# Simple tight approximations of the error performance for dual-hop MIMO relay systems in Rayleigh fading

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**Abstract**

In this paper we present tight approximations of the error performance for entire signal-to-noise ratio (SNR) range of practical interest for an amplify and forward (AF) relaying system with channel state information (CSI) at the relay and destination, which employs multiple antennas at the nodes and orthogonal space-time block coding (OSTBC) transmission in flat Rayleigh fading. Additionally, for high value SNR range, we simplified the tight approximation expression in the simple asymptotic expression. Comparison of the error performance obtained by those two approximation with the error performance obtained by numeric integration of Moment Generating Functions (MGF) and by Monte Carlo simulations show close matching of the results.

*Keywords:* Cooperative wireless communications, MIMO relay, OSTBC, Alamouti's coding, Dual-hop relay systems, Bit error performance, Rayleigh fading

## 1. Introduction

Multiple-input multiple-output (MIMO) technology is becoming commonplace in the contemporary communication systems since they offer significant performance improvements in terms of their capacity and reliability, achieved through exploiting the multipath propagation in the wireless medium. Wireless systems with 2 to 4 antennas are currently used for local area networks (802.11n) or developed for cellular systems such as 3GPP-LTE and LTE advanced.
Partly motivated by the MIMO concept, user cooperation has recently emerged as an additional breakthrough concept in wireless communications, called cooperative diversity, which has the potential to revolutionize the next generation

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communication systems by offering additional capacity and reliability improvements with small additional signal processing and cost [1]. Some cooperative (i.e. relay) techniques are already part of the standard for LTE-Advanced [2]. The neighboring wireless nodes (also called relays or partners) assist each other's communication process by dedicating some of their resources to transmit part (or all) of the partners' information. By properly coordinating different spatially distributed nodes in a wireless system, one can effectively synthesize a virtual antenna array to achieve spatial diversity, similarly as in MIMO. The distributed nature of such a communication process provides a unique opportunity for cooperation, distributed signal processing and of gaining the same advantages as those found in MIMO systems. The combination of MIMO and cooperative relaying schemes merge the benefits of MIMO system's diversity and multiplexing gains with benefits of relaying for overcoming shadowing, reducing unnecessary high transmission power and alleviating radio frequency interference to neighboring nodes.

In this paper, we study the error performance of dual hop relay system, consisted of a source, a half-duplex relay and a destination, each equipped with multiple antennas and utilizing an OSTBC (Orthogonal Space-Time Block Coding) transmission technique in Rayleigh fading environment.

At the relay we use a specific *amplify and forward* (AF) relaying scheme, based on which the relay decouples the OSTBC signal received from the source into multiple independent data streams, amplifies each of them separately and transmits them over the relay-destination hop. We arrive at an approximation for the error probability of this scheme, which proves to be extremely accurate in the SNR range of practical interest.

The remainder of this paper is organized as follows. Next section presents the system and channel model. In Section 3 we derive closed form expressions for very accurate approximation of error probability. In Section 4 we present an additional simple asymptotic approximation of the error probability, valid for high SNRs. The numerical analysis is presented in Section 5, and Section 6 concludes the article.

## 2. System And Channel Models

In this paper we analyze error performance of a dual-hop relay system (consisted of a source, a relay and a destination), with multiple antennas at the nodes that utilize OSTBC transmission (Fig.1). We consider two system configurations: $N$x$1$x$N$ configuration, where source and destination are equipped with $N$ antennas and the relay with single antenna and $N$x$N$x$N$ configuration, where source, relay and destination are each equipped with $N$ antennas. We assume that there is no spatial correlation between the signals transmitted or received in different antennas. The AF relay aplies variable-gain amplification of its input signal, which requires the instantaneous channel state information (CSI) of the source-relay hop being available to the relay [3]. The destination is also assumed to have a full CSI of the relay-destination hop for coherent demodulation. In Fig.1 we present MIMO dual-hop relay system (as the most general

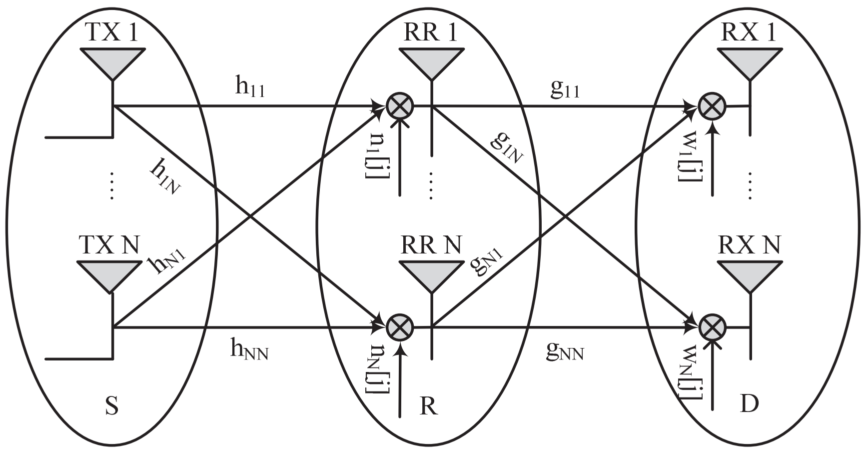


Figure 1: Dual-hop MIMO relay system.

