] tackles the problem of finding exact closed-form and asymptotic expressions of the outage and error performance in a single-antenna relay system where the source and the destination have multiple-antenna by using OSTBC transmissions in Nakagami-m fading. The analysis in [7] complement the analysis in [5] for a MIMO AF relay system using OSTBC transmissions in which the source, the relay, and the destination have two antennas each. The paper [8] extends the analysis of [5] and [7] by establishing an upper bound BEP analysis for a multiple-antenna source, relay and destination. The paper [9] provides error performance analysis for the MIMO AF relay system presented in [8] where each of its nodes is equipped with N antennas. It also provides an asymptotic error analysis and a comparison of the MIMO AF relay systems with multiple MIMO relays with and without relay selection. The paper [10] provides accurate expressions and closed-form bounds of outage probability of a MIMO AF relay system using OSTBC where each of its nodes is equipped with two antennas over a Nakagami-m fading environment. In the paper [11] lower bound error performance has been analyzed in a distributed space-time block code system in which there is a direct link between a dual-antennas source and destination and a indirect link through a multiple-antenna relay. The authors of [12] have provided thorough performance analysis of a dual-hop MIMO AF relay system using OSTBC where each of the nodes have multiple antennas and have developed exact closed-form BEP expressions for certain configurations of the system. The paper [13] provides tight and asymptotic approximations of the error performance of a dual-hop MIMO AF relay system using OSTBC where each of the nodes have multiple antennas.

The aforementioned papers for the implementation of the MIMO AF relaying system ([5]- [13]) use a specific *amplify and forward* (AF) relaying scheme called Decouple-and-Forward (DCF) relaying that has recently been proposed by Lee et al. in [7]. DCF is linear processing technique by which the relay converts multiple spatial streams of the received OSTBC signal into a single spatial stream signal without symbol decoding. After the relay decouples the OSTBC signal received from the source, it amplifies each of the data streams separately, with same amplification factor, then re-encodes the data streams with the same OSTBC code over the multiple output antennas, and then forwards them to the destination.

In this paper, we extend the analysis given in [13] with loose approximations of the error performance of a dual hop DCF relaying system, consisted of a source, a DCF half-duplex relay and a destination, each equipped with multiple antennas and utilizing an OSTBC transmission technique in a Rayleigh fading environment.

The remainder of this paper is organized as follows. Next section presents the system and channel model. In Section 3 we derive closed form expressions for a very accurate approximation of the error probability. In Section 4 we present simple loose approximation of the error probability. The numerical analysis is presented in Section 5, and Section 6 concludes the article.

II. System And Channel Models

We consider two system configurations (Fig.1): *N*x*1*x*N* configuration, where the source and the destination are equipped with *N* antennas and the relay with single antenna and *N*x*N*x*N* configuration, where the source, the relay and the destination are each equipped with *N* antennas. We assume that there is no spatial correlation between the signals transmitted or received in different antennas. The AF relay applies variable-gain amplification of its input signal, which requires the instantaneous channel state information (CSI) of the source-relay hop being available to the relay [13]. The destination is also assumed to have a full CSI of the relay-destination hop for coherent demodulation. The *S-R* hop and the *R-D* hop are modeled by respective channel matrices $\mathbf{H}$ and $\mathbf{G}$. The elements h_{ij} and g_{ij} of these matrices are the channel coefficients between the *i*-th transmit antenna and *j*-th receive antenna, considered as independent circularly-symmetric complex Gaussian random processes with zero mean and unit variance. Therefore, the squared envelope of signal transmitted over channel h_{ij} (g_{ij}) follows the exponentially decaying probability distribution function (PDF) [14] with same mean squared values $E\left[|h_{ij}|^2\right] = E\left[|g_{ij}|^2\right] = 1$. The source *S* transmit power is *P* and the direct communication between the source and the destination is unavailable. The utilized OSTBC codes are designated as three digit codes, *NKL*, where *N* is the number of antennas, *K* is the number of code symbols transmitted in a code block, and *L* represents the number of required time slots to transmit a single codeword [13].

We assume that the communication system operates in half-duplex mode, divided in two phases (phase 1 and phase 2). The source *S* transmits towards *R* during phase 1, then *R* transmits towards *D* during phase 2. Since we assume that the source *S* employs the OSTB encoding in the phase 1, group of *K* information symbols $\mathbf{X} = [x_1,\ x_2,\ ..x_K]^T$ are transmitted over the *N* transmit antennas in *L* successive time slots. During the phase 2 the *N*x1x*N* system's relay decouples, amplifies and transmits the *K* received symbols to the destination. In case of the *N*x*N*x*N* system, during the phase 2 the relay decouples, amplifies, OSTB encodes and transmits

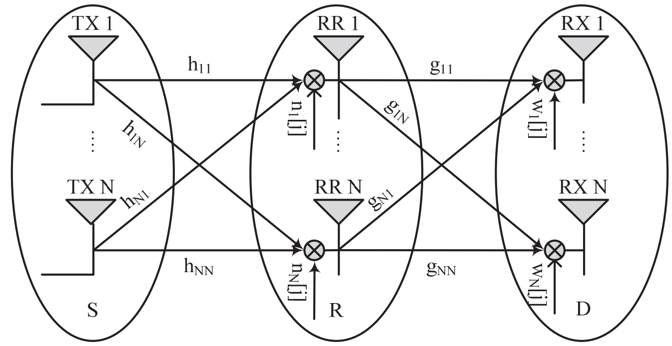

Fig. 1. Dual-hop MIMO relay system.

the *K* received symbols to the destination. The received signal in the single relay antenna at the end of the phase 1 is:

$$\mathbf{Y} = \sqrt{E}\,\mathbf{C}\,\mathbf{H} + \mathbf{N}\ ,\ \mathbf{Y} = [y_1,\ y_2,\ \ldots y_L]^T\ , \qquad (1)$$

where $\mathbf{C}$ is LxN codeword matrix of the OSTB code, $\mathbf{H}$ is Nx1 channel vector of the *S-R* hop $\mathbf{H} = [h_1,\ h_2,\ ..h_N]^T$ and $\mathbf{N} = [n_1,\ n_2,\ ..n_L]^T$ is *S-R* hop's Lx1 additive white Gaussian noise (AWGN) vector whose elements have zero mean and variance N_0. The superscript operator $\mathbf{T}$ denotes matrix transpose operation, and E is the average transmitted power per symbol. The particular decoupled symbol at each relay's antenna is given by [13]:

$$\tilde{x}_k = \sqrt{E}\ \|\mathbf{H}\|_F^2\ x_k + \xi_k, \quad k = 1, 2, ..K\ , \qquad (2)$$

where $\|\mathbf{H}\|_F^2 = \sum_{i=1}^{N} |h_i|^2$ is the squared Frobenius norm of the matrix $\mathbf{H}$, ξ is complex-valued AWGN random variable with zero mean and variance $\|\mathbf{H}\|_F^2\ N_0$. In case where the relay has *N* antennas (*N*x*N*x*N* system), the decoupled symbol at the relay is again given by (2) where $\|\mathbf{H}\|_F^2 = \sum_{i=1}^{N}\sum_{j=1}^{N} |h_{ij}|^2$. In order to satisfy the average power constraint at the relay we have chosen to amplify the decoupled symbol $\tilde{x}_k$ with the following amplification factor:

$$A = \sqrt{\frac{E_R}{\left(E_S\|\mathbf{H}\|_F^4 + \|\mathbf{H}\|_F^2\ N_0\right)}}\ , \qquad (3)$$