configuration that incorporates the two system configurations as special cases), where the source, the relay and the destination are denoted by $S$, $R$ and $D$, respectively. The relay system is designated as $N$x$1$x$N$ in the case when the relay $R$ has one antenna and $N$x$N$x$N$ when the relay $R$ has $N$ antennas. The $S$-$R$ hop and the $R$-$D$ hop are modeled as the independent MIMO Rayleigh channels with respective channel matrices $\mathbf{H}$ and $\mathbf{G}$. The elements $h_{ij}$ and $g_{ij}$ of these matrices are the channel coefficients between the $i$-th transmit antenna and $j$-th receive antenna, considered as independent circularly-symmetric complex Gaussian random processes with zero mean and unit variance. Therefore, the squared envelope of signal transmitted over channel $h_{ij}$ ($g_{ij}$) follows the exponentially decaying probability distribution function (PDF) [4] with same mean squared values $E\left[|h_{ij}|^2\right] = E\left[|g_{ij}|^2\right] = 1$. The source $S$ transmit power is $P$ and the direct communication between the source and the destination is unavailable. The utilized OSTBC codes at the source-relay link are designated as three digit codes, $NKL$, where $N$ is the number of antennas, $K$ is the number of code symbols transmitted in a code block, and $L$ represents the number of required time slots to transmit a single codeword [5]. We assume that the communication system operates in half-duplex mode, divided in two phases (phase 1 and phase 2). The source $S$ transmits towards $R$ during phase 1, then $R$ transmits towards $D$ during phase 2. Since we assume that the source $S$ employs the OSTB encoding in the phase 1, group of $K$ information symbols $\mathbf{X} = [x_1,\ x_2,\ ..x_K]^T$ are transmitted over the $N$ transmit antennas in $L$ successive time slots. During the phase 2 the $N$x1x$N$ system's relay decouples, amplifies and transmits the $K$ received symbols to destination. In case of the $N$x$N$x$N$ system, during the phase 2 relay decouples, amplifies, OSTB encodes and transmits the $K$ received symbols to destination. The received signal in the single relay antenna at the end of the phase 1 is:

$$\mathbf{Y} = \sqrt{E}\ \mathbf{C}\ \mathbf{H} + \mathbf{N}\ , \quad \mathbf{Y} = [y_1,\ y_2,\ \ldots y_L]^T\ , \tag{1}$$

where $\mathbf{C}$ is $L$x$N$ codeword matrix of the OSTB code, $\mathbf{H}$ is $N$x1 channel vector of the $S$-$R$ hop $\mathbf{H} = [h_1,\ h_2,\ ..h_N]^T$ and $\mathbf{N} = [n_1,\ n_2,\ ..n_L]^T$ is $S$-$R$ hop's $L$x1 AWGN vector whose elements have zero mean and variance $N_0$. The superscript operator $\mathbf{T}$ denotes matrix transpose operation, and $E$ is the average transmitted power per symbol. In [6] it is shown that the particular decoupled symbol at the single antenna at the relay is given by:

$$\tilde{x}_k = \ \sqrt{E}\ \|\mathbf{H}\|_F^2\ x_k + \xi_k, \quad k = 1, 2, ..K\ , \tag{2}$$

where $\|\mathbf{H}\|_F^2 = \ \sum_{\mathbf{i=1}}^{\mathbf{N}} |h_i|^2$ is the squared Frobenius norm of the matrix $\mathbf{H}$, $\xi$ is complex-valued AWGN random variable with zero mean and variance $\|\mathbf{H}\|_F^2\ N_0$. In case where the relay has $N$ antennas ($N$x$N$x$N$ system), the decoupled symbol at relay is again given by (2) where $\|\mathbf{H}\|_F^2 = \ \sum_{\mathbf{i=1}}^{\mathbf{N}} \sum_{\mathbf{j=1}}^{\mathbf{N}} |h_{ij}|^2$. We have chosen to amplify the decoupled symbol $\tilde{x}_k$ with the following amplification factor:

$$A = \sqrt{\frac{E_R}{\left(E_S \|\mathbf{H}\|_F^4 + \|\mathbf{H}\|_F^2\ N_0\right)}}\ . \tag{3}$$

where transmitted symbol energies from the source and the relay are equal i.e. $E_R = E_S = E$. For the $N$x*1*x$N$ system the destination $D$ combines the signals from its multiple antennas using (Maximum Ratio Combining) MRC. In order to compare the two systems fairly, we assume same transmit power from the relay in both cases. Therefore, for the $N$x*1*x$N$ system we should normalize the relay gain in (3) with power normalization factor $c$ which will increase the single antenna relay average transmit power proportionally to the number of the transmit antennas at $S$. Taking that in the consideration and approximating (3) for analytical tractability we define the relay gain as:

$$A \approx \frac{1}{\sqrt{b}\ \|\mathbf{H}\|_F^2}\ . \tag{4}$$

where $b$ is the power normalization factor. Namely, in (4) we select $b = c$ for the $N$x*1*x$N$ and $b = 1$ for the $N$x$N$x$N$ system. The decision variable for each of the decoupled symbols at the destination $D$ is expressed as:

$$\hat{x}_k = A\ \|\mathbf{G}\|_F^2\ \tilde{x}_k + \mu_k, \quad k = 1, 2, \ldots K \tag{5}$$

where $\|\mathbf{G}\|_{\mathrm{F}}^2 = \sum_{\mathbf{i=1}}^{\mathbf{N}} |g_i|^2$ and $\|\mathbf{G}\|_{\mathrm{F}}^2 = \sum_{\mathbf{i=1}}^{\mathbf{N}} \sum_{\mathbf{j=1}}^{\mathbf{N}} |g_{ij}|^2$ are the corresponding squared Frobenius norms of the channel matrix $\mathbf{G}$ for the $N$x*1*x$N$ and $N$x$N$x$N$ systems, and $\mu$ is complex-valued AWGN random variable with zero mean and variance $\|\mathbf{G}\|_{\mathrm{F}}^2\ N_0$. If we introduce (2) in (5) we obtain the expression that represents the decoupled symbols for both the $N$x*1*x$N$ and $N$x$N$x$N$ configurations at destination $D$:

$$\hat{x}_k = \sqrt{E}\ A\ \|\mathbf{H}\|_F^2\ \|\mathbf{G}\|_{\mathrm{F}}^2\ x_k + A\ \|\mathbf{G}\|_{\mathrm{F}}^2\ \xi_k + \mu_k\ . \tag{6}$$

The powers of the useful signal and of the noise are respectively given by:

$$\begin{aligned} P_S &= EA^2\|\mathbf{H}\|_F^4\|\mathbf{G}\|_{\mathrm{F}}^4 \ , \\ P_N &= A^2\|\mathbf{G}\|_{\mathrm{F}}^4\|\mathbf{H}\|_F^2 N_0+\|\mathbf{G}\|_{\mathrm{F}}^2 \ N_0 \ . \end{aligned} \tag{7}$$

Therefore, the end-to-end SNR per information symbol for both system configurations is:

$$\gamma = \frac{P_S}{P_N} = \frac{E}{N_0} \cdot \frac{A^2\|\mathbf{G}\|_{\mathrm{F}}^2 \ \|\mathbf{H}\|_F^4}{A^2\|\mathbf{G}\|_{\mathrm{F}}^2 \ \|\mathbf{H}\|_F^2+1} \ . \tag{8}$$

### 3. Error probability approximation for arbitrary SNR

The cumulative distribution function (CDF) of the analyzed systems is calculated by using of inverse Laplace transform of the Moment Generating Function (MGF) [4]:

$$F_\Gamma(\gamma) = \mathfrak{L}^{-1}\left[\mathrm{M}_\Gamma(-\mathrm{s})/\mathrm{s}\right] \ , \tag{9}$$

where $\mathfrak{L}^{-1}$ denotes the inverse Laplace transform, and $\gamma$ is instantaneous SNR of the signal before the receiver at $D$. Introducing (4) in (8) we can present the approximated end-to-end SNR ($\gamma$) for both systems in the following form [7]:

$$W = 1/\Gamma = 1/\left(\overline{\gamma}\ \|\mathbf{H}\|_F^2\right) + b/\left(\overline{\gamma}\ \|\mathbf{G}\|_F^2\right) = U + V \ , \tag{10}$$