where transmitted symbol energies from the source and the relay are equal i.e. $E_R = E_S = E$. For the *N*x*1*x*N* system the destination *D* combines the signals from its multiple antennas by using Maximum Ratio Combining (MRC). For the *N*x*N*x*N* system the relay decouples the OSTBC signal received from the source, it amplifies each of the data streams separately with same amplification factor, then re-encodes the data streams with the same OSTBC code over the multiple output antennas, and then forwards them to the destination. In order to compare the two systems fairly, we assume same transmit power from the relay in both cases. Therefore, for the *N*x*1*x*N* system we should normalize the relay gain in (3) with power normalization factor c which will increase the single-antenna relay average transmit power proportionally to the number of the transmit antennas at *S*. Taking that in the consideration and approximating (3) for analytical tractability we define the relay gain as:

$$A \approx \frac{1}{\sqrt{b}\ \|\mathbf{H}\|_F^2}\ . \tag{4}$$

where b is the power normalization factor. Namely, in (4) we select $b = c$ for the *N*x*1*x*N* and $b = 1$ for the *N*x*N*x*N* system. The decoupled symbols at the destination *D* are expressed as:

$$\hat{x}_k = A\ \|\mathbf{G}\|_F^2\ \tilde{x}_k + \mu_k, \quad k = 1, 2, \ldots K \tag{5}$$

where $\|\mathbf{G}\|_\mathrm{F}^2 = \sum_{\mathbf{i=1}}^{\mathbf{N}} |g_i|^2$ and $\|\mathbf{G}\|_\mathrm{F}^2 = \sum_{\mathbf{i=1}}^{\mathbf{N}} \sum_{\mathbf{j=1}}^{\mathbf{N}} |g_{ij}|^2$ are the corresponding squared Frobenius norms of the channel matrix $\mathbf{G}$ for the *N*x*1*x*N* and *N*x*N*x*N* systems, and μ is complex-valued AWGN random variable with zero mean and variance $\|\mathbf{G}\|_\mathrm{F}^2\ N_0$. If we introduce (2) in (5) we obtain the expression that represents the decoupled symbols for both the *N*x*1*x*N* and *N*x*N*x*N* configurations at the destination *D*:

$$\hat{x}_k = \sqrt{E}\ A\ \|\mathbf{H}\|_F^2\ \|\mathbf{G}\|_\mathrm{F}^2\ x_k + A\ \|\mathbf{G}\|_\mathrm{F}^2\ \xi_k + \mu_k\ . \tag{6}$$

The powers of the useful signal and of the noise are respectively given by:

$$\begin{aligned} P_S &= EA^2 \|\mathbf{H}\|_F^4 \|\mathbf{G}\|_\mathrm{F}^4\ , \\ P_N &= A^2 \|\mathbf{G}\|_\mathrm{F}^4 \|\mathbf{H}\|_F^2 N_0 + \|\mathbf{G}\|_\mathrm{F}^2\ N_0\ . \end{aligned} \tag{7}$$

III. Tight approximation of the average error probability

The end-to-end SNR per information symbol of the signal before the receiver at *D* is:

$$\gamma = \frac{P_S}{P_N} = \frac{E}{N_0} \cdot \frac{A^2 \|\mathbf{G}\|_\mathrm{F}^2\ \|\mathbf{H}\|_F^4}{A^2 \|\mathbf{G}\|_\mathrm{F}^2\ \|\mathbf{H}\|_F^2 + 1}\ . \tag{8}$$

Introducing (4) in (8) we can present the approximated end-to-end SNR (γ) for both systems in the following form [5]:

$$W = 1/\mathit{\Gamma} = 1/\left(\overline{\gamma}\ \|\mathbf{H}\|_F^2\right) + b/\left(\overline{\gamma}\ \|\mathbf{G}\|_F^2\right) = U + V\ , \tag{9}$$