where $W = 1/\Gamma$, $U = 1/(\overline{\gamma}\ \|\mathbf{H}\|_F^2)$, $V = b/(\overline{\gamma}\|\mathbf{G}\|_F^2)$, $\overline{\gamma} = E/N_0 = c\cdot\rho$ is average transmit SNR per symbol and $\rho$ is total average transmit SNR per symbol at $S$. The expression (10) is applicable for the $N$x$1$x$N$ if we substitute $b = c$ and for the $N$x$N$x$N$ system if we substitute $b = 1$. Given that instantaneous channel power follows exponential distribution, $\|\mathbf{H}\|_{\mathrm{F}}^2$ and $\|\mathbf{G}\|_{\mathrm{F}}^2$ follows the gamma distribution:

$$f\left(x\right) = \frac{x^{m-1}}{\theta^m\Gamma\left(m\right)}\ e^{-\frac{x}{\theta}} \ , \ \text{for } x > 0, \ \text{and } m, \theta > 0, \tag{11}$$

with scale parameter $\theta = 1$ and shape parameter $m$. Hence, the PDFs of the squared Frobenius norms are respectively given by:

$$f_{\|\mathrm{H}\|_{\mathrm{F}}^2}\left(x\right) = \frac{x^{m_1-1}}{\Gamma\left(m_1\right)}\ e^{-x} \ , \ f_{\|\mathrm{G}\|_{\mathrm{F}}^2}\left(x\right) = \frac{x^{m_2-1}}{\Gamma\left(m_2\right)}\ e^{-x} \tag{12}$$

where $x > 0$, $m = m_1 = m_2 = N$ for the $N$x$1$x$N$ system and $m = m_1 = m_2 = N^2$ for the $N$x$N$x$N$ system. By using the functional transformation of the random variable $U = 1/(\overline{\gamma}\ \|\mathrm{H}\|_F^2)$ and $V = b/(\overline{\gamma}\ \|\mathrm{G}\|_{\mathrm{F}}^2)$ it can be shown their PDFs are respectively given by:

$$f_U\left(u\right) = \frac{x^{-m-1}}{\overline{\gamma}^m\ \Gamma\left(m\right)}\ e^{-\frac{1}{x\ \overline{\gamma}}} \ , \quad f_V\left(x\right) = \frac{b^m\ x^{-m-1}}{\overline{\gamma}^m\ \Gamma\left(m\right)}\ e^{-\frac{b}{x\ \overline{\gamma}}} \ . \tag{13}$$

By means of [8, eq.(3.471.9)] it is easy to find MGFs of $U$ and $V$:

$$M_U(-s) = \frac{2}{\Gamma(m)} (\, s/\overline{\gamma})^{\frac{m}{2}}\ K_m\left(2\ \sqrt{s/\overline{\gamma}}\right)\ ,$$
$$M_V(-s) = \frac{2\ b^{m/2}}{\Gamma(m)} (\, s/\overline{\gamma})^{\frac{m}{2}}\ K_m\left(2\ \sqrt{b\ s/\overline{\gamma}}\right)\ , \tag{14}$$

where $K_m$ ([8, eq.(8.432.1)]) denote $m$-th order modified Bessel function of second kind. Since the *S*-*R* and *R*-*D* hops are subject to an independent Rayleigh fading, $U$ and $V$ are independent and the MGF of their sum is product of their MGF's:

$$M_w(-s) = \frac{4\ \sqrt{b^m}}{\Gamma^2(m)} \left(\frac{s}{\overline{\gamma}}\right)^m K_m\left(2\sqrt{\frac{s}{\overline{\gamma}}}\right)\ K_m\left(2\sqrt{\frac{b\ s}{\overline{\gamma}}}\right)\ . \tag{15}$$

For the $N$x*1*x$N$ system configuration, the MGF of the $1/\Gamma$ is expressed as (15), where $m = N$ and $b = c$, whereas, for the $N$x$N$x$N$ system configuration, $m = N^2$ and $b = 1$. Therefore, the CDF of the random variable $W = 1/\Gamma$ is obtained according to [9, eq.(31)]:

$$F_\Gamma(\gamma)\ = 1 - \mathfrak{L}^{-1}\left[M_w(-s)/s\right]\big|_{w=1/\gamma}\ . \tag{16}$$