where $W = 1/\mathit{\Gamma}$, $U = 1/(\overline{\gamma}\ \|\mathbf{H}\|_F^2)$, $V = b/(\overline{\gamma}\|\mathbf{G}\|_F^2)$, $\overline{\gamma} = E/N_0 = c \cdot \rho$ is the average transmit SNR per symbol and ρ is the total average transmit SNR per symbol at *S*. The expression (9) is applicable for the *N*x*1*x*N* if we substitute $b = c$ and for the *N*x*N*x*N* system if we substitute $b = 1$. U and V are random variables that follow the inverse gamma distribution [13]. Since the *S-R* and *R-D* hops are subject to an independent Rayleigh fading, U and V are independent and the MGF of their sum is product of their MGF's:

$$M_W(-s) = \frac{4\sqrt{b^m}}{\Gamma^2(m)} \left(\frac{s}{\overline{\gamma}}\right)^m K_m\left(\sqrt{\frac{4s}{\overline{\gamma}}}\right) K_m\left(\sqrt{\frac{4bs}{\overline{\gamma}}}\right)\ . \tag{10}$$

where K_m denote m-th order modified Bessel function of second kind [15, eq.(8.432.1)]. For the *N*x*1*x*N* system configuration, the MGF of the $1/\Gamma$ is expressed as (10), where $m = N$ and $b = c$, whereas, for the *N*x*N*x*N* system configuration, $m = N^2$ and $b = 1$. The CDF of the random variable $\mathit{\Gamma}$ is [13]:

$$F_{\mathit{\Gamma}}(\gamma) = 1 - \mathcal{L}^{-1}\left[M_w(-s)/s\right]\big|_{w=1/\gamma}\ , \tag{11}$$

where $\mathcal{L}^{-1}$ denotes the inverse Laplace transform. In order to find (11) in closed form [13], we approximated the function K_m using its power series expansion [15, eq.(8.446)]. For small arguments as $z \to 0$, the infinite sum in [15, eq.(8.446)] can be neglected and kept only the finite sum i.e.:

$$K_m(z) \approx \frac{1}{2} \cdot \left(\frac{2}{z}\right)^m \cdot \sum_{k=0}^{m-1} (-1)^k \cdot \frac{(m-k-1)!}{k!} \cdot \left(\frac{z}{2}\right)^{2\,k}\ . \tag{12}$$

After some algebraic manipulations [13], it can be shown that approximate CDF for the *N*x1x*N* and *N*x*N*x*N* systems is:

$$\begin{aligned} F_{\mathit{\Gamma}_\mathrm{a}}(\gamma) \approx 1 + \frac{1}{\Gamma(m)} \cdot \sum_{k=0}^{m-1} \sum_{n=0}^{m-1} \frac{(-1)^{m+k+n} \Gamma(m-k)}{\Gamma(m-n)} \\ \cdot \frac{(2\,k+n-m+1)_{m-n-1} (b+1)^n\ b^k}{\Gamma(k+1)\ \Gamma(n+1)} \\ \cdot \left(\frac{\gamma}{\overline{\gamma}}\right)^{n+2\,k} \cdot \exp\left(-\frac{(b+1)\gamma}{\overline{\gamma}}\right)\ . \end{aligned} \tag{13}$$

where $\mathit{\Gamma}(\ldots)$ is gamma function [15, 8.310.1], and $(\ldots)_{\ldots}$ is pochhammer symbol.
The expression (13) can be used to obtain the average error probability, based on the approach presented in [16]. According to [16, eq.(4)], under the assumption that error probability conditioned on the instantaneous SNR can be express in the form $Q\left(\sqrt{c\gamma}\right)$ where $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp\left(-t^2/2\right) dt$, the average error probability is given by:

$$Pe = \int_0^\infty F_\Gamma\left(\frac{x^2}{d}\right) \cdot \frac{e^{-\frac{x^2}{2}}}{\sqrt{2\ \pi}} dx, \tag{14}$$

where d is a constant determined by the modulation scheme, (e.g. for BPSK with coherent demodulation $d = 2$). Introducing approximate CDF from (13) into (14), using [15, eq.(3.461.2)] and [15, eq.(8.339.2)] we obtain an approximation for the average error probability for both system configurations:

$$\begin{aligned} P_{ea} \approx \frac{1}{2} + \frac{\sqrt{d\ \overline{\gamma}}}{2\ \sqrt{\pi}\ \Gamma(m)} \cdot \sum_{k=0}^{m-1} \sum_{n=0}^{m-1} \frac{(-1)^{m+k+n}\ 2^{n+2\,k}}{\Gamma(m-n)} \\ \cdot \frac{\Gamma(m-k)\, \Gamma\left(n+2\,k+\frac{1}{2}\right)}{\Gamma(k+1)\ \Gamma(n+1)} \\ \cdot \frac{(2\,k+n-m+1)_{m-n-1}\ (b+1)^n\ b^k}{(d\ \overline{\gamma}+2\ b+2)^{n+2\,k+\frac{1}{2}}}\ . \end{aligned} \tag{15}$$

Expressions (13) and (15) can be used for the *N*x*1*x*N* system by usage of the substitutions: $m = N$ and $b = c$ and for the *N*x*N*x*N* system by usage of the substitutions: $m = N^2$ and $b = 1$.

IV. Loose approximation of the average error probability

In order to simplify the equation (13) we take only the terms where $k = 0$. This simplification results in a loose approximation of the CDF:

$$F_{\Gamma_l}(\gamma) \approx 1 - \exp\left(-\frac{(b+1)\gamma}{\overline{\gamma}}\right) \cdot \sum_{n=0}^{m-1} \frac{(b+1)^n}{n!} \cdot \left(\frac{\gamma}{\overline{\gamma}}\right)^n . \tag{16}$$

The second therm of (16) can be expressed through gamma and upper incomplete gamma function [15, eq.(8.350.2)] by usage of [15, eq.(8.352.2)]:

$$\exp\left(-\frac{(b+1)\gamma}{\overline{\gamma}}\right) \cdot \sum_{n=0}^{m-1} \frac{(b+1)^n}{n!} \cdot \left(\frac{\gamma}{\overline{\gamma}}\right)^n = \frac{\Gamma\left(m, \frac{(b+1)\gamma}{\overline{\gamma}}\right)}{\Gamma(m)} \tag{17}$$

If we introduce (17) in (16) and use [17, eq.(6.5.3)] we obtain the following expression for the CDF:

$$F_{\Gamma_l}(\gamma) \approx 1 - \frac{\Gamma\left(m, \frac{(b+1)\gamma}{\overline{\gamma}}\right)}{\Gamma(m)} = \frac{\gamma\left(m, \frac{(b+1)\gamma}{\overline{\gamma}}\right)}{\Gamma(m)} , \tag{18}$$

where $\gamma(...)$ is lower incomplete gamma function [15, eq. (8.350.1)]. According [17, eq.(26.4.19)] the CDF function given in (18) is a CDF of the random variable that follows chi-squared distribution function, hence the instantaneous end-to-end SNR given with (8) may be approximated by random variable following chi-squared distribution with m degrees of freedom.

If we simplify (15) by taking the terms where $k = 0$ the loose approximation of the average error probability for both system configurations is:

$$P_{el} \approx \frac{1}{2} - \frac{\sqrt{d\,\overline{\gamma}}}{2\sqrt{\pi}} \cdot \sum_{n=0}^{m-1} \frac{2^n\,\Gamma\left(n+\frac{1}{2}\right)(b+1)^n}{\Gamma(n+1)\left(d\,\overline{\gamma}+2\,b+2\right)^{n+\frac{1}{2}}} . \tag{19}$$

Expression (19) can be used for the *N*x*1*x*N* system by usage of the substitutions: $m = N$ and $b = c$ and for the *N*x*N*x*N* system by usage of the substitutions: $m = N^2$ and $b = 1$.