It can easily be shown that $(m-1)$th derivate of $\mathfrak{L}^{-1}\left[M_w(-s)/s^m\right]$ equals $\mathfrak{L}^{-1}\left[M_w(-s)/s\right]$ therefore (16) can be rewritten in following form [7, eq.(9)]:

$$F_\Gamma(\gamma) = 1 - \left. d^{m-1}L(w)/d\ w^{m-1}\right|_{w=1/\gamma}\ , \tag{17}$$

where:

$$L(w) = \mathfrak{L}^{-1}\left[M_w(-s)/s^m\right]\ . \tag{18}$$

By using [10, eq.(3.16.6.6)] the $L(w)$ function for MGF given with (15) can be expressed as:

$$L(w) = \frac{2\ b^{m/2}\ e^{-\ \frac{b+1}{\overline{\gamma}\ w}}}{w\ \Gamma^2(m)\ \overline{\gamma}^m}\ K_m\left(\frac{2\ \sqrt{b}}{\overline{\gamma}\ w}\right)\ , \tag{19}$$

where $K_m$ is modified Bessel function of second kind. Furthermore, by replacing (19) in (17) the CDFs for a given value of $N$ can be presented in closed form for both system configurations. The PDF of end-to-end SNR can be found by taking derivate from the CDF given with (17) i.e. $f_\Gamma(\gamma) = dF_\Gamma(\gamma)/d\gamma$. In case of $N = 2$ the CDF and PDF can easily be found in closed form with some mathematical manipulation by using formula for derivate of modified Bessel functions [11, eq.(9.6.29)] ([7] [12]). For larger values of $N$ it is very difficult to find close form expression for the PDF and finding the average error performance with MGF approach given in [4] is even more difficult. Therefore we reverted to the approximation techniques. In order to apply (17), we approximated the function $K_m$ using its power series expansion [8, eq.(8.446)]. For small arguments as $z \to 0$, the infinite sum in [8, eq.(8.446)] can be neglected and kept only the finite sum i.e.:

$$K_m(z) \approx \frac{1}{2} \cdot \left(\frac{2}{z}\right)^m \cdot \sum_{k=0}^{m-1} (-1)^k \cdot \frac{(m-k-1)!}{k!} \cdot \left(\frac{z}{2}\right)^{2\ k} . \tag{20}$$

By combining (19) and (20) into (17), the CDF of $\gamma$ is approximated as:

$$\begin{aligned} F_{\Gamma_a}(\gamma) \approx 1 - &\frac{b^{m/2}}{\Gamma^2(m)\ \overline{\gamma}^m} \sum_{k=0}^{m-1} (-1)^k \cdot \frac{(m-k-1)!}{k!} \cdot \\ &\cdot \frac{d^{m-1}}{d\ w^{m-1}} \left[ \frac{\exp\left(-\frac{b+1}{\overline{\gamma}\ w}\right)}{w} \cdot \left(\frac{\overline{\gamma}\ w}{\sqrt{b}}\right)^{m-2\ k} \right] \Bigg|_{w=1/\gamma} . \end{aligned} \tag{21}$$

After some algebraic manipulations, it can be shown that approximate CDF for the $N$x$1$x$N$ and $N$x$N$x$N$ systems is:

$$\begin{aligned} F_{\Gamma_a}(\gamma) \approx 1 + &\frac{1}{\Gamma^2(m)} \cdot \sum_{k=0}^{m-1} \sum_{n=0}^{m-1} \frac{(-1)^{m+k+n} \Gamma(m-k)\ \Gamma(2\ k)}{\Gamma(2\ k+n-m+1)\ \Gamma(m-n)} \cdot \\ &\cdot \frac{\Gamma(m)(b+1)^n\ b^k}{\Gamma(k+1)\ \Gamma(n+1)} \cdot \left(\frac{\gamma}{\overline{\gamma}}\right)^{n+2\ k} \cdot \exp\left(-\frac{(b+1)\gamma}{\overline{\gamma}}\right) . \end{aligned} \tag{22}$$

Previous expression can be used to obtain the average error probability, based on the approach presented in [13]. According to [13, eq.(4)], the average error probability is given by:

$$Pe = \int_0^\infty F_\Gamma\left(\frac{x^2}{d}\right) \cdot \frac{e^{-\frac{x^2}{2}}}{\sqrt{2\ \pi}} dx, \tag{23}$$

where $d$ is a constant determined by the modulation and demodulation scheme (e.g. for BPSK with coherent demodulation $d = 2$). Introducing (22) into (23), using [8, eq.(3.461.2)] and [8, eq.(8.339.2)] we obtain an approximation for the average error probability for both system configurations:

$$\begin{aligned} P_{ea} \approx \frac{1}{2} + &\frac{\sqrt{d\ \overline{\gamma}}}{2\ \sqrt{\pi}\Gamma^2(m)} \cdot \sum_{k=0}^{m-1} \sum_{n=0}^{m-1} \frac{(-1)^{m+k+n}\ 2^{n+2k}}{\Gamma(2\ k+n-m+1)} \\ &\cdot \frac{\Gamma(m-k)\ \Gamma\left(n+2k+\frac{1}{2}\right)\ \Gamma(2\ k)\ \Gamma(m)(b+1)^n\ b^k}{\Gamma(m-n)\ \Gamma(k+1)\ \Gamma(n+1)\ (d\ \overline{\gamma}+2\ b+2)^{n+2k+\frac{1}{2}}} . \end{aligned} \tag{24}$$

Expressions (22) and (24) can be used for the $N$x$1$x$N$ system by usage of the substitutions: $m = N$ and $b = c$ and for the $N$x$N$x$N$ system by usage of the substitutions: $m = N^2$ and $b = 1$.

## 4. Error probability approximation for high SNRs

Based on [14, eq.(13)] the asymptotic approximation of PDF of the end-to-end SNR for high value SNR range can be presented in the following form:

$$P_{eas} = \frac{1}{2 \cdot d^m \cdot m!} \lim_{\gamma \to 0} \frac{d^m F_\Gamma(\gamma)}{d\gamma^m} \cdot \prod_{i=1}^{m} (2 \cdot i - 1) \ , \qquad (25)$$

where $m$ is product of the number of the transmitting and receiving antennas of the $S$-$R$ i.e. $R$-D hops and $d$ is constant determined by the modulation format used in (23). Replacing (22) in (25) it can be shown that asymptotic approximation of error probability for high value SNR is:

$$P_{eas} \approx \frac{\Gamma\left(2\ m+1\right) \cdot \left(b^m+1\right)}{2^{m+1} \cdot \Gamma^2\left(m+1\right) \cdot d^m \cdot \overline{\gamma}^m} \ . \qquad (26)$$

Previous expression can be used for calculating the asymptotic approximation of error probability for the $N$x*1*x$N$ system ($m = N$ and $b = c$) and for the $N$x$N$x$N$ system ($m = N^2$ and $b = 1$).

## 5. Numerical results

In this section we illustrate the accuracy of our approximations. At the beginning, we will validate our approximations of bit error probability (BEP) for different number of antennas $N$ by comparison with: (a) the exact values obtained by numerical integration of the appropriate integrals, and (b) exact values obtained by Monte Carlo simulations. On Fig.2 we present BEP performance results for the $N$x*1*x$N$ system with up to 7 antennas at the source and the destination for $E = P/N$ i.e. $c = 1/N$. Solid line curves represent results obtained by expresion for aproximation of error probability (24) and dotted lines represent results obtained by expression for asymptotic error probability approximation (26). It should be mentioned that respective curves for $N > 7$ could be easily obtained. In Fig.3 we present BEP results for the $N$x$N$x$N$ system with up to 6 antennas at the source, relay and the destination for $E = P/N$. Solid line curves represent results obtained by expression for error probability approximation (24), and dotted lines represent results obtained by expression for asymptotic error probability approximation (26). All results up to $N = 6$ are easily obtainable, and the acquisition of the results for $N > 6$ is limited by the precision of the numeric analysis package. Next, we focus on several practical OSTBC schemes, such as 222, 334 and 434, and establish their respective exact and approximate error probabilities when applied in our considered system (Fig.1). According to [5] and [15] the codeword matrices for these schemes are

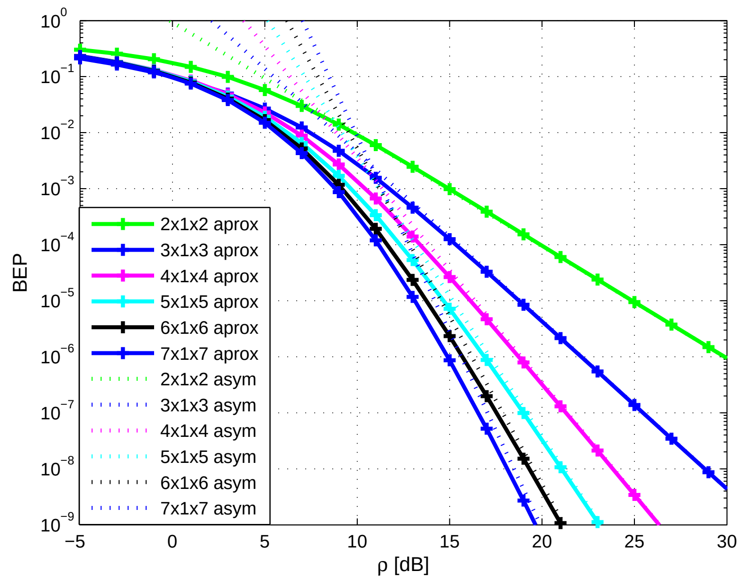


Figure 2: BEP of Dual-hop MIMO $N$x$1$x$N$ relay system mode.