V. Numerical results

In this section we illustrate the accuracy of our approximations. We will validate our approximations of BEP for different number of antennas *N* by comparison with: (a) the exact values obtained by numerical integration of the appropriate integrals, and (b) exact values obtained by Monte Carlo simulations. We focus on several practical OSTBC schemes, such as 222, 334 and 434, and establish their respective exact and approximate error probabilities when applied in the considered system (Fig.1). The used codeword matrices for these schemes are [13]:

$$\mathbf{C}_{222} = \begin{bmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{bmatrix},\ \mathbf{C}_{334} = \begin{bmatrix} x_1 & x_2 & x_3 \\ -x_2^* & x_1^* & 0 \\ x_3^* & 0 & -x_1^* \\ 0 & x_3^* & -x_2^* \end{bmatrix} ,$$

$$\mathbf{C}_{434} = \begin{bmatrix} x_1 & x_2 & \frac{x_3}{\sqrt{2}} & \frac{x_3}{\sqrt{2}} \\ -x_2^* & x_1^* & \frac{x_3}{\sqrt{2}} & \frac{-x_3}{\sqrt{2}} \\ \frac{x_3}{\sqrt{2}} & \frac{x_3}{\sqrt{2}} & \frac{(-x_1-x_1^*+x_2-x_2^*)}{2} & \frac{(-x_2-x_2^*+x_1-x_1^*)}{2} \\ \frac{x_3^*}{\sqrt{2}} & \frac{-x_3^*}{\sqrt{2}} & \frac{(x_2+x_2^*+x_1-x_1^*)}{2} & -\frac{(x_1+x_1^*+x_2-x_2^*)}{2} \end{bmatrix} . \tag{20}$$

For the OSTB codes given in (20) average power per symbol is calculated as:

$$E = P \cdot c, \qquad c = \frac{L}{K\ N} . \tag{21}$$

The received symbols in a single relay antenna for 222, 334 and 434 OSTB codes are decoupled by [13]:

$$\begin{aligned} &\widetilde{\mathbf{X}}_{222}^T = \left[y_1 h_1^* + y_2^* h_2,\ y_1 h_2^* - y_2^* h_1\right] , \\ &\widetilde{\mathbf{X}}_{334}^T = \left[y_1 h_1^* + y_2^* h_2 - y_3^* h_3,\ y_1 h_2^* - y_2^* h_1 - y_4^* h_3,\right. \\ &\left. y_1 h_3^* + y_3^* h_1 + y_4^* h_2\right] , \\ &\widetilde{\mathbf{X}}_{434} = \\ &\begin{bmatrix} y_1 h_1^* + y_2^* h_2 + \frac{(y_4-y_3)(h_3^*-h_4^*)}{2} - \frac{(y_3^*+y_4^*)(h_3+h_4)}{2} \\ y_1 h_2^* - y_2^* h_1 + \frac{(y_4+y_3)(h_3^*-h_4^*)}{2} + \frac{(y_4^*-y_3^*)(h_3+h_4)}{2} \\ \frac{(y_1+y_2)\ h_3^*}{\sqrt{2}} + \frac{(y_1-y_2)h_4^*}{\sqrt{2}} + \frac{(h_1+h_2)y_3^*}{\sqrt{2}} + \frac{(h_1-h_2)y_4^*}{\sqrt{2}} \end{bmatrix} . \end{aligned} \tag{22}$$

On Fig.2 we present the BEP for 2x1x2 system with Alamouti's coding [18], 3x1x3 with 334 OSTBC and 4x1x4 system with 434 OSTBC. We have compared the results obtained by Monte Carlo simulation, the tight approximation results obtained by (15), the loose approximation results obtained by (19) and exact results obtained by numeric integration of [14, eq.(9.15)] by using the expressions for MGF obtained by approach presented in [5]. The parameter ρ used on the horizontal axis of the figure represent the total average transmit SNR per symbol at *S*. The comparison has shown close match of the results obtained by tight approximation (15), exact result obtained by the numerical integration and simulation. Moreover, the comparison shows good matching of the loose approximation (19) and the exact results. It should be mentioned that the respective curves for $N > 4$ could easily be obtained.