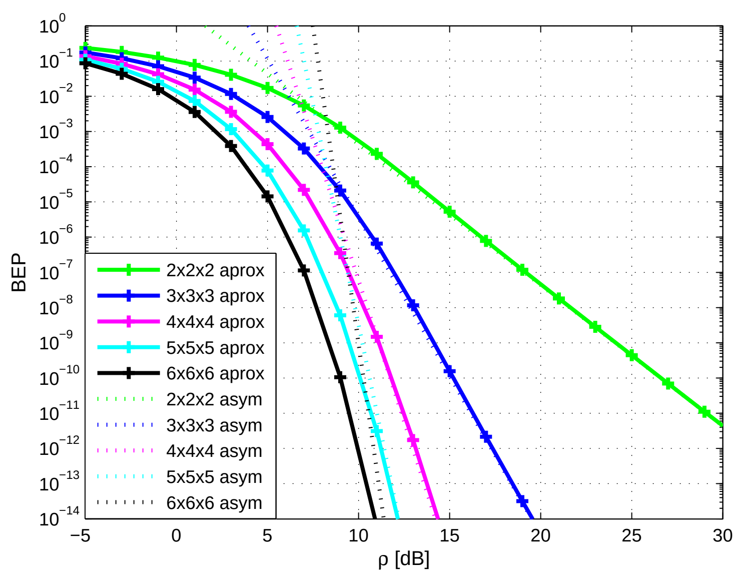


Figure 3: BEP of Dual-hop MIMO $N$x$N$x$N$ relay system model.

given by:

$$\mathbf{C}_{222} = \begin{bmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{bmatrix}, \ \mathbf{C}_{334} = \begin{bmatrix} x_1 & x_2 & x_3 \\ -x_2^* & x_1^* & 0 \\ x_3^* & 0 & -x_1^* \\ 0 & x_3^* & -x_2^* \end{bmatrix} ,$$
$$\mathbf{C}_{434} = \begin{bmatrix} x_1 & x_2 & x_3/\sqrt{2} & x_3/\sqrt{2} \\ -x_2^* & x_1^* & x_3/\sqrt{2} & -x_3/\sqrt{2} \\ x_3/\sqrt{2} & x_3/\sqrt{2} & \frac{(-x_1-x_1^*+x_2-x_2^*)}{2} & \frac{(-x_2-x_2^*+x_1-x_1^*)}{2} \\ x_3^*/\sqrt{2} & -x_3^*/\sqrt{2} & \frac{(x_2+x_2^*+x_1-x_1^*)}{2} & -\frac{(x_1+x_1^*+x_2-x_2^*)}{2} \end{bmatrix} . \tag{27}$$

For OSTB codes given in (27) average power per symbol is calculated as:

$$E = P \cdot c, \quad c = \frac{L}{K\ N} . \tag{28}$$

The received symbols in single relay antenna for 222, 334 and 434 OSTB codes are decoupled according to [5] [6]:

$$\widetilde{\mathbf{X}}_{222}^T = [y_1 h_1^* + y_2^* h_2,\ y_1 h_2^* - y_2^* h_1] ,$$
$$\widetilde{\mathbf{X}}_{334}^T = [y_1 h_1^* + y_2^* h_2 - y_3^* h_3,\ y_1 h_2^* - y_2^* h_1 - y_4^* h_3,\ y_1 h_3^* + y_3^* h_1 + y_4^* h_2] ,$$
$$\widetilde{\mathbf{X}}_{434} = \begin{bmatrix} y_1 h_1^* + y_2^* h_2 + \frac{(y_4-y_3)(h_3^*-h_4^*)}{2} - \frac{(y_3^*+y_4^*)(h_3+h_4)}{2} \\ y_1 h_2^* - y_2^* h_1 + \frac{(y_4+y_3)(h_3^*-h_4^*)}{2} + \frac{(y_4^*-y_3^*)(h_3+h_4)}{2} \\ \frac{(y_1+y_2)\ h_3^*}{\sqrt{2}} + \frac{(y_1-y_2)h_4^*}{\sqrt{2}} + \frac{(h_1+h_2)y_3^*}{\sqrt{2}} + \frac{(h_1-h_2)y_4^*}{\sqrt{2}} \end{bmatrix} . \tag{29}$$

On Fig.4 we present BEP for 2x1x2 system with Alamouti coding [16], 3x1x3 with 334 OSTBC and 4x1x4 system with 434 OSTBC. We have compared the results obtained by Monte Carlo simulation, the approximation results obtained by (26) and (24) for $m = N$ and $b = c$ and exact results obtained by numeric integration of [4, eq.(9.15)] by using the expressions for MGF obtained by approach presented in [7]. The comparison has shown close match of the results obtained by approximation (24), numerical integration and simulation. On Fig.5 we present BEP for 2x2x2, 3x3x3 and 4x4x4 systems using OSTBC given with (27). On the figure we present comparison of the results obtained by means of simulation, results obtained by usage of expression for error probability approximation (24), results obtained from the asymptotic error probability approximation (26) and results obtained by numeric integration of MGF given in [17, eq.(15)] by usage of [4, eq.(9.15)]. We have chosen to use MGF presented in [17] due to better numerical computational tractability. Again, the comparison has shown close match of the results obtained by approximation (24), numerical integration and simulation.

## 6. Conclusions

In this paper we have analyzed error performance of the dual-hop relay systems with multiple antennas at the source, relay and destination that utilize OSTBC and amplify-and-forward relaying schemes. For those systems we

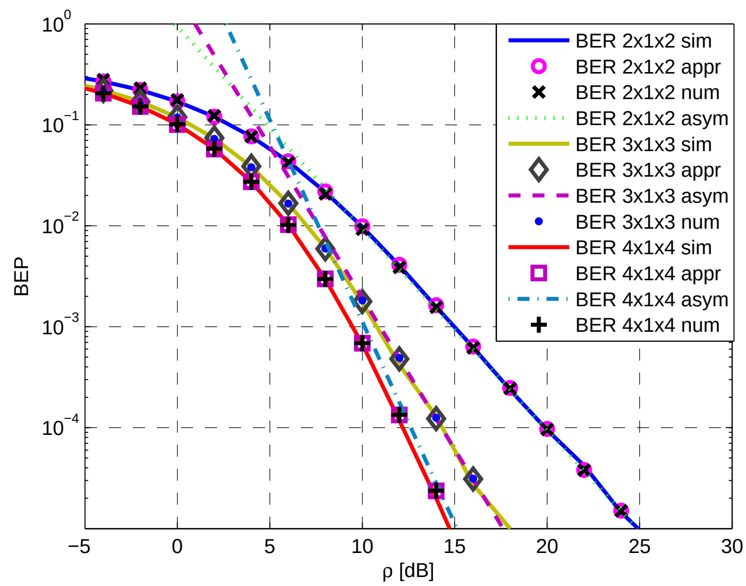


Figure 4: BEP for dual-hop MIMO 2x1x2/3x1x3/4x1x4 AF systems with BPSK and 222/334/434 OSTB coding.

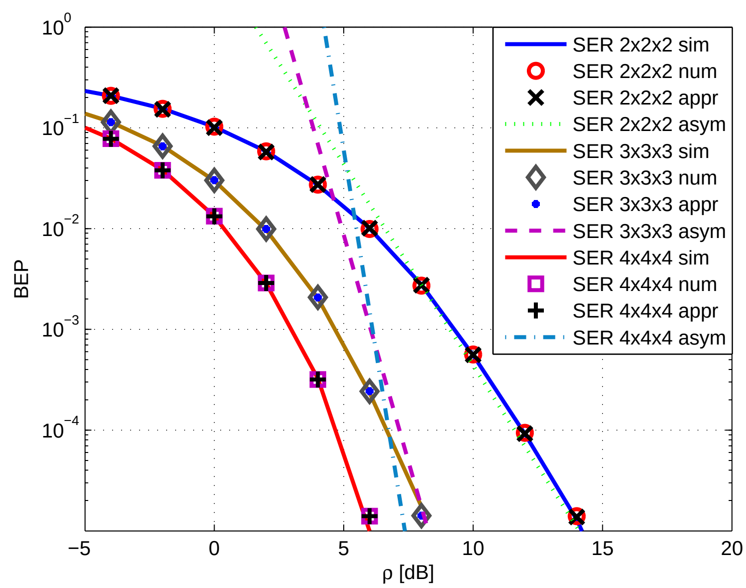


Figure 5: BEP for dual-hop MIMO 2x2x2/3x3x3/4x4x4 AF systems with BPSK and 222/334/434 OSTB coding.

have derived generalized closed form expression for very tight approximation of the error probability. Furthermore we have derived generalized asymptotic expression for error probability for high SNRs. We have shown that the results obtained with approximation closely match the exact values obtained by simulation and numerical integration of the appropriate integrals for all transmit SNRs of practical interest.

Apart from BPSK modulation schemes, this work can be easily extended for other types of modulation. For example for QAM one may find that [18, eq.(16)] is in full compliance with definition of (23). Therefore one can change corresponding values for $d$ in (24) and obtain error probability approximation for the considered systems utilizing QAM.

Due to the computational efficiency, the obtained expressions can be used as reference for easy check of the error performance during the analysis of other configurations of multi-hop MIMO systems or can be used as reference for quick assumption of error performance in future hardware implementation of such systems.


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