On Fig.3 we present the BEP for 2x2x2, 3x3x3 and 4x4x4 systems using OSTBC given with (20). On the figure we present a comparison of the results obtained by means of simulation, results obtained by usage of expression for tight error probability approximation (15), results obtained by usage of the expression for loose error probability approximation (19) and results obtained by numeric integration of MGF given in [9, eq.(15)] by usage of [14, eq.(9.15)]. We have chosen to use MGF presented in [9] due to better numerical

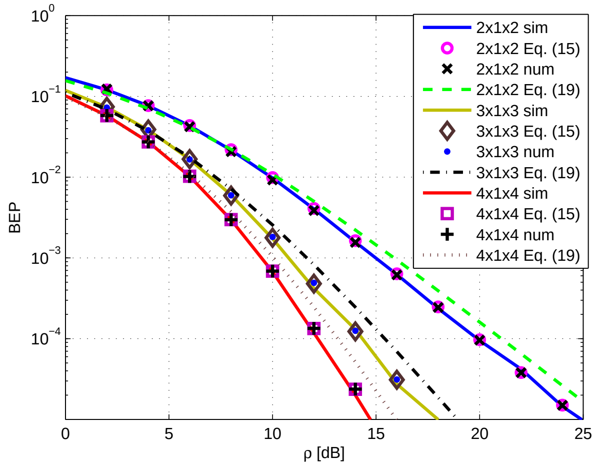

Fig. 2. BEP for dual-hop MIMO 2x1x2/3x1x3/4x1x4 AF systems with BPSK and 222/334/434 OSTB coding.

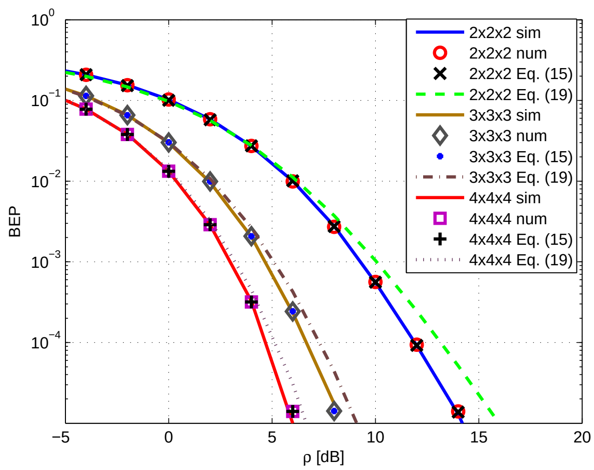

Fig. 3. BEP for dual-hop MIMO 2x2x2/3x3x3/4x4x4 AF systems with BPSK and 222/334/434 OSTB coding.

computational tractability. Again, the comparison has shown close match of the results obtained by tight approximation (15), exact result obtained by the numerical integration and simulation. Moreover, the comparison shows good matching of the loose approximation (19) and the exact results.

VI. Conclusions

In this paper we have analyzed error performance of the dual-hop relay systems with multiple antennas at the source, the relay and the destination that utilize OSTBCs and amplify-and-forward relaying schemes. For those systems we have derived generalized closed form expression for very tight approximation of the error probability (15). Furthermore we have derived a generalized closed form expression for loose approximation of the error probability (19). We have shown that the results obtained with the approximations match the exact values obtained by a simulation and numerical integration of the appropriate integrals for all transmit SNRs of a practical interest.

Apart from BPSK modulation schemes, this work can be easily extended for other types of modulation. For example, for QAM one may find that [19, eq.(16)] is in full compliance with definition of (14).

Due to the computational efficiency, the obtained expressions can be used as a reference for an easy check of the error performance during the analysis of the MIMO AF relaying systems using OSTBCs or can be used as a reference for quick assessment of the error performance in the future hardware implementation of such systems.

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